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The term structure of inflation expectations — Source link

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Abstract

We present estimates of the term structure of inflation expectations, derived from an affine model of real and nominal yield curves. The model features stochastic covariation of inflation with the real pricing kernel, enabling us to extract a time-varying inflation risk premium. We fit the model not only to yields, but also to the yields' variance-covariance matrix, thus increasing identification power. We find that model-implied inflation expectations can differ substantially from break-even inflation rates when market volatility is high. Our model's ability to be updated weekly makes it suitable for real-time monetary policy analysis.

Key words: affine term structure models, inflation expectations, stochastic volatility, asset pricing, monetary policy

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1 Introduction

Central banks monitor financial market developments and economic data releases at relatively high frequencies. In the U.S., for example, Federal Reserve economists brief monetary policy makers at a weekly frequency, and the Federal Reserve's trading desk monitors market developments in real time. Understanding the linkages between asset price movements and macroeconomic developments is one of the key objectives of these monitoring efforts.

The evolution of breakeven inflation rates—the difference between nominal and real yields at different maturities—is an especially important indicator for the conduct of monetary policy, as breakeven inflation rates can be interpreted as measures of inflation expectations. Since the seminal work of Kydland and Prescott (1977) and Barro (1983), monetary economists have emphasized the importance of longer term inflation expectations, and some studies suggest that the containment of long term inflation expectations is the most important objective in conducting monetary policy (see Woodford, 2003 and Bernanke et al., 2001 for summaries).

Breakeven inflation rates measure inflation expectations with a number of biases, some of which have been previously documented (see Barr and Campbell, 1997, Elsasser and Sack, 2004, and Gurkaynak, Sack, and Wright, 2007). First, inflation linked benchmark securities are typically less liquid than nominal on-the run Treasuries. Second, the coupons of nominal and real securities with similar maturities are often different, leading to differences in duration.

The focus of the current paper is the difference between expected inflation and breakeven inflation due to inflation risk. We analyze the term structure of breakeven inflation computed as the difference between a zero coupon, off-the-run nominal Treasury yield curve and a zero coupon, real TIPS curve.[1] This breakeven term structure is sometimes called the term structure of implied inflation. Implied inflation is a better measure of inflation expectations than breakeven inflation, as it adjusts for liquidity differences and for differences in durations.[2] However, breakeven inflation rates of zero-coupon off-the-run curves (i.e., implied inflation) still are not pure measures of inflation expectations. This is because the absence of arbitrage implies that the difference between zero-coupon nominal and real yields can be decomposed into three components:

$$\text{Breakeven Inflation} = \text{Expected Inflation} + \text{Inflation Risk Premium} + \text{Convexity}$$

The literature commonly adjust for the convexity effect (see Elsasser and Sack, 2004). However, the adjustment of breakeven inflation for the inflation risk premium requires the estimation of a term structure model.

In this paper, we develop an affine term structure model that captures the dynamics of real and nominal yields curves, as well as the evolution of their variance-covariance matrix. This is important, as the inflation risk premium is proportional to the conditional covariance of the real pricing kernel and inflation. In order to increase the power for identifying the inflation risk premium, we match both the term structure of the yield curves, and the term structure of variances and covariances.

We find a relatively small and stable inflation risk premium. The order of magnitude of the inflation risk premium is comparable to other recent estimates in studies that use inflation protected bonds over similar sample periods, but it is smaller and less

[1] The nominal term structure is from Gurkaynak, Sack, and Wright (2006), the real term structure is from Gurkaynak, Sack, and Wright (2007).

[2] In the remainder of the paper, we use the terminology "implied inflation" and "breakeven inflation" interchangably, as our yield curves are off-the-run zero coupon curves.

variable than estimates that use nominal bonds and inflation over longer time periods (see Buraschi and Jiltsov, 2005, and Ang, Bekaert, and Wei, 2006).

An additional contribution of this paper is to present estimates of an arbitrage free term structure model using weekly data. Much of the academic literature estimates term structure models at lower frequencies (using monthly, quarterly, or annual data). However, for the purpose of monetary policy decision making, it is desirable to have estimates of inflation expectations at higher frequencies. For example, in a particular week, payrolls could exceed expectations, the price of oil could decline, and the dollar depreciate. How such news changes inflation expectations at different maturities has a potentially important impact on policy decisions.

We use only real and nominal yields to estimate the term structure of inflation expectations. Our requirement to match the variance-covariance matrix of the real and nominal yield curves provides us with enough identification power to estimate expected inflation. In comparison, some recent work incorporates estimates of inflation expectations from survey data to achieve identification (see Joyce, Lildholdt, and Sorensen, 2007, Hördahl and Tristani, 2007, and D'Amico, Kim, and Wei, 2007). We view those papers as complementary identification strategies. The advantage of our approach is that our model provides estimates of inflation expectations using only financial market data. It can furthermore updated in real time.

We fit our model to an estimated variance-covariance matrix of the real and nominal yield curves. Alternatively, options data could be used to obtain information about second moments. For example, Goldstein and Collin-Dufresne (2002) use interest rate caps and floors to fit an affine term structure of the nominal yield curve that features stochastic volatility. The advantage of our approach is that our data is readily available. To obtain an implied covariance between nominal and real yields, one would have to have a time series of options on inflation swaps, which is less liquid, and not always traded.

The remainder of the paper is organized in five sections. In Section 2, we discuss the nominal and real yield curves and derive the relationship between breakeven inflation and expected inflation from no arbitrage. We present our model in Section 3, and our estimation results in Section 4. In section 5, we provide robustness checks, and Section 6 concludes.

2 Breakevens and the Inflation Risk Premium

2.1 The nominal and real yield curves

Breakeven inflation provides a crude measure of inflation expectations that can be tracked at high frequencies. For example, in Figure 1, we plot the difference between the 10-year on-the-run Treasury yields and the 10-year on-the-run TIPS yields in five minute intervals for the first three weeks of March 2007. In the sample period, we can see that breakeven inflation reacted to the release of nonfarm payrolls on March 9, and the release of the consumer price index on March 16.[3]

However, using breakevens to measure inflation expectations is problematic. On-the-run Treasuries are more liquid than benchmark TIPS. On average, the difference between the on-the-run and the off-the-run yield is 6 basis points since the beginning of 2003, with a daily standard deviation of 2.2% (see also Fleming, 2003, and Krishnamurthy, 2002, for analysis of the on-the-run/off-the-run Treasury yield spread). In periods of financial market turbulence, the on-the-run/off-the-run spread tends to

[3] Fleming and Remolona (1999) analyze the impact of economic news on nominal Treasury securities in a systematic fashion, and Beechey and Wright (2008) conduct a similar analysis for the TIPS market.

widen. However, financial market turbulences are times when inflation expectations potentially change, and when variances and covariances and hence risk premia tend to change. An increase in the on-the-run/off-the-run spread during those times tends to understate inflation expectations relative to breakeven inflation computed from on-the-run bonds.

In addition, coupons of nominal Treasuries and TIPS with similar maturities are often different. A 10-year breakeven spread typically has a duration that is shorter than ten years, and the difference in duration of the nominal and the real yield introduces a bias. The average wedge between par and zero coupon yields with a ten year maturity is 44 basis points since the beginning of 2004.

In this paper, we use the zero-coupon, off-the run term structure of nominal Treasury yields computed by Gurkaynak, Sack, and Wright (2006), and the zero coupon the real term structure computed from TIPS real yields by Gurkaynak, Sack, and Wright (2007). The yield data is daily and spans from January 3, 1999 to August 31, 2008. We estimate the model at a weekly frequency, using the last day of each week to construct our dataset, providing us with 505 weeks. We plot the yield curves of weekly real and nominal yields for maturities 3, 4, 5, 6, 7, 8, 9, and 10 year in Figure 1, and provide summary statistics in Table 1.

2.2 The inflation risk premium

We denote the maturity of a bond by τ. The price of a nominal bond that pays \$1 at time $t+\tau$ is P_t^τ. The price of a real bond that pays \$1 at time $t+\tau$ is R_t^τ. The Daily Reference Level of the CPI is denoted by Q_t. The absence of arbitrage implies that there exists a discount factor M_t such that (see, for example Dybvig and Ross, 1987, for an exposition of the Fundamental Theorem of Arbitrage Pricing):

$$P_t^\tau = E_t\left[\frac{M_{t+\tau}}{M_t}\frac{Q_t}{Q_{t+\tau}}\right], \qquad R_t^\tau = E_t\left[\frac{M_{t+\tau}}{M_t}\right] \tag{1}$$

We denote the continuously compounded yield of the nominal bond $y_t^\tau = -(1/\tau)\ln(P_t^\tau)$, and of the inflation linked bond by $r_t^\tau = -(1/\tau)\ln(R_t^\tau)$. We further denote the logarithmic inflation rate by π_t, and the rate of change of (the negative of) the pricing kernel by m_t:

$$m_{t+1} = -(\ln M_{t+1} - \ln M_t) \text{ and } \pi_{t+1} = \ln Q_{t+1} - \ln Q_t \tag{2}$$

Using these expressions (2) in equation (1) gives:

$$\tau y_t^\tau = -\ln E_t\left[\exp\left(-\Sigma_{s=1}^\tau m_{t+s} - \Sigma_{s=1}^\tau \pi_{t+s}\right)\right] \tag{3}$$

$$\tau r_t^\tau = -\ln E_t\left[\exp\left(-\Sigma_{s=1}^\tau m_{t+s}\right)\right] \tag{4}$$

We follow the affine term structure literature (see Piazzesi, 2003, and Singleton, 2006 for recent surveys) and assume that shocks to inflation and the real pricing kernel are conditionally normal. We can then use the properties of the moment generating function of the normal distribution to rewrite the last two equations as:

$$\underbrace{y_t^\tau - r_t^\tau}_{\substack{\text{Breakeven} \\ \text{Inflation}}} = \underbrace{\frac{1}{\tau} E_t \left(\Sigma_{s=1}^{\tau} \pi_{t+s} \right)}_{\substack{\text{Expected} \\ \text{Inflation}}} + \underbrace{\frac{1}{\tau} Cov_t \left(\Sigma_{s=1}^{\tau} \pi_{t+s}, -\Sigma_{s=1}^{\tau} m_{t+s} \right)}_{\substack{\text{Inflation Risk} \\ \text{Premium}}} - \underbrace{\frac{1}{2\tau} Var_t \left(\Sigma_{s=1}^{\tau} \pi_{t+s} \right)}_{\substack{\text{Convexity} \\ \text{Adjustment}}} \quad (5)$$

Breakeven inflation thus consists of three components: expected inflation, the inflation risk premium, and a convexity adjustment. Intuitively, the real bond is insulated from CPI inflation as it is indexed, so the nominal bond compensates investors for the expected inflation until maturity. In addition, investors are compensated for the risk that inflation varies in the future. The inflation risk premium is positive if inflation covaries negatively with the pricing kernel M. In a consumption based asset pricing framework, the pricing kernel is related to the growth rate of the marginal utility of consumption. Inflation tends to be high when consumption growth is high, and the marginal utility of consumption is low. So the negative of the pricing kernel tends to covary positively with inflation, and we would expect the inflation risk premium to be positive. The convexity adjustment is proportional to the conditional variance of inflation.

2.3 The term structure of yield second moments

Equation (5) shows that the inflation risk premium is proportional to the covariation between future inflation and the future real pricing kernel. In order to identify this covariation precisely, we match the variance-covariance matrix of nominal and real yields. Second moments cannot be observed directly, but they can be estimated precisely when the frequency of observed yields is high (see Merton, 1980). We use the multivariate GARCH model proposed by Engle and Kroner (1995) to estimate the dynamics of the variance-covariance matrix for nominal and real yields, for each maturity.

$$\begin{bmatrix} \left(\hat{\sigma}^y_{t+1}\right)^2 & \hat{\sigma}^{yr}_{t+1} \\ \hat{\sigma}^{yr}_{t+1} & \left(\hat{\sigma}^r_{t+1}\right)^2 \end{bmatrix} = A_0' A_0 + A_1' \begin{bmatrix} \left(\hat{\sigma}^y_t\right)^2 & \hat{\sigma}^{yr}_t \\ \hat{\sigma}^{yr}_t & \left(\hat{\sigma}^r_t\right)^2 \end{bmatrix} A_1 + A_2' \begin{bmatrix} \left(\hat{\varepsilon}^y_{t+1}\right)^2 & \hat{\varepsilon}^y_{t+1}\hat{\varepsilon}^r_{t+1} \\ \hat{\varepsilon}^y_{t+1}\hat{\varepsilon}^r_{t+1} & \left(\hat{\varepsilon}^r_{t+1}\right)^2 \end{bmatrix} A_2$$

where $\hat{\varepsilon}^y_{t+1}$ is the residual of a regression of nominal yields on lagged nominal and real yields (maturity by maturity), and $\hat{\varepsilon}^r_{t+1}$ are the residuals of a regression of real yields on lagged real and nominal yields (again maturity by maturity). Furthermore, $\left(\hat{\sigma}^y_t\right)^2 = Var_t\left(y_{t+1}\right)$, $\hat{\sigma}^{yr}_t = Cov_t\left(y_{t+1},\ r_{t+1}\right)$, $\left(\hat{\sigma}^r_t\right)^2 = Var_t\left(r_{t+1}\right)$. Because the evolution of the variance-covariance matrix is according to a quadratic form, it is assured to be positive definite. We provide summary statistics of the estimated variances and covariances in Table 2.

3 Modeling Inflation Expectations

3.1 State variables and pricing kernel specification

We allow for one inflation factor π_t and two real factors m^1_t and m^2_t such that $m_t = m^1_t + m^2_t$. We further model the stochastic evolution of the variance-covariance matrix of the factors explicitly, denoting the conditional variance of π_{t+1} by $(\sigma_t^\pi)^2$, the conditional covariance of π_{t+1} and m_{t+1}^1 by $\sigma_t^{\pi m_1}$, etc. The vector of state variables is:

$$X_t = \left[\begin{array}{cccccccc} \pi_t & m_t^1 & m_t^2 & (\sigma_t^\pi)^2 & \sigma_t^{\pi m_1} & \sigma_t^{\pi m_2} & (\sigma_t^{m_1})^2 & (\sigma_t^{m_2})^2 \end{array} \right]'$$

with dynamic evolution:

$$X_{t+1} = \mu + \Phi X_t + \Sigma_t \epsilon_{t+1} \text{ where } \epsilon_{t+1} \sim N(0, I_8) \tag{6}$$

$$vec(\Sigma_t \Sigma_t') = S_0 + S_1 X_t \tag{7}$$

We provide more detail of the specification in appendix A.

Following Duffie and Kan (1996), we model the pricing kernel allowing for flexible prices of risk. In particular, we specify the rate of change of (the negative of) the log pricing kernel m_t from equation (2) as:

$$m_{t+1} = r_t^1 + \frac{1}{2} \lambda_t' \Sigma_t^{-1} \Sigma_t'^{-1} \lambda_t + \lambda_t' \Sigma_t^{-1} \epsilon_{t+1} \tag{8}$$

where λ_t is the time-varying market price of risk, and r_t is the one-period real short rate, both of which are affine functions of state variables:

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \qquad r_t^1 = \delta_0 + \delta_1 X_t \tag{9}$$

3.2 No-arbitrage pricing restrictions

Because the evolution of state variables is conditionally Gaussian, and prices of risk are conditionally Gaussian, nominal and real yields y_t^τ and r_t^τ are affine functions of X_t. In particular, by replacing (6), (8), and (9) into (3), we show in appendix B that yields can be expressed as:

$$\begin{pmatrix} y_t^\tau \\ r_t^\tau \end{pmatrix} = \frac{1}{\tau} C_0^\tau + \frac{1}{\tau} C_1^\tau X_t \tag{10}$$

where the coefficient matrices C_0^τ and C_1^τ depend on the parameters that govern the dynamic evolution of the state variables. It directly follows from (10) that the conditional variance-covariance matrix of nominal and real yields is also affine:

$$vec \begin{bmatrix} Var_t\left(y_{t+1}^\tau\right) & Cov_t\left(y_{t+1}^\tau, r_{t+1}^\tau\right) \\ Cov_t\left(y_{t+1}^\tau, r_{t+1}^\tau\right) & Var_t\left(r_{t+1}^\tau\right) \end{bmatrix} = \frac{1}{\tau^2}\left(C_1^\tau \otimes C_1^\tau\right) S_0 + \frac{1}{\tau^2}\left(C_1^\tau \otimes C_1^\tau\right) S_1 X_t \tag{11}$$

The requirement for the term structure to be arbitrage free thus not only imposes consistency of pricing across the yield curve, but also consistency of the term structure of variance-covariance matrices across maturities.

3.3 Estimation method

We estimate the model via maximum likelihood and obtain the state variables from the Kalman filter. The state space representation of our model is:

$$\begin{aligned} Y_t &= C_0 + C_1 X_t + v_t && (12) \\ X_{t+1} &= \mu + \Phi X_t + \Sigma_t \epsilon_{t+1} && (13) \\ vec(\Sigma_{t+1}\Sigma_{t+1}') &= S_0 + S_1 X_t && (14) \end{aligned}$$

where $Y_t = \begin{bmatrix} y_t & r_t & Var_{t-1}(y_t) & Cov_{t-1}(y_t,\ r_t) & Var_{t-1}(r_t) \end{bmatrix}'$ and C_0 and C_1 stack coefficients of (10) and (11) across maturities. We treat the variance-covariance matrix of nominal / real yield pairs as observable. We assume that the pricing error υ_t is normally distributed with constant, diagonal covariance matrix R. Based on state space representation in (8), we filter the factors according to the Kalman filter:

$$\hat{X}_t = \mu + \Phi \hat{X}_{t-1} + K_t \left(Y_t - C_0 - C_1 \left(\mu + \Phi \hat{X}_{t-1} \right) \right) \tag{15}$$

where K_t is the Kalman gain (see Hamilton, 1994). Given estimates of the latent factors $\hat{X}_t$, the parameters $\Theta = \{\mu, \Phi, S_0, \delta_0, \delta_1, \lambda_0, \lambda_1\}$ can be estimated by maximum likelihood, based on the conditional distribution of $Y_t|Y_{t-1}$ for each observation. The conditional distribution of Y_t is $N\left(\hat{Y}_t|Y_{t-1}, \Omega_t^Y\right)$ with $\hat{Y}_t|Y_{t-1} = C_0 + C_1 \hat{X}_{t-1}|Y_{t-1}$ and $\Omega_t^Y = C_1 Var\left(X_t|Y_{t-1}\right) C_1 + R$, and we assume that the variance-covariance matrix of the observation errors, R, is constant and diagonal. The log likelihood function is then:

$$L(\Theta) = -\sum_{t=1}^{T} \left(\log \left| \Omega_t^Y \right| + \left(Y_t - \hat{Y}_t|Y_{t-1} \right) \left(\Omega_t^Y \right)^{-1} \left(Y_t - \hat{Y}_t|Y_{t-1} \right)' \right) \tag{16}$$

4 Estimating Inflation Expectations

4.1 Pricing

Table 3 presents the parameter estimates. We use annualized percent yields in the estimation, so the long-run mean of estimated inflation is the first element of $-\left(B_1\right)^{-1} B_0$, which is 1.9% annual. We reject a unit root for the inflation process, but do find inflation to be persistent. This corresponds to the average CPI inflation over the sample period (recall that we do not use any actual inflation data). The two factors of the real pricing kernel m^1 and m^2 correspond to the level and slope factors of the real term structure. We normalize m^2 to have zero mean.

We match yields and second moments well. This can be seen in Figures 3-7. In Figures 3 and 4, we plot the actual and fitted yields of the nominal and real term structures. In Figures 5, 6, and 7, we plot the actual and fitted (annualized) real and nominal variances and covariances. We give the summary statistics of pricing errors in Table 5. We find small errors for yields and the variance-covariance of yields.

In the existing literature, term structure models are usually fitted to match yields. In this paper, we the variance-covariance matrix of yields as well. This procedure provides us with greater confidence about the accuracy of our estimated inflation risk premium. If we do not impose the constraint that second moments should be matched, a three factor model (with two real and one nominal factor and constant second moments) produces small pricing errors. Instead of using estimated volatility to fit the model, some have included option price data to gauge information about second moments (see Goldstein and Collin-Dufresne, 2002 and Bikbov and Chernov, 2005). Unfortunately, option data on TIPS securities is not readily available.

4.2 Factors

The filtered factors of the pricing kernel are plotted in Figures 8 and 9. The real kernel m declined in 2001 and 2002, and again in the second half of 2007. In a simple consumption based asset pricing model, m is proportional to the growth rate of consumption. In more elaborate habit formation models, m is proportional to the deviation of consumption growth from a moving average of consumption growth (the habit). The latter model of the real kernel might be consistent with our estimated m, but we do not investigate this route further (see Wachter, 2006 for a term structure model with habit formation).

The filtered inflation factor π increases in 2003-2004, and declines from an average

2.4% in 2005 to an average of 1.8% since the beginning of 2007. In the period from 1999-2001, the inflation factor π is rather low—this is likely due to the low liquidity of the TIPS market in the first three years (see subsection 5.2 for further discussion).

The variance of the real pricing kernel σ_m^2 is higher than the variance of inflation σ_π^2, as can be seen in Figure 9. Real volatility was particularly high in 2001-2003, and then again towards the end of 2007, corresponding to periods of low m. Real volatility is thus high when m (and hence real interest rates) are low. Inflation variance is particularly high in the early part of the sample (1999-2002 with an average of 1%), and is only 30 basis points since the beginning of 2004. The higher volatility of the inflation factor again likely reflects some of the illiquidity of the TIPS market in the early sample. Average covariance of the real pricing kernel and inflation is negative, giving rise to a positive inflation risk premium (Equation 5).

4.3 Expected inflation and forward inflation

Expected inflation can be computed from the parameters of (6):

$$\frac{1}{\tau}E_t\left(\Sigma_{s=1}^{\tau}\pi_{t+s}\right) = [1\ 0\ 0...]\left(\tilde{C}_0^{\tau} + \tilde{C}_1^{\tau}X_t\right) \tag{17}$$

where $\tilde{C}_0^{\tau} = \tilde{C}_0^{\tau-1} + \left(I_8 + \tilde{C}_1^{\tau-1}\right)\mu$, $\tilde{C}_1^{\tau} = \left(I_8 + C_1^{\tau-1}\right)\Phi$, $\tilde{C}_0^0 = \tilde{C}_1^0 = 0$.

We plot the 5-10 year breakeven forward rates together with the expected inflation forward rates in Figure 10, and the 4-5 and 9-10 year breakeven and expected inflation forward rates in Figure 11. Expected forward inflation is less volatile than breakeven forward inflation, especially for longer maturities.

Differences of breakeven and expected inflation are more pronounced for forward rates. This can be seen in Figure 11, where we plot the 4-5 year, 5-10 year, and 9-

10 year forward inflation rates for breakevens and expected inflation. The difference between 5-10 year breakeven and 5-10 year inflation was 31 basis points in the second half of 2007, and 27 basis points since the beginning of 2003. The difference between expected inflation and breakeven inflation is less pronounced for shorter maturities such as the 4-5 year forward, as risk premia are smaller at shorter horizons.

We show a comparison of our estimated π to current core and total CPI inflation in Figure 12. Since 2003, it appears that the inflation that we extract from the term structure is closer to the core CPI than to total CPI. This reflects the fact that the difference between core and total CPI is transitory, so it does not affect expected inflation at the maturities above three years which we use in our analysis.

The adjustment to breakeven inflation (i.e. the difference between breakeven inflation and expected inflation) correlates highly with market based measures of implied volatility. This can be seen in Figures 14 and 15. In Figure 14, we plot the model implied adjustment to breakeven inflation together with the Merrill Lynch Option Implied Volatility Index (MOVE). The move index is a simple average of the Treasury implied volatility of 2-year, 5-year, 10-year, and 15-30-year exchange traded options. We can see that the breakeven adjustment is high when Treasury implied volatility is high. In Figure 15, we plot the model implied adjustment together with the S&P 500 implied volatility (VIX) computed by the Chicago Board Options Exchange (CBOE). We again find a high correlation between the adjustment, and implied volatility.

5 Robustness

5.1 Testing for the number of factors

Our baseline specification has eight factors: two factors of the real pricing kernel that capture the level and slope of the real term structure, one inflation factor to model inflation expectations, and the variances and covariances of the real and nominal variables. Compared to most results in the literature, this is a relatively large number of factors. However, the variances and covariances are pinned down by our requirement that the model fit the variance-covariance matrix of nominal and real yields across maturities.

We have estimated a number of alternative specifications. In Table 6, we report a test against an alternative model with five factors: one real factor, inflation, and the variances of the real kernel and inflation, as well as their covariance. We can see that the five factor model is rejected against the eight factor alternative at the 1% level. This result arises as one factor for the real term structure is not sufficient to model the dynamics of level and slope. Figure 2 shows that the real term structure has noticeable movements in both level and slope, and that those movements are not perfectly correlated. Thus yield pricing errors increase substantially when only one real factor is used.

We also estimate alternative models with ten factors: two real factors, two inflation factors, and their variances and covariances (not reported). We do not find that the pricing performance improves significantly in the ten factor model, and thus use the eight factor model in our baseline specification in order to preserve parsimony. We have also estimate specifications without time varying variances and covariances. We achieve a similar fit in terms of yield errors, but do not fit second moments, per construction.

5.2 Structural break tests

Elsasser and Sack (2004) point out that the TIPS market was relatively illiquid for a number of years. The liquidity in the TIPS market biases real rates upwards, thus artificially compressing breakeven inflation rates. In order to see how the illiquidity might change our estimates, we fit the model separately before 2002 and since the beginning of 2002 (thus splitting the sample into a three year and a six year period). We use 2002 as a break point as Elsasser and Sack argue that liquidity in the inflation protected market was comparable to the liquidity of the off-the-run Treasury market since then.

We do find a structural break at the beginning of 2002, as reported in Table 7. However, the changes in parameters in the two separate sample periods is small. Importantly, the filtered factors do not change substantially.

Our reason for using the 1999-2008 sample period in the baseline estimates—and not the shorter 2002-2008 sample—is that the longer period captures a whole business cycle. Recall that the U.S. economy was in a recession at the beginning of 2001. If we start estimation in 2002, we exclude the recession from the sample, and pick up a trend in the slope of the real curve (see Figure 2).

5.3 Inflation as observable

The indexation of TIPS to the Consumption Price Index (CPI) introduces a predictable component in yield changes. This predictable component is sometimes called "carry", and a carry adjustment of yields is undoing this predictability. The interest and principal payments for TIPS are linked to the non-seasonally adjusted urban CPI (CPI-UNSA) with a three-month lag. The CPI is published every month. The daily reference index (DRI) for TIPS payoff and pricing calculation is computed based on the CPIU-NAS values with two- and three-month lags (M2 and M3) as,

$$\text{Daily Reference Index} = \text{Three-Month CPI Lag}$$
$$+\frac{(\text{Today}-1)}{(\text{Number of Days in Month})}(\text{Two-Month CPI Lag} - \text{Three-Month CPI Lag}).$$

TIPS principal is adjusted by multiplying the principal at issuance by the DRI at maturity and then dividing it by the DRI at issuance date. The adjusted principal is paid at maturity. The principal payment at maturity is

$$\$\ \text{Par Value}\times\frac{\text{Daily Reference Index}_{\text{maturity date}}}{\text{Daily Reference Index}_{\text{issuance date}}}$$

where the ratio of the two DRIs is often referred to as the index ratio.

We estimate specifications where the DRI is included in the observation equations (estimation results of such specifications is not reported here). We adjust for the carry effect by modeling the indexation lag explicitly. We do not find substantial differences in our estimates of the term structure of expected inflation, so we omit it from the current paper.

6 Conclusion

We propose a novel methodology to extract the term structure of inflation expectations from the term structures of nominal and real interest rates. Our contribution is to fit an arbitrage free affine model not only to yields, but also their conditional variance-covariance matrix. We find that an eight factor model with two real factors, one inflation factor, and five variance-covariance factors fits both first and second moments of the term structures well.

Our model can be updated daily, making it suitable for market monitoring. We do find that there can be substantial differences between model implied inflation expectations, and breakeven inflation rates. These differences are highly correlated with market volatility measures such as the MOVE Treasury implied volatility index, or the VIX equity implied volatility index. Intuitively, as implied volatility increases, risk premia increase, and breakevens tend to overpredict inflation expectations.

A Compact state-space form of 8-factor model

Let $Z_t = [\pi_t \; m_t^1 \; m_t^2]'$. The transition equation for Z_t follows a VAR(1) with a conditional variance-covariance matrix Ω_t:

$$Z_{t+1} - Z_t = B_0 + B_1 Z_t + \sqrt{\Omega_t}\varepsilon_{t+1} \text{ with } \varepsilon_t \sim N(0, I_3) \tag{18}$$

We model the evolution of the variance-covariance matrix Ω_t of ε_t as a multivariate stochastic process:

$$vec(\Omega_{t+1}) = \tilde{B}_0' \otimes \tilde{B}_0' + \tilde{B}_1' \otimes \tilde{B}_1' vec(\Omega_t) + \tilde{S}_0' \otimes \tilde{S}_0' vec(\tilde{\varepsilon}_t) \text{ with } p \cdot vec(\tilde{\varepsilon}_t) \sim N(0, I_5) \tag{19}$$

where p and $pinv$ are selection matrices:

$$p = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad pinv = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that $p \cdot pinv = I_5$.

To obtain the compact state space (6), we stack equations (18) and (19):

$$X_t = [Z_t \;\; p \cdot \mathrm{vec}\,(\Omega_t)]' \qquad \Sigma_t = \begin{bmatrix} \sqrt{\Omega_t} & 0 \\ 0 & \sqrt{p\left(\tilde{S}_0' \otimes \tilde{S}_0'\right) pinv} \end{bmatrix}$$

$$\mu = \begin{bmatrix} B_0 \\ \left(\tilde{B}_0' \otimes \tilde{B}_0'\right) vec\,(I_3) \end{bmatrix} \qquad \Phi = \begin{bmatrix} B_1 + I_3 & 0 \\ 0 & p\left(\tilde{B}_1' \otimes \tilde{B}_1'\right) pinv \end{bmatrix}$$

and $\epsilon_t = \begin{bmatrix} \varepsilon_t \\ \tilde{\varepsilon}_t \end{bmatrix}$. We denote the elements of Ω_t by:

$$\Omega_t = \begin{bmatrix} (\sigma_t^{\pi})^2 & \sigma_t^{\pi m^1} & \sigma_t^{\pi m^2} \\ \sigma_t^{\pi m^1} & \left(\sigma_t^{m^1}\right)^2 & 0 \\ \sigma_t^{\pi m^2} & 0 & \left(\sigma_t^{m^2}\right)^2 \end{bmatrix} \tag{20}$$

where $(\sigma_t^{\pi})^2$ is the conditional variance of π_{t+1}, $\left(\sigma_t^{m^1}\right)^2$ the conditional variance of m_{t+1}^1, etc, so $p \cdot vec\,(\Omega_t) = \begin{bmatrix} (\sigma_t^{\pi})^2 & \sigma_t^{\pi m_1} & \sigma_t^{\pi m_2} & (\sigma_t^{m_1})^2 & (\sigma_t^{m_2})^2 \end{bmatrix}'$.

B No-arbitrage restrictions

We make the following guess for yields:

$$\begin{aligned} y_t^\tau &= \tfrac{1}{\tau} C_{0y}^\tau + \tfrac{1}{\tau} C_{1y}^\tau X_t \\ r_t^\tau &= \tfrac{1}{\tau} C_{0r}^\tau + \tfrac{1}{\tau} C_{1r}^\tau X_t \end{aligned} \tag{21}$$

For real yields, we have:

$$r_t^\tau = -\frac{1}{\tau} \ln E_t \left[\exp - m_{t+1} - (\tau - 1)\, r_{t+1}^{\tau-1} \right] \tag{22}$$

Replacing the guess for the yield function:

$$C_{0r}^\tau + C_{1r}^\tau X_t = -\ln E_t \left[\exp - r_t^1 - \frac{1}{2} \lambda_t' \Sigma_t^{-1} \Sigma_t'^{-1} \lambda_t - \lambda_t' \Sigma_{t\epsilon t+1}^{-1} - C_{0r}^{\tau-1} - C_{1r}^{\tau-1} X_{t+1} \right]$$

Using the properties of the moment generating function of the normal distribution and collecting terms gives:

$$\begin{aligned} C_{0r}^\tau + C_{1r}^\tau X_t &= \delta_0 + C_{0r}^{\tau-1} + C_{1r}^{\tau-1} (\mu - \lambda_0) - \frac{1}{2} C_{1r}^{\tau-1} \otimes C_{1r}^{\tau-1} S_0 \\ &\quad + \left[-\frac{1}{2} C_{1r}^{\tau-1} \otimes C_{1r}^{\tau-1} S_1 + C_{1r}^{\tau-1} (\Phi - \lambda_1) + \delta_1 \right] X_t \end{aligned}$$

Matching terms gives:

$$C_{0r}^\tau = C_{0r}^{\tau-1} + C_{1r}^{\tau-1} (\mu - \lambda_0) - \frac{1}{2} C_{1r}^{\tau-1} \otimes C_{1r}^{\tau-1} S_0 + \delta_0 \tag{23}$$

$$C_{1r}^\tau = C_{1r}^{\tau-1} (\Phi - \lambda_1) - \frac{1}{2} C_{1r}^{\tau-1} \otimes C_{1r}^{\tau-1} S_1 + \delta_1 \tag{24}$$

For nominal yields, we have:

$$\tau y_t^\tau = -\frac{1}{\tau} \ln E_t \left[\exp\left(-\Sigma_{s=1}^{\tau} m_{t+s} - \Sigma_{s=1}^{\tau} \pi_{t+s}\right)\right] \tag{25}$$

Or:

$$C_{0y}^\tau + C_{1y}^\tau X_t = -\ln E_t \left[\exp -m_{t+1} - \phi X_{t+1} - C_{0y}^{\tau-1} - C_{1y}^{\tau-1} X_{t+1}\right]$$

where $\phi = [1\ 0\ 0\ ...]$. Then we find:

$$\begin{aligned} C_{0y}^\tau + C_{1y}^\tau X_t &= \delta_0 + C_{0y}^{\tau-1} + \left(C_{1y}^{\tau-1} + \phi\right)(\mu - \lambda_0) - \frac{1}{2}\left(C_{1y}^{\tau-1} + \phi\right) \otimes \left(C_{1y}^{\tau-1} + \phi\right) S_0 \\ &\quad + \left[\left(C_{1y}^{\tau-1} + \phi\right)(\Phi - \lambda_1) - \frac{1}{2}\left(C_{1y}^{\tau-1} + \phi\right) \otimes \left(C_{1y}^{\tau-1} + \phi\right) S_1 + \delta_1\right] X_t \end{aligned}$$

Matching coefficients, and we find:

$$C_{0y}^\tau = C_{0y}^{\tau-1} + \left(C_{1y}^{\tau-1} + \phi\right)(\mu - \lambda_0) - \frac{1}{2}\left(C_{1y}^{\tau-1} + \phi\right) \otimes \left(C_{1y}^{\tau-1} + \phi\right) S_0 + \delta_0 \tag{26}$$

$$C_{1y}^\tau = \left(C_{1y}^{\tau-1} + \phi\right)(\Phi - \lambda_1) - \frac{1}{2}\left(C_{1y}^{\tau-1} + \phi\right) \otimes \left(C_{1y}^{\tau-1} + \phi\right) S_1 + \delta_1 \tag{27}$$

Note that if the yields are rescaled by a factor γ, say they are expressed as percent per annum so that $\gamma = 1200$, the quadratic term above must be multiplied by γ^{-1}. For notational convenience we stack:

$$\begin{aligned} C_0^\tau &= \left[\ C_{0y}^\tau \quad C_{0r}^\tau\ \right]' \\ C_1^\tau &= \left[\ C_{1y}^\tau \quad C_{1r}^\tau\ \right]' \end{aligned}$$

C References

Ang, Andrew, Geert Bekaert, Min Wei, 2007, "The Term Structure of Real Rates and Expected Inflation", *National Bureau of Economic Research Working Paper* 12930.

Bernanke, Ben S., Thomas Laubach, Frederic S. Mishkin, and Adam S. Posen, 2001, *Inflation Targeting: Lessons from the International Experience*, Princeton University Press.

Barr, David G., and John Y. Campbell, 1997, "Inflation, Real Interest Rates, and the Bond Market: A Study of UK Nominal and Index-Linked Government Bond Prices", *Journal of Monetary Economics* 39, pp. 361–383.

Barro, Robert J., 1983, "Inflationary Finance under Discretion and Rules", *Canadian Journal of Economics* 16, Issue 1, pp.1-17.

Beechey, Meredith, and Jonathan Wright, 2008, "The High-Frequency Impact of News on Long-Term Yields and Forward Rates: Is It Real?," unpublished manuscript, Johns Hopkins University.

Bibkov, Ruslan, and Mikhail Chernov, 2005, "Term Structure and Volatility: Lessons from the Eurodollar Markets," *Columbia University Working Paper.*

Buraschi, Andrea, and Alexie Jiltsov, 2005 "Inflation Risk Premia and The Expectations Hypothesis", *Journal of Financial Economics* 75, pp. 429 - 490.

Campbell, John Y., Adi Sunderam, and Luis Viceira, 2007, "Inflation bets of Deflation Hedges? The Changing Risks of Nominal Bonds", *Harvard University Working Paper.*

Chen, Ren-Raw, Bo Liu, and Xiaolin Cheng, 2005, "Inflation, Fisher Equation, and the Term Structure of Inflation Risk Premia: Theory and Evidence from TIPS," *SSRN Working Paper* 809346.

Collin-Dufresne, Pierre, and Robert S. Goldstein, 2001, "Do Bonds Span Fixed Income Markets? Theory and Evidence for 'Unspanned' Stochastic Volatility," *Journal of Finance*, pp. 1685-1730.

D'Amico, Stefania, Don Kim, and Min Wei, 2006, "Tips from TIPS: The Informational Content of Treasury Inflation-Protected Security Prices," *SSRN Working Paper* 960869.

Dai, Qiang, and Kenneth Singleton, 2000, "Specification Analysis of Affine Term Structure Models," *Journal of Finance* 55, pp. 1943-1978.

Dai, Qiang, and Kenneth Singleton, 2002, "Expectations Puzzle, Time-Varying Risk Premia, and Affine Models of the Term Structure," *Journal of Financial Economics* 63, pp. 415-441.

Dai, Qiang, and Kenneth Singleton, 2003, "Term Structure Dynamics in Theory and Reality", *Review of Financial Studies* 16, pp. 631-678.

Duffee, Gregory R., 2002, "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance* 57, pp. 405-443.

Duffie, Darrell and Rui Kan, 1996, "A Yield-factor Model of Interest Rates", *Mathematical Finance* 6, pp. 379- 406,

Durham, J. Benson, 2006, "An Estimate of the Inflation Risk Premium using a Three-Factor Affine Term Structure Model", *Board of Governors of the Federal Reserve Finance and Economics Discussion Series* 2006-42.

Dybvig, Philip H., and Stephen A. Ross, 1987, "Arbitrage," *The New Palgrave: a Dictionary of Economics*, pp. 100-106.

Elsasser, Robert, and Brian P. Sack, 2004, "Treasury Inflation-Indexed Debt: A Review of the U.S. Experience," *Economic Policy Review* 10, pp. 47-63.

Engle, Robert, and Kenneth Kroner, 1995, "Multivariate Simultaneous GARCH," *Econometric Theory* 11, pp. 122-150.

Evans, Martin, 2003, "Real Risk, Inflation Risk, and the Term Structure," *Economic Journal* 113, pp. 345-389.

Fleming, Michael J., 2003, "Measuring Treasury Market Liquidity," *Federal Reserve Bank of New York Economic Policy Review* 9, pp. 83-108.

Fleming, Michael J., and Eli M. Remolona, 1999, "Price Formation and Liquidity in the U.S. Treasury Market: The Response to Public Information," *Journal of Finance* 54, pp. 1901-15.

Gurkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2006, "The U.S. Treasury Yield Curve: 1961 to the Present," *Board of Governors of the Federal Reserve Finance and Economics Discussion Series* 2006-28.

Gurkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2007, "The TIPS Yield Curve and Inflation Compensation," *Board of Governors of the Federal Reserve Finance and Economics Discussion Series* 2007.

Gong, Frank F. and Eli M. Remolona 1997, "A Three-factor Econometric Model of the US Term Structure," *Federal Reserve Bank of New York Staff Reports* 19.

Hamilton, James D., 1994, *Time Series Analysis*, Princeton University Press.

Hördahl, Peter, and Oreste Tristani, 2007, "Inflation Risk Premia in the Term Structure of Interest Rates," *European Central Bank Working Paper* 734.

Joyce, Mike, Peter Lildholdt, and Steven Sorensen, 2007, "Extracting Inflation Expectations and Inflation Risk Premia from the Term Structure: a Joint Model of the UK Nominal and Real Yield Curves," *Bank of England Working Paper.*

Kim, Don, 2004, "Inflation and the Real Term Structure," *Stanford University Dissertation.*

Krishnamurthy, Arvind, 2002, "The Bond/Old-Bond Spread," *Journal of Financial*

Economics 66, pp. 463-506.

Kydland, Finn, and Edward C Prescott, 1977, "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy* 85(3), pp. 473-91

Merton, Robert C., 1980, "On Estimating the Expected Return on the Market: An Exploratory Investigation," *Journal of Financial Economics* 8, pp. 323-61.

Nelson, Daniel B., 1990, "ARCH Models as Diffusion Approximations," *Journal of Econometrics* 45, pp. 7-38.

Piazzesi, Monika, 2003, "Affine Term Structure Models," *Handbook of Financial Econometrics.*

Singleton, Kenneth J., 2006, *Empirical Dynamic Asset Pricing*, Princeton University Press.

Wachter, Jessica A., 2006, "A Consumption-Based Model of the Term Structure of Interest Rates," *Journal of Financial Economics* 79, pp. 365-399.

Woodford, Michael, 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

	Nominal Yields					Real Yields				
Maturity	Mean	Median	Std	Max	Min	Mean	Median	Std	Max	Min
3	3.97	4.02	1.30	6.74	1.39	1.94	1.94	1.27	4.23	-0.47
4	4.15	4.24	1.16	6.70	1.74	2.15	2.11	1.18	4.34	-0.16
5	4.31	4.38	1.04	6.69	2.08	2.30	2.20	1.10	4.37	0.12
6	4.47	4.45	0.95	6.71	2.40	2.43	2.25	1.04	4.38	0.38
7	4.61	4.53	0.87	6.72	2.70	2.50	2.28	0.98	4.37	0.61
8	4.74	4.61	0.81	6.77	2.96	2.56	2.32	0.94	4.36	0.81
9	4.86	4.69	0.76	6.81	3.20	2.62	2.35	0.90	4.36	0.99
10	4.97	4.77	0.72	6.83	3.41	2.66	2.37	0.86	4.35	1.14

Table 1: Summary Statistics for Yields.

The table reports summary statistics for nominal, zero coupon, off-the-run Treasury yields and real, zero coupon TIPS yields for maturities 3, 4, 5, 6, 7, 8, 9, and 10 years. The data is weekly from 1/11/1999-8/31/2008. Source: Board of Governors of the Federal Reserve. The nominal term structure is from Gurkaynak, Sack, and Wright (2006), the real term structure is from Gurkaynak, Sack, and Wright (2007).

	Nominal Yield Variance ($\times 10^2$)					Real Yield Variance ($\times 10^2$)				
Maturity	Mean	Median	Std	Max	Min	Mean	Median	Std	Max	Min
3	2.02	1.91	0.82	4.98	0.94	1.96	1.54	1.53	8.63	0.31
4	1.98	1.88	0.76	4.70	0.92	1.39	1.15	0.78	4.42	0.30
5	1.92	1.86	0.72	4.48	0.89	1.19	0.99	0.65	3.69	0.29
6	1.83	1.78	0.69	4.24	0.83	1.08	0.90	0.61	3.24	0.27
7	1.74	1.68	0.61	3.81	0.83	0.98	0.83	0.58	2.95	0.25
8	1.66	1.63	0.59	3.49	0.80	0.91	0.76	0.55	2.75	0.23
9	1.59	1.56	0.54	3.22	0.81	0.83	0.69	0.50	2.49	0.22
10	1.53	1.52	0.51	3.10	0.79	0.78	0.66	0.47	2.43	0.20
	Yield Covarianc ($\times 10^2$)									
Maturity	Mean	Median	Std	Max	Min					
3	1.31	1.12	0.77	4.18	0.33					
4	1.24	1.02	0.66	3.64	0.34					
5	1.17	0.93	0.61	3.26	0.34					
6	1.09	0.86	0.57	2.86	0.38					
7	1.01	0.81	0.52	2.57	0.34					
8	0.95	0.78	0.50	2.55	0.31					
9	0.88	0.73	0.46	2.40	0.28					
10	0.84	0.70	0.43	2.27	0.27					

Table 2: Summary Statistics for Variances and Covariances of Yields.

	Coefficient Estimate	Std. Err.
$B_0 =$	$\begin{pmatrix} 0.0077 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.0221 \\ . \\ . \end{pmatrix}$
$B_1 =$	$\begin{pmatrix} -0.0010 & 0.0003 & -0.0072 \\ 0.0031 & -0.0002 & 0 \\ 0.0036 & 0.0006 & -0.0116 \end{pmatrix}$	$\begin{pmatrix} 0.0085 & 0.0036 & 0.0100 \\ 0.0026 & 0.0019 & . \\ 0.0119 & 0.0047 & 0.0144 \end{pmatrix}$
$\tilde{B}_1 =$	$\begin{pmatrix} 0.5512 & 0 & 0 \\ 0 & 0.6760 & 0 \\ 0 & 0 & 0.4945 \end{pmatrix}$	$\begin{pmatrix} 0.2036 & . & . \\ . & 0.13436 & . \\ . & . & 0.3876 \end{pmatrix}$
$\tilde{S}_0 =$	$\begin{pmatrix} 0.06161 & 0 & 0 \\ 0 & -0.0037 & 0 \\ 0 & 0 & 0.0049 \end{pmatrix}$	$\begin{pmatrix} 1.0081 & . & . \\ . & 3.100 & . \\ . & . & 1.160 \end{pmatrix}$
$\delta_1 =$	$\begin{pmatrix} 0.0807 \\ 1.2922 \\ 0.9952 \\ 0_5 \end{pmatrix}$	$\begin{pmatrix} 0.0988 \\ 0.0801 \\ 0.1784 \\ . \end{pmatrix}$

Table 3: Parameter Estimates.

$$\lambda_0 = \begin{pmatrix} 0.0084 & -0.0039 & -0.0015 & 0.0021 & 0.0015 & -0.0051 & -0.0003 & 0.0025 \end{pmatrix}$$

$$Std.Err. = \begin{pmatrix} 0.0687 & 0.1482 & 0.1614 & 1.2515 & 6.4519 & 3.6793 & 9.3959 & 1.6002 \end{pmatrix}$$

$$\lambda_1 = \begin{pmatrix} -0.0058 & -0.0029 & 0.0048 & 0_{1\times5} \\ -0.0031 & -0.0020 & 0.0144 & 0_{1\times5} \\ -0.0030 & -0.0028 & 0.0028 & 0_{1\times5} \\ 0_{5\times1} & 0_{5\times1} & 0_{5\times1} & 0_{5\times5} \end{pmatrix}.$$

$$Std.Err. = \begin{pmatrix} 0.00844 & 0.0035 & 0.0103 & . \\ 0.00257 & 0.0019 & 0.0022 & . \\ 0.0118 & 0.00480 & 0.0143 & . \\ . & . & . & . \end{pmatrix}$$

Measurement Error

$$R_{\text{yield}} = 0.00046 \ \left(1.3. \times 10^{-5}\right)$$

$$R_{\text{vol}} = 1.9 \times 10^{-6} \ \left(8.5 \times 10^{-7}\right)$$

Table 4: Parameter Estimates (Continued).

	Nominal Yield			Real Yield		
Maturity	Mean	Median	Std	Mean	Median	Std
3	0.0068	0.0129	0.0990	-0.0339	-0.0344	0.1342
4	-0.0202	-0.0176	0.0691	-0.0069	-0.0068	0.0689
5	-0.0176	-0.0166	0.0476	0.00078	-0.0049	0.0652
6	-0.0031	-0.0004	0.0351	0.0023	-0.0037	0.0590
7	0.0113	0.0090	0.0343	0.0016	0.0015	0.0518
8	0.0185	0.0171	0.0429	0.0002	0.0013	0.0503
9	0.0156	0.0127	0.0565	-0.0017	0.0015	0.0554
10	0.0015	-0.004	0.0727	-0.0046	0.0004	0.0649

	Nominal Variance($\times 10^4$)			Real Variance($\times 10^4$)		
Maturity	Mean	Median	Std	Mean	Median	Std
3	11.11959	10.3784	5.3897	49.5037	22.6688	71.1979
4	1.6215	2.4298	8.7564	-4.5012	0.6644	17.8840
5	-2.5428	-0.9189	8.4537	-12.9125	-3.7495	24.6054
6	-5.1399	-5.0284	5.6064	-12.5788	-4.7463	21.2169
7	-5.5393	-4.4742	3.3265	-9.7451	-4.4325	15.8819
8	-2.6838	-2.8632	3.5931	-6.2577	-2.5856	12.2774
9	2.1412	1.7740	6.5962	-3.6854	-1.6542	9.3206
10	8.4337	6.4254	9.5207	0.6464	1.9233	7.5621

	Covariance($\times 10^4$)		
Maturity	Mean	Median	Std
3	-0.9134	-1.9248	16.4993
4	-5.8432	-4.1209	5.6884
5	-6.0529	-3.4845	7.4071
6	-4.8311	-3.7114	5.0572
7	-1.6575	-1.9004	2.8968
8	2.6626	1.1844	5.5238
9	6.4907	3.9956	8.9856
10	11.6731	7.7593	11.7283

Table 5: Pricing Errors.

Model	Maximum Likelihood Value (Mean)	Number of Parameters	Number of Observations
5-factor	-290.66	23	505
8-factor	111.22	37	505

Tests	Chi Square	Significance	Degrees of Freedom
8-factor / 5-factor	405898.80	***	14

Table 6: Testing for the Number of Factors.

Model	Maximum Likelihood Value (Mean)	Number of Parameters	Number of Observations
8-factor Model	111.22	37	505
8-factor Model Before 2002	105.17	37	157
8-factor Model After 2002	118.06	37	348
Tests	**Chi Square**	**Significance**	**Degrees of Freedom**
Structural Break in 2002	2860.33	***	37

Table 7: Testing for a Structural Break.

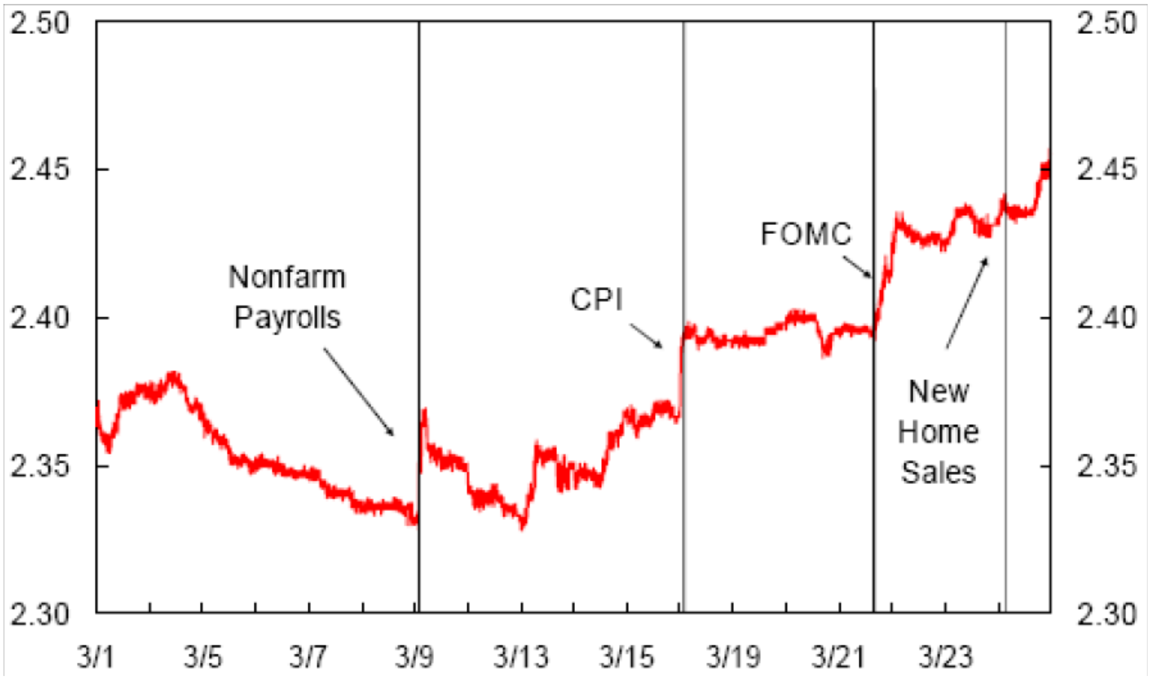

Figure 1: **Figure 1: On-the-Run 10-Year Breakeven Inflation.**

The figure plots the difference between the on-the-run 10-year Treasury and TIPS yields in five minute intervals for March 1, 2007 - March 24, 2007. Source: Bloomberg.

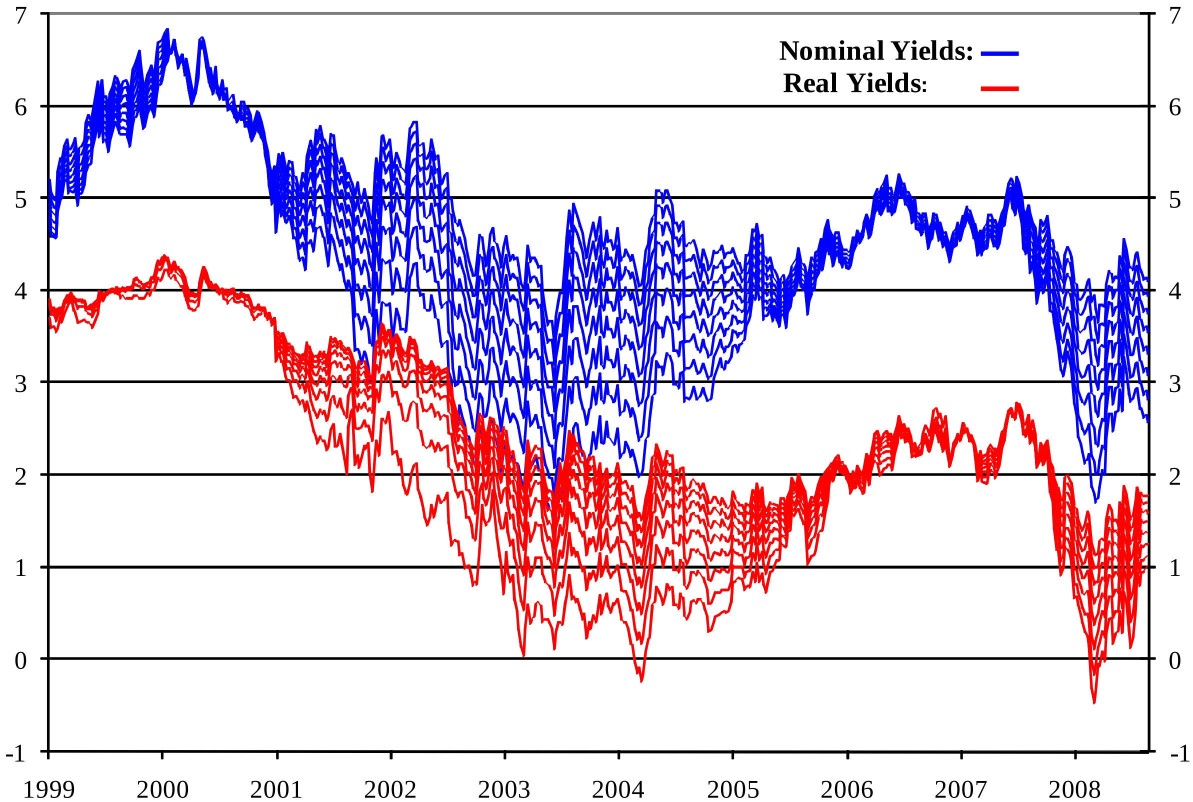

Figure 2: Nominal and Real Yields

The figure plots nominal, zero coupon, off-the-run Treasury yields and real, zero coupon TIPS yields for maturities 3, 4, 5, 6, 7, 8, 9, and 10 years. The data is weekly from 1/11/1999-8/31/2008. Source: Board of Governors of the Federal Reserve. The nominal term structure is from Gurkaynak, Sack, and Wright (2006), the real term structure is from Gurkaynak, Sack, and Wright (2007).

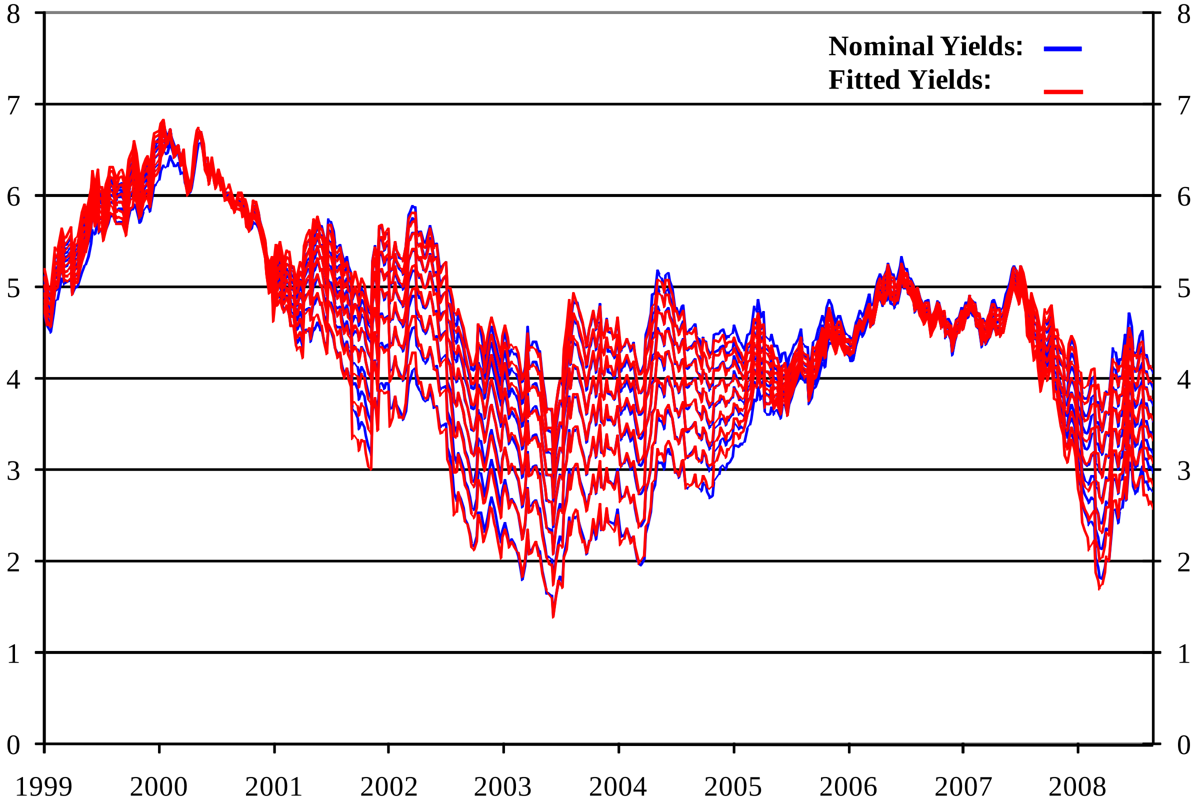

Figure 2: **Figure 3: Actual and Fitted Nominal Yields.**

The figure plots the nominal yields from Figure 2 together with the yields predicted by the term structure model.

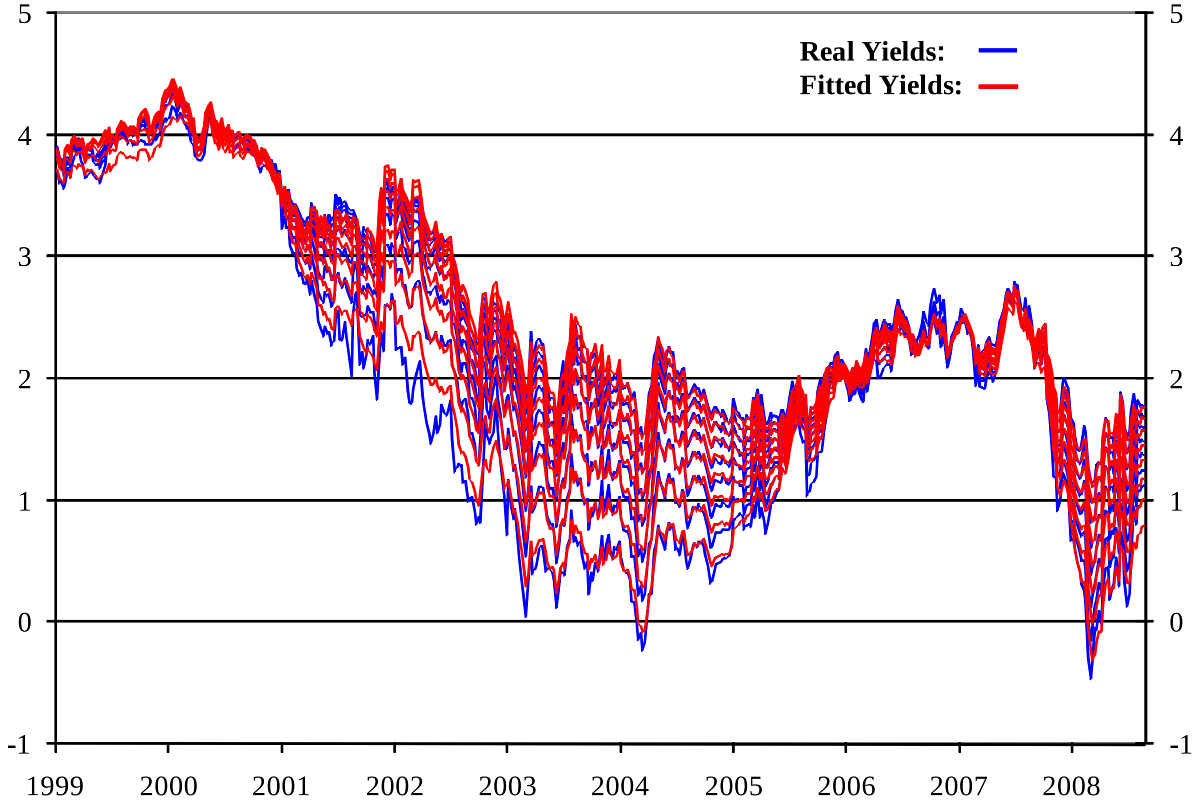

Figure 4: Actual and Fitted Real Yields.

The figure plots the real yields from Figure 2 together with the yields predicted by the term structure model.

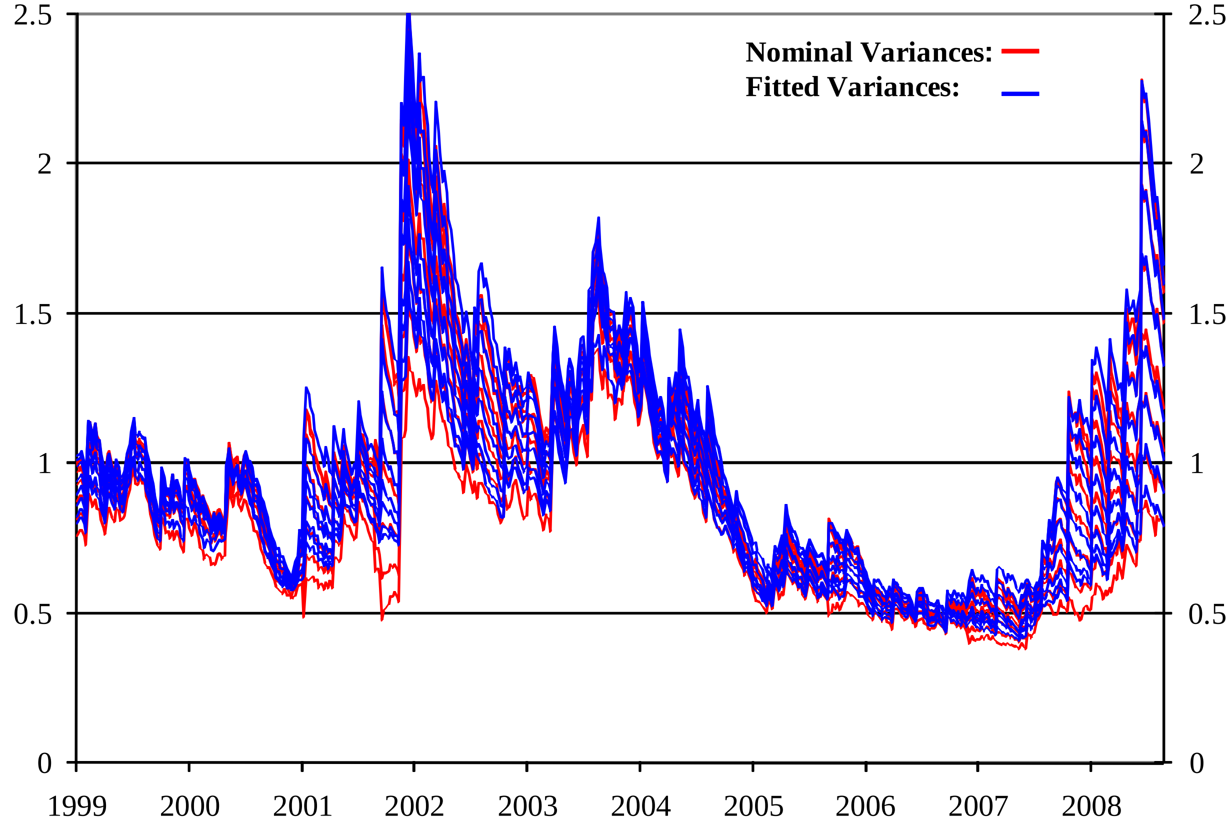

Figure 5: Actual and Fitted Conditional Variances of Nominal Yields.

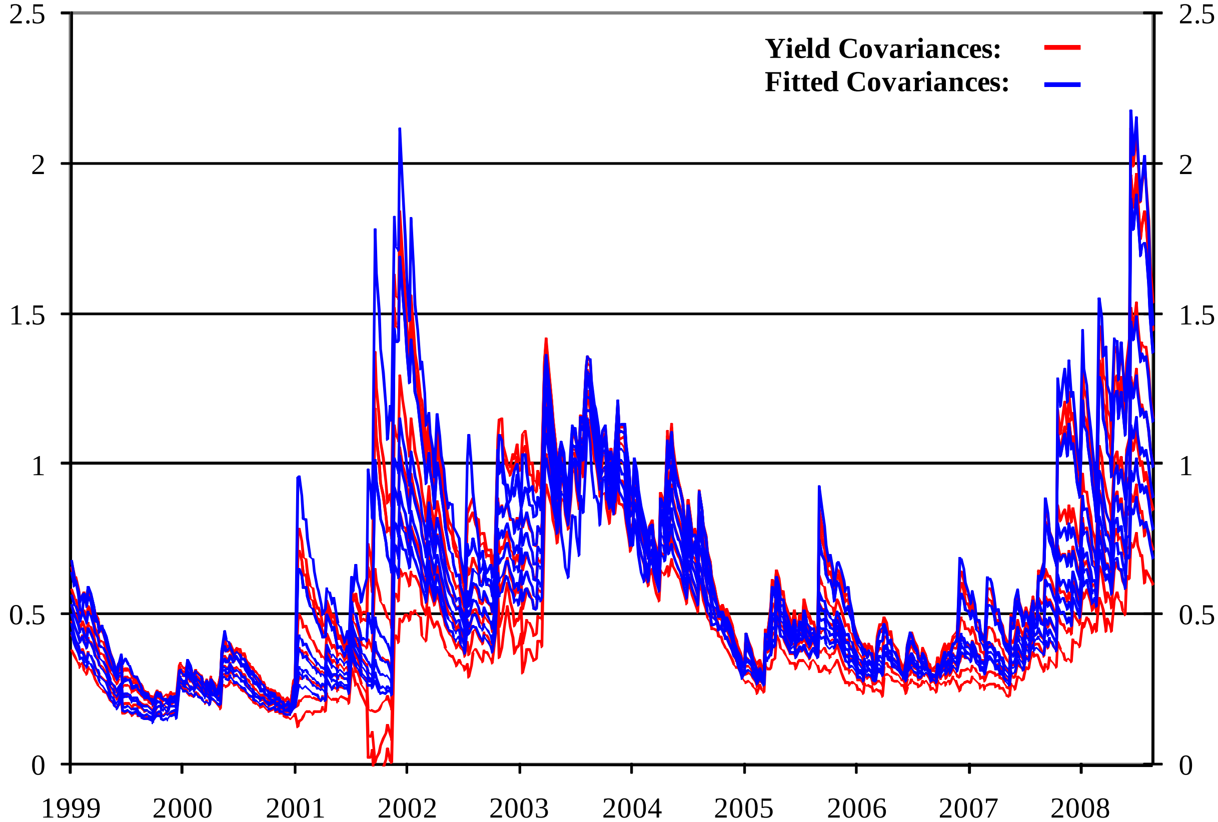

Figure 6: Actual and Fitted Conditional Variances of Real Yields.

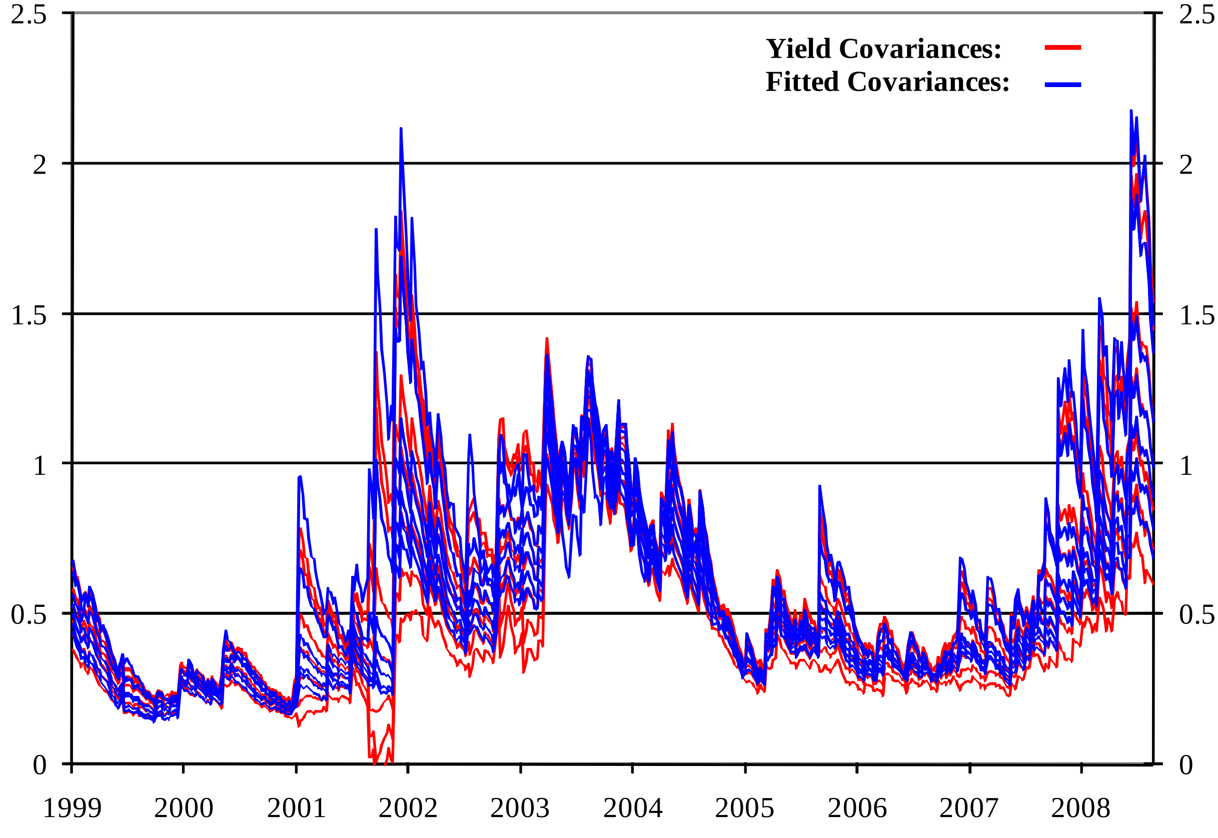

Figure 7: Actual and Fitted Conditional Covariance of Nominal and Real Yields.

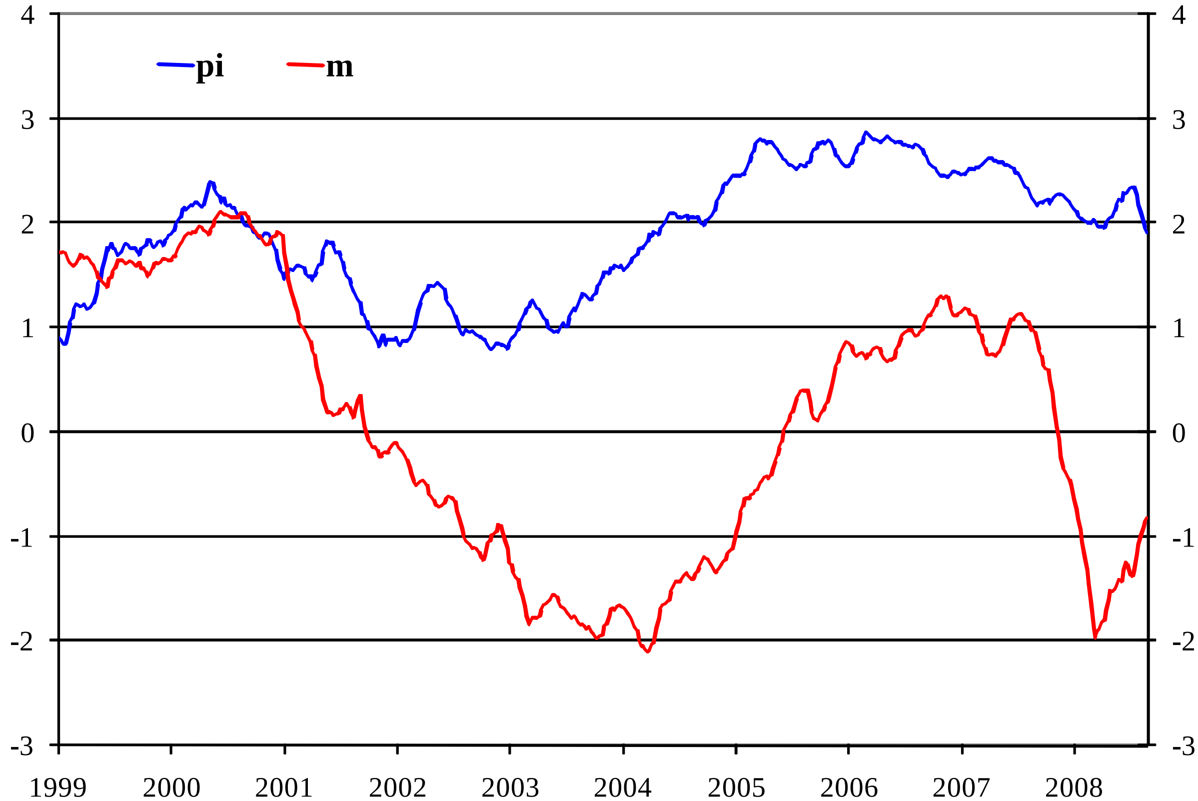

Figure 8: Estimated Inflation Factor π and Real Pricing Kernel m.

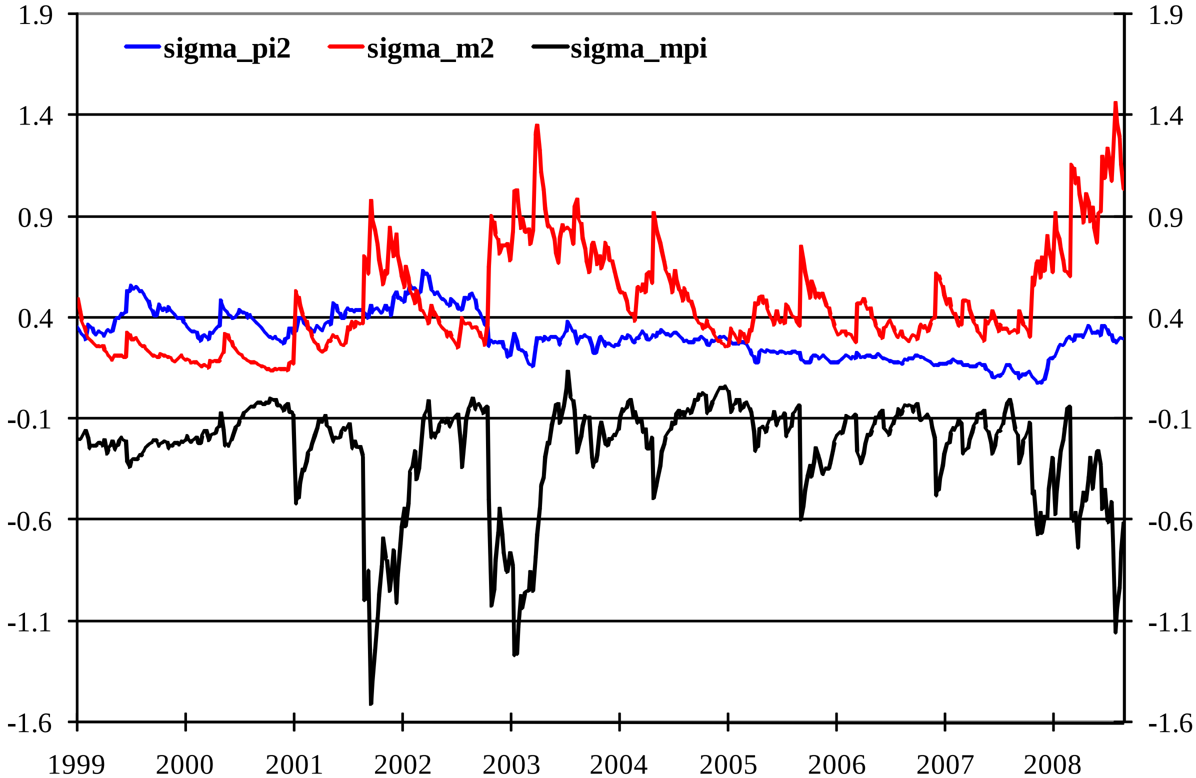

Figure 9: Pricing Factor Variances and Covariances (Annualized).

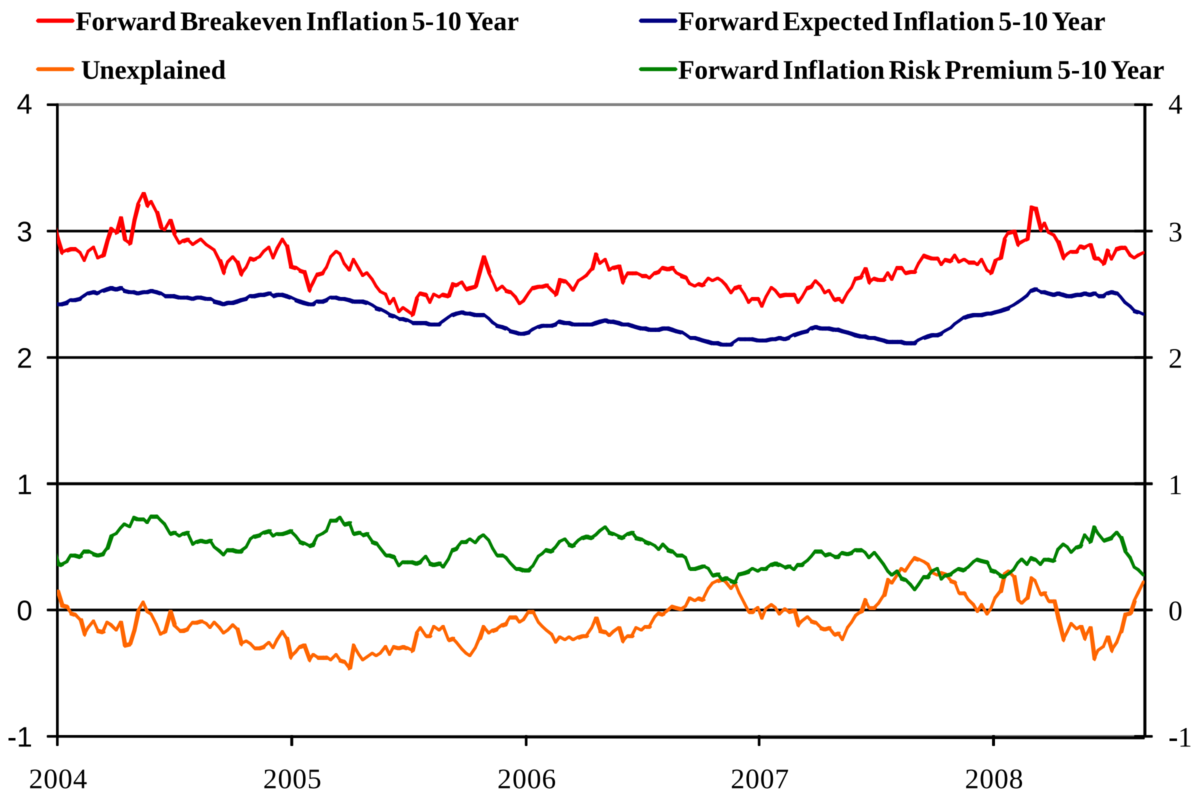

Figure 10: Forward Breakeven 5-10 Year and Forward Expected Inflation 5-10 Year.

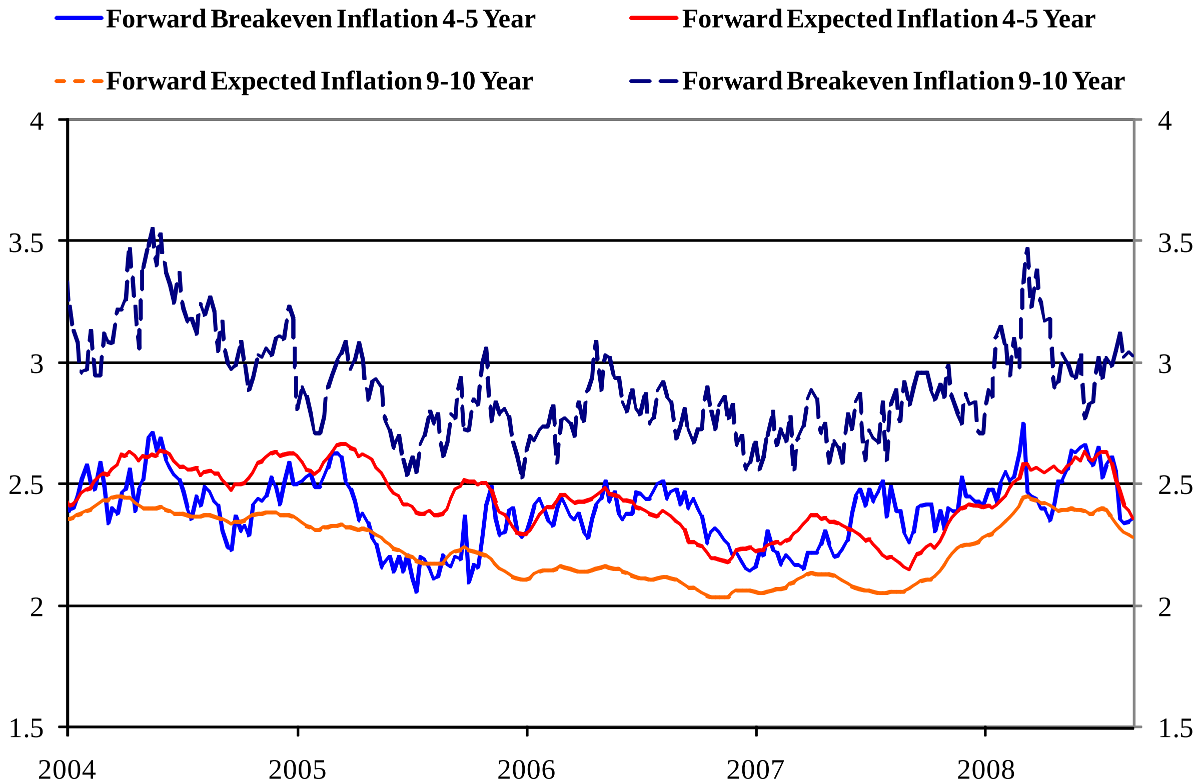

Figure 11: Forward Breakeven and Forward Expected Inflation 4-5 Year, 9-10 Year.

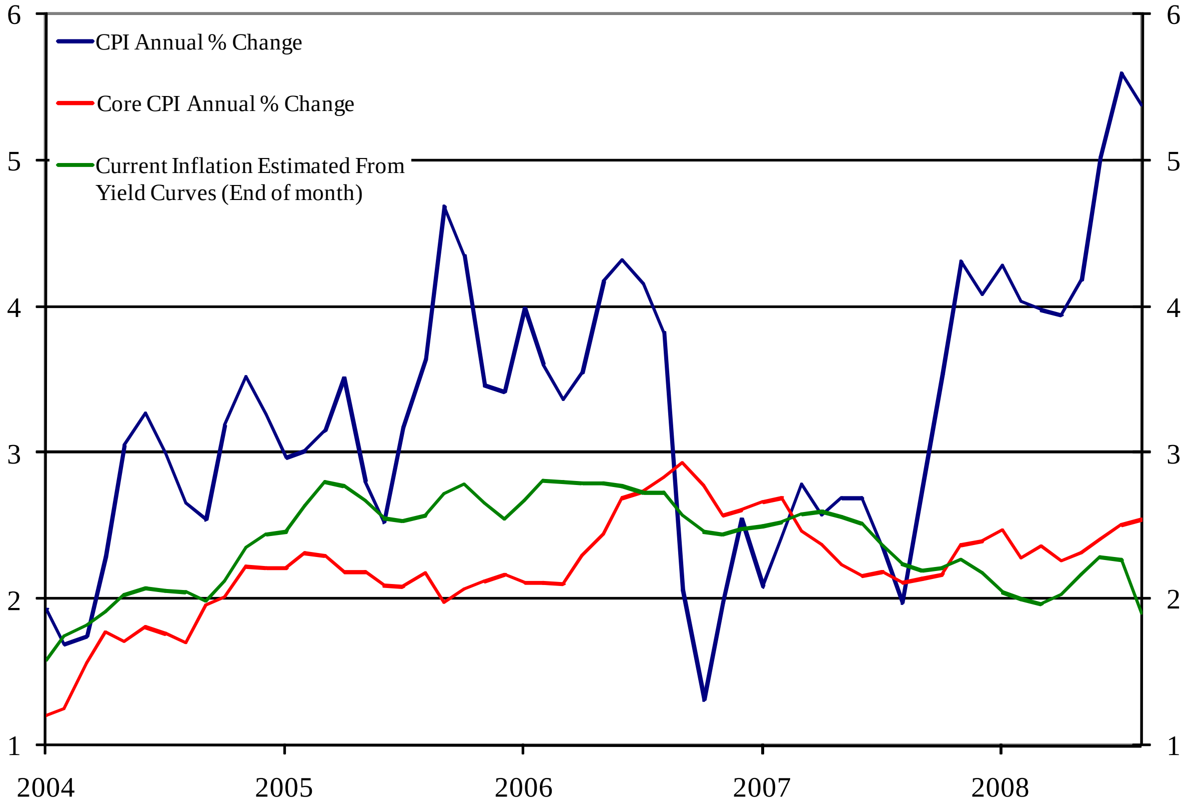

Figure 12: Comparison to Current Inflation.

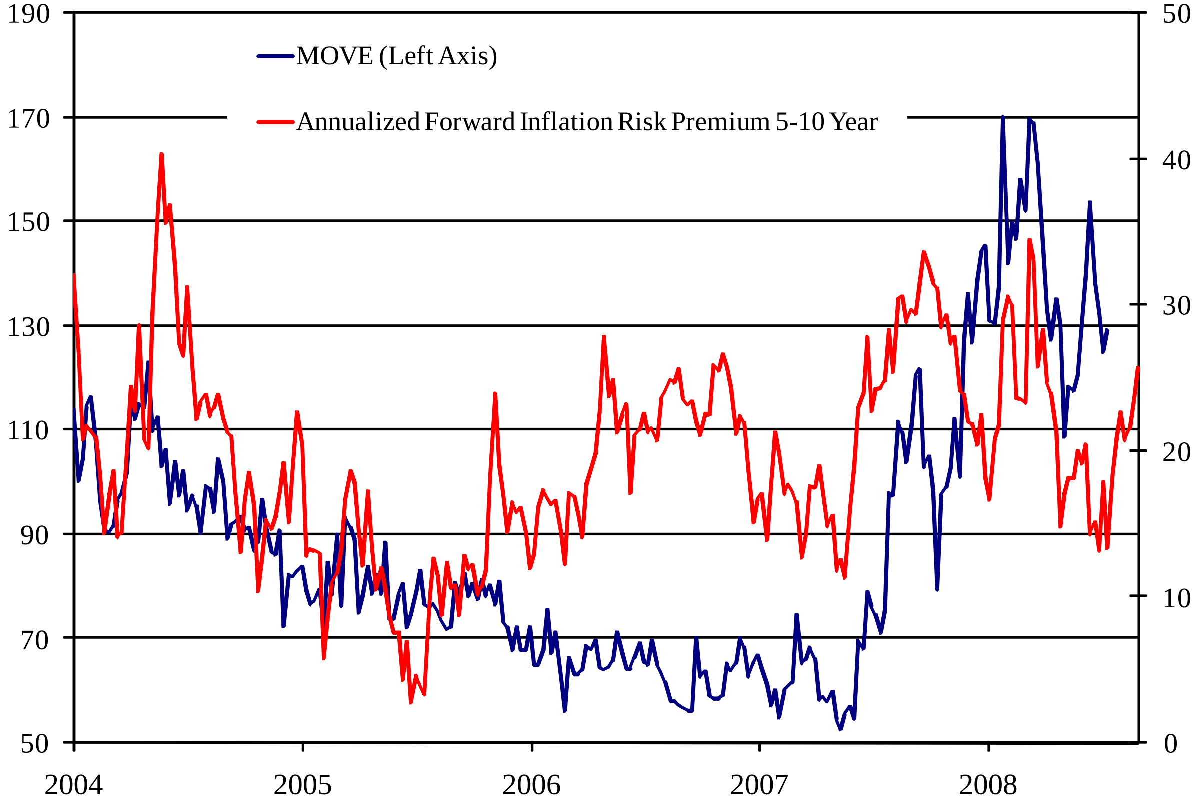

Figure 13: Breakeven Adjustment and MOVE Implied Volatility.

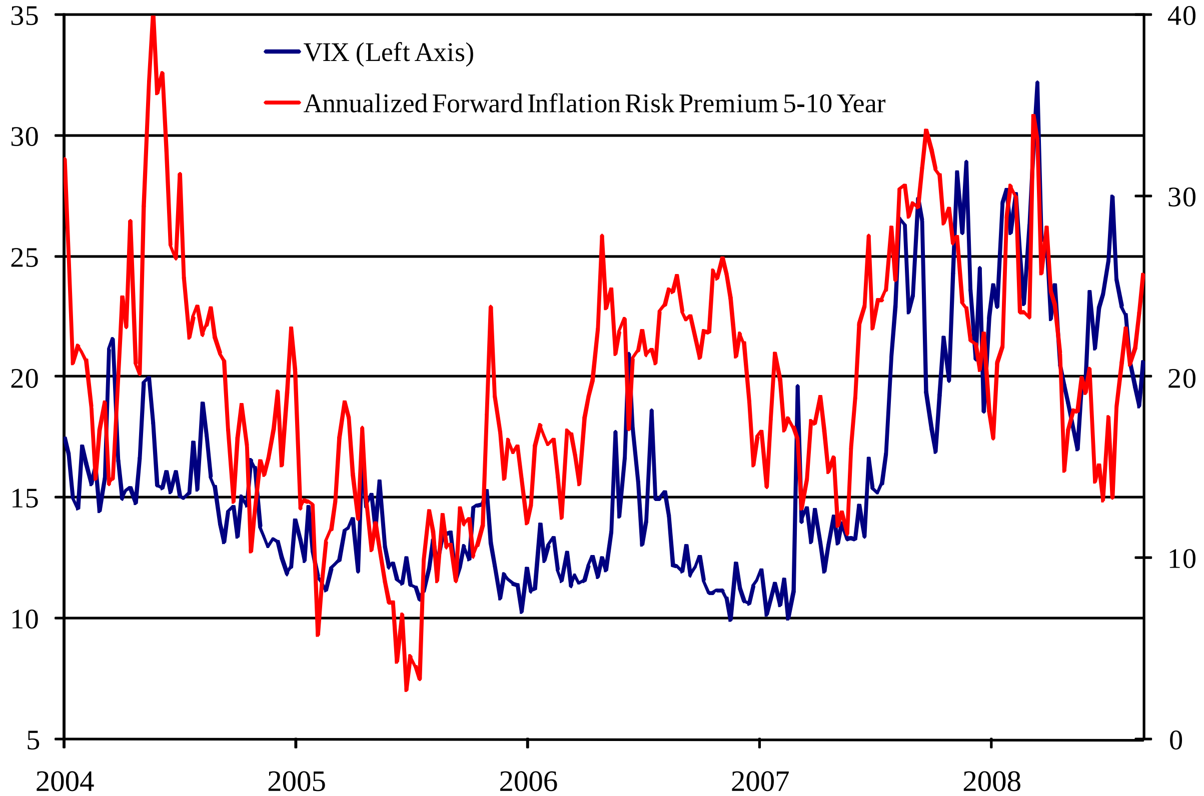

Figure 14: Breakeven Adjustment and VIX Implied Volatility.