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THE THEORY OF ARMATURE WINDINGS.

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SUMMARY.

In this paper an attempt is made to outline the theory of armature windings in heteropolar machines (i.e. machines with alternate north and south poles).

Section 1.—Attention is drawn to the fundamental importance of the relation between the number of slots and the number of poles, and, by the aid of simple vector diagrams, the condition is established for obtaining a winding in which the sum of all the induced pressures is zero at any instant. If all the coils in such a winding are joined in series to form a closed winding, no internal current can circulate. Following this, the condition is found for obtaining a number of similar symmetrical polyphase systems in such a winding, this being essential when parallel circuits are needed, as in most commutator machines. A winding fulfilling these conditions is called symmetrical, and an extension of the argument reveals the relationship that must exist between the number of slots and the number of poles in a symmetrical 3-, 4-, or 6-phase winding. Table 1 shows the number of similar circuits it is possible to have with various numbers of poles.

Section 2.—This deals solely with single-layer windings. The arrangement of the coils for various symmetrical and hemisymmetrical polyphase windings is discussed. The effect of the number of slots on the number of similar parts in each phase is shown, and the means indicated for suppressing tooth effects.

Section 3.—The arrangement of open and closed double-layer windings is illustrated, and the methods of loading closed windings are compared. The connecting rules for lap and wave windings are deduced, and the restrictions examined for making these windings symmetrical. These results are given in Tables 2 and 3, along with the slottings possible for N-phase, lap and wave windings. These tables enable the designer to see at once the number of coil-sides per slot, the number of slots, and the number of phases possible in any symmetrical winding.

Lastly, it is shown how to find the points where a winding must be tapped or opened to obtain phases, examples being given to illustrate the various cases.

The sections of the paper are subdivided as follows :—

DEFINITIONS.

The following definitions may be found useful in reading the paper :—

Symmetrical polyphase system.—A polyphase system is said to be symmetrical when all the pressures have the same amplitude and are equally displaced from one another in a period, i.e. when $\beta = 2\pi/N$, where N is the number of phases and β the angle between successive phases. In such a system, the sum of the N pressures is equal to zero at any instant, so that the N phases can be joined in series (mesh) without causing internal currents to flow.

Hemisymmetrical polyphase system.—A polyphase system is called hemisymmetrical when all the pressures have the same amplitude and are displaced from one another by 1/Nth of half a period. A hemisymmetrical, N-phase system is one-half of a symmetrical, 2 N-phase system.

Phase-pitch.—The displacement between two successive phases of a polyphase system is called the phase-pitch. The phase-pitch can be measured in any suitable unit in angular or linear measure, or expressed as a number of segments, slots, etc. When measured in radians, the angular phase-pitch is β.

Phase-spread.—The angle subtended in each pole-pitch by the group of coil-sides of a phase is called the angular phase-spread. This will be denoted by σ. According to whether the phase-spread is 1/Nth or 2/Nths of a pole-pitch, we have the narrow or wide phase-spread.

Symbols Used.

C = total number of coils in a winding, or segments in commutator.
I = R.M.S. value of current.
N = number of phases.
Q = number of slot-pitches per pole-pitch (or slots per pole) $= S/2p$.
S = total number of slots in periphery.
S' = number of slots in a symmetrical polyphase system $= S/a$.
$\frac{C}{S} = \frac{u}{2}$ = segments per slot.

a = any integer, 1, 2, 3, etc.
= number of similar parts (or circuits) in a phase of a symmetrical winding.
e = instantaneous value of electromotive force.
$\bar{e}$ = maximum value of electromotive force.
m = number of coils in a group.
n = any integer.
p = total number of pole-pairs in a machine.
p' = number of pole-pairs in a symmetrical polyphase system $= p/a$.
q = number of slots per pole and phase $= Q/N$.
q_o = number of wound slots per pole and phase.
u = number of coil-sides per slot.
y_c = commutator pitch measured in commutator segments (or coils).
y_b and y_f = back and front winding pitches measured in coil-sides.
y_p = equipotential (or potential) pitch $= C/a$ segments (or coils).
y_{ph} = phase-pitch $= C/aN$ segments (or coils) $= S/aN$ slots.

β = angular phase-pitch in radians.
γ = angular slot-pitch in radians.
σ = angular phase-spread in radians.
ψ = angular phase displacement between the pressures induced in successive coils in the winding.

1. Armature Windings.

In a heteropolar machine, that is, a machine with alternate north and south poles, the periphery of p pole-pairs represents $2\pi p$ radians, in electrical measure. Denoting the total number of slots in the uniformly-slotted periphery by S, the angular slot-pitch in radians is then

$$\gamma = \frac{2\pi p}{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The phase of the pressure induced in a conductor depends chiefly on the position in the field of the slot in which the conductor lies, and only to a small extent on the position of the conductor in the slot. If no flux whatever passed through the slot, the phase of the pressure induced in each conductor in the slot would be the same. Consequently, in a slotted armature, the position of the slots in the field, or, in other words, the relation between the number of slots and the number of poles, is of fundamental importance. To examine this relation, then, we need only consider the slots filled with coils arranged to form a uniform winding over the periphery. For this purpose we can take an ordinary two-layer winding with two coil-sides per slot—one in the top and the other in the bottom layer. In this winding, the number of coils C is equal to the number of slots S. These coils can be joined up in many different ways, according to whether the constant phase displacement ψ between the pressures induced in successive coils is to be greater than, equal to, or less than the slot-pitch γ, as can be readily understood from the rules for lap and wave connections, which we shall consider presently.

Ignoring any harmonics that may be present in the pressure wave, that is, assuming a sinusoidal flux-distribution, the pressures induced in the successive coils a, b, c, etc., can be represented by vectors displaced from one another by the angle ψ, as shown in Fig. 1 (a), whilst the resultant pressure obtained by joining m coils in series is shown in Fig. 1 (b). (Small letters are used to denote instantaneous

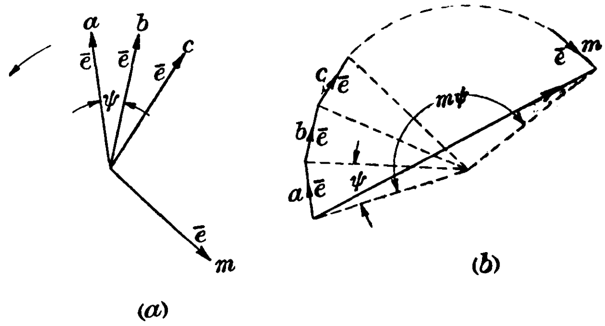

Fig. 1.—Pressures induced in successive coils of a winding.
(a) Vector diagram. (b) Resultant of vector diagram.

values, and a bar is added to denote maximum values.) If the m vectors in Fig. 1 (b) form a closed figure, as is the case whenever $m\psi = 2\pi, 4\pi, \ldots$, or, in general, $2\pi a$, where $a = 1, 2, 3$, etc., then it is clear that $\sum_a^m e = 0$, and if the m coils represented by these vectors are joined in series to form a closed winding, no internal current will circulate.

In the special case when $m\psi = 2\pi$, the m coils form, by definition, a symmetrical m-phase system. If a winding has a such systems all alike, that is, a similar sets of m coils each, it is said to be symmetrical. If like coils of the a similar parts of a symmetrical winding are joined in series or parallel, neither the phase nor the shape of the pressure wave will be affected in any way, nor can currents circulate between the several parts.

We shall now see how these conditions can be fulfilled in a winding.

(I) *Condition for obtaining a Closed Winding.*

A closed winding may be defined as a winding in which the sum of all pressures is zero at any instant, since no current of fundamental frequency can circulate in such a winding when all the coils are joined in series to form a closed winding.

In order that the sum of the pressures induced in the S coils in the winding may be zero, we must make the total phase displacement $S\psi = 2\pi a$, because only in this case do the S vectors form a closed figure.

We must have then

$$S\psi = 2\pi a,$$

so that the phase angle ψ must be (see Equation 1)

$$\psi = \frac{2\pi a}{S} = \frac{2\pi a}{2\pi p}\gamma = \frac{a}{p}\gamma. \quad . \quad . \quad . \quad . \quad (2)$$

Thus by making $\psi = a\gamma/p$, we obtain a winding in which $S\psi = 2\pi a$, and therefore $\sum_1^S e = 0$.

It will be seen presently that the rules for lap and wave connections are developed from the simple relation in Equation (2).

In the above argument, it is noticed that a is merely a number; that is, $a = 1, 2, 3$, etc.

(II) *Condition for obtaining a Symmetrical Winding.*

If a winding in which $S\psi = 2\pi a$ has a similar parts, it can be called symmetrical, because the coils in each part then form a symmetrical polyphase system, in which the sum of the pressures is zero at any instant.

Let a be the highest common factor (H.C.F.) of the number of slots S and the number of pole-pairs p. Then $S/a = S'$ and $p/a = p'$ are both integers, and the above formulæ become

$$S'\psi = 2\pi, \text{ and } S'\gamma = 2\pi p'.$$

Hence the phase angle ψ will now be

$$\psi = \frac{2\pi}{S'} = \frac{\gamma}{p'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Thus by making $\psi = \gamma/p'$, we obtain a winding in which $S'\psi = 2\pi$, and therefore $\sum_1^{S'} e = 0$. Each set of S′ vectors forms a closed polygon, and there are a such polygons exactly alike. Consequently such a winding has a similar parts, and each part forms a symmetrical S′-phase system, in which the sum of the S′ pressures is zero at any instant.

In this case, then, a has a definite meaning—it is the H.C.F. of S and p, and denotes the number of similar parts in the winding, or the number of times the slot positions in the field recur.

Since from each of a similar systems a similar circuit can be taken, it follows that a represents the number of similar circuits possible in a symmetrical winding. These a similar circuits can be united to form one phase of a polyphase system—the series or parallel connection being used according to the pressure and current required. The several phases can then be interlinked in star or mesh, as desired, unless they are already joined in mesh to form a closed winding. When the circuits are connected in series, the same current flows through all; when they are joined in parallel the current per circuit $I_a = I_p/a$, where I_p is the total current per phase.

In this way the common symmetrical polyphase windings are formed, as shown in sub-section (IV). The chief of these are

Symmetrical 2-phase winding: 2 phases at 180° (diametral tappings).
Symmetrical 3-phase winding: 3 phases at 120°.
Symmetrical 4-phase winding: 4 phases at 90°.
Symmetrical 6-phase winding: 6 phases at 60°.

When the coils of a symmetrical winding are joined in series to form a closed winding, there will always be a coils at the same potential. These a equipotential coils are $S' = S/a$ coils apart, and must be connected to a common terminal by means of tappings or commutator brushes, in order to load the a similar parts of the closed winding uniformly. For this reason we call the distance between successive equipotential coils the equipotential (or, briefly, the potential) pitch, i.e. the potential pitch $y_p = S/a$ slots or C/a coils. Obviously, equipotential connectors (or equalizing rings) can be joined to any of the corresponding S′ phases in the a similar parts of the winding, when desired.

An interesting case of the mesh connection with the a systems in parallel is the continuous-current machine, where connection is made to a closed winding by means of commutator brushes. In this case a brush does not make connection with two similar circuits, but with the two phases at 180° of a symmetrical 2-phase system. The total current I_p in each phase is then equal to half the continuous current I_c, so that the current per circuit $I_a = I_p/a = I_c/(2a)$.

The above conditions can be summarized as follows:—

(I) When the total phase displacement of the S coils $S\psi = 2\pi a$, where $a = 1, 2, 3$, etc., then $\sum_1^S e = 0$, and the S coils can be joined in series to form a closed winding.

(II) When in addition to $S\psi = 2\pi a$, a is the H.C.F. of S and p, then $S'\psi = 2\pi$, and therefore $\sum_1^{S'} e = 0$; so that the S coils form a similar symmetrical S′-phase systems.

A winding which satisfies conditions (I) and (II) is called symmetrical, whilst a winding which satisfies condition (I) but not condition (II) is called unsymmetrical.

These conditions show clearly how the number of similar parts or circuits in a winding depends on the relation between the number of poles and the number of slots. When we come to deal with the actual windings themselves in the latter part of the paper, it will be found that a certain amount of confusion exists due to a slotting being used for a number of circuits different from the H.C.F. of S and p. In practice this often leads to undesirable results, as will be seen when we refer to two of the commonest examples. In an alternator it is quite common to use a slotting which gives the number of similar parts a equal to the number of pole-pairs p when only one circuit per phase is needed, with the result that tooth effects are repeated throughout the phase, whereas when a is less than p (for example $a = 1$), the tooth effects are suppressed by joining dissimilar pressures in series. On the other hand, in a commutator machine where the a parts are joined in parallel at the commutator or at the slip-rings it is very important to make the winding symmetrical, that is, the a paths exactly similar, to prevent sparking and the circulation of internal currents.

(III) *Examples of Symmetrical and Unsymmetrical Windings.*

To illustrate the foregoing, let us choose at random different numbers of slots S and of pole-pairs p, and show what windings with 2 coil-sides per slot are possible when we make $a = 1, 2, 3$, etc. The possible lap and wave windings are found from the closing rules derived from Equation (2), and given on page 29. Though single-layer windings might also be included in this table, nothing would be gained by their addition, for the symmetry or dissymmetry of a winding depends fundamentally on the relation between the number of poles and the number of slots, and not on the type of winding used.

see under what conditions the slotting for a symmetrical winding can be divided into N equal parts at an angle $\beta = 2\pi/N$ radians apart to form a symmetrical N-phase system. In other words, we must see when it is possible to derive a symmetrical N-phase system from S' slots and p' pole-pairs.

First, with regard to the slots, let S' be exactly divisible

Pole-pairs p	Total Slots S	H.C.F. of S and p	p'	S'	Assumed a	$\psi = \frac{a}{p}\gamma$	$S\psi = 2\pi a$	Conditions satisfied	Possible Windings
2	39	1	2	39	1	$\gamma/2$	2π	I and II	Symmetrical wave
"	"	"	"	"	2	γ	4π	I	Unsymmetrical lap
"	"	"	"	"	3	$3\gamma/2$	6π	I	Unsymmetrical wave
2	36	2	1	18	1	$\gamma/2$	—	—	Impossible *
"	"	"	"	"	2	γ	4π	I and II	Symmetrical lap or wave
"	"	"	"	"	3	$3\gamma/2$	—	—	Impossible
3	42	3	1	14	1	$\gamma/3$	—	—	Impossible
"	"	"	"	"	2	$2\gamma\ 3$	—	—	"
"	"	"	"	"	3	γ	6π	I and II	Symmetrical lap or wave
3	41	1	3	41	1	$\gamma/3$	2π	I and II	Symmetrical wave
"	"	"	"	"	2	$2\gamma/3$	4π	I	Unsymmetrical wave
"	"	"	"	"	3	γ	6π	I	Unsymmetrical lap
3	40	1	3	40	1	$\gamma/3$	2π	I and II	Symmetrical wave
"	"	"	"	"	2	$2\gamma/3$	4π	I	Unsymmetrical wave
"	"	"	"	"	3	γ	6π	I	Unsymmetrical lap
4	48	4	1	12	1	$\gamma/4$	—	—	Impossible
"	"	"	"	"	2	$\gamma/2$	—	—	"
"	"	"	"	"	3	$3\gamma/4$	—	—	"
"	"	"	"	"	4	γ	8π	I and II	Symmetrical lap or wave
4	47	1	4	47	1	$\gamma/4$	2π	I and II	Symmetrical wave
"	"	"	"	"	2	$\gamma/2$	—	—	Impossible
"	"	"	"	"	3	$3\gamma/4$	6π	I	Unsymmetrical wave
"	"	"	"	"	4	γ	8π	I	Unsymmetrical lap
4	46	2	2	23	1	$\gamma/4$	—	—	Impossible
"	"	"	"	"	2	$\gamma/2$	4π	I and II	Symmetrical wave
"	"	"	"	"	3	$3\gamma/4$	—	—	Impossible
"	"	"	"	"	4	γ	8π	I	Unsymmetrical lap
4	45	1	4	45	1	$\gamma/4$	2π	I and II	Symmetrical wave
"	"	"	"	"	2	$\gamma/2$	—	—	Impossible
"	"	"	"	"	3	$3\gamma/4$	6π	I	Unsymmetrical wave
"	"	"	"	"	4	γ	8π	I	Unsymmetrical lap

* Cases are marked impossible when it is not possible to find windings, due to the impossibility of giving ψ the required values or to other reasons. For example, with 18 slots per pole-pair, it is clearly impossible to make ψ an odd multiple of $\gamma/2$.

Though windings are also possible when $a > p$, these have not been included in the above table, because, being unsymmetrical, they are not so important, and because the above table is merely intended to be illustrative, the complete tables being given later in the paper (see page 32).

(IV) *Conditions for obtaining a Symmetrical N-phase Winding.*

Though it is possible to have a similar S'-phase systems in a symmetrical winding, usually only a smaller number of phases $N = 3, 4$, or 6 is required. We must therefore by N. We then get N equal phases displaced from one another by an angle $S'\psi/N = 2\pi/N = \beta$ (since $S'\psi = 2\pi$, in a symmetrical winding), which is what is needed. We can call the integer S'/N the phase-pitch measured in slots, and denote it by y_{ph}, that is, $y_{ph} = S'/N$.

Coming now to the poles, it is clear that $2\pi p'/N = p'\beta$ must give the correct phase displacement $2\pi/N = \beta$. This is the case when $p' = nN \pm 1$, where $n = 0, 1, 2, 3$, etc., for then $p'\beta = (nN \pm 1)2\pi/N = \pm 2\pi/N = \pm\beta$. That is to say, when $p' = nN \pm 1$ the same positions are obtainable for N phases in the field with p' pole-pairs as with one pole-pair. This is not possible, however, if p' is a multiple of N, for when $p' = n\,\mathrm{N}$, $p'\,\beta = n\,\mathrm{N}(2\,\pi/\mathrm{N}) = 0$. The same result can also be deduced from the fact that in a symmetrical winding S′ and p' have no common factor greater than unity, and since S′/N is to be an integer, it follows that p'/N cannot be integral.

Since a symmetrical winding has a similar parts, it follows that each phase in the N-phase system will have a similar parts, and by connecting these a parts in parallel we get a circuits per phase.

Thus to get a symmetrical N-phase winding, we must have a symmetrical winding (as defined in Equation 3) such that

$$\left.\begin{aligned} v_{ph} &= \frac{\mathrm{S}}{\mathrm{N}} = \frac{\mathrm{S}}{a\,\mathrm{N}} = \text{integer} \\ \text{and} \qquad p' &= \frac{p}{a} = n\,\mathrm{N} \pm 1 \end{aligned}\right\} \quad . \; . \; . \; . \; . \quad (4)$$

where $a =$ H. C. F. of S and $p =$ number of similar parts (or circuits) per phase.

The numbers of slots that will satisfy Equation (4) for N = 3, 4, and 6, are determined in the following sections of the paper, and are tabulated for lap and wave windings on page 32, whilst the following table shows the relation between the number of phases N, the number of similar parts (or circuits, with parallel connection) per phase a and the number of pole-pairs p in the machine.

Table 1 shows that certain cases are not possible; for example, it is not possible to obtain a symmetrical 3-phase winding with only one or two similar parts per phase in a 6- or 12-pole machine, nor is it possible to obtain a symmetrical 4-phase pressure from an 8-pole machine unless we have four similar parts in each phase.

Unless a commutator is used, it is not necessary, and with some types of winding not possible, to wind all the slots in each of the N parts; but provided that all N phases are made equal and properly spaced, a symmetrical N-phase pressure is obtained.

An extension to the above rule for obtaining a similar circuits per phase enables us to get $2\,a$ parallel circuits per phase when N is even. For example, a symmetrical 6-phase system is equivalent to two symmetrical 3-phase systems at 180°, so that by joining opposite phases I and IV, III and VI, and V and II in parallel, we get a symmetrical 3-phase system with the same number of circuits as the 6-phase system from which it is derived. It this way it is possible to get two circuits per phase in a 2-pole machine, or in general to obtain as many circuits per phase as there are poles—a matter of great importance in practice. In a similar way it is possible to derive a hemisymmetrical 2-phase winding from a symmetrical 4-phase winding.

Obviously, the number of phases in an N-phase system, when N is even, can be halved by joining opposite phases in series, and this is a very common way of obtaining a 3-phase winding.

Before applying these conditions to single- and double-layer windings, occasion may be taken here to point out the importance of making polyphase windings completely symmetrical wherever possible. The nature of the effects of dissymmetry in alternating-current systems can be understood by considering what happens when polyphase synchronous machines work in parallel. When the pressures of the several machines are unequal, due to incorrect field excitation, equality is established by reactive currents, which maintain the fluxes in the several machines at such a value that the sum of all the pressures round any circuit is zero at every instant. When there is a phase displacement between a machine pressure and a line pressure, a power component of current flows, and according to whether the phase of the machine pressure is ahead or behind that of the line pressure, the machine acts as a generator or motor. In the same way, when the pressures in the phases of a winding are unequal, reactive currents will flow to equalize them, whilst dissimilar phase angles will give rise to power components.

Where continuous currents are concerned, experience has shown that sparking in rotary converters and flickering in the lights connected on each side of the middle wire

TABLE 1.

Possible Numbers of Similar Circuits per Phase in a Symmetrical N-phase Winding on a Machine with 2 p Poles.

Number of Pole-pairs in Machine p	Number of Similar Circuits per Phase a	$p' = \frac{p}{a}$	Possible Number of Phases N
1	1	1	3, 4, 6
2	1	2	3
	2	1	3, 4, 6
3	1	3	4
	3	1	3, 4, 6
4	1	4	3
	2	2	3
	4	1	3, 4, 6
5	1	5	3, 4, 6
	5	1	3, 4, 6
6	2	3	4
	3	2	3
	6	1	3, 4, 6
7	1	7	3, 4, 6
	7	1	3, 4, 6
8	1	8	3
	2	4	3
	4	2	3
	8	1	3, 4, 6
9	1	9	4
	3	3	4
	9	1	3, 4, 6
10	1	10	3
	2	5	3, 4, 6
	5	2	3
	10	1	3, 4, 6
11	1	11	3, 4, 6
	11	1	3, 4, 6
12	3	4	3
	4	3	4
	6	2	3
	12	1	3, 4, 6

of dynamos with static balancers have often been caused by unsymmetrical phases on the alternating-current side. Also, the trouble due to having the parallel circuits unlike one another in a commutator machine are well known. Of course, in some cases, like the rotor of a small induction motor, a certain amount of dissymetry may be harmless.

(V) *Phase-spread and Coil-span.*

In each pole-pitch the coil-sides of each phase are spread over a definite arc—this arc we shall call the phase-spread, and denote by σ (angular measure). According to the way the winding is arranged, the phase-spread can be made equal to 1/Nth or 2/Nths of the pole-pitch, or $\sigma = \pi/N$ or $2\pi/N$ radians. With a narrow phase-spread, $\sigma \leqq \pi/N$, and with a wide phase-spread $\sigma \leqq 2\pi/N$ radians.

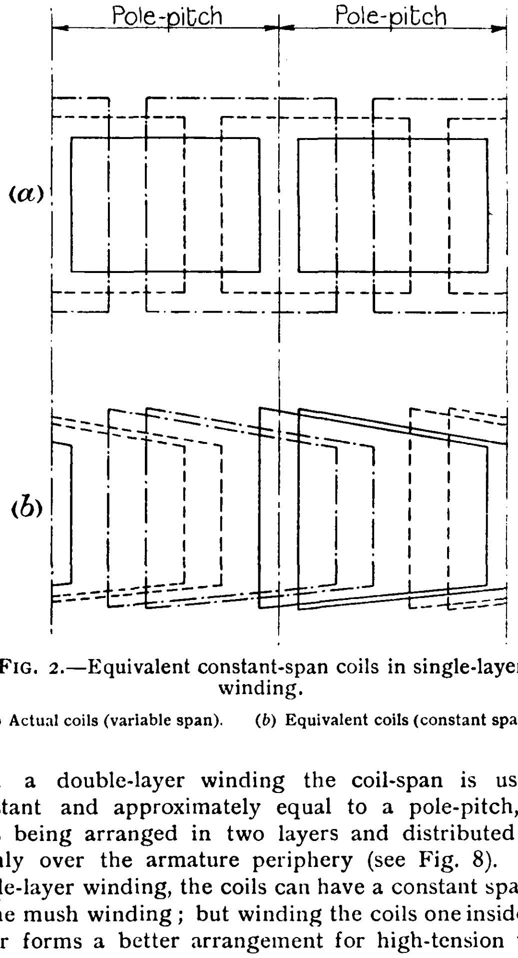

FIG. 2.—Equivalent constant-span coils in single-layer winding.

(*a*) Actual coils (variable span). (*b*) Equivalent coils (constant span).

In a double-layer winding the coil-span is usually constant and approximately equal to a pole-pitch, the coils being arranged in two layers and distributed uniformly over the armature periphery (see Fig. 8). In a single-layer winding, the coils can have a constant span, as in the mush winding; but winding the coils one inside the other forms a better arrangement for high-tension work and is preferred by some makers for all voltages. The variable coil-span, however, is due merely to the arrangement of the overhang, and by changing this we can obtain constant-span coils without affecting the position of the coil-sides in the slots in any way (see Fig. 2).

Both the phase-spread and the coil-span have a direct bearing on the output of the winding. To obtain the largest output from a given number of phases, the mean coil-span should equal the pole-pitch to make the coil-span factor unity, and the narrow phase-spread should be used to obtain a high distribution factor.* The influence of the phase-spread also explains why a larger output is obtained from an increased number of phases when the mean span of the coils in each phase is kept equal to the pole-pitch.

2. SINGLE-LAYER WINDINGS.

(I) *Arrangement of Single-layer Windings.*

The peculiarity of the single-layer winding is the use of one coil-side per slot, so that every coil fills two slots. Consequently if the coils of a phase are to lie in adjacent slots, as is usual in single-layer windings, each pole-pair must be divided into 2 N equal parts, two of which are monopolized by each phase. The phase-spread in a single-layer winding is then $\sigma = 2\pi/2N = \pi/N$ radians. It is also possible to obtain a phase-spread of $2\pi/N$ radians in a single-layer winding by spreading the coil-sides of a phase over the wider arc and filling the empty slots by the coil-sides of another phase; but owing to the reduced output obtainable with the wide phase-spread, this arrangement is only resorted to in special cases, as, for example, in a 3-phase winding when it is desired to suppress the third harmonic in the phase pressure.

In single-layer windings, then, it is standard practice to use the narrow phase-spread, and a number of such polyphase windings are represented diagrammatically in Fig. 3, where a circle denotes a pole-pair of 2π radians. The circle is divided into 2 N parts, and single- and double-dashed numerals are used to denote the two sets of coil-sides in each phase. The phases can be interlinked as desired—thus diagrams *a–e* represent hemisymmetrical windings having 1, 2, 3, 4, and 6 phases respectively; whilst *f, g, h,* and *k* represent symmetrical 2, 3, 4, and 6-phase windings.

It is seen in Fig. 3 that different conditions arise according to whether the number of phases N is even or odd. Diagrams *a–e* and *g* show that with the mean coil-span equal to a pole-pitch, it is possible to obtain any hemisymmetrical N-phase winding, but only a symmetrical N-phase winding when N is odd. When N is even, a symmetrical N-phase winding is obtainable by making the mean coil-span equal to $(N-1)/N$ times the pole-pitch, as shown in diagrams *f, h,* and *k.* The relative outputs given on the diagrams are calculated for uniformly distributed windings, that is, for a phase-spread $= \pi/N$.

The diagrams in Fig. 3 clearly show how two phases in which the pressures are equal and opposite can be joined in parallel or series to form a single phase. For example, in the symmetrical 6-phase winding in diagram *k*, the pressures in phases I and IV are equal and opposite, and by combining each such pair we get the hemisymmetrical 3-phase winding in diagram *c*, or the symmetrical 3-phase winding in diagram *g*. In diagram *k*, where $N = 6$ is even, the mean coil span is equal to $(N-1)/N = 5/6$ths of the pole-pitch; whilst in diagram *g*, where $N = 3$ is odd, the mean span equals the pole-pitch. Similarly the hemisymmetrical 2-phase winding with the mean span equal to the full-pitch in diagram *b* can be obtained from the symmetrical 4-phase winding with the mean coil-span

* S. P. SMITH and R. S. H. BOULDING: "The Shape of the Pressure Wave in Electrical Machinery." *Journal I.E.E.*, 1915, vol. 53, p. 205.

equal to three-quarters of the pole-pitch in diagram *h* by combining phases I and III to form phase I and phases II and IV to form phase II. Or, again, the 1-phase winding in diagram *a* can be derived from the 2-phase winding in diagram *f*, by joining phases I and II in series or parallel.

The advantage of obtaining an N-phase winding with full-pitch coils from a 2 N-phase winding with short-pitch coils is threefold. First, the two parts can be joined in parallel, thereby doubling the number of circuits in a phase; secondly, the overhang is often shorter, thereby reducing the copper and heating in this part of the winding; thirdly, the same output is obtained with half the number of phases. Of course, when the series connection is used it is not necessary to have the two parts which are joined together alike, and it is quite common to have one part with one coil more than the other. This is equivalent to having a 2 N-phase winding with 2 N empty slots per pole-pair, and winding the empty slots in the N-phase winding.

(II) *Number of Armature Slots.*

In single-layer windings it is common practice for a phase to have the same number of coils in each pole-pair. Let q_0 denote the number of coils per phase in each pole-pair. These q_0 coils require $2\,q_0$ slots, so that for the N phases there must be $2\,q_0\,\mathrm{N}$ slots per pole-pair, or in p' pole-pairs there must be $2\,q_0\,\mathrm{N}\,p'$ slots. The total number of slots S′ in the p' pole-pairs can be written

$$\mathrm{S}' = 2\,q_0\,\mathrm{N}\,p' + x,$$

where x denotes the number of slots in the p' pole-pairs not needed by the q_0 coils per group, and will be left empty when all groups are alike—see sub-section (*a*) below. If the x slots are wound, some groups will have more coils than others—see sub-section (*b*) below.

In order to obtain a symmetrical N-phase winding, the

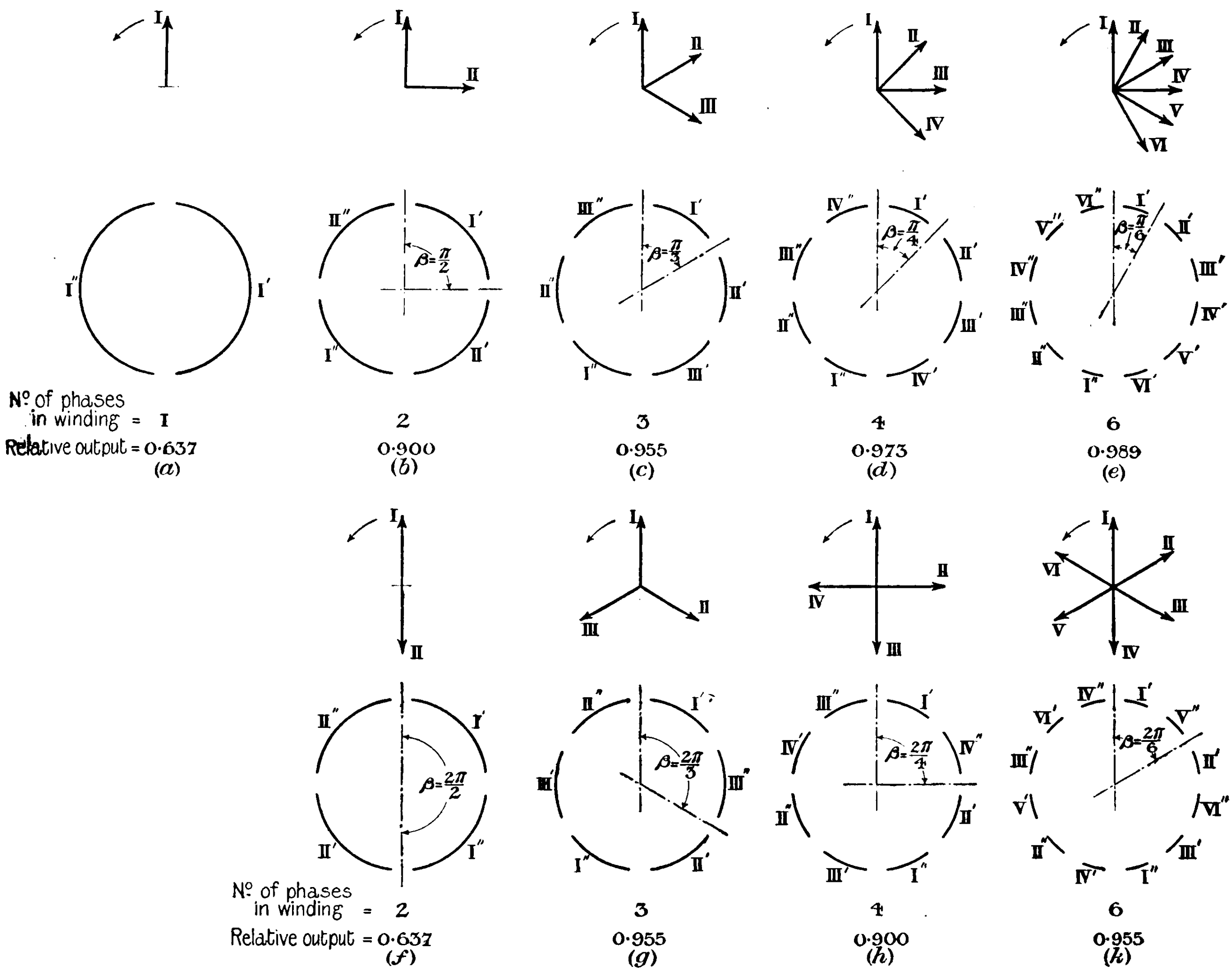

FIG. 3.—Diagrammatic arrangements of single-layer windings to obtain N phases with narrow phase-spread ($\sigma = \pi/\mathrm{N}$).

(*a*) to (*e*)—Hemisymmetrical systems ($\beta = \pi/\mathrm{N}$) with full-pitch coils. (*g*)—Symmetrical system ($\beta = 2\,\pi/\mathrm{N}$) with full-pitch coils. (*f*), (*h*), and (*k*)—Symmetrical systems ($\beta = 2\,\pi/\mathrm{N}$) with short-pitch coils.

number of slots S′ must satisfy Equation (4), that is, the phase-pitch must be

$$y_{ph} = \frac{S}{aN} = \frac{S'}{N} = 2\,q_0 p' + \frac{x}{N} \text{ slots} \quad . \quad . \quad . \quad (5)$$

where $p' = n\,N \pm 1 = p/a$.

Since y_{ph} must be a whole number and $2\,q_0 p'$ is integral, it follows that the number of extra slots x in the p' pole-pairs must be divisible by the number of phases N, and each phase-pitch contain x/N extra slots.

The total number of slots S in the machine is then

$$S = S'a = 2\,q_0 N p + x a \quad . \quad . \quad . \quad . \quad (5a)$$

In the common 3-phase single-layer winding, this becomes

$$S = 6\,q_0 p + x a \quad . \quad . \quad . \quad . \quad . \quad . \quad (5b)$$

where $x = 0$, 3, 6, etc., and $a = p/p' = p/(3\,n \pm 1)$ is an integer $\lesseqgtr p$.

As these expressions are so simple, there is no need to tabulate the slottings for single-layer windings, and we shall therefore only consider the general cases.

(*a*) *Whole number of slots per pole.*—In most single-layer windings, the number of slots per pole Q is an integer. In this case every pole-pair is alike and forms a complete N-phase system. Thus $S' = 2\,q_0 N + x$ and $Q = q_0 N + x/2$, where x can be zero or any multiple of N. The total number of slots $S = S'p$, since $a = p$ when $p' = 1$.

An interesting relation holds when N is even and the polyphase winding is symmetrical, as pointed out in connection with Fig. 3. In this case the mean coil-span equals $(N - 1)/N$ times the pole-pitch, but by halving the number of phases we double the number of circuits and make the mean span equal to the pole-pitch. The important case in practice is when a 3-phase winding has a whole number of slots per pole. In this case the slotting is always that of a symmetrical 6-phase winding. Thus when $N = 6$, $Q = 6\,q_0 + \frac{1}{2}x$. If now $x = 0$, then Q is a multiple of 6, that is an even multiple of 3, so that the 6-phase winding consists of two 3-phase windings, which can be joined in series or parallel. If $x = 6$, $Q = 6\,q_0 + 3$; that is, Q is an odd multiple of 3, so that we have a symmetrical 6-phase winding with 3 empty slots per pole, or a symmetrical 3-phase winding with $Q/3 = 2\,q_0 + 1$ slots per phase and pole. In the latter case there are not two equal circuits per phase per pole-pair when all slots are wound, but this is immaterial when only the series connection is required. It is thus seen that with a whole number of slots per pole, the slotting for a symmetrical 3-phase winding is the same as that for a symmetrical 6-phase winding, and it is due to this fact that it is possible to obtain a 3-phase single-layer winding with one circuit per pole and phase.

When the number of slots per pole is integral, the same slot positions recur under every pole, and the coil-sides of the whole winding can only occupy Q positions in the field. Consequently, any effect due to the spacing of the slots or the swinging of the flux is repeated under every pole and appears undiminished in the phase pressure in the form of a spacing ripple or a tooth ripple. Thus for obtaining a smooth wave for the phase pressure we have the worst possible conditions with a whole number of slots per pole. There are several ways of suppressing the effects of the slots and teeth, most of which make the winding equivalent to a uniformly distributed winding. Thus by skewing the slots or the pole-shoes by an amount equal to the slot-pitch this result is obtained. An equally effective method, and one that is often simpler and cheaper than the foregoing from the works' point of view, is to make the number of slots per pole fractional by adding extra slots.

(*b*) *Fractional number of slots per pole.*—When Q is a fraction, the coil-sides do not occupy the same position in the field under the different poles, consequently the effect of the teeth is not reproduced throughout the phase. Expressed mathematically, we can say that $2\,Q + x$ and $2\,Q - x$, where $x = 1$, 3, 5, etc., no longer represent harmonics when Q is a fraction, whereas they do so when Q is an integer, and have a high winding factor f_x, when x is small (see footnote on page 23).

We shall only consider the practical case of the symmetrical 3-phase winding. Inserting $N = 3$ in Equation (4), we have $y_{ph} = S/3a = S'/3$.

The coils will occupy the greatest number of positions in the field when $a = 1$ and $p' = p$, that is, when there can be only one circuit per phase. We can always make $a = 1$ unless p is a multiple of 3. If p is a multiple of 3, we must make a a multiple of 3 so that $p' = p/a = 3\,n \pm 1$, is not a multiple of 3, and so obtain a similar parts per phase—see Table 1. For example, when $p = 3$ we must take $a = 3$, so that $p' = 1$; with $p = 6$, we can take $a = 3$ to make $p' = 2$, or $a = 6$ to make $p' = 1$. With $a = 3$, the slot positions in the field repeat themselves 3 times; with $a = 6$, 6 times.

(1) *Use of empty slots.*—The simplest way to make Q fractional is to have a number of extra slots not needed by the winding. From Equation (5) we get, for $N = 3$, the phase-pitch $y_{ph} = 2\,q_0 p' + x/3$. Thus x must be a multiple of 3.

Taking $x = 3$ as the smallest number of empty slots that can be added, we have, when $p' = 3\,n \pm 1 = p$, $S' = 3\,y_{ph} = 6\,q_0 p' + 3 = S$. This gives the largest number of positions possible in p pole-pairs with a symmetrical 3-phase winding, for in this case the slot positions in the field never recur and there can only be one circuit per phase. Thus with $x = 3$ empty slots and $p = 3\,n \pm 1$, we have:

Total slots, $S = 6\,q_0 p + 3$,
Slots per pole, $Q = S/(2\,p) = 3\,q_0 + 3/(2\,p)$,
Slots per phase and pole, $q = Q/3 = q_0 + 1/(2\,p)$,
Phase-pitch in slots, $y_{ph} = 2\,q_0 p + 1$.

When $p' = 1$, or in a 2-pole machine, with three extra slots we get $S' = 6\,q_0 + 3 = 2\,Q$, so that $Q =$ an integer $+ \frac{1}{2}$. This also has the effect of suppressing the tooth or spacing ripples owing to the action of the coil-span (see footnote on page 23).

In some cases it is found desirable to make $x = 6$, that is, to use 6 empty slots, but it must be remembered that with an even number of pole-pairs this makes two a common factor of S and p, or $p' = p/2$ and $a = 2$, thus giving two similar parts and only half the maximum number of possible positions in the field for each phase.

(ii) *Use of unequal coil groups.*—It is clear that when the number of extra slots in the machine is an even multiple of 3, i.e. when $x = 6\,n$, these slots can be wound and an equal number of extra coils allotted to each phase. In this way the coil groups in the several pole-pairs are made unequal.

Taking the case of 6 extra slots in p' pole-pairs, we get one extra coil per phase in each circuit. Thus in a 4-pole machine there are then $S = 6q_0 \times 2 + 6 = 12q_0 + 6$ slots, that is an odd number of slots in each pole-pair, which is equivalent to a 2-pole machine with 3 empty slots. In one group there will be q_0 coils and in the other $q_0 + 1$, thus making $(q_0 + \frac{1}{2})$ slots per phase and pole. This arrangement can be used whenever the number of poles is a multiple of 4. In general, with 6 extra slots in the periphery there are in each phase $(p - 1)$ groups containing q_0 coils and one group with $(q_0 + 1)$ coils. According to whether $p = 3n \pm 1$ is even or odd, the number of slot positions in the field is increased p or $2p$ times.

Generally speaking, unequal coil groups are not desirable in practice, owing to the varying magnetomotive force from pole to pole and the constructional complications.

Example.—As an example of the use of extra slots, we can consider an 8-pole, 3-phase machine with a normal slotting for 2 slots per pole and phase, i.e. $q_0 = 2$. The total number of slots is then 48, and according to whether we make the mean coil-span equal to the pole-pitch or to 5/6ths of the pole-pitch, we get a 3- or 6-phase winding, so that a 3-phase winding with four or eight circuits per phase respectively is possible.

When extra slots are used, we get the following alternatives:—

No. of Similar Parts a	$p' = \frac{p}{a}$	Extra Slots in p' Pole-pairs x	Slots in p' Pole-pairs $S' = 6q_0p' + x$	Total Slots $S = S'a$	Extra Possible Coils per Phase
1	4	3	51	51	none
2	2	3	27	54	1
4	1	3	15	60	2

The phase-pitch in each case is $y_{ph} = S'/3$. We thus have no fewer than five alternatives for the above winding—two usual and three unusual. All the latter are effective in suppressing tooth effects—the suppression in the case of $S' = 15$ slots per pole-pair being due to the fact that the coil-span differs from the pole-pitch by an odd number of half slot-pitches (see footnote on page 23).

3. Double-layer Windings.

(1) *Arrangement of Double-layer Windings.*

When the sides of the coils are in two layers in the slots, one side of a coil lies in the top layer in one slot and in the bottom layer in some other slot, usually about a pole-pitch away. Generally all the coils are identical and arranged so that they can be joined in mesh, by means of lap or wave connections, to form a closed winding (see Fig. 8).

The smallest number of coil-sides per slot is 2, but any even number can be used, the commonest numbers being 4, 6, 8, and 10 coil-sides per slot. Since only one side of a coil has a place in either layer, the 2π radians need only be divided into N equal parts—one part being allotted to each phase. In this case the phase-spread $\sigma = 2\pi/N$ radians. When, however, a commutator is not used, it is usual and better to divide the 2π radians into $2N$ parts and so reduce the phase-spread to $2\pi/2N = \pi/N$, as in a single-layer winding, in order to obtain the larger output.

(*a*) *Phase tappings off closed windings.*—By observing the conditions developed in Equations (2) and (3) it is possible to obtain a lap or wave winding having a similar, symmetrical polyphase windings in each of which the sum of the phase angles between the pressures is 2π radians and the resultant pressure is zero. Consequently the coils can be joined in mesh to form a closed winding, and according to the relation between the number of coils and the pitch between successive coils, the a systems can be made independent of one another or can be interconnected. (To join the coils in mesh, the finish of one coil is joined to the start of the successive coil until the winding closes.)

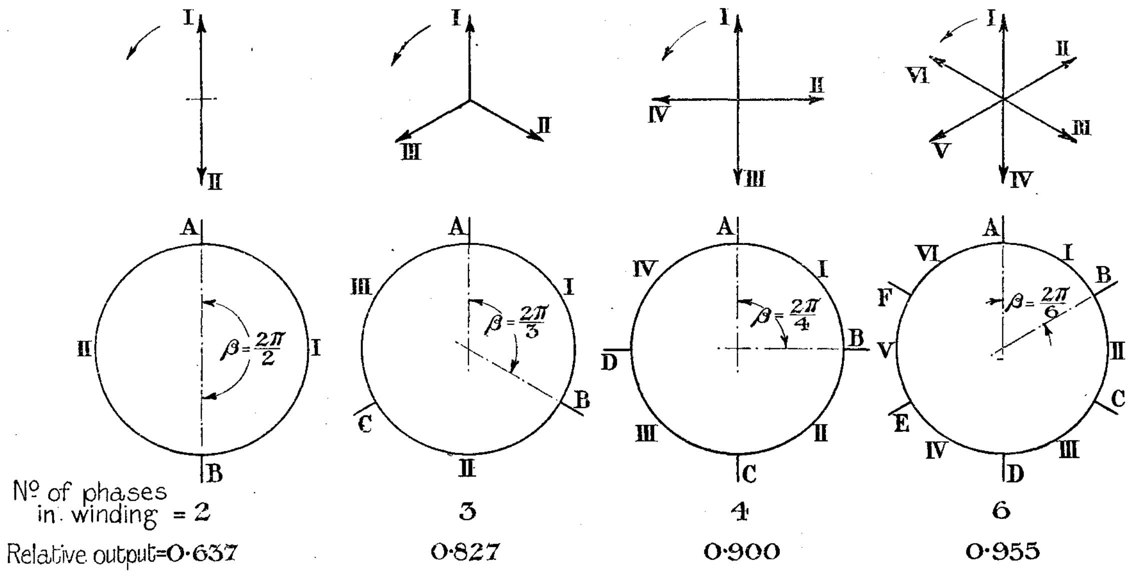

FIG. 4.—Diagrammatic system of closed double-layer windings with full-pitch coils to obtain N symmetrical mesh-connected phases with wide phase-spread ($\sigma = 2\pi/N$), by means of polygon tappings.

Permanent connections to a closed winding take the form of tappings—the corresponding equipotential points in each of the a systems being connected together to place the latter in parallel. Each point where a closed winding is tapped makes connection with the start of one phase and the finish of another.

In what follows, only one of the a similar polyphase systems need be considered, and again a circle will be used to represent the commutator bars or joints between the coils forming the field displacement of 2π radians. This circle has then to be split up to form the N phases. Pressures can be tapped off a closed winding in two ways: (i) polygon tappings; (ii) diametral tappings.

(i) *Polygon tappings.*—For an N-phase system, the circle is simply divided into N equal parts by means of N equidistant tappings, and the loads are placed across adjacent tappings. Tappings taken off a closed winding in this way can be conveniently called "polygon" tappings. The d is the common way of loading a rotary converter with 6 slip-rings.

It is easily shown that the method of diametral loading of a closed winding yields the same output as is obtained from an open winding with a phase-spread of π/N.

(*b*) *Open double-layer windings.*—If a commutator is not used, the windings in Fig. 4 can be opened at the points tapped, as shown in Fig. 6, where the top coil-sides are indicated by full and the bottom by dotted lines. Each phase has now its own start and finish and can be loaded independently, or the phases can be interlinked in star (the original closed winding with polygon tappings is the mesh connection in Fig. 4); but the phase-spread and phase-pitch remain unaltered, that is, $\sigma = \beta = 2\pi/N$.

When full-pitch coils are used, it is seen from Fig. 6 that phases r and $r + \frac{1}{2}N$ overlap completely when N is even,

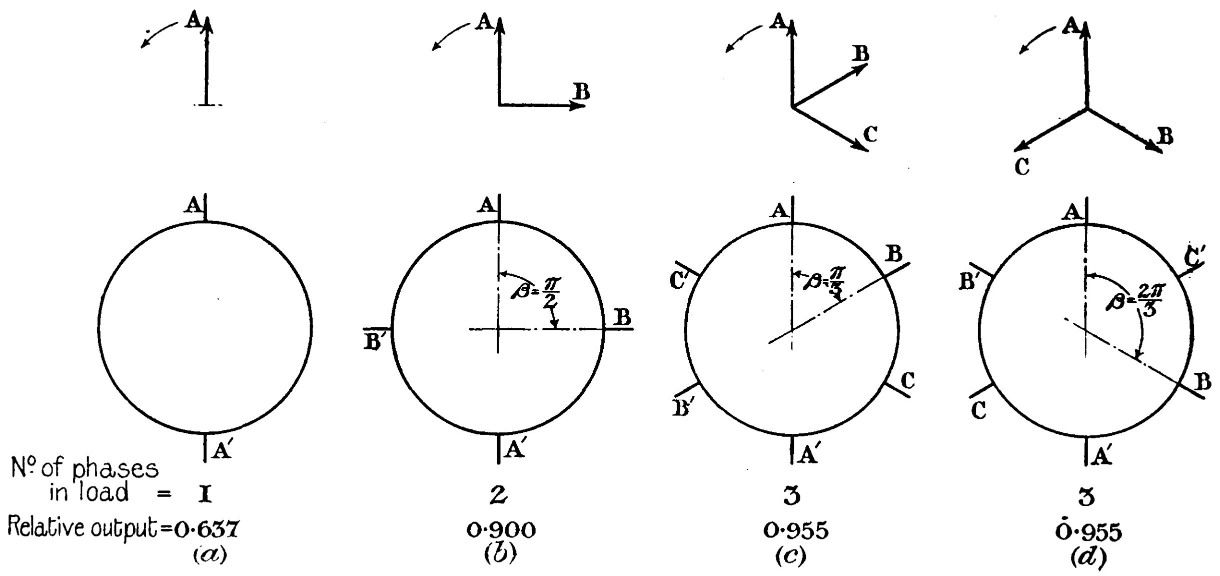

FIG. 5.—Diagrammatic arrangement of closed double-layer windings with full-pitch coils to obtain N phases with phase-spread $\sigma = \pi$, by means of diametral tappings.

(*a*) to (*c*)—Hemisymmetrical systems ($\beta = \pi/N$). (*d*)—Symmetrical system ($\beta = 2\pi/N$).

angular phase-pitch β is then equal to the angular phase-spread σ, that is, $\beta = \sigma = 2\pi/N$.

In this way it is possible to obtain any symmetrical N-phase system of pressures, as shown in Fig. 4. The pressures and currents in such a system are thus those in a mesh-connected system.

(ii) *Diametral tappings.*—Here the winding is tapped at twice as many points as the number of phases required, and each load phase is formed by placing two opposite winding phases in parallel by means of diametral tappings. Thus for an N-phase load, we have 2 N tappings—two exclusively for each phase.

The angle between successive load phases is then $2\pi/(2N) = \pi/N$ radians, so that the hemisymmetrical systems in Fig. 5 (*a–c*) are obtained. When N is odd, however, the system can be loaded symmetrically by connecting the phases as shown in Fig. 5 (*d*)—thus diagram

whilst two phases half overlap whenever N is odd. It is much better, however, to halve the phase-spread in order to obtain a larger output with the same number of phases in the load. To do this, the 2π radians are divided into 2 N parts, thereby making the phase-spread $\sigma = 2\pi/(2N) = \pi/N$, and the two parts which completely overlap are joined in parallel or series, as desired, to form a single phase. This is always possible, since 2 N is always even. To join the two equal and opposite parts in series, either both finishes or both starts must be joined together; whilst to join them in parallel, the start of one must be connected to the finish of the other, and conversely. The windings thus obtained are represented diagrammatically in Fig. 7, and it is seen that with full-pitch coils any hemisymmetrical system whatever can be obtained in this way, but a symmetrical system is only obtainable when N is odd. Thus, by opening a double-layer winding at 2 N equidistant points, an N-phase system is obtained with a phase-spread equal to π/N radians.

(II) *Connecting Rules for Closed Windings.*

Whereas single-layer windings are solely used for obtaining an N-phase system of pressures, the double-layer winding finds its chief application on continuous-current machines, though its use as a polyphase winding is also quite general. When used as a polyphase winding in conjunction with a commutator, a closed winding is employed, and the N pressures are obtained by means of tappings. When no commutator is needed, an open winding with a narrow phase-spread is used, and though it is customary to have the same winding pitches as with a closed winding, this is not necessary.

There are two common modes of connecting up the coils of double-layer windings—the lap type of connection being used when successive coils a and b lie under the same pole-pair, and the wave when they lie under adjacent pole-pairs (see Fig. 8). In order to obtain a closed winding, the total displacement in the field must be a multiple of 2π radians (see Equation 2). In cases where the number of coil-sides per slot u is greater than 2, for the purpose of connecting up the coils we can imagine there are as many

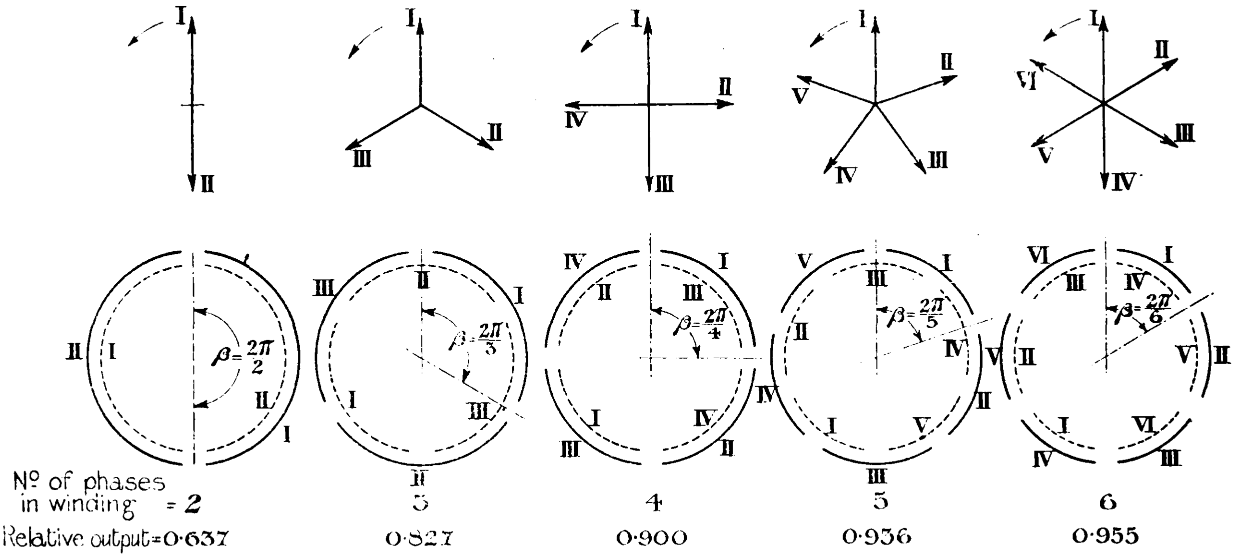

FIG. 6.—Diagrammatic arrangement of open double-layer windings with full-pitch coils to obtain N symmetrical phases with wide phase-spread ($\sigma = 2\pi/N$).

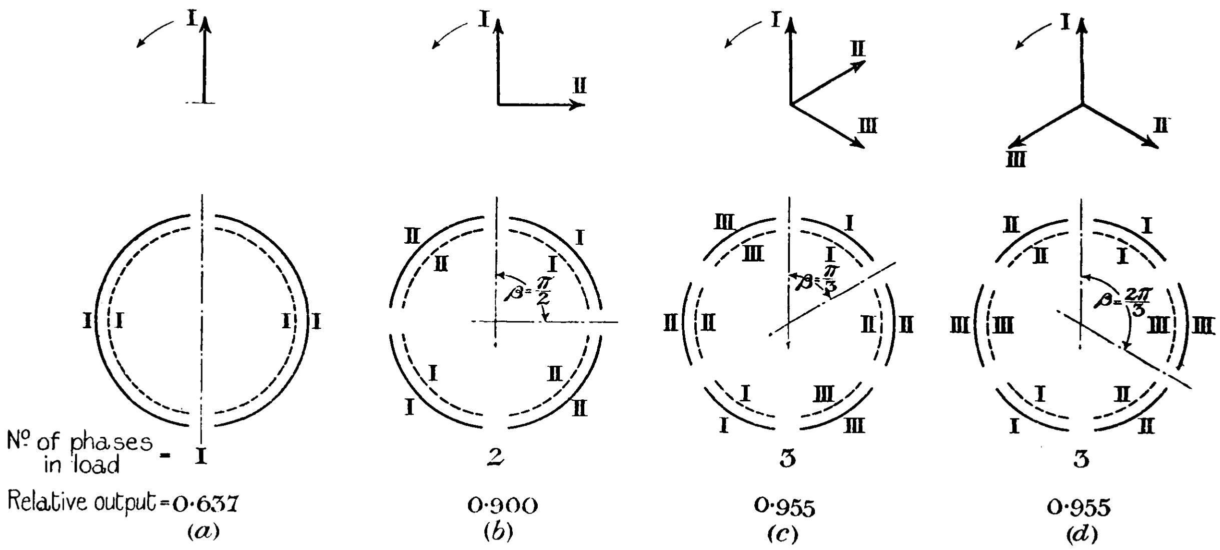

FIG. 7.—Diagrammatic arrangement of open double-layer windings with full-pitch coils to obtain N phases with narrow phase-spread ($\sigma = \pi/N$).

(a) to (c)—Hemisymmetrical systems ($\beta = \pi/N$). (d)—Symmetrical system ($\beta = 2\pi/N$).

slots as there are coils or commutator segments, i.e. we make $S = C$. Equation (2) then becomes

$$\left.\begin{aligned} C\,\psi &= 2\,\pi\,a \\ \text{and} \qquad \frac{\psi}{\gamma} &= \frac{a}{p} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where ψ is now the phase displacement between successive coils when $u = 2$, and γ the angular pitch of the commutator segments (or joints) or of the slots when $u = 2$.

The number of joints or segments between successive coils in the winding is called the commutator pitch, and will be denoted by y_c; thus y_c must contain an exact number of commutator-bar pitches γ.

Though this formula enables us to deduce the rules for closed lap and wave windings, i.e. windings in which $\Sigma\, e = 0$, so that no circulating currents of fundamental frequency can flow, it does not follow that such windings will have similar parts. When we apply the conditions for obtaining a similar circuits per phase with the actual number of slots $S = 2\,C/u$—for either a continuous or alternating pressure—we shall find that the number of symmetrical windings obtainable is much smaller than the number of closed windings.

(*a*) *Closing rule for lap windings.*—With the lap connection, the finish of one coil is joined to the start of another under the same pole-pair, so that the angle $y_c\,\gamma$ between successive coils on the armature is equal to the phase displacement ψ between them. Applying $y_c\,\gamma = \psi$ to Equation (6), we get

$$\left.\begin{aligned} y_c &= \frac{\psi}{\gamma} = \frac{a}{p} \\ \text{or} \qquad a &= y_c\,p = p,\ 2\,p,\ 3\,p, \text{etc.} \end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

Thus a closed lap winding can be obtained by making a a multiple of the number of pole-pairs p.

(*b*) *Closing rule for wave windings.*—With the wave connection, the finish of one coil is joined to the start of another under the next pole-pair, so that the angle $y_c\,\gamma$ between successive coils on the armature is equal to $2\,\pi \pm \psi$. Applying $y_c\,\gamma = 2\,\pi \pm \psi$ to Equation (6), we get

$$\left.\begin{aligned} y_c &= \frac{2\,\pi \pm \psi}{\gamma} = \frac{2\,\pi}{\gamma} \pm \frac{\psi}{\gamma} \\ &= \frac{C}{p} \pm \frac{a}{p} = \frac{C \pm a}{p} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (8)$$

where $C\,\gamma = 2\,\pi\,p$, when we make $S = C$.

Hence in a closed wave winding, $a = p\,y_c - C$, so that a can be greater than, equal to, or less than p.

It is easy to see that the H.C.F. of C and y_c determines the number of independent windings on the armature with either lap or wave connections.

(III) *Conditions for obtaining Symmetrical Lap and Wave Windings.*

The conditions for obtaining closed lap and wave windings are set forth in Equations (7) and (8), whilst the conditions for obtaining a similar circuits are given in Equation (3). We must now combine these to find the conditions for obtaining symmetrical lap and wave windings. To get a closed winding, the resultant of the vector diagram is merely a closed figure subtending $2\,\pi\,a$ radians at the centre, whereas in a symmetrical winding there are a similar polygons in the resultant diagram, each subtending an angle of $2\,\pi$ radians at the centre. It is only when the winding is symmetrical that there are a coils which are always at the same potential, and which therefore can be properly joined together by means of an equalizing or slip-ring or by a commutator brush.

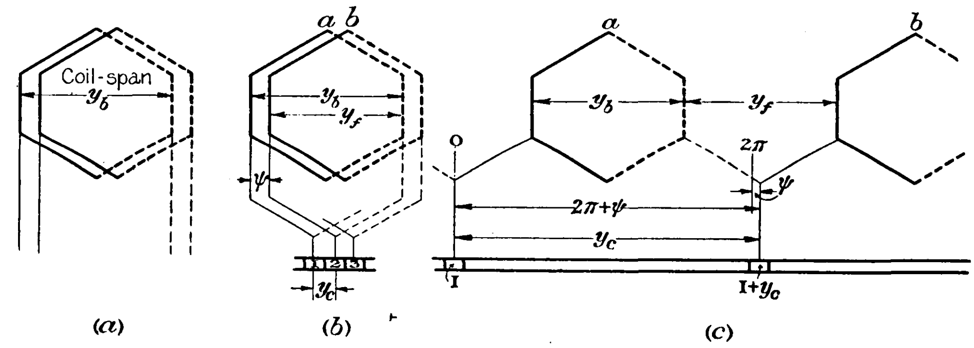

FIG. 8.—Double-layer windings.

(*a*) Coils of double-layer windings. (*b*) Lap connection. (*c*) Wave connection.

(*a*) *Symmetrical lap windings.*—In Equation (7) it is seen that a can be any multiple of p in a closed lap winding, whilst in Equation (3) it is seen that p/a must be integral in order to get a like parts. The only value of a which satisfies both these equations is obviously $a = p$. Further, since the number of slots in each of the a parts must also be integral, we must have $S/a = S/p$ a whole number, i.e. there must be a whole number of slots per pole-pair.

Hence it is only possible to obtain a symmetrical lap winding with as many similar parts (or circuits) as there are pole-pairs and with a whole number of slots per pole-pair, or

$$\left.\begin{aligned} a &= p, \quad \text{or} \quad p' = 1 \\ \text{and} \qquad S' &= \frac{S}{p} \quad \text{or} \quad S = p\,n \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where $n = S' =$ any integer.

The number of coil-sides per slot in a symmetrical lap winding is not restricted in any way, provided that we make $C = \frac{1}{2}\,u\,S$, i.e. avoid idle coils.

The $a = p$ equipotential joints or segments to be connected together at an equalizing or slip-ring are equidistant from one another on the periphery, and the potential pitch $y_p = C/p$.

The above particulars are summarized in Table 2, whilst illustrative examples are given in sub-section (V).

The condition that $p' = p/a$ and C/S must have no common factor greater than unity at once restricts the number of segments per slot C/S that can be used with given numbers of poles as shown below.

Segments per Slot in Symmetrical Wave Windings.

Pole-pairs p'	1	2	3	4	5	6	7	8	9	10	11	12
Segments per slot (C/S)	any	1, 3, 5, 7	1, 2, 4, 5, 7, 8	1, 3, 5, 7	1 to 4 6 to 8	1, 5, 7	1 to 6, 8	1, 3, 5, 7	1, 2, 4, 5, 7, 8	1, 3, 7	any	1, 5, 7

(*b*) *Symmetrical wave windings.*—In Equation (8), the commutator pitch in a closed wave winding $y_c = (C \pm a)/p$, whence we get the possible number of coils $C = p\, y_c \pm a$. Since there are to be no idle coils, i.e. the segments per slot $C/S = u/2$, we can write for the number of slots in a closed wave winding

$$S = \frac{C}{u/2} = \frac{C}{C/S} = \frac{p\, y_c}{C/S} \pm \frac{a}{C/S}.$$

Dividing throughout by a, we get

$$\frac{S}{a} = \frac{p}{a} \cdot \frac{y_c}{C/S} \pm \frac{1}{C/S}.$$

Introducing the conditions of symmetry from Equation (3) to obtain $\bar{a}$ identical S'-phase systems, we must have $S' = S/a$ and $p' = p/a$, that is, S/a and p/a must both be integers, or

$$S' = p' \frac{y_c}{C/S} \pm \frac{1}{C/S}.$$

Now by the ordinary rules of division we can write

$$\frac{y_c}{C/S} = n + \frac{x}{C/S} \quad \text{or} \quad y_c = n\frac{C}{S} + x,$$

where n is any integer and x is an integer less than C/S.

Substituting for y_c, we then get

$$\begin{aligned} S' &= \frac{p'}{C/S}\left(n\frac{C}{S} + x\right) \pm \frac{1}{C/S} \\ &= p'\, n + p' \frac{x}{C/S} \pm \frac{1}{C/S} \\ &= p'\, n + \frac{1}{C/S}(p'\, x \pm 1) \quad . \quad . \quad . \quad . \quad (10) \end{aligned}$$

To obtain a symmetrical wave winding, then, this expression must be a whole number. Now clearly $p'\, n$ is an integer; hence, for S' to be an integer, $p'\, x \pm 1$ must be exactly divisible by C/S. This will be obviously impossible if either p' or x and C/S have a common factor greater than unity, for then $\frac{p'\, x}{C/S}$ would have a smaller denominator than $\frac{1}{C/S}$.

The above table, then, shows the number of coil-sides per slot, $u = 2\,C/S$, up to 16, possible with any number of poles up to 24 in a symmetrical wave winding.

To examine the restriction that x and C/S must have no common factor > 1, let us write

$$\frac{1}{C/S}(p'\, x_1 + 1) = A \quad \text{and} \quad \frac{1}{C/S}(p'\, x_2 - 1) = B,$$

where x_1 and x_2 must be values of x which have no common factor with C/S greater than unity, and which make A and B whole numbers. We can now determine A and B and x_1 and x_2 as follows:—

Since A and B are integers, $A + B = \frac{p'}{C/S}(x_1 + x_2)$ must be integral. Now p' and C/S have no common factor > 1, therefore $x_1 + x_2$ must be divisible by C/S. But, by assumption, x_1 and x_2 are each less than C/S, so that their sum must be less than $2\,C/S$; hence it is only possible to have $x_1 + x_2 = C/S$. Substituting for $x_1 + x_2$, we get $A + B = p'$.

Inserting A and B in Equation (10), and disregarding any difference between n and $n + 1$ (since n can be any integer),

$$\begin{aligned} S' &= p'\, n + A, \quad \text{or} \quad p'\, n + B \\ &= p'\, n + A, \quad \text{or} \quad p'\,(n + 1) - A \\ &= p'\, n \pm A \\ &= p'\, n \mp B \end{aligned}$$

Since $p'\, n + A$ gives the same series of numbers as $p'\, n - B$, and $p'\, n - A$ the same series as $p'\, n + B$, we can write for the number of slots in a symmetrical wave winding

$$S' = p'\, n \pm K \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where $K = A$ or B, according to which value we choose to give it. In what follows, for the sake of simplicity we shall always make K equal to the smaller of these two integers.

We now draw up the following table for x_1 and x_2 for given values of C/S.

Values of x_1 and x_2 where $x_1 + x_2 = C/S$, and x_1, x_2 and C/S have no Common Factor greater than Unity.

C/S	1	2	3	4	5	6	7	8
x_1 and x_2	0 and 1	1 and 1	1 and 2	1 and 3	1 and 4 2 and 3	1 and 5	1 and 6 2 and 5 3 and 4	1 and 7 3 and 5

This enables us to evaluate K for given values of p' and C/S, for we have only to work out A and B and tabulate the lower value. The spaces denoted by a dash are the impossible cases of C/S with particular values of p', as given by Equation (10) and already determined.

Value of K in Equation (11).

Values of C/S.	Values of $p' = p/a$: 1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	–	1	–	2	–	3	–	4	–	5	–
3	1	1	–	1	2	–	2	3	–	3	4	–
4	1	–	1	–	1	–	2	–	2	–	3	–
5	1	1	1	1	–	1	3	3	2	–	2	5
6	1	–	–	–	1	–	1	–	–	–	2	–
7	1	1	1	1	2	1	–	1	4	3	3	5
8	1	–	1	–	2	–	1	–	1	–	4	–

Having found S′ in this way, $S = S'a$ for any value of a.

Table 3, showing the number of slots and coil-sides per slot in symmetrical wave windings can now be drawn up and, wherever possible, the designer should adhere strictly to these conditions.

The following example will illustrate how K is determined. Let $p' = 4$ and $C/S = 3$. Then $A + B = p' = 4$, and $x_1 + x_2 = C/S = 3$. The possible values of x_1 and x_2 are 1 and 2. Taking $x_1 = 2$, then $A = \frac{1}{3}(4 \times 2 + 1) = 3$; and taking $x_2 = 1$, $B = \frac{1}{3}(4 \times 1 - 1) = 1$. We thus take $K = B = 1$, and get for the possible numbers of slots $S' = p'n \pm K = 4n \pm 1$, as shown in Table 3.

If $a = 1$, as in the common wave winding, then $p' = p = 4$, and $S' = S$. Thus the total number of slots in an 8-pole machine with a symmetrical wave winding and 6 coil-sides per slot is $S = 4n \pm 1$.

Similarly, if $a = 2$, then $p = p'a = 4 \times 2 = 8$; and $S = aS' = 2S' = 2(4n \pm 1) = 8n \pm 2$. In this way, Table 3 has been drawn up for machines with any number of poles up to 24 and any number of coil-sides per slot up to 16.

The potential pitch, $y_p = C/a$. Each tapping forms the start of one phase and the finish of another. With diametral tappings, these two phases are equal and opposite, so that the current divides equally between them. In this way each tapping represents two parallel circuits, so that tapping a points by means of a commutator brush or a slip-ring makes connection with $2a$ circuits. This only holds, however, for the case when the phases are taken over 180°, as with diametral tappings or in a continuous-current machine, and corresponds with the case of a single-layer winding, where a 3-phase winding with $2a$ circuits per phase is obtained by joining in parallel the equal and opposite phases of a 6-phase winding.

(IV) *Conditions for obtaining Symmetrical N-phase Lap and Wave Windings.*

The conditions for obtaining a symmetrical N-phase system of pressures given in Equation (4) show that we must have a symmetrical winding where S′ is exactly divisible by N and $p' = nN \pm 1$.

(*a*) *Symmetrical N-phase lap windings.*—In a symmetrical lap winding, $p' = 1$, so that $p' = nN \pm 1$ is always satisfied in a symmetrical N-phase lap winding.

Further, in a symmetrical lap winding, $S' = S/p$ (Equation 9), hence to obtain a symmetrical N-phase winding, $S'/N = S/(pN)$ must be a whole number, or the number of slots

$$S = Npn \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This condition for 3-, 4-, and 6-phase lap windings is entered in Table 2.

Though this condition is apparently so simple, it is easy to leave it unsatisfied by ignoring it. For example, if we have a rotary converter with $C/p = 96$ segments per pole-pair and $u = 6$ coil-sides per slot, we shall have $S/p = 2C/(pu) = 96/3 = 32$ slots per pole-pair. This is not exactly divisible by 6, so that the 6-phase pressure will not be symmetrical. With 2 or 4 segments per slot ($u = 4$ or 8), however, a symmetrical 6-phase pressure is obtained.

(*b*) *Symmetrical N-phase wave windings.*—With wave windings, both p' and S′ are subjected to restrictions when we require an N-phase system.

(i) *Number of poles.*—The only symmetrical wave windings from which a symmetrical N-phase system of pressures can be obtained are those where $p' = nN \pm 1$. These are shown in Table 1, and the impossible cases are denoted by a dash in Table 3. The same result can be obtained from the condition that S′/N must be integral, for obviously

$$\frac{S'}{N} = \frac{p'}{N} \cdot \frac{y_c}{C/S} \pm \frac{1}{N} \cdot \frac{1}{C/S}$$

can only be a whole number when p' and N have no common factor greater than unity.

(ii) *Number of slots.*—To find what numbers of slots in a symmetrical winding are exactly divisible by N, we can re-write Equation (11) for S′ thus:

$$S' = p'n \pm K = p'n' \pm (p'n'' \pm K),$$

where $n = n' \pm n''$. Let n' and n'' be chosen so that $n' = n'''N$ and $p'n'' \pm K = kN$, where n''' and k are whole numbers. We can then write Equation (11) in the form

$$S' = p'Nn''' \pm kN = p'Nn \pm kN,$$

where $S'/N = p'n \pm k$ is clearly a whole number (n''', being merely a number, can be replaced by n). In this way we get the values of S′ which are divisible by N, and for the total number of slots we have

$$S = aS' = ap'Nn \pm akN \quad . \quad . \quad . \quad (13)$$

In this equation, then, $kN = p'n'' \pm K$ is merely a value of S′ divisible by N. For the sake of convenience, we shall always take kN as the lowest value of S′ divisible by N.

To show how the values of S for $N = 3$, 4, and 6 are obtained in Table 3, we can work out an example. Let $p' = 3$, $a = 3$, and $N = 4$. For $p' = 3$ we see $S' = 3n \pm 1$, and the lowest value of this divisible by N is $kN = 3 \times 1 \pm 1 = 4$. Hence the numbers of slots that can be used in a symmetrical 4-phase wave winding with $p' = 3$ and $a = 3$ are

$$S = ap'Nn \pm akN = 3 \times 3 \times 4n \pm 3 \times 4 = 36n \pm 12.$$

Permissible Numbers of Slots and Coil-sides per Slot in Lap and Wave, Double-layer Windings to obtain (i) "*a*" *Similar Parts or Circuits*, (ii) $N = 3, 4$, *or* 6 *Symmetrical Phases.*

TABLE 2.—*Lap Windings.*

Pole-pairs p'	Slots S'	Number of Similar Parts in Winding a	Number of Pole-pairs in Machine $p = p'a$	Number of Slots in Machine $S = S'a$	Number of Slots, S, in Machine when N = 3	N = 4	N = 6	Coil-sides per Slot u	Number of Coils in Winding C
1	n	p	p	pn	$3pn$	$4pn$	$6pn$	any even number	$u\,S/2$

Commutator Pitch : $y_c = 1$; Potential Pitch : $y_p = C/p$; Phase Pitch : $y_{ph} = C/(p\,N)$; no idle coils permissible ; n = any integer.

TABLE 3.—*Wave Windings.*

Pole-pairs p'	Slots S'	Permissible Coil-sides per Slot u	Number of Similar Parts in Winding a	Number of Pole-pairs in Machine $p = p'a$	Number of Slots in Machine $S = S'a$	Number of Slots, S, in Machine when N = 3	N = 4	N = 6
1	n	any even number	1	1	n	$3n$	$4n$	$6n$
			p	p	pn	$3pn$	$4pn$	$6pn$
2	$2n \pm 1$	2, 6, 10, 14	1	2	$2n \pm 1$	$6n \pm 3$	—	—
			2	4	$4n \pm 2$	$12n \pm 6$	—	—
			3	6	$6n \pm 3$	$18n \pm 9$	—	—
			4	8	$8n \pm 4$	$24n \pm 12$	—	—
			5	10	$10n \pm 5$	$30n \pm 15$	—	—
			6	12	$12n \pm 6$	$36n \pm 18$	—	—
3	$3n \pm 1$	2, 4, 8, 10, 14, 16	1	3	$3n \pm 1$	—	$12n \pm 4$	—
			2	6	$6n \pm 2$	—	$24n \pm 8$	—
			3	9	$9n \pm 3$	—	$36n \pm 12$	—
			4	12	$12n \pm 4$	—	$48n \pm 16$	—
4	$4n \pm 1$	2, 6, 10, 14	1	4	$4n \pm 1$	$12n \pm 3$	—	—
			2	8	$8n \pm 2$	$24n \pm 6$	—	—
			3	12	$12n \pm 3$	$36n \pm 9$	—	—
5	$5n \pm 1$	2, 8, 12	1	5	$5n \pm 1$	$15n \pm 6$	$20n \pm 4$	$30n \pm 6$
			2	10	$10n \pm 2$	$30n \pm 12$	$40n \pm 8$	$60n \pm 12$
	$5n \pm 2$	4, 6, 14, 16	1	5	$5n \pm 2$	$15n \pm 3$	$20n \pm 8$	$30n \pm 12$
			2	10	$10n \pm 4$	$30n \pm 6$	$40n \pm 16$	$60n \pm 24$
6	$6n \pm 1$	2, 10, 14	1	6	$6n \pm 1$	—	—	—
			2	12	$12n \pm 2$	—	—	—
7	$7n \pm 1$	2, 12, 16	1	7	$7n \pm 1$	$21n \pm 6$	$28n \pm 8$	$42n \pm 6$
	$7n \pm 2$	6, 8	1	7	$7n \pm 2$	$21n \pm 9$	$28n \pm 12$	$42n \pm 12$
	$7n \pm 3$	4, 10	1	7	$7n \pm 3$	$21n \pm 3$	$28n \pm 4$	$42n \pm 18$
8	$8n \pm 1$	2, 14	1	8	$8n \pm 1$	$24n \pm 9$	—	—
	$8n \pm 3$	6, 10	1	8	$8n \pm 3$	$24n \pm 3$	—	—
9	$9n \pm 1$	2, 16	1	9	$9n \pm 1$	—	$36n \pm 8$	—
	$9n \pm 2$	8, 10	1	9	$9n \pm 2$	—	$36n \pm 16$	—
	$9n \pm 4$	4, 14	1	9	$9n \pm 4$	—	$36n \pm 4$	—
10	$10n \pm 1$	2	1	10	$10n \pm 1$	$30n \pm 9$	—	—
	$10n \pm 3$	6, 14	1	10	$10n \pm 3$	$30n \pm 3$	—	—
11	$11n \pm 1$	2	1	11	$11n \pm 1$	$33n \pm 12$	$44n \pm 12$	$66n \pm 12$
	$11n \pm 2$	10, 12	1	11	$11n \pm 2$	$33n \pm 9$	$44n \pm 20$	$66n \pm 24$
	$11n \pm 3$	8, 14	1	11	$11n \pm 3$	$33n \pm 3$	$44n \pm 8$	$66n \pm 30$
	$11n \pm 4$	6, 16	1	11	$11n \pm 4$	$33n \pm 15$	$44n \pm 4$	$66n \pm 18$
	$11n \pm 5$	4	1	11	$11n \pm 5$	$33n \pm 6$	$44n \pm 16$	$66n \pm 6$
12	$12n \pm 1$	2	1	12	$12n \pm 1$	—	—	—
	$12n \pm 5$	10, 14	1	12	$12n \pm 5$	—	—	—

Number of Coils in Winding: $C = u\,S/2$; Commutator Pitch : $y_c = (C \pm a)/p$; Potential Pitch : $y_p = C/a$
Phase Pitch : $y_{ph} = C/(a\,N)$; no idle coils permissible ; n = any integer.

N.B.—$a = 1$ denotes the common wave winding: i.e. $y_c = (C \pm 1)/p$.

Attention may be drawn to the fact that, despite the restrictions on the number of poles, it is possible to obtain a symmetrical N-phase winding in most cases, though a modification in the design may be necessary. Thus, when a static balancer is to be used it is more important to have the N phases symmetrical than to insist on making $N = 3$ or $N = 4$. Consequently with a 4-pole wave winding we ought to make $N = 3$; whilst with a 6-pole winding we should make $N = 4$. When it is important to obtain a symmetrical N-phase winding on a machine with a large number of poles, it is often essential to alter the number of poles; for example, with $p = 6$, we can do nothing, but by making $p = 5$ or 7, we can get symmetrical 3-, 4-, or 6-phase pressures.

Similarly, it is necessary to select the number of coil-sides per slot suitably to make the N phases symmetrical.

In practice, p, a, and N, are known, whilst an approximate idea of the number of coils or segments is obtained from the preliminary design of the machine. Table 3 then shows the numbers of coil-sides per slot that are permissible, whence the value of S giving $C = \frac{1}{2} u S$ nearest to the approximate figure can be determined.

If a wave winding is closed, as it is when used with a commutator, the phases are obtained by means of tappings connected to slip-rings. When "polygon" loading is used, the phase-spread, $\sigma = \beta = 2\pi/N$; but by doubling the number of tappings and loading them diametrally, the output is increased to the same extent as when the phase-spread of an open winding is halved. Diametral loading, however, calls for no special treatment, for the tappings are merely taken to correspond. Thus, for a 3-phase diametral load we need to take the same tappings as for a 6-phase polygon load.

When a wave winding is not used with a commutator, as in the case of alternators and induction motors, the phases should be given the narrow phase-spread. This is done by opening the winding for 2 N phases and reconnecting the phases to form an N-phase winding, as explained earlier in this section and illustrated in the following examples:—

(V) *Location of Tappings and Openings.*

(*a*) *Tappings off symmetrical lap or wave windings.*—When $S/a N$ is an integer in any symmetrical lap or wave winding, a symmetrical N-phase system of pressures can be derived from it, and the points to be tapped for the phases can be written down at once after determining the potential-pitch, $y_p = C/a$ segments, and the phase-pitch, $y_{ph} = C/a N$ segments. This is simply due to the fact that this is the only way of dividing the C segments into $a N$ equal parts. Then we have,

$$\left.\begin{array}{l}
\text{Phase I taps segments or joints} \\
\qquad 1;\ 1 + y_p;\ 1 + 2y_p;\ \ldots\ 1 + (a-1)y_p \\
\text{Phase II taps segments or joints} \\
\qquad 1 + y_{ph};\ 1 + y_p + y_{ph};\ 1 + 2y_p + y_{ph};\ \ldots\ 1 + (a-1)y_p + y_{ph} \\
\text{Phase III taps segments or joints} \\
\qquad 1 + 2y_{ph};\ 1 + y_p + 2y_{ph};\ 1 + 2y_p + 2y_{ph};\ \ldots \\
\qquad\qquad 1 + (a-1)y_p + 2y_{ph} \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
\text{Phase N taps segments or joints} \\
\qquad 1 + (N-1)y_{ph};\ 1 + y_p + (N-1)y_{ph};\ 1 + 2y_p + (N-1)y_{ph};\ \ldots \\
\qquad\qquad 1 + (a-1)y_p + (N-1)y_{ph}
\end{array}\right\} \quad (14)$$

In windings with one turn per coil, the tappings can be taken equally well off the back of the winding, and this is usually done in practice for convenience.

Examples of the use of these equations will be found below.

(*b*) *Opened wave windings.*—In many alternators and induction motors it is necessary or desirable to have a fractional number of slots per pole and phase, when the number of conductors per slot is small or when it is desired to suppress ripples. This is a simple matter with the double-layer winding, and for this purpose the common wave winding ($a = 1$) is eminently suitable. It is always best, wherever possible, to design the winding for 2 N phases, open it, and join each pair of opposite phases in series or parallel in order to reduce the phase-spread and so obtain the increase in output, due to the higher winding factor. The important case is the 3-phase winding, although a 2-phase winding is occasionally obtained in this way. The number of coil-sides per slot with an opened wave winding seldom exceeds 8, and is usually 4 or 6.

When $a = 1$ (as Table 3 shows), it is only possible to get a symmetrical 6-phase system of pressures with machines having 2, 10, 14, and 22 poles, which are of little practical importance, although u is little restricted in these cases. It is possible, however, to get a symmetrical 3-phase system for all numbers of poles not a multiple of 3, and we shall show how the narrow phase-spread can be obtained by evolving the symmetrical 3-phase winding from an unsymmetrical 6-phase winding, a result which enables us to use the common series connection. With $a = 1$, and $N = 3$, however, it is seldom possible to make $u = 4$. The parallel connection is also possible by making a wave winding with $a = 2$, which gives two like parts.

These various points can best be illustrated by means of the following examples.

(*c*) *Illustrative examples.*—To illustrate the points discussed in this section we shall now work out a few typical examples to show how to locate the points where a winding is to be opened or tapped for phases.

Example 1.—A 12-pole lap winding has 432 coils and 6 coil-sides per slot. Find the tappings for 3-phase diametral loading: $p = a = 6$; $C = 432$; $u = 6$; and therefore $S = 2C/u = 144$.

In order to get 3-phase diametral loading, we must find tappings for 6-phase polygon loading. The potential-pitch, $y_p = C/a = C/p = 432/6 = 72$ coils, and the phase-pitch, $y_{ph} = C/a N = C/p N = 72/6 = 12$ coils. We can now write down the segments to be tapped, in accordance with Equation (14).

Slip-ring.

I taps segments or joints:	1	73	145	217	289	361
II " " "	13	85	157	229	301	373
III " " "	25	97	169	241	313	385
IV " " "	37	109	181	253	325	397
V " " "	49	121	193	265	337	409
VI " " "	61	133	205	277	349	421

For the diametral load, the phases A, B, C are taken between the rings thus:—

Phase A between slip-rings I and IV.
" B " " III " VI.
" C " " V " II.

Since $S/(aN) = S/(pN) = 144/(6 \times 6) = 4$ is an integer, the system of pressures is quite symmetrical.

Example 2.—A common wave winding ($a = 1$) has 10 poles, 144 coils, and 4 coil-sides per slot. Find

(α) Tappings for 6-phase polygon load;
(β) Tappings for 3-phase diametral load;
(γ) Openings for 3-phase winding with $\sigma = \beta/2$.

We have $p = 5$; $a = 1$; $C = 144$; $u = 4$; and $S = 72$. The commutator-pitch, $y_c = (C \pm 1)/p = (144 \pm 1)/5 = 29$ coils; take $y_b = y_f = y_c = 29$.

(α) *Tappings off closed windings for 6-phase polygon load.*—For a 6-phase pressure, $S = 30n + 12 = 30 \times 2 + 12 = 72$ (see Table 3).

The phase-pitch $y_{ph} = C/aN = 144/6 = 24$ coils, or $S/(aN) = 72/6 = 12$ slots. Therefore we have a symmetrical 6-phase winding, each phase containing 24 coils, the successive phases being 12 slot-pitches apart. The joints or segments to be tapped can be written down at once from Equation (14) by putting $a = 1$. Thus we have

Phase	I	II	III	IV	V	VI
Segment tapped	1	$1+y_{ph}$	$1+2y_{ph}$	$1+3y_{ph}$	$1+4y_{ph}$	$1+5y_{ph}$
	1	25	49	73	97	121

If the vector polygon for the C coils is drawn, it will be seen at once that these are the numbers that divide the polygon into 6 equal parts (see part (γ) of this example).

the 6 points where it is tapped for a 6-phase system, and phases x and $x + \frac{1}{2}N$ joined in series or parallel as desired. We can represent the vector polygon of the winding by the simple circle (a) and the winding itself by (b) and (c) as in Fig. 7. The joints to be opened corresponding with the vectors marked on the polygon are shown in the table, along with the positions of the coil-sides in the slots (see (d), Fig. 9).

Vector No. x	Joint No. $b=1+(x-1)y_c$	Top Coil-side $2b-1$		Bottom Coil-side $(2b-1)-y_f$	
1	1	1	a in slot 1	260	d in slot 65
25	121	241	,, ,, 61	212	,, ,, 53
49	97	193	,, ,, 49	164	,, ,, 41
73	73	145	,, ,, 37	116	,, ,, 29
97	49	97	,, ,, 25	68	,, ,, 17
121	25	49	,, ,, 13	20	,, ,, 5

In the above table it is noticed that the numbers of the vectors are the same as the numbers of the joints they represent. Obviously this must always be the case when

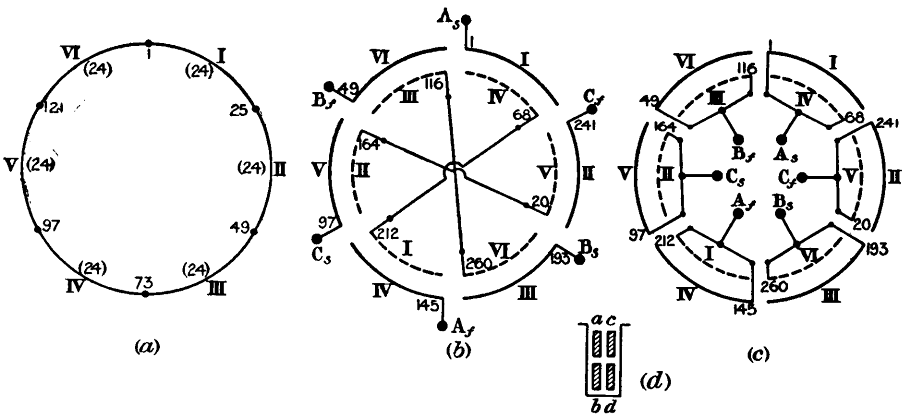

FIG. 9.—Symmetrical 6-phase wave winding with $a = 1$, opened to give a symmetrical 3-phase winding with narrow (60°) phase-spread. (Suitable for series or parallel connection.)
(a) Vector polygon (b) Series connection. (c) Parallel connection. (d) Numbering of coil-sides in a slot.

To show that the phases are 60° apart, we know that one slot-pitch in electrical degrees is equal to $p \times 360°/S = 5 \times 360°/72 = 25°$; hence 12 slot-pitches $= 300°$ forward, which, in the field, is the same as 60° backward, that is, $\beta = 2\pi/N = 60°$.

(β) *Tappings off closed winding for 3-phase diametral load.*—In this case the same tappings are taken as in (α), but are renumbered as follows:—

Phase	A	C′	B	A′	C	B′
Slip-ring	I	II	III	IV	V	VI
Segment tapped ...	1	25	49	73	97	121

(γ) *Openings for symmetrical 3-phase winding with phase-spread of* 60° (Fig. 9).—The winding is opened at C is exactly divisible by N, though the cyclic order, of course, may be different.

Since the slot-pitch is 25°, it will be seen from the position of the coil-side in the slots that, from A_s to B_s is $48 \times 25° = 1{,}200° = 120°$, and A_s to C_s is $24 \times 25° = 600° = 240°$, thus showing that the phases are 120° apart. The winding has $72/(10 \times 3) = 2{\cdot}4$ slots per pole and phase, a useful value between 2 and 3.

Example 3.—A common wave winding ($a = 1$) has 4 poles, 135 coils, and 6 coil-sides per slot. Find

(α) Tappings off a closed winding for 3-phase polygon load.
(β) Openings for symmetrical 3-phase winding with phase-spread of 60°.

We have $p=2$; $a=1$; $C=135$; $u=6$; and $S=45$. Therefore the commutator-pitch $y_c=(C\pm1)/p=\frac{1}{2}(135\pm1)=68$ or 67. Take $y_c=y_b=y_f=67$.

(a) *Tappings off closed winding for 3-phase polygon load.*—Since $S=6n\pm3=6\times7\pm3=45$ (Table 3), the phase-pitches $C/(aN)=135/3=45$ coils, and $S/(aN)=45/3=15$ slots, are whole numbers, so that it is possible to get a symmetrical 3-phase system of pressures from the winding. The segments to be tapped are then :

Phase ...	A	B	C
Segment tapped	1;	$1+y_{ph}=1+45=46$;	$1+2y_{ph}=46+45=91$

In the vector diagram in Fig. 10 it is seen that the three vectors with these numbers are 120° apart.

It is clearly not possible to obtain a symmetrical 3-phase system from this winding by diametrical loading, since C/a is not divisible by 6.

(β) *Openings for symmetrical 3-phase winding with phase-spread of* 60° (Fig. 10).—In Example 2 we obtained a symmetrical 3-phase winding with $\sigma=\frac{1}{2}\beta=60°$, from a symmetrical 6-phase winding. Table 3 shows it is seldom possible to obtain a symmetrical 6-phase system ; but, unless p' is a multiple of 3, an unsymmetrical 6-phase system can be obtained which will give a symmetrical 3-phase system when the unequal phases x and $x+\frac{1}{2}N$ are joined in series. The parallel connection is inadmissible, since opposite phases are dissimilar. The present winding is a case in point. We have $C/(aN)=135/6=22\frac{1}{2}$; and hence we take the 6 phases containing 23 and 22 coils alternately, as shown in Fig. 10, and draw up the following table :—

Vector No. x	Joint No. $b=1+(x-1)y_c$	Top Coil-side $2b-1$				Bottom Coil-side $(2b-1)-y_f$			
1	1	1	a	in slot	1	204	f	in slot	34
24	57	113	e	„	19	46	d	„	8
46	46	91	a	„	16	24	f	„	4
69	102	203	e	„	34	136	d	„	23
91	91	181	a	„	31	114	f	„	19
114	12	23	e	„	4	226	d	„	38

The slot-pitch is $p\times360°/S=2\times360°/45=16$ electrical degrees. Hence the phase displacement between A_s and B_s is 15 slot-pitches $=240°$, and between A_s and C_s 30 slot-pitches $=480°=120°$. Thus A_s, B_s, and C_s are at 120°, and we shall find that A_f, B_f, and C_f are at 120° also. Thus by placing phases I and IV, III and VI, and V and II, in series, we shall get the symmetrical 3-phase system A, B, C. In this example $q=45/(4\times3)=3{\cdot}75$ slots per pole and phase.

Example 4.—A wave winding with $a=2$ has 8 poles, 342 coils, and 6 coil-sides per slot. Find

(a) Tappings off closed winding for 3-phase polygon load.

(β) Openings for symmmetrical 3-phase winding with phase-spread of 60°.

In this case $p=4$, $a=2$, $C=342$, $u=6$, and $S=114$. Hence the commutator-pitch, $y_c=\dfrac{C\pm a}{p}=\dfrac{342\pm2}{4}=86$ or 85. Take $y_c=y_b=y_f=85$.

(a) *Tappings off closed winding for 3-phase polygon load.*—Since $S=12n\pm6=12\times10-6=114$ (Table 3), the phase-pitch is $C/(aN)=342/(2\times3)=57$ coils or $S/(aN)=114/(2\times3)=19$ slots. Both these are integers, showing that it is possible to obtain a 3-phase symmetrical system by polygon loading. The potential-pitch, $y_p=C/a=342/2=171$. The segments to be tapped are then written down from Equation (14) thus :—

Phase I taps segments or joints :	1	and 172
II „ „ „	58	„ 229
III „ „ „	115	„ 286

(β) *Openings for symmetrical 3-phase winding with phase-spread of* 60° (Fig. 11).—We can now open the winding

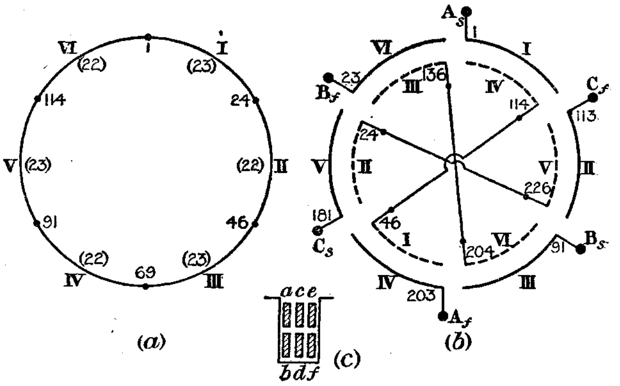

FIG. 10.—Unsymmetrical 6-phase wave winding with $a=1$, opened to give a symmetrical 3-phase winding with narrow (60°) phase-spread. (Suitable for series connection only.)

(a) Vector polygon. (b) Series connection.
(c) Numbering of coil-sides in a slot.

and get a 3-phase system by a method similar to that used in the last example. This case is often very useful, as the two parts of the windings are alike and can be put in parallel—a connection which cannot be employed with common wave windings when the phases x and $x+\frac{1}{2}N$ are unequal. Thus by making $a=2$ we can overcome the restriction met with in Example 3. The diagrams and table are drawn up below, each set of 57 coils being divided into 29 and 28 alternately.

Vector No. x	Joint No. $b=1+(x-1)y_c$	Top Coil-side $2b-1$			Bottom Coil-side $(2b-1)-y_c$		
1	1	1	a in	1	600	f in	100
30	72	143	e „	24	58	d „	10
58	58	115	a „	20	30	f „	5
87	129	257	e „	43	172	d „	29
115	115	229	a „	39	144	f „	24
144	186	371	e „	62	286	d „	48
172	172	343	a in	58	258	f in	43
201	243	485	e „	81	400	d „	67
229	229	457	a „	77	372	f „	62
258	300	599	e „	100	514	d „	86
286	286	571	a „	96	486	f „	81
315	15	29	e „	5	628	d „	105

It will be seen that the starts or the finishes of successive phases are 19 slot-pitches $= 4 \times 360^\circ \times 19/114 = 240^\circ$ apart in each polyphase system. From each polyphase system, a similar symmetrical 3-phase system is obtained, and they can be joined in series or parallel as desired. In this way, 2 similar circuits per phase are obtained.

In this example, $q = 114/(3 \times 8) = 4{\cdot}75$ slots per pole and phase.

The above examples will be sufficient to show the great variety of ways by which a symmetrical N-phase system can be obtained from double-layer windings with a fractional number of slots per pole. These are of immense importance in alternating-current machines.

(VI) *Wave Windings with a Whole Number of Slots per Pole.*

A double-layer winding which finds frequent use in practice is the wave winding with a whole number of slots per pole, or per pole and phase, as the case may be. In this winding, the pitch between successive coils is not uniform, as in the ordinary wave winding, connected up in accordance with Equation (8). The coil-span is usually made equal to a pole-pitch, likewise the pitch between successive coils, and after every p coils, i.e. after each tour of the periphery, the pitch is shortened or lengthened to miss the place already occupied.

Though the slotting is suitable for obtaining a number of similar parallel circuits, this form of wave connection is mostly used where only one circuit per phase is needed. For alternators and induction motors with 2, 4, 6, or 8 coil-sides per slot, this type of winding has many advantages over the equivalent single-layer winding, but the pronounced spacing and tooth ripples, which are always possible with a whole number of slots per pole, are not affected thereby. In other words, the use of wave connectors with a whole number of slots per pole does not make the winding in any way equivalent to a winding with a fractional number of slots per pole.

By joining all the coils in the winding in series in this way, we get a winding distributed uniformly over the circumference and in which the sum of the induced pressures is zero. Consequently the start and finish can be joined together to form a closed winding, from which a mesh-connected polyphase system can be obtained. In this way the slotting for a lap winding can be used for a wave winding, but here again it must be remembered that we may get pronounced ripples in the pressure curve.

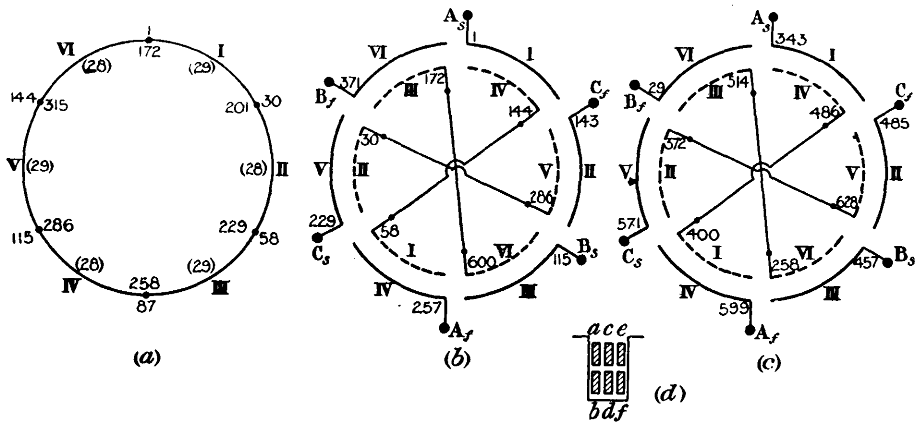

FIG. 11.—Unsymmetrical 6-phase wave winding with $a = 2$, opened to give a symmetrical 3-phase winding with narrow (60°) phase-spread. (Suitable for series or parallel connection.)

(*a*) Vector polygon. (*b*) and (*c*) Coils in the two similar polyphase systems. (*d*) Numbering of coil-sides in a slot.

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