


THE THEORY OF   
MAGNETISM  
MADE SIMPLE

An Introduction to Physical Concepts  
and to Some Useful Mathematical Methods

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 **World Scientific**

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