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THE THERMAL FUTURE OF THE UNIVERSE

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SUMMARY

The thermal behaviour of matter and electromagnetic radiation in the late stages of ever expanding cosmologies is examined. The cooling rate of ions is calculated, and the absorption of radiation discussed, leading to a condition for complete opaqueness.

There has been much discussion on the thermal history of the Universe, and Rees (1969) has also examined the thermal fate of astrophysical systems in oscillating cosmological models. In this paper the thermal future of the ever expanding cosmological models is traced through to the heat death as the cosmic time $t \to \infty$. We shall not be concerned with the dynamics of particular models, and merely assume the usual homogeneity and isotropy (cosmological principle), so that the models are described by a time dependent scale factor $R(t)$ that is simply taken to be of the form $R \propto t^n$, $n > 0$.

1. EMISSION PROCESSES

At the present epoch t_0, the thermal capacity of the radiation field vastly exceeds that of matter (10^8 photons per baryon), but only about 1 per cent of the energy of this radiation is due to emission processes such as starlight, the bulk being a black body background of apparently primeval origin. Eventually though, the starlight will accumulate, and may become sufficient to affect the thermal future of the Universe. For galaxies of constant luminosity density, the energy density of starlight is

$$\propto R^{-4}(t) \int^t R(\tau)\, d\tau,$$

while that of the primeval background radiation falls like $R^{-4}(t)$. The ratio of energies is therefore 1 per cent $\times (t/t_0)^{n+1}$, using the present values. The starlight becomes comparable after about $10^{2/(n+1)}\, t_0$, so that the faster the expansion, the shorter the equalization time. For the Dirac, Einstein–de Sitter and Milne models $R \propto t^{1/3}$, $t^{2/3}$ and t respectively, and the equalization times are $30t_0$, $16t_0$ and $10t_0$.

However, the luminosity density will not remain constant. Most stars in our Galaxy will have reached the white dwarf stage in 1 or $2t_0$, and thereafter drop steadily in brightness. Possibly new populations of stars will form, and new matter become accreted from the intergalactic medium. Certainly, only a small fraction of available nuclear fuel has been used up. The possibility that starlight may influence the thermal future of the Universe is then likely, especially for the

Friedmann models. This starlight will not be thermalized, though for rough calculations we shall assume that a meaningful temperature T_γ can be applied to the total radiation field.

2. THE FINAL COOLING

The radiation cools like $T_\gamma \propto R^{-1}(t)$ while the neutral matter is decoupled and cools independently like $R^{-2}(t)$. If there is an ionized intergalactic medium, it will start to recombine steadily when emission of radiation by galaxies ceases to be important. In some models the expansion rate is fast enough to prevent complete recombination. According to Bates & Dalgarno (1962) the recombination probability per unit time for a given electron is $\propto \rho_i T_i^{-0\cdot75}$ where ρ_i and T_i are the ion density and temperature respectively. For complete recombination

$$\int^{\infty} \rho_i T_i^{-0\cdot75}\, dt = \infty \tag{1}$$

that is, $\rho_i T_i^{-0\cdot75} \propto 1/t$ or slower. If $T_i \propto R^{-p}(1 \leqslant p \leqslant 2)$ this implies $3-\frac{3}{4}p \leqslant 1/n$. The limiting case is

$$3-\frac{3}{4}p = \frac{1}{n}. \tag{2}$$

The cooling rate of the ions (p) depends on the strength of the coupling to the radiation. The only important absorption process as $R \to \infty$ is inverse bremmstrahlung, which has a cross-section σ_b given by

$$\sigma_b \propto \frac{\rho_m}{T_i^{1/2}\omega^3}(1-\exp(-\omega/kT_i)) \tag{3}$$

where ρ_m is the heavy particle density, k is Boltzmann's constant and $\omega(\propto R^{-1}(t))$ is the photon frequency. Note that, as $\rho_m R^3$ is constant, ρ_m/ω^3 is constant also. Moreover, as $T_i \propto R^{-p}$ and $p \geqslant 1$, the exponential factor is either constant, or negligible as $R \to \infty$. Therefore we can replace (3) by

$$\sigma_b \propto T_i^{-1/2} \propto R^{p/2}. \tag{4}$$

A given ion will gain energy from the radiation at a rate $E_\gamma \sigma_b$, where E_γ is the radiation energy density ($c = 1$ here). It will lose energy by adiabatic expansion at a rate $\propto kT_i/t$. A dynamic equilibrium is attained when these rates balance out. This can only be so if the rates change in the same way as a function of R. Now $E_\gamma \sigma_b \sim R^{-4} \times R^{p/2} = R^{-(4-p/2)}$ and $kT_i/t \sim R^{-(p+1/n)}$, so equilibrium requires $4-\frac{1}{2}p = p+1/n$; that is

$$\frac{3}{2}p = 4-\frac{1}{n}. \tag{5}$$

Equation (5) does not apply when $n < \frac{2}{5}$ as it predicts $p < 1$, i.e. the ions heat up above the radiation spontaneously. This arises because we have neglected bremmstrahlung emission by ions. When $n \leqslant \frac{2}{5}$, the matter and radiation are in equilibrium at the same temperature. Because of the vastly greater thermal capacity of the photons, any ions would then follow the radiation cooling law in all these models ($T_i \propto R^{-1}(t)$).

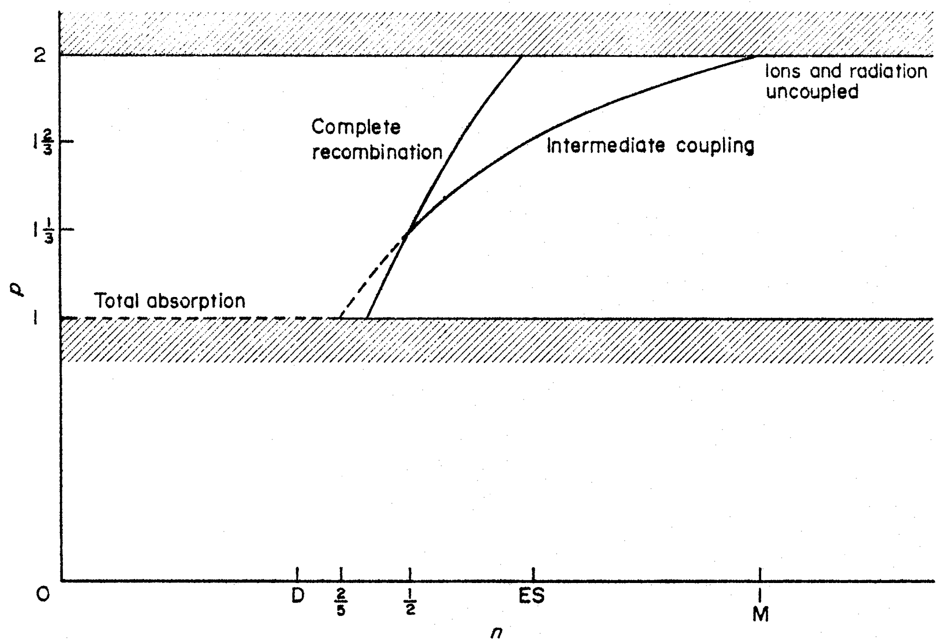

FIG. 1. *Ionic cooling rates.*

Actually, there are no ions left anywhere in such models. To show this we have plotted equations (2) and (5) on a graph of p against n (Fig. 1). The curves intersect at $n = \frac{1}{2}, p = \frac{4}{3}$. Therefore, all models which expand faster than $t^{1/2}$ (as do all the open Friedmann models) have some ions left at $t \to \infty$. The ions in models with $n \leqslant \frac{1}{2}$ completely recombine after sufficiently long times, so the ionic cooling rate is shown drawn with a broken line. For the Einstein–de Sitter (E–S) and Milne (M) models, we see that the cooling rate is $\propto R^{-5/3}$ and R^{-2} respectively. For all models with $n \geqslant 1$ the ions behave as though decoupled and cool like the neutral matter: $T_{\text{i}} \propto R^{-2}(t)$.

3. SCATTERING AND ABSORPTION PROCESSES

Only Thomson scattering need be considered. The cross-section σ_{T} is independent of ω, so that the product $\rho_{\text{i}}\sigma_{\text{T}} \propto R^{-3}$. As the process requires free ions, we must have $n > \frac{1}{2}$. This means $\rho_{\text{i}}\sigma_{\text{T}} \propto t^{-3/2}$ or faster, so that

$$\int^{\infty} \rho_{\text{i}}\sigma_{\text{T}}\, dt$$

always converges, rendering the process incomplete even for $t \to \infty$.

We have assumed until now that E_γ is constant after all emission processes by galaxies have ceased. This is a reasonable approximation when the photon/baryon ratio is so enormous, but we must check if it remains so as more and more photons are absorbed. Eventually (if $n < 1$), all the thermal energy of the ions is due to absorbed radiation, because in the absence of photons the ions would cool like R^{-2} to a negligible fraction of T_{i}. The total energy per unit comoving volume taken from the radiation field by an ion is therefore

$$\propto -\int^{0} k\, dT_{\text{i}},$$

which is finite. On the other hand, the total number of photons removed is

$$\propto -\int^{0} \frac{dT_{\mathrm{i}}}{\omega} \propto -\int^{0} \frac{dT_{\mathrm{i}}}{T_{\gamma}},$$

where T_γ is the radiation temperature, so that if T_{i} falls faster than T_γ (which is so for all $n > \frac{2}{5}$) then this integral also converges. Thus, the total number of photons removed is finite (and rather small), so that our approximation is valid. For models in which $T_{\mathrm{i}} = T_\gamma$ the integral diverges, implying that all photons are absorbed eventually, and the Universe is opaque. This would occur in all models with $n \leqslant \frac{2}{5}$ if the ionic recombination were not total. This could still be so if there was an electron or proton excess, for example.

Even in the absence of such an exotic hypothesis, absorption can still be complete, because it can take place by galaxies. Suppose for a moment that as $t \to \infty$, galaxies have a constant finite geometric cross-section. Absorption will take place by macroscopic objects independently of frequency ω, down to all wavelengths. A galaxy will then absorb photons at a rate proportional to the photon density ($\propto R^{-3}$), so that provided R increases slower than $t^{1/3}$ (Dirac model), the time integrated photon absorption diverges, i.e. the model is opaque.

Unfortunately, it is not clear how far this simple picture applies. Because of the complex structure of galaxies any discussion of their absorptive properties requires a knowledge of their evolution. Absorption may take place by gas and dust, stars, planets and black holes. Macroscopic objects will have an absorption cross-section equal to their geometric cross-section. After long times one would expect collisions between stars and their slow accretion by black holes, gradually reducing even the geometric cross-section. In the Dirac model, with $G \propto 1/t$, even the black hole radius $\to 0$ as $t \to \infty$. However, provided some matter escaped the black hole death, the $t^{1/3}$ Universe would still be opaque.

The cooling rate of galactic gas is more difficult to discover. It is not adiabatically expanded (being gravitationally bound), but can cool radiatively as far as its lowest bound state, though heavy impurities and molecular binding may allow this gas to reach very low temperatures. Any free ions would also cool the neutral gas, being coupled both to the radiation field and (via Coulomb forces) to the neutral matter. This would be exceedingly inefficient, though, because most galactic ions would recombine rapidly as soon as emission processes ceased.

Models which eventually absorb all photons are of interest in the Wheeler–Feynman theory of electromagnetism (Davies 1972). A further interesting consequence was suggested to me by P. Kafka. In the $n > \frac{1}{3}$ models there is no strict heat death, as a temperature difference always exists between the neutral matter and radiation, and perhaps the unrecombined ions also. A sufficiently resourceful intelligence could maintain life indefinitely (though with increasing difficulty) in this permanent thermodynamic disequilibrium, by using the radiation field as an energy source. However, in the opaque models, the photons would eventually disappear, the whole Universe reaching a uniform temperature (true heat death), and life would become impossible.

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REFERENCES

Bates, D. R. & Dalgarno, A., 1962. *Atomic and molecular processes*, Chap. 7, ed. D. R. Bates, Academic Press.

Davies, P. C. W., 1972. Is the Universe transparent or opaque? *J. Phys. A.*, in press.

Rees, M. J., 1969. *Observatory*, **89**, 193–8.