THE TAX EFFECT, AND THE RECENT BEHAVIOUR OF THE AFTER-TAX REAL RATE: IS IT TOO HIGH?

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I.

INTRODUCTION

Recent years have witnessed very high and volatile interest rates. This has stirred a debate among analysts as to whether observed interest rates are high by historical standards. Some analysts, focusing on the before-tax real rate, argue that if the observed nominal interest rate is corrected for the effect of expected (or actual) inflation, the ex ante (or ex post) real rate has been very high in recent years. Other analysts, however, note that it is also important to consider the effect of taxes on the behaviour of the nominal interest rate. Since real spending decisions in the economy are based on the after-tax real rate, it is more appropriate to focus on the behaviour of this latter real rate. Proponents of this view argue that the after-tax real interest rate observed since 1980 does not appear to be too high.[1]

At the center of the debate is an empirical issue of whether tax effects are fully recognized by investors. If the nominal interest rate does fully adjust to reflect the presence of an effective marginal tax rate on interest income, then it is more appropriate to look at the behaviour of the after-tax real interest rate. The theoretical proposition that nominal interest rates adjust to reflect the presence of taxes on interest income is intuitively appealing. As put by Michael R. Darby (1975), Martin Feldstein (1976) and Vito Tanzi (1976), the proposition states that, ceteris paribus, nominal interest rates will rise during an inflation by an amount which exceeds expected inflation enough to compensate lenders both for their expected loss of capital and for the taxation of interest income. Though this proposition is plausible, early empirical work failed to provide any firm empirical support for it. More recently, however, Joe Peek (1982) and Robert Ayanin (1983) were. able to produce empirical evidence supporting the presence of the tax effect on the nominal interest rate.

Despite recent empirical work implying that investors tend to adjust nominal interest rates for the presence of taxes on interest income, the question of whether nominal interest rates rise sufficiently to fully insulate expected real rates from the presence of an effective marginal tax rate on interest income has not been adequately investigated. This is an important issue because if nominal interest rates do not fully adjust, then the existence of the income tax on interest income will be another important source of variations in the after-tax real rates of interest.

This paper has two objectives. The first is to provide some further evidence supporting the existence of the tax effect on the nominal interest rate. In particular, the issue regarding whether nominal interest rates need to be fully adjusted for the presence of the effective marginal tax rate on the interest income is investigated. More specifically, the paper develops and applies a simple procedure to test this issue. The second objective is to focus on the behaviour of the after-tax expected short-term real rate. If one were to fully adjust the short-term nominal interest rate for effects of taxes and expected inflation, would the level and range of the real rate observed in recent years be high relative to the level and range observed during the period 1952-1979? This question is answered by deriving an after-tax expected real rate series over the period June 1952 to June 1983. In addition, to explain movements in the level of this short-term real rate over time, an empirical model of interest rate determination is presented and estimated.

The remainder of this paper is organized as follows: Section II reviews the early empirical work that investigated the presence of the tax effect and led to the inference that people have not considered the taxation of interest in determining the nominal

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[1] See "Rates Through Contrarian Eyes" in New York Times, October 9, 1983, page F29.

interest rate. It is argued that the conclusion regarding investor's ignorance of taxes could have been due to faulty interpretation of the econometric evidence. Section II also contains a discussion of the test procedure used by economists to infer the presence of the tax effect. Section III contains a discussion of the specific interest rate model that underlies the empirical work reported in this paper. Section IV presents and discusses various empirical estimates that underlie various conclusions. Section V contains main conclusions. Finally, the Appendix reviews and illustrates the J-test of non-nested regression models -a statistical procedure used in the previous work to test the presence of the tax effect on the nominal interest rate.

II.

BACKGROUND

The Fisher Equation and the Tax Effect

Since the studies of Darby (1975), Feldstein. (1976) and Tanzi (1976), economists have modified the standard Fisher relationship to incorporate the effects of taxation of' interest income. The standard formulation of the Fisher equation is

$$i = r + \pi, \qquad (1)$$

where i is the nominal interest rate, r is the real rate, and π the expected rate of inflation. This equation postulates that given r, an increase in the expected rate of inflation leads to an equivalent increase in the nominal interest rate. However, since interest is taxed, in order to leave the after-tax real interest rate unchanged we must have

$$i\,(1-T) = r^* + \pi, \qquad (2a)$$

or

$$i = r + (1/(1-T))\,\pi, \qquad (2b)$$

where r* is the after-tax real rate and T is the marginal tax rate on the interest income. Equation (2b) tells us that the size of the theoretical coefficient on the expected inflation rate is (1/(1-T)), and it exceeds unity for a nonzero tax rate. That is, in the presence of taxes on interest income, the nominal interest rate should rise during an inflation by an amount which, exceeds expected inflation sufficiently to compensate lenders both for their loss of capital and for the taxation of interest income.

It was this implication of equation (2b) that formed the basis of the early empirical work looking for the existence of the tax effect in the form of a greater-than-unitary coefficient in front of the expected inflation variable.[2] Moreover, in order to test whether nominal interest rates rise enough to fully insulate real rates from the effects of expected inflation and taxes, economists expected to find the estimated coefficient to equal the value implied by (1/(1-T)); if the marginal tax rate on the interest income equaled 32 percent, it implied a coefficient of approximately 1.47, i.e., (1/(1-.32)).

However, more often than not the estimated coefficient on the expected inflation variable was found to be close to or less than one. Initially, these empirical findings, were interpreted as providing little or no firm empirical support for the proposition that people have considered the taxation of interest in determining the nominal interest rate. For example, Vito Tanzi interpreted his estimated coefficient on expected inflation to be evidence that individuals ". . . have failed to see through the fiscal veil and thus have suffered from fiscal illusion" (p. 20). But the recent contributions of Levi and Makin (1978), Melvin (1982), and Makin, and Tanzi (1983) imply that a unitary or less-than-unitary response of the nominal interest rate to the expected inflation rate is not inconsistent with the presence of the tax effect. The basic point is that the Fisher equation is a reduced form relation. If we derive the Fisher equation from an explicitly specified structural macro model, the coefficient' in front of the expected inflation variable is a function of several structural parameters, and it Gill be equal to (1/(1-T)) only under specific restrictions on those parameters. In the absence of such restrictions, the response of the nominal interest rate to expected inflation is expected to be less than (1/(1-T)). (Section III demonstrates this in the context of a specific macro model that underlies the empircial work reported here.)

In fact, a more general analysis of the channels through, which expected inflation may influence the nominal interest rate suggests that it is very difficult to infer the presence of the tax effect by 'looking at the size of the estimated coefficient on the expected inflation variable in the interset rate equation based on equation (2b). The coefficient in front of the expected inflation variable may reflect, among other things, the influence of all or some of the following: (i) the Fisher effect, whereby the nominal interest rate rises by the full amount of a rise in expected inflation; (ii) the tax effect, whereby the nominal

[2] See Cargill (1977), and Tanzi (1980).

interest rate must rise by more than the rise in expected inflation to maintain a constant expected after-tax real return; (iii) the portfolio effect, whereby a rise in expected inflation, by raising the opportunity cost of holding money, causes people to shift from money to interest-bearing financial assets thereby restricting the rise in the nominal rate; and (iv) the Feldstein-Summers effect, whereby a rise in anticipated inflation depresses the expected after-tax profits and causes investment to fall. This latter effect, like the portfolio effect, tends to depress the real rate. In sum, tax-effects move the coefficient in front of expected inflation above unity, while the portfolio effect and the Feldstein-Summers effect both push it below unity. The net impact of all these on the coefficient in front of expected inflation is uncertain.[3] Hence the size of the estimated coefficient in front of the expected inflation proxy variable can not be used to reveal the presence or the degree of "fiscal illusion."

The Fisher Equation and the Magnitude of the Tax-Adjustment

Even if one focuses on the simple Fisher equation as formulated in equation (2b) and assumes the existence of the full Fisher effect, a general analysis of the tax effect on nominal interest rates suggests that there is no reason the market will always adjust the nominal interest rate for effects of expected inflation and income taxes by the full amount given by (1/(1-T)). Milton J. Ezrati (1982) points out that the tax effect on interest rates depends upon the tax status of market participants and the tax burden imposed on alternative uses of funds. The tax-adjusted inflation premium for nominal interest rates will equal (1/(1-T)) as suggested in equation (2b) only in the special case where the tax rate on alternative uses of funds equals zero and the tax rate on interest income is greater than zero.

In order to explain these results, let us explicitly discuss the alternative investment available to market participants. This alternative investment option pays some rate of return that can be compared with the interest rate. Markets are in equilibrium when the after-tax expected real returns are equal on these investment alternatives.[4] When these returns are not equal, wealth-maximizing investors will reallocate funds among these investment alternatives until the expected real returns are equalized on the after-tax basis. These considerations imply that in equilibrium, expected real returns must satisfy the following relationship :

$$i - (1/1-T))\,\pi = ia - (1/(1-Ta))\,\pi, \quad (3a)$$

where i, π, and T are defined as before and ia = nominal dollar rate of return on the alternative use of funds; Ta = marginal tax rate on income from the alternative use of funds. We can also express equation (3a) in the following way :

$$(i - ia) = ((T-Ta)/(1-T)(1-Ta))\,\pi. \quad (3b)$$

The equations (3a) and (3b) imply that even though after-tax expected real returns are equalized on the alternative uses of funds, nominal returns differ due to the interaction of differential tax rates and the inflation premium. If we augment the basic Fisher equation (1) with this "tax-differential" term, we get the following equation :

$$i = r + \pi + [(T-Ta)/(1-T)(1-Ta)]\,\pi, \quad (4a)$$

or

$$i = r + (1+c)\,\pi, \quad (4b)$$

where 1 in (4b) can be viewed as the inflation premium coefficient without tax considerations and c = the tax-differential adjustment = ((T-Ta)/(1-T)(1-Ta)).

From equation (4a), it is clear that if the two tax rates are equal (T=Ta), or if holders of funds are entirely tax exempt (T = Ta = 0), then the tax differential term is zero, and there is no tax effect in response of the nominal interest rate to expected inflation. This tax-differential term becomes positive when returns on the alternative uses of funds are taxed at a lower rate than returns on financial securities. We will get the tax-adjusted basic Fisher equation (2b) in the case where Ta equals zero, and T is greater than zero; in this special case, the tax-adjusted inflation premium equals (1/(1-T)).

In general, not all market participants in securities markets are tax exempt. There are taxless options open to individuals in "consumption" and to many investors in the purchase of tax-free securities ; in these cases, the tax rate on alternative investments falls short of the rate applied to interest income. Investment in real plant and equipment also offers relative tax breaks accorded by accelerated depreciation schedules, investment tax credits, etc. In view of these considerations, the estimated coefficient in front of the expected inflation variable even in the

[3] See Makin and Tanzi (1983).

[4] To be realistic, these perceived returns should be adjusted for risk.

simple Fisher equation (2b) may turn out to be smaller than (l/(1--T)). But that result in itself will not be indicative of the presence of "fiscal illusion".

An Alternative Test of the Presence of the Tax Effect

In view of the discussion in the previous two sections, it is clear that the test based on the magnitude of the estimated coefficient in front of the expected inflation variable could not be relied upon to reveal the presence of the tax effect on the nominal interest rate. Aware of this difficulty, Peek (1982) and Ayanin (1983) used tests not contingent on the magnitude of the estimated coefficient in front of the expected inflation variable. Peek finds evidence for the tax effect by showing that the forecasting performance of the nominal interest rate equation estimated to allow the full adjustment of the nominal interest rate for the presence of taxes is better than that of the same equation estimated ignoring the presence of taxes. Moreover, he also uses the non-nested J-test to reveal the presence of the tax effect (see Appendix for details).

In order to explain these tests as well as to motivate the empirical work reported in this paper, the Fisher equation could be written as

$$i\,(1-kT) = r^* + b\pi,\ 0 \leqq k \leqq 1, \qquad (5a)$$

or

$$i = (1/(1-kT)\ [r^* + b\pi] + u_t \qquad (5b)$$

where i, T, r* and π are defined as before and b = the inflation premium coefficient not necessarily equal to one ; u = the random error term; k = the tax-adjustment parameter. The procedure used in the early empirical work to test the presence of the tax effect could then be characterized as follows: estimate equation (5b) setting the parameter k to zero and then examine whether or not the estimated coefficient on the expected inflation variable is greater than one. Moreover, under the assumption that the population parameter b equals one, examine whether or not the value of this estimated coefficient exactly equals the value given by (1/(1-T)), where T is the average marginal tax rate on interest income. As observed before, more often than not the estimated coefficient on the expected inflation variable was found to be less than one.

In his empirical investigation of the tax effect, Peek argues the crucial question is really whether the estimation of the interest rate equation (5b) should proceed by dividing all the explanatory variables by (1-T) or not. Equivalently, should, the estimation of equation (5b) be carried out by setting k to zero or k to one? He shows that the forecasting performance of the tax-adjusted Fisher equation (equation (5b) estimated setting k to one) is better than that of the standard Fisher equation (equation (5b) estimated setting k to zero).[5]

However, the general analysis of the tax effect presented in the previous section implies that the nominal interest rate may only partially adjust for the presence of the effective marginal tax rate on interest income. In order. to investigate this possibility, the empirical work in this paper treats the tax-adjustment parameter k as an unknown parameter and estimates it along with other parameters. Since the parameter k is hypothesized to take values ranging from zero (no tax-adjustment) to one (complete tax-adjustment), the empirical procedure employed is to search for that value of k that minimizes the standard error of the regression. An estimated value of k which is less than one but greater than zero, could be interpreted to imply an incomplete adjustment of the nominal rate to the presence of taxes.

III.

THE MODEL OF INTEREST RATE DETERMINATION

As observed before, the Fisher equation (5b) should be viewed as a reduced form relation. In order to estimate it, we need a model to help identify the important determinants of the expected real rate and the expected inflation rate. Therefore, this section presents a simple IS-LM-Aggregate Supply model[6] which can be seen as providing the basis for the nominal interest rate equation (5b) estimated in this paper.

The linearized version of this model could be expressed as

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[5] The procedure used by Ayanin (1983) is entirely different; he does not estimate the Fisher equation. Instead, through regression technique, he examines the yield spread between taxable and tax-exempt bonds. He finds that the nominal yield on taxable bonds has risen sufficiently to compensate the lenders for the presence of an effective marginal tax rate on the interest income; his results imply an effective average marginal tax rate in the neighborhood of 40 percent.

[6] This macro model is in essence similar to the ones given in Peek (1982), Wilcox (1983), and Peek and Wilcox (1983).

$$\text{IS:}\quad i(1-T) - \pi = \alpha_0 + \alpha_1 (X - Y^n) - \alpha_2 (Y - Y^n) - \alpha_3 SS + \alpha_4 Z_t, \qquad (6)$$
$$\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0$$

$$\text{LM:}\quad i(1-T) = \frac{b_0}{b_2} + \frac{b_1}{b_2}(Y - Y^n) + \frac{1}{b_2}(P - M + Y^n)^7,\ b_1, b_2, > 0, \qquad (7)$$

$$\text{AS:}\quad P = c_0 + P^e + c_1(Y - Y^n) + c_2 SS, \qquad (8)$$
$$c_1, c_2 > 0,$$

where all the variables except i and Z are in natural logs and where Y is actual real output, Y^n is the natural real output, X is the exogenous component of real demand, M is the nominal money stock, P is the price level, P^e is the expected price level, i is the nominal interest rate, π is the expected inflation rate, SS is a supply shock variable measuring things like oil price disturbances, Z is the percentage change in the real output lagged one period, and T is the average marginal tax rate on interest income.

Figure 1 presents graphs of IS, LM, and aggregate supply (AS) equations. Equation (6) is the equation of the IS curve showing an inverse relationship between the after-tax nominal rate i(1-T) and real output* (Y-Y") ; its position depends upon the exogenous component of the real demand X, the expected inflation rate π, the lagged growth in real income Z, and the supply shock variable SS. Equation (7) is the equation of the LM curve showing a positive relationship between the after-tax nominal rate i(1-T) and real output (Y-Y^n) ; its position depends upon the price level P and the nominal money stock M. Equation (8) is the equation of the aggregate supply curve implying a positive relationship between the price level and real ouput; its position depends upon the expected price level P^e and the supply shock variable SS. The model as formulated above enables one to consider the short-run behaviour of the nominal interest rate as the economy deviates from its natural real output level.

Equation (6) through (8) can be combined to yield the following nominal interest rate equation:

$$i = (1/(1-T))[A_0 + A_1\pi + A_2 X + A_3 SS + A_4 M^1 + A_5 Z] \qquad (9)$$

where M^1 is $(M - P^e - Y^n)$,[9] and where A_1, A_2, A_3, A_4 and A_5 are the reduced form parameters. It can be easily shown that the latter are related to the structural equation parameters as follows:

$$A_1 = (b_1 + c_1)/d, \qquad (10.1)$$
$$A_2 = (\alpha_1 b_1 + \alpha_1 c_1)/d, \qquad (10.2)$$
$$A_3 = (c_2\alpha_2 - \alpha_3 b_1 - \alpha_3 c_1)/d, \qquad (10.3)$$
$$A_4 = (-\alpha_2)/d, \qquad (10.4)$$
$$A_5 = (\alpha_4 b_1 + \alpha_4 c_1)/d, \qquad (10.5)$$

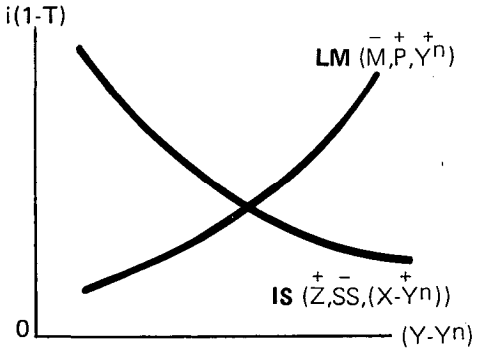
[7] The demand equation for real money balances underlying the LM curve is assumed to be $(M-P-Y^n)d = b_0 + b_1 (Y - Y^n) - b_2 i (1-T)$. Assuming that the money supply equals the money demand, we can solve the equilibrium expression for the after-tax nominal interest rate to get equation (7) of the text.

[8] Actual real output is measured relative to its natural level.

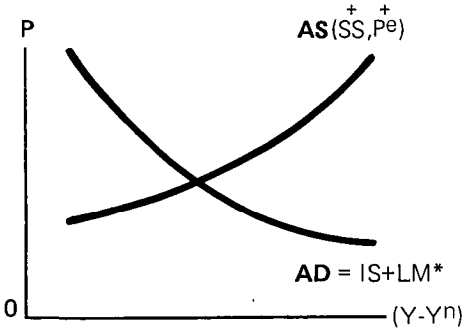
[9] In the empirical section of the paper, M1 is proxied by the variable LIQ; the latter is defined as the current growth rate of the nominal money stock relative to its most recent trend growth rate. See Wilcox (1983).

Figure 1

IS AND LM CURVES



AGGREGATE SUPPLY AND AGGREGATE DEMAND CURVES



*Aggregate demand curve (**AD**) is derived by combining the **IS** and **LM** curves.

where $d \equiv (b_1+c_1+b_2\alpha_2)$. In this model, the nominal interest rate responds positively to an increase in expected inflation ($A_1 > 0$), exogenous components of real demand ($A_2 > 0$), and real income ($A_5 > 0$). All of these variables lead to an upward shift in the IS curve and therefore to a rise in the nominal interest rate. The supply shock variable has a priori an uncertain effect. on the nominal interest rate ($A_3 \gtreqless 0$). An adverse supply shock, such as a rise in the relative price of energy, is assumed, at least in the short run, to reduce the demand for capital because capital and energy are complements in the production process. This reduction in the demand for capital implies reduction in investment which effect, by itself, tends to cause a decline in the nominal interest rate (see eq. (6)). However, an adverse supply shock at the same time tends to raise input costs and in so doing shifts upward the aggregate supply curve (see eq. (8)): This shift raises the price level, reduces the real money supply and thereby causes a rise in the nominal interest rate. The net impact of an adverse supply shock on the nominal interest rate, therefore, depends upon the relative importance of the investment effect and the input cost effect.[10] The coefficient in front of the monetary variable is expected to be negative (A4 < 0).

The interest rate equation (9) yields two interesting implications. First, the presence of taxes on the interest income ($T \neq 0$) in general affects not only the parameter in front of the expected inflation variable but also other parameters in the interest rate equation. Hence important changes in the tax policy can bring about changes in the response of the nominal interest rate to the determinants of the real rate and the expected inflation rate. Second, as mentioned above the coefficient in front of the expected inflation rate will equal (1/(1-T)) only under some special assumptions about the structure. This parameter, given in equation (10.1), can be expressed as

$$A_1 = (1/(1-T))\ [b_1 + c_1)/(b_1 + c_1 + b_2\alpha_2)].$$

In the context of this simple structural model, this coefficient will equal (1/1-T)) only if either b_2 or α_2 is zero (the LM curve is vertical or the IS curve is horizontal). In general, this coefficient will be less than (1/(1-T)). Therefore, the presence of the tax effect is not inconsistent with the findings of a smaller-than-unitary coefficient in front of the expected inflation variable.

[10] For details, see Wilcox (1983).

IV.

EMPIRICAL RESULTS

This section reports the evidence on the existence and the magnitude of the tax effect. The procedure employed here is to search for that value of the tax-adjustment parameter k that, produces the lowest standard error of the estimated interest rate equation. For different values of k between zero and one, the nominal interest rate equation (9) is estimated multiplying all the right hand side explanatory variables by (1/(1-kT)), where T is replaced by the actual values of the average marginal tax rate on interest income.

Table I reports the standard errors of the estimated interest rate equation for the full period 1952-1979 and for two subperiods. 1952-1970 and 1971-1979. It is clear that the nominal interest rate equation estimated under the assumption of the full tax-adjustment (assumed by setting k equal one in (1/(1-kT))) yields the lowest standard error of the regression (compare the standard errors of the

Table I

EVIDENCE ON THE MAGNITUDE OF THE TAX-ADJUSTMENT IN THE INTEREST RATE EQUATION

semiannual data

Tax-Adjustment Factor k in (1/(1−kT))	Standard Errors of the Regression 1952-1979 (1)	1952-1970 (2)	1971-1979 (3)
k = 0	.7339	.6272	.6164
k = .1	.7311	.6252	.6141
k = .2	.7280	.6232	.6117
k = .3	.7249	.6211	.6093
k = .4	.7216	.6189	.6067
k = .5	.7183	.6169	.6039
k = .6	.7147	.6145	.6012
k = .7	.7111	.6122	.5983
k = .8	.7074	.6098	.5952
k = .9	.7036	.6075	.5921
k = 1.0	.6996	.6052	.5889

Note: The entries in column (1) through (3) above list standard errors of the regression of the nominal interest rate equation estimated for different, sample periods under various hypothesized values about the magnitude of the tax-adjustment factor k (see note in Table II for a description of the interest rate equation estimated).

estimated interest rate equation in Table I).[11] This empirical result can be interpreted to imply that the nominal interest rate fully adjusts for the presence of an effective marginal tax rate on interest income. These findings support the assumption of complete tax-adjustment made by Peek (1982).

Table II presents estimates of the nominal interest rate equation (9). Row 1 presents estimates obtained ignoring the presence of income taxes on interest income (k=0), and row 2 presents estimates obtained assuming the full tax-adjustment (k=1). The estimates presented in rows 1 and 2 imply that all the explanatory variables have the expected influence on the behaviour of the nominal interest rate. That is, rises in expected inflation, exogenous components of aggregate demand, and lagged real income growth raise interest rates while positive supply shocks and accelerations in money growth lower them (see coefficients on PE12, X, SS, LIQ, and Z in Table II).

Given the above empirical results, Chart 1A graphs the behaviour of the after-tax short-term real rate for the period June 1952 to June 1983. The solid line displays the actual ex ante real rate and is computed as $\hat{i}\ (1-T)-PE12$. The dotted line displays the behaviour of the after-tax real rate predicted by the nominal interest rate equation. For the period June 1952 to December 1979, it is computed as $\hat{i}\ (1-T)-$ PE12, where $\hat{i}$ is the predicted value of the nominal interest rate equation estimated over the period 1952-1979. For the period June 1980 through June 1983, the predicted values are the simulated values from the interest rate equation estimated over the period 1952-1979.

This chart suggests some interesting inferences. The after-tax real rate that was positive in the '50s and '60s turned negative in the '70s. The level of the after-tax real rate observed during the period June 1981 to June 1983 is again positive but it is within the range observed in the '50s and the '60s. Therefore, when judged against that range, it cannot be considered unusually high. However, the real rate does appear high relative to the negative levels observed in the '70s.

[11] This result about the existence of complete tax-adjustment seems fairly robust with respect to the measure of inflation used and the estimation procedure employed. If some right hand side explanatory variables in the interest rate equation (9) are not strictly exogenous, then the ordinary least squares estimation procedure will provide inconsistent estimates of the parameters including k. Therefore, the nominal interest rate equation was also estimated by the instrumental variable estimation procedure treating both Z and LIQ as right hand side endogenous variables. Even here, the nominal interest rate equation estimated by setting k equal one in (1/(1-kT)) yielded the lowest standard error of the regression. Similarly, considering the possibility that the Livingston survey measure of inflationary expectations (PE12) may contain measurement errors and thereby produce biased estimates of the regression parameters, a two-step procedure as outlined in Lahiri (1976) was employed to estimate the nominal interest rate equation (9). Again, this estimation procedure yielded the same conclusion about the presence of the complete tax-adjustment. These results are available upon request from the author.

Table II

REDUCED FORM ESTIMATES FOR THE INTEREST RATE EQUATION

semiannual data, 1952-1979

	Coefficient on						
	PE12	X	SS	LIQ	Z	SER	$\bar{R}^2$
1. Non-tax-adjustment k = 0	.81 (20.4)	7.70 (3.2)	- 2.84 (-4.5)	- 17.1 (-2.9)	14.9 (2.5)	.7339	.891
2. Full tax-adjustment k = 1	.55 (20.7)	5.81 (3.6)	- 2.52 (- 5.9)	- 11.9 (- 2.9)	10.2 (2.5)	.6996	.981

Note: The nominal interest rate equation estimated and reported above is from the text (equation 9) and can be expressed, using proxy variables, as

$$i = (1/(1-kT))\,[A_0 + A_1\,PE12 + A_2\,X + A_3\,SS + A_4\,LIQ + A_5\,Z],\ 0 \leqq k \leqq 1,$$

where i is the average market yield on a one-year Treasury bill, X is the normalized value of real exports and real government expenditure, SS is the ratio of the deflator for imports and deflator for GNP adjusted for changes in the exchange rate, PE12 is the Livingston survey forecast for inflation over the 12-month horizon, LIQ is the annualized growth rate of the nominal money stock (M1B) over the last six months minus its annualized growth rate over the last three years, T is the series on the average marginal tax rate prepared by Joe Peek (1982), and Z is the lagged value of the rate of growth of the real GNP. The time series on the average marginal tax rate was kindly provided by Peek, and the one on PE12 by the Federal Reserve Bank of Philadelphia. The interest rate equation is estimated using semiannual observations corresponding to the Livingston survey data collected each June and December. SER is the standard error of the regression, and $\bar{R}^2$ is R^2 adjusted for degrees of freedom. The equations are estimated by the ordinary least squares estimation procedure.

Chart 1A

REAL AFTER-TAX INTEREST RATES
ACTUAL AND PREDICTED

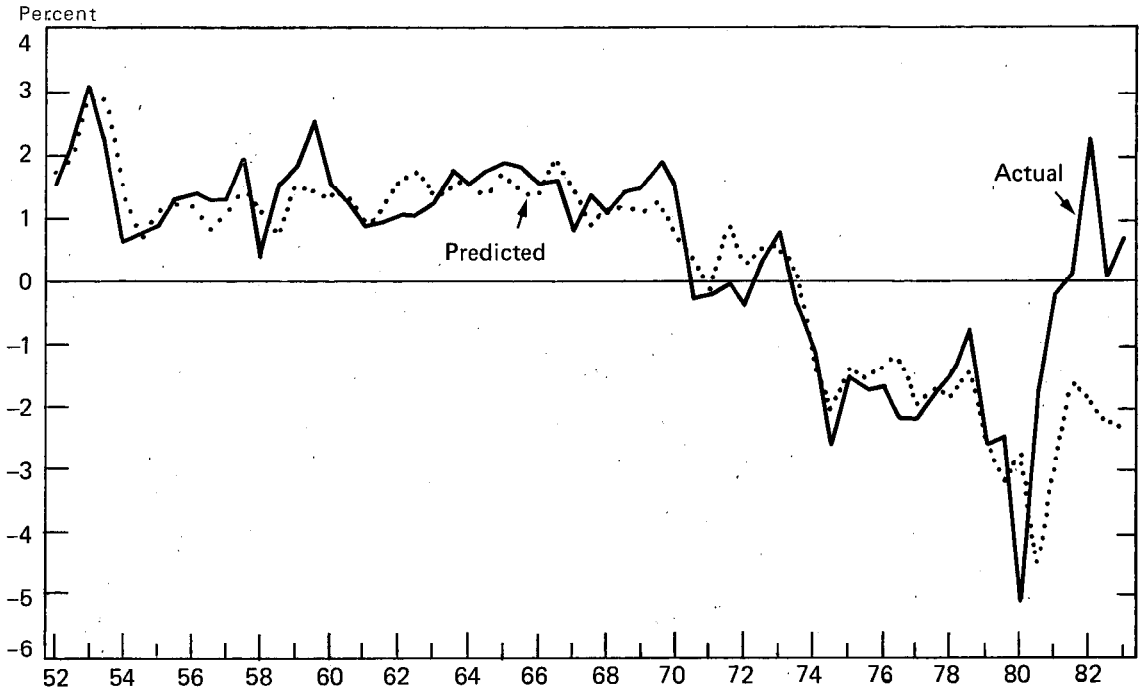
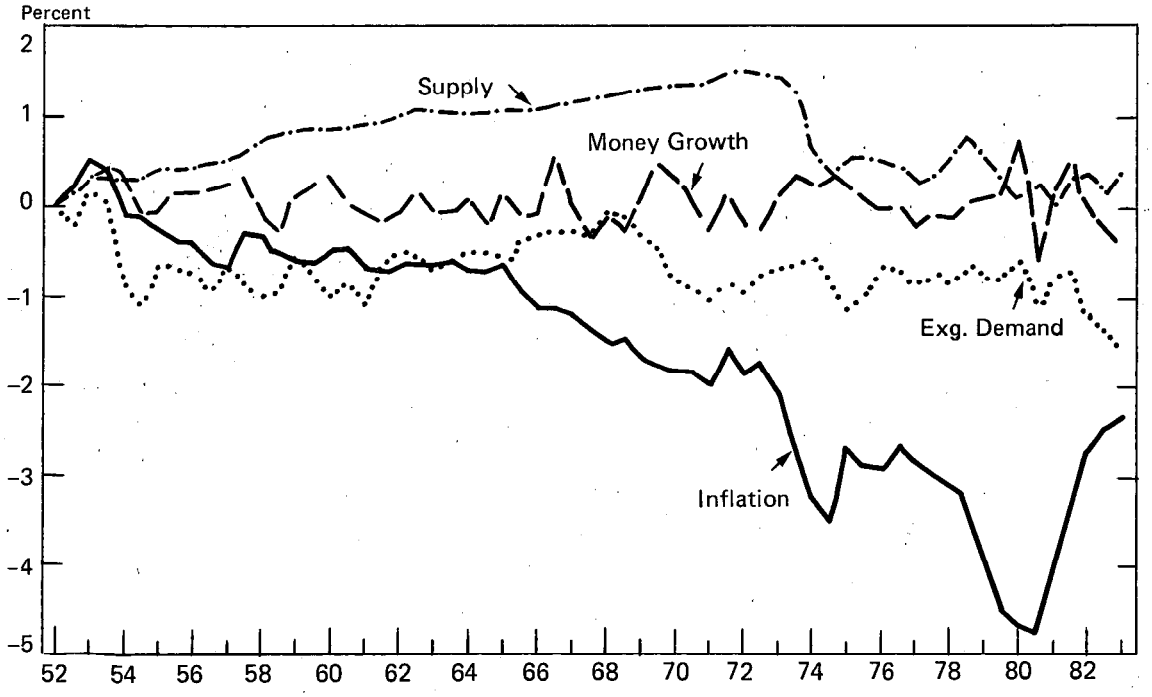


Chart 1B

EFFECT ON THE AFTER-TAX REAL RATE
OF CHANGING EXPLANATORY VARIABLES



These observations on the level of the after-tax short-term real rate raise one important question. Why did the after-tax real rate turn negative in the 1970s? In order to suggest an answer to this question, Chart 1B displays the effect on the after-tax real rate of changing explanatory variables like the expected inflation rate (PE12), predetermined components of aggregate demand (X and Z), supply conditions (SS), and money growth rate (LIQ). Each plotted series traces the impact of an explanatory variable on the after-tax real rate and is calculated as the product of the variable and its estimated coefficient from row 2 (in Table II), less the value of that product for the first observation of the sample. Thus, each measure's movement of the after-tax expected real rate is due to that explanatory variable from the June 1952 base. Consider the solid line depicting the effect on the real rate of changing expected inflation (PE12). The solid line shows that the effect of expected inflation has been to depress the real rate, and the magnitude of this depressing influence has been changing over time. Thus, the steadily rising expected inflation drove down the real rate by almost 2 percentage points from the early '50s until the end of the '60s. The magnitude of this depressing influence increased as the expected inflation rate accelerated in the late '70s; it reduced the real rate by almost 5 percentage points by the end of the '70s. In the early '80s, the reduction in the expected inflation rate did decrease the magnitude of this depressing influence. Other lines in Chart 1B can be similarly interpreted.

Overall, Chart 1B shows the rising expected inflation rate to be the most important factor that contributed to depress the real rate in the 1970s. The adverse supply shocks of the 1970s were another factor contributing to low real rates in this period (Wilcox (1983)). Both of these factors were responsible for producing excessively low real rates of interest in the 1970s.

Even though the interest rate model estimated here reasonably explains the behaviour of the after-tax real rate during the period 1952-1979, it does not explain very well the behaviour of the after-tax real rate in the post-1979 period. The recent drastic reduction in the level of inflationary expectations and the recent stability in oil prices were among the important factors contributing to the recent increase in the after-tax real rate; however, they alone cannot explain all of the recent rise in real rates (see Chart 1A). This suggests that an important change might have occurred in the response of nominal and real rates to various explanatory variables in the post-1979 period.

V.

SUMMARY REMARKS

One of the important issues arising as a result of the recent appearance of high and volatile real interest rates concerns the existence of the tax effect on the response of the nominal interest rate to expected inflation. Those who ignore the effect of taxes on the nominal interest rate tend to focus on the before-tax real rate of interest. The before-tax real rate may appear high by historical standards. However, there is growing evidence that the tax effect does exist, and this paper presented some further evidence on its full existence. The empirical results reported here imply that investors have fully recognized the effect of income taxes in reducing the after-tax expected real rates of interest and therefore have adjusted nominal interest rates to insulate real rates from the effect of taxes. In view of this, it is more appropriate to focus on the behaviour of the after-tax real rate of interest.

Several analysts, focusing both on short- and long-term real rates, have expressed the view that real rates are excessively high by historical standards. This view may not be entirely correct. For the evidence from an estimate of short-term real interest rates presented in this paper shows that the range of the after-tax real interest rate observed in recent years is not different from the range observed in the '50s and the '60s. The after-tax real interest rate was positive during the years 1952-1970 and turned negative in the '70s. Recently, it has been positive. Since the real rate has been positive in recent years, it does indeed appear excessive when compared with the negative real rate observed in the '70s. However, the level is well within the range experienced during the period of positive real yields.[12]

The simple interest rate equation reported and estimated in this paper suggests that accelerating expected inflation and, to some extent, adverse supply

[12] The result concerning short-term real rates does not imply that long-term real interest rates may not be high by historical standards. However, the evidence on the existence of the tax effect reported in this paper does imply that it might be appropriate to adjust long-term nominal interest rates for effects of expected inflation and taxes.

shocks produced abnormally low real rates of interest in the late '70s. Recent years, however, have witnessed a drastic reduction in the expected inflation rate and considerable stability in oil prices. These two factors together caused the real rate to rise from its severely depressed level of the late-'70s. However, the interest rate equation reported in this paper still cannot explain all of the recent rise in the real rate. But this observation notwithstanding, the level of the after-tax real rate observed in recent years falls well within the range experienced during the '50s and the '60s.

APPENDIX

THE J-TEST OF NON-NESTED REGRESSION MODELS: REVIEW AND AN APPLICATION

This Appendix reviews the J-test of non-nested regression models proposed by Davidson and MacKinnon (1981). This test is used by Joe Peek (1982) to prove the presence of the tax effect on nominal interest rates.

In applied econometric work, researchers very often face the problem of testing the specification of an econometric model in the presence of one or more other models which purport to explain the same phenomenon. The conventional techniques for hypothesis testing (such as the F-test) allow one to test the validity of a particular specification of an econometric model by testing restrictions on an alternative specification more general than the one being tested, conditional on the more general specification being valid. Since the specification whose validity is being tested (called the null hypothesis) can be obtained by imposing restrictions on the more general specification (called the alternative hypothesis), such hypotheses are said to be nested, i.e., the null hypothesis is nested within the alternative hypothesis.

However, in many cases, the alternative specifications suggested by economic theory are non-nested, meaning that any given specification whose validity we might be interested in testing is not nested within the alternative specification and could not be obtained by imposed restrictions on the latter. This is usually the case when each competing specification of the econometric model is characterized by the presence of some explanatory variables which are unique to that specification. Since the competing specifications are non-nested, the conventional F-test is not directly applicable. Recently, more powerful tests of non-nested hypotheses have been proposed,[13] and the J-test is one of them.

[13] Pesaran and Deaton (1978), Davidson and MacKinnon (1981), Pesaran (1982), and Davidson and MacKinnon (1983).

In order to illustrate how the J-test differs from the conventional F-test and how it is implemented, consider the simple model of interest rate determination discussed in the text. The nominal interest rate equation suggested by this model can be expressed as

$$i = (1/(1-T))\,[A_0 + A_1\pi + A_2X + A_3SS + A_4M + A_5Z] + u_1, \qquad (A1)$$

where all variables are defined as before and u_1 is the error term. Suppose one wants to test the hypothesis that one or more explanatory variables (say, Z and SS) suggested by the above model (A1) have no influence on the nominal interest rate. The conventional F-test sets up the following as the null (H_0) and the alternative (H_1) hypotheses

$$H_0:\ i = (1/(1-T))\,[A_0 + A_1\pi + A_2X + A_4M] + u_2, \qquad (A2)$$

$$H_1:\ i = (1/(1-T))\,[A_0 + A_1\pi + A_2X + A_3SS + A_4M + A_5Z] + u_1, \qquad (A3)$$

and then tests whether the restrictions implied by (A2) are correct, i.e., whether $A_3 = A_5 = 0$ in (A3). Note that the alternative specification (A3) is more general than the one being tested (A2) and that the latter is nested within the former.

Now suppose one wants to test the hypothesis that there is no tax effect on nominal interest rates. As explained in the text, the issue here is whether we should estimate equation (A1) by multiplying all the right hand side explanatory variables by (1/(1-T)) or not. So, the two competing specifications suggested by the tax issue can be expressed as

$$i = [A_0 + A_1\pi + A_2X + A_3SS + A_4M + A_5Z] + u_4, \qquad (A4)$$

$$i = [B_0 + B_1\pi^* + B_2X^* + B_3SS^* + B_4M^* + B_5Z^*] + u_1, \qquad (A5)$$

where the starred variables in (A5) are derived by multiplying the corresponding variables in (A4) by (1/(1---T)). Since the average marginal tax rate on interest income varies over time, we have entirely different values of the explanatory variables appearing in (A5). Therefore, one can view (AS) as an interest rate equation with a different specification of the right hand side explanatory variables. The specification (A4) implies that it is appropriate to estimate the interest rate equation ignoring the presence of taxes on interest income. The specification (A5) implies that it is appropriate to take into account the presence of an effective marginal tax rate on interest income and that tax effects are complete.

We can now see why the conventional F-test in this case is not directly applicable to the problem of testing the validity of a given specification (A4) (that there is no tax effect) against the alternative specification (A5) (that there is the full tax effect); the null hypothesis (A4) is not nested within the alternative hypothesis (AS) as the latter contains an entirely different set of explanatory variables. The alternative hypothesis here does not include the variables suggested by the null hypothesis, and one could not test the restrictions implied by the null hypothesis. However, there exists several non-nested test procedures which can be employed to test the validity of the alternative specifications of an econometric model.

The important point in the methodology of non-nested testing is that there is no presumption about the validity of any specification; each specification is on an equal footing with every other specification. This is so because the alternative specifications are non-nested by assumption and can not be ranked by the level of generality as can be done when the models are nested. To follow the non-nested test procedure, one takes the alternatives one at a time, assuming each one in turn to be true and inferring from the behaviour of the alternatives against the data whether or not the temporarily maintained or working alternative can or cannot explain the behaviour of the phenomenon one is interested in. One thus makes pair wise tests of each pair of alternatives and asks the question, is the performance of an alternative j against the data consistent with the truth of an alternative i?

In the present case, we have two alternative specifications (A4) and (A5). If one's working or currently maintained hypothesis is that (A4) is true, then one tests whether the performance of (A5) against the data is consistent with the truth of (A4). Similarly, if one's maintained hypothesis is that (A5) is true, then one tests whether the performance of (A4) against the data is consistent with the truth of (A5). In this procedure, it is conceivable both alternatives may be rejected, or that neither may be rejected. It is also conceivable that one may be rejected and the other may not be, in which case one would presumably want to choose the latter over the former. The case in which both specifications are rejected is interesting; it implies that there is some element of truth in both specifications, and that the researcher should expand the model to incorporate the important factors suggested by these competing non-nested specifications.

The J-test proposed by Davidson and MacKinnon (1981) can be implemented in two steps. The first step generates estimates of the regression parameters in (A4) and (A5) by using an estimation procedure that provides consistent estimates of the parameters. Since the error terms in (A4) and (A5) are assumed to be serially uncorrelated, homoscedastic, and uncorrelated with the right hand side explanatory variables, consistent estimates of the parameters of (A4) and (A5) are provided by the ordinary least squares estimation procedure. The estimated regression equations are then used to generate the within sample predictions of the dependent variable under the two alternative specifications. The second step consists of estimating two expanded regressions which can be expressed as

$$i = A_0 + A_1\pi + A_2X + A_3SS + A_4M + A_5Z + \gamma \hat{i} + u_6, \quad (A6)$$

$$i = B_0 + B_1\pi^* + B_2X^* + B_3SS^* + B_4M^* + B_5Z^* + \gamma \hat{\hat{i}} + u_7, \quad (A7)$$

where $\hat{i}$ and $\hat{i}$, respectively, are the predicted series for the dependent variable i from equations (A5) and (A4) estimated in step one. In the estimating equation (A6), the maintained hypothesis is (A4); one is testing the truth of it given the performance of the alternative (A5) against the data. If the specification (A4) is true, then the true value of *y* is zero. As shown by Davidson and MacKinnon (1981), one may validly test whether y=0 in (A6) by using a conventional t test or, equivalently, a likelihood ratio test. Thus, by testing the significance of the parameter y in (A6), one tests the truth of the maintained hypothesis (A4) given the performance of the alternative hypothesis (A5). The process is reversed in the estimating equation (A7); the maintained hypothesis here is (A5) and one tests the truth of it given the performance of the alternative (A4) against the data. Therefore, one tests the truth

of (A5) given the alternative (A4) by examining the significance of the parameter y in the estimating equation (A7).

Since the J-test uses t statistics from the expanded regression equations (A6) and (A7) to draw inferences about the truth of the alternative specifications, it is imperative that error terms in these regressions satisfy the important assumptions of the classical linear regression model, i.e., zero mean, homoscedastic variance, absence of serial correlation, and no correlation with the right hand side explanatory variables, etc. It is well known that t statistics are biased if error terms fail to possess some of these properties.[14]

Table III presents results of performing the J-test along the lines suggested above; it shows estimates of the relevant parameter y and the associated t-statistic from the estimating equations (A6) and (A7). Two sets of estimates are reported; the first set (labelled as y_1 and t-statistic$_1$) is based on the ordinary least squares estimates of equations (A6) and (A7) and the second set (labelled as y_2 and t-statistic$_2$) is based on the estimation of equations (A6) and (A7) assuming the presence of the first order serial correlation. Since the nature of serial correlation can differ across the alternative specifications, we let the serial correlation coefficient differ in equations (A6) and (A7).

Since the t-statistic is biased if the error term is serially correlated, we focus on the second set of estimates. These estimates are consistent with the following inferences : In the estimating equation (A6), the maintained hypothesis that there is no tax effect on the nominal interest rate is rejected (y_2 is significantly different from zero as evidenced by a significant t value) given the performance against the data of the alternative that the tax effect does exist. However, in the estimating equation (A7), the maintained hypothesis that the tax effect does exist is not rejected (y_2 is not significantly different from zero) given the performance against the data of the alternative that does not allow tax effects. These results together then imply that the specification of the interest rate model, which allows the existence of the full tax effect on the nominal interest rate, is the preferred specification when judged against the one which completely ignores the existence of taxes on interest income.

[14] See Davidson and MacKinnon (1983) and McAleer, Fisher, and Volker (1983) for an extension of the non-nested tests to cover the issues raised by the violation of some of these assumptions.

Table III

RESULTS OF THE J TEST

Sample Period 1952-1979

Maintained vs Alternative Hypotheses	Estimating Equation	No Correction for Serial Correlation		First Order Serial Correlation Correction		
		γ_1	t-statistic$_1$	γ_2	t-statistic$_2$	ρ
(A4) vs (A5)	(A6)	2.53	2.87*	2.78	2.52*	.3
(A5) vs (A4)	(A7)	−1.53	−1.74*	−1.78	−1.62	.3

* Significant at the .05 level; the two-tailed test.

Note: See the Appendix for on explicit description of various equations. ρ is the first order serial correlation coefficient; the equations (A6) and (A7) are estimated by the Hildreth-Lu estimation procedure.

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