The Theory of the Interaction of Elementary Particles

I. The Method of the Theory of Elementary Particles*)

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As is well known, the modern theory of elementary particles is composed of the following three stages. In the first place, a hypothesis concerning the model of the objects is postulated, with considerations as to what sort of elementary particles exist in nature, and what sort of interaction they are subjected to. Next comes the stage in which the field quantum theory of Heisenberg and Pauli is applied to this system; and finally the process of drawing conclusions comparable with experiment by means of suitable approximation methods (chiefly perturbation theory). Therefore, when, as is the case today, these conclusions deviate markedly from experimental results (as does the life-time of the meson), or involve contradictions within themselves (for example, the infinite self-energy), the source of the difficulties must be sought for with careful retrospection as to the three points, corresponding to the above mentioned three stages: (i) the property of the model, (ii) the limit of the applicablity of quantum theory, (iii) the validity of the approximation method. This last-mentioned question of the approximation method especially perturbation theory, has been studied in detail by Tomonaga and Wentzel and their respective co-workers, with many fruitful results. Also, the works of Heitler, Wilson and Sokolow are in accordance with this line, and have succeeded in removing the divergence difficulties in the high energy range. We have studied in particular the question of the model, and have shown that certain of the difficulties, for instance the problem of the life-time of mesons, can be solved by an alternation of the model.[1] But as concern various divergence difficulties appearing in field theory such as that of the self-energy of an electron inherent ever since Lorentz electron theory, the question seems to be connected with the limit of applicability of quantum theory, and hence a new theory has long been awaited. It is well known that Heisenberg, discussing this problem, considers it as a contradiction arising from the over-looking of a new *universal constant* with the dimension of length, which ought to be introduced upon unifying the theory of relativity and quantum theory. This has become a

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guiding principle with researchers of the theory of elementary particles to-day. We, too, trust that his anticipation will be confirmed in the future, but deem it necessary to dig down a little deeper in considering this problem.

If the law of the development of physics, governing the direction in which theory would make progress in the future, could be discovered, and this law be applied consciously to the analysis of concrete problems, then we could surely attain our end with far greater facility and certainty than depending upon incidental successes due to the gifted intuition of researchers. It should be highly appraised that Heisenberg has endeavoured to discover such a law within the process of development of the theory of relativity and quantum theory by tracing the process historically. However, his arguments, which are based solely upon analogy, cannot be *propter hoc* though they may be *post hoc*. It cannot be deduced from the fact that the sun rises every morning, that it will rise again tomorrow. Thus Heisenberg himself does not anticipate a universal length with absolute confidence, but simply states that such a consideration lies close at hand.

Taketani states in his "*Formation of Newtonian Mechanics*"[2] and other works that the following is the correct view. That is, we may look upon our cognition of Nature as a *dialectic logical* process proceeding spirally, repeating the circle corresponding to the "three stages of judgement" in Hegel's philosophy.[3] He calls these the *phenomenological*, *substantialistic*, and *essentialistic* stages. The first, that is the phenomenological stage, implies the *an sich stage* in which separate facts are described, and corresponds, in Newtonian mechanics, to that stage represented by the role which the works of Tycho Brahe plays. The second is the stage in which the substantial structure embodying the occurence of phenomena is learnt, and the descriptions of the phenomena are put in order and are moulded into laws. Here law is significant as a property of the entity. This is the *für sich stage* where it is stated that a particular structure possesses such-and-such a phenomenon under such-and-such conditions, and corresponds to the Keplerian stage. Lastly, the essentialistic stage is said of the *an und für sich* stage in which it is made clear what phenomenon an entity with a given structure causes under given conditions, which is the Newtonian stage. Taketani applies this view to the present status of meson theory and defines it to be in a state of searching for a path to sublimate itself from the substantialistic to the essentialistic stage, preliminarily setting things in order within substantialistic bounds.

Now, there are various theories postulated to cope with the contradictions in the present stage of the theory of elementary particles, most conspicuously appearing as the divergence difficulties, and we desire to find out the correct direction of development of the theory by classifying them from the above standpoint. Firstly, the cut-off method of Scherzer, March and others, the subtraction theories of Dirac, Bhabha, Heitler, Stueckelberg and others, and the λ-limiting process[4] of Wentzel and Dirac all attempt to remove the difficulty by subjecting the present theory to certain manipulations (which are relativistically invariant) in order to exclude the parts responsible for the divergences, and hence must be looked upon as belonging to the phenomenological stage. On the other hand, the new electromagnetic field theory of Mie, Born and Bopp[5] attempts to clarify the substantial structure of the interaction of elementary particles, so it may, together with Markov's theory which introduces non-commuting relations between the electromagnetic field quantities and the space co-ordinates of the electron, be classified in the substantialistic stage. It should, of course, be noted that such a classification must not be made merely formally.

According to the suggestion of Taketani in autumn 1945, we investigated in detail the structures of the Born-Bopp-sequence of theories which are in the substantialistic stage. Born's theory is famous for its non-linearity, and arouses much interest mathematically, but, as Bopp has pointed out, its essence does not lie in its non-linearity but in that its Lagrangian function involves derivatives higher than the second order of the potential. Bopp has in fact, succeeded in constructing a theory which, though linear, gives a finite self-energy. The quantization of this theory has been performed by Bopp himself but recently Podolsky and Kikuchi have also carried it out. With the same results as these, O. Hara[6] of our institute has proved, by the method of canonical transformations, that Bopp theory is a mixed field theory of the electromagnetic field and the neutral vector Yukawa field with negative energy. It is termed a mixed field theory because it is of the same type as that in which Møller and Rosenfeld considered, in order to remove the $1/r^3$ singularity of nuclear forces, a mixture of vector and pseudoscalar fields, and assumed particular numerical relations among the constants of interaction between them and the nucleons. As the static potential of an electromagnetic field has the from e^2/r, while that of a negative energy neutral vector Yukawa field is $-g^2e^{-\kappa r}/r$ (where e and g are the interaction constants of the respective fields, $h\kappa/c$ the meson mass, and r the distance), the total static potential for a mixed field with $e=g$ become $e^2[(1/r)-(e^{-\kappa r}/r)]$ and the self-energy takes a finite value. The original intention of Born and Bopp was a justification from the monistic standpoint, but we find it desirable, for the below-mentioned reason, to seek the essence of this theory in that it is a *mixed field* theory.

Hitherto, in field theory, the respective field interactions were taken out separately, and studied independently of others. But we wish to maintain that *it is the most effective method of developing the theory hereafter, and a promising path to the solution of the present difficulties, to investigate the internal correlations of the interactions between all the fields interacting with* *the same particles and all the particles acting as source of the same field.* The above-mentioned Bopp's theory and the mixed field theory of Møller and Rosenfeld certainly point to this direction. Also, the theory of Heitler and Ma, which assumes a multifariousness (various magnitudes of spin and charge) in the states of nucleons, which are the source of the nuclear field, and further Dirac's new quantization theory which leads to a mixture of positive and negative energy photon fields, both seem to verify the correctness of our standpoint. Dirac's electron theory started with the assumption that it was possible to evolve a relativistic quantum mechanics in connection with the one-body problem, but met with the negative energy difficulty, to cope with which it had, contradicting the first assumption, to pass over to the whole theory, which assumes the existence of infinitely many vacuum electrons. Is it not likely that the same applies to the divergence difficulty in the theory of elementary particles? We desire to look upon this divergence difficulty as indicating the narrow-mindedness of the *formal logical* method hitherto employed in complete negligence of the correlations between elementary particles as a whole. The attitude of studying the various aspects of nature separately, apart from their relations as a whole was an effective method in the early stages of the development of science, and a fundamental condition for the great progress of modern science. As a result, we have become used to the way of thinking in which nature is regarded as an accidental collection of mutually isolated and independent objects capable of being considered separately from one another. But once this *formal logical* method, which originates in Bacon, exceeds its limits of applicability, it becomes a one-sided, narrow-minded method, and leads to insoluble contradictions. This is because we concentrate our attention on separate objects and neglect their correlations, on their existence and forget growth and annihilation, on their stationary states and forget motion, seeing, as it were, only the trees and overlooking the forest. All the objects and phenomena occurring in nature are primarily correlated, depend on each other, and condition each other, so that if we take up any one phenomenon in an isolated form neglecting its correlation with environment, we are at once led to nonsensical results. The above-mentioned example of Dirac's electron theory is no special case, for whenever the *formal logical* method is applied inadequately and carried through to the end, we arrive at a point diametrically opposite to the starting-point. There are presumably very many confusions and contradictions in every branch of science, which arise because the majority of researchers are still not consciously aware of the limits of such a method. We desire to point out to the necessity, in the theory of elmentary particles too, of being ever awake to the limit or applicability of this method, and taking the dialectic viewpoint that all phenomena in the world are correlated with each other, and conditioned each other.

It is our intention, from this methodical viewpoint, to examine to what extent the difficulties of the theory of elementary particles may be removed by the mixed field theory. Also, we are investigating whether the methods of cut-off, subtraction, and λ-formalism which we mentioned above as being in the phenomenological stage may not, in some form or other, be corroborated by the mixed field theory.

However, such a theory as that of mixed fields, which merely postulates certain numerical relation between the interaction constants, is a theory in the phenomenological and substantialistic stages, involving an external and incidental cognition of the correlation between mutual interactions, and must, in future be elevated into the essentialistic stage, clarifying the more internal and inevitable correlations. Møller's 5-dimensional theory is note-worthy as a research tending in this direction, and we too must not forget that our final aim is to elevate our theory to a higher stage. In this case, most probably, as Taketani[7] has discussed in detail, the unification of the opposition between field and matter will play a fundamental role.[*] Also the existence of Heisenberg's "*universal length*" will be considered properly. The reason why we stress the necessity of a synthetical study of the mixed field theory including the multifariousness of the source is because we consider it an important work to set things in order within the phenomenological and substantialistic stages prior to passing on to the essentialistic stage. (to be continued)

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*) Interesting in this connection is Wheeler's recent work.[8] Previous theories of elementary particles not only considered all forces to be conveyed by fields, but also treated matter itself as field. In short, they were "theories of fields". But he attempts to replace fields by interactions between matter, from the standpoint of action at a distance. This is a "theory of matter", in contrast to the previous "theories of fields". Further, he holds the view that the action of all matter existing in nature must be taken into account even when discussing the motion of a single electron, and thus makes clear the intrinsic nature of the reaction of fields in previous theories. This coincides with the dialectic view that the world must be interacted in correlation as a whole.