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SHORTER NOTES

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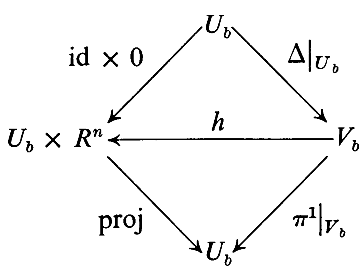
THE TANGENT MICROBUNDLE OF A SUITABLE MANIFOLD

RONALD J. STERN[1]

ABSTRACT. The purpose of this note is to generalize to the topological category the fact that a suitable differentiable manifold is parallelizable (Theorem 4 of [1]). This result has a "folk-theorem" status in some quarters, but I believe that in view of the recent interest in H-manifolds [2], it would be desirable to have the result on record.

Let M be an n-manifold. Define $\Delta: M \to M \times M$ to be the diagonal map, and $\pi^1, \pi^2: M \times M \to M$ to be the projections on the first and second factor respectively. Milnor [3] calls the diagram $\Delta: M \rightleftarrows M \times M : \pi^1$ the *tangent microbundle* of M, where for each point $b \in M$ there exists an open set U_b in M containing b, an open set V_b in $M \times M$ containing $\Delta(b)$, and a homeomorphism $h: V_b \to U_b \times R^n$ such that the following diagram commutes:



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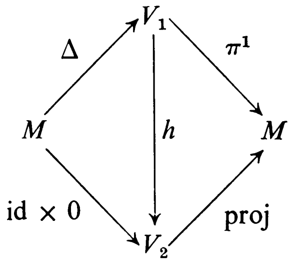
An n-manifold M is *topologically parallelizable* if there exists an open set V_1 in $M\times M$ containing $\Delta(M)$, an open set V_2 in $M\times R^n$ containing $M\times 0$, and a homeomorphism $h\colon V_1\to V_2$ so that the following diagram commutes:

(1)



Pick $e\in M$. M is *suitable* if there is a continuous map $\Phi\colon M\to G(M)$ such that $\Phi(x)(x)=e$ and $\Phi(e)=$identity, where $G(M)$ is the group of all homeomorphisms of M onto itself with the compact-open topology. By Theorem 2 of [**1**], M is suitable iff there exists a $\theta\in G(M\times M)$ such that

$$\theta(M\times(M-e))=\{(x,y)\in M\times M : x\neq y\} \quad \text{and} \quad \pi^1\theta=\pi^1.$$

Note that a suitable manifold supports an H-space structure [**1**].

THEOREM. *A suitable n-manifold M is topologically parallelizable.*

PROOF. Let U_b and V_b be as in the definition of the tangent microbundle of M. Let W be an open set in M such that $e\in W\subset \text{cl } W\subset U_e$. Choose the V_b's so that $\pi^2\theta^{-1}(x,y)\in W$ for $(x,y)\in V_b$. Let $k\colon U_e\to R^n$ be a co-ordinate map such that $k(e)=0$. Define $\lambda\colon M\to[0,1]$ so that λ is 1 on a neighborhood of cl W and 0 on a neighborhood of $M-U_e$.

Let $V=\bigcup_{b\in M} V_b$ and define $h\colon V\to M\times R^n$ by

$$h(x,y) = (x, \lambda(\pi^2\theta^{-1}(x,y))k(\pi^2\theta^{-1}(x,y))).$$

h is a local homeomorphism, i.e. for $b\in M$, $h\colon V_b\to$image $h|_{V_b}$ is a homeomorphism, for define h': image $h|_{V_b}\to V_b$ by

$$h'(x,r) = (x, \pi^2\theta(x, k^{-1}(r))).$$

Then on V_b, $\pi^2\theta^{-1}(x,y)\in W$ so $h'h=$id, and on image $h|_{V_b}$, $k^{-1}(r)\in W$ so $hh'=$id.

However, $h\colon\Delta(M)\to M\times 0$ homeomorphically, so by Lemma 4.1 of [**4**], there is a neighborhood V_1 in $M\times M$ of $\Delta(M)$ and a neighborhood V_2 in $M\times R^n$ of $M\times 0$ such that $h\colon V_1\to V_2$ is a homeomorphism. As h commutes in (1) this proves our result.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA, LOS ANGELES, CALIFORNIA 90024