The Systematic Structure-Change into the Molecule-like Structures in the Self-Conjugate $4n$ Nuclei

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The rotational bands with the diatomic-molecule-like structure in the self-conjugate $4n$ light nuclei, such as, α-α, α-^{12}C and α-^{16}O, appear systematically at near the threshold energy for the decay into the relevant subunit nuclei. The relations between the structure change into the molecule-like structure and the threshold energy for the decay into the subunit nuclei are discussed. According to this discussions, the diagram for the systematic structure changes into the molecule-like structures through the alpha particle release is presented as the function of the mass number and the energy. Upon this diagram the rotational bands with $K=0^{\pm}$ in light $4n$ nuclei can be summarized. The order of the degree of the polarization toward the separation of the subunit nuclei for the diatomic molecule-like structure case is discussed qualitatively.

§1. Introduction

Existence of the strongly tight alpha particle had led to the study of the alpha particle model[1)~4)] of the self-conjugate $4n$ nuclei which started from the rule[1)] of the constancy of the binding energy per bond. The assumption of the relative tightness of the alpha particles in whole system was, however, considered to be doubtful[2),5)] since alpha particles could easily dissolve into their constituents in the nucleus. The nucleus of ^{8}Be which should be basic for the alpha particle model was understood[5),6)] to have the high degree of dissolution of alpha particles, especially by the reason of the assignment of the low lying excited state with zero spin.

There is clear difference in the bond energy between this nucleus and the other $4n$-nuclei: Only the ground state of ^{8}Be appears as the quasi bound state which can decay into two alpha particles. Stressing this difference, Sekine, Taketani, Toyoda and Inoue[7)] reconsidered the alpha particle model of ^{8}Be and pointed out that the alpha particle model of ^{8}Be has no reasons to be refused from the theoretical point of view and also from the experimental facts, contrary to the previous criticism.[5)]

Experimentally, the rotational levels with large alpha decay widths and the large moment of intertia, and the absence of other levels up to the excitation energy of about 17 MeV with $T=0$ are now established.[8)] Theoretically,

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the interactions between alpha particles as the complex nuclei have been shown by Hokkaido-group[9] and others,[10] to be able to understand approximately in terms of an energy independent potential[11] with inner strong repulsive core due to the Pauli principle, outer weak attractive potential and the angular momentum dependence.

On the other hand, the ground state of ^{16}O shows the closed shell nature[12] which has been understood from the independent particle point of view. The simplest representation of the closed shell structure was given by the shell model with the closed shell configulations of $0s$ and $0p$ levels, where the four particle correlations are only incidental one of the spacial symmetry.[13] Much more importance of the four particle correlations in the ground state than expected by the shell model has been recognized since earlier time through the reexamination of the α-boson model[14] and the studies by the α cluster model.[15]~[17] The ground state correlations due to the residual forces which favor the spacial symmetry, however, are not strong enough to give rise the basic change[17] into α-particule condensations in the ground state. Such closed shell nature of the ground state of ^{16}O is thought mainly to be due to the effects of the Pauli principle which plays an essential role for the justification[18] of independent particle natures in the nuclear matter.

Recognitions of the two different kinds of the structures in the ground states of the $4n$ light nuclei lead us to an understanding that alpha particles lose its identity in the compact nucleus, where there remain the correlated characters like in ^{16}O, and, reversely, the correlated four particles in the whole system are able to tend to become alpha particles if the compactness of the whole nuclear system is released, such as the nucleus of ^{8}Be. When we accord to this understanding the changes of the structures can be expected to arise in the excited states of the $4n$ light nuclei, since the release of the compactness depends on not only the mass number but also the energy of the nuclear system.

More than ten years ago Morinaga[19] proposed an interesting idea concerning the change of the structures in the excited states of $4n$ light nuclei. In the ^{16}O case, when the four particles excite from the closed core, the structure with highly deformed configuration or the alpha chain like configuration may arise after the rearrangement of the whole nucleus and then rotational band with large moment of inertia can be expected. The rotational bands with these straight α-chain structures have been assigned[20] in ^{12}C to the rotational band upon the second excited zero plus state and reported to be observed in the higher excited region of the ^{16}O than considered before.[21]

It is necessary to the change from the ground state into the α-chain structure to release all alpha particles from the closed packed ground structure. The changes of structures are, however, possible to begin through an alpha particle-release from the ground structure, that is, through the change into

the diatomic-molecule-like structure[22] of an alpha particle plus the residual core $4n$-nucleus. We have recently reported[23] that the two rotational bands with $K=0^+$ and 0^- in $^{16}O^*$ and in ^{20}Ne can be naturally understood as "the twins" with the same molecule-like structures of an alpha particle plus residual nuclei of ^{12}C and ^{16}O, respectively.

Thus the many kinds of the structure changes can be expected to arise in the $4n$-light nuclei, starting from an alpha particle release up to all alpha particle-release. It is the purpose of this note to present an unified picture concerning the changes of structures into the molecule-like structures through the alpha particle releases in the self-conjugate $4n$ light nuclei.

In §2, the characteristic properties of the two rotational bands with $K=0^\pm$ in $^{16}O^*$ and in ^{20}Ne are summarized, from which we can know the systematic appearances of the molecule-like structures near the threshold energy for the decay into the subunit nuclei of an alpha particle and the residual $4n$-nucleus. In §3, the relations between the threshold energy and the molecule-like structures are discussed. In §4, the diagram for the systematic changes of structures into the molecule-like structures is presented. In §5, the order of degree of the polarization is discussed qualitatively.

§2. Systematic appearances of the diatomic-molecule-like structures near the threshold energy

The molecule-like structure with two alpha particles has been recognized[7),12] in the nuclear system of ^8Be. When such molecule-like structures exist in other $4n$-nuclei, the many rotational bands with similar characters to the ground rotational band of ^8Be should be observed. In the rotational bands in $^{16}O^*$ and $^{20}Ne^*$, the rotational bands with $K=0^-$ have been known to[22] have the characters similar to that of ^8Be. When the heteropolar diatomic molecule-like structure with an alpha particle plus residual core nucleus appears as the intrinsic structure of a rotational band with $K=0^-$, a rotational band with $K=0^+$ can exist as a twin of the same molecule-like structure with a small energy gap between these two rotational bands with $K=0^\pm$.

We briefly recapturate[23] the characteristic features of the two rotational bands with $K=0^\pm$, such as in ^{16}O ($K=0^+$ upon 6.06 MeV 0^+ state and $K=0^-$ upon 9.58 MeV 1^- state) and in ^{20}Ne ($K=0^+$ ground band and $K=0^-$ upon 5.80 MeV 1^- state);

i) The moments of inertia of the two bands are almost of the same value and very large.

ii) The alpha decay widths of the quasi-bound levels have the large fractions of the Wigner limit.[22),24)~26]

iii) These bands start from near the threshold energy for the decay of an alpha and residual $4n$-nucleus.

iv) Two bands with $K=0^{\pm}$ lie with a small gap of the excitation energy.

v) The energy level structures with positive parity upon the 6.06 MeV first excited state in ^{16}O can be well understood by the hypothesis of the weak coupling.[27]

We can see that characteristic features of the two rotational bands with $K=0^{\pm}$ show very similar features to the ground rotational band of ^{8}Be.

From the molecular viewpoint, we regulate the rotational level structure according to the following equations,

$$\begin{aligned} E_{+}(I) &= E_0 - \frac{\Delta E_0}{2} + \frac{\hbar^2}{2\mathcal{J}_{+}} I(I+1) \qquad \text{for} \quad K=0^{+}, \\ E_{-}(I) &= E_0 + \frac{\Delta E_0}{2} + \frac{\hbar^2}{2\mathcal{J}_{-}} I(I+1) \qquad \text{for} \quad K=0^{-}, \end{aligned} \tag{1}$$

where ΔE_0 is the splitting energy between the two rotational bands, E_0 is defined to be the intrinsic energy of the molecule-like structure and $\mathcal{J}_{\pm}$ are the moments of inertia of the two rotational bands with $K=0^{\pm}$ respectively. If we could assume the rigidity of two subunit nuclei just as in the real molecule, the splitting energy ΔE_0 should vanish. However, we do not assume the rigidity in the nuclear molecule-like structure since the two subunit nuclei interact with each other in one whole nuclear system. Then the rotational bands with $K=0^{\pm}$ naturally appear as the non-degenerate two bands, where the excitation energy of the symmetric part is lower than that of the anti-symmetric part; $\Delta E_0 > 0$. The values of E_0 and ΔE_0 are listed in Table I, together with the case of ^{8}Be.

We have the two sets of the energy values of E_0 and ΔE_0 from the experiments, which give us different informations about the molecule-like structures. One of the important facts is that E_0, the central energy between the $J=0^{+}$ state and the imaginary $J=0^{-}$ state, lies at near the threshold energy, especially in the case of $^{16}O^{*}$. Systematic appearances of the diatomic-

Table I. Characteristic value for each molecular band.

E_0 is the central energy between the $J=0^{+}$ and the imaginary $J=0^{-}$ state. Note especially that E_0 lies near the threshold energy E_{th}.

ΔE_0 is the splitting energy between the two rotational bands and is the measure of the degree of polarization. The smaller ΔE_0 is, the higher the degree is. Compare ΔE_0 of ^{16}O with ΔE_0 of ^{20}Ne. This connects with the difference of the degree of polarization between ^{16}O and ^{20}Ne.[23]

	^{8}Be	$^{16}O^{*}$	^{20}Ne
ΔE_0		3.14	5.53
E_0		7.62	2.76
E_{th}	−0.095	7.16	4.73

molecule-like structures, α-α for ^{8}Be, α-^{12}C for ^{16}O* and α-^{16}O for ^{20}Ne, "near the threshold energy" seem to give the characters of the structures changes in light $4n$-nuclei, which is discussed in §3. The magnitude of ΔE_0 represents the order of the softness of the subunit nuclei, which gives us the informations about the degree of the polarization toward the separation of the subunit nuclei, as was discussed in the previous paper.[23]

§3. The threshold energy for the decay into the subunit nuclei and the molecule-like structure

The molecular structure in the nuclear system has intrinsically the meaning as the relative concept since the interactions between relatively tight subunit nuclei are the same nuclear interactions as that between the nucleons in the subunit nuclei. We can therefore, define the molecule-like structure to have the high degree of the polarization toward the separation of the subunit nuclei and to have the weak degree of the dissolution of the subunit nuclei into the normal structure.

The ground state and the low-lying resonance levels of the nucleus of ^{8}Be have been recognized as with such molecule-like intrinsic structure of the very high degree of the polarization into two alpha particles. We can summarize the reasons of the realization of the molecule-like structure in the system of ^{8}Be as follows: i) the alpha particle is the strongly tight bound state in the isolated system and ii) the interaction between α particles is weak enough to trap two alpha particles scarecely in a quasi-bound state. In such situation, the Pauli principle between the nucleons of the different alpha particles plays a role as if against the dissolution.[9),10),28] These factors result in the binding energy of ^{8}Be which is a little smaller than the sum of the binding energies of two alpha particles.

Standing on the above understandings, we discuss the meaning of appearances of diatomic molecule-like structures near the threshold energy for the decay into relevant subunit nuclei. At first, we assume the relative tightness of the subunit nuclei of α, ^{12}C and ^{16}O.

As the natural consequence of the assumption and the appearances of diatomic molecule-like structures near the threshold energy, the strength of the interactions between subunit nuclei is expected to be the same order of the weak interactions between two alpha particles and the wave function for the relative motion stretches much more in the outside region where Pauli principle between the nucleons belonging to different subunit nuclei plays no role effectively. Since the dissolution of the subunit nuclei arises from the cooperative actions of the interactions with Pauli principle between them, the appearance of the diatomic-molecule-like structure near the threshold energy is the optimum condition in order to avoid the disturbance to the relative

tightness of the subunit nuclei. In such situations the attractive interactions between the subunit nuclei play the role mainly to keep the subunit nuclei in a scarce bound or quasi-bound state together with the Coulomb force and centrifugal potential, and also Pauli principle bears the role as if to act against the dissolutions.[9),10),28)]

If the interaction between subunit nuclei α, and ^{12}C or ^{16}O were not weak, the relative tightness of subunit nuclei in the whole nucleus could not be conserved since the cooperative actions of the interaction between them with Pauli principle will induce the rearrangement of the molecule-like structure. If not so, there arises the self-contradiction in the original assumption for the case of ^{16}O: Even if the interaction between α and ^{12}C is not weak, the relative tightness of subunit nuclei of α and ^{12}C could hypothetically be assumed. Then the total binding energy increases up to the binding energy of the ground state of ^{16}O and we get the inconsistent result that the independent unity of ^{16}O is equal to the molecule-like structure with α plus ^{12}C where the identity of α should be found in ^{16}O. Thus, we could not expect that the molecule-like structure appears in the appreciably lower energy region than the threshold energy so long as the ground state is not assumed to have the molecule-like structure.

We can understand that the appearance of the molecule-like structure near the threshold energy is, rather, necessary for its formation. In other words, when the molecule-like structure would appear, the rotational bands with the molecule-like structure should start near the threshold energy for the decay into the relevant subunit nuclei. In the above discussions we have only been concerned with the diatomic-molecule-like structure. However, since the subunit is also a $4n$-nucleus, the above discussions can be extended to the cases of the polyatomic molecule-like structures and the same conclusions as the case of diatomic-molecule-like structure might be obtained.

§4. The diagram for the systematic change of structure

In light $4n$-nuclei, the separation energy of an alpha particle is of an order of 7 MeV, which is due to the strong binding energy of the alpha particle. There arises the possibility of the structure change through an alpha particle release accompanied by the relatively large rearrangement of the ground state structure of light $4n$-nuclei at energy interval of the order of 7 MeV. Such changes of structures starting from an alpha particle release which results in the diatomic-molecule-like structure, may continue up to all alpha particle releases.

We hypothetically assume that the molecule-like structures with the subunit $4n$-light nuclei would appear at the threshold energy for the decay into the relevant subunit nuclei;

$$E_0 \approx E_{\text{th}}\,, \tag{2}$$

where E_0 is the excitation energy of the rotational band with the molecule-like structure and E_{th} the threshold energy for the decay into the relevant subunit nuclei. According to this relation of Eq. (2), we can suppose what kinds of the molecule-like structures have the possibility of appearance in the excited state systematically.

To make our picture more clear, a hypothetical diagram is presented in Fig. 1. The energy difference of each step to the vertical direction in Fig. 1 is taken to be the separation energy of an alpha particle from corresponding $4n$-nucleus. In the figure, we show the possible kinds of the molecule-like structure up to the nucleus of ^{24}Mg where the molecule-like structure of ^{12}C-^{12}C is included. It can be extended to the more heavier light $4n$-nuclei according to the above systematic arrangements.

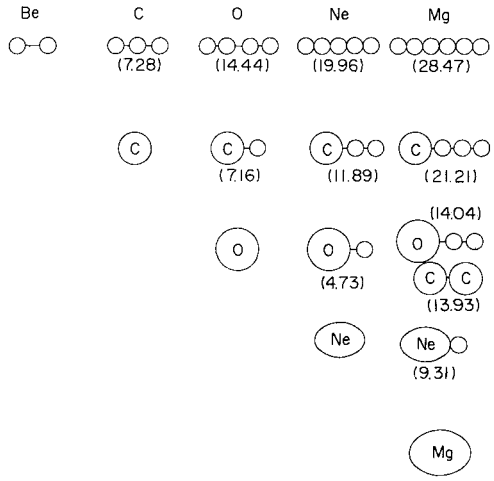

Fig. 1. Threshold energy for each decay mode. In the figure, the threshold energy for each decay mode is given in MeV. The systematics suggests the possible molecular nature around each energy. Some of the molecular states are already found and are represented in Fig. 2.

For the case of the diatomic-molecule-like structure, the loosely bound structure means properly the linear molecule-like structure. For the case of the polyatomic-molecule-like structure, it does not necessarily mean the linear structure. However, the linear molecule-like structure is assumed to have the plausible possibilities of realization because it can have a high degree of the polarization toward the separation of the subunit nuclei. If the alpha particles more than two contact with each other simultaneously, they have the chance to tend to become the ground configuration of ^{12}C or other $4n$-nuclei, which is the fundamental assumption of our picture shown in the figure. Therefore we may now consider the linear molecule-like structure. Generally, Coulomb force is noted to favour the linear molecule-like-structure, although the stability of linear polyatomic molecule-like structure is not so clear so long as we take into account only the effect of Coulomb force.

In the figure, we show only the sets of the subunit nuclei. We must note that when the different kinds of nuclei are included in one set of the subunit nuclei, there exist several molecule-like structures with different configurations of the subunit nuclei. We can distinguish them uniquely with

different linear arrays of the subunit nuclei where the mirror image of one configuration with a heteropolar polyatomic molecule-like structure is also taken into account; for example, α-^{12}C and ^{12}C-α for the case of diatomic-molecule-like structure, and α-α-^{12}C, ^{12}C-α-α and α-^{12}C-α for the case of triatomic-molecule-like structure.

We inspect the correspondence of the schematical diagram for the molecule-like structures with the experimental systematics. The origin of this diagram is ^{8}Be with diatomic molecule-like structure of two alpha particles, energy of which has a scarcely greater value than the threshold energy for the decay into two alpha particles. In ^{12}C, the rotational band with three alpha chain molecule-like structure has been expected to exist upon the second excited zero plus state[20] which lies at a little higher energy for the decay into an alpha particle and ^{8}Be or three alpha particles. In ^{16}O, the rotational bands with $K=0^{\pm}$ starting at near the threshold energy for the decay into an alpha particle and ^{12}C are considered to have the diatomic-molecule-like intrinsic structure (in §2). The rotational band with four alpha chain molecule-like structure has been observed at a little higher energy (about 2 MeV higher) than the threshold energy for the decay into two Beryllium.[21]

The excited rotational band with $K=0^{-}$ of ^{20}Ne starting at a little higher energy than the threshold energy[24] is plausibly considered to have the diatomic-molecule-like intrinsic structure with an alpha particle plus ^{16}O, the partner of which with $K=0^{+}$ corresponds just to the ground rotational band of ^{20}Ne.[23),24] The energy of the ground state of ^{20}Ne is, however, appreciably lower than one except from the relation of Eq. (3). This point will be discussed in §5. In the energy region between 7 MeV and 10 MeV of ^{20}Ne, several rotational bands with $K=0^{+}$ or 0^{+}-2^{+} levels have been observed.[24),26] The energies of these rotational bands are too low to expect that the molecule-like structures with two alpha particles and ^{12}C would be found in them, and so it remains as a question. There are the levels of the spins and parities 0^{+}-1^{-}-2^{+}-3^{-}[19] in the excited states of ^{24}Mg. These levels start from 11.75 MeV which is a little smaller (about 3 MeV) than the threshold energy for the decay into ^{8}Be and ^{16}O (and also into

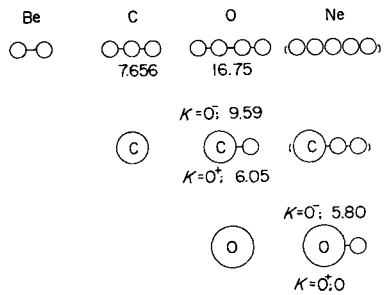

Fig. 2. Experimental systematics of molecular states.
In the figure the experimental band head energy is given in MeV. $K=0^{+}$ (6.06 MeV), $K=0^{-}$ (9.59 MeV) bands in ^{16}O will correspond to C-α configuration and $K=0^{+}$ (ground), $K=0^{-}$ (5.80 MeV) bands in ^{20}Ne will correspond to O-α configuration.[23] Molecular states in parentheses are not yet found.

^{12}C plus ^{12}C). It is not sure experimentally that the intrinsic structure of these levels could be assigned to be some kind of the molecule-like structure like as two alpha particles plus ^{16}O.

Fig. 2 shows the above mentioned rotational bands with $K=0^{\pm}$ of which the intrinsic structure can be assumed to be the melecule-like structures where the energies of the band heads are listed. For the reference the molecule-like structures which have the possibility of realization are also illustrated hypothetically at the corresponding threshold energy in Fig. 2. Since we now assume that the structure of the ground rotational band may belong to the diatomic-molecule-like structure, ^{20}Ne is not considered to be the subunit nucleus.

§5. Concluding remarks

The change into the molecule-like structures could be assumed to arise near the threshold energy for the decay into the relevant subunit nuclei (in §3). In the experimental data the excitation energies of the rotational bands with $K=0^{\pm}$ were examined (in §4) to take the values near the corresponding threshold energies. We summarize the deviations $\Delta\varepsilon$ from the relation of Eq. (2);

$$\Delta\varepsilon = E_0 - E_{\text{th}}\,, \tag{3}$$

where E_0 is the excitation energy and E_{th} the threshold energy for the decay into the relevant subunit nuclei: The experimental values of $\Delta\varepsilon$ are 0.09 MeV, -1.1 MeV and -4.7 MeV for the rotational bands with $K=0^{+}$ of ^{8}Be(α-α), ^{16}O*(α-^{12}C) and ^{20}Ne(α-^{16}O) respectively, and 2.0 MeV and 0.7 MeV for the rotational bands with $K=0^{-}$ of ^{16}O*(α-^{12}C) and ^{20}Ne*(α-^{16}O), respectively. For the two cases of the alpha chain-like structures in ^{12}C and ^{16}O, the values of $\Delta\varepsilon$ are 0.4 MeV and 2.5 MeV, respectively.

The upper limit of $\Delta\varepsilon$ is determined by the height of Coulomb barrier and also centrifugal barrier. The lower limit of $\Delta\varepsilon$ is thought to exist as a critical energy for the change into the lower order of the molecule-like structure. Experimental examples show such characteristics, that is, the values of $\Delta\varepsilon$ are positive or small negative values for all the cases except for the ground rotational band of ^{20}Ne. The energy dependent character appearing in the structure changes may have the wider generalities since the structure changes are caused by the alpha particle releases; i) The order of the degree of the polarization toward the separation of the subunit nuclei is thought to be the increasing function of the energy $\Delta\varepsilon$ within the energy region between the above two limits. ii) The higher order of the molecule-like structure (for example the alpha chain-like structure) may have the clearer and higher degree of the polarization into the subunit nuclei than the lower order of the molecule-like structures, (for example diatomic-molecule-like structure). The

measure concerning the order of the degree of the polarization can be directly known through the absolute magnitude of the reduced widths for the decay into subunit nuclei as was seen in the cases of the diatomic-molecule-like structures.

An appreciable deviation from the energy systematics can be seen in the ground rotational band of ^{20}Ne where $\Delta\varepsilon$ is equal to -4.73 MeV. This value is about half of the average value of separation energy of an alpha particle from $4n$-nuclei such as ^{16}O and ^{12}C. It seems that the ground state of ^{20}Ne is situated near the critical point of the structure change. To exhibit this remark, the part of the systematic diagram of Fig. 2 is illustrated in Fig. 3. It is seen that the ground state of ^{20}Ne lies between the excited zero plus state of ^{16}O with diatomic-molecule-like structure and the ground state of ^{24}Mg. In Fig. 3 the line from the ground state of ^{16}O to ^{20}Ne is extrapolated toward the molecule-like structure of α-α-^{16}O which has not, however, been confirmed.

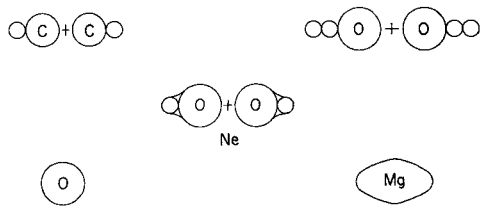

Fig. 3. Schematical picture of ^{20}Ne ground state. This picture indicates the intermediate nature of ^{20}Ne. ^{16}O excited state and ^{24}Mg excited state have the clearer molecular structure, whereas ^{16}O and ^{24}Mg ground states have shell and obscurer molecular structure respectively.

When the molecule-like structure is assumed to be formed in the two rotational bands with $K=0^{\pm}$ of ^{16}O* and ^{20}Ne, with the high degree of the polarization into subuint nuclei of an alpha particle plus the residual $4n$-nucleus, the energy gap can be understood as the result of the polarization effect of the subunit $4n$-nucleus into the alpha particle and the residual part due to the presence of the outer alpha particle coupled weakly with the subunit $4n$-nucleus. From this point of view, we have studied this energy gap and obtained the value of order of 2~3 MeV,[23] which corresponds to the value for the case of ^{16}O*. The value of the energy gap for the case of ^{20}Ne is also expected to be the same order of 3 MeV as for ^{16}O*. However the experimental value of the energy gap for the case of ^{20}Ne is 5.5 MeV, where we can again see the appreciable deviation of the molecule-like structure in the ground state of ^{20}Ne. We may take into consideration of the freedom of the alpha particle dissolution into its constituent nucleons which moves outside of the closed shell of ^{16}O.

The three alpha chain-like structure of ^{12}C is now considered to correspond to the intrinsic structure of the rotational band upon the second zero plus state. The rotational spectra is not, however, confirmed experimentally. The state with broad alpha decay width at 10 MeV could be assigned to

be 2^+ [20] which indicates that the moment of inertia is smaller than what is expected from a straight alpha chain. Under the assumption of adiabaticity for the rotational motion, we can estimate the restoring force due to the Coulomb force. The stability for the bending vibration cannot be understood completely since the restoring force is not enough to save satisfactorily the zero point oscillation for the bending mode.

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