The Theory of Learning in Games

Drew Fudenberg
and
David K. Levine

The MIT Press
Cambridge, Massachusetts
London, England

Contents

Series Foreword xi
Acknowledgments xiii

1 Introduction 1
1.1 Introduction 1
1.2 Large Populations and Matching Models 4
1.3 Three Common Models of Learning and/or Evolution 7
1.4 Cournot Adjustment 8
1.5 Analysis of Cournot Dynamics 9
1.6 Cournot Process with Lock-In 10
1.7 Review of Finite Simultaneous-Move Games 12
Appendix: Dynamical Systems and Local Stability 24
References 27

2 Fictitious Play 29
2.1 Introduction 29
2.2 Two-Player Fictitious Play 30
2.3 Asymptotic Behavior of Fictitious Play 32
2.4 Interpretation of Cycles in Fictitious Play 35
2.5 Multiplayer Fictitious Play 37
2.6 Payoffs in Fictitious Play 39
2.7 Consistency and Correlated Equilibrium in Games with Two Strategies 43
2.8 Fictitious Play and the Best-Response Dynamic 44
2.9 Generalizations of Fictitious Play 46
Appendix: Dirichlet Priors and Multinomial Sampling 48
References 49

3 Replicator Dynamics and Related Deterministic Models of Evolution 51
3.1 Introduction 51
3.2 Replicator Dynamics in a Homogeneous Population 53
3.3 Stability in the Homogeneous-Population Replicator Dynamic 55
3.4 Evolutionarily Stable Strategies 58
3.5 Asymmetric Replicator Models 63
3.6 Interpretation of the Replicator Equation 66
3.7 Generalizations of the Replicator Dynamic and Iterated Strict Dominance 73
3.8 Myopic Adjustment Dynamics 78
3.9 Set-Valued Limit Points and Drift 83
3.10 Cheap Talk and the Secret Handshake 89
3.11 Discrete-Time Replicator Systems 91
Appendix: Liouville's Theorem 95
References 96

4 Stochastic Fictitious Play and Mixed-Strategy Equilibria 101
4.1 Introduction 101
4.2 Notions of Convergence 102
4.3 Asymptotic Myopia and Asymptotic Empiricism 103
4.4 Randomly Perturbed Payoffs and Smoothed Best Responses 105
4.5 Smooth Fictitious Play and Stochastic Approximation 110
4.6 Partial Sampling 114
4.7 Universal Consistency and Smooth Fictitious Play 117
4.8 Stimulus-Response and Fictitious Play as Learning Models 121
4.9 Learning about Strategy Spaces 128
Appendix: Stochastic Approximation Theory 130
References 133

5 Adjustment Models with Persistent Randomness 137
5.1 Introduction 137
5.2 Overview of Stochastic Adjustment Models 138
5.3 Kandori-Mailath-Rob Model 142
5.4 Discussion of Other Dynamics 146
5.5 Local Interaction 149
5.6 Radius and Coradius of Basins of Attraction 154
5.7 Modified Coradius 157

5.8 Uniform Random Matching with Heterogeneous Populations 161
5.9 Stochastic Replicator Dynamics 163
Appendix A: Review of Finite Markov Chains 168
Appendix B: Stochastic Stability Analysis 171
References 173

6 Extensive-Form Games and Self-confirming Equilibrium 175
6.1 Introduction 175
6.2 An Example 176
6.3 Extensive-Form Games 178
6.4 A Simple Learning Model 179
6.5 Stability of Self-confirming Equilibrium 184
6.6 Heterogeneous Self-confirming Equilibrium 185
6.7 Consistent Self-confirming Equilibrium 186
6.8 Consistent Self-confirming Equilibria and Nash Equilibria 188
6.9 Rationalizable SCE and Prior Information on Opponents' Payoffs 191
References 199

7 Nash Equilibrium, Large Population Models, and Mutations in Extensive-Form Games 201
7.1 Introduction 201
7.2 Relevant Information Sets and Nash Equilibrium 202
7.3 Exogenous Experimentation 203
7.4 Learning in Games Compared to the Bandit Problem 207
7.5 Steady-State Learning 213
7.6 Stochastic Adjustment and Backward Induction in a Model of "Fast Learning" 216
7.7 Mutations and Fast Learning in Models of Cheap Talk 223
7.8 Experimentation and the Length of the Horizon 226
Appendix: Review of Bandit Problems 227
References 229

8 Sophisticated Learning 231
8.1 Introduction 231
8.2 Three Paradigms for Conditional Learning 233
8.3 Bayesian Approach to Sophisticated Learning 235
8.4 Interpreting the Absolute Continuity Condition 238
8.5 Choosing among Experts 241

8.6 Conditional Learning 243
8.7 Discounting 247
8.8 Categorization Schemes and Cycles 249
8.9 Introspective Classification Rules, Calibration, and Correlated Equilibrium 254
8.10 Sonsino's Model of Pattern Recognition 259
8.11 Manipulating Learning Procedures 260
References 266

Index 269