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THE STRUCTURE OF THE CRAB NEBULA—III
The Radio Filamentary Radiation

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SUMMARY

It is proposed that the 5 GHz continuum radiation from the optical filaments in the Crab Nebula is synchrotron radiation from an increased magnetic field. The field is circular (or spiral) about the filaments in agreement with Woltjer's suggestion that the filaments carry a current along their length. The radio prominence of filaments extended along the line of sight is a consequence of path length and effects of directional emission. The absence of enhanced optical continuum radiation from the filaments is attributable to the short half lives of the appropriate electrons against synchrotron losses. It is suggested that the circular field may originate from an interaction between the magnetic field of the pulsar and the ejected gas.

1. INTRODUCTION

This paper is the third in a series discussing high resolution maps of the Crab Nebula made with the Cambridge One-Mile Radio Telescope. The maps themselves were presented in Paper I (Wilson 1972a), and Paper II (Wilson 1972b) put forward a diffusion–loss model to account for the variation of size of the nebula with frequency and the behaviour of the integrated spectrum. Here I consider the process responsible for the enhancement of the 5 GHz radiation near the bright optical filaments (the ' radio filamentary radiation '). The observed properties of this component are briefly summarized in Section 2. Section 3 considers the basic mechanism and shows that the observations are consistent with Woltjer's suggestion that the filaments carry an electric current along their length.

2. SUMMARY OF OBSERVATIONS

The properties of the 5 GHz radiation from the filaments were described in detail in Section 3.3 of Paper I and are summarized here.

(i) The maxima of the radio emission and the Hα+[N II] line emission are generally very close.

(ii) The widths of the radio ridges are greater than the widths of the corresponding bright optical features visible in the usual emission line photographs.

(iii) The brightness of the emission is typically 200 K at 5 GHz.

(iv) The filaments which contribute the greatest brightness are those from which the optical polarization vectors diverge radially.

(v) The spectral indices of the brighter ridges between 2·7 and 5 GHz do not differ from that of the rest of the radiation by more than about $\pm 0{\cdot}4$.

(vi) There is evidence that the emission from some regions is weakly polarized at 5 GHz.

(vii) There is no apparent enhancement of the *optical* continuum intensity by the filaments.

3. DISCUSSION

3.1 *The radiation mechanism*

It can easily be shown that this radiation cannot be thermal in origin. The emission measures, E.M. = $\int N_e^2 dl$, of the bright filaments may be derived from their surface brightnesses in Hβ (Woltjer 1958) or, alternatively, from the electron densities, $N_e \sim 10^3\ \mathrm{cm}^{-3}$, determined by Osterbrock (1957) from the the intensity ratios of the λ 3727 Å doublet and the upper limits to the filamentary knot sizes of 0·02 pc. The values so found are E.M. $\leqslant 2\times 10^4\ \mathrm{cm}^{-6}$ pc. The thermal emission at 5 GHz from filaments with such emission measures is only about 2 per cent of the observed brightnesses implying that the radio radiation is non-thermal. The observed brightness of thermal radio radiation would in fact be even less because the measurements of Hβ intensity refer to bright filamentary knots of size 1″–3″ arc whereas the radio emission is averaged over the beam size (6″ × 16″ arc) and its brightness would be reduced by regions of lower emissivity. In addition, it is found that the radio radiation originates in a larger volume than that of the bright optical filaments. Although there may well be hot gas surrounding the bright filaments, it seems unlikely it could give rise to the radio emission by thermal bremsstrahlung without (a) being detectable in Hβ and (b) having such a high density as to produce strong depolarization at 5 GHz *around* the optically bright filaments.

The association of the radio emission with the magnetic field structure and the evidence for weak linear polarization suggest that the enhanced radio radiation may be ascribed to the synchrotron process arising in an increased magnetic field near the bright filaments. If the field is increased near the filaments over its ambient value by a factor m, the corresponding enhancement of emissivity will depend on (a) whether the adiabatic invariant $(\sin^2\theta)/H$ remains constant for any electron (e.g. Shklovskii 1968, p. 119) or whether the distribution of electrons remains isotropic in pitch angle, and (b) the relative densities of the relativistic electrons in the filamentary field and outside it. However, unless the density of relativistic particles near the filaments is considerably reduced, it is to be expected that enhancement of synchrotron emissivity will result from this process. From the observed size and brightness of the regions we may estimate that m lies in the range 1·2–3.

3.2 *The filamentary currents*

Woltjer (1958) has suggested that the field of the nebula is of the force-free type and if it is confined to a finite volume (that of the nebula itself), the boundary conditions require that it be surrounded by a surface current. He proposed that this current is, in fact, carried by the filaments because of the radially directed optical polarization vectors around some of them. Such filaments (e.g. M and P in Plate II and Fig. 10 of Paper I) are extended along the line of sight and the magnetic field is circular about them. Similarly Trimble (1968) has shown that the filaments in the regions J and K are a superposition of emitting material at the front and back of the object and it may well be that filaments here thread through the nebula and give rise to the radial polarization vectors observed. Furthermore, Münch (1958) noticed that some filaments run parallel to the optical polarization vectors and presumably these are current carrying filaments in the plane of the sky. The current is also a means of stabilizing the filaments by the pinch effect. It is natural, therefore, to explore the possibility that the increased field near the filaments is a consequence of such currents.

Woltjer's calculations show that if the current is distributed over five filaments, forming closed loops around the nebula, each filament should carry a current of 10^{15} amp in order to produce a proper boundary for a simple force-free field of magnitude 3×10^{-4} Gauss in the central parts of the nebula. 6″ arc away from such a filament the field is 10^{-3} Gauss, i.e. enhanced by a factor of ~ 3, which is close to the value of m derived from the observations (Section 3.1). The supposition that the current is only carried by five bright filaments is, of course, unrealistic (presumably the more diffuse gas also carries current) but the example serves to show that the idea of filamentary currents is broadly consistent with the radio observations.

A cylindrical self-pinched column of gas is unstable to a localized bend or constriction in the column (e.g. Jackson 1962, p. 326). A longitudinal component of magnetic field would, however, tend to stabilize the column.

3.3 *Synchrotron radiation from the filaments*

The filaments which give rise to the greatest enhancement of radio emission are those surrounded by radially directed optical polarization vectors (Section 2 of this paper and Fig. 10 of Paper I). We now show that this effect is a simple consequence of path length and the effects of directional emission. For the sake of definiteness we consider a very simple model of a long, straight, narrow filament carrying a current. We emphasize, however, that the effect discussed is essentially a function of the angle between the line of sight and the length of the filament and not much dependent on the actual variation of the strength of the magnetic field with distance from the filament.

3.3.1 *Synchrotron radiation from the magnetic field associated with a long, straight, current-carrying filament.* We restrict ourselves to two cases, when the filament is extended perpendicular and parallel to the line of sight.

(a) *Filament perpendicular to the line of sight.* The geometry is shown in Fig. 1(a) where the line of sight is in the plane of the paper and the filament is perpendicular to this plane. Clearly the influence of a current-carrying filament is noticeable only near the filament; the magnetic field further away in the centre is affected by all the filaments. We therefore compute the brightness distribution from a cylinder of radius R, coaxial with the wire and pervaded by a uniform and isotropic relativistic electron distribution with the usual power law spectrum in energy

$$N(E)\, dE = KE^{-\gamma}\, dE.$$

To examine the brightness distribution from this system we assume there is no emission from outside this cylinder. The brightness seen along the line of sight is

$$T_\mathrm{b} \propto 2\int_0^{\sqrt{R^2-x^2}} \epsilon(l)\, dl \qquad (3.1)$$

and

$$\epsilon \propto K(H \sin\theta)^{(\gamma+1)/2} \qquad (3.2)$$

where

$$H = 2I/r, \qquad dl = -x \sec^2\theta\, d\theta.$$

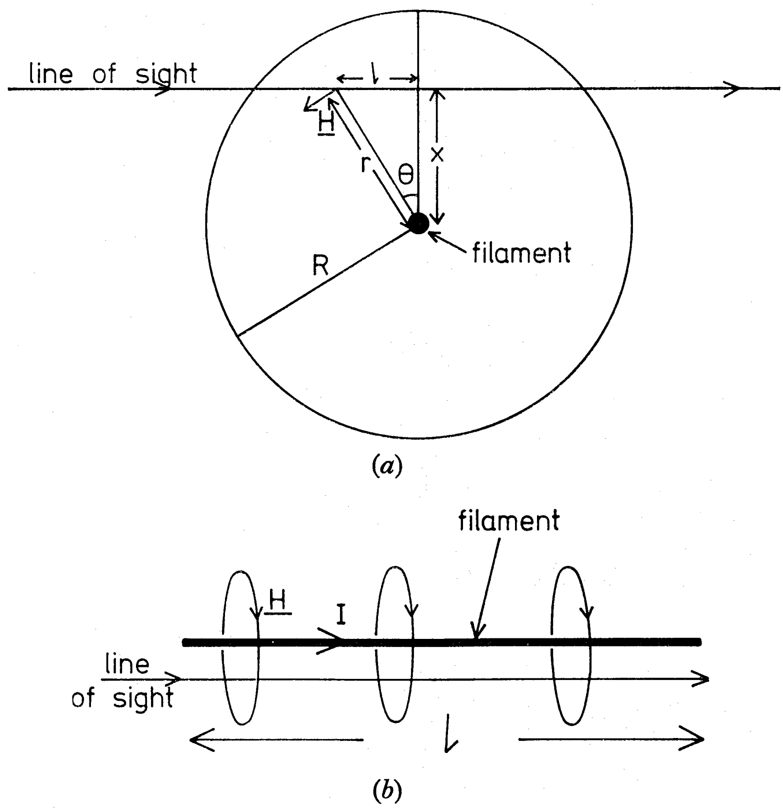

FIG. 1. *Geometry for deducing brightness of filamentary radiation. (a) Filament extended perpendicular to line of sight. (b) Filament extended along line of sight.*

Substituting equations (3.2) into equation (3.1) we have

$$T_b \propto 2I^{(\gamma+1)/2}\, Kx^{-(\gamma-1)/2} \int_0^{\theta_0} (\sin 2\theta)^{(\gamma+1)/2} \sec^2 \theta \, d\theta$$

where

$$\theta_0 = \cos^{-1}(x/R).$$

For the electrons giving radio emission, $\gamma = 1{\cdot}52$. The integral has been evaluated numerically for $\theta_0 = 20^\circ$, 40°, 60° and 80°, and the brightness distribution computed as a function of x, taking R to be about the size of the observed radio emission from the filaments, that is $20''$ arc $= 0{\cdot}2$ pc. The brightness distribution is shown as curve (a) in Fig. 2.

(b) *Filament parallel to the line of sight.* This case is simpler to analyse because the magnetic field is always perpendicular to the line of sight ($\sin\theta = 1$) and is constant along any given line of sight through the emitting material (Fig. 1(b)). We now have

$$T_b \propto 2^{(\gamma+1)/2}\, I^{(\gamma+1)/2}\, Kx^{-(\gamma+1)/2}\, l$$

where l is the length of the filament. Taking the conservative estimate of $l = 0{\cdot}2$ pc we have again calculated the variation of T_b with x (Fig. 2 curve (b)). It is clear that the brightness distribution of curve (b) will be more prominent than that of curve (a) and explains the correlation of the radio emission with the radial polarization vectors. The brightness does not, of course, increase indefinitely as x is decreased, because in the region of the current itself, which will be finite in extent, the magnetic field decreases with decreasing r. Inside a cylindrical wire carrying a uniform density of direct current, $H \propto r$, outside $H \propto 1/r$. Considering

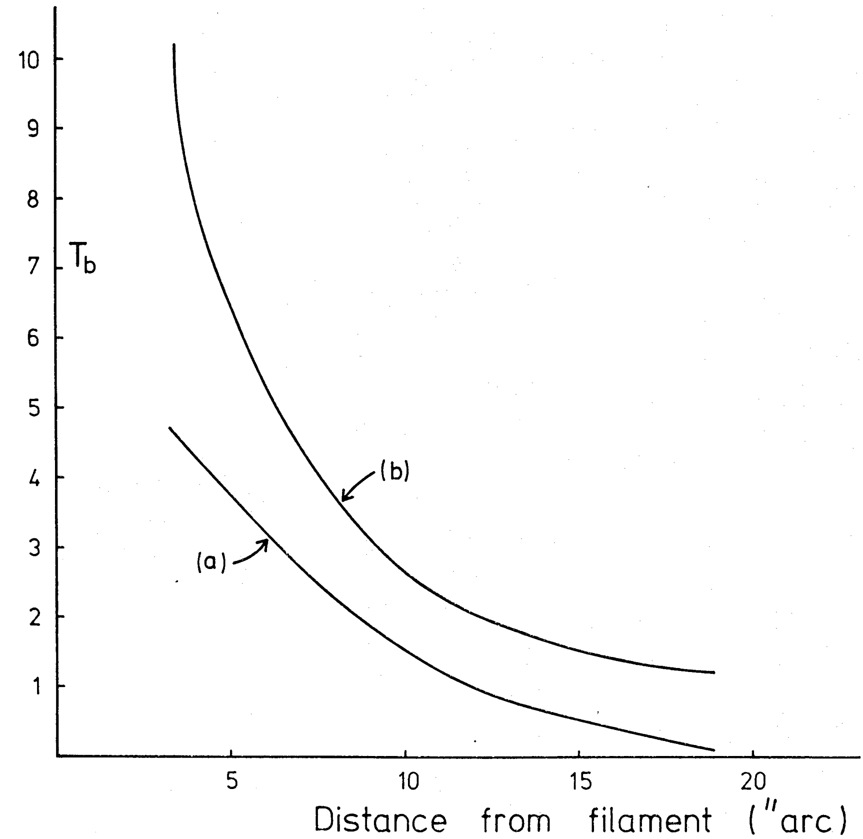

FIG. 2. *Theoretical variation of brightness of filamentary radiation with projected distance from filament. (a) Filament extended perpendicular to line of sight. (b) Filament extended along line of sight.*

the idealized nature of the theory and the effects of telescope beam smoothing, the brightness distributions of Fig. 2 are consistent with the observations shown in Fig. 9, Paper I.

3.4 *Lack of optical continuum*

We have already noted (Section 2) that although the percentage polarization of the optical continuum radiation is high near the regions of greatest radio enhancement and the direction of polarization is radial to the filament (implying there are *some* ' optical ' electrons near the filaments), there is no apparent increase in the *intensity* of the optical continuum. In this subsection we discuss the reason for this. At first sight it appears that the optical continuum should also be enhanced by the stronger field near the filaments. There are, however, at least two effects which will change the relative enhancement of emissivity at optical frequencies.

(1) It was shown in Paper II (Figs 3 and 4) that the emissivity at optical wavelengths decreases more rapidly towards the edge of the nebula than that at radio wavelengths. We interpreted this as a consequence of the depletion of the density of ' optical ' electrons by synchrotron losses. Thus there are relatively fewer ' optical ' electrons than ' radio ' electrons reaching the filaments, many of which are in the outer parts of the nebula. This effect will tend to reduce the enhancement at optical frequencies.

(2) On the other hand, the emissivity $\epsilon \propto H^{(\gamma+1)/2}$ where γ is the exponent in the electron energy spectrum. Because the electrons emitting optical radiation

have a larger γ than those emitting radio radiation, the enhancement of emissivity in the filament relative to its immediate surroundings will be greater at optical than at radio frequencies.

It may easily be shown that the combination of these two effects should still yield an observable enhancement of optical intensity by the filaments. Clearly, if the idea that the enhanced radio emission is caused by an increased magnetic field is to be tenable, some process must reduce the density of optical electrons in the filaments. Such a process might, again, be synchrotron energy loss. For example, it may be difficult for electrons to diffuse through the circular field structure associated with the filaments. The electrons in the filamentary field may then be those produced at about the time of the original explosion. Any 'optical' electrons will have lost most of their energy long ago, leaving only the low energy ones which radiate at radio frequencies.

3.5 *Speculation on the origin of the currents*

If there were relative motion between the magnetic field and a filament, the magnetic field could be distorted in the observed manner. The interaction with an 'end on' filament is shown schematically in Fig. 3. The field bends around the conducting gas (b) and may set up a current along the filament. Such a process could also explain the radial direction of the optical polarization vectors on the inner edge of the 'bays', if the 'bays' contain a large amount of conducting gas. Kardashev (1970) and Gunn & Rees (1972, private communication) suggest that the field of the Crab Nebula has been 'wound up' by the pulsar. Gunn & Rees consider that when this field meets a filament it will wrap round it in the way indicated in Fig. 3.

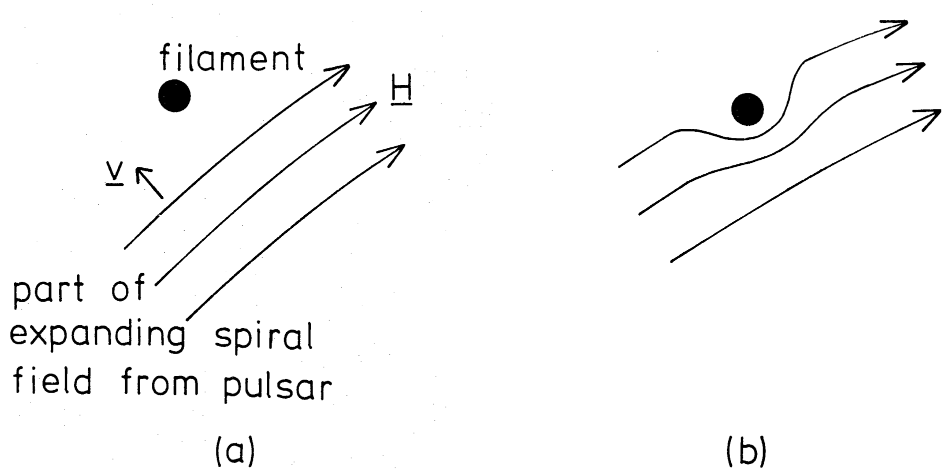

FIG. 3. *Origin of relation of magnetic field to filaments* (*Section* 3.5).

3.6 *Comparison with other supernova remnants*

Recent high resolution maps of some shell supernova remnants, such as the Cygnus Loop (Moffat 1971) and IC 443 (Hill 1972), have shown that the radio emission is closely associated with the optical filamentary structure although in others, such as Cas A, there seems to be no detailed correspondence between fine radio structure and optical filaments (Rosenberg 1970). The radio emission is probably synchrotron in all cases but the filamentary radiation from these supernovae and the Crab Nebula are not strictly comparable because:

(a) IC 443 and the Cygnus Loop are much older and their shells consist of material swept up from the interstellar medium while the filaments in the Crab Nebula are ejected material.

(b) In some regions of the Cygnus Loop and IC 443 the radio emission has its peak closer to the centre than the optical filaments whereas in the Crab Nebula it seems to be coincident with them.

(c) The radio polarization of the southern part of the Cygnus Loop suggests a predominantly circumferential magnetic field, that is, along the filaments, in contrast to the circular or helical field structure about the filaments in the Crab Nebula.

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