The Term Structure of Currency Carry Trade Risk Premia

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Abstract

We find that average returns to currency carry trades decrease significantly as the maturity of the foreign bonds increases, because investment currencies tend to have small local bond term premia. The downward term structure of carry trade risk premia is informative about the temporal nature of risks that investors face in currency markets. We show that long-maturity currency risk premia only depend on the domestic and foreign *permanent* components of the pricing kernels, since *transitory* currency risk is automatically hedged by interest rate risk for long-maturity bonds. Our findings imply that there is more cross-border sharing of permanent than transitory shocks.

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In this paper, we show that the term structure of currency carry trade risk premia is downward-sloping: the returns to the currency carry trade are much smaller for bonds with longer maturities than for Treasury bills. We derive a preference-free condition that links foreign and domestic long-term bond returns, expressed in a common currency, to the permanent components of the pricing kernels. The downward-sloping term structure of average carry trade returns is therefore informative about the temporal nature of risks that investors face in currency markets. While most of the risk priced in securities markets is very persistent, a large fraction of this risk is shared between countries. Currency markets only price unshared shocks and those are mostly transitory.

Carry trades at the short end of the maturity curve are akin to selling Treasury bills in funding currencies and buying Treasury bills in investment currencies. The exchange rate is here the only source of risk. The set of funding and investment currencies can be determined by the level of short-term interest rates or the slope of the yield curves, as noted by Ang and Chen (2010) and Berge, Jordà, and Taylor (2011). Likewise, carry trades at the long end of the maturity curve are akin to selling long-term bonds in funding currencies and buying long-term bonds in investment currencies. Each leg of the trade is subject to exchange rate and interest rate risk. The log return on a foreign bond position (expressed in U.S. dollars) in excess of the domestic (i.e., U.S.) risk-free rate is equal to the sum of the log excess bond return in local currency plus the return on a long position in foreign currency. Therefore, average foreign bond excess returns converted in domestic currency are the sum of a local bond term premium and a currency risk premium. Market completeness has clear theoretical implications for those two risk premia.

On the one hand, at the short end of the maturity curve, currency risk premia are high when there is less overall risk, be it temporary or permanent, in foreign countries' pricing kernels than at home (Bekaert, 1996; Bansal, 1997; and Backus, Foresi, and Telmer, 2001). High foreign interest rates and/or a flat slope of the yield curve mean less overall risk in the foreign pricing kernel. On the other hand, at the long end of the maturity curve, local bond term premia compensate investors for the risk associated with temporary innovations to the pricing kernel (Bansal and Lehmann, 1997; Hansen and Scheinkman, 2009; Alvarez and Jermann, 2005; Hansen, 2012; Hansen, Heaton, and Li, 2008; and Bakshi and Chabi-Yo, 2012).

In this paper, we combine those two insights to derive three preference-free theoretical results under the assumption of complete financial markets. First, the difference between domestic and foreign long-term bond risk premia, expressed in domestic currency terms, is pinned down by the difference in the entropies of the permanent components of the domestic and foreign stochastic discount factors (SDF). The long-term bond risk premia, expressed in domestic currency, are higher on foreign bonds than on domestic bonds when there is less permanent risk in foreign countries' pricing kernels than at home. The temporary components of SDFs do not account for this difference because the currency exposure completely hedges the exposure of the long-short strategy in long-term bonds to the 'unshared' temporary pricing kernel shocks. Second, when permanent shocks are fully shared across countries and therefore exchange rates are driven by temporary innovations and thus stationary, bond returns in dollars are identical across countries, date by date. We refer to this condition as the long-term uncovered bond return parity condition and test it in the data. Testing for long-term uncovered bond return parity, using the information encoded in long-term bonds, is an alternative to unit root tests of exchange rate stationarity, which have low power in small samples (Campbell and Perron, 1991). Third, we derive a lower bound on the covariance between the domestic and foreign permanent components of the pricing kernels when they are lognormal. The lower bound depends on the difference between the maximum log return and the return on a long-term bond in the domestic and foreign countries, as well as the volatility of the permanent component of exchange rate changes.

Before confronting these theoretical results to the data, we illustrate them in simple reduced-form models. Building on Backus, Foresi, and Telmer (2001), Lustig, Roussanov, and Verdelhan (2011) show that asymmetric exposure to global innovations to the pricing kernel are key to understanding the global currency carry trade premium at short maturities. They identify innovations in the volatility of global equity markets as candidate shocks, while Menkhoff, Sarno, Schmeling, and Schrimpf (2012) propose the volatility in global currency markets instead. However, these risk premia disappear at longer maturities unless the global risk priced in currency markets is permanent: building on the reduced-form model of Lustig, Roussanov, and Verdelhan (2011), we show that the foreign term premium in U.S. dollars is the same as the U.S. term premium if there is no asymmetry in the loadings on the permanent global shocks.

Turning to the data, we study the term structure of currency carry trade risk premia and the long-term uncovered bond return parity condition both in the cross-section and in the time-series of foreign bond returns. The theoretical results pertain to risk-free zero-coupon bonds with infinite maturity: those characteristics are not available in practice, and thus we rely on long-term government bonds of developed countries. Our data pertain to either long time-series of G10 sovereign coupon bond returns over the 12/1950–12/2012 sample, or a shorter sample (12/1971–12/2012) of G10 sovereign zero-coupon yield curves. Although we do not observe infinite maturity bonds in either case, we find significant differences in carry trade returns across maturities.

Using zero-coupon bonds, Figure 1 offers a first glimpse at the term structure of currency carry trade risk premia. The figure, which is studied in details later in the paper, shows the dollar log excess returns as a function of the bond maturities, using the same set of funding and investment currencies. Investing in short-term bills of countries with flat yield curves (mostly high short-term interest rate) while borrowing at the same horizon in countries with steep yield curves (mostly low short-term interest rate countries) leads to positive excess returns on average. This is the classic carry trade, whose average excess return is represented here on the left hand side of the graph. Investing and borrowing in long-term bonds of the same countries, however, deliver negative excess returns on average. As the maturity of the bonds increases, the average excess return decreases.

Between 12/1950 and 12/2012, the portfolio of flat-slope (mostly high short-term interest rate) currencies yields a one-month currency risk premium of 3.0% and a local term premium of −1.8% per annum (which sum to a bond premium of 1.2%). Over the same period, the portfolio of steep-slope (mostly low short-term interest rate) currencies yields a currency risk premium of 0.05% and a local term premium of 4.0% (which sum to a bond premium of 4.05%). The average spread in dollar Treasury bill returns between the low slope and high slope portfolios

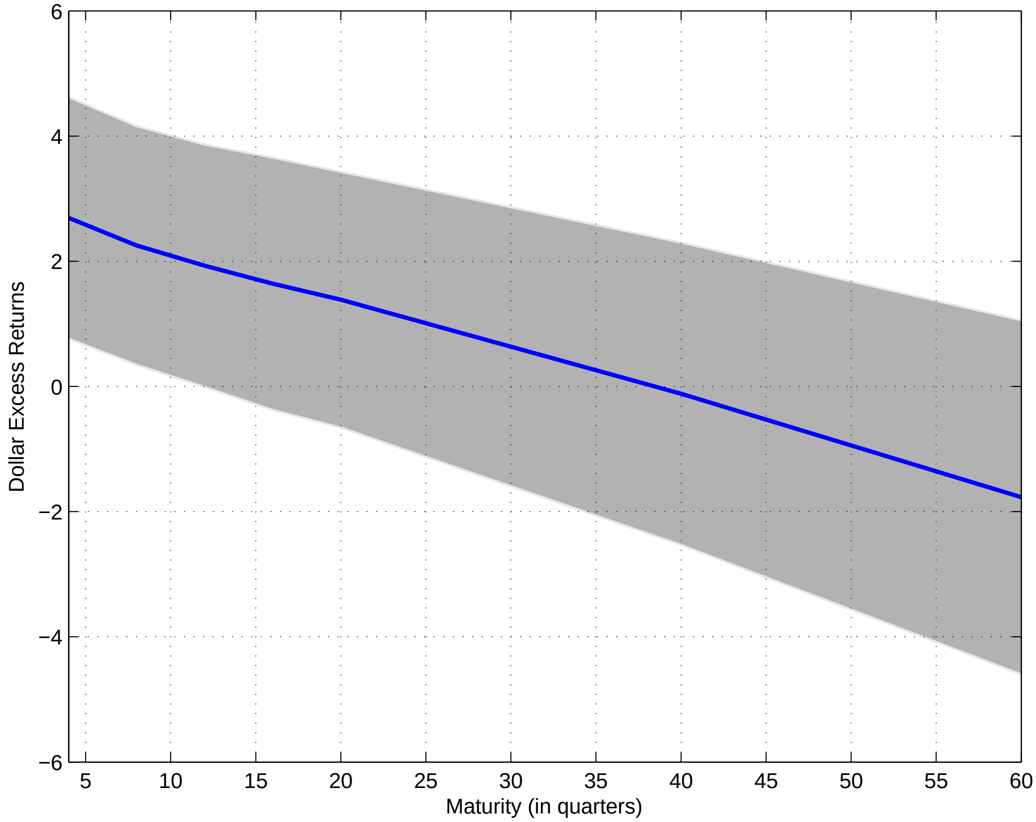

Figure 1: Term Structure of Dollar Bond Risk Premia — The figure shows the dollar log excess returns as a function of the bond maturities. Dollar excess returns correspond to the holding period returns expressed in U.S. dollars of investment strategies that go long and short foreign bonds of different countries. The unbalanced panel of countries consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. At each date t, the countries are sorted by the slope of their yield curves into three portfolios. The first portfolio contains countries with flat yield curves (mostly high interest rate) while the last portfolio contains countries with steep yield curves (mostly low interest rate countries). The first portfolio correspond to the investment currencies while the third one corresponds to the funding currencies. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The holding period is one quarter. The returns are annualized. The shaded areas correspond to two standard deviations above and below each point estimates. Standard deviations are obtained by bootstrapping. Zero-coupon data are monthly, and the sample window is 4/1985–12/2012.

is thus 2.95% (3.0% − 0.05%) for Treasury bills, but it is −2.85% (1.2% − 4.05%) for the 10-year bond portfolios. Countries with a high currency risk premium tend to have a low bond term premium. The profitable bond strategy therefore involves shorting the usual carry trade investment currencies and going long in the funding currencies. High foreign interest rates or a flat slope signal less transitory risk abroad, but not less permanent risk. We obtain similar results when sorting countries by the level of their short-term interest rates: the risk premia at the long end of the maturity curve are significantly smaller that those at the short end, as the difference in local currency bond term premia largely offsets the currency risk premium. As a result, the average returns on foreign long-term bonds, once converted into U.S. dollars are small and rarely statistically different from the average return on U.S. long-term bonds, consistent with the long-term uncovered bond return parity condition.

The long-run uncovered bond parity condition, however, is a better fit in the cross-section than in the time series. In the post-Bretton Woods period, an 1% increase in U.S. long-term bond returns increases foreign bond returns in dollars by an average of 0.4%, not 1% as the bond parity condition implies. The exchange rate exposure accounts for almost a third of this effect: the dollar appreciates on average against a basket of foreign currencies when the U.S. bond returns are lower than average, and vice-versa, except during flight-to-liquidity episodes. While we reject the long-run uncovered bond return parity condition in the time series, we do find a secular increase in the sensitivity of foreign long-term bond returns to U.S. bond returns over time, consistent with an increase in the correlation of permanent shocks in international financial markets. After 1991, an 1% increase in U.S. long-term bond returns increases foreign bond returns in dollars by 0.5% on average. Since bond returns expressed in dollars do not move one for one in the time-series, permanent shocks to the foreign and domestic pricing kernels must not be perfectly shared.

To shed additional light on the correlation of pricing kernels across countries, we therefore decompose exchange rates into their permanent and transitory components. Hansen and Scheinkman (2009), Alvarez and Jermann (2005), Hansen, Heaton, and Li (2008), and Bakshi and Chabi-Yo (2012) have explored the implications of such a decomposition of domestic pricing kernels for asset prices. Given that exchange rates express differences in pricing kernels across countries, the pricing kernel decomposition implies that exchange rate changes can be also broken into two components, one that encodes cross-country differences in the permanent SDF components and one that reflects differences in the transitory components. Data on long-maturity bond returns allow us to extract the time series for two exchange rate components for a cross-section of exchange rates. The characteristics of the permanent components of exchange rates are then a key ingredient to compute our lower bound on the covariance between the domestic and foreign permanent components of the pricing kernels.

We find that the two exchange rate components contribute about equally to the volatility of exchange rate changes, implying that internationally unshared pricing kernel transitory shocks are equally important to unshared permanent shocks for exchange rate determination. This finding contrasts with previous results obtained on domestic markets. From the relative size of the equity premium (large) and the term premium (small), Alvarez and Jermann (2005) infer that almost all the variation in stochastic discount factors arises from permanent fluctuations. Since permanent fluctuations are an order of magnitude larger than transitory fluctuations, the similar volatilities of the permanent and transitory components of exchange rates imply that permanent shocks are much more correlated across countries than transitory shocks. Indeed, we find that the implied correlation of the transitory stochastic discount factor components, although positive, is much lower than the implied correlation of state prices, as calculated in Brandt, Cochrane, and Santa-Clara (2006). Using our theoretical lower bound, we show that the correlation between the permanent components of SDFs is at least equal to 0.9.

The high correlation of SDFs can be interpreted in terms of risk-sharing if and only if the domestic and foreign agents consume the same baskets of goods and participate in complete financial markets. To the contrary, variation in relative prices of different consumption baskets can drive a wedge between the pricing kernels even in the case of perfect risk sharing across borders. Likewise, when markets are segmented, as in Alvarez, Atkeson, and Kehoe (2002, 2009), the correlation of SDFs does not imply risk-sharing of the non-participating agents. Therefore, under the additional assumptions that agents consume the same goods and participate in financial markets, our findings that permanent components of the SDFs are highly correlated across countries imply that the bulk of permanent shocks are shared across countries.

Finally, we also find that the two exchange rate components are negatively correlated with each other: permanent innovations that raise the state price of a given country relative to that of a foreign country tend to be partly offset by unshared transitory innovations. This is line with the results from statistical decompositions of the underlying fundamentals. For example, Morley, Nelson, and Zivot (2003) find a strong, negative correlation in the trend and cycle component of U.S. GDP. Using only exchange rates and bond prices, we document the same result in the country-specific part of marginal utility.

Our paper is related to three large strands of the literature: the international correlation of SDFs, the carry trade returns, and the term premia across countries.

The high correlation of the SDF permanent components extends previous results in a key dimension. Brandt, Cochrane, and Santa-Clara (2006) show that the combination of relatively smooth exchange rates (10% per annum) and much more volatile stochastic discount factors (50% per annum) implies that state prices are highly correlated across countries. It is important to show that the permanent, not the temporary, SDF components are correlated because the welfare gains from removing all aggregate consumption uncertainty come almost exclusively from the low frequency component in consumption, not the business cycle component (Alvarez and Jermann, 2004). Recently, Chabi-Yo and Colacito (2013) generalize the lower bound on the comovement of domestic and foreign permanent SDFs to pricing kernels that are not necessarily lognormal, study the term structure of this comovement, and consider the ability of several international finance models to address the empirical properties of that SDF decomposition.

Our paper builds on the vast literature on uncovered interest rate parity condition (UIP) and the currency carry trade [Engel (1996) and Lewis (2011) provide recent surveys]. We are the first to derive general conditions under which long-run unconditional UIP follows simply from market completeness: if all permanent shocks to the pricing kernel are common, then foreign and domestic yield spreads in dollars on long maturity bonds will be equalized, regardless of the properties of the pricing kernel.

Our focus is on the cross-sectional relation between the slope of the yield curve, interest rates and exchange rates. We study whether investors earn higher returns on foreign bonds from countries in which the slope of the yield curve is higher than the cross-country average. Prior work, from Campbell and Shiller (1991) to Bekaert and Hodrick (2001) and Bekaert, Wei, and Xing (2007), focus mostly on the time series, testing whether investors earn higher returns on foreign bonds from a country in which the slope of the yield curve is currently higher than average for that country. Chinn and Meredith (2004) document some time-series evidence that supports a conditional version of UIP at longer holding periods, while Boudoukh, Richardson, and Whitelaw (2013) show that past forward rate differences predict future changes in exchange rates.

The rest of the paper is organized as follows. In Section 1, we derive the no-arbitrage, preference-free theoretical restrictions imposed on currency and term risk premia. In Section 2, we provide three simple theoretical examples. In Section 3, we examine the cross-section of bond excess returns in local currency and in U.S. dollars and we contrast it with the cross-section of currency excess returns. In Section 4, we test the uncovered bond return parity condition in the time-series. In Section 5, we decompose exchange rate changes into a permanent and a temporary component and we link their properties to the extent of risk-sharing. In Section 6, we present concluding remarks. The Appendix contains all proofs and an Online Appendix contains supplementary material not presented in the main body of the paper.

1 The Term Premium and the Currency Risk Premium

We begin by defining notation and then deriving our main theoretical results.

1.1 Notation

In order to state our main results, we first need to introduce the domestic and foreign pricing kernels, stochastic discount factors, and bond holding period returns.

Pricing Kernel, Stochastic Discount Factor, and Bond Return The nominal pricing kernel is denoted $\Lambda_t(\varpi)$; it corresponds to the marginal value of a dollar delivered at time t in some state of the world ϖ. The nominal SDF is the growth rate of the pricing kernel: $M_{t+1} = \Lambda_{t+1}/\Lambda_t$. The price of a zero-coupon bond that matures k periods into the future is given by:

$$P_t^{(k)} = E_t\left(\frac{\Lambda_{t+k}}{\Lambda_t}\right).$$

The one-period return on the zero-coupon bond with maturity k is $R_{t+1}^{(k)} = P_{t+1}^{(k-1)}/P_t^{(k)}$. The log excess returns, denoted $rx_{t+1}^{(k)}$, is equal to $\log R_{t+1}^{(k)}/R_t^f$, where the risk-free rate is $R_t^f =$

$R_{t+1}^{(0)} = 1/P_t^{(1)}$. The expected log excess return on the zero-coupon bond with maturity k, or term premium, is:

$$E_t\left[rx_{t+1}^{(k)}\right] = E_t\left[\log R_{t+1}^{(k)}/R_t^f\right].$$

The yield spread is the log difference between the yield of the k-period bond and the risk-free rate: $y_t^{(k)} = -\log\left(R_t^f/(P_t^{(k)})^{1/k}\right)$.

Entropy Bond returns and SDFs are volatile, but not necessarily normally distributed. In order to measure the time-variation in their volatility, it is convenient to use entropy.[1] The conditional volatility of any random variable X_{t+1} is thus measured through its conditional entropy L_t, defined as:

$$L_t\left(X_{t+1}\right) = \log E_t\left(X_{t+1}\right) - E_t\left(\log X_{t+1}\right).$$

The conditional entropy of a random variable is determined by its conditional variance, as well as its higher moments; if $var_t\left(X_{t+1}\right) = 0$, then $L_t\left(X_{t+1}\right) = 0$, but the reverse is not generally true. If X_{t+1} is conditionally lognormal, then the entropy is simply the half variance of the log variable: $L_t\left(X_{t+1}\right) = (1/2)var_t\left(\log X_{t+1}\right)$. The relative entropy of the permanent and transitory components of the domestic and foreign SDFs turns out to be key to understanding the term structure of carry trade risk.

Permanent and Transitory Innovations Following Alvarez and Jermann (2005), Hansen, Heaton, and Li (2008), and Hansen and Scheinkman (2009), we decompose each pricing kernel into a transitory ($\Lambda_t^{\mathbb{T}}$) component and a permanent ($\Lambda_t^{\mathbb{P}}$) component with:

$$\Lambda_t = \Lambda_t^{\mathbb{P}}\Lambda_t^{\mathbb{T}}, \text{ where } \Lambda_t^{\mathbb{T}} = \lim_{k\to\infty}\frac{\delta^{t+k}}{P_t^{(k)}}.$$

The constant δ is chosen to satisfy the following regularity condition: $0 < \lim_{k\to\infty}\frac{P_t^{(k)}}{\delta^k} < \infty$ for all t. We also assume that, for each $t+1$, there exists a random variable x_{t+1} with finite $E_t(x_{t+1})$

[1] Backus, Chernov, and Zin (2014) make a convincing case for the use of entropy in assessing macro-finance models.

such that a.s. $\frac{\Lambda_{t+1}}{\delta^{t+1}}\frac{P^{(k)}_{t+1}}{\delta^k} \leq x_{t+1}$ for all k. Under those regularity conditions, the infinite maturity bond return is then:

$$R^{(\infty)}_{t+1} = \lim_{k\to\infty} R^{(k)}_{t+1} = \lim_{k\to\infty} P^{(k-1)}_{t+1}/P^{(k)}_t = \frac{\Lambda^{\mathbb{T}}_t}{\Lambda^{\mathbb{T}}_{t+1}}.$$

The permanent component, $\Lambda^{\mathbb{P}}_t$, is a martingale.[2] It is an important component of the pricing kernel. Alvarez and Jermann (2005) derive a lower bound on its volatility, and, given the size of the equity premium relative to the term premium, conclude that the permanent component of the pricing kernel is large and accounts for most of the risk.[3] In other words, a lot of persistence in the pricing kernel is needed to deliver a low term premium and a high equity premium. In the absence of arbitrage, Alvarez and Jermann (2005) show that the local term premium in local currency is given by:

$$E_t\left[rx^{(\infty)}_{t+1}\right] = \lim_{k\to\infty} E_t\left[rx^{(k)}_{t+1}\right] = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right).$$

Hansen, Heaton, and Li (2008), Hansen and Scheinkman (2009), and Borovicka, Hansen, Hendricks, and Scheinkman (2011) provide examples of similar factorizations in affine models. The SDF decomposition defined here is subject to important limitations that need to be highlighted. Hansen and Scheinkman (2009) point out that this decomposition is not unique in general and provide parametric examples in which uniqueness fails. In addition, the temporary (or transient) and permanent components are potentially highly correlated, which complicates their interpretation.[4] Despite these limitations, we show that this decomposition proves to be particularly useful when analyzing foreign bond returns at longer maturities.

[2]Note that $\Lambda^{\mathbb{P}}_t$ is equal to:

$$\Lambda^{\mathbb{P}}_t = \lim_{k\to\infty} \frac{P^{(k)}_t}{\delta^{t+k}}\Lambda_t = \lim_{k\to\infty} \frac{E_t(\Lambda_{t+k})}{\delta^{t+k}}.$$

The second regularity condition ensures that the expression above is a martingale.

[3]Alvarez and Jermann (2005) derive the following lower bound:

$$L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) \geq E_t\left(\log R_{t+1}\right) - E_t\left(\log R^{(\infty)}_{t+1}\right),$$

where R_{t+1} denotes any positive return and $R^{(\infty)}_{t+1}$ is the return on a zero-coupon bond of infinite maturity.

[4]The authors thank Lars Hansen for a detailed account of these issues.

Exchange Rates The nominal spot exchange rate in foreign currency per U.S. dollar is denoted S_t. When S increases, the U.S. dollar appreciates. Similarly, F_t denotes the one-period forward exchange rate, and f_t its log value. When markets are complete, the change in the exchange rate corresponds to the ratio of the domestic to foreign SDFs:

$$\frac{S_{t+1}}{S_t} = \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\Lambda_t^*}{\Lambda_{t+1}^*},$$

where * denotes a foreign variable. The no-arbitrage definition of the exchange rate comes directly from the Euler equations of the domestic and foreign investors, for any asset R^* expressed in foreign currency: $E_t[M_{t+1} R_{t+1}^* S_t / S_{t+1}] = 1$ and $E_t[M_{t+1}^* R_{t+1}^*] = 1$. When markets are complete, the SDF is unique, and thus the change in exchange rate is the ratio of the two SDFs. The log currency excess return corresponds to:

$$rx_{t+1}^{FX} = \log \left[\frac{S_t}{S_{t+1}} \frac{R_t^{f,*}}{R_t^f} \right] = (f_t - s_t) - \Delta s_{t+1},$$

when the investor borrows at the domestic risk-free rate, R_t^f, and invests at the foreign risk-free rate, $R_t^{f,*}$, and where the forward rate is defined through the covered interest rate parity condition: $F_t/S_t = R_t^{f,*}/R_t^f$. As Bekaert (1996) and Bansal (1997) show, in a lognormal model, the log currency risk premium equals the half difference between the conditional volatilities of the log domestic and foreign SDFs. Higher order moments are critical for understanding currency returns.[5] When higher moments matter and markets are complete, the currency risk premium is equal to the difference between the entropy of the domestic and foreign SDFs (Backus, Foresi, and Telmer, 2001):

$$E_t\left[rx_{t+1}^{FX}\right] = (f_t - s_t) - E_t(\Delta s_{t+1}) = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right).$$

[5]The literature on disaster risk in currency markets concurs. In earlier work, Brunnermeier, Nagel, and Pedersen (2009) show that risk reversals increase with interest rates. Jurek (2008) provides a comprehensive empirical investigation of hedged carry trade strategies. Gourio, Siemer, and Verdelhan (2013) study a real business cycle model with disaster risk. Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2013) estimate a no-arbitrage model with crash risk using a cross-section of currency options. Chernov, Graveline, and Zviadadze (2011) study jump risk at high frequencies. Gavazzoni, Sambalaibat, and Telmer (2012) show that lognormal models cannot account for the cross-country differences in carry returns and interest rate volatilities.

Following the decomposition of the pricing kernel discussed above, exchange rate changes can also be decomposed into a permanent and a transitory component, defined below:

$$\frac{S_{t+1}}{S_t} = \left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\frac{\Lambda_t^{\mathbb{P},*}}{\Lambda_{t+1}^{\mathbb{P},*}}\right)\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\frac{\Lambda_t^{\mathbb{T},*}}{\Lambda_{t+1}^{\mathbb{T},*}}\right) = \frac{S_{t+1}^{\mathbb{P}}}{S_t^{\mathbb{P}}}\frac{S_{t+1}^{\mathbb{T}}}{S_t^{\mathbb{T}}}.$$

Exchange rate changes capture the differences in both the transitory and the permanent component of the two countries' SDFs. In this paper, we use returns on long term bonds to implement this decomposition in the data.

Term Premium on Foreign Bonds The log return on a foreign bond position (expressed in U.S. dollars) in excess of the domestic (i.e., U.S.) risk-free rate is denoted $rx_{t+1}^{(k),\$}$. It can be expressed as the sum of the log excess return in local currency plus the return on a long position in foreign currency:

$$rx_{t+1}^{(k),\$} = \log\left[\frac{R_{t+1}^{(k),*}}{R_t^f}\frac{S_t}{S_{t+1}}\right] = \log\left[\frac{R_{t+1}^{(k),*}}{R_t^{f,*}}\frac{R_t^{f,*}}{R_t^f}\frac{S_t}{S_{t+1}}\right] = rx_{t+1}^{(k),*} + rx_{t+1}^{FX}.$$

The first component of the foreign bond excess return is the excess return on a bond in foreign currency, while the second component represents the log excess return on a long position in foreign currency, given by the forward discount minus the rate of depreciation. Taking expectations, the total term premium in dollars thus consists of a foreign bond risk premium, $E_t[rx_{t+1}^{(k),*}]$, plus a currency risk premium, $(f_t - s_t) - E_t\Delta s_{t+1}$.

1.2 Main Theoretical Results

In this section, we present our four key theoretical results on (i) the term structure of carry trade premia; (ii) the decomposition of exchange rates into permanent and transitory components; (iii) the long-term bond return parity condition; and (iv) a lower bound on the risk-sharing of permanent shocks.

Carry Trade Term Premia We begin with a characterization of carry trade risk premia at long maturities.

Proposition 1. *The foreign term premium in dollars is equal to the domestic term premium plus the difference between the domestic and foreign entropies of the permanent components of the pricing kernels:*

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + (f_t - s_t) - E_t[\Delta s_{t+1}] = E_t\left[rx_{t+1}^{(\infty)}\right] + L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right) - L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}}\right).$$

In case of an adverse temporary innovation to the foreign pricing kernel, the foreign currency appreciates, but this is exactly offset by the capital loss suffered on the longest maturity zero-coupon bond, as a result of the increase in foreign interest rates. Hence, interest rate exposure completely hedges the temporary component of the currency risk exposure, and the only source of priced currency risks in the foreign bond positions are the permanent innovations.

In order to produce a currency risk premium at longer maturities, entropy differences in the permanent component of the pricing kernel are required. If there are no such differences and domestic and foreign pricing kernels are identically distributed, then high local currency term premia coincide with low currency risk premia and vice-versa and dollar term premia are identical across currencies.

The unconditional version of this result is equivalent to uncovered interest rate parity for very long holding periods. Note that the k-period holding return in excess of the U.S. risk-free rate on a foreign k-period bond denoted in dollars is $k(y_t^{k,*}) - \Delta s_{t\to t+k} + k(f_t - s_t)$. The unconditional version of long-run UIP implies that, as $k \to \infty$, $E[y_t^{k,*}] - E[\lim_{k\to\infty} \frac{1}{k}\sum_{j=1}^{k} \Delta s_{t+j}] + E[f_t - s_t] = E[y_t^k]$.

Corollary 1. *The average foreign yield minus the average rate of depreciation equals the average domestic yield at very long horizons plus the average difference between the domestic and foreign entropies of the permanent components of the pricing kernels:*

$$E[y_t^{(\infty),*}] - E[\lim_{k\to\infty} \frac{1}{k}\sum_{j=1}^{k} \Delta s_{t+j}] + E[f_t - s_t] = E[y_t^{(\infty)}] + E\left[L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right) - L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}}\right)\right].$$

On average, deviations from long-run uncovered interest rate parity are driven only by differences in entropy of the permanent components. If there are no differences, then no arbitrage implies that long-run UIP holds but only unconditionally. Examining the conditional moments of one-period holding period returns on long maturity bonds, the focus of our paper, is not equivalent to studying the moments long bond yields.

Permanent Component of Exchange Rates The valuation of long-maturity bonds thus encodes information about the nature of shocks that drive the changes in exchange rates. Using the prices of long-maturity bonds in the domestic and foreign countries, under the regularity conditions defined previously, we can decompose the changes in the bilateral spot exchange rate into two parts: a part that captures cross-country differences in the transitory components of the pricing kernel and a part that encodes differences in the permanent components of the pricing kernel.

Proposition 2. *When markets are complete, the ratio of the domestic and foreign infinite maturity bond returns, expressed in the same currency, measures the permanent component of exchange rate changes:*

$$\lim_{k \to \infty} \frac{S_t}{S_{t+1}} \frac{R_{t+1}^{(k),*}}{R_{t+1}^{(k)}} = \frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}} \frac{\Lambda_t^{\mathbb{P}}}{\Lambda_{t+1}^{\mathbb{P}}} = \frac{S_t^{\mathbb{P}}}{S_{t+1}^{\mathbb{P}}}.$$

The left-hand side of this equality can be approximated by long term bonds, thus leading to a measure of the permanent component of exchange rates. Since exchange rates are observed, the temporary component of the exchange rates can also be easily obtained.

Long-Term Bond Return Parity Condition The exchange rate decomposition above implies an uncovered long-bond return parity condition when countries share permanent innovations to their SDFs. In this polar case, even if most of the innovations to the pricing kernel are highly persistent, the shocks that drive exchange rates are not, because the persistent shocks are shared across countries. When bond parity holds, the exchange rate is a stationary process.

Corollary 2. *If the domestic and foreign pricing kernels have common permanent innovations, $\Lambda_{t+1}^{\mathbb{P}}/\Lambda_t^{\mathbb{P}} = \Lambda_{t+1}^{\mathbb{P},*}/\Lambda_t^{\mathbb{P},*}$ for all states, then the one-period returns on the foreign longest maturity bonds in domestic currency are identical to the domestic ones: $R_{t+1}^{(\infty),*}\frac{S_t}{S_{t+1}} = R_{t+1}^{(\infty)}$ for all states.*

Hau and Rey (2006) and Pavlova and Rigobon (2007) propose and test uncovered equity parity conditions in specific models in international economics. Our novel international parity condition pertains instead to the bond markets and is model-free. This condition can be used to revisit the purchasing power parity and international risk-sharing literatures.

Purchasing Power Parity So far we have discussed the implications for nominal bonds, nominal exchange rates and the nominal pricing kernel, but we can re-interpret these results in real terms, introducing the returns on long-term real bonds and the real exchange rate instead. If the permanent components of the real pricing kernels are common across countries, then the log of the real exchange rate equals the difference of the temporary components of the real pricing kernels. As a result, the real exchange rate is stationary and a weak version of the purchasing power parity condition (PPP) holds in the long run (see, e.g., Rogoff [1996] for a survey of the PPP literature).

Hence, our result suggests a test of exchange rate stationarity that simply uses the returns on (long) indexed bonds and (real) exchange rates, without resorting to unit root tests of (real) exchange rates which have low power in short samples (Enders, 1988). Instead, we learn about the anticipated long-run behavior of (real) exchange rates from the information encoded in long-term (real) bonds. The real version of this test is a test of PPP.

Risk-Sharing The nature and magnitude of international risk sharing is an important and open question in macroeconomics (see, for example, Cole and Obstfeld, 1991; van Wincoop, 1994; Lewis, 2000; Gourinchas and Jeanne, 2006; Lewis and Liu, 2012; Coeurdacier, Rey, and Winant, 2013; Didier, Rigobon, and Schmukler, 2013; as well as Colacito and Croce, 2011, and Stathopoulos, 2012, on the high international correlation of state prices). The exchange rate decomposition also sheds light on the nature of cross-country risk-sharing, but only under the assumption that domestic and foreign agents have identical consumption baskets and participate in complete financial markets. Again, in a multi-good world, variation in the relative prices of these goods drives a wedge between the pricing kernels, even in the case of perfect risk sharing (Cole and Obstfeld, 1991).

Brandt, Cochrane, and Santa-Clara (2006) show that the combination of relatively smooth exchange rates and much more volatile SDFs implies that state prices are very highly correlated across countries. A 10% volatility in exchange rate changes and a volatility of marginal utility growth rates of 50% imply a correlation of at least 0.98. We can derive a specific bound on the covariance of the permanent component across different countries.

Proposition 3. *If the permanent SDF component is unconditionally lognormal, the cross-country covariance of the SDF' permanent components is bounded below by:*

$$cov\left(\log\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}}, \log\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right) \quad\geq\quad E\left(\log\frac{R_{t+1}^{*}}{R_{t+1}^{(\infty),*}}\right) + E\left(\log\frac{R_{t+1}}{R_{t+1}^{(\infty)}}\right) - \frac{1}{2}var\left(\log\frac{S_{t+1}^{\mathbb{P}}}{S_t^{\mathbb{P}}}\right).$$

for any positive returns R_{t+1} and R_{t+1}^{*}. A conditional version of the expression holds for conditionally lognormal permanent pricing kernel components.

We can compute the variance of the permanent component of exchange rates, $var\left(\log\frac{S_{t+1}^{\mathbb{P}}}{S_t^{\mathbb{P}}}\right)$, in the data; the contribution of the last term will typically be on the order of 1% or less — this value will be discussed in our empirical work. Given the large size of the equity premium compared to the term premium (a 7.5% difference according to Alvarez and Jermann, 2005), and the relatively small variance of the permanent component of exchange rates, this bound implies a large correlation of the permanent components. In Figure 2, we plot the implied correlation of the permanent component against the volatility of the permanent component in the symmetric case for two different scenarios. The dotted line is for $Std\left(\log S_t^{\mathbb{P}}/S_{t+1}^{\mathbb{P}}\right) = 10\%$, and the diamond line is for $Std\left(\log S_t^{\mathbb{P}}/S_{t+1}^{\mathbb{P}}\right) = 16\%$. In both cases, the implied correlation of the permanent components of the domestic and foreign pricing kernels is clearly above 0.9.

Our results extend the insights of Brandt, Cochrane, and Santa-Clara (2006) and, given the assumption of identical consumption baskets across countries and market participation, directly

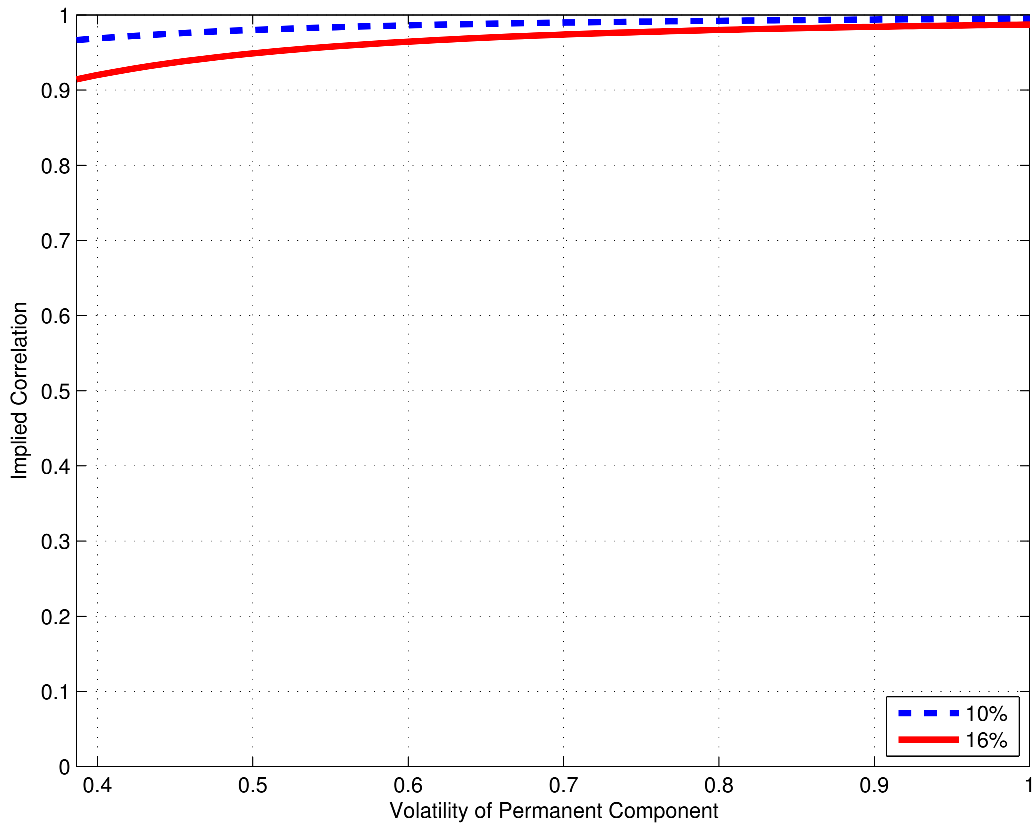

Figure 2: Permanent Risk Sharing — In this figure, we plot the implied correlation of the domestic and foreign permanent components of the SDF against the standard deviation of the permanent component of the SDF. The dotted line is for $Std\left(\log S_t^{\mathbb{P}}/S_{t+1}^{\mathbb{P}}\right) = 10\%$. The straight line is for $Std\left(\log S_t^{\mathbb{P}}/S_{t+1}^{\mathbb{P}}\right) = 16\%$. Following Alvarez and Jermann (2005), we assume that the equity minus bond risk premia are 7.5% in the domestic and foreign economies.

speak to the properties of international risk sharing. Earlier papers have studied cross-country SDF properties, but have not focused on either risk sharing or the decomposition of the SDF.

1.3 Special Case: No Permanent Innovations

Let us now consider the special case in which the pricing kernel is *not* subject to permanent innovations, i.e., $\lim_{k\to\infty} \frac{E_{t+1}[\Lambda_{t+k}]}{E_t[\Lambda_{t+k}]} = 1$. For example, the Markovian environment recently considered by Ross (2013) to derive his recovery theorem satisfies this condition. Building on this work, Martin and Ross (2013) derive closed-form expressions for bond returns in a similar environment. Alvarez and Jermann (2005) show that this case has clear implications for domestic returns: if the pricing kernel has no permanent innovations, then the term premium on an infinite maturity bond is the largest risk premium in the economy.[6]

The absence of permanent innovations also has a strong implication for the term structure of the carry trade risk premia. When the pricing kernels do not have permanent innovations, the foreign term premium in dollars equals the domestic term premium:

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + (f_t - s_t) - E_t[\Delta s_{t+1}] = E_t\left[rx_{t+1}^{(\infty)}\right].$$

The proof here is straightforward. In general, the foreign currency risk premium is equal to the difference in entropy. In the absence of permanent innovations, the term premium is equal to the entropy of the pricing kernel, so the result follows. More interestingly, a much stronger result holds in this case. Not only are the risk premia identical, but the returns on the foreign bond position are the same as those on the domestic bond position state-by-state, because the foreign bond position automatically hedges the currency risk exposure. As already noted, if the domestic and foreign pricing kernels have no permanent innovations, then the one-period returns on the longest maturity foreign bonds in domestic currency are identical to the domestic ones:

$$\lim_{k\to\infty} \frac{S_t}{S_{t+1}} \frac{R_{t+1}^{(k),*}}{R_{t+1}^{(k)}} = 1.$$

In this class of economies, the returns on long-term bonds expressed in domestic currency are equalized:

$$\lim_{k\to\infty} rx_{t+1}^{(k),*} + (f_t - s_t) - \Delta s_{t+1} = rx_{t+1}^{(k)}.$$

In countries that experience higher marginal utility growth, the domestic currency appreciates but is exactly offset by the capital loss on the bond. For example, in a representative agent economy, when the log of aggregate consumption drops more below trend at home than abroad, the domestic currency appreciates, but the real interest rate increases, because the representative agent is eager to smooth consumption. The foreign bond position automatically hedges the

[6] If there are no permanent innovations to the pricing kernel, then the return on the bond with the longest maturity equals the inverse of the SDF: $\lim_{k\to\infty} R_{t+1}^{(k)} = \Lambda_t/\Lambda_{t+1}$. High marginal utility growth translates into higher yields on long maturity bonds and low long bond returns, and vice-versa.

currency exposure.

2 Three Theoretical Lognormal Examples

This section provides three lognormal examples of the theoretical results derived above. We start with a simple homoscedastic SDF suggested by Alvarez and Jermann (2005), and then turn to a heteroscedastic SDF like the one proposed by Cox, Ingersoll, and Ross (1985). Building on these two preliminary examples and on Lustig, Roussanov, and Verdelhan (2011), the section ends with a model featuring global permanent and transitory shocks. This model illustrates the necessary conditions for the cross-sections of carry and term premia.

2.1 Homoskedastic SDF

Alvarez and Jermann (2005) propose the following example of an economy without permanent shocks: a representative agent economy with power utility investors in which the log of aggregate consumption is a trend-stationary process with normal innovations.

Example 1. *Consider the following pricing kernel (Alvarez and Jermann, 2005):*

$$\log \Lambda_t = \sum_{i=0}^{\infty} \alpha_i \epsilon_{t-i} + \beta \log t,$$

with $\epsilon \sim N(0, \sigma^2), \alpha_0 = 1$. *If* $\lim_{k \to \infty} \alpha_k^2 = 0$*, then the SDF has no permanent component. The foreign SDF is defined similarly.*

In this example, Alvarez and Jermann (2005) show that the term premium equals one half of the variance: $E_t \left[rx_{t+1}^{(\infty)} \right] = \sigma^2/2$, the highest possible risk premium in this economy, because the returns on the long bond are perfectly negatively correlated with the stochastic discount factor. When marginal utility is [temporarily] high, the representative agent would like to borrow, driving up interest rates and lowering the price of the long-term bond.

In this case, we find that the foreign term premium in dollars is identical to the domestic term premium:

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + (f_t - s_t) - E_t[\Delta s_{t+1}] = \frac{1}{2}\sigma^2 = E_t\left[rx_{t+1}^{(\infty)}\right].$$

This result is straightforward to establish: recall that the currency risk premium is equal to the half difference in the domestic and foreign SDF volatilities. Currencies with a high local currency term premium (high σ^2) also have an offsetting negative currency risk premium, while those with a small term premium have a large currency risk premium. Hence, U.S. investors receive the same dollar premium on foreign as on domestic bonds. There is no point in chasing high term premia around the world, at least not in economies with only temporary innovations to the pricing kernel. Currencies with the highest local term premia also have the lowest (i.e., most negative) currency risk premia.

Building on the previous example, Alvarez and Jermann (2005) consider a log-normal model of the pricing kernel that features both permanent and transitory shocks.

Example 2. *Consider the following pricing kernel (Alvarez and Jermann, 2005):*

$$\begin{aligned}
\log \Lambda_{t+1}^{\mathbb{P}} &= -\frac{1}{2}\sigma_P^2 + \log \Lambda_t^{\mathbb{P}} + \varepsilon_{t+1}^P, \\
\log \Lambda_{t+1}^{\mathbb{T}} &= \log \beta^{t+1} + \sum_{i=0}^{\infty} \alpha_i \varepsilon_{t+1-i}^T,
\end{aligned}$$

where α is a square summable sequence, and ε^P and ε^T are i.i.d. normal variables with mean zero and covariance σ_{TP}. A similar decomposition applies to the foreign SDF.

In this case, Alvarez and Jermann (2005) show that the term premium is given by the following expression: $E_t\left[rx_{t+1}^{(\infty)}\right] = \sigma_T^2/2 + \sigma_{TP}$. Only the transitory risk is priced in the market for long bonds. When marginal utility is temporarily high, interest rates increase because the representative agent wants to borrow, and long bonds suffer a capital loss. Permanent shocks to marginal utility do not have this effect. In this economy, the foreign term premium in dollars is:

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + (f_t - s_t) - E_t[\Delta s_{t+1}] = \frac{1}{2}\left(\sigma^2 - \sigma_P^{2,*}\right).$$

Provided that $\sigma_P^{2,*} = \sigma_P^2$, the foreign term premium in dollars equals the domestic term premium:

$$E_t\left[rx_{t+1}^{(\infty),*}\right] + (f_t - s_t) - E_t[\Delta s_{t+1}] = \frac{1}{2}\sigma_T^2 + \sigma_{TP} = E_t\left[rx_{t+1}^{(\infty)}\right].$$

2.2 Heteroskedastic SDFs

We turn now to a workhorse model in the term structure literature: the Cox, Ingersoll, and Ross (1985) model (denoted CIR).

Example 3. *The CIR model is defined by the following two equations:*

$$\begin{aligned} -\log M_{t+1} &= \alpha + \chi z_t + \sqrt{\gamma z_t} u_{t+1}, \\ z_{t+1} &= (1-\phi)\theta + \phi z_t - \sigma\sqrt{z_t} u_{t+1}. \end{aligned}$$

In this model, log bond prices are affine in the state variable z: $p_t^{(n)} = -B_0^n - B_1^n z_t$, where B_0^n and B_1^n are the solution to difference equations. The expected log excess return of an infinite maturity bond is then:

$$E_t\left[rx_{t+1}^{(\infty)}\right] = \left[-\frac{1}{2}\left(B_1^\infty\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^\infty\right] z_t = \left[B_1^\infty(1-\phi) - \chi + \frac{1}{2}\gamma\right] z_t,$$

where B_1^∞ is defined implicitly in the following second-order equation: $B_1^\infty = \chi - \gamma/2 + B_1^\infty\phi - \left(B_1^\infty\right)^2\sigma^2/2 + \sigma\sqrt{\gamma}B_1^\infty$. The first component, $-\left(B_1^\infty\right)^2\sigma^2/2$, is a Jensen term. The term premium is driven by the second component, $\sigma\sqrt{\gamma}B_1^\infty z_t$. In the CIR model, there are no permanent innovations to the pricing kernel provided that $B_1^\infty(1-\phi) = \chi$. In this case, the permanent component of the pricing kernel is constant:

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = \beta^{-1}e^{-\alpha-\chi\theta}.$$

In the case of no permanent innovations in the CIR model, the expected long-run term premium is simply $E_t\left[rx_{t+1}^{(\infty)}\right] = \gamma z_t/2$, one half of the variance of the SDF. Hence, in the symmetric case, the foreign term premium in dollars is equal to the domestic term premium:

$$E_t\left[rx_{t+1}^{(\infty),*}\right]+(f_t-s_t)-E_t[\Delta s_{t+1}]=\frac{1}{2}\gamma z_t=E_t\left[rx_{t+1}^{(\infty)}\right].$$

Naturally, this implies that, on average, U.I.P holds at long holding periods:

$$E[y_t^{(\infty),*}]+E[f_t-s_t]-E[\lim_{k\to\infty}\frac{1}{k}\sum_{j=1}^{k}\Delta s_{t+j}]=E[y_t^{(\infty)}]$$

Lustig, Roussanov, and Verdelhan (2011) show that the CIR model with (i) global shocks and (ii) heterogeneity in the SDFs' loadings on those global shocks can replicate the empirical evidence on currency excess returns.[7] Building on their work, we turn now to a version of the CIR model with two global components: a persistent component and a transitory component. We show that heterogeneity in the SDFs' loadings on the *permanent* global shocks is key to obtaining a cross-section of foreign term premia expressed in U.S. dollars.

Example 4. *The LRV (2011) model is defined by the following set of equations:*

$$\begin{aligned}
-\log M_{t+1} &= \alpha+\chi z_t+\sqrt{\gamma z_t}u_{t+1}+\tau z_t^{\mathbb{P}}+\sqrt{\delta z_t^{\mathbb{P}}}u_{t+1}^{\mathbb{P}},\\
z_{t+1} &= (1-\phi)\theta+\phi z_t-\sigma\sqrt{z_t}u_{t+1},\\
z_{t+1}^{\mathbb{P}} &= (1-\phi^{\mathbb{P}})\theta^{\mathbb{P}}+\phi^{\mathbb{P}}z_t^{\mathbb{P}}-\sigma^{\mathbb{P}}\sqrt{z_t^{\mathbb{P}}}u_{t+1}^{\mathbb{P}},
\end{aligned}$$

where z_t is the transitory factor, and $z_t^{\mathbb{P}}$ is the permanent factor.

Note that the model abstracts from the country-specific shocks and state variables; they can be added easily as in Lustig, Roussanov, and Verdelhan (2011).

The nominal log zero-coupon n-month yield of maturity in local currency is given by the standard affine expression $y_t^{(n)}=-\frac{1}{n}\left(A_n+B_n z_t+C_n z_t^{\mathbb{P}}\right)$, where the coefficients satisfy second-order difference equations. The nominal log risk-free interest rate is an affine function of the persistent and transitory factors: $r_t^f=\alpha+\left(\chi-\frac{1}{2}\gamma\right)z_t+\left(\tau-\frac{1}{2}\delta\right)z_t^p$. In this model, the expected

[7] The model parameters must satisfy the two following constraints: interest rates must be pro-cyclical with respect to the state variables, and high interest rate countries must load less on the global shocks than low interest rate countries.

log excess return on an infinite maturity bond is:

$$E_t[rx_{t+1}^{(\infty)}] = \left[B_\infty(1-\phi) - \chi + \frac{1}{2}\gamma\right] z_t - \left[C_\infty(1-\phi^p) + \tau - \frac{1}{2}\delta\right] z_t^{\mathbb{P}}.$$

To give content to the notion that z_t is transitory, we impose that $B_\infty(1-\phi) = \chi$. This restriction implies that the permanent component of the pricing kernel is not affected by the transitory factor z_t, as can easily be verified. In this case, the permanent component of the SDF reduces to:

$$\frac{M_{t+1}^{\mathbb{P}}}{M_t^{\mathbb{P}}} = \frac{M_{t+1}}{M_t}\left(\frac{M_{t+1}^{\mathbb{T}}}{M_t^{\mathbb{T}}}\right)^{-1} = \beta^{-1}e^{-\alpha-\chi\theta}e^{-C_\infty\left[(\phi^{\mathbb{P}}-1)(z_t^{\mathbb{P}}-\theta^{\mathbb{P}})-\sigma^{\mathbb{P}}\sqrt{z_t^{\mathbb{P}}}u_{t+1}\right]},$$

which does not depend on z_t. Given this restriction, the bond risk premium is equal to:

$$E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\gamma z_t - \left[\tau - \frac{1}{2}\delta + C_\infty(1-\phi^{\mathbb{P}})\right] z_t^{\mathbb{P}}.$$

Both factors are common across countries, but, following Lustig, Roussanov, and Verdelhan (2011), we allow for heterogeneous loadings on these common factors. The foreign SDF is therefore defined as:

$$-\log M_{t+1}^* = \alpha + \chi z_t + \sqrt{\gamma^* z_t}u_{t+1} + \tau z_t^{\mathbb{P}} + \sqrt{\delta^* z_t^{\mathbb{P}}}u_{t+1}^{\mathbb{P}}.$$

The log currency risk premium is equal to: $E_t[rx_{t+1}^{FX}] = (\gamma-\gamma^*)z_t/2 + (\delta-\delta^*)z_t^{\mathbb{P}}/2$. This implies that the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to:[8]

$$E_t[rx_{t+1}^{(\infty),\$}] = E_t[rx_{t+1}^{(\infty),*}] + E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma z_t - \left[\tau - \frac{1}{2}\delta + C_{\infty,*}(1-\phi^{\mathbb{P}})\right] z_t^{\mathbb{P}}.$$

Hence, the difference between the foreign and the domestic term premium is driven by: $(C_{\infty,*} - C_\infty)(1-\phi^{\mathbb{P}})z_t^{\mathbb{P}}$. In the symmetric case in which $\delta = \delta^*$, then $C_{\infty,*} = C_\infty$, and the foreign term premium

[8]The coefficient $C_{\infty,*}$ is defined by the following second-order equation: $C_{\infty,*} = -(\tau - \delta^*/2) + C_{\infty,*}\left(\phi^p + \sigma^p\sqrt{\delta^*}\right) + \left(C_{\infty,*}\sigma^p\right)^2/2$. Therefore, if $\delta = \delta^*$, then $C_{\infty,*} = C_\infty$.

in dollars equals the domestic term premium. In this case, a cross-section of currency risk premia exists, but term premia in dollars are all the same across countries. If $\gamma > \gamma^*$, there is a large positive foreign currency risk premium (equal here to $E_t[rx_{t+1}^{FX}] = (\gamma - \gamma^*)z_t/2)$, but that is exactly offset by a smaller foreign term premium. This model thus illustrates our main theoretical findings: chasing high currency risk premia does not necessarily imply high term premia. If there is no heterogeneity in the loadings of the permanent global component of the SDF, then the foreign term premium, once converted to U.S. dollars is the same as the U.S. term premium.

Since carry trade returns are base-currency-invariant, heterogeneity in the exposure of the pricing kernel to a *global* component of the pricing kernel is required to explain the carry trade premium (Lustig, Roussanov, and Verdelhan, 2011). Clearly, a carry trade premium at longer maturities exists only with heterogeneous exposure to a permanent global component.

3 The Cross-Section of Long-Term Bond Returns

The empirical experiment is guided by the main theoretical results presented in Section 1. For the reader's convenience, we summarize them here in the following three equations:

$$
\begin{aligned}
E_t\left[rx_{t+1}^{FX}\right] &= (f_t - s_t) - E_t(\Delta s_{t+1}) = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right) && (1)\\
E_t\left[rx_{t+1}^{(\infty),*}\right] &= \lim_{k\to\infty} E_t\left[rx_{t+1}^{(k),*}\right] = L_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right) - L_t\left(\frac{\Lambda_{t+1}^{*,\mathbb{P}}}{\Lambda_t^{*,\mathbb{P}}}\right) && (2)\\
E_t\left[rx_{t+1}^{(\infty),*}\right] + E_t\left[rx_{t+1}^{FX}\right] &= E_t\left[rx_{t+1}^{(\infty)}\right] + L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right) - L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}}\right). && (3)
\end{aligned}
$$

Equation (1) shows that the currency risk premium is equal to the difference between the entropy of the domestic and foreign SDFs (Backus, Foresi, and Telmer, 2001). Equation (2) shows that the term premium is equal to the difference between the total entropy of the SDF and the entropy of its permanent component (Alvarez and Jermann, 2005). Equation (3) shows that the foreign term premium in dollars is equal to the domestic term premium plus the difference in the entropy of the permanent component of the pricing kernel of the domestic and the foreign country. Our empirical work thus focuses on three average excess returns: the currency risk premium, the term premium in foreign currency, and the term premium in U.S. dollars. To test the predictions of the theory, we sort currencies into portfolios based on variables that can be used to predict bond and currency returns: the slope of the yield curve and then the level of short-term interest rates. The slope of the yield curve is a natural measure of the local term premium, governed by the entropy of the transitory shocks to the pricing kernel. Returns are computed over horizons of one, three, and twelve months. In all cases, portfolios formed at date t only use information available at that date. Portfolios are rebalanced monthly.

In closely related work on the cross-section, Koijen, Moskowitz, Pedersen, and Vrugt (2012) and Wu (2012) examine the currency-hedged returns on 'carry' portfolios of international bonds, sorted by a proxy for the carry on long-term bonds, but they do not examine the interaction between currency and term risk premia, the topic of our paper. We focus on portfolios sorted by interest rates, as well as yield spreads, since it is well-know since Campbell and Shiller (1991) that yield spreads can predict excess returns on bonds. Ang and Chen (2010) and Berge, Jordà, and Taylor (2011) have shown that yield curve variables can also be used to forecast currency excess returns. These authors do not examine the returns on foreign bond portfolios. On the other hand, Dahlquist and Hasseltoft (2013) study international bond risk premia in an affine asset pricing model and find evidence for local and global risk factors. Jotikasthira, Le, and Lundblad (2012) report similar findings. Our paper revisits the empirical evidence on bond returns without committing to a specific term structure model.

3.1 Samples

The benchmark sample consists of a small homogeneous panel of developed countries with reasonably liquid bond markets. This G-10 panel includes Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The domestic country is the United States. It only includes one country from the eurozone, Germany. For robustness checks, we consider two additional sets of countries: first, a larger sample of 20 developed countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the U.K.), and second, a large sample of 30 developed and emerging countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, India, Ireland, Italy, Japan, Mexico, Malaysia, the Netherlands, New Zealand, Norway, Pakistan, the Philippines, Poland, Portugal, South Africa, Singapore, Spain, Sweden, Switzerland, Taiwan, Thailand, and the U.K.).

In order to build the longest time-series possible, we obtain data from Global Financial Data. The dataset includes a 10-year government bond total return index for each of our target countries in dollars and in local currency and a Treasury bill total return index. The 10-year bond returns are a proxy for the bonds with the longest maturity. The log excess returns on currency (rx^{FX}) and the log returns on the bond portfolio in local currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$) are first obtained at the country level. Then, the portfolio-level excess returns are obtained by averaging these log excess returns across all countries in a portfolio. The benchmark sample is summarized by three portfolios while the other two samples are summarized by four and five portfolios. For each set of countries, we report averages over the 12/1950–12/2012 and 12/1971–12/2012 periods. The main text focuses on the benchmark sample, while the Online Appendix reports detailed results for the robustness checks.

While Global Financial Data offers, to the best of our knowledge, the longest time-series of government bond returns available, the series have three key limits. First, they pertain to discount bonds, while the theory pertains to zero-coupon bonds. Second, they include default risk, while the theory focuses on default-free bonds. Third, they only offer 10-year bond returns, not the entire term structure of bond returns. To address these issues, we use zero-coupon bonds obtained from the estimation of term structure curves using government notes and bonds and interest rate swaps of different maturities; the time-series are shorter and dependent on the term structure estimations. In contrast, bond return indices, while spanning much longer time-periods, offer model-free estimates of bond returns. Our results turn out to be similar in both samples.

Our zero-coupon bond dataset consists of a panel of the benchmark sample of countries from

12/1971 to 12/2012. To construct our sample, we use the entirety of the dataset in Wright (2011) and complement the sample, as needed, with sovereign zero-coupon curve data sourced from Bloomberg. The panel is unbalanced: for each currency, the sample starts with the beginning of the Wright (2011) dataset.[9] Yields are available at maturities from 3 months to 15 years, in 3-month increments. We also construct an extended version of this dataset which, in addition to the countries of the benchmark sample, includes the following countries: Austria, Belgium, the Czech Republic, Denmark, Finland, France, Hungary, Indonesia, Ireland, Italy, Malaysia, Mexico, the Netherlands, Poland, Portugal, Singapore, South Africa, and Spain. The data for the aforementioned extra countries are sourced from Bloomberg.[10]

3.2 Sorting Currencies by the Slope of the Yield Curve

Let us start with portfolios of countries sorted by the slope of their yield curve. Recall that the slope of the yield curve, a measure of the term premium, is largely determined by the entropy of the temporary component of the pricing kernel. As this entropy increases, the local term premium increases as well. However, the dollar term premium only compensates investors for the relative entropy of the permanent component of the U.S. and the foreign pricing kernel, because the interest rate risk associated with the temporary innovations is hedged by the currency risk. In the extreme case in which all permanent shocks are common, the dollar term premium should equal the U.S. term premium.

Benchmark Sample In the data, there is substantial turnover in the portfolios, more so than in the usual interest rate-sorted portfolios, but the typical currencies in Portfolio 1 (flat yield curve currencies) are the Australian and New Zealand dollar and the British pound, whereas the typical currencies in Portfolio 3 (steep yield curve currencies) are the Japanese yen and the

[9] The starting dates for each country are as follows: 2/1987 for Australia, 1/1986 for Canada, 1/1973 for Germany, 1/1985 for Japan, 1/1990 for New Zealand, 1/1998 for Norway, 12/1992 for Sweden, 1/1988 for Switzerland, 1/1979 for the U.K., and 12/1971 for the U.S. For New Zealand, the data for maturities above 10 years start in 12/1994.

[10] The starting dates for the additional countries are as follows: 12/1994 for Austria, Belgium, Denmark, Finland, France, Ireland, Italy, the Netherlands, Portugal, Singapore, and Spain, 12/2000 for the Czech Republic, 3/2001 for Hungary, 5/2003 for Indonesia, 9/2001 for Malaysia, 8/2003 for Mexico, 12/2000 for Poland, and 1/1995 for South Africa.

German mark. Table 1 reports the annualized moments of log returns on the three slope-sorted portfolios.

We start by discussing the results obtained at the one-month horizon over the whole sample (12/1950–12/2012). Clearly, the uncovered interest rate parity condition fails in the cross-section. For example, in Portfolio 1 the foreign interest rate exceeds the U.S. interest rate by 3.03%, but the USD appreciates only by 0.01%. Average currency excess returns decline from 3.02% per annum on Portfolio 1 to 0.06% per annum on the Portfolio 3. Therefore, a long-short position of investing in steep-yield-curve currencies and shorting flat-yield-curve currencies delivers an excess return of -2.97% per annum and a Sharpe ratio of -0.47. Our findings basically confirm those of Ang and Chen (2010). The slope of the yield curve predicts currency excess returns very well. The entropy of the temporary component plays a large role in determining currency risk premia. Turning to the returns on local bonds, as expected, Portfolio 3 produces large bond excess returns of 4% per annum, compared to -1.82% per annum on Portfolio 1. Hence, a long-short position produces a spread of 5.82% per annum.

A natural question is whether U.S. investors can "combine" the bond risk premium with the currency risk premium. To answer this question, we compute the dollar bond excess returns $rx^{(10),\$}$ by adding the currency excess returns rx^{FX} and the local currency bond returns $rx^{(10),*}$. In dollars, the aforementioned 5.82% spread is reduced to 2.85%, because of the partly offsetting pattern in currency risk premia. What is driving these results? The low slope currencies tend to be high interest rate currencies, while the high slope currencies tend to be low interest rate currencies: Portfolio 1 has an interest rate difference of 3.03% relative to the U.S., while Portfolio 3 has a negative interest rate difference of -0.77% per annum. Thus, the flat slope currencies are the investment currencies in the carry trade, whereas the steep slope currencies are the funding currencies.

For long maturities, global bond investors want to reverse the standard currency carry trade. They can achieve a return of 2.85% per annum by investing in the (low interest rate, steep curve) funding currencies and shorting the (high interest rate, flat slope) carry trade currencies. This difference is statistically significant. Importantly, this strategy involves long positions in bonds

Table 1: Slope-Sorted Portfolios: Benchmark Sample

Portfolio		1	2	3	$3-1$	1	2	3	$3-1$	1	2	3	$3-1$
Horizon		1-Month				3-Month				12-Month			
						Panel A: 12/1950–12/2012							
$-\Delta s$	Mean	-0.01	0.77	0.83	0.84	0.01	0.39	1.18	1.17	-0.09	0.55	1.09	1.18
$f-s$	Mean	3.03	0.41	-0.77	-3.81	2.96	0.42	-0.71	-3.68	2.76	0.46	-0.55	-3.31
rx^{FX}	Mean	3.02	1.18	0.06	-2.97	2.97	0.81	0.47	-2.50	2.67	1.01	0.54	-2.13
	s.e.	[0.97]	[0.94]	[0.94]	[0.81]	[1.08]	[1.03]	[0.95]	[0.87]	[1.14]	[1.07]	[1.13]	[0.86]
	Std	7.59	7.37	7.36	6.30	8.25	7.75	7.60	6.84	9.05	8.31	8.39	6.65
	SR	0.40	0.16	0.01	-0.47	0.36	0.10	0.06	-0.37	0.30	0.12	0.06	-0.32
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.13]	[0.15]	[0.14]	[0.13]	[0.13]	[0.14]
$rx^{(10),*}$	Mean	-1.82	1.61	4.00	5.82	-0.86	1.33	3.33	4.19	-0.22	1.20	2.79	3.01
	s.e.	[0.50]	[0.46]	[0.51]	[0.54]	[0.58]	[0.51]	[0.58]	[0.60]	[0.62]	[0.69]	[0.65]	[0.58]
	Std	3.97	3.67	4.09	4.29	4.60	4.24	4.65	4.67	5.10	4.88	5.29	4.90
	SR	-0.46	0.44	0.98	1.35	-0.19	0.31	0.72	0.90	-0.04	0.25	0.53	0.61
	s.e.	[0.12]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.11]
$rx^{(10),\$}$	Mean	1.21	2.79	4.06	2.85	2.12	2.14	3.80	1.68	2.45	2.21	3.33	0.88
	s.e.	[1.09]	[1.07]	[1.12]	[0.99]	[1.19]	[1.17]	[1.18]	[1.08]	[1.28]	[1.18]	[1.31]	[1.12]
	Std	8.61	8.36	8.84	7.76	9.34	8.98	9.42	8.14	10.45	9.57	10.29	8.67
	SR	0.14	0.33	0.46	0.37	0.23	0.24	0.40	0.21	0.23	0.23	0.32	0.10
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.14]	[0.13]	[0.13]	[0.12]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-0.30	1.28	2.55	2.85	0.60	0.62	2.28	1.68	0.91	0.66	1.79	0.88
	s.e.	[1.28]	[1.14]	[1.21]	[0.99]	[1.42]	[1.29]	[1.14]	[1.08]	[1.51]	[1.25]	[1.37]	[1.12]
						Panel B: 12/1971–12/2012							
$-\Delta s$	Mean	-0.08	1.21	1.03	1.11	-0.08	0.61	1.51	1.60	-0.30	0.76	1.47	1.77
$f-s$	Mean	3.40	0.54	-1.02	-4.42	3.31	0.54	-0.94	-4.25	3.08	0.57	-0.72	-3.80
rx^{FX}	Mean	3.32	1.75	0.01	-3.32	3.23	1.15	0.58	-2.65	2.78	1.33	0.75	-2.03
	s.e.	[1.45]	[1.41]	[1.37]	[1.16]	[1.65]	[1.55]	[1.44]	[1.26]	[1.71]	[1.62]	[1.66]	[1.22]
	Std	9.29	8.95	8.80	7.40	10.09	9.38	9.16	8.15	11.04	10.05	10.12	7.95
	SR	0.36	0.20	0.00	-0.45	0.32	0.12	0.06	-0.32	0.25	0.13	0.07	-0.26
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.18]	[0.17]	[0.16]	[0.16]	[0.17]
$rx^{(10),*}$	Mean	-1.68	1.94	4.56	6.24	-0.52	1.70	3.64	4.16	0.10	1.71	2.98	2.87
	s.e.	[0.74]	[0.69]	[0.72]	[0.76]	[0.85]	[0.75]	[0.81]	[0.85]	[0.88]	[1.02]	[0.91]	[0.83]
	Std. Dev.	4.67	4.37	4.63	4.84	5.45	5.03	5.20	5.26	5.95	5.73	5.86	5.54
	SR	-0.36	0.44	0.98	1.29	-0.10	0.34	0.70	0.79	0.02	0.30	0.51	0.52
	s.e.	[0.15]	[0.15]	[0.17]	[0.15]	[0.16]	[0.16]	[0.17]	[0.15]	[0.16]	[0.17]	[0.20]	[0.12]
$rx^{(10),\$}$	Mean	1.64	3.69	4.56	2.93	2.70	2.85	4.22	1.51	2.88	3.04	3.73	0.84
	s.e.	[1.63]	[1.59]	[1.62]	[1.41]	[1.81]	[1.74]	[1.73]	[1.56]	[1.89]	[1.77]	[1.91]	[1.63]
	Std	10.45	10.13	10.46	8.97	11.30	10.79	11.15	9.59	12.54	11.33	12.15	10.23
	SR	0.16	0.36	0.44	0.33	0.24	0.26	0.38	0.16	0.23	0.27	0.31	0.08
	s.e.	[0.16]	[0.16]	[0.16]	[0.15]	[0.16]	[0.16]	[0.16]	[0.15]	[0.17]	[0.17]	[0.17]	[0.15]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-0.87	1.18	2.06	2.93	0.17	0.32	1.69	1.51	0.32	0.47	1.16	0.84
	s.e.	[1.85]	[1.63]	[1.69]	[1.41]	[2.09]	[1.88]	[1.62]	[1.56]	[2.23]	[1.87]	[1.98]	[1.63]

Notes: The table reports the average change in exchange rates (Δs), the average interest rate difference ($f-s$), the average log currency excess return (rx^{FX}), the average log foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$), as well as the difference between the average foreign bond log excess return in U.S. dollars and the average U.S. bond log excess return ($rx^{(10),\$}-rx^{US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The annualized monthly log returns are realized at date $t+k$, where the horizon k equals 1, 3, and 12 months. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the one-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns.

issued by countries like Germany and Japan. These are countries with fairly liquid bond markets and low sovereign credit risk. As a result, credit and liquidity risk differences are unlikely candidate explanations for the return differences. Of course, at the one-month horizon, this strategy involves frequent trading. At the 12-month horizon, these excess returns are essentially gone. The local term premium almost fully offsets the carry trade premium.

Our findings confirm that currency risk premia are driven to a large extent by temporary shocks to the pricing kernel. When we sort currencies by the yield curve slope, an approximate measure of the entropy of the temporary SDF component, we find large differences in currency risk premia: the largest currency risk premia correspond to the lowest bond risk premia. This finding is consistent with the theory, as total SDF entropy is negatively related to currency risk premia, but positively related to bond risk premia. However, those differences in temporary pricing kernel risk do not appear to produce significant cross-sectional differences in the quantity of permanent risk: carry trade premia at the 10-year maturity, which are associated with differences in the entropy of the permanent SDF component, are modest. Notably, the behavior of long-maturity dollar bond returns suggests that local investors in carry trade countries are less exposed to temporary risk than those in funding currencies, but more exposed to permanent risk.

We get similar findings when we restrict our analysis to the post-Bretton Woods sample. More generally, as can be verified from Figure 3, the difference in slope-sorted bond returns is rather stable over time, although the difference in local bond premia is smaller in the last part of the sample. For example, between 1991 and 2012, the difference in currency risk premia at the one-month horizon between Portfolio 3 and Portfolio 1 is -4.20% per annum, compared to a 3.29% spread in local term premia. This adds up to a -0.91% return on a long position in the steep-sloped bonds and a short position in the flat-sloped bonds. However, this difference is not statistically significant, as the standard error is 1.52% per annum.

Robustness Checks: Developed Countries and Whole Sample In the sample of developed countries, the steep-slope (low yielding) currencies are typically countries like Germany, the Netherlands, Japan, and Switzerland, while the flat-slope (high-yielding) currencies are typically

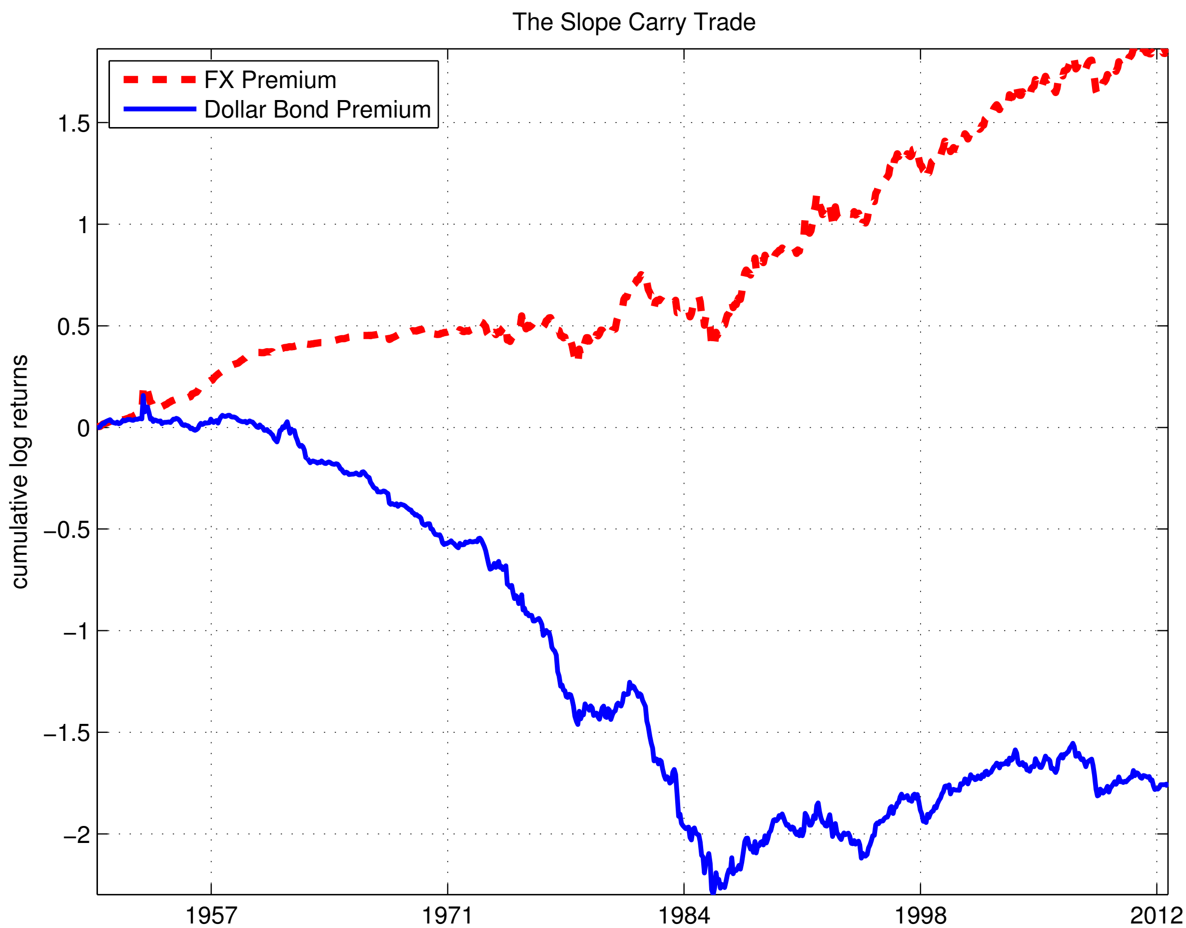

Figure 3: The Carry Trade and Term Premia: Conditional on the Slope of the Yield Curve – The figure presents the cumulative one-month log returns on investments in foreign Treasury bills and foreign 10-year bonds. The benchmark panel of countries includes Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. Countries are sorted every month by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the spread between the 10-year bond yield and the one-month interest rate. The returns correspond to an investment strategy going long in Portfolio 3 and short in the Portfolio 1. The sample period is 12/1950–12/2012.

Australia, New Zealand, Denmark and the U.K. At the one-month horizon, the 2.42% spread in currency excess returns obtained in this sample is more than offset by the 5.90% spread in local term premia. This produces a statistically significant 3.48% return on a position that is long in the low yielding, high slope currencies and short in the high yielding, low slope currencies. These results are essentially unchanged in the post-Bretton-Woods sample. At longer horizons, the currency excess returns and the local risk premia almost fully offset each other.

In the entire sample of countries, including the emerging market countries, the difference in currency risk premia at the one-month horizon is 6.11% per annum, which is more than offset by a 11.45% difference in local term premia. As a result, investors earn 5.33% per annum on a long-short position in foreign bond portfolios of slope-sorted currencies. As before, this involves shorting the flat-yield-curve currencies, typically high interest rate currencies, and going long in the steep-slope currencies, typically the low interest rate ones. The annualized Sharpe ratio on this long-short strategy is 0.60.

3.3 Sorting Currencies by Interest Rates

Let us turn now to portfolios of countries sorted by their short-term interest rates. To save space and because the results are very similar to those obtained on slope-sorted portfolios, we summarize our findings rapidly here; detailed results are presented in the Online Appendix.

In our benchmark sample, as in our large samples, average currency excess returns increase from low- to high-interest-rate portfolios. But local currency bond risk premia decrease from low- to high-interest-rate portfolios. The decline in the local currency bond risk premia partly offsets the increase in currency risk premia. As a result, the average excess return on the foreign bond expressed in U.S. dollars measured in the high-interest-rate portfolio is only slightly higher than the average excess returns measured in the low-interest-rate portfolio. As the holding period increases from 1 to 3 and 12 months, the differences in local bond risk premia between portfolios shrink, but so do the differences in currency risk premia. Even at the 12-month horizon, there is no evidence of statistically significant differences in dollar bond risk premia across the currency portfolios.

Overall, our findings suggest that investors in high interest rate countries are less exposed to overall pricing kernel risk than those in low interest countries, but these differences are mostly about temporary pricing kernel risk, not persistent pricing kernel risk.

3.4 The Term Structure of Currency Carry Trade Risk Premia

The previous results focus on the 10-year maturity and show that currency risk premia offset local currency term premia for coupon bond returns. We now turn to a full set of returns in the maturity spectrum, using the zero-coupon bond dataset. Table 2 reports summary statistics on one-quarter holding period returns on zero-coupon bond positions with maturities from 4 (1 year) to 60 quarters (15 years).

The term structure of currency carry trade risk premia is downward sloping: currency carry trade strategies that yield positive risk premia for short-maturity bonds yield lower (or even negative) risk premia for long-maturity bonds. This is due to the offsetting relationship between currency premia and term premia. As we move from the 4-quarter maturity to the 60-quarter maturity, the difference in the dollar term premium between Portfolio 1 (flat yield curve) currencies and Portfolio 3 (steep yield curve) currencies decreases from 2.69% to −1.77%. While investing in flat yield curve currencies and shorting steep yield curve currencies provides significant gains in the short end of the term structure, it yields negative returns in the long end.

Figure 4 shows the local currency excess returns (in logs) in the top panel, and the dollar excess returns (in logs) in the bottom panel. The top panel in Figure 4 shows that countries with the steepest local yield curves (Portfolio 3, center) exhibit local bond excess returns that are higher, and increase faster with the maturity than the flat yield curve countries (Portfolio 1, on the left-hand side). This effect is strong enough to undo the effect of the level differences in yields at the short end: the steep-slope currencies are typical funding currencies with low yields in levels at the short end of the maturity curve while the flat-slope currencies typically have high yields at the short end. At the 4-quarter maturity, Table 2 reports a −0.31% interest rate difference with the U.S. in Portfolio 3, compared to a 3.15% interest rate difference with the U.S. in Portfolio 1. Thus, ignoring the effect of exchange rates, investors should invest in

Table 2: The Maturity Structure of Returns in Slope-Sorted Portfolios

Maturity		4-Quarters				20-Quarters				40-Quarters				60-Quarters			
Portfolio		1	2	3	$3-1$	1	2	3	$3-1$	1	2	3	$3-1$	1	2	3	$3-1$
$-\Delta s$	Mean	2.80	2.51	2.96	0.16	2.80	2.51	2.96	0.16	2.80	2.51	2.96	0.16	2.75	2.55	2.87	0.13
$f-s$	Mean	3.15	0.79	-0.31	-3.46	3.15	0.79	-0.31	-3.46	3.15	0.79	-0.31	-3.46	3.12	0.73	-0.36	-3.48
rx^{FX}	Mean	5.95	3.30	2.64	-3.31	5.95	3.30	2.64	-3.31	5.95	3.30	2.64	-3.31	5.87	3.29	2.52	-3.35
	s.e.	[1.19]	[1.06]	[0.99]	[0.97]	[1.19]	[1.05]	[0.99]	[0.96]	[1.18]	[1.04]	[0.98]	[0.97]	[1.18]	[1.06]	[0.99]	[0.97]
	Std	10.98	9.61	9.00	8.88	10.98	9.61	9.00	8.88	10.98	9.61	9.00	8.88	11.00	9.68	9.09	8.94
	SR	0.54	0.34	0.29	-0.37	0.54	0.34	0.29	-0.37	0.54	0.34	0.29	-0.37	0.53	0.34	0.28	-0.38
	s.e.	[0.12]	[0.12]	[0.11]	[0.12]	[0.12]	[0.12]	[0.11]	[0.12]	[0.12]	[0.12]	[0.11]	[0.12]	[0.12]	[0.12]	[0.11]	[0.12]
$rx^{(k),*}$	Mean	-0.16	0.36	0.45	0.61	1.39	2.62	3.31	1.92	2.27	4.34	5.69	3.42	2.65	5.52	7.78	5.13
	s.e.	[0.11]	[0.09]	[0.08]	[0.11]	[0.61]	[0.51]	[0.51]	[0.50]	[1.06]	[0.88]	[0.90]	[0.79]	[1.43]	[1.22]	[1.28]	[1.13]
	Std	1.01	0.80	0.73	0.97	5.43	4.54	4.67	4.57	9.61	7.96	8.23	7.19	12.95	11.06	11.67	10.39
	SR	-0.16	0.45	0.62	0.63	0.26	0.58	0.71	0.42	0.24	0.55	0.69	0.48	0.20	0.50	0.67	0.49
	s.e.	[0.11]	[0.11]	[0.12]	[0.12]	[0.11]	[0.12]	[0.12]	[0.11]	[0.11]	[0.12]	[0.12]	[0.11]	[0.11]	[0.12]	[0.12]	[0.11]
$rx^{(k),\$}$	Mean	5.79	3.66	3.10	-2.69	7.34	5.92	5.95	-1.39	8.22	7.64	8.34	0.12	8.52	8.81	10.30	1.77
	s.e.	[1.19]	[1.05]	[1.01]	[0.96]	[1.35]	[1.10]	[1.18]	[1.02]	[1.60]	[1.26]	[1.39]	[1.21]	[1.84]	[1.50]	[1.63]	[1.42]
	Std	11.00	9.53	9.19	8.82	12.33	9.98	10.70	9.44	14.72	11.56	12.68	10.99	16.87	13.64	15.00	13.03
	SR	0.53	0.38	0.34	-0.31	0.60	0.59	0.56	-0.15	0.56	0.66	0.66	0.01	0.51	0.65	0.69	0.14
	s.e.	[0.12]	[0.12]	[0.11]	[0.12]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.11]	[0.12]	[0.11]
$rx^{(k),\$}-rx^{(k),US}$	Mean	8.77	6.64	6.08	-2.69	7.22	5.81	5.84	-1.39	5.71	5.13	5.83	0.12	4.43	4.72	6.20	1.77
	s.e.	[1.19]	[1.06]	[0.98]	[0.96]	[1.29]	[1.08]	[1.00]	[1.02]	[1.47]	[1.23]	[1.14]	[1.21]	[1.73]	[1.49]	[1.37]	[1.42]

Notes: The table reports summary statistics on annualized log returns realized on zero coupon bonds with maturity varying from $k=4$ to $k=60$ quarters. The holding period is one quarter. The table reports the average change in exchange rates ($-\Delta s$), the average interest rate difference ($f-s$), the average currency excess return (rx^{FX}), the average foreign bond excess return in foreign currency ($rx^{(k),*}$) and in U.S. dollars ($rx^{(k),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(k),\$}-rx^{(k),US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns. Data are monthly, from the zero-coupon dataset, and the sample window is 4/1985–12/2012.

the short-term and long-term bonds of steep yield curve currencies.

However, considering the effect of currency fluctuations by focusing on dollar returns radically alters the results. Figure 4 shows that the dollar excess returns of Portfolio 1 are higher than those of Portfolio 3 at the short end of the yield curve, consistent with the carry trade results of Ang and Chen (2010). Yet, an investor that would attempt to replicate the short-maturity carry trade strategy at the long end of the maturity curve would incur losses on average: the long-maturity excess returns of flat yield curve currencies are lower than those of steep yield curve currencies, as currency risk premia more than offset term premia. This result is apparent in the lower panel on the right, which is the same as Figure 1 in the introduction. The term structure of currency carry trade risk premia slopes downwards.

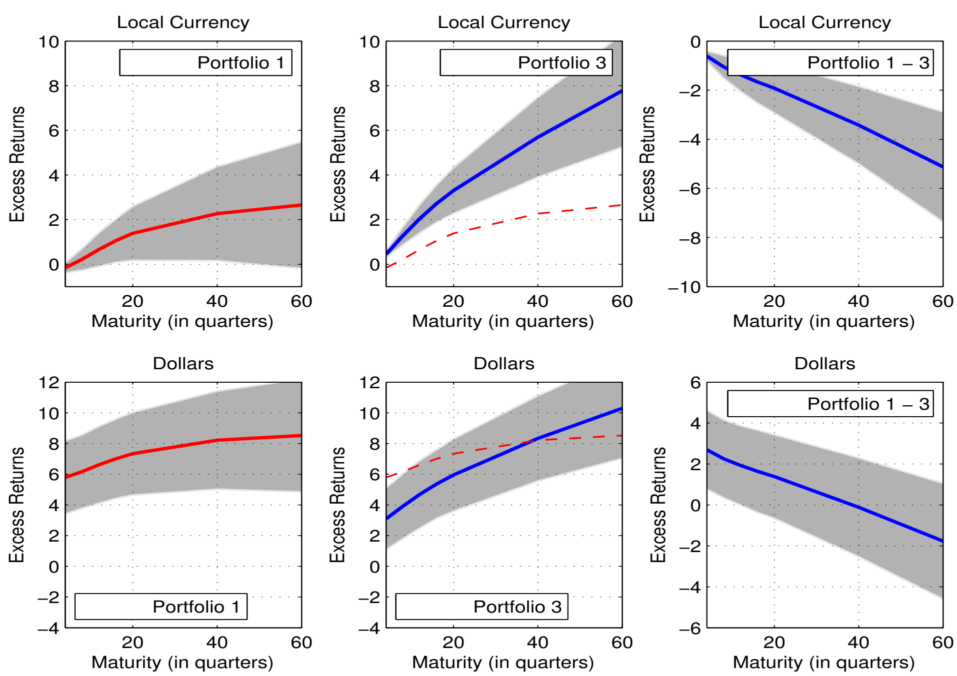

Figure 4: Term Structure of Dollar Bond Risk Premia — The figure shows the local currency log excess returns in the top panel, and the dollar log excess returns in the bottom panel as a function of the bond maturities. The left panel focuses on Portfolio 1 (flat yield curve currencies) excess returns, while the middle panel reports Portfolio 3 (steep yield curve currencies) excess returns. The middle panels also report the Portfolio 1 excess returns in dashed lines for comparison. The right panel reports the difference. Data are monthly, from the zero-coupon dataset, and the sample window is 4/1985–12/2012. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into three portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The holding period is one quarter. The returns are annualized. The shaded areas correspond to two standard deviations above and below each point estimates. Standard deviations are obtained by bootstrapping.

Finally, we check the robustness of our findings by repeating our analysis using the extended zero-coupon dataset. The detailed results are reported in the separate Appendix. It can be easily seen that considering a large cross-section of countries does not change our main result. An investor who buys the one-year bonds of flat-yield curve currencies and shorts the one-year bonds of steep-yield-curve currencies realizes a dollar excess return of 4.10% per year on average. However, at the long end of the maturity structure this strategy generates negative excess returns: the average annualized dollar excess return of an investor who pursues this strategy using 15-year bonds is -0.42%. The term structure of currency carry trade risk premia remains downward-sloping. We turn now to time-series tests of the uncovered bond return parity.

4 Time-Series Tests of the Uncovered Bond Return Parity

We first examine the correlation and volatility of foreign bond returns and then test the uncovered bond return parity condition.

4.1 The Correlation and Volatility of Dollar Bond Returns

If international risk sharing is mostly due to countries sharing their permanent pricing kernel fluctuations, holding period returns on zero-coupon bonds, once converted to a common currency (the U.S. dollar, in particular), should become increasingly similar as bond maturities approach infinity. To determine whether this hypothesis has merit, Figure 5 reports the correlation coefficient between three-month returns on foreign zero-coupon bonds (either in local currency or in U.S. dollars) and corresponding returns on U.S. bonds for bonds of maturity ranging from 1 year to 15 years. All foreign currency yield curves exhibit the same pattern: correlation coefficients for U.S. dollar returns start from very low (often negative) values and increase monotonically with bond maturity, tending towards one for long-term bonds. The clear monotonicity is not observed on local currency returns. The local currency three-month return correlations do not exhibit any discernible pattern with maturity, implying that the convergence of U.S. dollar return correlations towards the value of one results from exchange rate changes that partially offset differences in local currency bond returns. Similar results hold true for volatility ratios (instead of correlations); we report those in the Online Appendix.

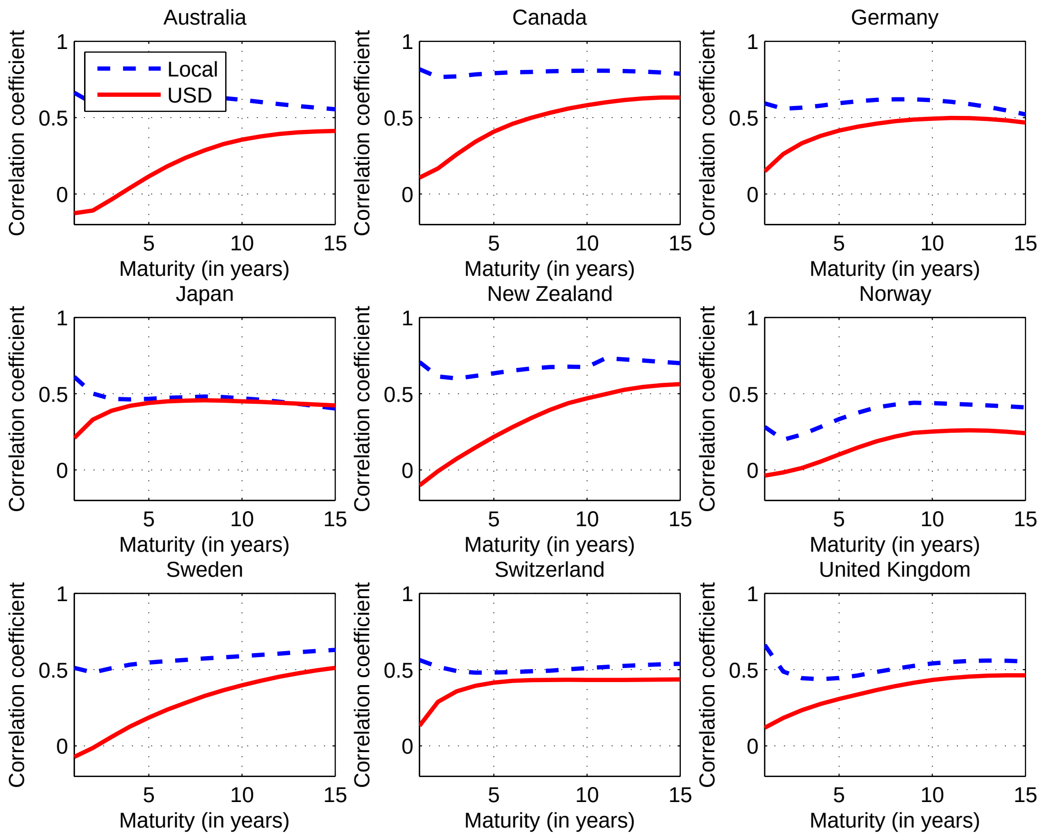

Figure 5: The Maturity Structure of Bond Return Correlations — The figure presents the correlation of foreign bond returns with U.S. bond returns. The time-window is country-dependent. Data are monthly. The holding period is three-months.

In sum, the behavior of U.S. dollar bond returns and local currency bond returns differs markedly as bond maturity changes. While U.S. dollar bond returns become more correlated and roughly equally volatile across countries as the maturity increases, the behavior of local currency returns does not appear to change when bond maturity changes.

4.2 Testing Uncovered Bond Return Parity in the Time-Series

We now turn to tests of the bond return parity condition in the time series. To the extent the 10-year bond is a reasonable proxy for the infinite-maturity bond, uncovered long-bond parity implies that the domestic and foreign 10-year bond returns are not statistically different across countries, once converted into a common currency. To determine whether exchange rate changes completely eliminate differences in countries' permanent SDF components, nominal U.S. dollar holding period returns on 10-year foreign bonds are thus regressed on the corresponding U.S. dollar returns on 10-year U.S. bonds:

$$r_{t+1}^{(10),\$} = \alpha + \beta r_{t+1}^{(10)} + \epsilon_{t+1},$$

where small letters denote the log of their capital letter counterpart. Uncovered long bond parity implies $\alpha = 0$ and $\beta = 1$. Table 3 reports the regression results, as well as those obtained with each component of the foreign bonds' dollar return, i.e., the local currency bond return $r_{t+1}^{(10),*}$ and the change in the log exchange rate. The sum of the local currency bond return beta and the exchange change beta equals the total dollar bond return beta. Section I of Table 3 uses discount bonds, while Section II uses zero-coupon bonds.

Individual Countries Panel A of Table 3 reports the results for the benchmark sample of discount bonds. The slope coefficient for dollar returns is positive and, with the exception of New Zealand, statistically significant for all the countries in the benchmark sample. The slope coefficient ranges from 0.08 (New Zealand) to 0.69 (Canada); on average, it is 0.38. The cross-sectional average of the exchange rate coefficient is 0.11, so it accounts for almost one-third of the overall effect. Hence, exchange rates actively enforce long-run uncovered bond return parity: when U.S. bond returns are high, the dollar tends to depreciate relative to other currencies, whereas when dollar returns are low, the U.S. dollar tends to appreciate. The exceptions are the Australian dollar and the New Zealand dollar: we find negative slope coefficients for those two currencies. These are positive carry currencies (with high average interest rates) of countries that are commodity exporters. To the extent that high U.S. bond returns are associated with a run to quality in times of global economic stress, the depreciation of the Australian and New Zealand dollars is consistent with the model of Ready, Roussanov, and Ward (2013), which illustrates the relative riskiness of the currencies of commodity-producing countries.

Currency Portfolios Panels B and C of Table 3 report the regression coefficients for slope-sorted and interest-rate-sorted currency portfolios, respectively. There are interesting differences in the slope coefficient across these portfolios. As evidenced in Panel B, the bond returns of countries with flat yield curves have lower dollar betas than the returns of steep yield curve countries. Furthermore, Panel C reveals that the long-maturity bond returns of low interest rate countries comove more with U.S. bond returns than the returns of high interest rate countries. Thus, it looks like that there is more sharing of permanent innovations between the U.S. and countries with low interest rates and steep yield curves.

Time Variation To understand the time-variation in the regression coefficients, we expand the sample period. Specifically, we consider an equally-weighted portfolio of all the currencies in the developed country sample and regress its dollar return and its components on the U.S. bond return from 12/1950 to 12/2012. We run the following regressions of local returns, exchange rate changes and dollar returns on U.S. bond returns over rolling 60-month windows:

$$\begin{aligned}
\frac{1}{N}\sum_i r_{i,t+1}^{(10),*} &= \alpha + \beta^{local} r_{t+1}^{(10)} + \epsilon_{t+1}, \\
\frac{1}{N}\sum_i -\Delta s_{i,t+1} &= \alpha + \beta^{fx} r_{t+1}^{(10)} + \epsilon_{t+1}, \\
\frac{1}{N}\sum_i r_{i,t+1}^{(10),\$} &= \alpha + \beta^{dollar} r_{t+1}^{(10)} + \epsilon_{t+1}.
\end{aligned}$$

Figure 6 plots the 60-month rolling window of the regression coefficients. We note large increases in the dollar beta after the demise of the Bretton-Woods regime, mostly driven by increases in the exchange rate betas. The same is true around the early 1990s. Furthermore, there is a secular increase in the local return beta over the entire sample.

The exchange rate coefficient is positive during most of our sample period, providing evidence that the currency exposure hedges the interest rate exposure of the foreign bond position. There are two main exceptions: the Long Term Capital Management (LTCM) crisis in 1998 and the recent financial crisis. During these episodes, the dollar appreciated, despite the strong performance of the U.S. bond market, weakening the comovement between foreign and local

Table 3: Tests of the Uncovered Bond Return Parity Condition

	Return in dollars ($r^{\$}$)			Return in local currency (r^*)			Change in exchange rate ($-\Delta s$)			Obs.
	β	s.e.	$R^2(\%)$	β	s.e.	$R^2(\%)$	β	s.e.	$R^2(\%)$	
					Section I: Discount Bonds					
					Panel A: Individual Countries					
Australia	0.32	[0.08]	3.77	0.37	[0.06]	16.93	-0.05	[0.07]	-0.08	493
Canada	0.69	[0.06]	31.19	0.68	[0.04]	57.58	0.01	[0.04]	-0.19	493
Germany	0.62	[0.08]	14.16	0.38	[0.03]	25.97	0.23	[0.07]	2.61	493
Japan	0.51	[0.09]	8.09	0.24	[0.04]	8.46	0.27	[0.06]	3.55	493
New Zealand	0.08	[0.10]	-0.02	0.16	[0.05]	1.87	-0.08	[0.08]	0.11	493
Norway	0.19	[0.07]	1.83	0.11	[0.03]	3.06	0.08	[0.06]	0.23	493
Sweden	0.28	[0.08]	3.55	0.16	[0.04]	5.25	0.12	[0.07]	0.63	493
Switzerland	0.43	[0.08]	7.39	0.15	[0.02]	15.97	0.28	[0.08]	3.21	493
United Kingdom	0.29	[0.07]	4.02	0.18	[0.03]	8.81	0.11	[0.07]	0.62	493
					Panel B: Slope-sorted Portfolios					
Portfolio 1	0.27	[0.07]	4.45	0.21	[0.03]	14.38	0.06	[0.06]	0.05	493
Portfolio 2	0.45	[0.06]	13.79	0.30	[0.03]	30.02	0.15	[0.06]	1.82	493
Portfolio 3	0.42	[0.06]	11.05	0.30	[0.03]	28.31	0.12	[0.05]	1.04	493
					Panel C: Interest-rate-sorted Portfolios					
Portfolio 1	0.47	[0.07]	11.98	0.27	[0.02]	28.97	0.20	[0.06]	2.89	493
Portfolio 2	0.39	[0.05]	11.90	0.29	[0.03]	29.92	0.10	[0.05]	0.77	493
Portfolio 3	0.28	[0.07]	4.48	0.25	[0.03]	16.13	0.03	[0.06]	-0.15	493
					Section II: Zero-Coupon Bonds					
					Panel A: Individual Countries					
Australia	0.54	[0.11]	12.39	0.77	[0.10]	37.78	-0.23	[0.07]	2.96	308
Canada	0.72	[0.09]	33.56	0.81	[0.06]	65.07	-0.10	[0.05]	1.30	321
Germany	0.63	[0.08]	24.21	0.46	[0.04]	37.57	0.17	[0.06]	2.79	477
Japan	0.69	[0.12]	20.00	0.36	[0.06]	21.76	0.33	[0.09]	7.36	333
New Zealand	0.77	[0.10]	21.76	0.84	[0.07]	45.21	-0.08	[0.10]	-0.03	273
Norway	0.38	[0.12]	5.81	0.44	[0.07]	18.70	-0.06	[0.14]	-0.37	177
Sweden	0.61	[0.11]	15.39	0.68	[0.09]	34.34	-0.07	[0.10]	-0.09	238
Switzerland	0.61	[0.09]	18.43	0.37	[0.05]	25.79	0.23	[0.10]	3.01	297
United Kingdom	0.58	[0.08]	18.44	0.52	[0.07]	28.97	0.06	[0.06]	0.22	405
					Panel B: Slope-sorted Portfolios					
Portfolio 1	0.68	[0.11]	21.22	0.56	[0.07]	32.98	0.12	[0.08]	0.96	333
Portfolio 2	0.53	[0.07]	21.03	0.51	[0.06]	39.19	0.02	[0.07]	-0.24	333
Portfolio 3	0.74	[0.07]	35.03	0.57	[0.06]	47.61	0.17	[0.08]	3.35	333
					Panel C: Interest-rate-sorted Portfolios					
Portfolio 1	0.70	[0.09]	28.34	0.44	[0.05]	40.44	0.26	[0.07]	6.81	333
Portfolio 2	0.70	[0.07]	32.73	0.60	[0.07]	51.72	0.10	[0.09]	0.88	333
Portfolio 3	0.60	[0.09]	20.76	0.60	[0.07]	39.94	0.00	[0.08]	-0.30	333

Notes: The table reports regression results obtained when regressing the log return on foreign bonds (expressed in U.S. dollars) $r^{\$}$, or the log return in local currency r^*, or the log change in the exchange rate Δs on the log return on U.S. bonds in U.S. dollars. Section I uses discount bonds. Returns are monthly and the sample period is 12/1971–12/2012. Standard errors are obtained with a Newey-West approximation of the spectral density matrix with two lags. Section II uses zero-coupon bonds. Returns are quarterly (sampled monthly) and the sample period is 12/1971–12/2012 (or available subsample) for individual currencies and 4/1985–12/2012 for currency portfolios. Standard errors are obtained with a Newey-West approximation of the spectral density matrix with six lags.

bond returns.

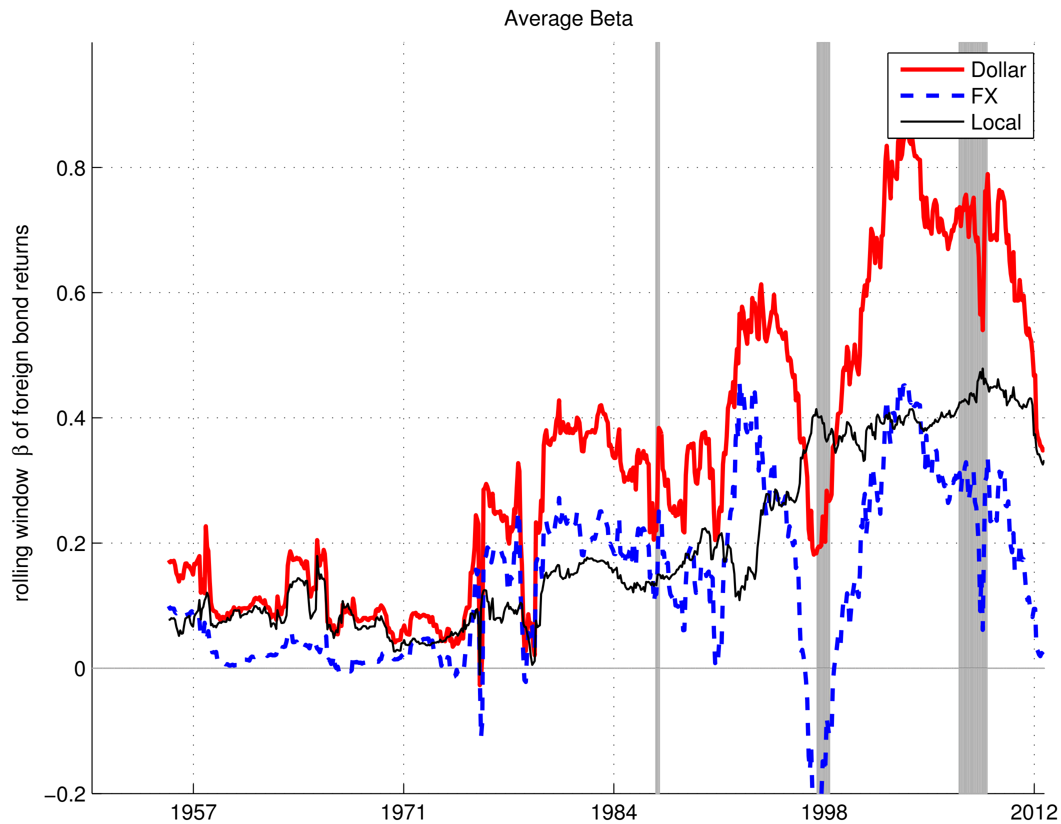

Figure 6: Foreign Bond Return Betas — This figure presents the 60-month rolling window estimation of beta with respect to US bond returns for the equal-weighted average of log bond returns in local currency, the log change in the exchange rate and the log dollar bond returns for the benchmark sample of countries. The panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. The sample is 12/1950–12/2012. The dark shaded areas represent the 1987 crash, the 1998 LTCM crisis and the 2007-2008 U.S. financial crisis.

Robustness Check To check the robustness of our results, we run the bond return parity regressions on zero-coupon bonds. The results are reported in the Section II of Table 3. They are broadly consistent with the previous findings. Specifically, the cross-sectional average of the dollar return slope is 0.61, implying significant comovement between foreign and U.S. dollar bond returns. This is due to the fact that the dataset is biased towards the recent period, when dollar betas are historically high, as shown in Figure 6.

5 The Properties of Exchange Rate and SDF Components

The rejection of the long-term bond parity in the time-series implies that the permanent component of exchange rate changes exists and is volatile. In this section, we decompose exchange rate changes into their transitory and permanent components and report their properties. The volatilities and correlation of these components provide insights on the SDF components. The decomposition offers a new perspective on international risk-sharing.

5.1 The Properties of Exchange Rate and SDF Components

Alvarez and Jermann (2005) show that asset prices are almost exclusively determined by the properties of the permanent SDF component, which accounts for almost all of the variation of the nominal SDF. Our results indicate that this is not true for exchange rates. Table 4 shows that the internationally unshared parts of the two components of the nominal SDF contribute roughly equally to the volatility of the nominal SDF. The annualized volatility of the 3-month exchange rate changes ranges from 7% to 12%. Although the two exchange rate components contribute to exchange rate volatility about equally, the transitory component tends to be the relatively smoother component, with its volatility ranging from 6% to 12%. The permanent component is slightly more volatile than the overall exchange rate, ranging in volatility from 10% to 16%.

The two components are negatively correlated for all exchange rates. Across the countries of the benchmark sample, the unconditional correlation of the two exchange rate components ranges from -0.38 to -0.73. In unreported results, a similar decomposition of 6-month and 12-month exchange rate changes produces quantitatively similar results. This implies that changes in the unshared part of the permanent, asset-pricing component of the SDF are partly offset by changes in the unshared part of the transitory component. This offsetting does not affect asset prices, since the permanent SDF component is an order of magnitude larger than the transitory component. However, it can potentially have large effects in the behavior of exchange rates, as the size of the internationally non-shared parts of the two SDF components does not differ much.

Table 4: Properties of SDF and Exchange Rate Components

Moment	DEM	GBP	JPY	CAD	AUD	CHF	NZD	SEK	NOK
	Panel A: Exchange rate changes, Δs								
Mean	-0.02	0.01	-0.04	-0.01	-0.02	-0.02	-0.01	-0.01	-0.02
Std	0.12	0.11	0.12	0.07	0.12	0.12	0.12	0.12	0.12
Skewness	0.08	0.65	-0.39	0.30	1.02	0.15	0.34	0.64	1.03
Kurtosis	3.02	5.53	3.35	7.20	8.99	2.75	5.30	4.99	7.04
AR(1) coef.	0.70	0.72	0.70	0.66	0.72	0.70	0.76	0.69	0.70
	Panel B: Transitory exchange rate changes, $\Delta s^{\mathbb{T}}$								
Mean	-0.00	0.01	-0.04	0.00	0.02	-0.04	0.01	0.01	-0.01
Std	0.10	0.12	0.09	0.06	0.09	0.08	0.08	0.09	0.09
Skewness	0.24	0.55	0.39	-0.12	0.70	-0.10	0.14	-0.22	0.37
Kurtosis	4.30	5.48	5.11	3.44	4.92	3.41	4.16	4.52	3.75
AR(1) coef.	0.63	0.55	0.58	0.55	0.59	0.64	0.64	0.68	0.56
	Panel C: Permanent exchange rate changes, $\Delta s^{\mathbb{P}}$								
Mean	-0.01	-0.00	0.00	-0.02	-0.04	0.02	-0.02	-0.02	-0.01
Std	0.14	0.16	0.14	0.10	0.14	0.12	0.13	0.13	0.14
Skewness	-0.01	-0.09	-0.40	0.56	0.08	0.32	0.36	0.13	0.35
Kurtosis	3.23	4.68	4.56	4.22	3.96	3.03	3.33	3.15	4.10
AR(1) coef.	0.65	0.65	0.63	0.62	0.62	0.63	0.67	0.66	0.65
	Panel D: Exchange Rate Correlations								
$corr(\Delta s, \Delta s^{\mathbb{T}})$	0.15	0.00	0.13	-0.14	0.15	0.30	0.18	0.18	0.14
$corr(\Delta s, \Delta s^{\mathbb{P}})$	0.74	0.68	0.75	0.81	0.74	0.77	0.78	0.75	0.75
$corr(\Delta s^{\mathbb{T}}, \Delta s^{\mathbb{P}})$	-0.55	-0.73	-0.55	-0.70	-0.55	-0.38	-0.48	-0.52	-0.55
	Panel E: Transitory SDF								
Std	0.09	0.12	0.08	0.10	0.12	0.07	0.11	0.11	0.09
s.e.	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.01]	[0.01]	[0.01]
$corr(m^{\mathbb{T},US}, m^{\mathbb{T},*})$	0.61	0.54	0.47	0.81	0.62	0.51	0.67	0.59	0.44
s.e.	[0.04]	[0.06]	[0.07]	[0.02]	[0.05]	[0.06]	[0.04]	[0.07]	[0.07]

Notes: The table reports the mean, standard deviation, skewness, kurtosis, and autocorrelation of three-month changes in exchange rates, as well as the moments of the transitory and permanent components of exchange rates. 10-year zero-coupon nominal bonds are used as proxy of infinite-maturity bonds in order to decompose nominal exchange rate changes into their permanent and transitory components. Means and standard deviations are annualized. The last panel reports the standard deviations of the transitory component of the nominal SDF, along with its correlation with the transitory component of the U.S. SDF. Standard errors are obtained from block bootstrapping with blocks of four periods (10,000 replications). Monthly zero-coupon data from 12/1971 to 12/2012, or subset available.

This negative correlation between the cyclical and trend components of the exchange rate is sensible based on what we know about the dynamics of GDP growth. Morley, Nelson, and Zivot (2003) study the correlation of the cyclical (temporary) and trend component in U.S. GDP growth. They document that there is a strong, negative correlation of the cycle and trend component. That is not surprising. Suppose the arrival of a new technology (e.g., the internet) raises the trend of GPP growth upon arrival, but not actual GDP. Then the cyclical component will initially be negative and slowly revert back to the mean. In a standard, representative agent asset pricing model, the pricing kernel would inherit the same properties as aggregate consumption growth. Of course, we document these properties of the country-specific cyclical and trend component using only asset pricing data, not a statistical decomposition of the underlying fundamentals.[11] To illustrate the difference of our decomposition, which aims to capture differences in the trend and cycle components of pricing kernels across countries using bond returns, with standard statistical decompositions of the nominal exchange rate time-series, we also perform a Beveridge and Nelson (1981) trend-cycle decomposition of nominal exchange rates. In unreported results, we find that the trend and cycle components of nominal exchange rates are negatively correlated; notably, the trend component of nominal exchange rates is positively correlated with the permanent component of our decomposition (with the average correlation coefficient being around 0.5), whereas the cyclical component is uncorrelated with the transitory component of our decomposition.

Cross-sectionally, the correlation between overall exchange rate changes and their transitory component ranges from -0.14 to 0.30, whereas the correlation between exchange rate changes and their permanent component is much higher, ranging from 0.68 to 0.81.

5.2 The Implications for International Risk Sharing

Our findings imply that fluctuations in the permanent SDF component are internationally shared to a significantly larger extent than fluctuations in the transitory SDF component. For each country's transitory SDF component, we calculate the annualized standard deviation and the

[11] We would like to thank Ian Dew-Becker for pointing out this connection to us.

correlation coefficient with the transitory component of the U.S. SDF and report the results in Table 4. We also report the bootstrap standard error for each moment.

Table 4 shows that for all countries, the annualized standard deviation of the transitory SDF component is significantly smaller than the typically calculated Hansen-Jagannathan lower bound of total SDF volatility. For the 3-month pricing kernel changes, the volatility ranges from 7% (for Switzerland) to 12% (for a number of countries), whereas conventional estimates of the SDF lower bound exceed 50%. Those results are consistent with the Alvarez and Jermann (2005) findings that the transitory SDF component is second-order. However, by Brandt, Cochrane, and Santa-Clara (2006) logic, the much smaller volatility of the transitory SDF component, coupled with the fact that the transitory component of exchange rate changes is about as volatile as the total exchange rate changes, implies that the cross-country correlation of the transitory SDF component is significantly lower than the cross-country correlation of the overall SDF. Indeed, the last panel of Table 4 reports that, for 3-month pricing kernel changes, correlations range from 0.44 (between the U.S. and Norway) to 0.81 (between the U.S. and Canada). Those correlation coefficients represent much smaller common variation than the almost perfectly correlated SDFs reported in Brandt, Cochrane, and Santa-Clara (2006). It follows that there is much more internationally common variation regarding the permanent SDF component than the transitory SDF component. In unreported results, we find that our conclusions are robust for six-month and 12-month pricing kernel changes.

5.3 Real Exchange Rate Decomposition

In order to understand whether our results on the decomposition of nominal exchange rate changes reflect the dynamics of real variables or are primarily driven by inflation, we also study the components of real exchange rate changes. To do so requires estimates of the returns on real bonds. We either proxy real bonds by inflation-indexed bonds, or synthetically construct real bonds by hedging the inflation exposure of nominal bonds using inflation swaps. Due to the limited data availability of inflation-indexed bond and inflation swap data, our analysis spans a limited set of currencies (the U.S. dollar, the U.K pound, the Euro and the Japanese yen) and sample periods. We focus here on the main results and report the details in the Appendix.

Our findings are quantitatively similar with those on nominal exchange rates. The main quantitative differences are that the two components of real exchange rate changes are more negatively correlated than those of nominal exchange rate changes and that the transitory components of real stochastic discount factors are less correlated across countries than their nominal SDF counterparts.

The similarity of the real and nominal exchange rate change decompositions is due to the relatively small size of inflation swap rate changes. To illustrate this point, we derive the following mapping from the nominal to the real exchange rate decomposition, based on the k-period zero-coupon inflation swap rates $f_t^{(k)}$.

Proposition 4. *The transitory component of real exchange rate changes equals the transitory component of nominal exchange rate changes scaled by the relative change in the inflation swap rates:*

$$\frac{S_{t+1}^{real,\mathbb{T}}}{S_t^{real,\mathbb{T}}} = \frac{S_{t+1}^{\mathbb{T}}}{S_t^{\mathbb{T}}} \lim_{k\to\infty} \left(\frac{(1+f_{t+1}^{*,(k-1)})^{k-1}}{(1+f_t^{*,(k)})^k} \frac{(1+f_t^{(k)})^k}{(1+f_{t+1}^{(k-1)})^{k-1}} \right).$$

At long horizons, the changes in foreign inflation swap rates relative to the U.S. are too small to modify the properties of the transitory and permanent changes in nominal exchange rates.

6 Conclusion

In this paper, we derive a novel theoretical uncovered bond return parity condition. If permanent shocks to the pricing kernels are perfectly shared, then long-term bond returns, once expressed in a common currency, should be equalized across countries. If permanent shocks are not perfectly shared, then the difference between the domestic and foreign long-term bond risk premia, again expressed in a common currency, reflects the difference in the entropy of the permanent components of the stochastic discount factor. We decompose exchange rate changes into a component that reflects cross-country differences in permanent pricing kernel innovations and one that encodes differences in transitory innovations. This decomposition is informative on the correlation of the permanent components of stochastic discount factors. Our results are preference-free and rely only on market completeness.

In the cross-section, we find that the term structure of currency risk premia is downward sloping. While carry trade strategies based on the three-month Treasury bills are highly profitable, carry trade strategies using long-maturity bonds are not. In the time-series, bond excess returns expressed in a common currency appear highly correlated, albeit not perfectly. While Alvarez and Jermann (2005) find that domestic equity and bond markets' risk premia imply that the pricing kernels are mostly driven by permanent shocks, we find that the permanent components of the stochastic discount factors are highly correlated across countries.

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Appendix

Section A reports the proofs of the theoretical results presented in the main text. Section B details the decomposition of the SDF in two affine term structure models.

A Proofs

This Section gathers all the proofs of the theoretical results in the paper.

- Proof of Proposition 1:

  Proof. The proof builds on some results in Backus, Foresi, and Telmer (2001) and Alvarez and Jermann (2005). Specifically, Backus, Foresi, and Telmer (2001) show that the foreign currency risk premium is equal to the difference between domestic and foreign total SDF entropy:

$$(f_t - s_t) - E_t[\Delta s_{t+1}] = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right).$$

  Furthermore, Alvarez and Jermann (2005) establish that total SDF entropy equals the sum of the entropy of the permanent SDF component and the expected log term premium:

$$L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) = L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right) + E_t\left(\log\frac{R^{(\infty)}_{t+1}}{R^f_t}\right).$$

  Applying the Alvarez and Jermann (2005) decomposition to the Backus, Foresi, and Telmer (2001) expression yields the desired result.

  To derive the Backus, Foresi, and Telmer (2001) expression, consider a foreign investor who enters a forward position in the currency market with payoff $S_{t+1} - F_t$. The investor's Euler equation is:

$$E_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}(S_{t+1} - F_t)\right) = 0.$$

  In the presence of complete, arbitrage-free international financial markets, exchange rate changes equal the ratio of the domestic and foreign stochastic discount factors:

$$\frac{S_{t+1}}{S_t} = \frac{\Lambda_{t+1}}{\Lambda_t}\frac{\Lambda^*_t}{\Lambda^*_{t+1}},$$

  Dividing the investor's Euler equation by S_t and applying the no arbitrage condition, the forward discount is:

$$f_t - s_t = \log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - \log E_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right).$$

  The second component of the currency risk premium is expected foreign appreciation; applying logs and conditional expectations to the no arbitrage condition above leads to:

$$E_t[\Delta s_{t+1}] = E_t\left(\log\frac{\Lambda_{t+1}}{\Lambda_t}\right) - E_t\left(\log\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right).$$

  Combining the two terms of the currency risk premium leads to:

$$(f_t - s_t) - E_t[\Delta s_{t+1}] = \log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - E_t\left(\log\frac{\Lambda_{t+1}}{\Lambda_t}\right) - \log E_t\left(\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right) + E_t\left(\log\frac{\Lambda^*_{t+1}}{\Lambda^*_t}\right)$$

  Applying the definition of conditional entropy in the equation above yields the Backus, Foresi, and Telmer (2001) expression.

To derive the Alvarez and Jermann (2005) result, first note that since the permanent component of the pricing kernel is a martingale, its conditional entropy can be expressed as follows:

$$L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right) = -E_t\left(\log \frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right).$$

The definition of conditional entropy implies the following decomposition of total SDF entropy:

$$L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) = \log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - E_t\left(\log \frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right)$$

or, using the above expression for the conditional entropy of the permanent SDF component:

$$L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) = -\log R_t^f - E_t\left(\log \frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right) + L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right).$$

The Alvarez and Jermann (2005) result hinges on:

$$\lim_{k\to\infty} R_{t+1}^{(k)} = \Lambda_t^{\mathbb{T}}/\Lambda_{t+1}^{\mathbb{T}}.$$

Under the assumption that $0 < \lim\limits_{k\to\infty} \frac{P_t^{(k)}}{\delta^k} < \infty$ for all t, one can write:

$$\lim_{k\to\infty} R_{t+1}^{(k)} = \lim_{k\to\infty} \frac{E_{t+1}\left(\frac{\Lambda_{t+k}}{\Lambda_{t+1}}\right)}{E_t\left(\frac{\Lambda_{t+k}}{\Lambda_t}\right)} = \frac{\lim\limits_{k\to\infty} \frac{E_{t+1}(\Lambda_{t+k}/\delta^{t+k})}{\Lambda_{t+1}}}{\lim\limits_{k\to\infty} \frac{E_t(\Lambda_{t+k}/\delta^{t+k})}{\Lambda_t}} = \frac{\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_{t+1}}}{\frac{\Lambda_t^{\mathbb{P}}}{\Lambda_t}} = \Lambda_t^{\mathbb{T}}/\Lambda_{t+1}^{\mathbb{T}}.$$

Thus, the infinite-maturity bond is exposed only to transitory SDF risk. □

- Proof of Corollary 1:

Proof. Start from Proposition 1. The proof from there relies on Proposition 5 in Alvarez and Jermann (2005). They show that, when the limits of the k-period bond risk premium and the yield difference between the k-period discount bond and the one-period riskless bond (when the maturity k tends to infinity) are well defined and the unconditional expectations of holding returns are independent of calendar time, then the average term premium $E\left[\lim_{k\to\infty} rx_{t+1}^{(k),*}\right]$ equals the average yield spread $E[\lim_{k\to\infty} y_t^{(k),*}]$. Substituting for the term premiums in this equation leads to:

$$E[y_t^{(\infty),*}] + E(f_t - s_t) - E[\Delta s_{t+1}] = E[y_t^{(\infty),*}] + E\left[L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right) - L_t\left(\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}}\right)\right].$$

Under regularity conditions, $E[\lim_{k\to\infty} \frac{1}{k}\sum_{j=1}^{k} \Delta s_{t+j}] = \lim_{k\to\infty} \frac{1}{k}\sum_{j=1}^{k} E[\Delta s_{t+j}]$ converges to $E[\Delta s_{t+1}]$. Using this result produces the corollary. □

- Proof of Proposition 2:

Proof. As shown inAlvarez and Jermann (2005) (see the proof of Proposition 1), the return of the infinite maturity bond reflects the transitory SDF component:

$$\lim_{k\to\infty} R_{t+1}^{(k)} = \Lambda_t^{\mathbb{T}}/\Lambda_{t+1}^{\mathbb{T}}.$$

The result of Proposition 2 follows directly from the no-arbitrage expression for the spot exchange rate when markets are complete:

$$\frac{S_{t+1}}{S_t} = \frac{\Lambda_{t+1}}{\Lambda_t}\frac{\Lambda_t^*}{\Lambda_{t+1}^*}.$$

In this case,

$$\lim_{k\to\infty}\frac{S_t}{S_{t+1}}\frac{R^{(k),*}_{t+1}}{R^{(k)}_{t+1}}=\frac{S_t}{S_{t+1}}\frac{\lim_{k\to\infty}R^{(k),*}_{t+1}}{\lim_{k\to\infty}R^{(k)}_{t+1}}=\frac{S_t}{S_{t+1}}\frac{\Lambda^{\mathbb{T}}_t}{\Lambda^{\mathbb{T}}_{t+1}}\frac{\Lambda^{\mathbb{T},*}_{t+1}}{\Lambda^{\mathbb{T},*}_t}=\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t}\frac{\Lambda^{\mathbb{P}}_t}{\Lambda^{\mathbb{P}}_{t+1}}=\frac{S^{\mathbb{P}}_t}{S^{\mathbb{P}}_{t+1}},$$

using the decomposition of exchange rate changes into a permanent and a transitory component:

$$\frac{S_{t+1}}{S_t}=\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\frac{\Lambda^{\mathbb{P},*}_t}{\Lambda^{\mathbb{P},*}_{t+1}}\right)\left(\frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t}\frac{\Lambda^{\mathbb{T},*}_t}{\Lambda^{\mathbb{T},*}_{t+1}}\right)=\frac{S^{\mathbb{P}}_{t+1}}{S^{\mathbb{P}}_t}\frac{S^{\mathbb{T}}_{t+1}}{S^{\mathbb{T}}_t}.$$

The exposure of the domestic and foreign infinite-maturity bonds to transitory risk fully offsets the transitory component of exchange rate changes, so only the exposure to the permanent part remains. □

- Proof of Proposition 3

Proof. Alvarez and Jermann (2005) establish that the lower bound of the entropy of the permanent component of the SDF is given by the distance of the expected return of the infinite-maturity bond from the maximum expected return in the economy:

$$L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)\geq E_t\left(\log R_{t+1}\right)-E_t\left(\log R^{(\infty)}_{t+1}\right).$$

Intuitively, the larger the difference between the maximum risk premium in the economy (which compensates investors for exposure to all shocks) and the term premium (which compensates investors only for transitory shocks), the larger the significance of permanent fluctuations.
To construct this bound, first use the concavity of the logarithmic function to show that for any return R_{t+1}:

$$E_t\left(\log\frac{\Lambda_{t+1}}{\Lambda_t}\right)+E_t\left(\log R_{t+1}\right)=E_t\left(\log\frac{\Lambda_{t+1}}{\Lambda_t}R_{t+1}\right)\leq\log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}R_{t+1}\right)=0,$$

where the last equality follows from the assumption of no arbitrage. Using the definition of conditional entropy, this inequality implies that the maximum risk premium in the economy establishes a lower bound for total SDF entropy:

$$L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)=\log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)-E_t\left(\log\frac{\Lambda_{t+1}}{\Lambda_t}\right)\geq E_t\left(\log R_{t+1}\right)-\log R^f_t.$$

To understand the implications of that bound for the amount of permanent risk in the economy, decompose total SDF entropy. Recall that (see the proof of Proposition 1) total SDF entropy can be decomposed into the sum of permanent SDF entropy and the term risk premium:

$$L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)=L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)+E_t\left(\log R^{(\infty)}_{t+1}\right)-\log R^f_t.$$

Since the infinite-maturity bond is solely exposed to transitory SDF fluctuations, ceteris paribus a large term premium implies significant transitory risk. Applying this decomposition to the total SDF entropy bound above yields the Alvarez and Jermann (2005) bound on the entropy of the permanent SDF component.

The Alvarez and Jermann (2005) bound is used to construct a bound for the covariance of the log permanent component of two countries' stochastic discount factors. First, by the definition of the permanent component of exchange rates, its variance reflects the variance of the difference of the two countries' log permanent SDF components:

$$var_t\left(\log\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}-\log\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t}\right)=var_t\left(\frac{S^{\mathbb{P}}_{t+1}}{S^{\mathbb{P}}_t}\right).$$

The variance of the difference of two random variables implies that:

$$cov_t\left(\log\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t},\log\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)=\frac{1}{2}\left[var_t\left(\log\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t}\right)+var_t\left(\log\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)-var_t\left(\log\frac{S^{\mathbb{P}}_{t+1}}{S^{\mathbb{P}}_t}\right)\right].$$

Given the assumption of conditional lognormality of the permanent component of pricing kernels, this expression can be rewritten in terms of conditional entropy as follows:

$$cov_t\left(\log\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t},\log\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)=L_t\left(\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t}\right)+L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)-\frac{1}{2}var_t\left(\log\frac{S^{\mathbb{P}}_{t+1}}{S^{\mathbb{P}}_t}\right).$$

In effect, this expression applies the logic of the Brandt, Cochrane, and Santa-Clara (2006) decomposition to permanent SDF components: the implied covariance of the log permanent SDF components is increasing in the permanent SDF entropy of the two countries and decreasing in the variance of the permanent component of exchange rate changes.
Finally, the covariance above relates to risk premia by using the Alvarez and Jermann (2005) bound for $L_t\left(\frac{\Lambda^{\mathbb{P},*}_{t+1}}{\Lambda^{\mathbb{P},*}_t}\right)$ and $L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)$. This yields the expression in Proposition 3.

For the unconditional version of Proposition 3, first follow Alvarez and Jermann (2005) and establish a bound for the unconditional entropy of the permanent SDF component:

$$L\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)\geq E\left(\log R_{t+1}\right)-E\left(\log R^{(\infty)}_{t+1}\right).$$

Applying unconditional expectations on the two sides of the previously established conditional SDF entropy bound leads to:

$$L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\geq E_t\left(\log R_{t+1}\right)-\log R^f_t,$$

using the following property of entropy: for any admissible random variable X, it holds that

$$E\left[L_t(X_{t+1})\right]=L(X_{t+1})-L\left[E_t(X_{t+1})\right].$$

After some algebra, the following bound for the unconditional entropy of the SDF is obtained:

$$L\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\geq L\left(\frac{1}{R^f_t}\right)+E\left(\log\frac{R_{t+1}}{R^f_t}\right).$$

To derive an expression for the unconditional entropy of the permanent SDF component, one needs to decompose the unconditional SDF entropy. To do so, start with the decomposition of the conditional SDF entropy:

$$L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)=L_t\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)+E_t\left(\log R^{(\infty)}_{t+1}\right)-\log R^f_t,$$

and apply unconditional expectations on both sides of the expression in order to obtain:

$$L\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)=L\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)+L\left(\frac{1}{R^f_t}\right)+E\left(\log\frac{R^{(\infty)}_{t+1}}{R^f_t}\right),$$

using the fact that the permanent component of the pricing kernel is a martingale:

$$L\left(E_t\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)=0.$$

Using the above decomposition of unconditional SDF entropy, the unconditional entropy bound can be written as follows:

$$L\left(\frac{\Lambda^{\mathbb{P}}_{t+1}}{\Lambda^{\mathbb{P}}_t}\right)+L\left(\frac{1}{R^f_t}\right)+E\left(\log\frac{R^{(\infty)}_{t+1}}{R^f_t}\right)\geq L\left(\frac{1}{R^f_t}\right)+E\left(\log\frac{R_{t+1}}{R^f_t}\right).$$

The Alvarez and Jermann (2005) unconditional bound follows immediately by rearranging the terms in the expression above. Considering the unconditional covariance of the domestic and foreign permanent SDF components and using this bound yields the unconditional expression of Proposition 3:

$$cov\left(\log\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}},\log\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}\right)\geq E\left(\log\frac{R_{t+1}^{*}}{R_{t+1}^{(\infty),*}}\right)+E_t\left(\log\frac{R_{t+1}}{R_{t+1}^{(\infty)}}\right)-\frac{1}{2}var\left(\log\frac{S_{t+1}^{\mathbb{P}}}{S_t^{\mathbb{P}}}\right).$$

This result relies on the assumption that $\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}}$ and $\frac{\Lambda_{t+1}^{\mathbb{P},*}}{\Lambda_t^{\mathbb{P},*}}$ are unconditionally lognormal and the entropy property

$$L(X)=\frac{1}{2}var(\log X)$$

for any lognormal random variable X. □

- Proof of Proposition 4:

Proof. Consider the pricing equation for an inflation swap

$$0=E_t\left[\frac{\Lambda_{t+k}}{\Lambda_t}\left(\frac{CPI_{t+k}}{CPI_t}-(1+f_t^{(k)})^k\right)\right]=E_t\left[\frac{\Lambda_{t+k}^{real}}{\Lambda_t^{real}}\right]-(1+f_t^{(k)})^kE_t\left[\frac{\Lambda_{t+k}}{\Lambda_t}\right]$$

where $f_t^{(k)}$ is the inflation swap rate at t for k periods into the future. Rearranging the pricing equation, the conditional expectation of the nominal pricing kernel is derived as a function of the conditional expectation of the real pricing kernel, the price level and the inflation swap rate:

$$E_t\left[\Lambda_{t+k}\right]=\frac{\Lambda_t}{\Lambda_t^{real}}\frac{1}{(1+f_t^{(k)})^k}E_t\left[\Lambda_{t+k}^{real}\right]=\frac{E_t\left[\Lambda_{t+k}^{real}\right]}{CPI_t(1+f_t^{(k)})^k}. \quad (4)$$

The relative USD holding period return of foreign and domestic nominal bonds with maturity k periods into the future is given by:

$$\frac{S_t}{S_{t+1}}\frac{R_{t+1}^{(k),*}}{R_{t+1}^{(k)}}=\frac{E_{t+1}\left(\Lambda_{t+k}^{*}\right)}{E_t\left(\Lambda_{t+k}^{*}\right)}\frac{E_t\left(\Lambda_{t+k}\right)}{E_{t+1}\left(\Lambda_{t+k}\right)}$$

Using Equation (4) then leads to:

$$\frac{S_t}{S_{t+1}}\frac{R_{t+1}^{(k),*}}{R_{t+1}^{(k)}}=\frac{(1+f_t^{*,(k)})^k}{(1+f_{t+1}^{*,(k-1)})^{k-1}}\frac{(1+f_{t+1}^{(k-1)})^{k-1}}{(1+f_t^{(k)})^k}\frac{CPI_t^{*}}{CPI_{t+1}^{*}}\frac{CPI_{t+1}}{CPI_t}\frac{E_{t+1}\left[\Lambda_{t+k}^{*,real}\right]}{E_t\left[\Lambda_{t+k}^{*,real}\right]}\frac{E_t\left[\Lambda_{t+k}^{real}\right]}{E_{t+1}\left[\Lambda_{t+k}^{real}\right]}$$

Given the relationship between the nominal and the real exchange rate,

$$\frac{S_t^{real}}{S_{t+1}^{real}}=\frac{S_t}{S_{t+1}}\frac{CPI_{t+1}^{*}}{CPI_t^{*}}\frac{CPI_t}{CPI_{t+1}},$$

the ratio of holding period returns is:

$$\frac{R_{t+1}^{(k),*}}{R_{t+1}^{(k)}}=\frac{(1+f_t^{*,(k)})^k}{(1+f_{t+1}^{*,(k-1)})^{k-1}}\frac{(1+f_{t+1}^{(k-1)})^{k-1}}{(1+f_t^{(k)})^k}\frac{S_{t+1}^{real}}{S_t^{real}}\left[\frac{E_{t+1}\left[\Lambda_{t+k}^{*,real}\right]}{E_t\left[\Lambda_{t+k}^{*,real}\right]}\frac{E_t\left[\Lambda_{t+k}^{real}\right]}{E_{t+1}\left[\Lambda_{t+k}^{real}\right]}\right]$$

and since the terms inside the brackets equal the relative USD return of foreign and domestic real bonds, given by $\frac{S_t^{real}}{S_{t+1}^{real}}\frac{R_{t+1}^{real,(k),*}}{R_{t+1}^{real,(k)}}$, the ratio of holding period returns can be rewritten as:

$$\frac{R_{t+1}^{(k),*}}{R_{t+1}^{(k)}}=\frac{(1+f_t^{*,(k)})^k}{(1+f_{t+1}^{*,(k-1)})^{k-1}}\frac{(1+f_{t+1}^{(k-1)})^{k-1}}{(1+f_t^{(k)})^k}\frac{R_{t+1}^{real,(k),*}}{R_{t+1}^{real,(k)}}$$

Therefore, the relative return of nominal bonds (in terms of their local currency) equals the relative local currency return of real bonds scaled by a term that reflects swap rates. Rearranging the expression above and focusing on returns of infinite maturity bonds leads to the link between the transitory components of real and nominal exchange rate changes:

$$\lim_{k\to\infty}\left(\frac{R^{real,(k),*}_{t+1}}{R^{real,(k)}_{t+1}}\right)=\lim_{k\to\infty}\left(\frac{R^{(k),*}_{t+1}}{R^{(k)}_{t+1}}\right)\lim_{k\to\infty}\left(\frac{(1+f^{*,(k-1)}_{t+1})^{k-1}}{(1+f^{*,(k)}_{t})^{k}}\frac{(1+f^{(k)}_{t})^{k}}{(1+f^{(k-1)}_{t+1})^{k-1}}\right).$$

□

B Factorization in Affine Term Structure Models

This Section focuses on the Cox, Ingersoll, and Ross (1985) and Lustig, Roussanov, and Verdelhan (2011) models.

B.1 Cox, Ingersoll, and Ross (1985) Model

The Cox, Ingersoll, and Ross (1985) model (denoted CIR) is defined by the following two equations:

$$\begin{aligned} -\log M_{t+1} &= \alpha+\chi z_t+\sqrt{\gamma z_t}u_{t+1}, \\ z_{t+1} &= (1-\phi)\theta+\phi z_t-\sigma\sqrt{z_t}u_{t+1}, \end{aligned} \tag{5}$$

where M denotes the stochastic discount factor.

Bond Prices Log bond prices are affine in the state variable z:

$$p^{(n)}_t = -B^n_0-B^n_1 z_t.$$

The price of a one period-bond is:

$$P^{(1)}=E_t(M_{t+1})=e^{-\alpha-(\chi-\frac{1}{2}\gamma)z_t}.$$

Bond prices are defined recursively by the Euler equation: $P^{(n)}_t=E_t(M_{t+1}P^{(n-1)}_{t+1})$. Thus the bond price coefficients evolve according to the following second-order difference equations:

$$\begin{aligned} B^n_0 &= \alpha+B^{n-1}_0+B^{n-1}_1(1-\phi)\theta, \\ B^n_1 &= \chi-\frac{1}{2}\gamma+B^{n-1}_1\phi-\frac{1}{2}\left(B^{n-1}_1\right)^2\sigma^2+\sigma\sqrt{\gamma}B^{n-1}_1. \end{aligned} \tag{6}$$

Decomposition The temporary pricing component of the pricing kernel is:

$$\Lambda^{\mathbb{T}}_t = \lim_{n\to\infty}\frac{\beta^{t+n}}{P^{(n)}_t}=\lim_{n\to\infty}\beta^{t+n}e^{B^n_0+B^n_1 z_t},$$

where the constant β is chosen in order to satisfy Assumption 1 in Alvarez and Jermann (2005):

$$0<\lim_{n\to\infty}\frac{P^{(n)}_t}{\beta^n}<\infty.$$

The limit of $B^n_0-B^{n-1}_0$ is finite: $\lim_{n\to\infty}B^n_0-B^{n-1}_0=\alpha+B^{\infty}_1(1-\phi)\theta$, where B^{∞}_1 is defined implicitly in a second-order equation above. As a result, B^n_0 grows at a linear rate in the limit. We choose the constant β to offset the growth in B^n_0 as n becomes very large. Setting

$$\beta=e^{-\alpha-B^{\infty}_1(1-\phi)\theta}$$

guarantees that Assumption 1 in Alvarez and Jermann (2005) is satisfied. The temporary pricing component of the SDF is thus equal to:

$$\frac{\Lambda^{\mathbb{T}}_{t+1}}{\Lambda^{\mathbb{T}}_t}=\beta e^{B^{\infty}_1(z_{t+1}-z_t)}=\beta e^{B^{\infty}_1\left[(\phi-1)(z_t-\theta)-\sigma\sqrt{z_t}u_{t+1}\right]}.$$

As a result, the martingale component of the SDF is then:

$$\frac{\Lambda_{t+1}^{\mathbb{P}}}{\Lambda_t^{\mathbb{P}}} = \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\Lambda_{t+1}^{\mathbb{T}}}{\Lambda_t^{\mathbb{T}}}\right)^{-1} = \beta^{-1}e^{-\alpha-\chi z_t-\sqrt{\gamma z_t}u_{t+1}}e^{-B_1^{\infty}\left[(\phi-1)(z_t-\theta)-\sigma\sqrt{z_t}u_{t+1}\right]}. \tag{7}$$

Bond Risk Premia The holding period return on a zero-coupon bond of maturity n between date t and $t+1$ is $R_{t+1}^{(n)} = P_{t+1}^{(n-1)}/P_t^{(n)}$. Let $r_{t+1}^{(n)}$ denote the corresponding log holding period return.

$$\begin{aligned} r_{t+1}^{(n)} &= B_0^n - B_0^{n-1} - B_1^{n-1}z_{t+1} + B_1^n z_t, \\ &= B_0^n - B_0^{n-1} - B_1^{n-1}(1-\phi)\theta \\ &+ [\chi - \frac{1}{2}\gamma - \frac{1}{2}\left(B_1^{n-1}\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^{n-1}]z_t \\ &+ B_1^{n-1}\sigma\sqrt{z_t}u_{t+1}. \end{aligned}$$

Hence it follows that the log holding period return in excess of the risk-free rate is given by:

$$\begin{aligned} rx_{t+1}^{(n)} &= r_{t+1}^{(n)} - \alpha - (\chi - \frac{1}{2}\gamma)z_t, \\ &= [-\frac{1}{2}\left(B_1^{n-1}\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^{n-1}]z_t + B_1^{n-1}\sigma\sqrt{z_t}u_{t+1}. \end{aligned}$$

The expected log excess return is thus given by:

$$E_t[rx_{t+1}^{(n)}] = [-\frac{1}{2}\left(B_1^{n-1}\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^{n-1}]z_t.$$

The expected log excess return of an infinite maturity bond is then:

$$\begin{aligned} E_t[rx_{t+1}^{(\infty)}] &= [-\frac{1}{2}\left(B_1^{\infty}\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^{\infty}]z_t, \\ &= [B_1^{\infty}(1-\phi) - \chi + \frac{1}{2}\gamma]z_t. \end{aligned}$$

The $-\frac{1}{2}\left(B_1^{\infty}\right)^2\sigma^2$ is a Jensen term. The term premium is driven by $\sigma\sqrt{\gamma}B_1^{\infty}z_t$, where B_1^{∞} is defined implicitly in the second order equation $B_1^{\infty} = \chi - \frac{1}{2}\gamma + B_1^{\infty}\phi - \frac{1}{2}\left(B_1^{\infty}\right)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^{\infty}$.

No Permanent Shocks Consider the special case of $B_1^{\infty}(1-\phi) = \chi$. In this case, the expected term premium is simply $E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\gamma z_t$, which is equal to one-half of the variance of the log stochastic discount factor.

Foreign Pricing Kernel Suppose that the foreign pricing kernel is specified as in Equation (6) with the same parameters. However, the foreign country has its own factor $z^{\star}$. As a result, the difference between the domestic and foreign log term premia is equal to the log currency risk premium, which is given by $E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma(z_t - z_t^*)$. In other words, the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is equal to the U.S. term premium: $E_t[rx_{t+1}^{(\infty),*}] + E_t[rx_{t+1}^{FX}] = \frac{1}{2}\gamma z_t$.

This special case corresponds to the absence of permanent shocks to the SDF: when $B_1^{\infty}(1-\phi) = \chi$, the permanent component of the stochastic discount factor is constant. To see this result, let us go back to the implicit definition of B_1^{∞} in Equation (7):

$$\begin{aligned} 0 &= \frac{1}{2}\left(B_1^{\infty}\right)^2\sigma^2 + (1-\phi-\sigma\sqrt{\gamma})B_1^{\infty} - \chi + \frac{1}{2}\gamma, \\ 0 &= \frac{1}{2}\left(B_1^{\infty}\right)^2\sigma^2 - \sigma\sqrt{\gamma}B_1^{\infty} + \frac{1}{2}\gamma, \\ 0 &= \left(\sigma B_1^{\infty} - \sqrt{\gamma}\right)^2. \end{aligned}$$

In this special case, $B_1^{\infty} = \sqrt{\gamma}/\sigma$. Using this result in Equation (7), the permanent component of the SDF reduces to:

$$\frac{M_{t+1}^{\mathbb{P}}}{M_t^{\mathbb{P}}} = \frac{M_{t+1}}{M_t}\left(\frac{M_{t+1}^{\mathbb{T}}}{M_t^{\mathbb{T}}}\right)^{-1} = \beta^{-1}e^{-\alpha-\chi z_t-\sqrt{\gamma z_t}u_{t+1}}e^{-B_1^{\infty}\left[(\phi-1)(z_t-\theta)-\sigma\sqrt{z_t}u_{t+1}\right]} = \beta^{-1}e^{-\alpha-\chi\theta},$$

which is a constant.

The same structure exists in the foreign economy. All foreign variables are denoted with a *. We do not impose the parameters to be the same. We define the log changes in exchange rates as the log difference in the SDFs.

B.2 Lustig, Roussanov, and Verdelhan (2011) Model

Suppose we have a version of the CIR model with two common components: a persistent and a transitory component. This model is defined by the following set of equations:

$$\begin{aligned} -\log M_{t+1} &= \alpha + \chi z_t + \sqrt{\gamma z_t}u_{t+1} + \tau z_t^{\mathbb{P}} + \sqrt{\delta z_t^{\mathbb{P}}}u_{t+1}^{\mathbb{P}}, \\ z_{t+1} &= (1-\phi)\theta + \phi z_t - \sigma\sqrt{z_t}u_{t+1}, \\ z_{t+1}^{\mathbb{P}} &= (1-\phi^{\mathbb{P}})\theta^{\mathbb{P}} + \phi^p z_t^{\mathbb{P}} - \sigma^{\mathbb{P}}\sqrt{z_t^{\mathbb{P}}}u_{t+1}^{\mathbb{P}}, \end{aligned}$$

where z_t is the transitory factor, and $z_t^{\mathbb{P}}$ is the permanent factor.

Bond Prices The nominal log zero-coupon yield of maturity n months in the currency of country i is given by the standard affine expression:

$$y_t^{(n),\$} = -\frac{1}{n}\left(\tilde{A}_n + \tilde{B}_n z_t + \tilde{C}_n z_t^{\mathbb{P}}\right),$$

where the coefficients satisfy the second-order difference equation:

$$\begin{aligned} \tilde{A}_n &= -\alpha + \tilde{A}_{n-1} + \tilde{B}_{n-1}(1-\phi)\theta + \tilde{C}_{n-1}\left(1-\phi^{\mathbb{P}}\right)\theta^{\mathbb{P}}, \\ \tilde{B}_n &= -\left(\chi - \frac{1}{2}\gamma\right) + \tilde{B}_{n-1}(\phi + \sigma\sqrt{\gamma}) + \frac{1}{2}\left(\tilde{B}_{n-1}\sigma\right)^2, \\ \tilde{C}_n &= -\left(\tau - \frac{1}{2}\delta\right) + \tilde{C}_{n-1}\left(\phi^{\mathbb{P}} + \sigma^{\mathbb{P}}\sqrt{\delta}\right) + \frac{1}{2}\left(\tilde{C}_{n-1}\sigma^p\right)^2. \end{aligned} \tag{8}$$

The nominal log zero-coupon price of maturity n months in the currency of country i is given by the standard affine expression

$$p_t^{(n),\$} = \left(\tilde{A}_n + \tilde{B}_n z_t + \tilde{C}_n z_t^{\mathbb{P}}\right).$$

Bond prices are defined recursively by the Euler equation: $P_t^{(n)} = E_t(M_{t+1}P_{t+1}^{n-1})$, and the price of a one period-bond is given by:

$$P^{(1)} = E_t(M_{t+1}) = e^{-\alpha-(\chi-\frac{1}{2}\gamma)z_t-\left(\tau-\frac{1}{2}\delta\right)z_t^{\mathbb{P}}},$$

which implies that the nominal risk-free interest rate (in logarithms) is given by this affine function of the persistent component and the transitory component:

$$r_t^f = \alpha + \left(\chi - \frac{1}{2}\gamma\right)z_t + \left(\tau - \frac{1}{2}\delta\right)z_t^{\mathbb{P}}.$$

Bond Risk Premia The log of the holding period return on a zero-coupon bond of maturity n between date t and $t+1$ is :

$$
\begin{aligned}
r_{t+1}^{(n)} &= -A_n + A_{n-1} + B_{n-1} z_{t+1} - B_n z_t + C_{n-1} z_{t+1}^{\mathbb{P}} - C_n z_t^{\mathbb{P}} \\
&= -A_n + A_{n-1} + B_{n-1}[(1-\phi)\theta + \phi z_t - \sigma\sqrt{z_t} u_{t+1}] + C_{n-1}[(1-\phi^{\mathbb{P}})\theta^{\mathbb{P}} + \phi^{\mathbb{P}} z_t^{\mathbb{P}} - \sigma^{\mathbb{P}} \sqrt{z_t^{\mathbb{P}}} u_{t+1}] \\
&- [-\left(\chi - \frac{1}{2}\gamma\right) + \tilde{B}_{n-1}\left(\phi + \sigma\sqrt{\gamma}\right) + \frac{1}{2}\left(\tilde{B}_{n-1}\sigma\right)^2] z_t, \\
&- [-\left(\tau - \frac{1}{2}\delta\right) + \tilde{C}_{n-1}\left(\phi^{\mathbb{P}} + \sigma^{\mathbb{P}}\sqrt{\delta}\right) + \frac{1}{2}\left(\tilde{C}_{n-1}\sigma^{\mathbb{P}}\right)^2] z_t^{\mathbb{P}} \\
&= -A_n + A_{n-1} + B_1^{n-1}(1-\phi)\theta + C_{n-1}(1-\phi^{\mathbb{P}})\theta^{\mathbb{P}} \\
&+ \left[\left(\chi - \frac{1}{2}\gamma\right) - \tilde{B}_{n-1}\left(\phi + \sigma\sqrt{\gamma}\right) - \frac{1}{2}\left(\tilde{B}_{n-1}\sigma\right)^2\right] z_t, \\
&+ \left[\left(\tau - \frac{1}{2}\delta\right) - \tilde{C}_{n-1}\left(\phi^{\mathbb{P}} + \sigma^{\mathbb{P}}\sqrt{\delta}\right) - \frac{1}{2}\left(\tilde{C}_{n-1}\sigma^{p}\right)^2\right] z_t^{p} \\
&- B_{n-1}\sigma\sqrt{z_t} u_{t+1} - C_{n-1}\sigma^{\mathbb{P}} \sqrt{z_t^{p} u_{t+1}^{\mathbb{P}}}.
\end{aligned}
$$

Thus, the log holding period return minus the risk-free rate is:

$$
\begin{aligned}
rx_{t+1}^{(n)} &= r_{t+1}^{(n)} - (\alpha + \left(\chi - \frac{1}{2}\gamma\right) z_t + \left(\tau - \frac{1}{2}\delta\right) z_t^{\mathbb{P}}), \\
&= -A_n + A_{n-1} + B^{n-1}(1-\phi)\theta + C^{n-1}(1-\phi^{\mathbb{P}})\theta^{\mathbb{P}} - \alpha \\
&- \left[\tilde{B}_{n-1}\left(\sigma\sqrt{\gamma}\right) + \frac{1}{2}\left(\tilde{B}_{n-1}\sigma\right)^2\right] z_t - C_1^{n-1}\sigma\sqrt{z_t} u_{t+1}, \\
&- \left[\tilde{C}_{n-1}\left(\sigma^{\mathbb{P}}\sqrt{\delta}\right) + \frac{1}{2}\left(\tilde{C}_{n-1}\sigma^{p}\right)^2\right] z_t^{\mathbb{P}} - C_1^{n-1}\sigma\sqrt{z_t^{\mathbb{P}}} u_{t+1}^{\mathbb{P}}.
\end{aligned}
$$

The expected log excess return on an n-maturity zero coupon bond is thus:

$$
\begin{aligned}
E_t[rx_{t+1}^{(n)}] &= -\left[\frac{1}{2}\left(B_1^{n-1}\right)^2 \sigma^2 + \sigma\sqrt{\gamma} B_1^{n-1}\right] z_t, \\
&- \left[\frac{1}{2}\left(C_1^{n-1}\right)^2 (\sigma^{\mathbb{P}})^2 + \sigma\sqrt{\gamma} C_1^{n-1}\right] z_t^{\mathbb{P}}.
\end{aligned}
$$

The expected log excess return on an infinite maturity bond is thus:

$$
\begin{aligned}
E_t[rx_{t+1}^{(\infty)}] &= -\left[\frac{1}{2}\left(B_1^{\infty}\right)^2 \sigma^2 + \sigma\sqrt{\gamma} B_1^{\infty}\right] z_t, \\
&- \left[\frac{1}{2}\left(C_1^{\infty}\right)^2 (\sigma^{\mathbb{P}})^2 + \sigma\sqrt{\gamma} C_1^{\infty}\right] z_t^{\mathbb{P}}.
\end{aligned}
$$

Using the expression for B_1^{∞} and C_1^{∞} implicit in (9), this equation can be restated as follows:

$$
\begin{aligned}
E_t[rx_{t+1}^{(\infty)}] &= -\left[\left(B_1^{\infty}\right)(1-\phi) + \chi - \frac{1}{2}\gamma\right] z_t, \\
&- \left[\left(C_1^{\infty}\right)(1-\phi^{\mathbb{P}}) + \tau - \frac{1}{2}\delta\right] z_t^{\mathbb{P}}.
\end{aligned}
$$

To give content to the notion that z_t is transitory, we impose that $B_1^{\infty}(1-\phi) = \chi$. This restriction implies that the permanent component of the pricing kernel is not affected by the transitory factor z_t, as can easily be verified:

Using this result in expression 7, the permanent component of the stochastic discount factor reduces to:

$$
\begin{aligned}
\frac{M_{t+1}^{\mathbb{P}}}{M_t^{\mathbb{P}}} &= \frac{M_{t+1}}{M_t}\left(\frac{M_{t+1}^{\mathbb{T}}}{M_t^{\mathbb{T}}}\right)^{-1} \\
&= \beta^{-1}e^{-\alpha-\chi z_t-\sqrt{\gamma z_t}u_{t+1}-\tau z_t^{\mathbb{P}}-\sqrt{\delta z_t^{p}}u_{t+1}^{\mathbb{P}}}e^{-B_1^{\infty}\left[(\phi-1)(z_t-\theta)-\sigma\sqrt{z_t}u_{t+1}\right]} \\
&\times\ e^{-C_1^{\infty}\left[(\phi^{\mathbb{P}}-1)(z_t^{\mathbb{P}}-\theta^{\mathbb{P}})-\sigma^{\mathbb{P}}\sqrt{z_t^{\mathbb{P}}}u_{t+1}\right]} \\
&= \beta^{-1}e^{-\alpha-\chi\theta}e^{-C_1^{\infty}\left[(\phi^{\mathbb{P}}-1)(z_t^{\mathbb{P}}-\theta^{\mathbb{P}})-\sigma^{\mathbb{P}}\sqrt{z_t^{\mathbb{P}}}u_{t+1}\right]},
\end{aligned}
$$

which does not depend on z_t. Given this restriction, the bond risk premium is given by:

$$
E_t[rx_{t+1}^{(\infty)}] = \frac{1}{2}\gamma z_t - \left[\tau - \frac{1}{2}\delta + (C_1^{\infty})(1-\phi^{p})\right] z_t^{\mathbb{P}}.
$$

Foreign Pricing Kernel Both factors are common across countries, but we allow for heterogeneous factor loadings on these common or global factors following Lustig, Roussanov, and Verdelhan (2011). The foreign SDF is given by:

$$
\begin{aligned}
-\log M_{t+1}^{*} &= \alpha + \chi z_t + \sqrt{\gamma^{*} z_t}u_{t+1} + \tau z_t^{\mathbb{P}} + \sqrt{\delta^{*} z_t^{\mathbb{P}}}u_{t+1}^{\mathbb{P}}, \\
z_{t+1} &= (1-\phi)\theta + \phi z_t - \sigma\sqrt{z_t}u_{t+1}, \\
z_{t+1}^{\mathbb{P}} &= (1-\phi^{\mathbb{P}})\theta^{\mathbb{P}} + \phi^{\mathbb{P}} z_t^{\mathbb{P}} - \sigma^{\mathbb{P}}\sqrt{z_t^{\mathbb{P}}}u_{t+1}^{\mathbb{P}}
\end{aligned}
$$

where z_t is the transitory factor, and $z_t^{\mathbb{P}}$ is the permanent factor. As can easily be verified, the log currency risk premium is given by: $E_t[rx_{t+1}^{FX}] = \frac{1}{2}(\gamma-\gamma^{*})z_t + \frac{1}{2}(\delta-\delta^{*})z_t^{\mathbb{P}}$. This implies that the expected foreign log holding period return on a foreign long bond converted into U.S. dollars is :

$$
\begin{aligned}
E_t[rx_{t+1}^{(\infty),\$}] &= E_t[rx_{t+1}^{(\infty),*}] + E_t[rx_{t+1}^{FX}], \\
&= \frac{1}{2}\gamma z_t - \left[\tau - \frac{1}{2}\delta + (C_1^{\infty,*})(1-\phi^{\mathbb{P}})\right] z_t^{\mathbb{P}}
\end{aligned}
$$

where $\tilde{C}_{\infty,*}$ is defined by the following equation $\tilde{C}_{\infty,*} = -\left(\tau - \frac{1}{2}\delta^{*}\right) + \tilde{C}_{\infty,*}\left(\phi^{\mathbb{P}} + \sigma^{\mathbb{P}}\sqrt{\delta^{*}}\right) + \frac{1}{2}\left(\tilde{C}_{\infty,*}\sigma^{\mathbb{P}}\right)^2$. Hence, the difference between the foreign and the domestic term premium is given by: $(C_1^{\infty,*} - C_1^{\infty})(1-\phi^{\mathbb{P}})z_t^{\mathbb{P}}$. In the symmetric case in which $\delta = \delta^{*}$, the foreign term premium in dollars equals the domestic term premium. If $\gamma > \gamma^{*}$, there is a large positive foreign currency risk premium $E_t[rx_{t+1}^{FX}] = \frac{1}{2}(\gamma-\gamma^{*})z_t$, but that is exactly offset by a smaller foreign term premium.

Online Appendix for "The Term Structure of Currency Carry Trade Risk Premia" —Not For Publication—

This online appendix describes additional empirical results on the cross-section of currency and term risk premia. Section A reports additional results on portfolios of countries sorted by the slope of the yield curves. Section B reports similar results for portfolios of countries sorted by the short-term interest rates. Section C reports additional time-series tests of the uncovered bond return parity condition. Section D focuses on the decomposition of real exchange rates into a permanent and a temporary component.

A Sorting Currencies by the Slope of the Yield Curve

Figure 7 presents the composition over time of portfolios of the 9 currencies of the benchmark sample sorted by the slope of the yield curve.

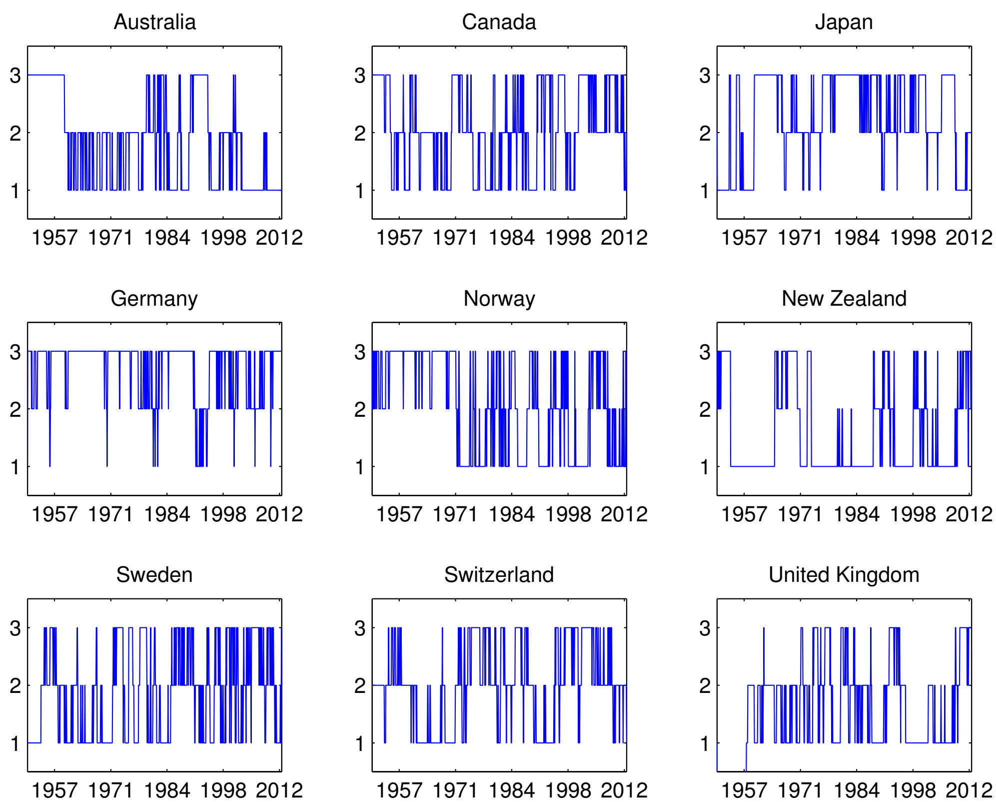

Figure 7: Composition of Slope-Sorted Portfolios — The figure presents the composition of portfolios of the currencies in the benchmark sample sorted by the slope of their yield curves. The portfolios are rebalanced monthly. The slope of the yield curve is measured as the 10-year interest rate minus the one-month Treasury bill rates. Data are monthly, from 12/1950 to 12/2012.

Table 5 reports the results of sorting on the yield curve slope on the sample of developed countries, whereas Table 6 reports the results obtained from using the entire cross-section of countries, including emerging countries. The results are commented in the main text.

Table 5: Slope Sorted Portfolios: Developed sample

Portfolio		1	2	3	4	4 − 1	1	2	3	4	4 − 1	1	2	3	4	4 − 1
Horizon		1-month					3-month					12-month				
Panel A: 1950-2012																
$-\Delta s$	Mean	-0.78	0.14	0.05	0.67	1.45	-0.94	0.25	-0.06	0.68	1.62	-0.79	0.13	-0.01	0.48	1.28
$f-s$	Mean	3.69	1.64	0.82	-0.18	-3.87	3.60	1.63	0.85	-0.12	-3.71	3.33	1.62	0.90	0.06	-3.27
rx^{FX}	Mean	2.91	1.78	0.87	0.49	-2.42	2.66	1.88	0.79	0.56	-2.10	2.54	1.74	0.89	0.54	-1.99
	s.e.	[0.96]	[1.01]	[1.05]	[1.03]	[0.63]	[1.09]	[1.07]	[1.11]	[1.05]	[0.64]	[1.22]	[1.07]	[1.23]	[1.15]	[0.65]
	Std	7.62	7.92	8.16	8.08	5.03	8.33	8.08	8.59	8.09	4.95	9.32	8.68	9.44	8.67	4.88
	SR	0.38	0.22	0.11	0.06	-0.48	0.32	0.23	0.09	0.07	-0.42	0.27	0.20	0.09	0.06	-0.41
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]
$rx^{(10),*}$	Mean	-1.96	0.27	2.27	3.95	5.90	-1.29	0.95	1.88	3.15	4.44	-0.33	1.08	1.60	2.20	2.52
	s.e.	[0.51]	[0.52]	[0.51]	[0.74]	[0.84]	[0.61]	[0.58]	[0.61]	[0.78]	[0.86]	[0.68]	[0.86]	[0.67]	[1.08]	[1.01]
	Std	4.05	4.08	4.02	5.84	6.60	4.89	4.72	4.67	6.43	7.01	5.86	5.82	5.68	6.87	6.97
	SR	-0.48	0.07	0.56	0.68	0.89	-0.26	0.20	0.40	0.49	0.63	-0.06	0.19	0.28	0.32	0.36
	s.e.	[0.13]	[0.13]	[0.13]	[0.14]	[0.16]	[0.13]	[0.13]	[0.13]	[0.14]	[0.15]	[0.13]	[0.14]	[0.13]	[0.13]	[0.18]
$rx^{(10),\$}$	Mean	0.95	2.05	3.14	4.44	3.48	1.37	2.83	2.67	3.71	2.34	2.21	2.82	2.49	2.74	0.53
	s.e.	[1.09]	[1.15]	[1.15]	[1.39]	[1.10]	[1.22]	[1.18]	[1.28]	[1.40]	[1.12]	[1.36]	[1.35]	[1.37]	[1.59]	[1.30]
	Std	8.59	9.06	8.97	10.91	8.72	9.51	9.23	9.72	11.27	9.09	10.86	10.37	11.18	11.36	9.45
	SR	0.11	0.23	0.35	0.41	0.40	0.14	0.31	0.27	0.33	0.26	0.20	0.27	0.22	0.24	0.06
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-0.56	0.53	1.63	2.93	3.48	-0.15	1.31	1.15	2.19	2.34	0.67	1.28	0.95	1.20	0.53
	s.e.	[1.25]	[1.23]	[1.19]	[1.46]	[1.10]	[1.40]	[1.24]	[1.26]	[1.42]	[1.12]	[1.47]	[1.35]	[1.43]	[1.63]	[1.30]
Panel B: 1971-2012																
$-\Delta s$	Mean	-0.96	0.18	0.29	0.84	1.80	-1.20	0.37	0.00	0.83	2.03	-1.18	0.27	0.05	0.54	1.73
$f-s$	Mean	4.29	1.89	1.03	-0.20	-4.49	4.18	1.87	1.06	-0.11	-4.29	3.87	1.86	1.12	0.13	-3.74
rx^{FX}	Mean	3.33	2.07	1.32	0.64	-2.69	2.98	2.24	1.06	0.72	-2.26	2.69	2.13	1.18	0.68	-2.01
	s.e.	[1.44]	[1.51]	[1.52]	[1.55]	[0.94]	[1.61]	[1.60]	[1.62]	[1.58]	[0.95]	[1.82]	[1.62]	[1.82]	[1.75]	[0.95]
	Std	9.22	9.71	9.75	9.90	5.99	10.13	9.87	10.35	9.90	5.94	11.35	10.55	11.43	10.61	5.90
	SR	0.36	0.21	0.14	0.07	-0.45	0.29	0.23	0.10	0.07	-0.38	0.24	0.20	0.10	0.06	-0.34
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]	[0.16]	[0.18]
$rx^{(10),*}$	Mean	-1.89	0.75	2.56	4.76	6.65	-1.10	1.72	2.32	3.50	4.60	0.07	1.81	2.03	2.17	2.10
	s.e.	[0.74]	[0.73]	[0.75]	[1.09]	[1.22]	[0.89]	[0.86]	[0.87]	[1.14]	[1.26]	[0.95]	[1.26]	[0.95]	[1.62]	[1.52]
	Std	4.77	4.75	4.77	6.92	7.79	5.77	5.58	5.50	7.61	8.33	6.85	6.77	6.69	8.12	8.33
	SR	-0.40	0.16	0.54	0.69	0.85	-0.19	0.31	0.42	0.46	0.55	0.01	0.27	0.30	0.27	0.25
	s.e.	[0.16]	[0.16]	[0.15]	[0.17]	[0.18]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.16]	[0.18]	[0.17]	[0.16]	[0.19]
$rx^{(10),\$}$	Mean	1.44	2.82	3.87	5.40	3.96	1.88	3.96	3.38	4.21	2.34	2.76	3.94	3.21	2.85	0.09
	s.e.	[1.60]	[1.70]	[1.66]	[2.09]	[1.63]	[1.77]	[1.75]	[1.85]	[2.07]	[1.66]	[1.97]	[1.97]	[1.97]	[2.40]	[1.95]
	Std	10.29	10.98	10.67	13.24	10.35	11.40	11.14	11.60	13.61	10.87	12.91	12.27	13.29	13.67	11.38
	SR	0.14	0.26	0.36	0.41	0.38	0.16	0.36	0.29	0.31	0.22	0.21	0.32	0.24	0.21	0.01
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.16]
$rx^{(10),\$}-rx^{(10),US}$	Mean	-1.06	0.31	1.37	2.90	3.96	-0.66	1.43	0.85	1.68	2.34	0.19	1.37	0.64	0.28	0.09
	s.e.	[1.76]	[1.74]	[1.66]	[2.14]	[1.63]	[2.01]	[1.79]	[1.80]	[2.08]	[1.66]	[2.15]	[1.96]	[2.06]	[2.43]	[1.95]

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 21 currencies sorted every month by the slope of the yield curve (10-year yield minus T-bill rate) at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

Table 6: Slope Sorted Portfolios: Whole sample

Portfolio		1	2	3	4	5	$5-1$	1	2	3	4	5	$5-1$	1	2	3	4	5	$5-1$
Horizon		1-month						3-month						12-month					
		Panel A: 1950-2012																	
$-\Delta s$	Mean	-2.15	-0.71	-0.18	0.27	-0.47	1.67	-2.47	-0.53	0.02	-0.11	-0.32	2.14	-2.19	-0.59	-0.15	0.02	-0.55	1.64
$f-s$	Mean	4.63	2.06	1.28	0.52	-0.08	-4.71	4.45	2.04	1.30	0.54	0.50	-3.95	4.12	1.99	1.30	0.74	0.27	-3.85
rx^{FX}	Mean	2.49	1.35	1.10	0.79	-0.55	-3.04	1.99	1.51	1.32	0.43	0.18	-1.81	1.93	1.40	1.15	0.76	-0.28	-2.21
	s.e.	0.94	0.91	0.98	1.03	0.82	0.72	1.04	0.97	1.05	1.08	0.84	0.73	1.16	1.02	1.11	1.18	0.89	0.81
	Std	7.48	7.10	7.70	8.00	6.37	5.65	8.11	7.66	7.92	8.14	8.91	8.56	8.90	8.16	8.76	9.70	6.87	6.06
	SR	0.33	0.19	0.14	0.10	-0.09	-0.54	0.24	0.20	0.17	0.05	0.02	-0.21	0.22	0.17	0.13	0.08	-0.04	-0.36
	s.e.	0.14	0.13	0.13	0.13	0.13	0.16	0.14	0.13	0.13	0.13	0.13	0.16	0.13	0.13	0.13	0.13	0.13	0.14
$rx^{(10),*}$	Mean	-3.32	-0.82	1.46	2.56	5.05	8.37	-2.56	-0.03	1.43	2.11	3.83	6.38	-1.32	0.32	1.50	1.55	3.07	4.40
	s.e.	0.53	0.49	0.46	0.49	0.66	0.81	0.62	0.54	0.56	0.59	0.68	0.82	0.58	0.69	0.75	0.66	0.96	0.89
	Std	4.14	3.88	3.65	3.91	5.20	6.31	4.87	4.42	4.40	4.62	8.46	9.18	5.05	5.42	5.57	6.80	6.06	6.36
	SR	-0.80	-0.21	0.40	0.66	0.97	1.33	-0.53	-0.01	0.32	0.46	0.45	0.70	-0.26	0.06	0.27	0.23	0.51	0.69
	s.e.	0.11	0.12	0.13	0.14	0.15	0.15	0.11	0.13	0.13	0.13	0.15	0.16	0.14	0.14	0.14	0.14	0.12	0.17
$rx^{(10),\$}$	Mean	-0.83	0.53	2.56	3.35	4.50	5.33	-0.57	1.48	2.74	2.54	4.00	4.57	0.61	1.72	2.65	2.31	2.80	2.19
	s.e.	1.09	1.06	1.08	1.17	1.16	1.14	1.24	1.07	1.16	1.28	1.18	1.18	1.28	1.16	1.38	1.34	1.33	1.20
	Std	8.80	8.23	8.53	9.17	9.08	8.95	9.84	8.71	9.04	9.73	9.35	9.63	10.70	9.59	10.73	10.83	9.29	9.32
	SR	-0.09	0.06	0.30	0.37	0.50	0.60	-0.06	0.17	0.30	0.26	0.43	0.47	0.06	0.18	0.25	0.21	0.30	0.23
	s.e.	0.13	0.13	0.13	0.13	0.14	0.12	0.13	0.13	0.13	0.13	0.13	0.12	0.13	0.13	0.13	0.13	0.13	0.13
$rx^{(10),\$}-rx^{(10),US}$	Mean	-2.34	-0.99	1.04	1.84	2.99	5.33	-2.09	-0.04	1.22	1.02	2.48	4.57	-0.94	0.18	1.11	0.76	1.25	2.19
	s.e.	1.32	1.19	1.19	1.19	1.33	1.14	1.50	1.21	1.22	1.27	1.33	1.18	1.53	1.21	1.51	1.30	1.45	1.20
		Panel B: 1971-2012																	
$-\Delta s$	Mean	-2.91	-0.62	-0.10	0.13	-0.96	1.95	-3.30	-0.43	-0.07	-0.08	-0.85	2.45	-2.73	-0.65	0.03	-0.08	-1.05	1.68
$f-s$	Mean	5.54	2.37	1.53	0.66	-0.11	-5.65	5.30	2.35	1.55	0.68	0.05	-5.25	4.94	2.29	1.55	0.74	0.33	-4.61
rx^{FX}	Mean	2.63	1.75	1.44	0.79	-1.06	-3.70	2.01	1.91	1.49	0.60	-0.80	-2.81	2.20	1.65	1.58	0.66	-0.72	-2.93
	s.e.	1.39	1.32	1.41	1.37	1.13	1.06	1.53	1.41	1.45	1.48	1.16	1.13	1.60	1.50	1.55	1.67	1.24	1.11
	Std	8.95	8.39	9.02	8.74	7.27	6.80	9.72	9.07	9.38	9.11	7.25	7.19	10.49	9.75	10.37	10.24	7.80	7.02
	SR	0.29	0.21	0.16	0.09	-0.15	-0.54	0.21	0.21	0.16	0.07	-0.11	-0.39	0.21	0.17	0.15	0.06	-0.09	-0.42
	s.e.	0.17	0.16	0.16	0.16	0.16	0.20	0.16	0.16	0.16	0.16	0.16	0.18	0.17	0.16	0.16	0.16	0.16	0.17
$rx^{(10),*}$	Mean	-3.73	-0.56	1.40	3.81	6.13	9.85	-2.73	0.47	1.54	2.93	5.15	7.87	-1.19	0.72	2.04	2.20	3.54	4.73
	s.e.	0.77	0.72	0.66	0.70	0.90	1.11	0.89	0.78	0.84	0.80	0.93	1.15	0.82	1.00	1.12	0.89	1.32	1.23
	Std	4.90	4.61	4.30	4.51	5.81	7.10	5.72	5.29	5.36	5.34	6.18	7.55	5.85	6.31	6.64	6.49	6.68	7.10
	SR	-0.76	-0.12	0.33	0.85	1.06	1.39	-0.48	0.09	0.29	0.55	0.83	1.04	-0.20	0.11	0.31	0.34	0.53	0.67
	s.e.	0.14	0.15	0.16	0.16	0.18	0.18	0.14	0.16	0.16	0.16	0.18	0.18	0.16	0.18	0.19	0.18	0.16	0.20
$rx^{(10),\$}$	Mean	-1.10	1.19	2.84	4.60	5.06	6.16	-0.72	2.39	3.03	3.53	4.35	5.07	1.02	2.37	3.62	2.86	2.82	1.80
	s.e.	1.63	1.52	1.56	1.59	1.60	1.61	1.80	1.55	1.61	1.77	1.63	1.72	1.77	1.70	1.94	1.89	1.87	1.74
	Std	10.49	9.74	10.03	10.17	10.31	10.25	11.69	10.33	10.66	11.09	10.49	11.16	12.56	11.25	12.53	12.62	10.38	10.66
	SR	-0.10	0.12	0.28	0.45	0.49	0.60	-0.06	0.23	0.28	0.32	0.41	0.45	0.08	0.21	0.29	0.23	0.27	0.17
	s.e.	0.15	0.16	0.16	0.16	0.17	0.15	0.15	0.16	0.16	0.16	0.16	0.15	0.16	0.17	0.16	0.17	0.16	0.17
$rx^{(10),\$}-rx^{(10),US}$	Mean	-3.60	-1.32	0.33	2.09	2.56	6.16	-3.25	-0.14	0.50	1.00	1.82	5.07	-1.55	-0.20	1.06	0.29	0.25	1.80
	s.e.	1.89	1.63	1.65	1.56	1.78	1.61	2.17	1.68	1.64	1.74	1.77	1.72	2.17	1.72	2.12	1.78	2.00	1.74

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 30 currencies sorted every month by the slope of the yield curve (10-year yield minus T-bill rate) at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, India, Ireland, Italy, Japan Mexico, Malaysia, the Netherlands, New Zealand, Norway, Pakistan, the Philippines, Poland, Portugal, South Africa, Singapore, Spain, Sweden, Switzerland, Taiwan, Thailand, and the United Kingdom.

Figure 8hows the local currency log excess returns in the top panel, and the dollar log excess returns in the bottom panel as a function of the bond maturities for zero-coupon bonds of our extended sample of developed countries. The results are also commented in the main text.

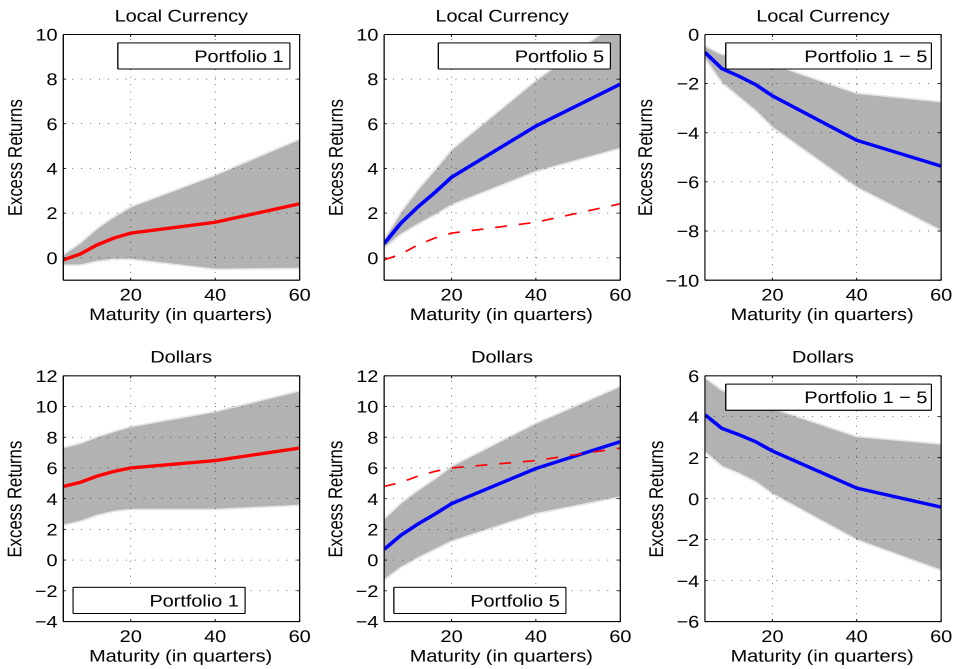

Figure 8: Term Structure of Dollar Bond Risk Premia: Extended Sample — The figure shows the local currency log excess returns in the top panel, and the dollar log excess returns in the bottom panel as a function of the bond maturities. The left panel focuses on Portfolio 1 (flat yield curve currencies) excess returns, while the middle panel reports Portfolio 5 (steep yield curve currencies) excess returns. The middle panels also report the Portfolio 1 excess returns in dashed lines for comparison. The right panel reports the difference. Data are monthly, from the zero-coupon dataset, and the sample window is 5/1987–12/2012. The unbalanced sample includes Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Germany, Hungary, Indonesia, Ireland, Italy, Japan, Malaysia, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into five portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The holding period is one quarter. The returns are annualized.

B Sorting Currencies by Interest Rates

Benchmark Sample Figure 9 plots the composition of the three interest rate-sorted portfolios of the currencies of the benchmark sample, ranked from low to high interest rate currencies. Typically, Switzerland and Japan (after 1970) are funding currencies in Portfolio 1, while Australia and New Zealand are the carry trade investment currencies in Portfolio 3. The other currencies switch between portfolios quite often.

Table 7 reports the annualized moments of log returns. The structure of the table is the same as of Table 1. As expected [see Lustig and Verdelhan (2007) for a detailed analysis], the average excess returns increase from Portfolio 1 to Portfolio 3. The average excess return on Portfolio 1 is -0.24% per annum, while the average excess return on Portfolio 3 is 3.26%. The spread between Portfolio 1 and Portfolio 3 is 3.51% per annum. The volatility of these returns increases only slightly from the first to the last portfolio. As a result, the Sharpe ratio (annualized) increases from -0.03 on Portfolio 1 to 0.40 on the Portfolio 3. The Sharpe ratio on a long position in Portfolio 3 and a short position in the Portfolio 1 is 0.49 per annum. The results for the post-Bretton-Woods sample are very similar. Hence, the currency carry trade is profitable at the short end of the maturity spectrum.

Table 7: Interest Rate-Sorted Portfolios: Benchmark Sample

Portfolio		1	2	3	3 − 1	1	2	3	3 − 1	1	2	3	3 − 1
Horizon		1-Month				3-Month				12-Month			
						Panel A: 12/1950–12/2012							
$-\Delta s$	Mean	1.43	0.35	-0.18	-1.62	1.65	0.23	-0.28	-1.93	1.78	0.22	-0.42	-2.20
$f - s$	Mean	-1.68	0.78	3.45	5.13	-1.64	0.81	3.39	5.03	-1.55	0.91	3.19	4.74
rx^{FX}	Mean	-0.24	1.13	3.26	3.51	0.00	1.03	3.11	3.10	0.23	1.12	2.77	2.54
	s.e.	[1.00]	[0.84]	[1.03]	[0.92]	[1.02]	[0.91]	[1.20]	[1.03]	[1.10]	[1.03]	[1.25]	[0.98]
	Std	7.92	6.64	8.10	7.22	8.29	6.91	8.74	7.97	9.20	7.32	9.59	8.03
	SR	-0.03	0.17	0.40	0.49	0.00	0.15	0.36	0.39	0.03	0.15	0.29	0.32
	s.e.	[0.13]	[0.13]	[0.14]	[0.14]	[0.13]	[0.13]	[0.14]	[0.15]	[0.13]	[0.13]	[0.14]	[0.15]
$rx^{(10),*}$	Mean	2.39	1.67	-0.21	-2.60	2.17	1.31	0.41	-1.76	1.98	1.07	0.85	-1.13
	s.e.	[0.45]	[0.47]	[0.57]	[0.57]	[0.50]	[0.53]	[0.64]	[0.61]	[0.58]	[0.65]	[0.72]	[0.65]
	Std	3.50	3.65	4.50	4.47	4.01	4.34	5.10	4.72	4.41	5.22	5.67	5.05
	SR	0.68	0.46	-0.05	-0.58	0.54	0.30	0.08	-0.37	0.45	0.20	0.15	-0.22
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.13]	[0.12]	[0.15]	[0.14]	[0.13]	[0.12]
$rx^{(10),\$}$	Mean	2.15	2.81	3.06	0.91	2.17	2.34	3.52	1.35	2.21	2.19	3.62	1.41
	s.e.	[1.18]	[0.98]	[1.16]	[1.09]	[1.22]	[1.06]	[1.31]	[1.21]	[1.26]	[1.14]	[1.42]	[1.23]
	Std	9.33	7.70	9.21	8.61	10.00	8.22	9.94	9.32	10.64	8.83	11.08	9.95
	SR	0.23	0.36	0.33	0.11	0.22	0.28	0.35	0.14	0.21	0.25	0.33	0.14
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.14]
$rx^{(10),\$} - rx^{(10),US}$	Mean	0.63	1.30	1.55	0.91	0.65	0.82	2.00	1.35	0.67	0.65	2.08	1.41
	s.e.	[1.22]	[1.09]	[1.32]	[1.09]	[1.16]	[1.18]	[1.51]	[1.21]	[1.33]	[1.22]	[1.61]	[1.23]
						Panel B: 12/1971–12/2012							
$-\Delta s$	Mean	2.16	0.30	-0.29	-2.45	2.39	0.14	-0.49	-2.89	2.61	0.05	-0.74	-3.35
$f - s$	Mean	-2.03	1.07	3.87	5.90	-1.99	1.11	3.80	5.78	-1.89	1.23	3.60	5.49
rx^{FX}	Mean	0.13	1.38	3.57	3.44	0.41	1.25	3.30	2.90	0.72	1.28	2.86	2.14
	s.e.	[1.52]	[1.26]	[1.50]	[1.35]	[1.56]	[1.37]	[1.75]	[1.52]	[1.67]	[1.55]	[1.82]	[1.42]
	Std	9.66	8.11	9.57	8.46	10.12	8.42	10.46	9.52	11.22	8.88	11.59	9.68
	SR	0.01	0.17	0.37	0.41	0.04	0.15	0.32	0.30	0.06	0.14	0.25	0.22
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.16]	[0.17]	[0.17]	[0.18]
$rx^{(10),*}$	Mean	2.82	2.12	-0.13	-2.95	2.60	1.66	0.56	-2.05	2.48	1.30	1.01	-1.47
	s.e.	[0.64]	[0.68]	[0.81]	[0.79]	[0.72]	[0.77]	[0.92]	[0.87]	[0.84]	[0.94]	[1.01]	[0.87]
	Std	4.12	4.33	5.17	5.09	4.69	5.12	5.85	5.33	5.17	6.06	6.31	5.45
	SR	0.68	0.49	-0.02	-0.58	0.55	0.32	0.10	-0.38	0.48	0.21	0.16	-0.27
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.15]	[0.19]	[0.18]	[0.16]	[0.15]
$rx^{(10),\$}$	Mean	2.95	3.49	3.45	0.50	3.01	2.90	3.86	0.85	3.20	2.58	3.87	0.67
	s.e.	[1.78]	[1.46]	[1.71]	[1.59]	[1.85]	[1.60]	[1.89]	[1.77]	[1.91]	[1.66]	[2.03]	[1.75]
	Std	11.33	9.34	10.82	10.06	12.09	9.91	11.68	10.95	12.85	10.44	13.01	11.70
	SR	0.26	0.37	0.32	0.05	0.25	0.29	0.33	0.08	0.25	0.25	0.30	0.06
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.16]	[0.17]	[0.18]	[0.16]
$rx^{(10),\$} - rx^{(10),US}$	Mean	0.44	0.99	0.94	0.50	0.48	0.37	1.33	0.85	0.64	0.01	1.31	0.67
	s.e.	[1.77]	[1.55]	[1.88]	[1.59]	[1.70]	[1.71]	[2.16]	[1.77]	[1.99]	[1.78]	[2.32]	[1.75]

Notes: The table reports the average change in exchange rates (Δs), the average interest rate difference ($f - s$), the average currency excess return (rx^{FX}), the average foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*}$) and in U.S. dollars ($rx^{(10),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(10),\$} - rx^{US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The annualized monthly log returns are realized at date $t + k$, where the horizon k equals 1, 3, and 12 months. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. The countries are sorted by the level of their short term interest rates into three portfolios. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns.

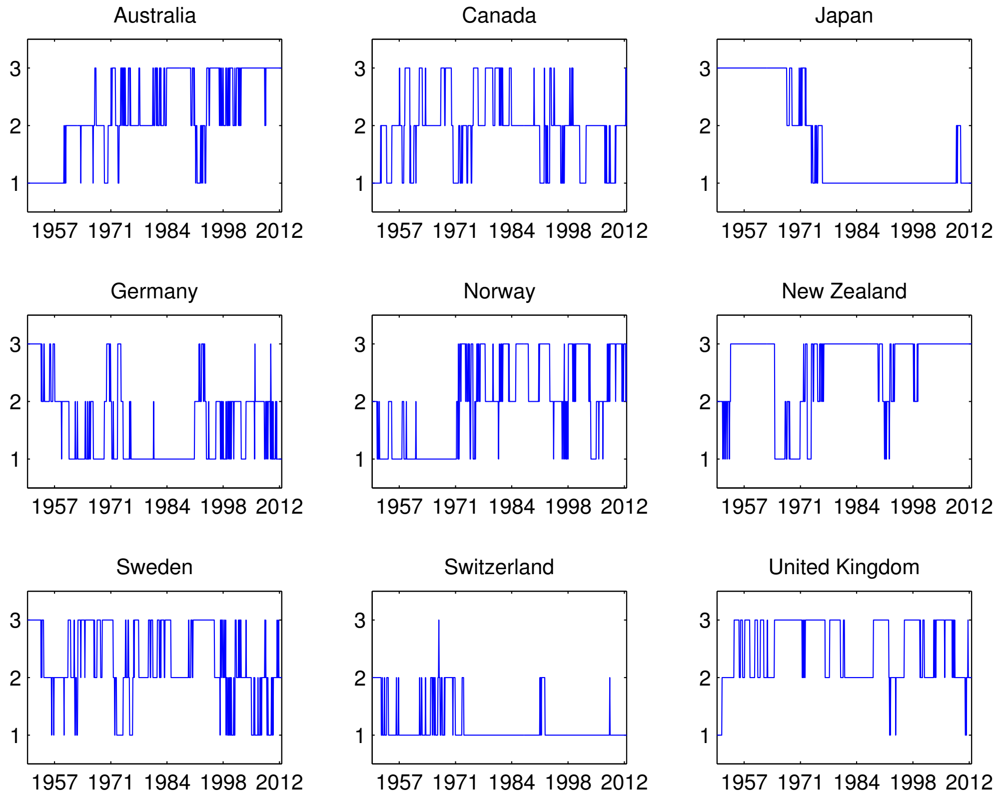

Figure 9: Composition of Interest Rate-Sorted Portfolios — The figure presents the composition of portfolios of 9 currencies sorted by their short-term interest rates. The portfolios are rebalanced monthly. Data are monthly, from 12/1950 to 12/2012.

Recall that the absence of arbitrage implies a negative relationship between the equilibrium risk premium for investing in a currency and the SDF entropy of the corresponding country. Therefore, given the pattern in currency risk premia, high interest rate currencies have low entropy and low interest rate currencies have high entropy. As a result, sorting by interest rates (from low to high) seems equivalent to sorting by pricing kernel entropy (from high to low). In a log-normal world, entropy is just one half of the variance: high interest rate currencies have low pricing kernel variance, while low interest rate currencies have volatile pricing kernels.

Table 7 shows that there is a strong decreasing pattern in local currency bond risk premia. The average excess return on Portfolio 1 is 2.39% per annum and its Sharpe ratio is 0.68. The excess return decreases monotonically to -0.21% on Portfolio 3. Thus, there is a 2.60% spread per annum between Portfolio 1 and Portfolio 3.

If all of the shocks driving currency risk premia were permanent, then there would be no relation between currency risk premia and term premia. To the contrary, we find a very strong negative association between local currency bond risk premia and currency risk premia. Low interest rate currencies tend to produce high local currency bond risk premia, while high interest rate currencies tend to produce low local currency bond risk premia. The decreasing term premia are consistent with the decreasing entropy of the total SDF from low (Portfolio 1) to high interest rates (Portfolio 3) that we had inferred from the foreign currency risk premia. Furthermore, it appears that these are not offset by equivalent decreases in the entropy of the permanent component of the foreign pricing kernel.

The decline in the local currency bond risk premia partly offsets the increase in currency risk premia. As a result, the average excess return on the foreign bond expressed in U.S. dollars measured in Portfolio 3 is only 0.91% per annum higher than the average excess returns measured in Portfolio 1. The Sharpe ratio on a long-short

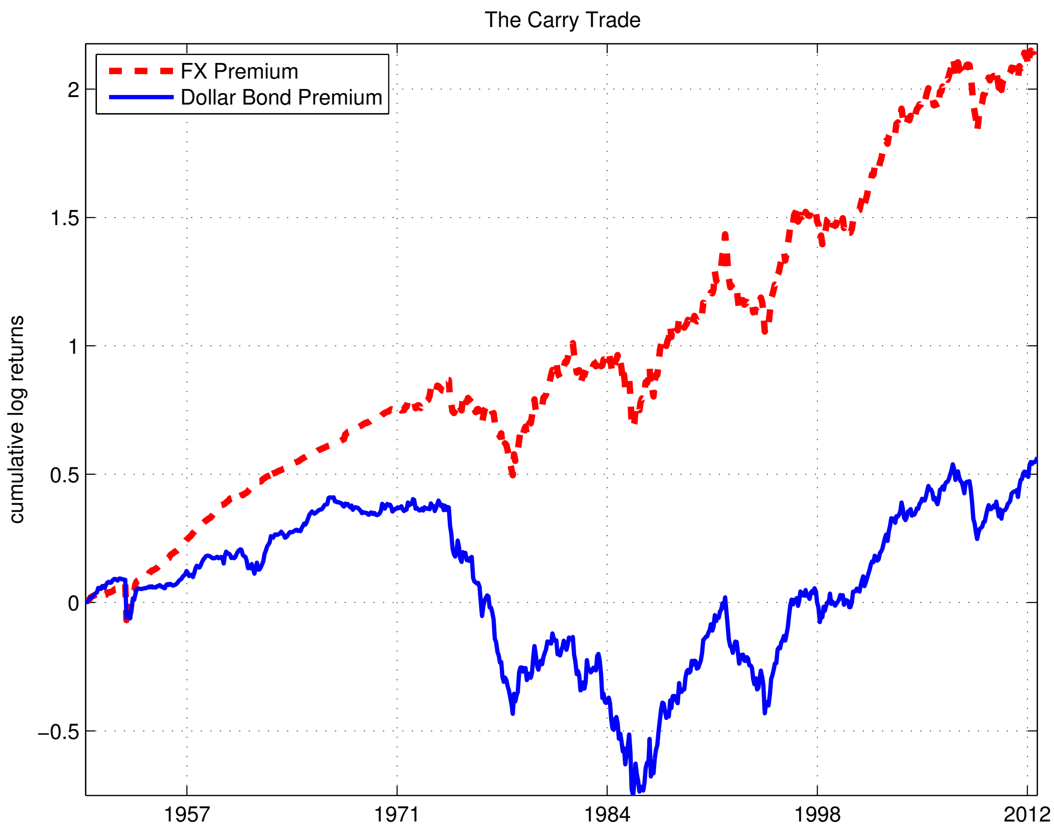

Figure 10: The Carry Trade and Term Premia – The figure presents the cumulative one-month log returns on investments in foreign Treasury bills and foreign 10-year bonds. The benchmark panel of countries includes Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the U.K. Countries are sorted every month by the level of their one-month interest rates into three portfolios. The returns correspond to a strategy going long in the Portfolio 3 and short in Portfolio 1. The sample period is 12/1950–12/2012.

position in bonds of Portfolio 3 and Portfolio 1 is only 0.11. U.S. investors cannot simply combine the currency carry trade with a yield carry trade, because these risk premia roughly offset each other. Interest rates are great predictors of currency excess returns and local currency bond excess returns, but not of dollar excess returns. To receive long-term carry trade returns, investors need to load on differences in the quantity of permanent risk, as shown in Equation (3). The cross-sectional evidence presented here does not lend much support to these differences in permanent risk.

Table 7 shows that the results are essentially unchanged in the post-Bretton-Woods sample. The Sharpe ratio on the currency carry trade is 0.41, achieved by going long in Portfolio 3 and short in Portfolio 1. However, there is a strong decreasing pattern in local currency bond risk premia, from 2.82% per annum in Portfolio 1 to −0.13% in the Portfolio 3. As a result, there is essentially no discernible pattern in dollar bond risk premia.

Figure 10 presents the cumulative one-month log returns on investments in foreign Treasury bills and foreign 10-year bonds. Most of the losses are concentrated in the 1970s and 1980s, and the bond returns do recover in the 1990s. In fact, between 1991 and 2012, the difference is currency risk premia at the one-month horizon between Portfolio 1 and Portfolio 3 is 4.54%, while the difference in the local term premia is only 1.41% per annum. As a result, the un-hedged carry trade in 10-year bonds still earn about 3.13% per annum over this sample. However, this difference of 3.13% per annum has a standard error of 1.77% and, therefore, is not statistically significant.

As we increase holding period k from 1 to 3 and 12 months, the differences in local bond risk premia between portfolios shrink, but so do the differences in currency risk premia. Even at the 12-month horizon, there is no evidence of statistically significant differences in dollar bond risk premia across the currency portfolios.

Robustness Checks: Developed Countries and Whole Sample In the robustness tests, very similar patterns of risk premia emerge using larger sets of countries. In the sample of developed countries, we sort currencies in four portfolios. Figure 12 plots the composition of the four interest rate-sorted currency portfolios. Finally, Table 9 reports the results of sorting all the currencies in our sample, including those of emerging countries, into portfolios according to the level of their interest rate, ranked from low to high interest rate currencies. Switzerland and Japan (after 1970) are funding currencies in Portfolio 1, while Australia and New Zealand are carry trade investment currencies in Portfolio 4. Credit risk seems to be concentrated in Portfolios 3 and 4.

Table 8 reports the results of sorting the developed country currencies into portfolios based on the level of their interest rate, ranked from low to high interest rate currencies. Essentially, the results are very similar to those obtained on the benchmark sample of developed countries. There is no economically or statistically significant carry trade premium at longer maturities. The 2.98% spread in the currency risk premia is offset by the negative 3.03% spread in local term premia at the one-month horizon against the carry trade currencies.

In the sample of developed and emerging countries, the pattern in returns is strikingly similar, but the differences are larger. At the one-month horizon, the 6.66% spread in the currency risk premia is offset by a 5.15% spread in local term premia. A long-short position in foreign bonds delivers an excess return of 1.51% per annum, which is not statistically significantly different from zero. At longer horizons, the differences in local bond risk premia are much smaller, but so are the carry trade returns.

As in the previous samples, the rate at which the high interest rate currencies depreciate (2.99% per annum) is not high enough to offset the interest rate difference of 6.55%. Similarly, the rate at which the low interest rate currencies appreciate (0.43% per annum) is not high enough to offset the low interest rates (3.52% lower than the U.S. interest rate). Uncovered interest rate parity fails in the cross-section. However, the bond return differences (in local currency) are closer to being offset by the rate of depreciation. The bond return spread is 4.63% per annum for the last portfolio, compared to an annual depreciation rate of 6.55%, while the spread on the first portfolio is −0.29%, compared to depreciation of −0.43%. In Figure 11, we plot the rate of depreciation against the interest rate (bond return) differences with the U.S. The vertical distance from the 45-degree line is an indication of how far exchange rates are from the uncovered interest rate parity or long-run uncovered bond parity. The size of the marker indicates the number of the portfolio. Especially for the first and last portfolios, long-run uncovered bond parity is a much better fit for the data than the uncovered interest rate parity. The currency exposure hedges the interest rate exposure in the bond position. High returns are offset by higher depreciations. As a result, foreign bond portfolios are almost hedged against foreign country-specific interest rate risk, while Treasury bill portfolios are not.

The Term Structure of Currency Carry Trade Risk Premia At the short end of the maturity spectrum, it is profitable to invest in flat-yield-curve currencies and short the currencies of countries with steep yield curves: the annualized dollar excess return on that strategy using 1-year bonds is 4.10%. However, this excess return monotonically declines as the bond maturity increases: it is 2.33% using 5-year bonds and only 0.52% using 10-year bonds. At the long end of the maturity spectrum, this strategy delivers negative dollar excess returns: an investor who buys the 15-year bond of flat-yield-curve currencies and shorts the 15-year bond of steep-yield-curve currencies loses 0.42% per year on average. The term structure of currency carry trade risk premia is downward-sloping.

C Time-Series Tests of the Uncovered Bond Return Parity

To further test whether USD bond returns of different countries become increasingly similar as bond maturity increases, Figure 13 reports the ratio of foreign to domestic U.S. dollar bond returns; the k-year maturity volatility ratio is given by: $VolR^{(k),\$} = \sigma\left(r_{t+1}^{(k),*} - \Delta s_{t+1}\right)/\sigma\left(r_{t+1}^{(k)}\right)$ obtained on three-month returns. For comparison, Figure 13 reports also the corresponding volatility ratio for local currency returns, given by $VolR^{(k)} = \sigma\left(r_{t+1}^{(k),*}\right)/\sigma\left(r_{t+1}^{(k)}\right)$ for $k = 1, 2, ...15$ years. The pattern is unambiguous: the unconditional volatility of the U.S. dollar 3-month foreign returns is much higher than that of the corresponding volatility of U.S. bond

Table 8: Interest Rate Sorted Portfolios: Developed sample

Portfolio		1	2	3	4	4 − 1	1	2	3	4	4 − 1	1	2	3	4	4 − 1
Horizon		1-month					3-month					12-month				
Panel A: 1950-2012																
$-\Delta s$	Mean	1.30	0.58	0.06	-1.17	-2.47	1.40	0.37	0.19	-1.23	-2.63	1.54	0.38	-0.13	-1.13	-2.68
$f - s$	Mean	-1.41	0.39	1.51	4.03	5.44	-1.38	0.42	1.52	3.97	5.35	-1.26	0.53	1.56	3.79	5.05
r^{FX}	Mean	-0.11	0.97	1.56	2.86	2.98	0.02	0.79	1.71	2.74	2.72	0.28	0.91	1.43	2.65	2.37
	s.e.	[1.02]	[1.04]	[1.02]	[0.97]	[0.62]	[1.06]	[1.10]	[1.11]	[1.10]	[0.65]	[1.12]	[1.17]	[1.11]	[1.22]	[0.66]
	Std	8.02	8.26	7.96	7.67	4.87	8.36	8.43	8.28	8.18	5.25	9.27	8.76	8.79	9.24	5.36
	SR	-0.01	0.12	0.20	0.37	0.61	0.00	0.09	0.21	0.33	0.52	0.03	0.10	0.16	0.29	0.44
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.13]	[0.13]	[0.13]	[0.13]	[0.15]	[0.13]	[0.13]	[0.13]	[0.13]	[0.17]
$rx^{(10),*}$	Mean	3.00	1.90	1.05	-0.02	-3.03	2.46	1.59	1.08	0.48	-1.98	1.98	1.02	0.93	1.03	-0.95
	s.e.	[0.53]	[0.56]	[0.56]	[0.53]	[0.62]	[0.60]	[0.62]	[0.64]	[0.62]	[0.63]	[0.71]	[0.89]	[0.80]	[0.76]	[0.61]
	Std	4.15	4.40	4.41	4.14	4.81	4.53	4.84	5.06	4.95	4.91	5.00	5.81	6.24	5.88	4.52
	SR	0.72	0.43	0.24	-0.01	-0.63	0.54	0.33	0.21	0.10	-0.40	0.39	0.18	0.15	0.18	-0.21
	s.e.	[0.12]	[0.14]	[0.13]	[0.13]	[0.11]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]	[0.13]	[0.13]	[0.13]	[0.13]	[0.12]
$rx^{(10),\$}$	Mean	2.89	2.87	2.62	2.84	-0.05	2.48	2.38	2.79	3.22	0.74	2.26	1.93	2.36	3.68	1.42
	s.e.	[1.22]	[1.24]	[1.18]	[1.09]	[0.91]	[1.29]	[1.28]	[1.26]	[1.19]	[0.93]	[1.32]	[1.47]	[1.34]	[1.40]	[0.88]
	Std	9.59	9.86	9.26	8.62	7.13	10.24	10.05	9.74	9.22	7.45	10.74	10.66	10.91	10.60	7.60
	SR	0.30	0.29	0.28	0.33	-0.01	0.24	0.24	0.29	0.35	0.10	0.21	0.18	0.22	0.35	0.19
	s.e.	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.13]	[0.14]	[0.12]	[0.14]
$rx^{(10),\$} - rx^{(10),US}$	Mean	1.38	1.36	1.11	1.33	-0.05	0.96	0.86	1.27	1.70	0.74	0.71	0.38	0.81	2.14	1.42
	s.e.	[1.27]	[1.31]	[1.21]	[1.24]	[0.91]	[1.23]	[1.31]	[1.30]	[1.34]	[0.93]	[1.33]	[1.51]	[1.39]	[1.46]	[0.88]
Panel B: 1971-2012																
$-\Delta s$	Mean	1.86	0.68	0.28	-1.41	-3.27	1.95	0.40	0.36	-1.49	-3.44	2.18	0.41	-0.17	-1.42	-3.60
$f - s$	Mean	-1.64	0.47	1.66	4.63	6.27	-1.59	0.51	1.68	4.56	6.15	-1.46	0.66	1.74	4.35	5.81
r^{FX}	Mean	0.22	1.16	1.94	3.22	3.00	0.36	0.91	2.04	3.07	2.71	0.71	1.07	1.57	2.93	2.22
	s.e.	[1.55]	[1.59]	[1.50]	[1.46]	[0.91]	[1.58]	[1.63]	[1.63]	[1.61]	[0.95]	[1.70]	[1.76]	[1.62]	[1.84]	[0.97]
	Std	9.80	10.13	9.54	9.30	5.83	10.23	10.32	9.98	9.89	6.26	11.33	10.69	10.65	11.16	6.45
	SR	0.02	0.11	0.20	0.35	0.51	0.03	0.09	0.20	0.31	0.43	0.06	0.10	0.15	0.26	0.34
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.16]	[0.17]	[0.16]	[0.17]	[0.19]
$rx^{(10),*}$	Mean	3.67	2.58	1.20	0.44	-3.23	2.95	2.16	1.25	1.07	-1.88	2.46	1.25	1.05	1.67	-0.80
	s.e.	[0.79]	[0.83]	[0.81]	[0.75]	[0.89]	[0.87]	[0.90]	[0.92]	[0.88]	[0.91]	[1.04]	[1.31]	[1.16]	[1.09]	[0.89]
	Std	4.98	5.29	5.17	4.77	5.62	5.40	5.75	5.95	5.76	5.76	5.92	6.82	7.39	6.79	5.28
	SR	0.74	0.49	0.23	0.09	-0.57	0.55	0.38	0.21	0.19	-0.33	0.42	0.18	0.14	0.25	-0.15
	s.e.	[0.15]	[0.17]	[0.16]	[0.16]	[0.14]	[0.16]	[0.16]	[0.16]	[0.15]	[0.15]	[0.16]	[0.16]	[0.16]	[0.17]	[0.15]
$rx^{(10),\$}$	Mean	3.89	3.73	3.14	3.66	-0.23	3.31	3.07	3.29	4.14	0.83	3.18	2.32	2.61	4.60	1.42
	s.e.	[1.85]	[1.87]	[1.74]	[1.62]	[1.33]	[1.92]	[1.90]	[1.83]	[1.73]	[1.35]	[1.96]	[2.17]	[1.92]	[2.03]	[1.31]
	Std	11.67	12.04	11.04	10.30	8.44	12.43	12.21	11.62	10.92	8.81	12.96	12.74	13.02	12.46	9.04
	SR	0.33	0.31	0.28	0.36	-0.03	0.27	0.25	0.28	0.38	0.09	0.25	0.18	0.20	0.37	0.16
	s.e.	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.16]	[0.17]	[0.17]	[0.16]	[0.17]
$rx^{(10),\$} - rx^{(10),US}$	Mean	1.38	1.23	0.63	1.15	-0.23	0.78	0.53	0.76	1.61	0.83	0.61	-0.24	0.05	2.03	1.42
	s.e.	[1.86]	[1.91]	[1.72]	[1.80]	[1.33]	[1.78]	[1.89]	[1.84]	[1.91]	[1.35]	[1.97]	[2.24]	[1.99]	[2.15]	[1.31]

Annualized monthly log returns realized at $t + k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 21 currencies sorted every month by T-bill rate at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

Table 9: Interest Sorted Portfolios: Whole sample

Portfolio		1	2	3	4	5	$5-1$	1	2	3	4	5	$5-1$	1	2	3	4	5	$5-1$
Horizon		1-month						3-month						12-month					
		Panel A: 1950-2012																	
$-\Delta s$	Mean	0.43	-0.05	0.49	-0.63	-2.99	-3.41	0.64	0.05	0.29	-0.67	-3.09	-3.73	0.87	0.04	-0.06	-0.71	-2.95	-3.82
$f-s$	Mean	-1.81	-0.15	0.87	2.09	5.70	7.51	-1.72	0.45	0.89	2.11	5.59	7.31	-1.54	0.00	1.09	2.12	5.32	6.86
rx^{FX}	Mean	-1.38	-0.20	1.36	1.46	2.72	4.10	-1.08	0.50	1.17	1.44	2.50	3.58	-0.66	0.04	1.04	1.41	2.37	3.04
	s.e.	0.82	0.94	0.94	0.91	0.84	0.63	0.84	1.03	0.96	0.97	0.95	0.68	0.87	1.08	1.04	1.03	1.05	0.68
	Std	6.44	7.40	7.43	7.14	6.68	4.98	6.66	11.04	7.38	7.60	7.26	5.46	7.47	8.20	8.71	8.08	8.14	5.71
	SR	-0.22	-0.03	0.18	0.20	0.41	0.82	-0.16	0.05	0.16	0.19	0.35	0.66	-0.09	0.00	0.12	0.18	0.29	0.53
	s.e.	0.13	0.13	0.13	0.13	0.14	0.14	0.13	0.13	0.13	0.13	0.14	0.16	0.13	0.13	0.13	0.14	0.14	0.19
$rx^{(10),*}$	Mean	3.02	1.86	1.45	1.16	0.44	-2.58	2.56	1.05	1.26	1.14	0.99	-1.58	1.95	1.18	1.04	1.04	1.64	-0.31
	s.e.	0.46	0.52	0.48	0.54	0.54	0.64	0.47	0.65	0.55	0.55	0.61	0.65	0.63	0.82	0.59	0.64	0.71	0.65
	Std	3.59	4.05	3.79	4.17	4.29	5.09	3.96	9.22	4.28	4.57	5.00	5.35	4.33	5.24	6.62	5.25	5.52	5.19
	SR	0.84	0.46	0.38	0.28	0.10	-0.51	0.65	0.11	0.29	0.25	0.20	-0.30	0.45	0.22	0.16	0.20	0.30	-0.06
	s.e.	0.13	0.13	0.13	0.13	0.13	0.12	0.12	0.13	0.13	0.13	0.13	0.12	0.13	0.13	0.13	0.14	0.14	0.12
$rx^{(10),\$}$	Mean	1.64	1.66	2.81	2.62	3.16	1.52	1.49	1.56	2.43	2.58	3.49	2.01	1.29	1.22	2.07	2.45	4.02	2.73
	s.e.	0.99	1.12	1.08	1.09	1.05	0.97	1.00	1.26	1.11	1.08	1.16	1.01	1.03	1.35	1.24	1.21	1.25	0.95
	Std	7.81	8.79	8.54	8.51	8.28	7.60	8.23	9.48	8.65	9.00	9.16	8.22	8.76	9.65	9.57	9.72	10.12	8.14
	SR	0.21	0.19	0.33	0.31	0.38	0.20	0.18	0.16	0.28	0.29	0.38	0.24	0.15	0.13	0.22	0.25	0.40	0.34
	s.e.	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.14	0.13	0.15
$rx^{(10),\$}-rx^{(10),US}$	Mean	0.12	0.15	1.30	1.11	1.65	1.52	-0.04	0.04	0.91	1.06	1.97	2.01	-0.26	-0.33	0.53	0.91	2.47	2.73
	s.e.	1.18	1.23	1.14	1.21	1.30	0.97	1.12	1.31	1.20	1.22	1.45	1.01	1.11	1.45	1.39	1.31	1.45	0.95
		Panel B: 1971-2012																	
$-\Delta s$	Mean	0.74	0.09	0.46	-0.24	-3.78	-4.52	0.99	0.05	0.41	-0.67	-3.78	-4.77	1.17	0.15	-0.03	-0.74	-3.61	-4.78
$f-s$	Mean	-2.13	-0.16	1.03	2.62	6.91	9.04	-2.02	-0.12	1.07	2.64	6.74	8.76	-1.86	0.02	1.16	2.64	6.40	8.26
rx^{FX}	Mean	-1.39	-0.08	1.50	2.37	3.13	4.52	-1.03	-0.07	1.48	1.97	2.96	3.99	-0.69	0.17	1.13	1.90	2.79	3.48
	s.e.	1.17	1.37	1.28	1.26	1.20	0.88	1.19	1.46	1.39	1.42	1.38	0.95	1.27	1.52	1.37	1.44	1.54	1.07
	Std	7.47	8.75	8.27	8.10	7.76	5.57	7.76	8.97	8.64	8.71	8.47	6.29	8.76	9.44	8.97	9.33	9.80	6.62
	SR	-0.19	-0.01	0.18	0.29	0.40	0.81	-0.13	-0.01	0.17	0.23	0.35	0.63	-0.08	0.02	0.13	0.20	0.28	0.53
	s.e.	0.16	0.16	0.16	0.17	0.16	0.17	0.16	0.16	0.16	0.16	0.17	0.19	0.16	0.16	0.16	0.17	0.18	0.27
$rx^{(10),*}$	Mean	3.73	2.31	2.28	1.80	-0.04	-3.77	3.12	2.13	1.79	1.73	0.81	-2.31	2.52	1.60	1.56	1.19	1.73	-0.79
	s.e.	0.67	0.74	0.68	0.70	0.71	0.85	0.73	0.88	0.78	0.82	0.79	0.89	0.85	1.15	0.82	0.98	1.00	0.82
	Std	4.23	4.77	4.39	4.49	4.53	5.47	4.72	5.54	4.93	5.26	5.37	5.85	4.96	6.16	5.87	6.47	6.19	5.57
	SR	0.88	0.48	0.52	0.40	-0.01	-0.69	0.66	0.38	0.36	0.33	0.15	-0.40	0.51	0.26	0.27	0.18	0.28	-0.14
	s.e.	0.15	0.17	0.16	0.16	0.16	0.15	0.15	0.17	0.17	0.17	0.16	0.15	0.17	0.16	0.17	0.17	0.18	0.16
$rx^{(10),\$}$	Mean	2.34	2.24	3.77	4.17	3.09	0.75	2.09	2.06	3.27	3.70	3.77	1.68	1.83	1.77	2.69	3.08	4.52	2.69
	s.e.	1.44	1.64	1.51	1.47	1.47	1.33	1.47	1.78	1.58	1.56	1.66	1.43	1.51	1.90	1.65	1.70	1.84	1.46
	Std	9.13	10.43	9.74	9.37	9.45	8.48	9.71	10.96	10.01	9.97	10.49	9.43	10.29	11.13	10.84	11.14	12.07	9.49
	SR	0.26	0.21	0.39	0.45	0.33	0.09	0.22	0.19	0.33	0.37	0.36	0.18	0.18	0.16	0.25	0.28	0.37	0.28
	s.e.	0.16	0.16	0.16	0.16	0.16	0.16	0.15	0.16	0.16	0.16	0.16	0.16	0.16	0.17	0.17	0.18	0.18	0.20
$rx^{(10),\$}-rx^{(10),US}$	Mean	-0.17	-0.27	1.27	1.67	0.59	0.75	-0.44	-0.47	0.74	1.17	1.24	1.68	-0.73	-0.79	0.13	0.52	1.96	2.69
	s.e.	1.61	1.66	1.53	1.58	1.83	1.33	1.54	1.82	1.58	1.67	2.07	1.43	1.51	2.01	1.77	1.87	2.11	1.46

Annualized monthly log returns realized at $t+k$ on 10-year Bond Index and T-bills for k from 1 month to 12 months. Portfolios of 30 currencies sorted every month by T-bill rate at t. The unbalanced panel consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, India, Ireland, Italy, Japan Mexico, Malaysia, the Netherlands, New Zealand, Norway, Pakistan, the Philippines, Poland, Portugal, South Africa, Singapore, Spain, Sweden, Switzerland, Taiwan, Thailand, and the United Kingdom.

Table 10: The Maturity Structure of Returns in Slope-Sorted Portfolios: Extended Sample

Maturity		4-quarters						20-quarters						40-quarters						60-quarters					
Portfolio		1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1	1	2	3	4	5	5 − 1
$-\Delta s$	Mean	0.90	0.59	0.29	2.34	-0.30	-1.19	0.90	0.59	0.29	2.34	-0.30	-1.19	0.90	0.59	0.29	2.34	-0.30	-1.19	0.94	0.43	0.35	2.38	-0.39	-1.33
$f-s$	Mean	3.99	1.99	0.99	0.39	0.36	-3.63	3.99	1.99	0.99	0.39	0.36	-3.63	3.99	1.99	0.99	0.39	0.36	-3.63	3.93	2.00	0.95	0.32	0.32	-3.62
rx^{FX}	Mean	4.89	2.58	1.28	2.73	0.07	-4.82	4.89	2.58	1.28	2.73	0.07	-4.82	4.89	2.58	1.28	2.73	0.07	-4.82	4.87	2.43	1.30	2.70	-0.07	-4.94
	s.e.	[1.26]	[1.18]	[1.19]	[1.16]	[0.98]	[0.91]	[1.26]	[1.19]	[1.19]	[1.17]	[0.98]	[0.90]	[1.26]	[1.18]	[1.20]	[1.16]	[0.98]	[0.91]	[1.26]	[1.18]	[1.19]	[1.19]	[0.99]	[0.91]
	Std	11.00	10.36	10.44	10.24	8.56	7.95	11.00	10.36	10.44	10.24	8.56	7.95	11.00	10.36	10.44	10.24	8.56	7.95	11.01	10.34	10.44	10.44	8.65	7.99
	SR	0.44	0.25	0.12	0.27	0.01	-0.61	0.44	0.25	0.12	0.27	0.01	-0.61	0.44	0.25	0.12	0.27	0.01	-0.61	0.44	0.24	0.12	0.26	-0.01	-0.62
	s.e.	[0.13]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.11]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]
$rx^{(k),*}$	Mean	-0.09	0.11	0.32	0.33	0.63	0.72	1.11	2.12	2.56	2.73	3.60	2.50	1.59	2.69	4.16	4.32	5.90	4.31	2.42	2.84	6.36	4.78	7.78	5.36
	s.e.	[0.12]	[0.10]	[0.10]	[0.10]	[0.10]	[0.13]	[0.59]	[0.54]	[0.59]	[0.56]	[0.62]	[0.64]	[1.06]	[0.97]	[1.05]	[1.00]	[1.02]	[0.96]	[1.45]	[1.46]	[1.43]	[1.36]	[1.44]	[1.31]
	Std	1.04	0.88	0.88	0.84	0.88	1.15	5.13	4.70	5.22	4.96	5.42	5.53	9.34	8.56	9.19	8.78	9.01	8.44	12.67	12.82	12.55	11.92	12.70	11.76
	SR	-0.09	0.12	0.37	0.39	0.72	0.63	0.22	0.45	0.49	0.55	0.67	0.45	0.17	0.31	0.45	0.49	0.65	0.51	0.19	0.22	0.51	0.40	0.61	0.46
	s.e.	[0.11]	[0.12]	[0.11]	[0.12]	[0.11]	[0.11]	[0.12]	[0.12]	[0.12]	[0.13]	[0.12]	[0.11]	[0.12]	[0.12]	[0.12]	[0.13]	[0.13]	[0.11]	[0.12]	[0.12]	[0.12]	[0.13]	[0.12]	[0.11]
$rx^{(k),\$}$	Mean	4.80	2.69	1.60	3.06	0.70	-4.10	6.00	4.70	3.84	5.46	3.67	-2.33	6.48	5.27	5.45	7.05	5.97	-0.52	7.29	5.27	7.66	7.47	7.71	0.42
	s.e.	[1.26]	[1.17]	[1.17]	[1.16]	[1.00]	[0.91]	[1.35]	[1.26]	[1.23]	[1.30]	[1.22]	[1.05]	[1.59]	[1.48]	[1.50]	[1.55]	[1.48]	[1.26]	[1.87]	[1.84]	[1.78]	[1.82]	[1.80]	[1.54]
	Std	11.05	10.29	10.32	10.27	8.72	7.99	11.87	11.07	11.02	11.50	10.71	9.17	14.10	13.13	13.25	13.84	13.14	11.24	16.45	16.10	15.74	16.04	15.96	13.97
	SR	0.43	0.26	0.16	0.30	0.08	-0.51	0.51	0.42	0.35	0.47	0.34	-0.25	0.46	0.40	0.41	0.51	0.45	-0.05	0.44	0.33	0.49	0.47	0.48	0.03
	s.e.	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]	[0.12]
$rx^{(k),\$}-rx^{(k),US}$	Mean	7.66	5.56	4.47	5.92	3.57	-4.10	6.28	4.99	4.13	5.75	3.96	-2.33	4.89	3.68	3.86	5.46	4.38	-0.52	4.67	2.65	5.04	4.85	5.09	0.42
	s.e.	[1.26]	[1.16]	[1.16]	[1.14]	[0.97]	[0.91]	[1.34]	[1.23]	[1.13]	[1.20]	[1.13]	[1.05]	[1.54]	[1.41]	[1.33]	[1.39]	[1.34]	[1.26]	[1.77]	[1.76]	[1.57]	[1.63]	[1.66]	[1.54]

Notes: The table reports summary statistics on annualized log returns realized on zero coupon bonds with maturity varying from $k = 4$ to $k = 60$ quarters. The holding period is one quarter. The table reports the average change in exchange rates ($-\Delta s$), the average interest rate difference ($f - s$), the average currency excess return (rx^{FX}), the average foreign bond excess return in foreign currency ($rx^{(k),*}$) and in U.S. dollars ($rx^{(k),\$}$), as well as the difference between the average foreign bond excess return in U.S. dollars and the average U.S. bond excess return ($rx^{(k),\$} - rx^{(k),US}$). For the excess returns, the table also reports their annualized standard deviation (denoted Std) and their Sharpe ratios (denoted SR). The unbalanced panel consists of Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Germany, Hungary, Indonesia, Ireland, Italy, Japan, Malaysia, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, South Africa, Spain, Sweden, Switzerland, and the U.K. The countries are sorted by the slope of their yield curves into five portfolios. The slope of the yield curve is measured by the difference between the 10-year yield and the 3-month interest rate at date t. The standard errors (denoted s.e. and reported between brackets) were generated by bootstrapping 10,000 samples of non-overlapping returns. Data are quarterly and the sample window is 5/1987–12/2012.

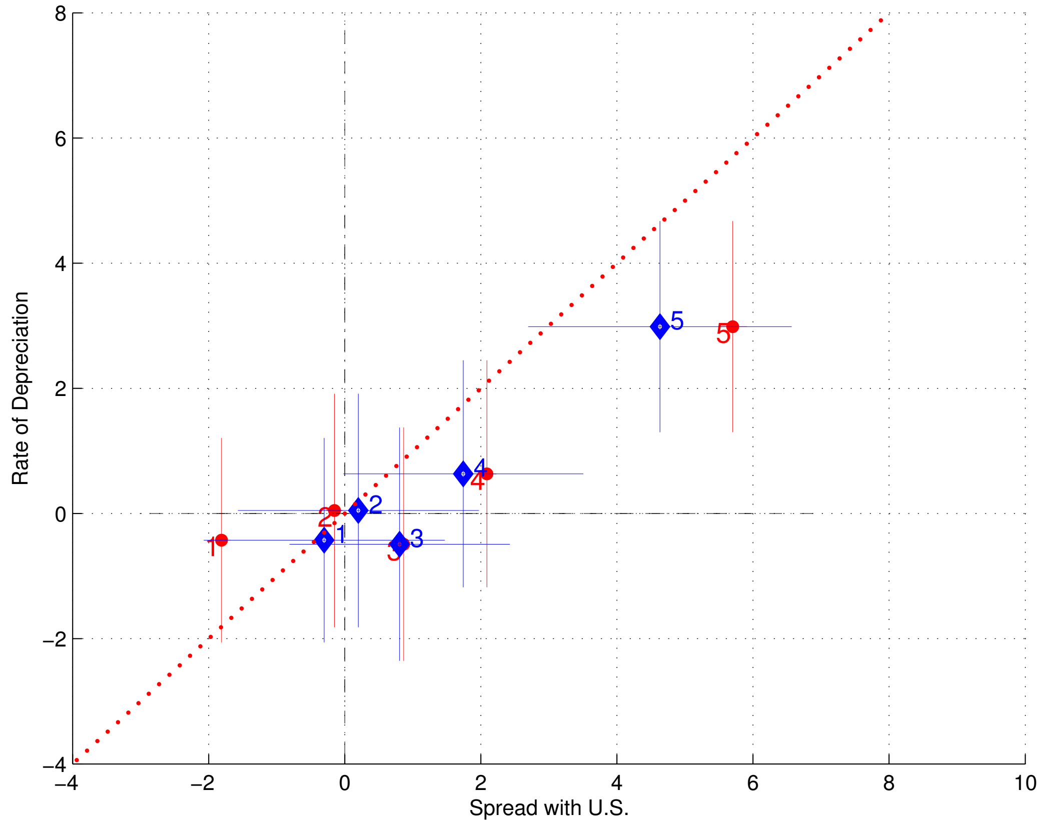

Figure 11: Uncovered Interest Rate Parity and Uncovered Bond Return Parity: All Sample Countries Sorted by Interest Rates – This figure presents, with red dots, the average exchange rate changes in five portfolios against the average interest rate differences between the foreign country and the U.S. for the same five portfolios. The figure also presents, with blue diamonds, the average exchange rate changes in five portfolios against the local currency bond return spread with the US, defined as the difference between the average bond return in foreign currency and the U.S. bond return in U.S. dollars. The lines indicate two standard errors around the point estimate. Countries are sorted by their short-term interest rates and allocated into five portfolios. The portfolios are rebalanced every month. The monthly returns are annualized. The sample period is 12/1950–12/2012. The sample includes developed and emerging countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, India, Ireland, Italy, Japan, Mexico, Malaysia, the Netherlands, New Zealand, Norway, Pakistan, the Philippines, Poland, Portugal, South Africa, Singapore, Spain, Sweden, Switzerland, Taiwan, Thailand, and the U.K.

returns for small maturities, but the volatility ratio falls sharply for higher maturities and is close to one for 15-year bonds. In contrast with the observed pattern for $VolR_k$, the local currency volatility ratio $VolR_k^{(k)}$ is virtually flat with maturity, implying that the convergence in U.S. dollar return bond volatility is due to the properties of the nominal exchange rate. Of course, even if exchange rates followed a random walk and exchange rate innovations are uncorrelated with returns, we could still observe this pattern, simply the exchange rates account for a smaller share of overall return volatility at longer maturities. However, we show that exchange rates actually hedge interest rate risk.

The results are robust to an increase of the holding period. Specifically, in unreported results, 6-month and 12-month returns produce the same patterns: for both holding periods and for virtually all currencies, there is an almost monotonic relationship between correlation coefficients of U.S. dollar returns and bond maturity. Furthermore, 6-month and 12-month local currency return correlations are not sensitive to maturity, the U.S. dollar return volatility ratio is very high for short maturities, but quickly converges towards one, and the local currency return volatility ratio is flat with maturity.

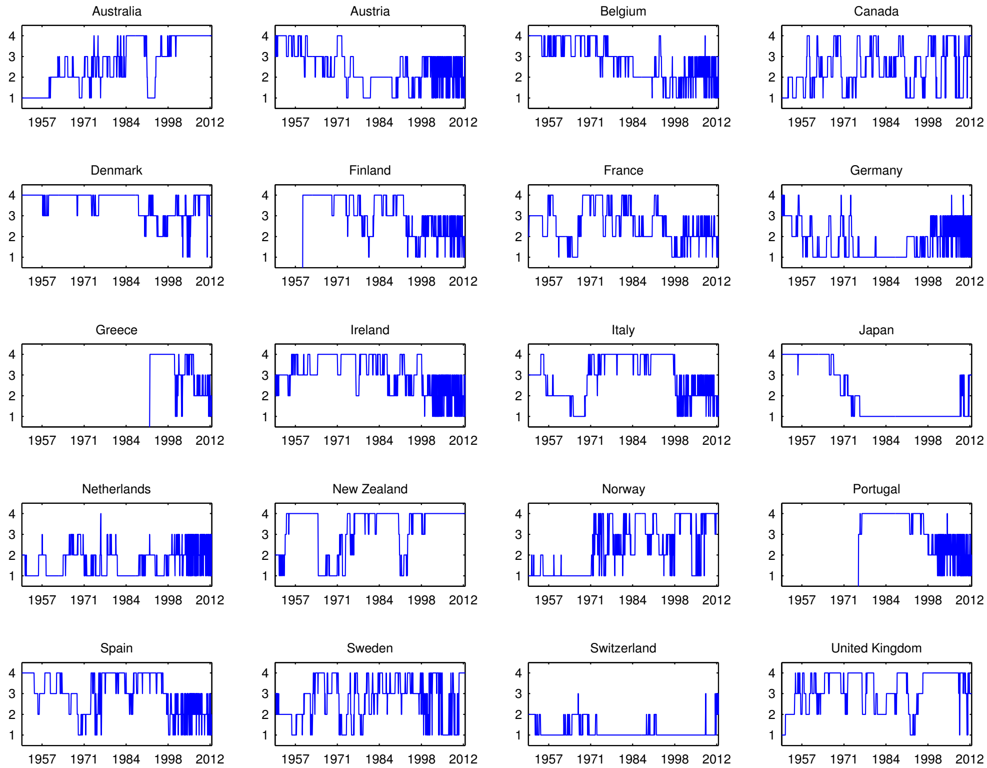

Figure 12: Composition of Interest Rate-Sorted Portfolios — The figure presents the composition of portfolios of 20 currencies sorted by their short-term interest rates. The portfolios are rebalanced monthly. Data are monthly, from 12/1950 to 12/2012.

D Real Exchange Rate Decomposition

We decompose real exchange rate changes using returns on real bonds. In the first part of our analysis, we proxy real bonds by inflation-indexed bonds. In the second part, we synthesize real bonds by coupling nominal bonds with inflation swaps.

D.1 Results using inflation-indexed bonds

Due to the scarcity of available data on inflation-indexed bonds, our sample includes only two countries, the United States and the United Kingdom. In particular, we use monthly sovereign zero-coupon yield curve data for annual maturities from 01/1999 to 12/2012, sourced from the Federal Reserve Board and the Bank of England. To construct the time-series for the end-of-month real exchange rate between the two countries, we multiply the nominal exchange rate by the ratio of the non-seasonally adjusted monthly price indices of the two countries. For each country, we use the reference index for the inflation-indexed sovereign bonds: the city-average, all items CPI for the United States and the RPI for the United Kingdom.

Our decomposition of real exchange rate changes is similar to our nominal exchange rate change decomposition. In particular, we rely on the observation that the one-period return of a real zero-coupon bond that matures k periods into the future is given by

$$R_{t+1}^{real,(k)} = \frac{P_{t+1}^{real,(k-1)}}{P_t^{real,(k)}} = \frac{E_{t+1}\left(\Lambda_{t+k}^{real}\right)}{\Lambda_{t+1}^{real}} \frac{\Lambda_t^{real}}{E_t\left(\Lambda_{t+k}^{real}\right)}.$$

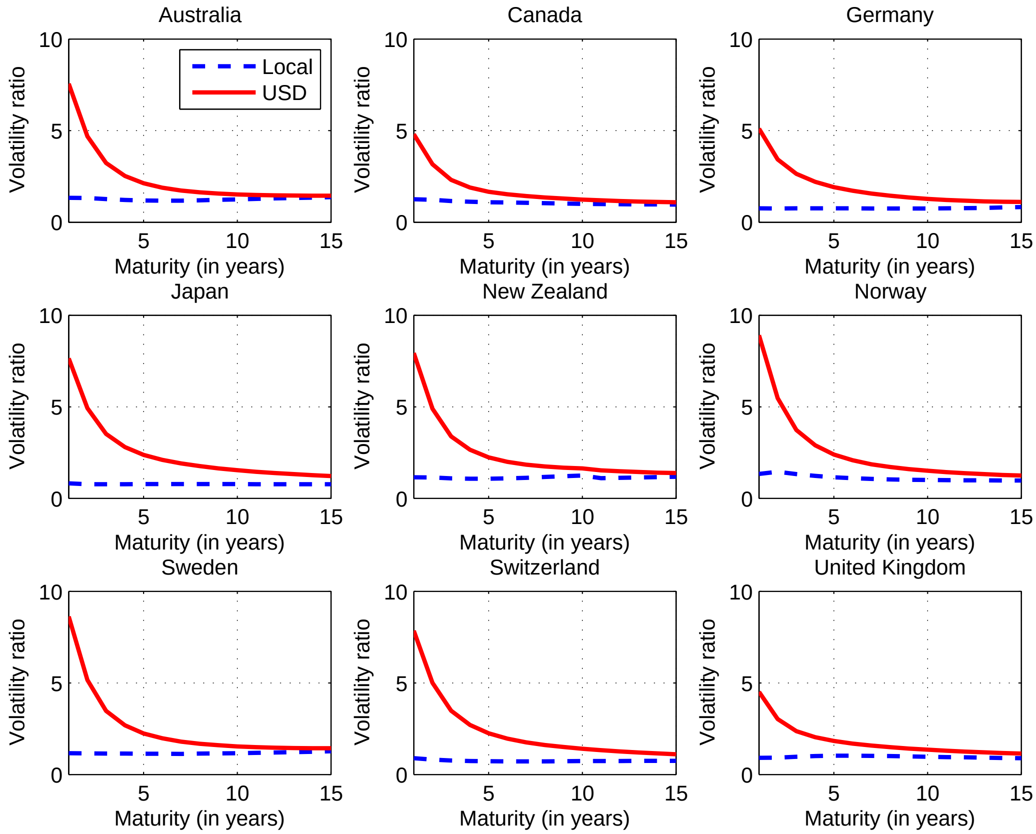

Figure 13: The Maturity structure of Bond Return Volatility — Volatility of Foreign and U.S. bond returns. The time-window is country-dependent. Data are monthly. The holding period is 3-months.

Thus the ratio of the USD holding period returns of two countries' infinite maturity real bonds is identically equal to the permanent component of their real exchange rate:

$$\lim_{k\to\infty}\left(\frac{S_t^{real}}{S_{t+1}^{real}}\frac{R_{t+1}^{real,(k),*}}{R_{t+1}^{real,(k)}}\right)=\frac{S_t^{real}}{S_{t+1}^{real}}\left(\frac{\Lambda_t^{real,*,\mathbb{T}}}{\Lambda_{t+1}^{real,*,\mathbb{T}}}\frac{\Lambda_{t+1}^{real,\mathbb{T}}}{\Lambda_t^{real,\mathbb{T}}}\right)=\frac{S_t^{real,\mathbb{P}}}{S_{t+1}^{real,\mathbb{P}}}.$$

Table 11 presents the properties of real exchange rate change components and real stochastic discount factors retrieved using real bond returns, with the holding period set equal to a year. For comparison, we also report the corresponding statistics resulting from the decomposition of nominal exchange rate changes. Notably, our findings on the decomposition of nominal exchange rate changes carry through to real exchange rate changes. This is due to the fact that inflation rate differentials contribute very little to the fluctuations of the nominal USD/GBP exchange rate. In particular, we find that the two components of real exchange rate changes have the same volatility as the nominal exchange rate components and are similarly negatively correlated. Real exchange rate changes have slightly lower skewness and kurtosis than their nominal counterparts, which is also true for their permanent component. On the other hand, the transitory component of real exchange rate changes is more skewed and fat-tailed than its nominal counterpart. Finally, the transitory component of the UK real SDF is slightly less volatile and less correlated with the transitory component of the US SDF than its nominal counterpart.

Table 11: Properties of SDF and Exchange Rate Components Using Inflation-Indexed Bonds

Moment	GBP nominal	GBP real
	Panel A: Exchange rate changes, Δs	
Mean	0.00	0.00
Std	0.10	0.10
Skewness	1.21	1.13
Kurtosis	4.84	4.51
AR(1) coef.	0.94	0.94
	Panel B: Transitory exchange rate changes, $\Delta s^{\mathbb{T}}$	
Mean	-0.02	-0.03
Std	0.04	0.05
Skewness	0.16	0.23
Kurtosis	2.21	3.16
AR(1) coef.	0.84	0.85
	Panel C: Permanent exchange rate changes, $\Delta s^{\mathbb{P}}$	
Mean	0.02	0.02
Std	0.10	0.10
Skewness	1.09	0.87
Kurtosis	4.33	3.24
AR(1) coef.	0.90	0.92
	Panel D: Exchange Rate Correlations	
$corr(\Delta s, \Delta s^{\mathbb{T}})$	0.34	0.31
$corr(\Delta s, \Delta s^{\mathbb{P}})$	0.90	0.90
$corr(\Delta s^{\mathbb{T}}, \Delta s^{\mathbb{P}})$	-0.10	-0.13
	Panel E: Transitory SDF	
Std	0.06	0.05
s.e.	[0.01]	[0.01]
$corr(m^{\mathbb{T},US}, m^{\mathbb{T},*})$	0.75	0.64
s.e.	[0.05]	[0.11]

Notes: The table reports the mean, standard deviation, skewness, kurtosis, and autocorrelation of annual changes in exchange rates, as well as the moments of the transitory and permanent components of exchange rates. 10-year zero-coupon nominal (inflation-indexed) bonds are used as proxy of infinite-maturity nominal (real) bonds in order to decompose nominal (real) exchange rate changes into their permanent and transitory components. The last panel reports the standard deviations of the transitory component of the SDF, along with its correlation with the transitory component of the U.S. SDF. Standard errors are obtained from block bootstrapping with blocks of 14 periods (10,000 replications). Monthly zero-coupon bond data from 1/1999 to 12/2012.

D.2 Results using inflation swaps

In addition to using using inflation-indexed bonds as a proxy for real bonds, we also synthetically construct real bonds by hedging the inflation exposure of nominal bonds using inflation swaps. In particular, we use inflation swaps for three foreign currencies (EUR, GBP and JPY) and the USD. Our data are monthly, from July 2004 (March 2007 for the JPY) to December 2012. Inflation swap data are available for annual maturities; in order to calculate quarterly returns, we generate a grid of quarterly inflation swap rates using linear interpolation of the inflation discount factors.

Table 12 presents the decomposition of quarterly real exchange rate changes. In particular, we present two sets of results, fixing the sample period for each currency. The first three columns present the decomposition of nominal exchange rate changes, while the last three columns decompose real exchange rate changes. Furthermore, we present the properties of the relative inflation swap rate change $\frac{(1+f_{t+1}^{*,(k-1)})^{k-1}}{(1+f_t^{*,(k)})^k}\frac{(1+f_t^{(k)})^k}{(1+f_{t+1}^{(k-1)})^{k-1}}$ in Panel D.

Table 12: Properties of SDF and Exchange Rate Components Using Inflation Swaps

	Nominal FX changes			Real FX changes		
Moment	EUR	GBP	JPY	EUR	GBP	JPY
	Panel A: Exchange rate changes, Δs					
Mean	-0.01	0.01	-0.07	0.00	0.00	-0.05
Std	0.11	0.11	0.11	0.11	0.11	0.11
Skewness	0.85	1.22	-0.10	0.65	1.01	-0.36
Kurtosis	4.32	7.17	2.96	3.68	6.09	3.40
AR(1) coef.	0.70	0.80	0.63	0.67	0.79	0.65
	Panel B: Transitory exchange rate changes, $\Delta s^{\mathbb{T}}$					
Mean	-0.01	0.00	-0.06	-0.01	-0.01	-0.03
Std	0.05	0.05	0.08	0.06	0.06	0.10
Skewness	0.49	0.42	-0.51	1.22	0.26	-1.33
Kurtosis	4.87	3.32	3.67	7.20	6.13	6.08
AR(1) coef.	0.51	0.51	0.57	0.54	0.55	0.70
	Panel C: Permanent exchange rate changes, $\Delta s^{\mathbb{P}}$					
Mean	0.00	0.02	0.00	0.00	0.01	-0.01
Std	0.11	0.11	0.09	0.10	0.12	0.12 0
Skewness	0.16	0.90	-0.30	0.18	1.90	0.45
Kurtosis	3.15	6.99	3.94	3.03	11.79	5.39
AR(1) coef.	0.68	0.75	0.51	0.61	0.70	0.58
	Panel D: Relative inflation swap rate changes					
Mean	-	-	-	0.00	-0.01	0.03
Std	-	-	-	0.04	0.05	0.07
Skewness	-	-	-	-0.23	-0.56	-2.37
Kurtosis	-	-	-	4.61	5.12	13.12
AR(1) coef.	-	-	-	0.49	0.53	0.52
	Panel E: Exchange Rate Correlations					
$corr(\Delta s, \Delta s^{\mathbb{T}})$	0.30	0.10	0.52	0.38	0.09	0.40
$corr(\Delta s, \Delta s^{\mathbb{P}})$	0.88	0.91	0.71	0.85	0.84	0.63
$corr(\Delta s^{\mathbb{T}}, \Delta s^{\mathbb{P}})$	-0.18	-0.31	-0.23	-0.17	-0.46	-0.47
	Panel F: Transitory SDF					
Std	0.07	0.08	0.03	0.06	0.08	0.06
s.e.	[0.01]	[0.01]	[0.00]	[0.00]	[0.01]	[0.02]
$corr(m^{\mathbb{T},US}, m^{\mathbb{T},*})$	0.80	0.85	0.72	0.67	0.65	0.19
s.e.	[0.05]	[0.04]	[0.06]	[0.06]	[0.08]	[0.18]

Notes: The table reports the mean, standard deviation, skewness, kurtosis, and autocorrelation of quarterly changes in exchange rates, as well as the moments of the transitory and permanent components of exchange rate changes. 10-year zero-coupon nominal bonds are used as proxy of infinite-maturity nominal bonds in order to decompose nominal exchange rate changes into their permanent and transitory components. For the decomposition of real exchange rate changes, 10-year zero-coupon nominal bonds hedged by inflation swaps are used. Means and standard deviations are annualized. The last panel reports the standard deviations of the transitory component of the SDF, along with its correlation with the transitory component of the U.S. SDF. Standard errors are obtained from block bootstrapping with blocks of four periods (10,000 replications). Monthly data from 7/2004 to 12/2012 (JPY from 3/2007).