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The Theory and Measurement of Partial Discharge Transients

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ABSTRACT

A theoretical approach to partial discharge transients is presented. This approach is based on the relationship between the charge induced on the measurement electrode by those created in the inter-electrode volume during partial discharge activity. The starting point of the analysis is the formulation of the measured-transient/induced-charge relationship. Thereafter the general relationship between the induced and the inducing charge is derived. This latter relationship is discussed with respect to both the $\vec{D}$ field and the $\vec{P}$ field. The $\vec{D}$-field approach is the more relevant from a practical point of view, but the $\vec{P}$-field approach provides a greater insight into the molecular physics of the phenomenon. An exposition of the currents related to these transient phenomena is then undertaken. The theory of void partial discharge transients based on the $\vec{D}$-field approach is thereafter reviewed and extended. This theory has allowed the influence of all relevant void parameters to be quantitatively assessed. A general derivation of the measured transients associated with the time dependence of the induced charge is presented, and the application to multiple electrode systems of practical interest is illustrated. A discussion of the salient features and practical aspects of the theory concludes the work.

1. INTRODUCTION

PARTIAL discharges in voids in a solid dielectric are often discussed in terms of a simple equivalent capacitive circuit known as the abc-model. The void is represented by a capacitance and discharges are simulated by the discharge of this capacitance [1–4]. An equivalent circuit is any circuit which can generate, as faithfully as possible, the signals which are manifest at the terminals of the actual system, and in this respect the abc-model has been very successful in promoting the development of

methods for partial discharge detection [5]. It is important to realize, however, that the operation of an equivalent circuit need in no way correspond to the physical processes that occur in the actual system. This is clearly evident in the case of the abc-model, as this model represents a physical phenomenon, which is inherently a field problem, in terms of lumped circuit parameters.

To envisage a void as a capacitor is an erroneous interpretation of the concept of capacitance. This concept is intrinsically related to conducting electrodes between

which a space-charge-free electrostatic field can be established. This means that the field must be Laplacian. Strict proportionality will consequently exist between the charges Q and $-Q$ on the electrodes and the applied voltage U between these electrodes, i.e.

$$Q = CU \tag{1}$$

where C is the capacitance. A meaningful application of (1) to the field within a void is not possible since the void wall is not an equipotential surface [6] and, in addition, once space charges are present, the field is no longer Laplacian, which again negates a meaningful use of (1), e.g. see [7].

The transients which are manifest at the electrodes of a system during partial discharge activity are related to the charges which are electrostatically induced on the electrodes. The primary sources for these induced charges are the charges which, as a result of partial discharge activity, are distributed within the voids of the insulating medium. These charge relationships will now be derived, and their application to the measurement of partial discharge transients discussed.

2. MEASURED TRANSIENT/INDUCED CHARGE RELATIONSHIP

THE charge Q_i on the i-th electrode in a space-charge-free system consisting of N electrodes is given by

$$Q_i = \sum_{j=0}^{N} C_{ij}(U_i - U_j) \tag{2}$$

with

$$\sum_{j=0}^{N} Q_j = 0 \tag{3}$$

in which $j = 0$ refers to ground. U_i and U_j are the potentials of the i-th and the j-th electrodes, and C_{ij} is the partial capacitance between these electrodes. If space charge is present in the interelectrode volume, a charge q_i will be induced on the i-th electrode. In addition, the original charge on the electrode may change from Q_i to $Q_{\rho i}$ and the potential may drop from U_i to $U_i - \Delta U_i$; see remarks in [8]. The total charge on the i-th electrode in the presence of space charge is the sum of the induced charge and the charge which is linked with the partial capacitances and the new potentials of the electrodes, viz.

$$Q_{\rho i} = q_i + \sum_{j=0}^{N} C_{ij}[(U_i - \Delta U_i) - (U_j - \Delta U_j)] \tag{4}$$

or

$$Q_{\rho i} = q_i + Q_i - \sum_{j=0}^{N} C_{ij}(\Delta U_i - \Delta U_j) \tag{5}$$

If $Q_{\rho i}$ differs from Q_i a charge ΔQ_i given by

$$\Delta Q_i = Q_{\rho i} - Q_i \tag{6}$$

will have been added to the charge on the i-th electrode. The induced charge q_i is thus given by

$$q_i = \Delta Q_i + \sum_{j=0}^{N} C_{ij}(\Delta U_i - \Delta U_j) \tag{7}$$

In principle the additional charge ΔQ_i can consist of two components, one delivered from the external source through the lead to the electrode and the other resulting from a direct exchange of charge between the surface of the electrode and the adjacent dielectric. Whereas the former component is directly accessible for measurements, the latter generally remains unknown. A direct measurement of ΔQ_i is possible, therefore, only in the absence of any exchange of charge between electrode and dielectric. This condition is normally fulfilled when the induced charge is related to discharges in a void which is completely embedded in a solid dielectric. In such cases (7) connects the induced charge with quantities which are manifest at the electrodes and which can be measured. This relationship is, therefore, the fundamental basis for partial discharge detection equipment. It does, however, not contain any quantity which directly connects the induced charge with its primary source, namely the space charge of the partial discharge. Further, it contains no information on the location and the size of any void in which partial discharges are active. The manner in which this important link can be established is discussed in the following.

3. INDUCED CHARGE/SPACE CHARGE RELATIONSHIP

THE induced charge can be expressed as the difference between the charge on an electrode following discharge activity and the charge on the same electrode prior to this activity [9]. The direct implementation of a method based on this approach can be rather cumbersome as it requires the solution of Poisson's equation. A more straightforward approach is possible through an application of the principle of superposition. This can be done in two different ways depending on whether the analysis is based on the $\vec{D}$ field [10, 11] or on the $\vec{P}$ field [12]. The former, the Maxwellian description, is convenient for practical application, whereas the latter, the quasi-molecular description, is suitable for fundamental studies of the molecular physics of the phenomenon.

3.1 MAXWELLIAN DESCRIPTION

The induced charge depends in a unique way on the location and magnitude of the space charge. This induced charge is independent of the electrode potentials if the permittivities are independent of the electric field. An infinitesimal charge dQ located somewhere in the inter-electrode region will induce charges on all the electrodes. The induced charge dq_i on the i-th electrode will, in view of the principle of superposition, be proportional to dQ, i.e.

$$dq_i = -\lambda_i\, dQ \tag{8}$$

The parameter λ_i is a dimensionless positive scalar function which depends on the location of dQ only. The entire induced charge on the i-th electrode from a distribution of space charges can thus be expressed in the form

$$q_i = -\iiint \lambda_i \rho\, d\Omega - \iint \lambda_i \sigma\, dS \tag{9}$$

in which ρ is the volume charge density in the volume element $d\Omega$ and σ is the surface charge density on the surface element dS of an interface between two dielectrics. The volume integral is extended over all space and the surface integral over all dielectric interfaces.

In a study of partial discharges in voids in solid dielectrics the space charges will be located within the voids and on the walls of these voids, and thus both terms in (9) have to be utilized. In contrast, the use of a probe to measure insulator surface charges [13] is concerned with only the second integral in (9).

The response function λ_i can be found, as shown by Maxwell [14], by applying Green's reciprocal theorem to the system in the following way. If all electrodes are held at ground potential all charges linked with the partial capacitances will be zero. The only charges left on the electrodes will then be the induced charges associated with space charges deposited in the space subtended by these electrodes. We compare this with the situation when $\rho = 0$ and $\sigma = 0$ everywhere, the potential of the i-th electrode is U_{ci} and all other electrodes are at zero potential. Applying Green's reciprocal theorem [15] to these two situations yields

$$U_{ci} q_i + \iiint V_{ci} \rho\, d\Omega + \iint V_{ci} \sigma\, dS = 0 \tag{10}$$

$$q_i = -\iiint \frac{V_{ci}}{U_{ci}} \rho\, d\Omega - \iint \frac{V_{ci}}{U_{ci}} \sigma\, dS \tag{11}$$

V_{ci} denotes the scalar potentials at $d\Omega$ and dS when the i-th electrode is at the potential U_{ci}, all other electrodes

are at zero potential, and the system is space charge free. Comparing (9) and (11) shows that

$$\lambda_i = \frac{V_{ci}}{U_{ci}} \tag{12}$$

As V_{ci} is the solution to Laplace's equation

$$\vec{\nabla} \cdot (\varepsilon \vec{\nabla} V_{ci}) = 0 \tag{13}$$

λ_i can be determined from

$$\vec{\nabla} \cdot (\varepsilon \vec{\nabla} \lambda_i) = 0 \tag{14}$$

in which ε denotes the permittivity. The boundary conditions are $\lambda_i = 1$ at the surface of the i-th electrode and $\lambda_i = 0$ at the surfaces of all other electrodes. In addition, the following condition must be fulfilled at all dielectric interfaces such as the walls of voids, *viz.*

$$\varepsilon_+ \left[\frac{\partial \lambda_i}{\partial n} \right]_+ = \varepsilon_- \left[\frac{\partial \lambda_i}{\partial n} \right]_- \tag{15}$$

where λ_i is differentiated in the direction normal to the interface and the signs $+$ and $-$ refer to each side of the interface, respectively. An alternative derivation of (14) based directly on Maxwell's equations is given in [13].

Since (14) is Laplace's equation any standard method for the calculation of space-charge-free electrostatic fields can be used to evaluate λ_i. It must be emphasized, however, that U_{ci} and V_{ci} are introduced solely for the purpose of calculation. U_{ci} can be given any arbitrarily chosen value, i.e. U_{ci} is not in any way synonymous with the potential of the i-th electrode during discharge activities for which totally different boundary conditions may exist.

3.2 QUASI-MOLECULAR DESCRIPTION

The polarization $\vec{P}$ in the dielectrics is a significant property when discussing the molecular aspects of the induced charge in a system which contains polarizable materials. The importance of the polarization is, however, not evident from the preceding analysis. The reason is that the effect of the polarization is included in the λ function. This is a definite advantage when the analysis is applied to practical systems, whereas it will be unsuitable in studies of the molecular physics of the phenomenon.

The effect of the polarization $\vec{P}$ can be taken into account by adopting a quasi-molecular description [11]. This means that the entire space subtended by the electrodes is viewed as vacuum in which the dielectric is represented by a distribution of dipoles with a dipole moment density equal to $\vec{P}$. The induced charge on an electrode is

then considered to consist of two parts. One which is linked solely with the space charge distribution created by partial discharge activities, and a second part which is related to the molecular dipoles in the dielectric, that is to changes in the polarization $\vec{P}$ caused by the presence of this space charge.

The induced charge related to a dipole can be found by visualizing a dipole as two charges, Q and $-Q$, separated by an infinitesimal distance $d\vec{r}$. The dipole moment is therefore

$$d\vec{\mu} = Q\,d\vec{r} \tag{16}$$

Let ϕ_i denote the λ_i function for vacuum, then

$$\phi_i(\vec{r} + d\vec{r}) = \phi(\vec{r}) + d\vec{r} \cdot \vec{\nabla}\phi_i \tag{17}$$

where $\vec{r}$ is the radius vector indicating the position of the dipole. The induced charge on the i-th electrode thus becomes

$$dq_i = -d\vec{\mu} \cdot \vec{\nabla}\phi_i \tag{18}$$

The response function for vacuum ϕ_i is given by Laplace's equation for vacuum, *i.e.*

$$\vec{\nabla} \cdot \vec{\nabla}\phi_i = 0 \tag{19}$$

or

$$\nabla^2 \phi_i = 0 \tag{20}$$

The boundary conditions are $\phi_i = 1$ at the surface of the i-th electrode and $\phi_i = 0$ at all other electrodes. No condition is imposed on ϕ_i at the dielectric interfaces. Any available method of electrostatic field calculation in vacuum can thus be applied to obtain ϕ_i from the equations

$$\nabla^2 V_{cvi} = 0 \tag{21}$$

and

$$\phi_i = \frac{V_{cvi}}{U_{ci}} \tag{22}$$

V_{cvi} is the scalar potential in a point of the space-charge-free electrostatic field in vacuum when the potential of the i-th electrode is U_{ci} and all other electrodes are at zero potential. As with the λ function determination, these potentials are computational quantities.

The polarization $\vec{P}$ in a point of the solid dielectric will depend on the applied voltages and on the field related to the space charge formed by partial discharges, *i.e.*

$$\vec{P} = \vec{P}_a + \Delta\vec{P} \tag{23}$$

where $\vec{P}_a$ is linked with the applied voltages and $\Delta\vec{P}$ is related solely to the effect of the inter-electrode space charge. The resulting induced charge on the i-th electrode

as a result of partial discharge activities will thus be given by

$$q_i = -\iiint (\phi_i \rho + \Delta\vec{P} \cdot \vec{\nabla}\phi_i)\,d\Omega - \iint \phi_i \sigma\,dS \tag{24}$$

The volume integral is extended over the entire space subtended by the electrodes and the surface integral over all dielectric interfaces, that is the walls of the voids.

In spite of the apparent simplicity of the ϕ function this approach is immensely more complicated than an analysis based on the λ function.

4. CURRENT PULSES IN THE LEADS

THE current I_i flowing in the lead towards the i-th electrode when the system is discharge free is given by

$$I_i = \frac{dQ_i}{dt} \tag{25}$$

During the periods of time in which partial discharges are active, a transient current I_{pi} is superimposed upon I_i. In addition, a transfer of charge can occur from the electrodes into the inter-electrode space; this is likely to take place if a void is located at an interface between an electrode and the solid dielectric. Such a transfer of charge can be accounted for by a current I_{ti} flowing away from the i-th electrode into the dielectric. The principle of conservation of charge then requires that

$$I_i + I_{pi} = I_{ti} + \frac{dQ_{pi}}{dt} \tag{26}$$

Combining this with (5) and (25) leads to

$$I_{pi} = I_{ti} + \frac{dq_i}{dt} - \sum_{j=0}^{N} C_{ij} \left[\frac{d\Delta U_i}{dt} - \frac{d\Delta U_j}{dt} \right] \tag{27}$$

Differentiating (9) with respect to the time t yields

$$\frac{dq_i}{dt} = -\iiint \lambda_i \frac{\partial\rho}{\partial t}\,d\Omega - \iint \lambda_i \frac{\partial\sigma}{\partial t}\,dS \tag{28}$$

which by means of the continuity equations

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial\rho}{\partial t} = 0 \qquad \vec{n} \cdot \Delta\vec{J} + \frac{\partial\sigma}{\partial t} = 0 \tag{29}$$

can be written in the form

$$\frac{dq_i}{dt} = \iiint \lambda_i \vec{\nabla} \cdot \vec{J}\,d\Omega + \iint \lambda_i \vec{n} \cdot \Delta\vec{J}\,dS \tag{30}$$

$\vec{J}$ is the current density at a point within a void during discharge activities. $\vec{n} \cdot \Delta\vec{J}$ is the interface divergence defined in the following manner [16],

$$\vec{n} \cdot \Delta\vec{J} = \vec{n} \cdot (\vec{J}_+ - \vec{J}_-) \tag{31}$$

where the signs $+$ and $-$ refer to each side of the interface, respectively, and $\vec{n}$ is a unit vector normal to the interface

and directed away from the positive side. It is assumed in this analysis that the effect of the surface conductivity of the interfaces, *i.e.* possible transient charge flow along the walls of the voids, can be neglected. Introducing the identities

$$\vec{\nabla} \cdot (\lambda_i \vec{J}) = \lambda_i \vec{\nabla} \cdot \vec{J} + \vec{J} \cdot \vec{\nabla} \lambda_i \qquad (32)$$

and

$$\lambda_i \vec{n} \cdot \Delta \vec{J} = \vec{n} \cdot \Delta(\lambda_i \vec{J}) \qquad (33)$$

then leads to

$$\frac{dq_i}{dt} = \iiint \vec{\nabla} \cdot (\lambda_i \vec{J}) \, d\Omega$$
$$+ \iint \vec{n} \cdot \Delta(\lambda_i \vec{J}) \, dS - \iiint \vec{J} \cdot \vec{\nabla} \lambda_i \, d\Omega \qquad (34)$$

Applying the extended divergence theorem of Gauss [17] to the vector field $\lambda_i \vec{J}$ reveals that

$$I_{ti} + \iiint \vec{\nabla} \cdot (\lambda_i \vec{J}) \, d\Omega + \iint \vec{n} \cdot \Delta(\lambda_i \vec{J}) \, dS = 0 \quad (35)$$

This means that

$$\frac{dq_i}{dt} = -I_{ti} - \iiint \vec{J} \cdot \vec{\nabla} \lambda_i \, d\Omega \qquad (36)$$

Inserting (36) in (27) shows that the additional transient current flowing in the lead towards the i-th electrode owing to discharge activities is given by

$$I_{\rho i} = - \iiint \vec{J} \cdot \vec{\nabla} \lambda_i \, d\Omega - \sum_{j=0}^{N} C_{ij} \left[\frac{d\Delta U_i}{dt} - \frac{d\Delta U_j}{dt} \right] \quad (37)$$

It should be noted that if $\vec{J}$ and $\vec{\nabla} \lambda_i$ are orthogonal, the value of the volume integral in (37) is zero, and thus a finite $\vec{J}$ does not inherently contribute to the lead current.

Since the λ_i function is determined by Laplace's equation, see (14), the gradient of λ_i can be found from

$$\vec{\nabla} \lambda_i = -\frac{\vec{E}_{ci}}{U_{ci}} \qquad (38)$$

in which $\vec{E}_{ci}$ is the field strength in the Laplacian electrostatic field between the electrodes of the system when the potential of the i-th electrode is U_{ci} and all other electrodes are at zero potential. It should again be emphasized that U_{ci} and $\vec{E}_{ci}$ are introduced solely for computational purposes. For this reason, $\vec{E}_{ci}$ and U_{ci} should not be inserted in the expression for the current: (38) should be evaluated independently of (37) to avoid any confusion with applied field terms because boundary conditions may be quite different.

Should it be desirable to include the polarization in the analysis, a similar procedure can be applied to (24). This

leads to the following expression for the transient current in the lead to the i-th electrode

$$I_{\rho i} = - \iiint \left[\vec{J} + \frac{\partial \Delta \vec{P}}{\partial t} \right] \cdot \vec{\nabla} \phi_i \, d\Omega$$
$$- \sum_{j=0}^{N} C_{ij} \left[\frac{d\Delta U_i}{dt} - \frac{d\Delta U_j}{dt} \right] \qquad (39)$$

The gradient of ϕ_i can be calculated from

$$\vec{\nabla} \phi_i = -\frac{\vec{E}_{cvi}}{U_{ci}} \qquad (40)$$

where $\vec{E}_{cvi}$ is the field strength in the electrostatic field associated with U_{ci} when the entire inter-electrode space is vacuum and all other electrodes are at zero potential. As with $\vec{\nabla} \lambda_i$, a similar precaution with the evaluation of $\vec{\nabla} \phi_i$ should be exercised.

The current density $\vec{J}$, see (37) and (39), is related to the motion of electrons and ions within the voids during discharge activities, *viz.*

$$\vec{J} = \sum_{k=0}^{m} \rho_k \vec{u}_k \qquad (41)$$

where ρ is the charge density and $\vec{u}$ the drift velocity, $k = 0$ refers to electrons and m is the number of possible species of positive and negative ions which are participating in the discharge process.

A direct application of (24) and (39) to practical systems would be rather complicated because of the explicit occurrence of the polarization in these equations. A closer analysis shows that the polarization plays an important role, from a molecular point of view, and that this effect becomes more dominant the more oblate the void is with respect to the direction of the gradient of ϕ_i [12].

Formulae similar to those derived above, but for systems in which polarizable materials are absent, have been given by many authors, see *e.g.* von Engel and Steenbeck [18], Shockley [19], Ramo [20]. These formulae are sometimes referred to as the Ramo-Shockley theorem [21]. It should be remembered, however, that these formulae were readily available in the standard literature prior to the publication of the papers by Ramo and Shockley, and that a quantitative treatment based on Faraday's concept of induced charge dates back at least to Maxwell [14].

Although the analysis given above is discussed with reference to partial discharges in voids, the sets of formulae are of general validity. Therefore these can be applied to electrode systems in which currents in the external leads

depend on charges in motion in the inter-electrode space; with respect to gas discharge studies, see [22].

5. APPLICATION TO VOID PARTIAL DISCHARGES

A discharge in a void results in a deployment of charges on the surface S of the void. The surface-charge density σ will attain such values that the electric field within the void will reduce until the discharge is quenched. As the detection of partial discharges refers to signals which are manifest at a specific electrode, we will omit subscripts. The resulting *final* value of the induced charge q on this measurement electrode is, in view of (9), given by

$$q = - \iint_S \lambda \sigma \, dS \qquad (42)$$

where σ is the final value of the surface-charge density on S.

The dimensions of any void are normally small relative to the system dimensions. In (42) λ can therefore, within the void, be replaced by its Taylor expansion. Let $\vec{r}$ denote the position vector for a fixed point inside the void and $\vec{s}$ denote a variable vector linking the end of $\vec{r}$ with dS. The value of λ at dS can now be written in the form of the Taylor expansion

$$\lambda(\vec{r} + \vec{s}) = \lambda(\vec{r}) + \sum_{n=1}^{\infty} \frac{s^n}{n!} \frac{\partial^n \lambda(\vec{r})}{\partial s^n} \qquad (43)$$

in which $\partial^n \lambda(\vec{r})/\partial s^n$ is the value of the n-th order directional derivative of λ for the fixed point given by $\vec{r}$ [23]. Combining (42) and (43) means that the induced charge q can be written as an infinite series

$$q = \sum_{n=0}^{\infty} q_n \qquad (44)$$

in which each term is related to the multipole expansion of the charges deployed on the walls of the void as a result of a partial discharge. The zero-order term q_0 is the monopole term

$$q_0 = -\lambda(\vec{r}) \iint_S \sigma \, dS \qquad (45)$$

or

$$q_0 = -\lambda(\vec{r}) Q_S \qquad (46)$$

where Q_S is the net charge deployed on S by the partial discharge. The first-order term q_1 is the dipole term given by

$$q_1 = -\mu_x \frac{\partial \lambda(\vec{r})}{\partial x} - \mu_y \frac{\partial \lambda(\vec{r})}{\partial y} - \mu_z \frac{\partial \lambda(\vec{r})}{\partial z} \qquad (47)$$

in which μ_x, μ_y, and μ_z are the Cartesian components of the dipole moment of the charge distribution, *viz.*

$$\vec{\mu} = \iint_S \vec{s}\sigma \, dS \qquad (48)$$

The second-order term q_2 is related to the quadrupole moment and is given by

$$q_2 = -\frac{1}{2}\mu_{xx}\frac{\partial^2 \lambda(\vec{r})}{\partial x^2} - \frac{1}{2}\mu_{yy}\frac{\partial^2 \lambda(\vec{r})}{\partial y^2} - \frac{1}{2}\mu_{zz}\frac{\partial^2 \lambda(\vec{r})}{\partial z^2}$$
$$- \mu_{xy}\frac{\partial^2 \lambda(\vec{r})}{\partial x \partial y} - \mu_{yz}\frac{\partial^2 \lambda(\vec{r})}{\partial y \partial z} - \mu_{zx}\frac{\partial^2 \lambda(\vec{r})}{\partial z \partial x} \qquad (49)$$

in which the six components of the quadrupole moment tensor are given [24] by

$$\mu_{xx} = \iint_S s_x^2 \sigma \, dS \qquad \mu_{xy} = \iint_S s_x s_y \sigma \, dS$$
$$\mu_{yy} = \iint_S s_y^2 \sigma \, dS \qquad \mu_{yz} = \iint_S s_y s_z \sigma \, dS \qquad (50)$$
$$\mu_{zz} = \iint_S s_z^2 \sigma \, dS \qquad \mu_{zx} = \iint_S s_z s_x \sigma \, dS$$

s_x, s_y, and s_z denote the Cartesian components of $\vec{s}$. The third-order term q_3 is related to the octupole moment, and progressively q_n to the n-th-order multipole. However, since the dimensions of the void are small, the second and all higher order terms can normally be neglected. Moreover, since in most cases the net charge Q_S will be zero, *i.e.* $q_0 = 0$, we are normally left with the dipole term only. In most cases therefore, the induced charge q will be given by (47). This expression can be written in vector form as

$$q = -\vec{\mu} \cdot \vec{\nabla} \lambda(\vec{r}) \qquad (51)$$

where the dipole moment $\vec{\mu}$ is given by (48).

In general, the gradient of λ can be determined only in such cases where the location and the geometrical form of the void are known. This difficulty can to some extent be circumvented by replacing λ with the value which the λ function would attain if we assume that the entire insulating system is completely free from voids. Let λ_0 denote this idealized function. The gradients of λ and λ_0 are connected by

$$\vec{\nabla} \lambda = h \vec{\nabla} \lambda_0 \qquad (52)$$

If the application is restricted to simple geometries, such as spheroids and to isotropic dielectrics, we can consider the parameter h to be a scalar. Generally, however, h is a tensor. Based on the mathematical analogies between

the λ function and an electrostatic field it can be deduced that

$$1 < h < \varepsilon_r \qquad (53)$$

where ε_r is the relative permittivity of the bulk dielectric. The lower limit applies to voids which are very prolate with respect to the direction of $\vec{\nabla}\lambda_0$. The upper limit is approached for a very oblate void.

The introduction of λ_0 leads to the following expression for the total induced charge on the measurement electrode

$$q = -h\vec{\mu} \cdot \vec{\nabla}\lambda_0 \qquad (54)$$

An assessment of the dipole moment requires a knowledge of the shape and location of the void. The nature of the gas within the void must also be known. All these particular data will not be available in connection with partial discharge testing of commercial HV equipment. The dipole moment and the parameter h will, however, remain constant if we consider discharges in a number of voids of fixed volume and form containing the same gas, but placed at different locations within the insulating system. The induced charge will in such cases vary with the location of the void in the same way as the gradient of λ_0 or, in view of the analogy with an electrostatic field, as indicated by the variation of the equivalent electric field strength for the idealized void-free system. It must be emphasized that, with reference to the measurement electrode, the associated equivalent electric field is *not* the same as the applied electric field. Although both fields are solutions of Laplace's equation, they are quite different solutions because, in general, each fullfils quite different boundary conditions.

Based on the above theory, a quantitative analysis related to ellipsoidal and spheroidal voids has been given by Crichton *et al.* [11].

It should be noted that in this particular Section, reference is made to the final, or total value of the induced charge, see (42). To deduce the temporal variation of $q(t)$, where t is the time, it is necessary to consider not only $\sigma(t)$ at the void wall, but also $\rho(t)$ within the void and thus both integrals in (9) must be evaluated.

6. TRANSIENTS RELATED TO INDUCED-CHARGE

THE detectable quantities from which the induced charge can be deduced are current pulses in the lead to, and transients in the potential of a measurement electrode. This electrode could be a suitable probe inserted in the system, a segment of one of the terminal electrodes or

simply one of the terminals. The relation between these transients and the induced charge is given by (7). To emphasize the transient nature of the phenomena we rewrite this expression in the form

$$q_i(t) = \Delta Q_i(t) + \sum_{j=0}^{N} C_{ij}[\Delta U_i(t) - \Delta U_j(t)] \qquad (55)$$

where t is the time. Since ΔQ_i is accessible for measurements only in the absence of any charge exchange between the electrode itself and the adjacent dielectric this condition is considered fulfilled in the following discussion.

The values of ΔU_i and ΔU_j are zero before and after the transient. The final value of the induced charge will, therefore, always be equal to the final value of the charge which is delivered to the measurement electrode from the external source; *viz.*

$$\lim_{t \to \infty} q_i(t) = \lim_{t \to \infty} \Delta Q_i(t) \qquad (56)$$

The delivered charge ΔQ_i is found in practice by integrating the transient current $I_{\rho i}$ flowing in the lead when a partial discharge is active, see (37) and (39). These expressions contain the current density $\vec{J}$ which is related to the motion of electrons and ions within a void, see (41). This current density is, however, unknown. A relationship between $I_{\rho i}$ and the transients at the electrode can be obtained from (55) by noting that, in the absence of a charge transfer between electrode and dielectric, the transient current is given by

$$I_{\rho i} = \frac{d\Delta Q_i}{dt} \qquad (57)$$

or

$$I_{\rho i} = \frac{dq_i}{dt} - \sum_{j=0}^{N} C_{ij}\left[\frac{d\Delta U_i}{dt} - \frac{d\Delta U_j}{dt}\right] \qquad (58)$$

The first term, which relates to the induced charge, is normally of very short duration as it is coincident with the creation of the partial discharge. The last term, which depends strongly on the impedances of the external circuit, will therefore in most cases dominate the recorded transient.

6.1 APPLICATION TO A THREE-ELECTRODE SYSTEM

Let us consider a three-electrode system, *i.e.* $N = 3$. Electrode-1 is the terminal to which the HV is applied, and electrode-2 is at ground potential. The 3rd electrode is the electrode at which the transients are recorded. This measurement electrode can be either a segment in one of

the other electrodes, or it can be a separate electrode used as a probe. The induced charge q_3 on this measurement electrode is then according to (55) given by

$$q_3 = \Delta Q_3 + C_{30}\Delta U_3 + C_{31}(\Delta U_3 - \Delta U_1) + C_{32}\Delta U_3 \quad (59)$$

or

$$q_3 = \Delta Q_3 + C_3\Delta U_3 - C_{31}\Delta U_1 \quad (60)$$

in which $C_3 = C_{30} + C_{31} + C_{32}$ is the total capacitance of electrode-3. The related transient current as given by (58) becomes

$$I_{\rho 3} = \frac{dq_3}{dt} - C_3\frac{d\Delta U_3}{dt} + C_{31}\frac{d\Delta U_1}{dt} \quad (61)$$

In order to study the relation between the induced charge q_3 and the space charges within a void, λ_3 or λ_{03} must be found. The former can be calculated only if the location and shape of the void are known, whereas λ_{03} can be determined without this knowledge, since it refers to a void-free system. The boundary conditions for λ_{03} are $\lambda_{03} = 1$ at the surface of electrode-3 and $\lambda_{03} = 0$ at the two other electrodes. In addition the condition given by (15) must be fulfilled at all interfaces between different dielectrics in the system, but with void interfaces excluded.

If an available method or program for electrostatic field calculation is used, the boundary conditions to be used in the calculating procedure for the electrode potentials become $U_{c1} = 0$, $U_{c2} = 0$, and $U_{c3} \neq 0$. It is evident, therefore, that the field calculated in this case is entirely different from that obtained with the voltage applied to electrode-1.

6.2 APPLICATION TO A TWO-ELECTRODE SYSTEM

The test object is often a two-electrode system, $N = 2$, with one terminal directly at ground potential. The transients related to the induced charge are, when using conventional detection equipment, referred to the HV terminal. Let $i = 1$ refer to this electrode and let electrode-2 be directly grounded. The induced charge q_1 thus becomes

$$q_1 = \Delta Q_1 + C_{10}\Delta U_1 + C_{12}\Delta U_1 \quad (62)$$

or

$$q_1 = \Delta Q_1 + C_1\Delta U_1 \quad (63)$$

where $C_1 = C_{10} + C_{12}$ is the total capacitance of electrode-1. The transient current becomes

$$I_{\rho 1} = \frac{dq_1}{dt} - C_1\frac{d\Delta U_1}{dt} \quad (64)$$

Should it be desirable to measure the transients at the grounded electrode, *i.e.* at electrode-2, an impedance must

be inserted between this electrode and ground; *i.e.* a voltage transient ΔU_2 is now manifest at this electrode. The induced charge q_2 becomes

$$q_2 = \Delta Q_2 + C_{20}\Delta U_2 + C_{21}(\Delta U_2 - \Delta U_1) \quad (65)$$

or

$$q_2 = \Delta Q_2 + C_2\Delta U_2 - C_{21}\Delta U_1 \quad (66)$$

in which $C_2 = C_{20} + C_{21}$ is the total capacitance of electrode-2.

It should be noted that all ΔU values are defined in this theory to be drops in potential. This means that ΔU_2 is negative if a voltage transient of positive polarity is recorded at electrode-2.

Similarly, we obtain for the transient current towards electrode-2 the expression

$$I_{\rho 2} = \frac{dq_2}{dt} - C_2\frac{d\Delta U_2}{dt} + C_{21}\frac{d\Delta U_1}{dt} \quad (67)$$

It is seen that q_2 and $I_{\rho 2}$ depend, in contrast to q_1 and $I_{\rho 1}$, on a voltage transient which occurs at the other terminal, in this case the HV terminal. If this additional transient is not recorded, an accurate assessment of the temporal variation cannot be achieved. However, the *final* value of the induced charge can still be correctly assessed, see (55) and (56).

7. DISCUSSION

7.1 SPACE CHARGE/INDUCED CHARGE ASPECTS

7.1.1 INDUCED CHARGE CONCEPT

THE currently accepted philosophy concerning partial discharge transients is based on the assumption that the capacitance of the system is affected by the discharge activity. This is, however, at variance with the concept of capacitance. The key to partial discharge transients is the concept of induced charge. As this charge arises via the process of electrostatic induction, the appropriate nomenclature is induced charge, a concept which dates back to Faraday. Consequently, the continued use of the term 'apparent charge', which is associated with the change-in-capacitance philosophy [1–5], is not recommended. It should be remembered that the partial capacitances of the system remain constant under partial discharge activity and that, as a consequence, the recorded transients cannot be related to a change in capacitance.

7.1.2 $\vec{D}$-FIELD AND $\vec{P}$-FIELD APPROACH

Based on the induced charge concept, analytical expressions can be derived for the charges induced on the measurement electrode of a system. This can be done in two different ways: *viz.* the Maxwellian description in which the field within the dielectrics is related to the $\vec{D}$-field, and the quasi-molecular description in which the polarization $\vec{P}$ plays a dominant role. The former is the more convenient method for practical applications. A direct implementation of the latter method to an actual system would be very cumbersome if at all possible. The reason is that, in the $\vec{P}$-field approach, parameters related to the ambient solid dielectric appear explicitly in the analytical expressions for the partial discharge transients, and their magnitudes are dependent on the unknown field sources. In the $\vec{D}$-field approach, these effects are all embodied in the λ function, and this function can be determined by well-known standard procedures.

7.1.3 THE λ FUNCTION

This function represents the proportionality factor between the free charge in the inter-electrode volume and the induced charge on the measurement electrode, *i.e.* λ is a dimensionless scalar function. The λ function is a solution of the general Laplace equation for the boundary conditions $\lambda = 1$ at the measurement electrode and $\lambda = 0$ at all other electrodes.

7.1.4 THE λ_0 FUNCTION

With respect to void partial discharge transients, it is advantageous to relate the λ function to the associated void-free system as this greatly simplifies the calculation of the induced charge. The corresponding function is designated λ_0. The variation in the induced charge with void location is then given by the gradient of λ_0. If, for example, the complete HV electrode is used for measurement purposes, then for a simple disk-type spacer in a coaxial system the induced charge is proportional to the inverse of the distance from the axis of this electrode to the center of the void [11].

For practical systems, which are always associated with nonuniform fields, the calculation of λ_0 (void free) is essentially a trivial problem in comparison to the evaluation of λ (void present). A fuller discussion of the $\vec{\nabla}\lambda/\vec{\nabla}\lambda_0$ relationship is to be found in [11].

7.1.5 THE ϕ FUNCTION

Unlike the λ function, the ϕ function is a solution of the reduced Laplace equation and thus it is simpler to evaluate. This advantage is lost however when the ϕ function is employed to determine the induced charge and currents in a system containing polarizable materials. This situation arises from the need to evaluate also the change in the polarization due to the field produced by the partial discharge space charge.

The ϕ function is adequate in traditional gas-discharge studies [22], owing to the absence of polarizable materials.

7.2 INDUCED CHARGE/MEASURED TRANSIENT ASPECTS

7.2.1 MEASUREMENT ELECTRODE

Both the λ function and the partial capacitances are strongly dependent upon the location and geometry of the measurement electrode. Consequently, the choice of this electrode will have a dominant influence on the actual transients recorded. The present analysis indicates the manner in which this influence can be assessed for each particular situation. The selection of the measurement electrode configuration can then be optimized.

7.2.2 MEASURED TRANSIENTS

Partial discharge measurements are undertaken to gain information about the partial discharge activity in the insulation. As a first step this requires a knowledge of the induced charge. This parameter is however related to effectively two components, see (55). The first of these, $\Delta Q_i(t)$, is associated with the transient current and hence will be strongly dependent on the circuit impedances.

The second component is related to the transient potentials of the remaining $(N-1)$ electrodes. This situation arises because the interelectrode space charge induces charges on all N electrodes in the system, such that we have

$$\iiint \rho \, d\Omega + \iint \sigma \, dS + \sum_{j=0}^{N} q_j = 0 \qquad (68)$$

Depending on the electrode in question, these induced charges can produce changes in the electrode potentials. Through the partial capacitances, these changes are registered at the measuring electrode. Consequently, because

the 'potential' component is closely related to the creation of the space charge, *i.e.* to the duration of the partial discharge, this component will have a very short risetime. The associated decay will again be controlled by the circuit impedance.

7.2.3 INTERPRETATION

As the induced charge is derived from the integral of a product, it is evident that the same value of induced charge could be related to an infinite number of space charge distributions and locations. Consequently, to obtain a unique interpretation of the induced charge signal requires an exact knowledge of void location and geometry, gas pressure and gas composition. These restrictions should, however, not prohibit a sound qualitative assessment of the insulation to be made on the basis of such measurements. The XIPD technique [25] offers distinct advantages in this area.

8. CONCLUSION

FROM a study of the relationship between the space charge, the induced charge and the measured transient, a theory of partial discharge transients has been developed. In so doing, the concept of induced charge has been examined from both a Maxwellian and quasi-molecular approach. The former approach, via the $\vec{D}$ field, provides an elegant method to address the problem of partial discharge transients. The quasi-molecular approach highlights the contribution of the bulk polarization to the partial discharge transients. This latter approach is, however, exceedingly cumbersome to apply in practice.

The $\vec{D}$-field approach has been applied to the problem of partial discharges in voids, and it is concluded that the induced charge on any one electrode is essentially proportional to the dipole moment of the partial discharge space charge. With this void partial discharge theory, a correct assessment of the influence of all relevant void parameters can be made.

In addition, the present analysis has clearly established the influence of both the measurement electrode and overall electrode configuration upon the partial discharge transients which are actually recorded. This allows a correct quantitative correlation between the measured transient and the induced charge to be undertaken. Through the λ and λ_0 functions, the space charge in the inter-electrode volume can be related to this induced charge in a quantitative manner.

The analysis presented has linked clearly the fundamental features of the generation, measurement and interpretation of partial discharge transients.

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