THE TEACHING OF STATISTICS[1]

By Harold Hotelling

The very great increase in the teaching of statistics since the First World War has been associated on one hand with the development of statistical theory. This important series of discoveries has made available more and more powerful and accurate statistical methods, and has also acquired an intellectual interest of its own as embodying the modern version of the most important part of inductive logic and as providing scope for mathematical and logical ingenuity of high order. The increased teaching of statistics has also been associated with the rapidly growing applications of statistics in innumerable fields, made possible by the development of the theory, by the availability of persons having some knowledge of the theory, and by an increasing realization of the possibilities of application. Doubtless most students of statistics enter upon the subject, not for its intrinsic interest, but with the idea of applying statistical methods as a tool to some particular end. This object may be scientific research, or to fulfill a requirement for a degree, but is often connected with some purely practical pursuit offering the ready prospect of a remunerative job. But it would be a mistake to ignore those whose interest is more purely intellectual, who desire an insight into the peculiar problems of probable inference and the structure of empirical knowledge, who wish to get a fundamental acquaintance with one of the most fundamental of subjects, to see and understand fully the mathematical derivations underlying so much practical and scientific activity, and perhaps to make their own contributions.

Of the magnitude of the demand for statisticians there can be no doubt. The realization of what statistical methods can do in a multitude of fields has gradually led the administrators of government agencies, directors of scientific organizations and research institutes, and business men, to employ rapidly increasing numbers of persons with some knowledge of statistical methods, and to accord an unusual degree of recognition and promotion in many such cases. The uses of statistical methods, and especially of sampling theory, are so varied that it is scarcely possible in a brief space to give any sort of survey of them. They enter, in one form or another, into the research work of the physicist, the chemist, the astronomer, the biologist, the psychologist, the anthropologist, the medical investigator, the economist, and the sociologist. Meteorology, which has lately acquired greatly increased importance, both civil and military, is with its masses of numerical observations very much a statistical matter. The engineer needs modern statistical methods both in the physical and in the

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economic aspects of his plans. The work of W. A. Shewhart has made clear the central importance of sampling theory in the economic control of quality of manufactured articles. Business men who use sampling surveys to test the markets for their products and the effectiveness of their advertising, who employ statisticians to make up index numbers and forecasts of business conditions, and whose manufacturing costs and quality are controlled with the help of recently devised statistical methods, are finding more and more uses for statisticians. Indeed, it seems as if the exploitation of the business and manufacturing possibilities of statistical methods has only begun, and that limitless further fields are coming into view. Insurance has of course always been essentially dependent on statistics.

But the most rapidly growing large class of positions for statisticians is at present in governmental activities. For some facts regarding the employment of statisticians by the federal government I am indebted to Dr. J. M. Thompson. It appears that it has about one hundred agencies using statistics, with almost eight hundred positions broadly classified as statistical or mathematical, in addition to more than six thousand generally classified as economists. The title "economist" covers many types of work, but much of it is largely statistical. The nature of the government's statistical work is varied and extensive. It includes such work as forecasting revenue from taxes, prices and production of agricultural commodities, general demand conditions, and weather. Some of the work consists in analyzing the effects of various taxes on other programs. In connection with proposed legislation, statisticians serving the lawmakers often attempt to outline the probable results of the legislation, as well as to assist in setting up definite formulae for carrying out the general policies aimed at in Acts of Congress. Administrators as well as lawmakers require statistical activities of a high order, exemplified in the Bureau of the Census, the Bureau of Agricultural Economics, and others. The scientific activities of the government, the work of the War Department, and many others that do not at first sight appear at all statistical, require the services of mathematical statisticians of high order. Even the judicial activities call for statistical theory of some of the most recently discovered kinds, as for instance in the investigation recently made of parole procedures. Cities and states, school and port authorities, employ numerous statisticians for other and widely diverse purposes.

The growing need, demand and opportunity have confronted the educational system of the country with a series of problems regarding the teaching of statistics. Should statistics be taught in the department of agriculture, anthropology, astronomy, biology, business, economics, education, engineering, medicine, physics, political science, psychology, or sociology, or in all these departments? Should its teaching be entrusted to the department of mathematics, or to a separate department of statistics, and in either of these cases should other departments be prohibited from offering duplicating courses in statistics, as they are often inclined to do? To what students, and at what stage of their advancement, should a course in statistics be administered?

Should there be mathematical or other prerequisites? How much of an investment in a statistical laboratory is warranted? Should courses be primarily theoretical and mathematical, or should they be made as practical as possible, equipping the student in the shortest possible time for a job as statistician, or for statistical work in the field with which a particular department is concerned? What about degrees in statistics? Eclipsing all these in importance, though it seems to have received too little of the attention of college and university administrative officers is the question, What sort of persons should be appointed to teach statistics?

To pressing practical problems answers are sure to be given either by considered policy or by processes of historical evolution. The latter are the more prominent in explaining the statistical teaching we have had. A synoptic picture of the origins, not many decades ago, of a good deal of it would perhaps be something like this. A university Department of X, where X stands for economics, psychology, or any one of numerous other fields, begins to note toward the end of the pre-statistical era that some of the outstanding work in its field involves statistics. The quantity and importance of such work are observed to increase, while at the same time its intelligibility seems to diminish. Evidently students turned out with degrees in the field of X who do not know something about statistics are going to be handicapped, and are not likely to reflect credit on Alma Mater. The department therefore resolves that its students must acquire at least an elementary knowledge of the fundamentals of statistics. To implement this principle, it perhaps inserts some acquaintance with statistics among the requirements for a degree. This situation naturally calls for the introduction of a course in statistics. Accordingly the head of the Department of X, in preparing the next Announcement of Courses, writes:

> "X 82. Elements of Statistics. An elementary but thorough course designed to acquaint students of X with the fundamental concepts of statistics and their applications in the field of X. The viewpoint will be practical throughout. Second semester, MWF at 10.
>
> "Instructor to be announced."

The problem now arises of finding someone to teach the new course. The few well-known statisticians in the country have positions elsewhere from which it would be impossible to dislodge them with the bait to be offered; for though the department wishes to have statistics taught as an auxiliary to the study of X, it feels that there must be no question of the tail wagging the dog, and that economy is appropriate in this connection. The members of the department of professorial rank do not respond favorably to the suggestion that they should themselves undertake to teach the new and unfamiliar course. But every university department has a bright graduate student whose placement is an immediate problem. Young Jones has already demonstrated a quantitative turn of mind in the course on Money and Banking, or in the Ph.D. thesis on which he has already made substantial progress, dealing with The Proportion of Public School Yard Areas Surfaced with Gravel. He may even recall having had a high-school course in trigonometry. His personality is all that might be desired. He is a white, Protestant, native-born American. And so the "Instructor to be announced" materializes as Jones.

This earnest young scholar now finds that, in addition to completing his thesis, he must look up the literature of statistics and prepare a course in the subject. His attention is directed by older members of the department to some of the research papers in the field of X involving statistics. He pursues "statistics" through the library card catalog and the encyclopedias. He reads about census and vital statistics, price statistics, statistical mechanics. Perhaps he encounters probable errors. Eventually he learns that Karl Pearson is the great man of statistics, and that Biometrika is the central source of information. Unfortunately most of the papers in Biometrika and of Pearson's writings, while not lacking in vigor, trail off into mathematical discourse of a kind with which young Jones feels ill at ease. What he wants is a textbook, couched in simple language and omitting all mathematics, to make the subject clear to a beginner. Perhaps he finds the impressive books of Yule and Bowley, but decides that they are too abstruse. Elderton's "Frequency Curves and Correlation" is far too mathematical. Jones decides that a simple book on statistics must be written, and that he will do it if he can ever succeed in mastering the subject. In the meantime, he contents himself perforce with the less mathematical writings of Karl Pearson, with applied examples in the field of X, and with such nonmathematical textbooks as may have been written by other young men who have earlier trod the same path as that on which Jones is now beginning. Somehow or other he gets the class through the course. After doing this two or three times, Jones is an experienced teacher of statistics, and his services are much in demand. His course expands, takes on a settled form, and after a while crystallizes into a textbook. At the same time he may be getting out some research, consisting of studies in the field of X in which statistical methods play a part. His promotion is rapid. He becomes a Professor of Statistics, and perhaps an officer in a national association. His textbook has a large sale, and is used as a source by other young men writing textbooks on statistics.

The textbooks written in this way form an interesting literary cycle. Measures of "central tendency" and of dispersion are introduced, and the use of one as against another of these measures is debated on every ground except the criterion that modern research has shown to be the important one, the sampling stability. Sampling considerations, indeed, get little attention. The urge to simplify by leaving out the more difficult parts of the subject, and especially the mathematical parts, is accompanied by pride in the great number of examples drawn from real life, that is, actual data that have been collected.

But the most fascinating feature of this literary cycle is the opportunity it offers for research by the standard methods of literary investigation, tracing the influence of one author upon another through parallelism of passages, and so forth. This study is facilitated by the accumulation of errors with repeated copying. One outstanding example is in certain formulae connected with the rank correlation coefficient, derived originally by Karl Pearson in 1907 and copied from textbook to textbook without adequate checking back. As one error after another was introduced in this process, the formulae presented to students (and apparently made the basis of class exercises involving numerical substitution) became less and less like Pearson's original equations. Incidentally, in trying to check this original work of Pearson's, recent investigation has raised the suspicion that it is erroneous; at any rate, he does not give a fully adequate argument. Thus it may be that the errors in copying, which are so useful in examining the history of statistics, never did any harm. The formulae in which the students were drilled may have been no worse than they would have been if all the copying had been done with more care.

While this process has been going on in the Department of X, the Y and Z Departments have likewise evolved the teaching of statistics. There is some interchange of ideas between the various statisticians on the campus, and there is a catholicity in the copying of textbooks. But by and large, statistics is regarded in the Economics Department as a branch of economics, in the Psychology Department as a part of psychology, and so forth. The astronomer is inclined to resent the suggestion that his students should be called upon to study their least squares with anyone but an astronomer. Medical and biological investigators suspect Economics and Psychology of charlatanry, and do not look with favor on the idea of turning their own students over to such departments for instruction in statistics. Most unthinkable of all would be putting the Department of Education in charge of an essential part of the training of scientific students. Thus the courses multiply.

The fact that it is essentially the same fundamental subject that is being taught under various names and with various kinds of notation in different departments is often concealed by including the teaching of statistical theory in a course whose title and prospectus are more suggestive of applications. A case in point is that of an economist of my acquaintance, not primarily engaged in teaching, who some years ago was invited to give a course in Price Forecasting in the Economics Department of a leading university. He carefully prepared a series of lectures on this subject, which had been the center of some extended research he had conducted. A large class enrolled for the course. But soon after beginning his series of lectures the economist noticed that the class was growing restive. Upon inquiring what was amiss, he learned that his discourse was unintelligible to many of them because he was using technical statistical terms and concepts with which they were not familiar. He thereupon undertook to use simpler language, and when this did not suffice to convey his meaning, to explain the statistical notions involved in his work on price forecasting. More and more his lectures came to deal with the elements of statistics, and less and less with price forecasting. At the end of the term he felt that he had given the students some elementary knowledge of statistical theory, for which they had not enrolled and for which he did not feel particularly well qualified, but had taught them virtually nothing about price forecasting. When the invitation was repeated the next year, the economist suggested imposing a course in statistics as a prerequisite for the course in Price Forecasting. This however was vetoed by the head of the Economics Department, who did not believe in prerequisites. The Price Forecasting course was not repeated.

This incident illustrates the evolution of a good deal of statistical teaching. At the beginning, the idea is to teach some application, but the teacher soon finds himself engaged at much more length than expected with the fundamentals of statistical theory and methods. In this way it has come about that a large number of persons are teaching theoretical statistics who initially had no intention of doing so, but were concerned with particular applications. The teaching of statistical theory has been undertaken belatedly and inexpertly because it was necessary to a discussion of some application originally in view. Thus it happens that a good deal of teaching of statistics, even of mathematical statistics, masquerades as something else.

The obvious inefficiency of overlapping and duplicating courses given independently in numerous departments by persons who are not really specialists in the subject leads to the suggestion that the whole matter be taken over by the Department of Mathematics. This is a promising solution, but it is doomed to failure if, as has sometimes happened, it means that the teaching of statistics is put under the jurisdiction of those who have no real interest in it. Moreover the teaching of statistics cannot be done appreciably better by mathematicians ignorant of the subject than by psychologists or agricultural experimenters ignorant of the subject. The latter indeed have a certain advantage in that the problems seem more real and definite to them; they can sense the difference between the important and the unimportant questions, even if they cannot express the questions in clear mathematical language, and can sometimes arrive intuitively at a correct result that leaves the mathematician puzzled. Also, they can understand more readily than can the mathematician the examples, drawn largely from biological material, which play so important a part in some of the leading expository work on statistics, such as R. A. Fisher's *Statistical Methods for Research Workers*. The pure mathematician has only one advantage over the non-mathematical worker in empirical fields: he is able to set about reading the serious literature of statistical theory. But he must still find this scattered literature, sort it out from a mass of rubbish, fallacies, and false starts, and trace it back historically until he can understand the notation and the presuppositions. He must also contend with the fact that a good deal that is important in statistics is still a matter of oral tradition, and some consists of laboratory techniques. In short, he needs a teacher before he himself sets out to teach the subject. When a Department of Mathematics calls in a young Ph.D., however brilliant, to teach statistics as a part or all of his program, the best thing it can do, if he has not already had a training in modern statistics, is to give him a furlough for a year or two to enable him to go where he can acquire such a training.

Qualifications of a good teacher of statistics include, first and foremost, a thorough knowledge of the subject. This statement seems trivial, but it has been ignored in such a way as to bring about the present unfortunate situation. Mathematicians and others, who deplore the tendency of Schools of Education to turn loose on the world teachers who have not specialized in the subjects they are to teach, would do well to consider their own tendency to entrust the teaching of statistics to persons who not only have not specialized in the subject, but have no sound knowledge of it whatever. A knowledge of theoretical statistics is not easy to obtain. There is no comprehensive treatise on the subject, starting from first principles, and proceeding by sound deductions and well-chosen definitions to the methods that need to be used in practice. (I have been trying for years to write such a treatise, but it has turned out to be a bigger task than at first appeared. This is partly because some things formerly thought to have been proved turn out, on critical examination, not to be sound, and much new research has been necessary.) The literature is scattered through journals pertaining primarily to many kinds of applications, and it is only in recent years that any large proportion of the current contributions to statistical theory and methods have been gathered into a few periodicals devoted to statistical theory. On the other hand, the seeker after truth regarding statistical theory must make his way through or around an enormous amount of trash and downright error. The great accumulation of published writings on statistical theory and methods by authors who have not sufficiently studied the subject is even more dangerous than the classroom teaching by the same people.

A good teacher of statistics needs of course a mathematical background, including at least an acquaintance with the theory of functions and n-dimensional euclidean geometry. A good deal of additional algebra and analysis are likely to be helpful, as well as some differential geometry. But no amount of such mathematics constitutes by itself any approach to sufficiency in the qualifications of a teacher of statistics. The most essential thing is that the man shall know the theory of statistics itself thoroughly from the ground up, including the mathematical derivations of proper methods and a clear knowledge of how to apply them in various empirical fields. In addition to the pure mathematics and the knowledge of statistical theory, a competent statistician or teacher of statistics needs a really intimate acquaintance with the problems of one or more empirical subjects in which statistical methods are applied. This is quite important. Sometimes excellent mathematicians have wasted time and misled students through failure to get that feeling for applications that is necessary for proper statistical work.

The theory of statistics has been making advances so rapid and so fundamental that some of the first things that need to be said in an elementary course, even for prospective practical statisticians, are affected by some of the most recent researches. So elementary a question as "What definition is it wise to give to the term 'standard deviation'?", which must be faced by every teacher of Statistics 1, requires for an intelligent answer a rather thorough understanding of modern sampling theory and techniques. The answer, it now seems, is *not* the definition given in most textbooks. In the selection of a statistic to represent a parameter, for example in fitting frequency curves or in linkage estimation in genetics, the fundamental consideration is connected with the sampling distribution, as R. A. Fisher showed in founding the modern theory of estimation. This is ignored in most of the current teaching of statistics, with the result that innumerable students are sent out to waste the money and time of their employers by demanding larger samples than are necessary for the purposes in view, wasting costly information by calculating inefficient statistics and using tests that are not the most powerful. On the other hand, students of statistics who are taught rule-of-thumb methods without their derivations are never quite conscious of the exact limitations and assumptions involved, and may make unwarranted inferences from samples that are too small or in some way violate the conditions underlying the derivations of the formulae.

A good teacher of statistics must be thoroughly familiar with these recent advances. He must examine very critically textbook statements unsupported by full proofs. Even though the students are not capable of following the complete mathematical argument—indeed, especially if the students are not to examine it—the instructor needs to give it a critical study. The custom of omitting proofs, which would not be tolerated in pure mathematics beyond a very limited extent, is common in the teaching of statistics, and is excused on the ground that the students do not know enough mathematics to understand the proofs. Perhaps in some cases a better reason is that the teachers, and the authors of the textbooks, do not understand the proofs. In some instances no proofs exist, and in some instances no genuine proofs can exist, because the methods taught are demonstrably wrong. The custom prevalent in the teaching of mathematics of going over each proof carefully in the class is, among other things, a safeguard against infiltration of false propositions. This safeguard is missing from most of the teaching of statistics, and there has been an infiltration of errors. Since it is accepted that a great many students need to learn something about statistical methods without learning enough mathematics to understand the proofs, it follows that the elementary teaching of statistics to these students must, if the perpetuation of gross errors is to be avoided, be in the hands of really competent mathematical statisticians. This is perhaps the greatest reform needed in the teaching of statistics today. Until the *elementary* teaching of statistics is conducted by those with a thorough and critical knowledge of current research in statistical theory, of a sort that seems virtually inseparable from participation in that research, there is likely to be a continuation of the laborious drilling of thousands of students in methods that ought never to be used. Here, of all places, is the great need for participation of research workers in elementary teaching.

Teachers and textbook writers might well abandon the idea of telling what statistical methods are used, and say instead what methods ought to be used. But before they can do this with confidence they must have a very close acquaintance with the research of the last three decades in statistical theory.

How can an appointing officer know whether a prospective teacher of statistics knows his subject? This question requires no answer peculiar to statistics in distinction from other subjects. Publication of research, constituting a contribution to the particular field, has always been accepted as the best proof. A substantial contribution to fundamental statistical theory, which is to be distinguished from the mere application of known statistical methods to empirical data, is the best indication of the kind of scholarship appropriate to a teacher of statistics.

Participation in research is not novel as a criterion of what constitutes a good teacher of a college or university subject, if the subject is Greek literature, physics, chemistry, biology, or indeed any of those departments that have been long enough established to attain with respect to the organization of their teaching a state approximating equilibrium. The more reputable institutions of higher learning have long maintained the principle, though with occasional violations in practice, that the Ph.D. degree or its equivalent, representing among other things the completion of a piece of scholarly research, is a minimum condition for a regular faculty appointment. It has usually been maintained also that the Ph.D. thesis should be a new contribution of a strictly scholarly character to the field of the scholar's competence, and not merely a routine application of known methods to an extraneous field. Thus a thesis offered for the Ph.D. degree in mathematics would be judged by its contribution to mathematics, rather than to physics or accounting. Moreover the regard in which universities have held members of their faculties has been intimately connected with their output of scholarly research. Other criteria of excellence have not been ignored, but research has been recognized in a fairly consistent manner. Some say that there has been an over-emphasis on research, and that more attention ought to be given to other qualities related to teaching. However this may be, the facts remain that scholarly research is something capable of a reasonably objective evaluation by scholars in the field, that it offers the main hope of fundamental progress, and that familiarity with current research is a necessary, though not sufficient, condition for the most important teaching in institutions of higher learning.

A peculiarity of the teaching of statistics, of which in practice the theory of statistics is an essential even if unacknowledged part, is that a good deal of it has been conducted by persons engaged in research, not of a kind contributing to statistical theory, but consisting of the application of statistical methods and theory to something else. A similar situation would exist if the teaching of mathematics were in the hands of an assortment of various kinds of engineers, or if zoology and botany were taught by practicing physicians. The teaching of mathematics and of elementary biology might perhaps gain in liveliness and concreteness by such arrangements, with the accompanying emphasis on the particular applications of the fundamental sciences. Moreover the engineer might in the course of such teaching refresh his own knowledge of elementary mathematics, while the physician might gain by renewing his acquaintance with elementary biology. Such arrangements might occasionally be made with profit. But if they were the general rule the advantages of specialization would be lost; the fundamental sciences would not be developed in so well-rounded a manner as they are by specialists in them, while the special skills and knowledge of the physician and engineer could not be utilized to the full in their respective professions. Statistical theory is a big enough thing in itself to absorb the full-time attention of a specialist teaching it, without his going out into applications too freely. Some attention to applications is indeed valuable, and perhaps even indispensable as a stage in the training of a teacher of statistics and as a continuing interest. But particular applications should not dominate the teaching of the fundamental science, any more than particular diseases should dominate the teaching of anatomy and bacteriology to pre-medical students. These subjects are not ordinarily taught by practicing physicians, but by anatomists and bacteriologists respectively.

In medical education the principle has been accepted, after a long struggle, that a medical school should have full-time professors engaged primarily in teaching and research, and that such professors should not treat patients except in cases of unusual interest from the standpoint of the science or art of medicine. An analogous principle would be that an institution offering extensive instruction in statistics should have full-time professors engaged in the teaching of and research in statistical theory and methods, without spending time over applied statistical problems excepting insofar as such problems might present novel features calling for the development of new statistical methods or theoretical extensions having interest going beyond the immediate case. Sometimes the complaint is heard in medical schools that the teaching tends to become too theoretical on account of detachment from clinical practice, and a similar difficulty might conceivably develop in connection with statistics; but in neither case does the trouble seem to be beyond the ability of the personnel involved to cure if they have the right background.

A specialist in statistics on a university faculty has a threefold function. In addition to the usual duties of teaching and research, there is a need for him to advise his colleagues, and other research workers, regarding the statistical methods appropriate to their various investigations. The advisory function is a highly important one for the activities of the university as a whole, and should be taken into consideration in adjusting the teaching load. Probably every university statistician is visited from time to time by earnest research workers, deeply engrossed in their respective specialities, speaking technical jargons unfamiliar to the statistician, and seeking his advice on matters concerning which he has a sinking feeling of lack of comprehension. After some hours of psychoanalyzing his visitor the statistician may be able to ascertain what it is he *really* wants to know, and thereafter either refer him to some standard formula, or more often, undertake a piece of new mathematical research designed to fit the particular problem, and very possibly having value also for a more extended class of problems. The statistician is then very likely to find himself embarked on a co-operative research venture in a field that is new to him.

To function well in this third, the consultative or co-operative function, he must have an unusually large store of general information. No one stands in greater need than he of that knowledge of "something about everything and everything about something" that was once said to be the goal of a liberal education. In planning the education of statisticians and teachers of statistics these considerations point to a somewhat wider diffusion of studies among various fields than is customary in many institutions, especially in graduate work. The co-operation, and their other work, would also be facilitated if research workers in general were more strongly urged to get a training in mathematical statistics at an early stage in their careers.

The problem of departmental organization is secondary to that of getting men having the requisite qualities of extensive mathematical preparation, a thorough knowledge of modern theoretical statistics, an understanding of some fields at least in which statistical methods can be applied, and the type of inquiring mind sometimes described as a "research outlook." A Department of Mathematics may well handle the fundamental teaching in statistics, provided it has men properly qualified for such teaching. If it does not have such men, its teaching of statistics and its inability to provide the needed statistical advice will inevitably tempt the other departments to set up again their own duplicating courses in what amounts essentially to statistical theory and methods, and to repeat the mistakes of the past.

A separate Department of Statistics, if competently staffed, could very well provide advice for the whole institution as well as conducting elementary instruction in statistical methods and theory, both for students having calculus and for those without it, and should certainly carry on advanced teaching and research in statistical theory and methods. But for efficient functioning of the institution as a whole it should be agreed that the Department of Statistics or the Department of Mathematics should do *all* the elementary instruction in statistics, and that courses in statistics in other departments should be confined to applications of the basic theory. Normally such courses in applied statistics in the other departments should require as a prerequisite one or more of the basic courses in the Department of Statistics, or of Mathematics. The basic course to be required as a prerequisite to others should be the one which itself requires calculus as a prerequisite wherever this is practicable. It is practicable for students of engineering, physics, astronomy, and mathematical economics, since these students must have calculus anyhow. Moreover the value of the sequence consisting of calculus, statistical theory and applied statistics, in this order, is so great that many other students are likely to avail themselves of it when it is once established and the true nature and value of statistics are more widely understood.

Exactly how far a Department of Statistics should go in particular applications would have to be decided anew from time to time by its members in the light of changing conditions and interests. It cannot teach everything that goes by the name of statistics. This problem may be exemplified by the case of population and vital statistics. This is a field with close connections with sociology, biology, medicine and insurance. It is cultivated in conjunction with each of these subjects in various places. Some of its most interesting and important phases make use of quite advanced mathematics, as in the work of A. J. Lotka, and in addition there is extensive use, and more extensive need, of the statistical methods centered around sampling theory which are the appropriate domain of a Department of Statistics. Should the study of population and vital statistics be included in a Department of Statistics? I think not, except as a temporary arrangement, or in a small institution, in spite of the history of the word "statistics," which originated in connection with material of this kind, and in one of its meanings is still applied to it. (My use of the unqualified word "statistics" in this paper is in the sense of theory and methods, not in the sense of statistical facts such as those found by the census.) Medical, biological and sociological considerations are prominent in the problems of vital statistics, and one of these departments might well handle the subject. But the vital statistician, like other research workers, should have acquired in the course of his training an intimate familiarity with the statistical theory and methods which are the appropriate province of a Department of Statistics. He also needs mathematics through integral equations, if he is to understand and extend the contributions of Lotka and Volterra. Students of vital statistics should have had an elementary course in statistical theory in the Department of Statistics, preferably the course requiring calculus.

A course in price statistics should be taught by an economist, presumably in the Department of Economics, but might well require as a prerequisite the same elementary courses in statistical theory and methods as would be required in psychology, medicine and other fields. In addition, there are problems of time series analysis whose treatment calls for a mathematical statistician having some acquaintance with both economic and meteorological data. A course on the treatment of time series might appropriately be included in the Department of Statistics, requiring the general elementary course as a prerequisite, and itself serving as a prerequisite for courses in economic and meteorological statistics.

One of the chief obstacles to efficient organization of teaching is the habit of not prescribing prerequisites outside one's own department. But when once the elementary courses in statistics have become established in the hands of well-equipped specialists in statistical theory and methods, in whose competence general confidence can be reposed, the various departments of application will lose their motive for establishing their own duplicating courses, and will be able to cultivate more intensively their respective specialities.

The detection of biases and the details of practical statistical work vary greatly from one application to another. These, consequently, are matters for the departments concerned with applications rather than with the fundamentals of statistics, and should not be the chief features of a course in elementary statistical methods and theory. The work of a Department of Statistics should be concerned largely with sampling theory, and should emphasize the unity of statistical methods and theory, regardless of the field of application. It should deal with statistics as a coherent science of inductive inference, of the preparation of observations for inference, and of the planning of investigations so as to yield observations from which inferences can best be made.

The question what mathematical prerequisites should be established for the fundamental course in statistical theory must be answered by a compromise between the ideal and what is expedient at a particular time and place. In Europe a large number of students have had a year of calculus before coming to universities, that is, before reaching the age of eighteen. If a university were willing to restrict its entrants to such students (thus automatically solving the problem of overcrowding) it could give them another year of calculus, mixed perhaps with advanced algebra and geometry, and then in their sophomore year give them a thorough course in elementary statistics and probability, based on calculus. These students would then be ready to tackle advanced statistics in the third year in a really effective way. If the teaching of economic theory, physics, chemistry and astronomy were geared to this program in such a way as to make real use of the calculus, the work in these subjects could be made far more efficient, in the sense that more material could be covered effectively in the allotted time, or an equivalent amount of material in less time. If, in addition, all the many departments in which statistical methods and theory are used required these statistical courses as prerequisites, and actually used the materials of these courses in their work, there would be a further huge gain in efficiency. The baccalaureate degree of such an institution would represent a far more thorough knowledge, and command of the tools of research, than is possible without an arrangement putting in this way the fundamentals first.

Institutions unwilling to undertake such a drastic improvement must face more or less delay and inadequacy in the acquisition by their students of the fundamentals of mathematics and of statistics. A division of the students into groups according to mathematical ability ought to be undertaken, and followed by a corresponding division of the elementary statistics course. Students having high mathematical ability could begin the study of statistics after completing calculus, and could look forward to rising ultimately to greater heights in pursuits involving mathematical or statistical knowledge than those of lesser mathematical talents. For these latter there would still be the possibility of acquiring, even without calculus, useful statistical tools; but it is essential that this should be done under the guidance of instructors thoroughly familiar with the mathematics of statistics. The task of leading the blind must not be turned over to the blind. Students possessing the ability to master the calculus should be encouraged to begin the study of statistics with the course having calculus as a prerequisite, and should not be put into the necessarily slower group not having the calculus. I believe that these elementary courses should begin with the theory of probability, but should go on to the chief distribution functions used in practice, and should include applied problems and work on calculating machines.

Putting a sound program of statistical teaching into effect will take time, partly because of the scarcity of suitable teachers of statistics. Nevertheless, the process is well under way, and the prospects are good for substantial improvements in the teaching of statistics. A body of able young research men possessing the requisite knowledge of statistical fundamentals is now in existence and is growing. Some of the recent textbooks represent striking improvements. The Institute of Mathematical Statistics itself, with the *Annals of Mathematical Statistics*, is perhaps the best evidence of a changed view making for better things.

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DISCUSSION OF PROFESSOR HOTELLING'S PAPER

By W. Edwards Deming

It is a pleasure to endorse Professor Hotelling's recommendations; in fact we have been following them pretty closely in the courses in the Graduate School of the Department of Agriculture. As a matter of fact, he has indirectly played an influential part in building up this set of courses, because some of our best instructors are his former students.

Listening to Professor Hotelling's paper, I was thinking of the possibility that some of his recommendations might be misunderstood. I take it that they are not supposed to embody all that there is in the teaching of statistics, because there are many other neglected phases that ought to be stressed. In the Bureau of the Census the population division alone has augmented its force by approximately 3500 statistical clerks during the past six months. They come from diverse schools and it has been interesting to observe how many of them have the idea that all the problems of sampling and inference from data can be solved by what are commonly known as modern statistical techniques—correlation coefficients, rank correlation coefficients, chi-square, analysis of variance, confidence limits, and the like. Most of them are shocked to learn that many of the so-called modern "theories of estimation" are not theories of estimation at all, but are rather theories of distribution and are a disappointment to one who is faced with the necessity of making a prediction from his data, i.e., of basing