The Theory of Finite Degree of Freedom and the Structure of Nucleons

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§1. Structure and extension

In each stratum of Nature, the extension of matter has an important meaning characterizing the physical theory which has exploited the stratum. The extension of an atom is a quantum mechanical property—owes to Planck constant h—of the electron moving around an atomic nucleus. The extension of the nucleus is due to the saturation property of nuclear force between constituting nucleons. The extension of the nucleon is a property of the pion cloud around a core—we call it "nucleonic nucleus"—predicted by Yukawa's meson theory. The theory exploiting a new stratum has given an extended structure of matter, introducing new constituting particles and a new element in dynamics. The structure of the nucleonic nucleus has not yet been established. The theory of hadronic matter proposed previously[1] gives an extended structure of the nucleonic nucleus, introducing basic particles and a constant having the dimension of length relating to the property of the basic particle.

§2. Point and divergence

The constituting particle is represented by a point in space in each stratum of physical theory. This does not mean that the particle does not have extension. The degree of freedom of the particle is represented by a point in a stratum and the extension of the particle is given by its structure introduced in the next stratum. It is a feature of strata of physical theories corresponding to strata of Nature emphasized by Sakata.[2]

The physical theory is described by a certain number (degree of freedom) of things (dynamical variables). The degree of freedom has a role only to distinguish the dynamical variables, so that it is represented by a set of points just sufficiently. The dynamical variable in field theory is the field being the function of points in space. Hence the degree of freedom is a set of points in space. In a Minkowskian space — the stage of the theory of special relativity — a finite region cannot be specified without referring to a special coordinate system. Due to this fact, the relativistic quantum field theory contains divergences and cannot be a closed theory.

§3. Finiteness principle and the theory of finite degree of freedom

The description of the relativistic theory requires an infinite set. The infinity exists in our thinking but, when we need infinite things to describe a theory, can we say that we can describe the theory? We hypothesize that the physical theory should be described by a finite number of things. We call it "finiteness principle".[3] The purpose of the theory of finite degree of freedom[3),4)] is to investigate whether the finiteness principle is supported or rejected by experimental facts.

According to the finiteness principle, the degree of freedom of the dynamical system should be a finite set. The theory of finite degree of freedom assumes that the momentum degree of freedom of the basic particle is a finite set of points on the hyper-surface $p_0=(p_1^2+p_2^2+p_3^2+M^2)^{1/2}$ (M is the mass) in a Minkowskian space $\{p_\mu\}=\{p_0, p_1, p_2, p_3\}$ distributed with the density such as

$$d\rho=QM(d^3p/p_0)F(\Lambda(N\cdot p)), \tag{1}$$

where N_μ is a universal time-like unit vector, Q and Λ are constants having dimensions of (length)^3 and (length), respectively, and F is a decreasing function of $\Lambda(N\cdot p)$ so as to be $\int d\rho<\infty$, e.g., $F=\exp[-\Lambda^2(N\cdot p)^2]$. (We take $\hbar=c=1$.) The dynamical variables are the creation and annihilation operators, $a(p, s)$ and $a^*(p, s)$ ($b(p, s)$ and $b^*(p, s)$) of the particle (antiparticle), where s represents the degree of freedom other than the momentum p. The dynamical principle is given by the total energy-momentum operators P_μ as a relation between the state vectors:

$$\Psi(\xi)=\exp[P\cdot\xi]\Psi(0), \tag{2}$$

$$P_\mu=\sum_{s,p}p_\mu\{a^*(p, s)a(p, s)+b^*(p, s)b(p, s)\}+N_\mu I, \tag{3}$$

where I represents the interaction and is an invariant function of dynamical variables. And, ξ_μ are undetermined constants which we call "unknowables".[3] They are parameters corresponding to the space-time displacements of the observer between two succeeding measurements. The relative velocity between the reference system in which $N_\mu=(1, 0, 0, 0)$ and our reference system on earth can be assumed to be $\beta=(v/c)\ll 1$ in a cosmological model.[5]

§4. Non-spatio-temporal description

The formulation of dynamical principle, (2) and (3), mentioned in the preceding section might seem featureless, but it achieves a generalization of the usual field theory by the reason mentioned in the following and allows the total momentum operators such as given by (1) and (3). We call it "non-spatio-temporal description". (Detailed discussions are given in Ref. 3).)

We can use the operators

$$\psi(x)=[(2\pi)^3 Q]^{-(1/2)}\sum_{p,s}\{a(p,s)u(p,s)e^{ipx}+b^*(p,s)v(p,s)e^{-ip\cdot x}\},\quad x\in\sigma_N, \tag{4}$$

as dynamical variables, instead of $a(p, s)$, etc., σ_N is the hyperplane perpendicular to N_μ in the Minkowskian x_μ-space being conjugate to p_μ-space. (u and v are appropriate functions.) The operators $\psi(x)$ of all points on σ_N are not all independent. When we replace F in (1), for simplicity's sake, by $F=1$ for $|\boldsymbol{p}|\leq(1/\Lambda)$ and $F=0$ for $|\boldsymbol{p}|>(1/\Lambda)$, where $\boldsymbol{p}$ is the component of p_μ perpendicular to N_μ, then only the points on σ_N distributed one per

$$v=6\pi^2\Lambda^3 \tag{5}$$

give the independent $\psi(x)$. (v is derived from $(4\pi/3)\ (1/\Lambda)^3 v=(2\pi)^3$.) These lattice points give the degree of freedom equivalent to the simplest case of the momentum degree of freedom in (1). (The lattice space extends infinitely in the limit $Q\rightarrow\infty$. When Q is finite, the lattice space is also finite.)

In the non-spatio-temporal description, the ξ-space in (2) and the x-space in (4) are set up independently and connected subsequently by the dynamical principle. In the usual field theory — spatio-temporal description — starting from the existence of space-time, ξ and x spaces are identical. This identification restricts P_μ in (2) strictly. The non-spatio-temporal description can loosen this restriction. In the theory of finite degree of freedom, the x-space and ξ-space have not only different roles but also different mathematical properties. (ξ_μ are parameters in the probabilistic law and considered to be continuous. x-space has a lattice structure. Physically, the concepts represented by ξ and x belong to different strata.[3])

§5. Hadronic matter

We imagine a basic particle of mass M, having spin (1/2) and unitary spin, as the constituting particle of the nucleonic nucleus and assume that the basic particle obeys the theory of finite degree of freedom. The operator I in (3) is taken to be of the form

$$I=l^2\int d\sigma_N\int d\sigma_N'\sum_i\int(\bar{\psi}(x)O^i\psi(x))g(x-x')(\bar{\psi}(x')O^i\psi(x')) \tag{6}$$

where $\psi(x)$ is the operator in (4) and $g(x-x')$ is a form factor of extension of the order of v in (5). (I is assumed to be invariant under Lorentz and SU_3 groups.) Due to the interaction I, the system of a small number of particles is unstable and the stable system is accompanied with a number of particle-antiparticle pairs. In this case, owing to the Pauli's exclusion principle, only a restricted number f of particles can occupy the volume v in (5). Taking account of spin and unitary spin, we take $f=12$. Therefore, if the mean number of basic particles in the nucleonic nucleus (generally, hadronic nucleus) is N, the nucleonic nucleus has the extension, on the average,

$$V=(N/f)v. \tag{7}$$

We can estimate N by the Tomonaga intermediate coupling method[6] (a variation method) under simplified conditions as a function of three parameters, l, M and Λ, reflecting our basic assumptions. We take

$$\Lambda \approx (1/M) \approx l \approx 10^{-15}\text{cm}. \tag{8}$$

In this case, N is the order of 10^4 and the radius of the nucleonic nucleus turns out about the nucleon Compton wave-length. We call the extended matter made of basic particles, such as the nucleonic nucleus, 'hadronic matter'.[1]

§6. The model of the interaction between hadronic nuclei

The property of heavy atomic nuclei is very difficult to derive from the solution of the Schrödinger equation initially assumed for the nuclei due to the complicated many-body problem. In these circumstances, we try to comprehend it referring to experimental facts by a model being considered compatible with the Schrödinger equation. We are now in a similar situation for the consideration of the property of hadronic matter.

According to the calculation of the intermediate coupling method, the hadronic nucleus is a superposition of degenerated Fermi gases of various numbers of basic particles (the number distribution is approximately Poisson with the mean number $N \approx 10^4$), having the mean volume $(4\pi/3)(1/M_N)^3$, where M_N is the nucleon mass. Hadron resonances and H-quantum (constant mass fireball) are explained as excited states of surface and volume vibrations, respectively.[1]

A nucleonic nucleus absorbs or emits a mesonic nucleus by the interaction, I in (6), between basic particles. This should give the Yukawa interaction. However, since it occurs at extremely high order of I, the direct derivation of the Yukawa interaction from I is very difficult. Then we estimate the coupling strength of the Yukawa interaction in our picture by a model which seems reasonable in view of the properties of hadronic matter. To settle the model, we take into consideration the following four characteristic properties of hadronic matter.[1]

(i) The particle density does not exceed (f/v) where f is the number in (7). The density is almost uniform and takes this maximum value, except for the vicinity of circumference.

(ii) The state with the mean particle number (hence, the mean volume by (7)) smaller or larger than the mean particle number $N \approx 10^4$ (hence, the mean volume) of the stable state is not a stationary state and transits instantly to the stable state. The velocity of this expansion or contraction is considered of the order of the mean particle velocity. It is estimated about 85% of the light velocity from the Fermi energy $\approx(1/\Lambda)$ of the hadronic matter.

(iii) The surface tension is very large. The surface energy per unit area ε of Fermi gas consists of potential and kinetic parts. If we assume they are proportional to the Fermi energy, ε of hadronic matter is 500 times larger than that of atomic nuclei. This value gives $\Delta E \equiv 500$ MeV for the quantum of the surface vibration of $L=2$, being about the same with the spacing ΔE for $\Delta J=2$ in the Chew-Frautchi plot.[7]

(iv) The moving hadronic matter receives the Lorentz contraction. This is not inconsistent with that a constant number f of particles always occupy the volume v in the reference system $N_\mu=(1, 0, 0, 0)$, as we can see from Fig. 1.

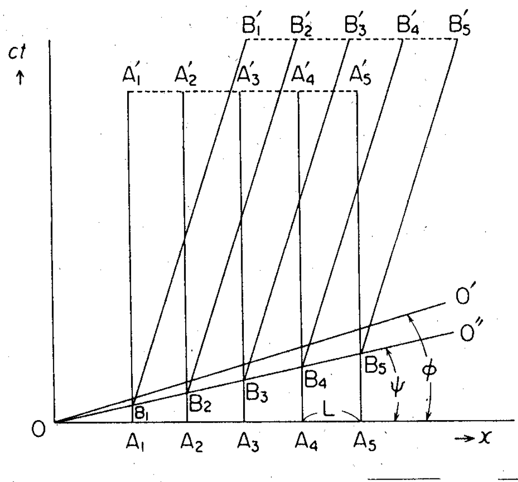

Fig. 1 Lorentz contraction of hadronic matter. $\{\overline{A_iA_i'}\}$ and $\{\overline{B_iB_i'}\}$ represent the world lines of lattice points in x-space and those of the basic particles of moving hadronic matter with the velocity β, respectively, in the reference system where $N_\mu=(1, 0, 0, 0)$. $\{\overline{B_iB_i'}\}$ coincides with $\{\overline{A_iA_i'}\}$, when $\beta=0$. (The complicated motions relative to the center-of-mass of matter are neglected.) Each cell ($L=\overline{A_iA_{i+1}}$) of lattice space contains only one representative particle ($\overline{B_iB_i'}$) at all times, in spite of the Lorentz contraction $\overline{B_1'B_5'}=\sqrt{1-\beta^2}\,\overline{A_1'A_5'}$. $\overline{OO'}$ is the line connecting simultaneous points for the moving matter: $\tan\phi=\beta$. $\overline{OO''}$ is the line connecting intersecting points of $\{\overline{A_iA_i'}\}$ and $\{\overline{B_iB_i'}\}$: $\tan\psi=\gamma\beta/(\gamma+1)$, $\gamma=1/\sqrt{1-\beta^2}$.

The model of the interaction which we use is as follows (c.f. Fig. 2.). The interaction I in (6) creates a particle-antiparticle pair near the circum-

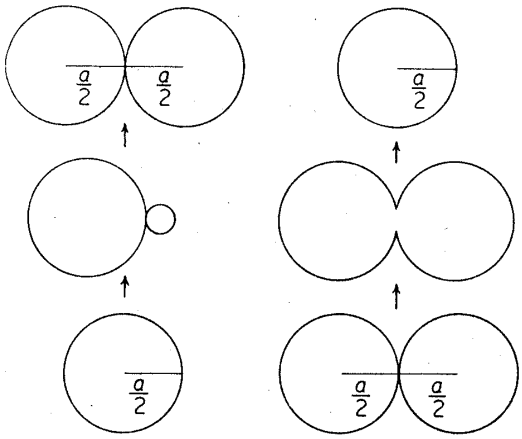

Fig. 2. The model of interaction. (1) Emission. (2) Absorption.

ference of a nucleonic nucleus. (By the property (i), it is difficult to create a pair in the interior part of the nucleonic nucleus. Since the extension of the form factor $g(x-x')$ in I is v, the pair is created near the circumference.) The created pair grows instantly into a mesonic nucleus by the property (ii). (This is considered to be the higher order effect of I.) When the circumferences of a nucleonic nucleus and a mesonic nucleus approach, they are connected by the interaction I. (The property (i) prevents the overlapping of two hadronic nuclei.) The connected state contracts instantly into a nucleonic nucleus by the properties (ii) and (iii).

§7. The interaction between nucleonic and mesonic nuclei

When we take into account only the degree of freedom x of the center-of-mass coordinate for the nucleonic and mesonic nuclei, without taking their structures into consideration, the interaction is expressed effectively by the energy of the form

$$H=g\int d^3xd^3x'\overline{\Psi}_N(x)\Psi_N(x)G(x-x')\Phi_M(x'). \tag{9}$$

(The scaler meson is assumed for simplicity's sake.) Due to the model of interaction, mentioned in the preceding section, the form factor $G(x-x')$ appears which has a maximum value at the distance $|x-x'|$ equal to two times the radius of hadronic nucleus (c.f. Fig. 2). The origin of this form factor is the finite degree of freedom restricted by Λ and the Pauli's exclusion principle.

The form factor $G(x-x')$ restricts the relative momentum k of interacting particles. When $G(x-x')=G(r)$, $r=|x-x'|$, the form factor in momentum space is

$$F(k) \propto \int G(r) e^{ik\cdot x} d^3x = (4\pi/k) \int_0^\infty r dr G(r) \sin kr. \tag{10}$$

Since $G(r)$ has a maximum value at the vicinity of $r=2(1/M_N)\equiv a$, where $(1/M_N)$ is the radius of hadronic nucleus, we may estimate $F(k)$, putting, e.g., $G(r)=r^2\exp[-(r/a)^2]$, then we have a cutoff function $|F(k)|^2$ which decreases as

$$|F(k)|^2 \propto \exp[-(ak)^2/2] \tag{11}$$

for large k.

The moving nucleonic nucleus receives the Lorentz contraction (the property (iv) in the preceding section), so that the above mentioned $|F(k)|^2$ should be the expression in the center-of-mass system of the nucleonic nucleus. In the general reference system, it is expressed as

$$|F(k)|^2 \propto \exp[-(a^2/2)[k-(p/M_N)((p/M_N)\cdot k)]^2], \tag{12}$$

where p_μ is the momentum of the nucleonic nucleus. In the reference system of $p_\mu=(M,0,0,0)$, (12) coincides with (11). In the reference system of $p_\mu=(M/\sqrt{1-\beta^2},\ -\beta M/\sqrt{1-\beta^2},\ 0, 0)$, (12) gives

$$|F(k)|^2 \propto \exp[-(a^2/2)[(1-\beta^2)k_1^2+k_2^2+k_3^2]]. \tag{13}$$

This means that, for the moving nucleonic nucleus with a large velocity, the cutoff acts only for transverse components, k_2 and k_3, of the mesonic nucleus so as to be $k \lesssim (\sqrt{2}/a) \approx (3/4)M_N$. As for the longitudinal component, $k_1 \lesssim (3/4)M_N/\sqrt{1-\beta^2}$. The cutoff (13) has a form similar to the Wataghin's cutoff.[8)]

When we replace the distribution of momentum degree of freedom in (1) approximately by the straight cutoff as we have done in this paper (The tail of cutoff is important and indispensable for extremely high energy phenomena.[7)]), the maximum momentum of hadronic nuclei turns out to be $N(1/\Lambda)\approx 100$ TeV, so that, for the phenomena of the energy much smaller than 100 TeV, we need not take into account the restriction for momenta of hadronic nuclei. In this case, $\Psi_N(x)$ and $\Phi_M(x)$ in (9) are expressed by the same form with $\psi(x)$ in (4), replacing M by M_N and M_M, respectively, and taking the limit $\Lambda\to 0$ and $Q\to\infty$. (M_M is the meson mass. We assume that the hadronic nucleus carries the most part of the mass of the hadron.) Further, we multiply the factor $(2\,M_M)^{-1/2}$ to $\Phi_M(x)$ in order to have the conventional one for the boson field.

The magnitude of the matrix element of H in (9) for the creation of a mesonic nucleus is estimated from that the extensions of the hadronic nucleus and the form factor $G(x-x')$ are of the order of $(1/M_N)^3$. It is of the order of

$$\langle H \rangle \sim g M_N. \tag{14}$$

From that the extensions of the basic particle and the form factor $g(x-x')$ are of the order of $v=(4\pi/3)(1/M)^3$, the order of the magnitude of the matrix element of I in (6) for the creation of a pair is estimated as

$$\langle I\rangle \sim l^2/v \sim M. \tag{15}$$

According to the model of the interaction mentioned in the preceding section, $\langle H\rangle$ and $\langle I\rangle$ are considered to be of the same order of magnitude and we have, using the values of parameters in (8), the magnitude of g of the order of 10, being reasonable for the coupling constant of the strong interaction.

§8. The structure of nucleons

The clouds of various mesonic nuclei are generated, by the interaction in (9), around a nucleonic nucleus. Each cloud extends to the region of the radius of the order of its meson Compton wave-length. According to the intermediate coupling method, the mean number of particles (mesonic nuclei) in the cloud turns out $\lesssim 1$,[6] being much smaller than the mean number of basic particles ($\approx 10^4$) in the nucleonic nucleus. This is due to the large extension of $G(x-x')$ in (9) (cutoff length $\sim(1/M_N)$) compared with that of $g(x-x')$ in (6) (cutoff length $\sim(1/M)$).

There are many meson resonances of masses larger than about $3\,M_\pi$ (M_π is the pion mass). The clouds of these mesons will overlap and form a very dense cloud around the nucleonic nucleus. The density of the basic particles in the cloud, however, should not exceed the maximum value (f/v), due to the property (i) of hadronic matter mentioned in §6. Then it will be reasonable to assume as a model of nucleons that the nucleonic nucleus has a dense cloud of the radius of the order of $(1/3\,M_\pi)$, the density of which is nearly the maximum value (f/v), and the thin pion cloud of the radius of the order of the pion Compton wave-length. This model of nucleons, infered from the properties of hadronic matter, is compatible with the following experimental facts.

(1) The characteristic features of Regions I, II and III of nuclear force[9] are explained by the pion cloud, the dense cloud and the nucleonic nucleus, respectively. The clouds are exchanged between the nucleonic nuclei by the interaction (9), and contribute to Regions I and II. The core of nuclear force (Region III) comes out from the property (i) preventing the overlapping of the nucleonic nuclei. It is a soft core, since the nucleonic nucleus has no sharp boundary (The distribution of volume is Poisson as mentioned in § 6.) and is compressible.[10] (The core of the nucleonic nucleus of the volume v will give a hard core.)

(2) The dense cloud, together with the nucleonic nucleus, is considered responsible for the electromagnetic form factor of nucleons. In this case,

the property (iv) of Lorentz contraction in §6 is essential.[11)]

(3) The fact, that the asymptotic value of the cross section of nucleon-nucleon reactions is considerably larger than $\pi(1/M_N)^2$ and is of the order of $\pi(1/M_\pi)^2$, seems to support the existence of the dense cloud.

(4) By the reason mentioned in §9, the model can explain the smallness of transverse momentum of multiply produced pions in high-energy nucleon-nucleon collisions.

The nucleonic nucleus has been obtained as a stable stationary state under the original interaction I in (6). The stationary states (hadronic nuclei) under I interact by H in (9), being considered a residual interaction derived from I. The physical nucleon is a stable stationary state under H and more stable than the nucleonic nucleus. In this case, the meson cloud in the nucleon is considered a correction to the nucleonic nucleus (zeroth approximation) by the residual interaction H. The effective coupling strength of H is much smaller than that of I, due to the much smaller cutoff momentum. (The dense cloud comes out not due to the strong coupling strength of H but due to a large number of kinds of heavy mesons.)

The hadronic matter is a superposition of various number n of basic particles. The average number $\langle n\rangle$ of hadronic nuclei is estimated by a variation method (the intermediate coupling method) to be $N\approx10^4$. We call hadronic matter with $\langle n\rangle\gg N$ "giant hadronic matter"[1)] which is very unstable (the property (ii) in §6). The dense cloud is not a giant hadronic matter. It is a superposition of various number of many kinds of mesonic nuclei (hadronic matter with $\langle n\rangle=N$). (In the dense cloud, the hadronic nuclei move with velocities comparable to the light velocity.)

In our picture, the nuclear force is resulted from the exchange of mesonic nuclei between two nucleonic nuclei by the interaction (9), in contrast to the production process of hadrons mentioned in the next section.

§9. The nucleon-pion interaction

Using a model of interaction similar to the model we have used to derive H, we can derive the interaction H' between a nucleon and a pion which are stationary states under H. A mesonic nucleus in the dense cloud in a necleon interacts with a pionic nucleus, as well as the nucleonic nucleus, since it is considered that the interaction between mesonic and pionic nuclei exists with the coupling strength of the order of that of H (with a similar form factor to $G(x-x')$), since all interactions between hadronic nuclei are derived from the basic interaction I. Then, a pionic nucleus is created by H near the circumference of the dense cloud of a nucleon. (The creation in the interior part of the dense cloud is supressed by the property (i) in §6, since the density of basic particles in the cloud is about the maximum value.) The created pionic nucleus is not a stationary state and grows into a pion instantly by a higher order effect of H. (c.f. Fig. 2.) When the circumferences of the dense clouds of a nucleon and a pion approach (The property (i) prevents the overlapping.), they are connected into a large dense cloud. This state is not a stationary state under H and shrinks instantly into a stable nucleon.

According to the above mentioned model of the nucleon-pion interaction infered from the properties of hadronic matter, we express the interaction as

$$H'=g'\int d^3xd^3x'\bar{\Psi}'_N(x)\Psi'_N(x)G'(x-x')\Phi'_\pi(x'), \tag{16}$$

where the states of nucleon and pion are represented only by their center-of-mass coordinates, x and x', and the effect of their structures is taken into account by the form factor $G'(x-x')$. Since the interaction occurs in the vicinity of the circumferences of the dense cloud, $G'(x-x')$ has a maximum value at $|x-x'|\approx 2(3M_\pi)^{-1}$. (We assume that all hadrons have dense clouds similar to the dense cloud of the nucleon.) Accordingly, the form factor $G'(x-x')$ in (16) is considered to have a similar form to that of $G(x-x')$ in (9) in which M_N is replaced by 3 M_π. Then, by the same argument with that mentioned in § 7, we have the cutoff function $|F'(k)|^2$ for the pion momentum, being of the form of $|F(k)|^2$ in (13) with $a=(2/3M_\pi)$. The cutoff suppresses the transverse momentum of pion larger than $\sim(9/4)M_\pi\approx$ 300 MeV.

The mean transverse momentum $\langle k_T\rangle$ observed in the multiparticle production is about 350 MeV. The fireball model attributes the small $\langle k_T\rangle$ to the low temperature of the fireball. The H-quantum hypothesis asserts that the multi-particle productions occurs through the production of a certain number (depending on energy) of fireballs of constant mass ((2$\sim$3) BeV). An H-quantum decays, on the average, into 6 pions and their $\langle k_T\rangle$ (the average value of the magnitudes of their momenta in the c.m. system of the H-quantum) is about 300 MeV.[12)] In this case, the mean transverse momentum of produced H-quanta should be much smaller than $\sim$6(350–300) MeV in order to have $\langle k_T\rangle\approx$350 MeV for all produced pions. (In our picture, the H-quantum is considered to be a hadron with an excited hadronic nucleus, the radius of which is about $(1/M_N)$ being the same with that of nucleonic nucleus.) The small $(\ll M_N)$ transverse momentum of produced pions favors the form factor $G'(x-x')$, resulted from the dense cloud, in the interaction between hadrons. Also in the multi-peripheral model, the small $\langle k_T\rangle$ is obtained by the form factor $G'(x-x')$ inserted into all vertices of hadronic interactions.

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