Published by Institute of Physics Publishing for SISSA/ISAS

RECEIVED: March 16, 2004 ACCEPTED: April 28, 2004

# The $t\bar{t}$ cross-section at 1.8 and 1.96 TeV: a study of the systematics due to parton densities and scale dependence<sup>\*</sup>

## Matteo Cacciari

Dipartimento di Fisica, Università di Parma Italy, and LPTHE, Université Paris 6 France E-mail: cacciari@lpthe.jussieu.fr

# Stefano Frixione and Giovanni Ridolfi

INFN, Sezione di Genova
Italy
E-mail: Stefano.Frixione@cern.ch, Giovanni.Ridolfi@ge.infn.it

## Michelangelo L. Mangano

CERN, Theoretical Physics Division Switzerland E-mail: Michelangelo.Mangano@cern.ch

## **Paolo Nason**

INFN, Sezione di Milano Italy E-mail: Paolo.Nason@mib.infn.it

ABSTRACT: We update the theoretical predictions for the  $t\bar{t}$  production cross-section at the Tevatron, taking into account the most recent determinations of systematic uncertainties in the extraction of the proton parton densities.

KEYWORDS: Heavy Quarks Physics, QCD, Hadronic Colliders.



<sup>\*</sup>This work was supported in part by the EU Fourth Framework Programme "Training and Mobility of Researchers", Network "Quantum Chromodynamics and the Deep Structure of Elementary Particles", contract FMRX–CT98–0194 (DG 12 - MIHT).

# Contents

1.	Introduction	1
2.	Outline of the uncertainty estimate	2
	2.1 Scale uncertainty	2
	2.2 PDF uncertainty	2
3.	Results	3
4.	Conclusions	7

# 1. Introduction

We present in this note an update of the predictions for the top quark production crosssection at the Tevatron. These predictions are based on two complementary ingredients:

- 1. the evaluation of the parton-level cross-sections, carried out in perturbative QCD with the inclusion of the full next-to-leading-order (NLO) matrix elements [1], possibly improved with the resummation to all orders of perturbation theory of classes of large soft logarithms [2, 3];
- 2. the proton parton densities (PDFs), which are typically extracted comparing existing data with NLO calculations available for the relevant processes, and extrapolated to the relevant region of  $Q^2$  using the NLO evolution equations (more recently, accurate estimates of the exact NNLO results have also become available [4], based on partial evaluations of the three-loop splitting functions).

The numbers we present here are based on the theoretical framework introduced in [5] and [6], where the complete NLO calculation of the  $t\bar{t}$  cross-section was improved with the resummation of leading [5] and next-to-leading [6] soft logarithms appearing at all orders of perturbation theory. The introduction of resummation turns out to have only a mild impact on the overall rates (the effects at NLL are typically of the order of a few percent), but improves the stability of the predictions with respect to changes of the renormalization scales. While no progress has occurred since 1998 in the calculation itself, significant development has taken place in the determination of the PDFs. In addition to much improved data from HERA and from fixed-target DIS experiments at FNAL, and to the implementation of Tevatron jet and W production data in the fits, progress has occurred in the assessment of the true uncertainties associated with the global fits to these data. This issue, which recently received considerable attention (Giele, Keller and

Kosower [7], CTEQ [8, 9], MRST [10], Botje [11], Alekhin [12]), has led to sets of PDF parameterizations which should provide a meaningful estimate of the uncertainty deriving from PDFs to be associated to any calculations of hard processes in hadronic collisions.

The introduction of these PDF sets "with uncertainties" relaxes the much constrained predictions which used to be anchored to predefined functional parametrizations, and it is natural to anticipate that the range of predictions for a given hard cross-section will be increased.

## 2. Outline of the uncertainty estimate

We shortly outline here the details of our calculation, before presenting the numerical results. Unless explicitly denoted as  $\sigma_{NLO}$ , all of our results are obtained using the NLL-improved formalism of ref. [6].

### 2.1 Scale uncertainty

The evaluation of the purely theoretical uncertainty is based on the standard exploration of the cross-section dependence on the renormalization ( $\mu_R$ ) and factorization ( $\mu_F$ ) scales used in the perturbative calculation. In this work, we follow the standard convention of considering the range  $m_{\rm top}/2 < \mu < 2m_{\rm top}$ , setting  $\mu_R = \mu_F \equiv \mu$ . A justification for this choice can be found in [6], where it was shown that  $\mu \sim m_{\rm top}/2$  corresponds to a point of minimal sensitivity, providing a maximum of the cross-section in the range  $0.1 < \mu/m_{\rm top} < 10$ . In the range of mass consistent with the current data, and for the two CM energy values of run I and run II (1.8 and 1.96 TeV, respectively), the relative scale uncertainty at NLO is of the order of  $\pm 10\%$ , independent to good approximation of  $\sqrt{S}$ ,  $m_{\rm top}$  and PDF sets. In this region of parameters, the maximum value is obtained for  $\mu \sim m_{\rm top}/2$ , and the minimum for  $\mu = 2m_{\rm top}$ . The inclusion of NLL resummation corrections reduces the uncertainty to the level of approximately  $\pm 5\%$  [6]<sup>1</sup>. This is the effect of very small NLL corrections to the NLO result for small values of  $\mu$ , where the NLO rate is largest, and bigger corrections for large  $\mu$ .

For completeness, we also considered the possibility of varying independently the value of renormalization and factorization scale. These were chosen in the range  $0.5 < \mu_R/\mu_F < 2$ , with  $0.5 < \mu_{R,F}/m_{top} < 2$ . We verified (see later) that within this range the results obtained using the choice  $\mu_R = \mu_F$  are not altered significantly, leading only to a small increase of the upper estimate.

### 2.2 PDF uncertainty

In the framework of [8]–[10], PDFs with uncertainties come in sets of  $n_{PDF}$  pairs, where  $n_{PDF}$  is the number of parameters used in the fits. Each pair corresponds to the fit obtained

<sup>&</sup>lt;sup>1</sup>This number, as well as all numerical estimates presented in this document, correspond to the choice A = 2, where A is the parameter introduced in [6] to parameterize the uncertainty about subleading higher order terms. In that paper, it was found that A = 2 gives a better estimate of the higher order uncertainties. A = 0, for example, would reduce the scale dependence to only  $\pm 2.5\%$ , without changing significantly the central value of the resummed cross-section

$\sqrt{S}$	$\mu = m_{\rm top}/2$		$\mu = m_{\rm top}$		$\mu = 2m_{\rm top}$	
(GeV)	$\sigma_{ m NLO}$	$\sigma_{\rm res}$	$\sigma_{ m NLO}$	$\sigma_{\rm res}$	$\sigma_{ m NLO}$	$\sigma_{\rm res}$
1800	5.17	5.19	4.87	5.06	4.32	4.69
1960	6.69	6.71	6.31	6.56	5.61	6.11

Table 1: Cross-section predictions (in pb) for the 1998 MRSR2 PDF and  $m_{\text{top}} = 175 \text{ GeV}$ .

by varying of  $\pm 1\sigma$  the value of the fit parameter eigenvalues, after diagonalization of the correlation matrix. By construction, the systematic uncertainty obtained for the observable  $\mathcal{O}$  is given by:

$$\Delta \mathcal{O} = \frac{1}{2} \sqrt{\sum_{i=1, n_{\text{PDF}}} \left(\mathcal{O}_{i+} - \mathcal{O}_{i-}\right)^2}, \qquad (2.1)$$

where  $\mathcal{O}_{i\pm}$  is the value obtained using the PDF set corresponding to the variation of the *i*th eigenvalue within its error range.<sup>2</sup> The central value of the prediction is obtained using a reference PDF set, typically labelled with i = 0. We explore in this work the sets in the CTEQ6 [9] parameterizations ( $n_{\text{CTEQ}} = 20$ , corresponding to 40 sets, plus 1 reference set) and in the MRST 2001E [10] compilation ( $n_{\text{MRST}} = 15$ , corresponding to 30 sets, plus 1 reference set). All sets in the CTEQ compilation have  $\alpha_s(M_Z) = 0.118$ , while those in the MRST one have  $\alpha_s(M_Z) = 0.119$ . The CTEQ sets are labeled as follows: **6M** for the default set, and **101-140** for the 20  $\pm 1\sigma$  variations. The MRST sets are labeled as **0** for the reference set, and **1-30** for the 15  $\pm 1\sigma$  variations. In both cases, CTEQ and MRST, we use the default values of *tolerances* chosen by the two groups to best represent the uncertainty. In particular, CTEQ selects  $\Delta \chi^2 = 100$ , while MRST selects  $\Delta \chi^2 = 50$ .

In addition, we shall also consider three sets obtained by the MRST group in 2001 [14], where the values of  $\alpha_s$  was frozen to  $\pm 1\sigma$  from the central world average. We shall label these sets as **A01L** for the low- $\alpha_s$  ( $\alpha_s(M_Z) = 0.117$ ) fit [14], **A01H** for the high- $\alpha_s$  ( $\alpha_s(M_Z) = 0.121$ ) fit [14], **J01** for a fit based on Tevatron jet data ( $\alpha_s(M_Z) = 0.121$ )[14].

# 3. Results

Table 1 summarizes the results obtained with the PDF sets used in 1998, when the work in ref. [6] appeared. The numbers agree with what appears in table 1 of that document.

Table 2 gives the central value and error for the CTEQ sets, for three values of the top mass (170, 175 and 180 GeV) and the two CM energies of interest ( $\sqrt{S} = 1800$  and 1960 GeV). We list the results obtained at the three reference values of the mass scale  $r_{\mu} = \mu/m_{top} = 0.5, 1, 2$ . Table 3 provides the same information for the MRST sets.

Figure 1 shows the contour plots of the NLL cross-section when  $\mu_R$  and  $\mu_F$  are varied independently. The region defined by the oblique solid lines corresponds to  $0.5 < \mu_R/\mu_F < 2$ . It shows that within this domain the range of NLL rates is compatible with the range obtained using  $\mu_R = \mu_F$ .

<sup>&</sup>lt;sup>2</sup>An improved definition of  $\Delta O$  was proposed in [13]. In our case, the results obtained following the two procedures are consistent.

$\sqrt{S}$	m	m	$\sigma_{\rm ref}({\bf 6M})$	$\Delta \sigma$
	$m_{\rm top}$	$r_{\mu}$		
1800	170	0.5	6.22	0.42
1800	170	1	6.10	0.40
1800	170	2	5.66	0.37
1800	175	0.5	5.29	0.35
1800	175	1	5.19	0.33
1800	175	2	4.81	0.31
1800	180	0.5	4.52	0.29
1800	180	1	4.43	0.28
1800	180	2	4.11	0.26
1960	170	0.5	7.97	0.57
1960	170	1	7.83	0.54
1960	170	2	7.29	0.49
1960	175	0.5	6.82	0.47
1960	175	1	6.70	0.45
1960	175	2	6.23	0.42
1960	180	0.5	5.86	0.40
1960	180	1	5.75	0.38
1960	180	2	5.35	0.35

**Table 2:** Range of cross-section predictions (in pb) for the CTEQ6 family of PDFs at a fixed scale  $r_{\mu} = \mu/m_{\text{top}}$ .  $\sigma_{\text{ref}}$  refers to the central value, using the **6M** set, and  $\Delta\sigma$  is the error, as defined in eq. (2.1).

In principle one should combine in quadrature the uncertainty due to PDFs and that due to the scale choice. We prefer to add them linearly, since the scale uncertainty is not really a systematic error in the strict sense. We therefore quote our range for the top cross-section as

$$\sigma(r_{\mu}=2) - \Delta\sigma_{\rm PDF}(r_{\mu}=2) < \sigma < \sigma \left(r_{\mu}=\frac{1}{2}\right) + \Delta\sigma_{\rm PDF}\left(r_{\mu}=\frac{1}{2}\right).$$
(3.1)

The corresponding values are given in table 4. The similar results for the MRST compilation are provided in table 5.

Three comments are in order:

1. the uncertainty ranges obtained using the CTEQ sets, for a fixed choice of scale, are almost twice as large as those for the MRST sets. We understand this is the result of the different tolerance criteria used by the two groups (see [9, appendix B4] and [10, section 6] for some discussion). The MRST range increases however if we include in the analys the 2001 sets with varying  $\alpha_s$ . This is shown in table 6. In this case the lowest predictions are obtained from the 2001 **A01L** fit, with the low value of  $\alpha_s$ , while the highest prediction comes from the 2001 jet-based **J01** fit. After the  $\alpha_s$  variation is included, the MRST range becomes compatible with that of CTEQ's.

$\sqrt{S}$	$m_{\mathrm{top}}$	$r_{\mu}$	$\sigma_{ m ref}(0)$	$\Delta \sigma$
1800	170	0.5	6.25	0.19
1800	170	1	6.14	0.18
1800	170	2	5.69	0.17
1800	175	0.5	5.32	0.16
1800	175	1	5.22	0.15
1800	175	2	4.84	0.14
1800	180	0.5	4.54	0.13
1800	180	1	4.45	0.12
1800	180	2	4.12	0.11
1960	170	0.5	8.05	0.27
1960	170	1	7.91	0.26
1960	170	2	7.35	0.24
1960	175	0.5	6.88	0.22
1960	175	1	6.76	0.21
1960	175	2	6.28	0.19
1960	180	0.5	5.89	0.19
1960	180	1	5.79	0.18
1960	180	2	5.38	0.16

**Table 3:** Range of cross-section predictions (in pb) for the MRST family of PDFs at a fixed scale  $r_{\mu} = \mu/m_{\text{top}}$ .  $\sigma_{\text{ref}}$  refers to the central value, using the **0** set, and  $\Delta\sigma$  is the error, as defined in eq. (2.1).

$\sqrt{S}$	$m_{\mathrm{top}}$	$\sigma_{\min}$	$\sigma_{\rm ref}({f 6M})$	$\sigma_{\rm max}$
1800	170	5.29	6.10	6.63
1800	175	4.51	5.19	5.64
1800	180	3.85	4.43	4.81
1960	170	6.79	7.83	8.54
1960	175	5.82	6.70	7.30
1960	180	5.00	5.75	6.25

**Table 4:** Full range of cross-section predictions (in pb) for the CTEQ6 family of PDFs, as defined in eq. (3.1).  $\sigma_{\rm ref}$  refers to the choice of **6M** and  $\mu = m_{\rm top}$ .

- 2. the central values obtained today for the top cross-section are about 3% larger than those obtained in 1998. At  $\sqrt{S} = 1.8 \text{ TeV}$  and  $\mu = m_{\text{top}} = 175 \text{ GeV}$  we had 5.06 pb with the set MRSR2 ( $\alpha_s(M_Z) = 0.119$ ). We now have 5.19 pb with CTEQ6M, and 5.21 pb with MRST0.
- 3. the contribution of the PDF systematics to the uncertainty range is large. In the case of the CTEQ sets, it is of the order of 6-7%, larger than that due to the choice of scale. This is a result of the large sensitivity of the top cross-section to the large-x gluon content of the proton, which is still poorly known. For CTEQ the largest

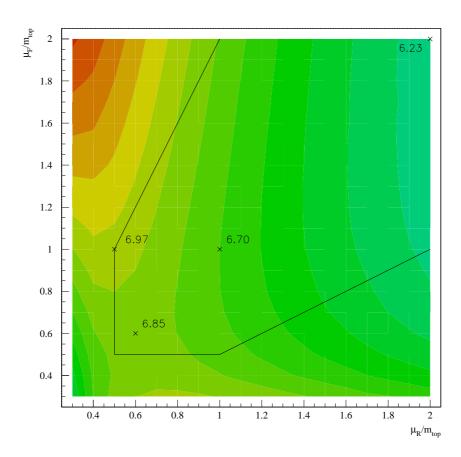


Figure 1: Contour plot of the NLL cross-section, in the  $\mu_F - \mu_R$  plane. The oblique solid line defines the region  $0.5 < \mu_R/\mu_F < 2$ .

$\sqrt{S}$	$m_{ m top}$	$\sigma_{\min}$	$\sigma_{ m ref}(0)$	$\sigma_{\rm max}$
1800	170	5.52	6.13	6.44
1800	175	4.69	5.21	5.47
1800	180	4.00	4.44	4.67
1960	170	7.11	7.90	8.31
1960	175	6.08	6.76	7.10
1960	180	5.21	5.79	6.08

**Table 5:** Full range of cross-section predictions (in pb) for the MRST family of PDFs, as defined in eq. (3.1).  $\sigma_{\rm ref}$  refers to the choice of set **0** and  $\mu = m_{\rm top}$ .

contribution to the error comes from the two sets **129** and **130**.<sup>3</sup> For these two sets, we find the contribution of the gg channel to be respectively 11% and 21% of the total rate. For comparison, the contributions of the  $q\bar{q}$  production channel for sets **129** and **130** are the same to within 1%. In other words, the PDF uncertainty on the top rate is mostly driven by the poorly known gluon density, whose luminosity in this kinematic range varies by up to a factor of 2 within the 1- $\sigma$  PDF range.

<sup>&</sup>lt;sup>3</sup>This is consistent with what found in a recent study of jet production at the Tevatron [15].

$\sqrt{S}$	$m_{\mathrm{top}}$	$\sigma_{\min} \ (r_{\mu} = 2, \ \mathbf{A01L})$	$\sigma_{\rm ref}(r_{\mu}=1,0)$	$\sigma_{\max} \ (r_{\mu} = 0.5,  \mathbf{J01})$
1800	170	5.48	6.13	6.72
1800	175	4.66	5.21	5.71
1800	180	3.98	4.44	4.86
1960	170	7.04	7.90	8.69
1960	175	6.03	6.76	7.41
1960	180	5.17	5.79	6.34

**Table 6:** Full range of cross-section predictions (in pb) for the MRST family of PDFs.  $\sigma_{\text{ref}}$  refers to the choice of **0** and  $\mu = m_{\text{top}}$ .  $r_{\mu} = \mu/m_{\text{top}}$  and **PDF** give the scale factor and PDF set at which the minimum and maximum rates are attained.

$\sqrt{S}$	$m_{\mathrm{top}}$	$\sigma_{\min}$	$\sigma_{ m ref}({f 6M})$	$\sigma_{\rm max}$
1800	170	5.29	6.10	6.72
1800	175	4.51	5.19	5.71
1800	180	3.85	4.43	4.86
1960	170	6.79	7.83	8.69
1960	175	5.82	6.70	7.41
1960	180	5.00	5.75	6.34

**Table 7:** Full range of cross-section predictions (in pb) for the combined study of CTEQ6, MRST and MRST with  $\alpha_s$  variation. The central values are taken from CTEQ6M. The minimum rates arise from CTEQ6, while the upper values arise from MRST set **J01**. These numbers should be quoted as "BCMN [6], as updated in this work."

While the overall production rate has a large relative uncertainty of approximately  $\pm 15\%$ , it is important to point out that the ratio of cross-sections at  $\sqrt{S} = 1.96$  TeV and  $\sqrt{S} = 1.8$  TeV is extremely stable. In the case of the CTEQ sets, for example, we found  $\sigma(1.96)/\sigma(1.8) = 1.295 \pm 0.015$  after scanning over the set of scale choices and for  $170 < m_{\rm top} < 180$  GeV. The error is about 1%. We therefore consider the prediction of the relative cross-section at the two energies to be a very stable one.

For reference, we collect the full set of cross-sections (at  $\sqrt{S} = 1.96 \text{ TeV}$  and  $m_{\text{top}} = 175 \text{ GeV}$ ) for all CTEQ sets and scale choices in table 8. Here, for the sake of documentation, we provide the NLO rates and the NLL-improved ones separately.

# 4. Conclusions

We reiterate here the main findings of this study. The inclusion of the full PDF systematics, made possible by the recent works of several groups, leads to a more realistic estimate of the top cross-section uncertainty. The latest MRST and CTEQ sets give rise to crosssections which are typically 3% larger than what obtained with sets available at the time of Run I. In addition to the increase in rate, the size of the uncertainty range has also increased, to a value of the order of  $\pm 15\%$ , dominated by the PDF and  $\alpha_s$  uncertainties. The leading source of PDF uncertainty comes from the (lack of) knowledge of the gluon luminosity at large values of x. The gg contribution can in fact change through the PDF

CTEQ6	$\mu = m_{\rm top}/2$		$\mu = m_{\rm top}$		$\mu = 2m_{\rm top}$	]
•	$\sigma_{\rm NLO}$	$\sigma_{\rm res}$	$\sigma_{\rm NLO}$	$\sigma_{ m res}$	$\sigma_{ m NLO}$	$\sigma_{\rm res}$
6M	6.81	6.82	6.47	6.70	5.76	6.23
101	6.94	6.95	6.60	6.83	5.88	6.35
102	6.68	6.69	6.35	6.57	5.65	6.11
103	6.79	6.81	6.46	6.69	5.75	6.22
104	6.82	6.83	6.49	6.71	5.78	6.25
105	6.80	6.82	6.47	6.70	5.76	6.23
106	6.81	6.83	6.48	6.70	5.77	6.24
107	6.67	6.69	6.34	6.57	5.64	6.11
108	6.95	6.96	6.61	6.84	5.89	6.36
109	6.89	6.91	6.53	6.77	5.81	6.30
110	6.74	6.75	6.42	6.64	5.73	6.18
111	6.80	6.81	6.47	6.69	5.76	6.22
112	6.81	6.83	6.47	6.70	5.76	6.24
113	6.80	6.82	6.47	6.70	5.77	6.23
114	6.81	6.82	6.47	6.70	5.76	6.23
115	6.80	6.82	6.46	6.69	5.75	6.23
116	6.87	6.88	6.54	6.76	5.82	6.29
117	6.75	6.76	6.41	6.64	5.71	6.18
118	6.92	6.93	6.59	6.81	5.87	6.34
119	6.83	6.84	6.51	6.72	5.80	6.26
120	6.80	6.82	6.46	6.69	5.74	6.23
121	6.75	6.77	6.42	6.64	5.72	6.18
122	6.85	6.87	6.51	6.74	5.79	6.27
123	6.71	6.73	6.38	6.60	5.67	6.14
124	6.68	6.69	6.35	6.57	5.65	6.11
125	6.73	6.74	6.40	6.62	5.69	6.16
126	6.82	6.83	6.48	6.71	5.76	6.24
127	6.85	6.86	6.51	6.74	5.80	6.27
128	6.87	6.88	6.53	6.76	5.82	6.29
129	6.56	6.58	6.28	6.47	5.61	6.03
130	7.36	7.37	6.94	7.21	6.14	6.70
131	6.70	6.71	6.36	6.59	5.66	6.13
132	6.67	6.68	6.34	6.56	5.64	6.11
133	6.63	6.64	6.31	6.52	5.62	6.07
134	6.79	6.80	6.44	6.67	5.73	6.21
135	6.86	6.87	6.52	6.75	5.81	6.28
136	6.86	6.87	6.52	6.75	5.81	6.28
137	6.94	6.95	6.58	6.82	5.84	6.34
138	6.75	6.77	6.43	6.65	5.73	6.19
139	6.83	6.85	6.49	6.72	5.78	6.26
140	6.79	6.80	6.46	6.68	5.75	6.21

**Table 8:** Full set of predictions for the CTEQ family of PDFs, and for  $m_{\text{top}} = 175 \text{ GeV}$ , at  $\sqrt{S} = 1.96 \text{ TeV}$ .  $\sigma_{\text{NLO}}$  is the NLO rate, while  $\sigma_{\text{res}}$  is the NLL improved result, according to [6]. All rates are in pb.

sets by up to a factor of 2 (from 10% to 20% of the total rate at 1.96 TeV). We find that the MRST sets give rise to a smaller PDF uncertainty, a result we ascribe to the tighter tolerances required by MRST in defining the range of the eigenvalues. The MRST

uncertainty increases however to values consistent with CTEQ's once the sets obtained from a  $\pm 1\sigma$  change of  $\alpha_s(M_Z)$  are included. This underscores the importance of including the  $\alpha_s$  uncertainty into the PDF fits in a more systematic fashion. On the same footing, the impact of higher order corrections, as well as of the treatment of higher twist effects in the fitting of low- $Q^2$  data, may need some more study before a final tabulation of the PDF uncertainties is achieved [10].

We collect in table 7 our final results. This summary table includes the CTEQ6M set and  $\mu = m_{top}$  as central values, and the most extreme rates extracted from tables 4, 5 and 6 as lower (with  $\mu = 2m_{top}$ ) and upper values (with  $\mu = m_{top}/2$ ).

In spite of the large overall uncertainty, the ratio of cross-sections at 1.96 and 1.8 TeV is extremely stable, being equal to  $1.295 \pm 0.015$  over the mass range  $170 < m_{\rm top} < 180$  GeV.

### Acknowledgments

We thank S. Catani and J. Huston for useful comments and discussions on the topic of this work.

# References

- P. Nason, S. Dawson and R.K. Ellis, The total cross-section for the production of heavy quarks in hadronic collisions, Nucl. Phys. B 303 (1988) 607;
   W. Beenakker, H. Kuijf, W.L. van Neerven and J. Smith, QCD corrections to heavy quark production in pp̄ collisions, Phys. Rev. D 40 (1989) 54.
- G. Sterman, Summation of large corrections to short distance hadronic cross-sections, Nucl. Phys. B 281 (1987) 310.
- [3] S. Catani and L. Trentadue, Resummation of the QCD perturbative series for hard processes, Nucl. Phys. B 327 (1989) 323; Comment on QCD exponentiation at large x, Nucl. Phys. B 353 (1991) 183.
- W.L. van Neerven and A. Vogt, Non-singlet structure functions beyond the next-to-next-to leading order, Nucl. Phys. B 603 (2001) 42 [hep-ph/0103123]; NNLO evolution of deep-inelastic structure functions: the singlet case, Nucl. Phys. B 588 (2000) 345 [hep-ph/0006154].
- [5] S. Catani, M.L. Mangano, P. Nason and L. Trentadue, The resummation of soft gluon in hadronic collisions, Nucl. Phys. B 478 (1996) 273 [hep-ph/9604351]; The top cross section in hadronic collisions, Phys. Lett. B 378 (1996) 329 [hep-ph/9602208].
- [6] R. Bonciani, S. Catani, M.L. Mangano and P. Nason, NLL resummation of the heavy-quark hadroproduction cross- section, Nucl. Phys. B 529 (1998) 424 [hep-ph/9801375].
- W.T. Giele, S.A. Keller and D.A. Kosower, Parton distribution function uncertainties, hep-ph/0104052;
  W.T. Giele and S. Keller, Implications of hadron collider observables on parton distribution function uncertainties, Phys. Rev. D 58 (1998) 094023 [hep-ph/9803393].
- [8] D. Stump et al., Uncertainties of predictions from parton distribution functions, I. The lagrange multiplier method, Phys. Rev. D 65 (2002) 014012 [hep-ph/0101051];

J. Pumplin et al., Uncertainties of predictions from parton distribution functions, II. The hessian method, Phys. Rev. D 65 (2002) 014013 [hep-ph/0101032].

- [9] J. Pumplin et al., New generation of parton distributions with uncertainties from global QCD analysis, J. High Energy Phys. 07 (2002) 012 [hep-ph/0201195].
- [10] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Uncertainties of predictions from parton distributions, I. Experimental errors, Eur. Phys. J. C 28 (2003) 455 [hep-ph/0211080].
- [11] M. Botje, A QCD analysis of HERA and fixed target structure function data, Eur. Phys. J. C 14 (2000) 285 [hep-ph/9912439].
- S.I. Alekhin, Statistical properties of the estimator using covariance matrix, hep-ex/0005042; Parton distributions from deep-inelastic scattering data, Phys. Rev. D 68 (2003) 014002
   [hep-ph/0211096].
- [13] Z. Sullivan, Fully differential W' production and decay at next-to- leading order in QCD, Phys. Rev. D 66 (2002) 075011 [hep-ph/0207290].
- [14] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, MRST2001: partons and α<sub>s</sub> from precise deep inelastic scattering and Tevatron jet data, Eur. Phys. J. C 23 (2002) 73 [hep-ph/0110215].
- [15] D. Stump et al., Inclusive jet production, parton distributions and the search for new physics, J. High Energy Phys. 10 (2003) 046 [hep-ph/0303013].