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The Submarine Equilibrium Profile: A Physical Model

Paul Zi-Fang Lee

Florida Department of Environmental Regulation
2600 Blair Stone Road
Tallahassee, FL 32399, U.S.A.

ABSTRACT

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The Submarine Profile (SP) is the sea floor profile normal to the coast line, located between the surf zone and wave base, and formed by wave action on the clastic sediment. The Submarine Equilibrium Profile (SEP) is a SP where geometry and sediment movement are in a quasi equilibrium status with offshore nominal waves (the large swell dominating this area). The geometry of the SEP has been identified qualitatively and conceptually as a concave upward smooth curve. The slope of SEP becomes gentler and gentler offshore until it is almost horizontal at the so-called wave base. Some empirical equations have been proposed and are used widely in engineering. This report summarizes previous field observations, lab experiments and theories concerning mechanisms involved in the formation of this geometry. A series of physical equilibrium conditions, including the major mechanical forces involved in particle movement, are used to derive an analytical model. This model expresses the geometry of the SEP in an explicit form where x is a function of y, in relation with the offshore wave period (or wave length). The basic idea has been qualitatively tested in a wave tank experiment. The final equation was then tested and calibrated on a number of submarine profiles plotted from three marine basins around the U.S. using a non-linear regression algorithm. The profiles located on the U.S. coasts along the Gulf of Mexico and on southeast U.S. Atlantic coasts were selected following geological and visual criteria which previous researches used to characterize the SEP concept. The comparison between the theoretical equation and the selected Gulf of Mexico and Atlantic profiles by a non-linear computer regression process showed a satisfactory match with high correlation coefficients. The Pacific profiles, located in an area with ongoing active tectonic movement are generally not in SEP status and have more features deviating from the conceptual and SEP physical models. The wave periods obtained from the nonlinear regression process for some SEP's on the Gulf of Mexico coast are further compared with the available hindcast offshore wave data published in 1989. The results of these comparisons are generally encouraging, except those comparisons involving SP's on the U.S. Pacific coast.

ADDITIONAL INDEX WORDS: *Breaker zone, shore profile, submarine equilibrium profile, submarine profile, wave tank experiments.*

INTRODUCTION

The *submarine profile* (SP) and the *submarine equilibrium profile* (SEP) are widely accepted terms and concepts (FENNEMAN, 1902; JOHNSON, 1919; KUENEN, 1950; PRICE, 1954; FISCH, 1960; BRUUN, 1962; ZENKOVITCH, 1967; DEAN, 1977, 1983, 1987; FRIEDMAN and SANDERS, 1978; SWIFT and PALMER, 1976).

The author divides a SP into four large sections, according to the major mechanical forces involved in forming their geometries (cf Figure 3).

(1) The breaker section, the geometry of which is mainly determined by the bottom shear force posed by the shoaling waves, plunging and breaking actions of the breakers and swash and back wash actions;

(2) and (3) The shoaling section and the platform section, which are beyond the breaker zone and above so-called 'wave base'. On these SP sections, fluidization of the water-saturated top layer of sediment, caused by the vibratory field of the wave action, tends to cancel the friction and dilatancy to form a perfectly horizontal interface between water and sediment. In the meantime, the shoreward shear force caused by the Cornaglia effect is imposed on the fluidized surface to form a slope.

(4) The repose section, which is the section located beyond the influence of the local swells, where the sediment forms an almost constant repose slope caused by the interactions between gravity, dilatancy and friction between the grains.

In addition, it is important to keep in mind that the general sediment movement direction on the whole section is from the source on the land toward the marine basin.

The SEP model in this article, especially the mathematic expression, is limited in reference to the SP geometry on sections 2 and 3, where clastic material movement has reached a dynamic balance so that the geometry is undergoing a minimal

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change; in other words, it is in equilibrium with the local offshore waves traveling toward the shore. The summary of the past researches in terms of descriptive features related to the general geological settings and the geometry of SEP can be regarded as the conceptual model of SEP, and presented as follows:

(1) The SEP is located in areas with minimal present tectonic movements.
(2) The SEP is not located in areas associated with strong erosional and depositional factors, such as a major river mouth, under an erosional sea cliff, etc.
(3) The SEP has a balanced longshore and onoffshore sediment budget; therefore, no net accretion or erosion occurs on the profile.
(4) The SEP is indefinitely under the constant activity of a swell with constant wave parameters.
(5) The SEP is smooth and concave upward, flattening out towards the offshore direction and reaches an almost horizontal platform at a depth of the so-called wave base.
(6) The depth of the wave base is somehow related to the wave length of local large swells.
(7) The SEP geometry is also related to the grain size of the clastic sediment material; the finer the grain size, the gentler the profile and its SEP extends farther from the shore.

The previous researchers found that SP's with these features are likely to be in an equilibrium condition and should be called SEP's.

Most of the submarine profiles in the real world are in either depositional or erosional status, or some sections are under erosional process while the others are receiving sediment. The vertical structure of the wave activity implies that the upper section of a profile receives larger amounts of wave activity and may more easily reach equilibrium with the recent wave condition than the deeper part of the same profile. An ideal equilibrium profile must be a very rare case, in which the whole profile is in equilibrium with a unique wave condition.

In the past, a number of investigators have conducted extensive field and laboratory observations and others have suggested empirical equations to describe the wave condition changes due to bathymetric changes (BAGNOLD, 1940, 1963; BASCOM, 1951; BRUUN, 1962, 1983; CHEONG, 1983; COLLINS, 1976; CORNAGLIA, 1887; DAVIS, 1976; DEAN, 1983, 1987, 1990; EAGLESON, 1958, 1961, 1963; FISHER, 1977; HORIKAWA, 1967; INMAN, 1966; IPPEN, 1955; IWAGAKI, 1963; JOHNSON, 1919; KING, 1972; KOMAR, 1973, 1975, 1983; KURIAN, 1950; LONGUET-HIGGINS, 1953; MANOBAR, 1955; MAY, 1973, 1974; McLEAN, 1969; MURRAY, 1967; PRICE, 1954, 1969; RANCE, 1969; RECTOR, 1954; SAVILLE, 1957; SHEPARD, 1950b; SHORT, 1979; SILVESTER, 1971; SONU, 1973; SOULSBY, 1987; SWIFT, 1976; TANNER, 1974, 1988; WATTS, 1954; ZENKOVICH, 1967) or the profile response to the sea level change (PER BRUUN, 1962). However, what are the basic mechanisms involved in the formation of the original SEP? Why is it concave upward? Why is it a smooth curve? Why does it become horizontal at a certain depth? Is it possible to devise an equation to express the profile involving some physical wave conditions? Per Bruun's model about the profile upshift associated with sea level rise is based on kinetic symmetry. Per Bruun's rule is always valid despite the original geometry of SEP.

The profile including the breaker zone has been studied by many authors. In many cases, when the geometry was attempted by a simple equation, the beach processes acting in the breaker zone and on the submarine profile were not distinguished or well defined. Dean's equation and Per Bruun's profile, also called Beach Equilibrium Profile, are expressed with a continuous and smooth curve—both include the breaker zone.

Dean's equation

$$y = Ax^{\text{[illegible]}}$$

where y is the depth of any point on the bottom profile,
x is the horizontal distance from the shoreline to the point,
A is a constant related to grain size,

representing a beach equilibrium profile, is an empirical model. This model was applied with an assumed criterion that the equilibrium condition is established when the wave energy dissipation rate per unit volume of water stays constant during the shoaling process.

However, the hypothesis to relate the wave energy dissipation with a particle-movement-controlled-geometry is a vague one because energy is a scalar (non directional) quantity, not a vector. A random disturbance of sand and water may dissipate a great amount of energy without any necessary bottom geometry change (especially how the dynamic energy carried by water waves would

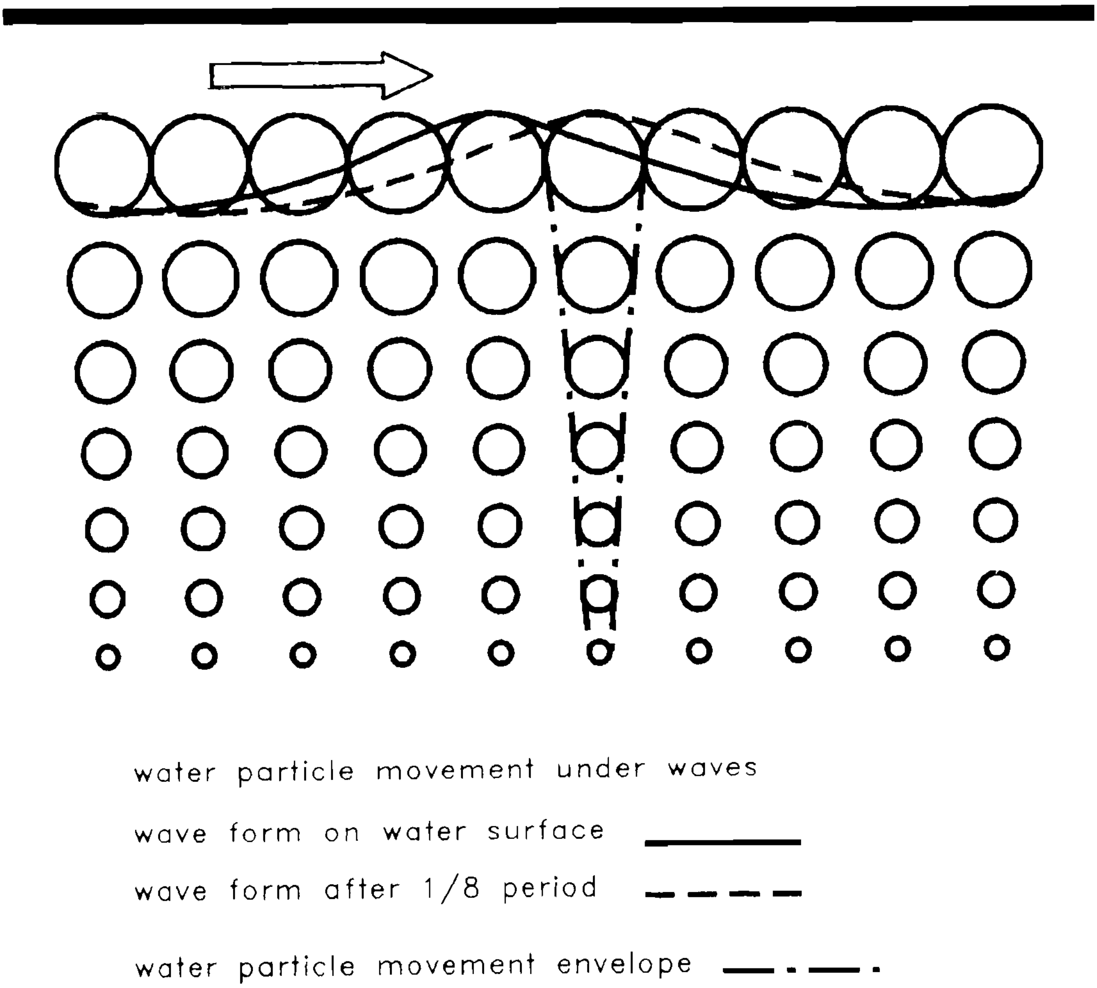


Figure 1. Airy wave theory.

transfer to the top of the sand particle aggregation has not been explicitly explained). Similarly, any symmetrical cyclic movement within a fluid may be attenuated by dissipating its energy into heat, without changing the shape of its 'container' (the shape confining the volume, in this case the air–water interface, consists of one side of the container, and the bottom profile consists of the other side of the container) into another definite shape.

Therefore, a dynamic model of SEP may be more reasonable than an energetic model because the former is based on the balance of forces which are vectors which relate to the shape of the profile with definite mechanisms. Through these mechanisms, the shape and the forces interact. Whether the dissipation rate of wave energy changes or keeps constant is the result of the shoaling process upon the equilibrium profile, but not the cause.

This article proposes a model to relate the geometry of the equilibrium profile as a whole with the basic wave theory in a logical and semi-quantitative fashion.

THE ANALYTICAL MODEL

Airy's wave theory teaches that the water particles in traveling waves are making cyclic movement in a vertical plane along the direction of wave propagation. The water particle orbital movement diameter attenuates through the increasing depth (Figure 1). For deep water waves,

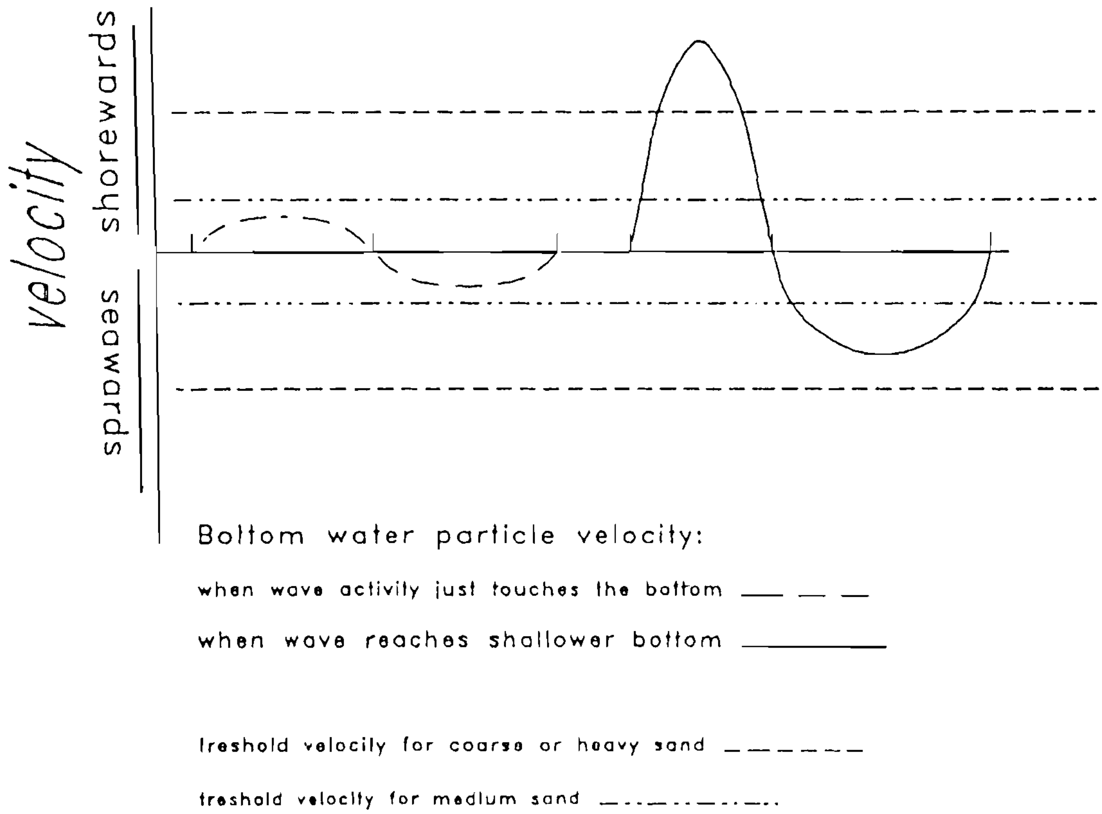


Figure 2. Cornaglia effect.

the orbital diameter at a depth of y is expressed as:

$$H_y = H_0 e^{ky} \qquad (1)$$

where H_y is the water particle orbital diameter at the depth of y,
H_0 is the wave height, the water particle orbital diameter at the water surface,
e is the base of natural logarithm,
y is the depth, always negative,
k is a constant representing $2\pi/L$, where L is wave length.

On a vertical plane parallel to the wave ray direction, the water particle orbital movement diameter can be expressed as a horizontal vector plotted on a vertical axis y of the corresponding depth, forming a wedge shaped curve. This is called an orbital diameter envelope. The width of the wedge at any depth represents the intensity of wave activity at that depth.

When the water particle movement reaches the marine floor, the circular movement is flattened and becomes an elliptical, then a back-and-forth oscillating movement. Under waves traveling toward the shore, the back-and-forth movement is not symmetrical. The onshore movement is more powerful but lasts a shorter time, whereas the offshore movement is weaker but lasts longer. The net effect of this asymmetrical interaction between the water and the sediment causes a shoreward shear force on the bottom which is known as the Cornaglia Effect (CORNAGLIA, 1887) (Figure 2).

The null point hypothesis theory based on the Cornaglia Effect suggested that a sediment grain of a certain size will stay at a certain depth on the profile, and the grains of the same size will move onshore if it is located on the onshore side of the null point and move offshore if it is located on the offshore side of the null point.

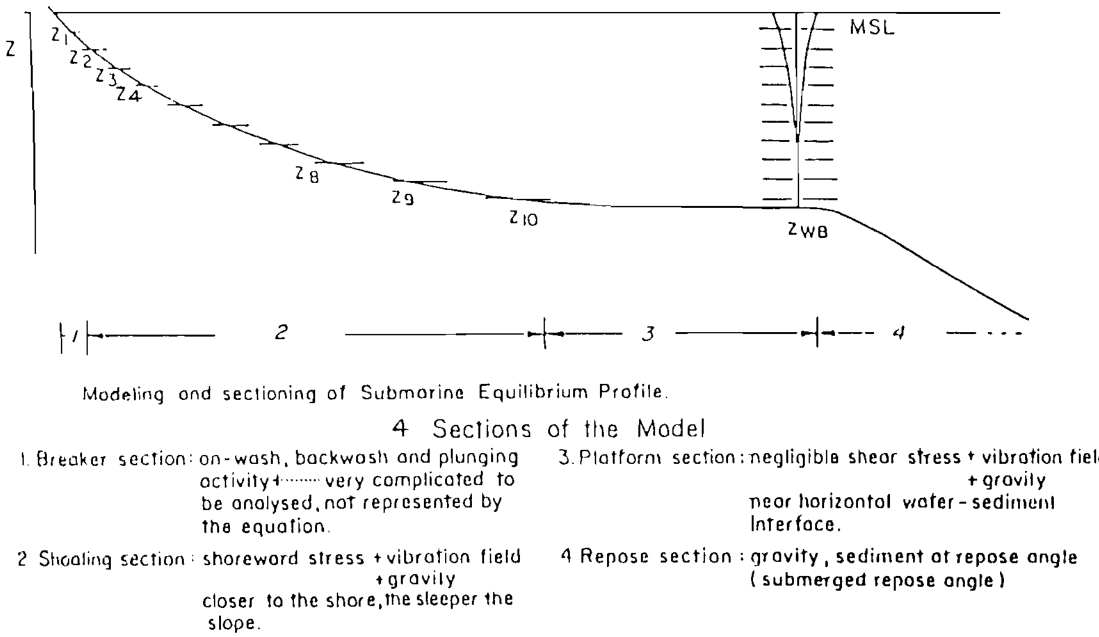


Figure 3. Overall sectional model.

The author disagrees with part of the null point hypothesis. He reasons that if a submarine profile is formed on a single sized sediment, any net effect of asymmetrical back-and-forth movement of individual sand grains will eventually reach an equilibrium with the geometry of the marine floor. Simply stated, any onshore movement of sand will form a steeper slope until the shear force in the onshore (up-slope) direction available at that point is balanced by the offshore (down-slope) force derived from gravity on a slope. On the other hand, any offshore movement of sediment will reduce the steepness of the slope at the point. Therefore, the entire geometry in balance must relate to the slope with the depth at each and every one of these points. This can be easily explained with the vertical structure of the wave activity. In the meantime, this oscillating movement fluidizes the very top layer of the water saturated sediment. The fluidization alone tends to form a horizontal interface between the water and the bottom sediment. However, if the shoreward shear force is applied on the bottom at the same time, the two effects will form a slope. According to basic physics, if the shear force is in balance with the down-slope component of gravity, the sine of the slope angle should be proportional to the shear force. A reasonable first approximation to relate the wave action with the shear force at any depth can be hypothesized as proportional to the water particle orbital diameter at that depth.

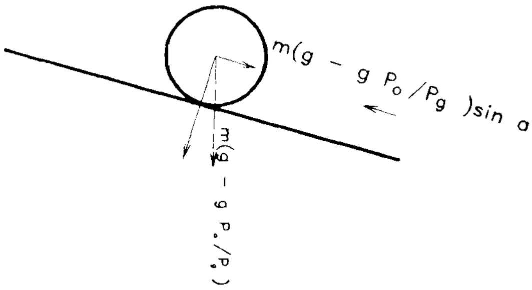
For a grain or a layer of grains on the top of the SEP, the force needed to keep them on a slope against gravity (because of fluidization, friction and dilatancy alone cannot keep the grains on any slope) should be proportional to the sine of the slope angle (Figure 4). When the slope angle is very small, the sine value approaches the tangent value of the slope angle which is dy/dx or y′, the first derivative of y.

The above balance condition leads to a differential equation. At any point on the SEP, the relationship expressed by the following differential equation is valid.

$$\frac{dy}{dx} = kH_0 e^{\frac{2\pi}{L}y} \qquad (2)$$

where dy/dx is the slope at any point on the profile,

y is the depth, always negative,



On a slope, the asymmetrical Cornaglia Effect results as a net shear force against gravity, and keeps the grain on the slope. In equilibrium state, the force needed is proportional to the sine of the slope. The force available from the waves at the corresponding depth is proportional to the intensity of the wave activity at the depth.

Figure 4. Analytical model derivation.

- x is the horizontal distance from the shore,
- H_0 is the wave height of the offshore swell,
- e is the base of natural logarithm,
- K is a proportionality, a constant,
- L is the wave length of the offshore swell.

For a wave with a constant offshore wave height,

$$KH_0 = K_1$$

where K_1 is another proportionality, a constant.

Assign $B = 2\pi/L$. Rearrange the differential equation:

$$e^{By}\,dy = K_1\,dx \qquad (3)$$

After integrating both sides, we get:

$$\frac{1}{B}e^{By} = K_1x + C_0 \qquad (4)$$

or,

$$\frac{1}{K_1B}e^{By} - \frac{C_0}{K_1} = x \qquad (5)$$

If we assign $1/K_1B = A$, and $-C_0K_1 = C$, then we get:

$$x = Ae^{By} + C \qquad (6)$$

As a boundary condition when depth is zero, the SEP should intercept the shoreline and X should be zero, too. Then the value of By becomes zero and the value of e^{By} becomes 1. Applying the boundary condition, y = 0 and X = 0, to equation (6),

$$0 = A + C, \quad \text{or} \quad -C = A$$

is established. Replacing C with −A following this relationship, we get the solution of the differential equation which represents a functional relationship between x and y versus the offshore wave length.

$$x = A(e^{By} - 1) \qquad (7)$$

where A is a constant, named extension factor, which may be related to grain size. Dimension = L.

B is a parameter assigned as $-2\pi/L$, where L is the offshore wave length, or a function of a corresponding offshore wave period T. Since $L = gT^2/2\pi$, $B = -4\pi^2/gT^2$. Dimension = L^{-1}.

The construction of Equation 7 takes into account all of the involved mechanical forces (gravity, Cornaglia Effect, and fluidization by the periodical vibratory field derived from the wave which works against dilatancy and friction) through a logical reasoning process. It is based on a physical condition that the net shoreward shear force imposed on the fluidized sediment surface and available at a certain depth is proportional to the intensity of the water particle orbital movement at the corresponding depth. So far, the model has not been involved with any statistics.

The thresholds for X should range from zero to positive infinity, and y should range from zero to negative infinity. In geological reality, y should be no deeper than the depth at which the local large swell is unable to effectively fluidize the sediment there. This limit of depth may be viewed as the threshold limit of the SEP model and the beginning of the repose section, where the slope of the profile is controlled by repose angle of the water-saturated sediment, not by the onshore shear force. The repose angle is steeper than the near horizontal slope at the end of the SEP where the influence of the wave action can barely fluidize the sediment, but no significant shear force is available at that depth. The slope at that transition zone will demonstrate a sudden change or a kink. This is perhaps where some of the submarine landslides and mudflows occur.

Since parameter B is rigidly connected with the wave length, $B = -2\pi/L$, by using a fixed arbitrarily selected parameter A value, we can examine the general shape of the theoretical profile. With different wave length values, we can create a series of curves in a X-Y coordinate system. We observe that:

(1) These curves are smooth and continuous.
(2) The X and Y relationship is monotonous; *i.e.*, the further the distance from the shore (larger X value), the deeper the sea floor.
(3) These curves are upwardly concave with a steeper and steeper slope when approaching the shore line. They approach near horizontal at a distance from the shore.
(4) If we assume all the offshore swells have the same range of wave steepness, we can also assume that the wave height is linearly connected with wave length. Then, the wave energy will be a function of the wavelength. The waves with a higher value of wavelength (larger L, smaller absolute value of B) will carve more deeply into the sea floor and create a steeper slope, when compared with the profile in an area of lower wave energy with smaller wave length.

All these features match quite well with the general concept of SEP.

Theoretically, the waves with a uniform suite of wave parameters should act on a SEP for an indefinite time. In the real world, the waves characterized by parameter B and responsible for forming the SEP may be those "very frequent and quite large swells" (SOTHEBY, 1987) occurring in the area.

NONLINEAR REGRESSION PROCESS

Every SEP should have a fixed pair of parameter A and parameter B. Using a nonlinear regression to match some real world conceptually recognized SEP's with the equation would reveal how close the connections are between Equation 7 and the conceptual SEP. In the meantime, a pair of A and B will be obtained for each profile; B can be compared with the field measured mean peak wave period in the area where the profile is located. Through these two procedures, the model would be verified and calibrated.

The nonlinear regression uses a SIMPLEX algorithm (CACECI *et al.*, 1984) re-written in GWBASIC by the author. The SIMPLEX algorithm follows the "least sum of the squared residuals" criterion to determine the best fit between the field data and the curve created by the equation, while trying different values of A and B. The regression was performed on an IBM compatible personal computer. The result of the regression was printed in graphic form (Figures 5 and 6).

B is a function of offshore wavelength or wave period, but the physical relationship between parameter A and grain size and/or other physical parameters is pending further studies.

The regression process illustrated good match between the equation and some profiles located in the Gulf of Mexico and the southeast U.S. Atlantic coast ($R^2 = 0.95$–0.99). These profiles are selected following the SEP conceptual model. The wave periods translated from the B parameters are in the same order of magnitude as the available measured mean peak wave periods in the Gulf area (HUBERTZ, 1989) (Table 1 and Figure 8).

Profiles from the U.S. Pacific coast were plotted only for comparison purposes; they do not meet most of the selection criteria for SEP. These profiles carry significant erosional features. The tectonic activities along the Pacific coast do not allow enough time to form SEP's. Besides, the sediment amount on the narrow continental shelf does not allow a clastic platform to build up. The profiles appearing as almost straight lines predominantly represent the repose section. The application of non-linear regression to these profiles yields a very small absolute value of B (less than 0.001); the corresponding wave length should be over several thousands of feet which has not been observed. These conditions plus the limited detail of the available Pacific coastal bathymetric maps excluded most of the Pacific profiles from the SEP model. Perhaps only in some pockets of the shoreline, certain Pacific profiles may demonstrate the state of SEP; *i.e.*, those that extend only a short distance from the shoreline, then are immediately followed by the repose section. For these reasons, the U.S. Pacific profiles do not qualify for this SEP model.

WAVE TANK EXPERIMENT

The fundamental mechanism from which the model is derived is whether the vertical attenuation of wave activity is related to the available shear force along the depth in the water body. In other words, the shear force should attenuate with the increasing depth. Further, the slope on which

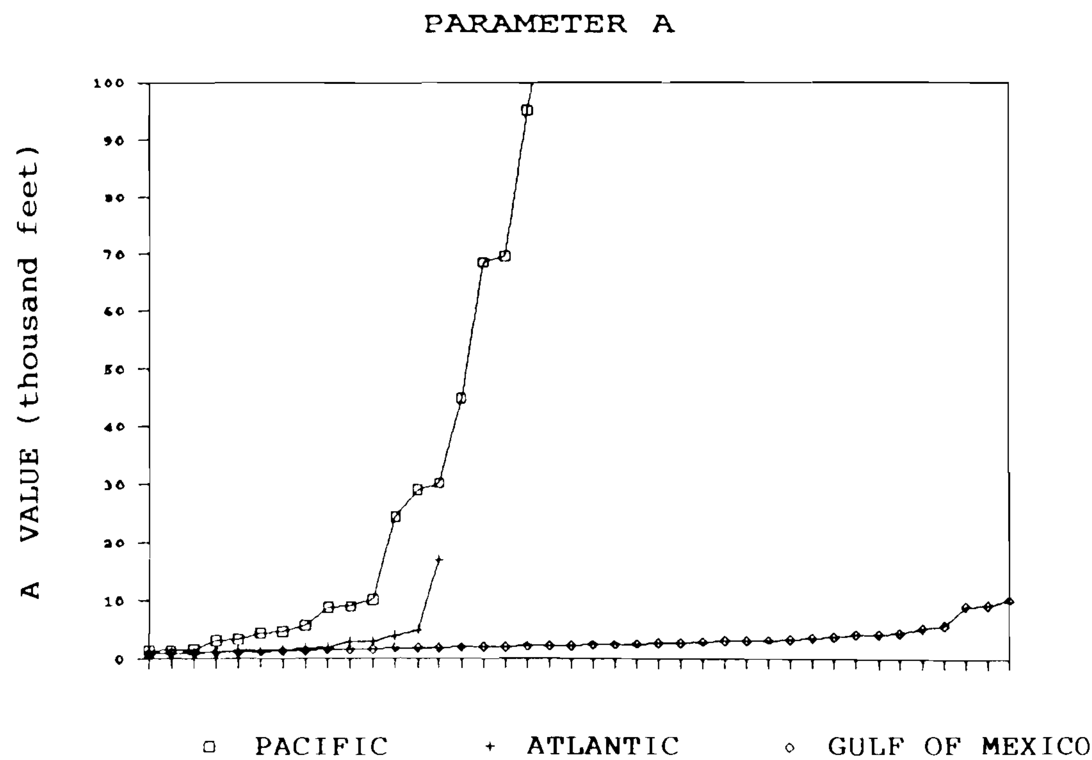


Figure 5. Comparison of parameter A for profiles from different marine basins around U.S.

the sand is in balance with the wave activity should be related to the depth. At a deeper location, this slope of balance should be gentler.

A 16 ft (W) × 32 ft (L) × 5.5 ft (D) wave tank was built for the purpose. On one end, there is a wave generator driven by a 1.5 H.P. motor. The experiment is conducted on a submerged adjustable platform. The slope and the depth of the platform can be adjusted independently (Figure 7). A patch of sand (1–3 grams) as an almost single layer was put on the platform and the net movement of sand was observed under the influence of traveling waves when the platform was set on different slopes and at different depths. The results of the experiment are shown in Tables 2 and 3. The experiments exclude breaker condition.

The results illustrated that:

(1) When a horizontal platform is lifted from the deepest position under a traveling wave, the sand grains start to move shoreward at a certain depth. Beyond the threshold depth, the sand grains are not affected by the traveling waves.

(2) The shoreward movement of sand becomes more and more significant with decreasing depth. The sand movement is always shoreward on a horizontal platform under shoreward traveling waves.

(3) When a tilted platform with a constant slope through the lifting process is lifted from the deepest position, the threshold depth at which the sand grains start to move shoreward becomes shallower. This means that a stronger shear force is needed to move sand grains upslope against the gravity. Under the same waves, the steeper the slope setting is, the shallower the threshold depth becomes.

(4) On a platform tilted at an even steeper slope, the sand grains first move toward the offshore direction when the platform is gradually lifted from the deepest position. This means that the sand grains under the influence of the vibratory disturbance of waves will move off-

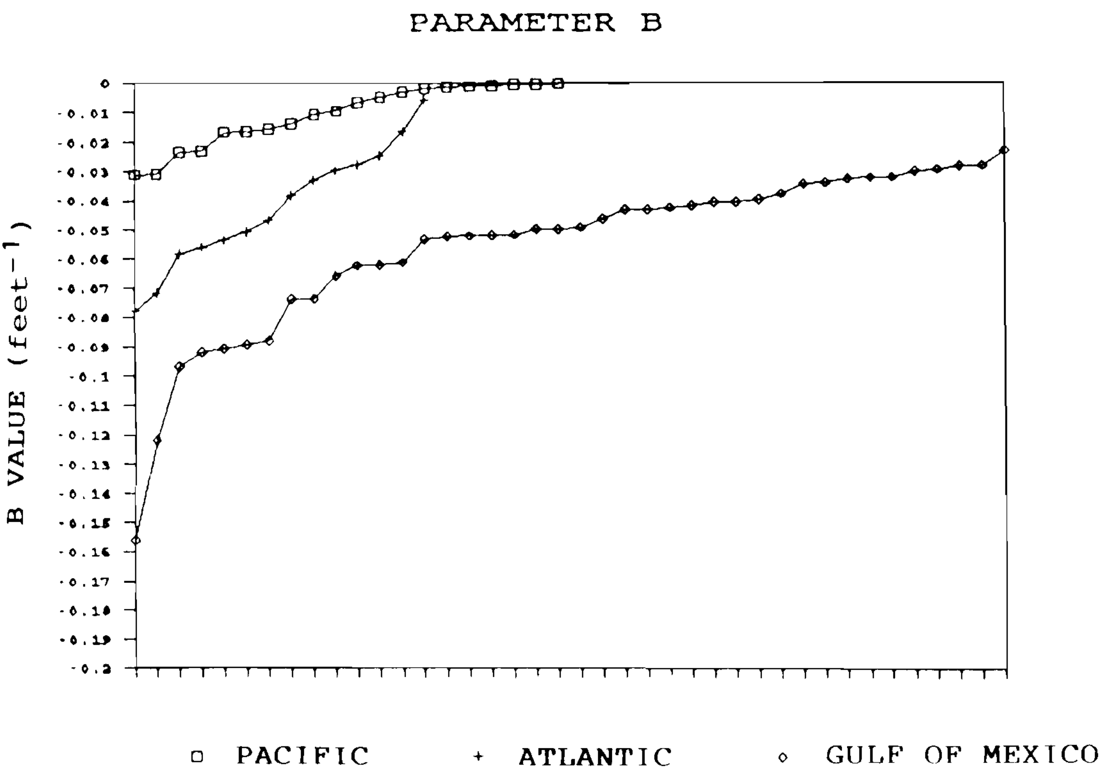


Figure 6. Comparison of parameter B for profiles from different marine basins around U.S.

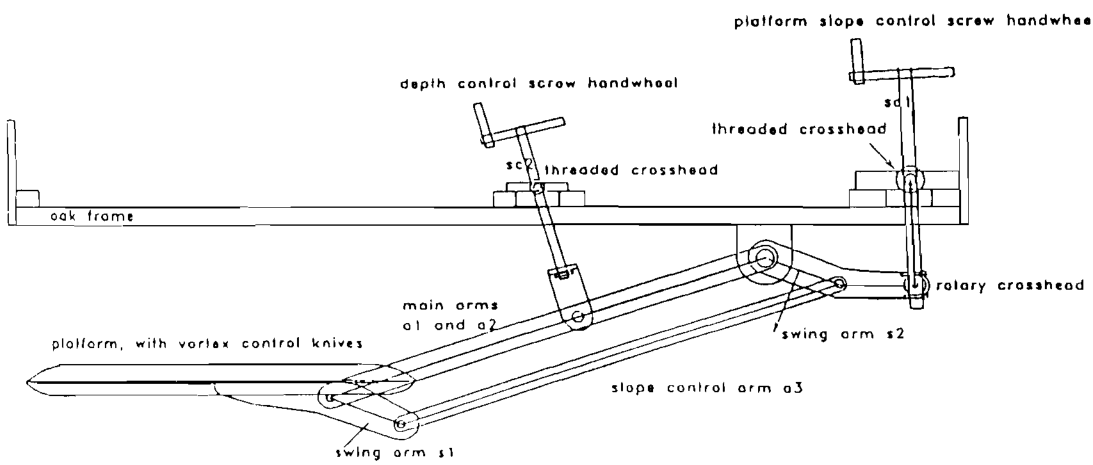


Figure 7. Depth and slope independently adjustable platform for wave tank experiment.

Table 1. *Comparison of wave periods.*

Profile Name	WIS Station No.	B Value	TB (sec) Calcltd. from B	Tp (sec) Field Measrd.
ARANSAS2	5	−0.028098	6.61	6.3
CHOC4	30	−0.042578	5.37	5.4
CHOC6	29	−0.043182	5.33	5.2
GALVA2	11	−0.062226	4.44	4.8
GALVA3	11	−0.073823	4.08	4.8
LUIS3	10	−0.032012	6.19	6.2
LUIS4	9	−0.049127	5.00	5.7
MADRE1	3	−0.034525	5.96	6.6
MADRE4	2	−0.033918	6.01	6.8
MATABAY1	9	0.049839	4.96	5.7
MATABAY2	9	−0.053377	4.79	5.7
ROLL1	14	−0.061358	4.47	5.2

Note: The profile names are based on the names of NOAA maps. Each profile is plotted between two points which are recorded with their latitudes and longitudes. The profile names do not have any physical significance to the readers. Readers can choose their own profiles according to the SEP concept model and obtain regression results by running the computer program. The SIMPLEX program written in GWBASIC is available to the readers. Please mail a formatted blank 1.44 MB floppy disk and an envelope with return address and postage stamp to: Dr. Paul Lee, Dept. of Environmental Regulation, 2600 Blair Stone Road, Tallahassee, FL 32399-2400. B parameter values obtained through regression carry six decimal points. This is due to the conventions set up in GWBASIC. The maximum number of effective digits through such a process would be no better than the original bathymetric data which have 2–3 effective digits.

shore if the shear stress available at that depth cannot compensate for the downslope component of gravity at that slope. When the platform of the same steeper slope is gradually lifted, the sand grain movement direction changes from offshore to standstill to onshore.

(5) These observations lead us to conclude that the balance condition where a sand grain remains on a position must never be referred to as only the depth, or only to onshore/offshore direction, it must be referred to the combination of a definite depth with a definite slope.

(6) For a given set of wave parameters, there is no shear force at certain depth. With decreasing depth (lifting the platform), the shear force available increases until the depth reaches the breaker condition.

These findings justified the construction of the differential equation. The adjustable platform eliminates the need for a large amount of sand, in the order of tons, to conduct a wave tank experiment. This experiment broke the paradox of null point hypothesis and the dilemma raised by MURRAY (1967). MURRAY concluded that "... in all but two of the twenty-one cases, the net transport was onshore, ..." and "the stronger the near-bottom orbital motions under the waves (calculated), the lesser the tendency for onshore movement of all grain sizes." These conclusions made without referring to the original slope and depth where the involved sediment particles are located are misleading. If, in almost all the cases, the net transport was onshore, it implies that all the beaches should always be under accretion and the source of sediment is from offshore. As a result, marine basins such as the Gulf of Mexico would never have any deposition (>3,000 ft of thickness in reality), which is obviously not true.

The wave tank experiment reveals a series of phenomena strongly supporting the analysis of mechanisms involved in this physical model. However, the wave tank experiment has some limitations. First, the simulation of swells is impossible in a wave tank of this size, as the wave generator is only 15 feet from the platform. Because of the different proportions between the physical parameters of the seas versus swells, direct calibration of the model with the wave tank experiment becomes impossible. Secondly, after the wave generator has produced about 6 to 10 wave periods, the whole wave tank will establish a standing wave regime. The sand behavior under such a condition will be irrelevant to our SEP model. Therefore, it is impossible to obtain an equilibrium with an extremely fine-tuned delicacy that would enable us to see an equilibrium as the end result of an indefinitely long period of time under the same waves.

CONCLUSIONS

(1) This research attempts to establish a theoretical physical model for SEP based on an analysis of the major mechanical forces involved in forming an SEP and the equilibrium conditions. This first order linear approach and simplified basic physical model of SEP yields an explicit equation of SEP.

(2) This physical model attempts to relate the wave parameters with the geometry of the SEP.

(3) Through a non-linear regression computer program written in BASIC following a SIMPLEX algorithm, the theoretical equation has been compared with real world SEP's selected from the U.S. Gulf of Mexico coast and the southeast U.S. Atlantic coast according to the criteria derived from the conceptual model of SEP. The regression

Table 2. *Wave tank experiment report.*

Depth (in.)	Platform Angle (°)									
	0	1.6	3.2	4.0	6.3	7.9	9.5	11.0	12.6	14.1
−3.0	+	+	+	+	+	+	+	+	+	+
−3.5	+	+	+	+	+	+	+	+	+	+
−4.0	+	+	+	+	+	+	+	+	+	+
−4.5	+	+	+	+	+	+	+	+	#	#
−5.0	+	+	+	+	#	+	#	#	#	–
−5.5	+	+	+	#	#	#	#	–	–	–
−6.0	+	+	+	#	#	–	–	–	–	–
−6.5	+	+	+	#	–		–	–	–	–
−7.0	+	+	+		–	–		–	–	–
−7.5	+	+	+			–	–	–	–	–
−8.0	+	+	+	#	–		–	–	–	–
−8.5	+	+	+	#	#	–	#	#	–	–
−9.0	+	+	+	#	#	#	#	#	#	–
−9.5	+	+	+	#	#	#	#	0	#	#
−10.0	+	+	+	#	0	#	#	0	0	0
−10.5	+	+	+	#	0	0	0	0	0	0
−11.0	+	+	+	0	0	0	0	0	0	0
−11.5	+	+	+	0	0	0	0	0	0	0
−12.0	+	+	+	0	0	0	0	0		
−12.5	+	+	+	0	0					
−13.0	+	+	0	0						
−13.5	+	+	0	0						
−14.0	+	0	0	0						
−14.5	0	0	0							
−15.0	0	0	0							

Wave height: 5 ± 0.5 in. Wave period: 1.36 ± 0.2 seconds. Adjustable platform: Smooth, red. Sand used: Medium-coarse quartz sand. Time: December 1989 to March 1990. Legends for sand movement: 0 = no movement, + = onshore net movement, – = offshore net movement, # = oscillating, no obvious net movement.

Table 3. *Wave tank experiment report.*

Depth (in.)	Platform Slope (°)										
	0	1.6	3.2	4.0	6.3	7.4	9.5	11.0	12.6	14.1	15.7
−3.0	+	+	+	+	+	+	+	+	+	+	+
−3.5	+	+	+	+	+	+	+	+	+	+	+
−4.0	+	+	+	+	+	+	+	+	+	+	+
−4.5	+	+	+	+	+	+	+	+	+	+	#
−5.0	+	+	+	+	+	+	+	#	#	#	#
−5.5	+	+	+	+	+	#	#	#	#	–	–
−6.0	+	+	+	+	+	#	#	–	–	–	–
−6.5	+	+	+	+	#	#	–	–	–	–	–
−7.0	+	+	+	+	#	#	–	–		–	–
−7.5	+	+	+	#	#	#	#	#	–	#	–
−8.0	0	+	#	#	0	0	#	0	#	#	#
−8.5	0	0	0	0	0	0	#	0	0	0	0
−9.0	0	0	0	0	0	0	0	0	0	0	0
−9.5	0	0	0	0	0	0	0				
−10.0	0	0									
−10.5	0										
−11.0											

Wave height: 5 ± 0.5 in. Wave period: 1.36 ± 0.2 seconds. Adjustable platform: Roughened with sand layers with epoxy, red. Sand used: Medium-coarse quartz sand. Time: March 1990. Legends for sand movement: 0 = no movement, + = onshore net movement, – = offshore net movement, # = oscillating, no obvious net movement.

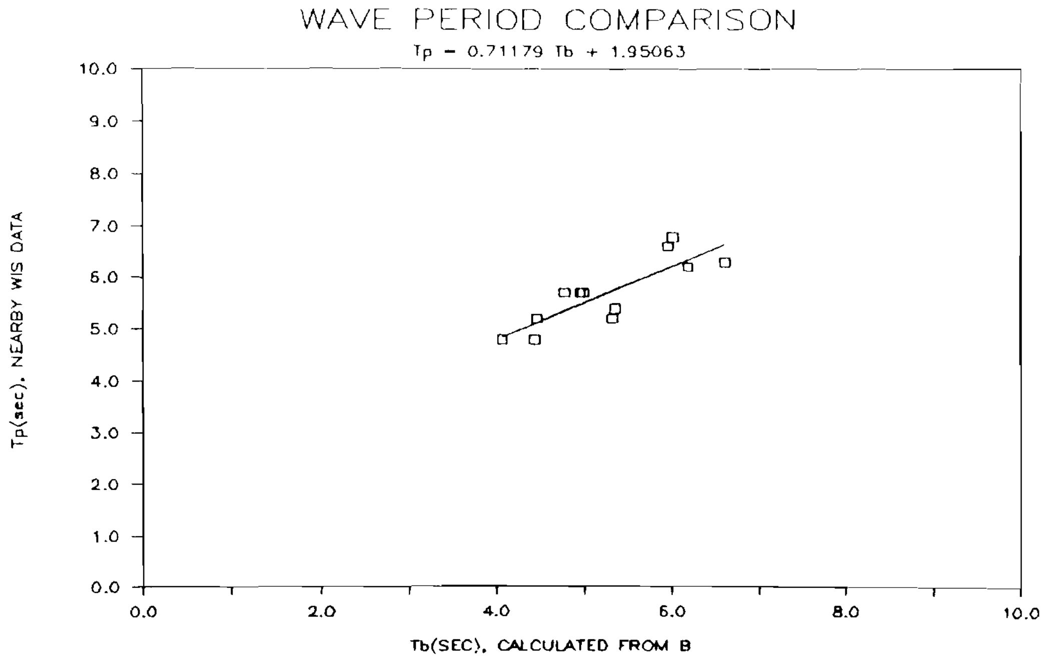


Figure 8. Correlation analysis of wave periods obtained by calculation from parameter B and by field observation.

process demonstrates a good fit between the theoretical equation and the selected SEP's (Figures 8–11 are examples), and the parameter B's calibrated through this process are within the reasonable range reflecting the physical wave conditions in these areas.

(4) Some SP profiles on the U.S. Pacific coast, though deviating from the conceptual SEP model, are also treated with the regression process. That the parameters obtained through the regression process are out of range from the observed wave parameters further indicates that with the given data and scale, in general, the SP's along the U.S. Pacific coast are not compatible with this SEP model.

(5) Some of the offshore wave lengths (periods) obtained through regression of the profiles in the Gulf of Mexico (which were completed in 1988) are then compared with the field data published in 1989 measured in some areas in the Gulf of Mexico by NOAA through the past 20 years. The result of the comparison is quite encouraging. The comparison of the two sets of data has one significant advantage because they are independent, one from the marine floor profile geometry and the other from observation of the waves through the field automatic stations. The comparison of the two sets of data also has one significant disadvantage: the locations of the wave stations and the profiles are not coordinated. Therefore, not all the data in one set are comparable with data in the other set. The locations of wave stations do not keep a consistent distance from the shore; some profiles are located nowhere near any wave station that can provide relevant wave information. Also, the distance between the profiles and the available nearby stations are not consistent.

(6) A wave tank and a unique adjustable platform were designed and constructed for this research. The wave tank experiments strongly support the mechanisms in structuring this physical and analytical model. The design of the adjustable platform provides a new tool to facilitate future studies on sediment/wave interaction.

(7) This physical model is a simplified, first approach, linear model. The model provides some insight into how the surface of the bottom sediment would behave under the influence of gravity, friction, dilatancy, vibratory field and shear force posed by the waves. However, the model does not

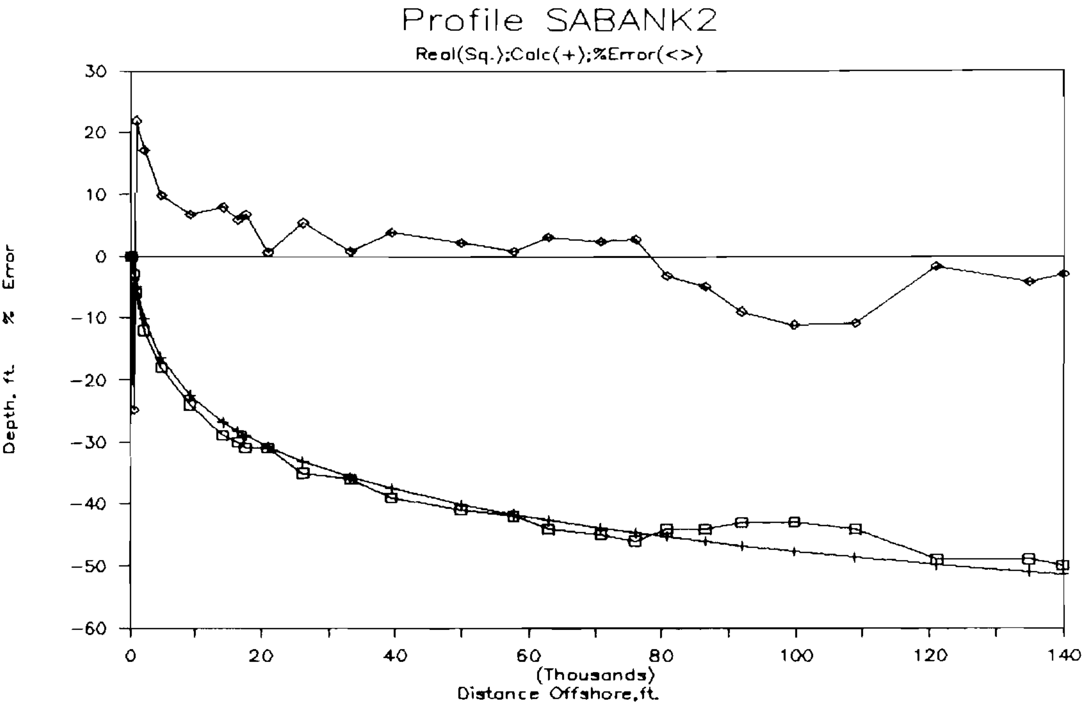


Figure 9. A regression match between the physical model of SEP and a submarine profile. R = 0.985420.

involve the grain size change and wave parameter change on the shoaling process.

FUTURE STUDIES

(1) Among the physical parameters which may have decisive influence on the geometry of the SEP, the following have been identified by previous investigators: wave length, wave height (if the offshore wave steepness is assumed a constant, then wave height and wave length are interlocked), grain size (assuming uniform shape and size), and the mass density of the sand (assuming quartz sand, then mass density is not a variable).

Therefore, in Equation 7 derived from this model,

$$x = A(e^{By} - 1)$$

B is known as a function of wave length, then A must be a function of grain size and may include wave length. If we compare Equation 7 with Dean's equation, we will find certain similarities, especially parameter A in Equation 7, and factor A in Dean's equation. Both are multiplication factors between a function of x and a function of y.

Therefore, calibration efforts for parameter A may be reasonable to concentrate on for data collection on the grain size.

(2) To test the validity of this dynamics model with any known energetics models, the wave energy attenuation rates on some real world SEP's should be collected and compared with the energy attenuation rates on an idealized SEP. The energetics criterion used by Dean and Per Bruun on the nearshore profile section including the surfing zone is that the wave energy attenuation rate per unit volume of water should stay constant (DEAN, 1990). The preliminary assumption must be such that during surfing, because of the extreme chaos in the water body, the energy can only be assumed homogeneously distributed at any depths within the water body. This attenuation rate is also not clear, whether it is based on attenuation rate referred to unit time or referred to unit distance the wave travels. More important is the fact when the wave propagates onshore from deep water, the wave energy contained in unit volume is very different at different depths. The calculation is as follows:

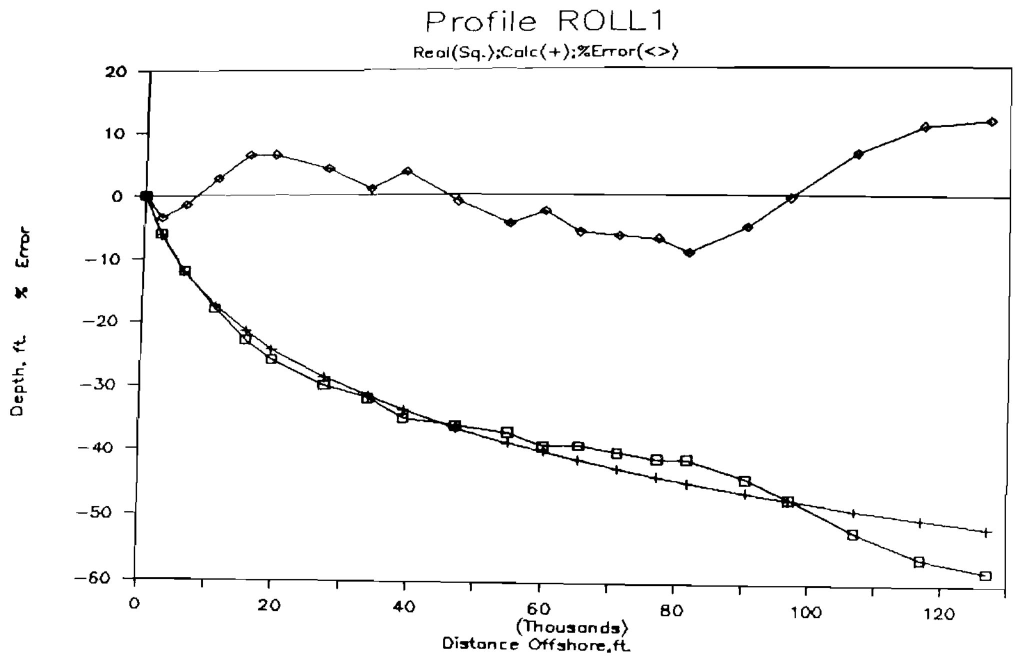


Figure 10. A regression match between the physical model of SEP and a submarine profile, $R^2 = 0.971477$.

The wave energy density E is the total wave energy contained in a column of water within a unit square area of infinite depth.

$$E = {}^1\!/\!_8\,\rho g H_0^2$$

where ρ is the water mass density,
g is the gravitational acceleration,
H_0 is the wave height,
E is energy density, energy per unit area on the surface of the sea and with an infinite depth.

The energy contained in a unit thickness of water body within a unit volume at a depth of y should be:

$$E_y = Q(H_y)^2$$

where E_y is the energy contained in a unit thickness of a unit volume of water body at a depth of y,
Q is a proportionality,
H_y is the water orbital diameter at the depth of y under a wave with a wave height of H_0.

The integration of E_y from minus infinity to zero should be the total energy density.

$$\frac{1}{8}\rho g H_0^2 = \int_{-\infty}^{0} Q(H_y)^2\,dy$$

Because $H_y = H_0 e^{2\pi y/L}$, where L is offshore wavelength,

$$\frac{1}{8}\rho g H_0^2 = \int_{-\infty}^{0} Q H_0^2 e^{\frac{4\pi y}{L}}\,dy$$

From this equation, we obtain

$$Q = \frac{\pi \rho g}{2L}$$

Energy contained in unit volume at the depth of y must be:

$$E_y = Q H_y^2 = \frac{\pi \rho g}{2L} H_y^2$$

or,

$$E_y = \frac{\pi \rho g}{2L} H_0 e^{\frac{2\pi y}{L}}$$

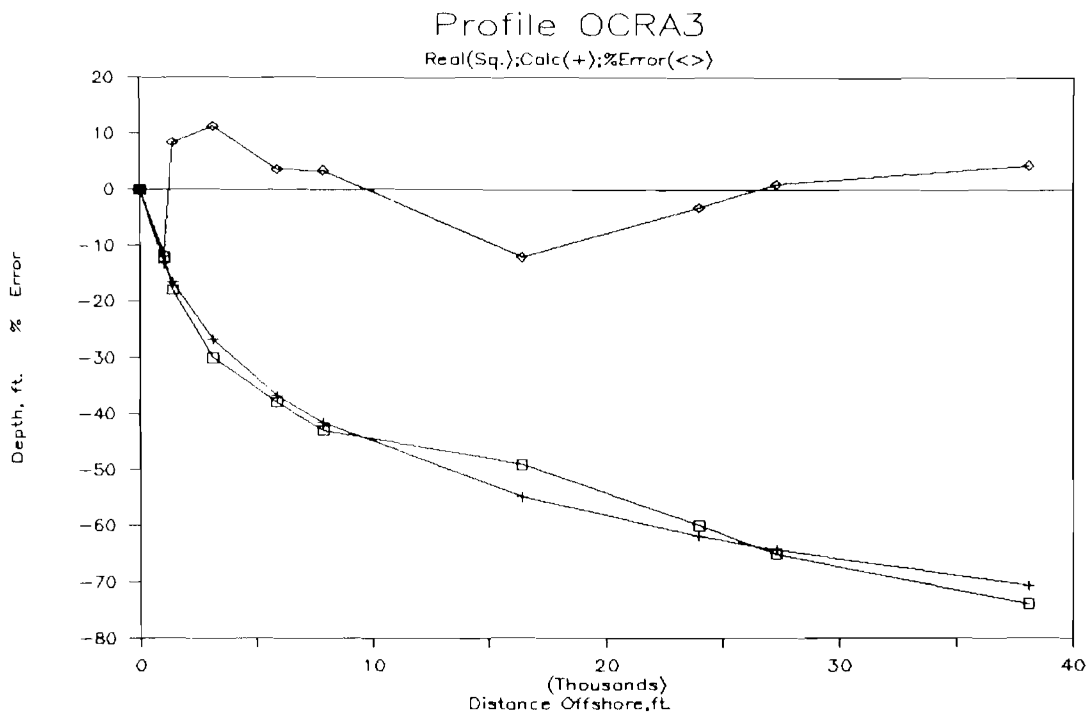


Figure 11. A regression match between the physical model of SEP and a submarine profile, $R^2 = 0.987572$.

It is very difficult to test a dynamic model by using an energetics criterion based on an assumption that an energetics model better reflects the natural situation. This is especially unfavorable when the energetics model has no solid physical reasoning by itself. We may only compare each with empirical approaches.

(3) Applications

(a) Calibration of parameter A can be achieved numerically through the nonlinear regression process, empirically using different profiles with known grain size characteristics, or through theoretical derivation following the hydrodynamic condition for balance on a slope.

(b) Assuming that Equation 7 really represents the SEP in balance with the waves having a wave length of L and with a fixed nominal offshore wave steepness (which states that the wave height is a simple function of the wave length), then using nonlinear regression, parameter A of the specific profile of interest can be obtained (calibrated). With this now available parameter A, idealized SEP's in different wave conditions can be predicted for this specific location. A real time sequential model, therefore, is possible to predict the responses of a real world profile to a wave condition. A computer program can be written to process such responses in a time sequence in which details of wave conditions in each time interval would be reasonably predictable from the meteorological forecast. The trilogy expressing the relationship of wind condition–wave condition–sediment movement has long been the goal of modern sedimentology studies. This model may bring this goal a step closer to reality.

(c) The idealized SEP can be used as a reference for assessment of profile stability and for predicting which sections of a profile may be undergoing deposition or erosion by the influence of the forecast wave conditions.

(d) The same principles applied to (c) can be applied to help site selection for offshore dredge and fill.

(e) A ring-shaped new wave flume with a synchronized regenerative wave generator and an adjustable platform is proposed for future study of the behavior of loose sand on different slopes and at different depths under the influence of continuous traveling waves. This wave tank can provide a complete suite of information on sand behavior under the waves following a systematic variation of parameters, such as grain size, grain mass density, wave height, wave length, slope, depth and other bottom features such as the influence of ripple marks, the origin of ripple marks, etc.

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