THE THERMIONIC TRIODE AS RECTIFIER.

By E. B. MOULLIN, M.A., and L. B. TURNER, M.A., Member.

(*Paper received 3rd February, and read before the* WIRELESS SECTION *1st March*, 1922.)

SUMMARY.

The performances of the triode rectifier arrangements used in wireless telegraphy, with signals of various strengths, and with or without a superposed local heterodyne oscillation, are investigated theoretically and experimentally. The theoretical results throughout are illustrated numerically by reference to a certain well-known pattern of triode, and in the more fundamental cases the theoretical deductions are compared with precise experimental measurements made at low frequency. The agreement is found to be very close. Experimental curves for this pattern of triode are given also in cases where it has not seemed practicable to evolve theoretical formulæ.

TABLE OF CONTENTS.

1. INTRODUCTION.

In wireless telegraphic practice hitherto the final indicating instrument has nearly always been of a type incapable of being actuated by the high-frequency current in the receiving antenna, or by any magnified copy of it, and therefore requiring some form of rectifier. A rectifier, in wireless parlance, is a conductor which more readily allows current to pass in one direction than in the other; it is an asymmetric conductor such that an alternating P.D. whose mean value is zero produces a current whose mean value is not zero. Any non-return valve device, such as the Dunlop valve of a pneumatic tyre, is of course a rectifier, and is sometimes termed a "perfect" rectifier because negative pressure difference is accompanied by *no* negative flow. Electrical valves of this type are available for use with large E.M.F.'s, as when diodes are used for obtaining a high-tension direct current from an a.c. source; but for the feeble signal E.M.F.'s of wireless receivers no such "perfect" rectifiers have been produced, and rectification depends upon the excess of the current in one direction over the current in the other direction (the latter no longer being sensibly zero).

With the recent development of the thermionic tube all other forms of asymmetric conductor (rectifier) have given place to it or are rapidly doing so, and the constancy and determinate nature of the thermionic conductor, as compared with (say) electrolytic or crystal detectors, are a great stimulus towards the elucidation of the mathematical theory of rectification. On the other hand, recent developments have complicated and widened the investigation in several ways. The high-frequency alternating E.M.F. due to the signal is now never applied alone to the rectifier circuit, but always added to a steady E.M.F.; it cannot always be treated as indefinitely small (as when relay replaces telephone reception); it is usually not applied in the circuit of the indicating instrument, but in an associated circuit thermionically related with it (as in triode rectification); and, finally, the circuits are sometimes such that the rectified current at any instant does not depend only on the signal at that instant but also upon the course of the signal in past instants (as in cumulative grid rectification).

Since it must cover so many different arrangements and conditions, the complete theory of rectification as at present practised in wireless receivers is somewhat extensive. It does not seem to have been fully worked out in previous publications, and, more especially, such theory as has been given has not been adequately verified experimentally.

2. SCOPE OF PAPER.

In the present paper, the performances of the various triode rectifying arrangements have been calculated fairly exhaustively in terms of the parameters of the triode as exhibited in its static characteristic curves. The paper does not deal with the physics of the triode upon which the characteristics depend; but the shapes of these characteristics are to some extent analysed, and formulæ are developed from which the rectifying performances of any given triode can be calculated and the best circuit values determined. The theoretical results obtained are in all cases illustrated by numerical calculations for a single typical pattern of triode, the well-known "R" triode used so largely during and since the war; and they have been experimentally checked in the more fundamental cases by precise measurements with signal E.M.F.'s of low frequency (90 periods per sec.).

The operative E.M.F. applied to the rectifier depends on the local heterodyne oscillator, when there is one, as well as on the far-off transmitter. Accordingly, the types of signal dealt with are classified as :

(S1). Sustained C.W. (continuous wave) signal E.M.F. $e = a \sin pt$;
(S2). The same with added heterodyne E.M.F. $b \sin qt$;
(S3). Damped train E.M.F. $e = a\epsilon^{-mt} \sin pt$.

The rectifier arrangements investigated with these signals are classified as follows :

(R1). Rectification by curvature of anode characteristic, called " anode rectification " ;
(R2). Rectification by curvature of grid characteristic, called " grid rectification " ;
(R3). The same with grid condenser, called " cumulative grid rectification."

In all cases the signal is supposed to be applied in the grid-filament circuit of a triode, and in all except one introductory case the indicating instrument (e.g. telephone or relay) is supposed to be inserted in the anode circuit ; the operative current in it, called the " signal current," is the change of mean anode current consequent on the arrival of the signal. In the case of heterodyne reception, where the high-frequency mean anode current fluctuates at an acoustic frequency, the term " signal current " is taken to connote the range of the acoustic fluctuations of the mean. Making use of the above S and R classification, the cases investigated are shown in Table 1.

TABLE 1.

Case	Strength of signal	Strength of heterodyne	Dealt with particularly in Sections
S1, R1 .. S1, R2 ..	General and weak	Nil	5, 6
S1, R1	Strong	Nil	7
S1, R3	Weak and moderate	Nil	8, 9
S1, R3	Strong	Nil	9
S2, R1	Weak and moderate	Moderate	11
S2, R1	General	Strong	12
S2, R3	Weak	Weak	13
S2, R3	General	Strong	13
S3, R1	Moderate	Nil	15

The investigations refer, in the first instance, to circuits in which the signal currents do not produce sensible changes of anode potential, although when indicating apparatus of sensible impedance is used, change of potential must occur. In Section 16 the correction required on account of the change of anode potential is calculated for the cases of sustained continuous-wave signal (S1 and S2).

3. THE " R " TRIODE.

Typical static characteristic curves for the triode used in most of the measurements, from which slopes and higher differentials can be calculated, are given in Figs. 1 and 2. This triode was in no way a peculiar one, and although different specimens of the same pattern will obviously have somewhat different characteristic curves, it was found that a number of specimens of the same make (" Osram ") exhibited curves of nearly identical shape. It may be assumed that calculations for this particular specimen will approximately hold for any other specimen of the same pattern ; that the formulæ developed will equally apply to other patterns of high-vacuum triode, the relevant values of the parameters being taken ; and

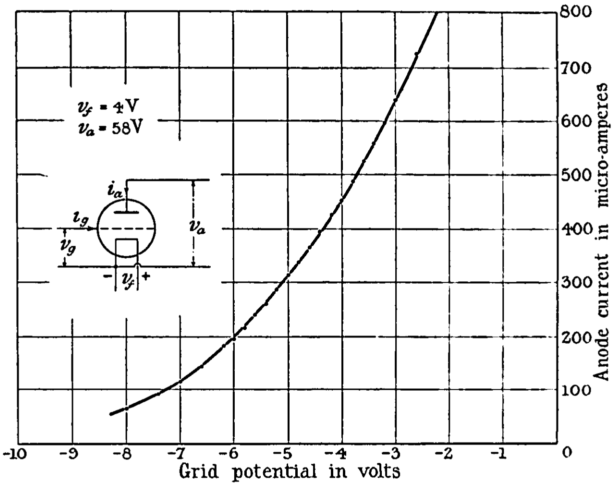

FIG. 1.—Anode-current/grid-potential curve for " R " triode.

that if the formulæ are found to agree with observation in the case of several specimens, there is every probability that they are generally true.

4. EXPERIMENTAL METHOD.*

Provided that within the triode there is nothing in the nature of sluggishness or temperature hysteresis making itself perceptible during the high-frequency changes occurring in actual use—which is probably the case in high-vacuum receiving tubes—it does not matter whether the predictions from the static characteristic as to behaviour with alternating current are put to the test at 50 or a million periods per second. The use of a commercial a.c. power supply offers great advantages in respect of ease and accuracy of adjustment and measurement of the applied signal E.M.F. Consequently the town supply at 90 periods was utilized in these tests, any desired low voltage being obtained by means of step-down transformers and potential dividers of resistance form.

In most rectifier measurements the steady non-signal current is vastly larger than the changes in it constituting the rectified current or signal current, and it is therefore necessary to employ a balance method of measuring the signal current. This was done

* The tests were carried out in the Engineeering Laboratory, Cambridge, with facilities put at our disposal by Prof. C. E. Inglis.

very simply in the way indicated in Fig. 3, which shows the typical circuit for measuring the rectified current in an S1, R1 arrangement (see Table 1). The potential divider P is set to give zero deflection of the galvanometer G (sometimes a "Unipivot" micro-ammeter and sometimes a reflecting galvanometer) in the absence of the signal; then in the presence of the signal the galvanometer reads an easily calculated fraction of the change of anode current.

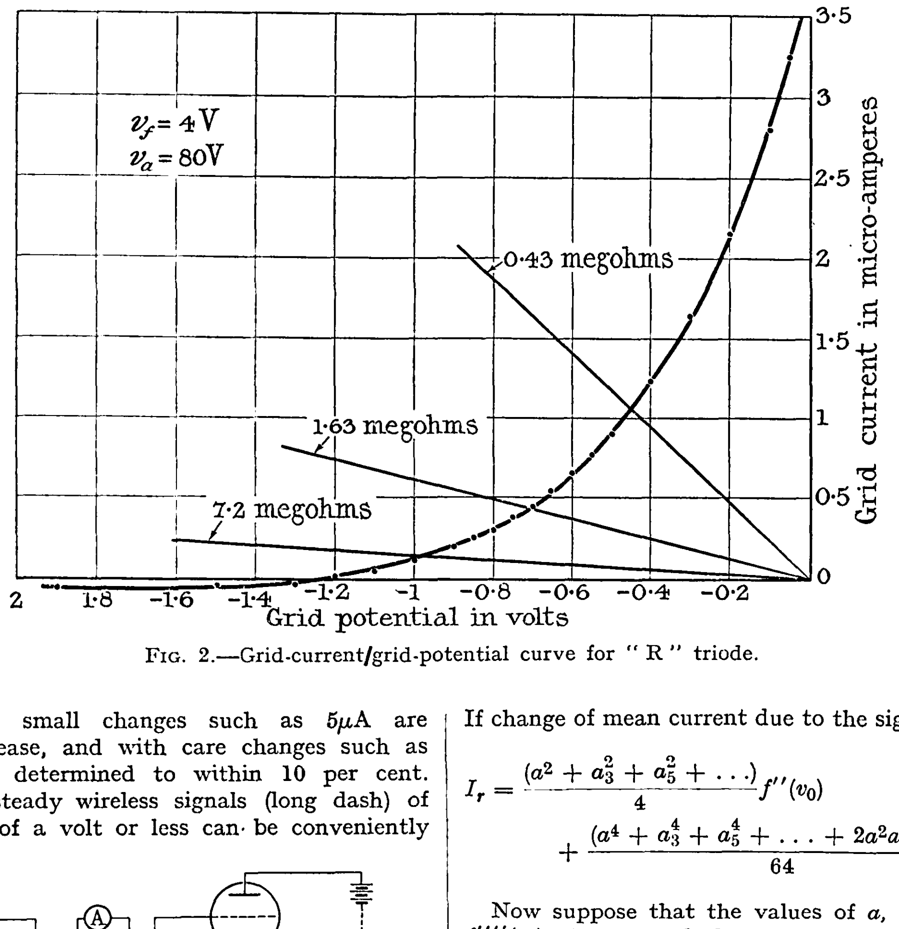

FIG. 2.—Grid-current/grid-potential curve for "R" triode.

By this means small changes such as 5μA are measured with ease, and with care changes such as $0{\cdot}25\mu$A can be determined to within 10 per cent. Indeed, actual steady wireless signals (long dash) of (say) one-tenth of a volt or less can be conveniently

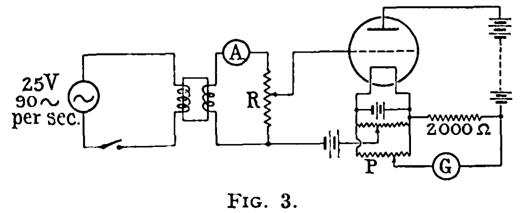

FIG. 3.

measured by means of the rectified current in this way.

5. Theory of Rectification of Sustained C.W. Signals by an Asymmetric Conductor.

The theory is well known, but may be stated briefly as follows (Fig. 4): Let the asymmetric conductor (the rectifier) have a curved characteristic of the general form $i = f(v)$, and suppose, moreover, that it is possible to represent this function by an infinite series.

Then $v = v_0 + e$,

and $i = f(v_0) + ef'(v_0) + \frac{e^2}{2!}f''(v_0) + \frac{e^3}{3!}f'''(v_0) + \dots$

Let $e = a \sin pt + a_3 \sin 3pt + a_5 \sin 5pt + \dots$

then $i = f(v_0) + (a \sin pt + a_3 \sin 3pt + a_5 \sin 5pt + \dots)f'(v_0)$

$$+\frac{1}{2!}(a \sin pt + a_3 \sin 3pt + a_5 \sin 5pt + \dots)^2 f''(v_0) + \dots$$

If change of mean current due to the signal e is I_r,

$$I_r = \frac{(a^2 + a_3^2 + a_5^2 + \dots)}{4} f''(v_0) + \frac{(a^4 + a_3^4 + a_5^4 + \dots + 2a^2a_3^2 + \dots)}{64} f''''(v_0)$$

Now suppose that the values of a, a_3, a_5, etc., and $f''''(v_0)$, etc., are such that

$$\frac{(a^4 + a_3^4 + a_5^4 + \dots + 2a^2a_3^2 + \dots)}{16} f''''(v_0)$$

is small compared with $(a^2 + a_3^2 + a_5^2 + \dots)f''(v_0)$; then

$$I_r = \frac{\text{mean-square } e}{2} f''(v_0) \equiv \frac{\mathcal{E}^2}{2}\frac{d^2}{dv^2} \quad . \quad . \quad (1)$$

Hence, unless the characteristic curve is such that d^4i/dv^4 is very large, the rectified current from a very weak signal is proportional to the rate of change of the slope of the characteristic, and to the square of the strength of the signal P.D., whether the latter contains harmonics or not—a point not always noticed.

This theory is, of course, applicable to a diode; or to a triode used as a diode by omitting to utilize either grid or anode; or to a triode in which the signal is applied in the grid circuit and the signal current is in the anode circuit. In the last case, the relevant i, v characteristic is that between anode cur-

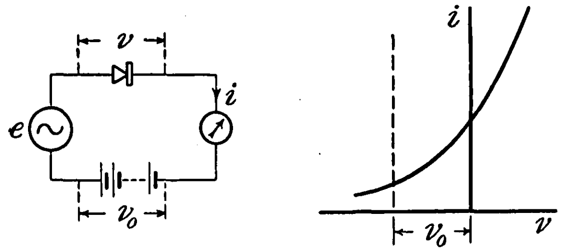

FIG. 4.

rent and grid potential for constant anode potential, as in Fig. 1.

6. EXPERIMENTAL VERIFICATION OF SECTION 5.

In Figs. 5 (*a*), 5 (*b*), 5 (*c*) are shown the curves for anode rectification relating observed rectified current

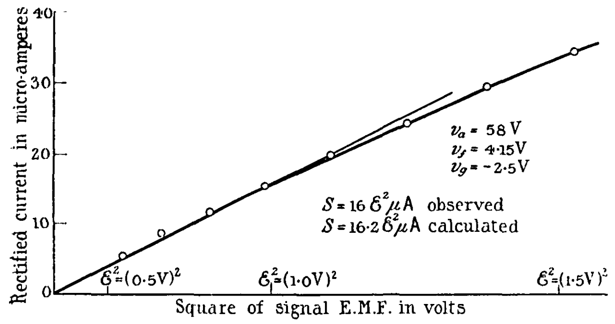

FIG. 5 (*a*).—Anode rectification.

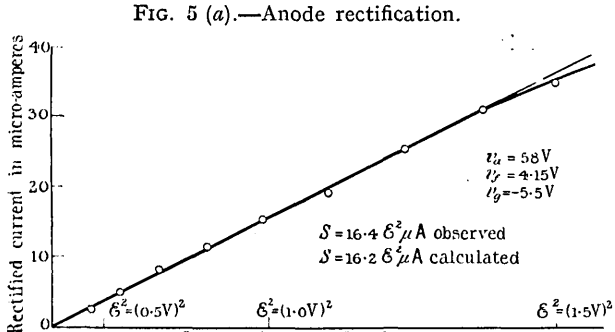

FIG. 5 (*b*).—Anode rectification.

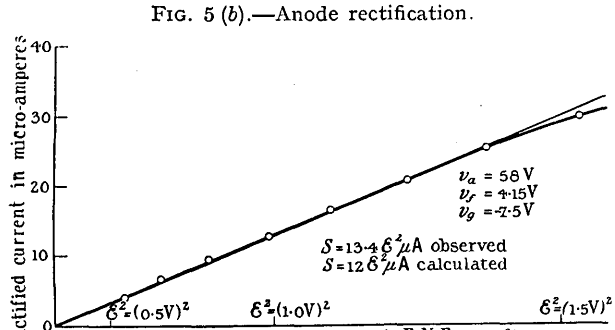

FIG. 5 (*c*).—Anode rectification.

with the square of the virtual value of the signal E.M.F.* It is seen that they are all straight lines so long as the applied E.M.F. does not exceed about 1·3 volts (R.M.S.). At greater signal strengths the observed values lie below this straight line: this agrees with the fact, shown by the static characteristic, that d^4i_a/dv_g^4 though small is not zero; it has a negative value, which makes the a^4 term of the expansion in Section 5 above increasingly significant as the signal increases in strength. On each curve the observed slope of the straight line is marked, and below it, for comparison, the calculated slope measured from the derived static characteristics.

Fig. 6 shows how the rectified current from a 1-volt signal depends upon the mean grid potential. Over a wide range it is almost independent of mean grid potential, and the rectified current is nearly $16\mathcal{E}^2$ micro-amperes. Three other "R" triodes of the same make tested in this way gave rectified currents differing by not more than $\pm$ 5 per cent from this figure, and consequently the relation $S = 16\mathcal{E}^2$ micro-amperes may be taken with fair accuracy to represent the anode rectification curve of any "R" triode.

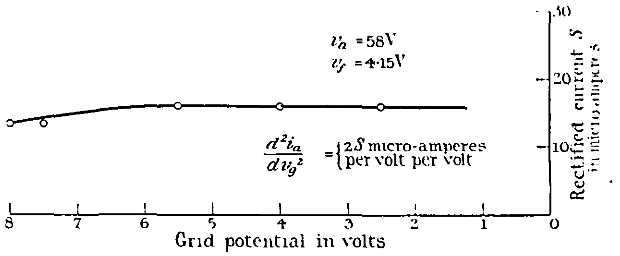

FIG. 6.—Anode rectification. $\mathcal{E}$ = constant = 1 volt (R.M.S.).

Analysis of static characteristic by means of these results.—As the mean grid potential is varied between $-$ 2·5 volts and $-$ 5·5 volts the value of d^2i_a/dv_g^2 is seen to lie between 32 and 33 micro-amperes per volt per volt; also when the applied signal E.M.F. is 1·5 volts (R.M.S.) the observed rectified current is not more than 3 per cent below the value given by the expression $S = 16\mathcal{E}^2$. Now the expansion in Section 5 will allow us, with the help of these numerical values, to calculate the approximate values of d^4i_a/dv_g^4. For

$$i = \frac{\mathcal{E}^2}{2}\frac{d^2 i_a}{dv_g^2}\left(1 + \frac{\mathcal{E}^2}{8}\cdot\frac{d^4 i_a}{dv_g^4}\Big/\frac{d^2 i_a}{dv_g^2}\right) \text{ approx.};$$

consequently $$\frac{(1\cdot5)^2 d^4i_a/dv_g^4}{8\,d^2i_a/dv_g^2} = 0\cdot03$$

that is, $$\frac{d^4 i_a}{dv_g^4} = 0\cdot1\frac{d^2 i_a}{dv_g^2}$$

Also Fig. 6 has shown that over a very large range d^2i_a/dv_g^2 varies by only about 3 per cent. Hence the anode current characteristic of an "R" triode is evidently very nearly a parabola. If, therefore, it is desired to calculate the instantaneous anode current, so long as the mean grid potential gives a state point on the characteristic suitable for rectification, the anode characteristic may be taken as given by the equation

$$i_a = \alpha + \beta v_g + \gamma v_g^2$$

* Throughout the paper E.M.F.'s are quoted in volts.

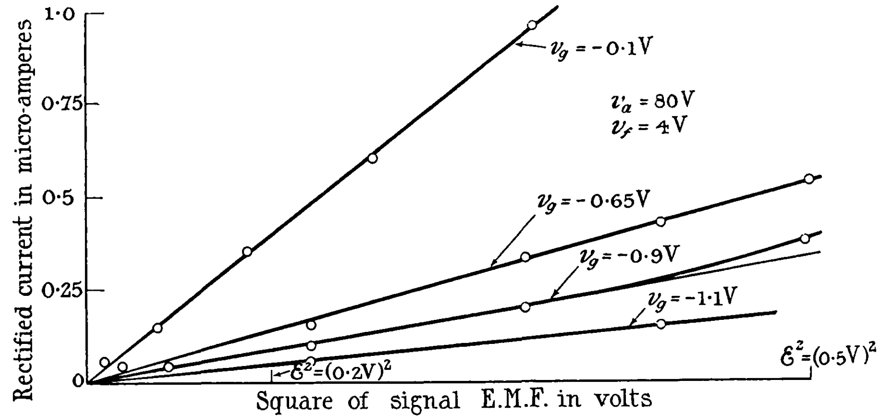

FIG. 7 (*a*).

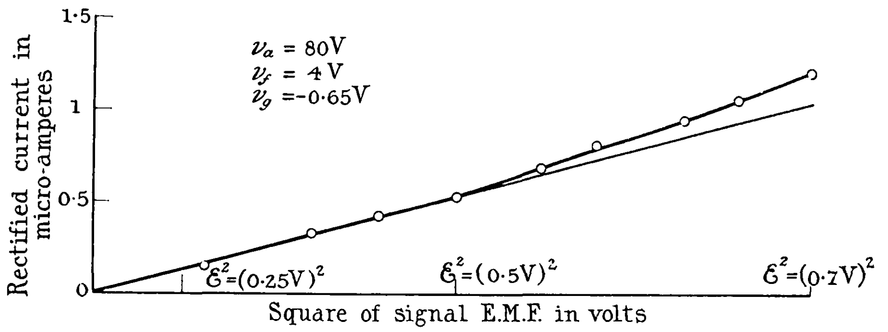

FIG. 7 (*b*).—Extension of one curve in Fig. 7 (*a*).

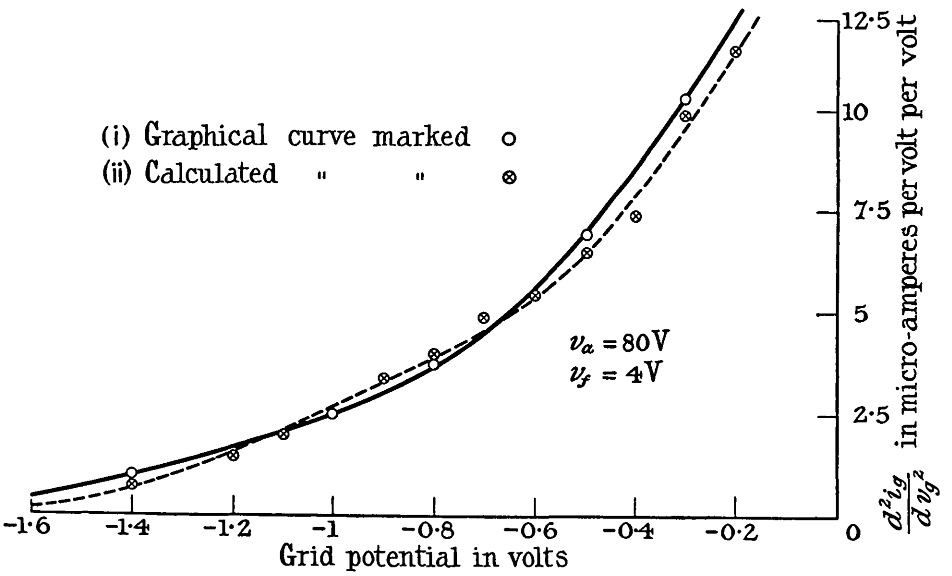

FIG. 8.—Curves of d^2i_g/dv_g^2, (i) derived graphically from static characteristic, and (ii) calculated from formula $S = \frac{1}{2}\mathcal{E}^2(d^2i_g/dv_g^2)$ with $\mathcal{E}$ = constant = 0·38 V (R.M.S.).

This is a fact that may be of use in analysing the behaviour of a triode in various arrangements, and is actually applied later in the paper in the examination of reception with weak heterodyne (Section 11).

Rectification by curvature of grid current characteristic.—In this case the rectified current is not very much smaller than the mean grid current, and in measuring the rectified current it is not necessary to resort to the balance method of Fig. 3.

In Fig. 7 observed rectified current is plotted against the square of the applied signal P.D. with four different mean grid potentials. It is seen from these graphs that if the signal much exceeds about 0·5 volt (R.M.S.) the terms involving $\mathcal{E}^4 . d^4 i_a/dv_g^4$, etc., assume importance; but that if the signal is less than 0·5 volt the rectified current varies very closely as the square of the signal strength. If, therefore, a constant weak signal is applied and the rectified current is measured with various mean grid potentials, it should be possible to deduce the value of $d^2 i_a/dv_g^2$ for any mean grid potential. Fig. 8 shows the curve connecting v_g and $d^2 i_a/dv_g^2$ obtained experimentally in this way, using a constant signal of 0·38 volt (R.M.S.), together with the same curve derived graphically from the static characteristic. It is seen that the two curves very nearly coincide—a striking confirmation of the adequacy of the theoretical analysis of this type of rectification. It follows that the grid current curve does not, like the anode current curve, approximate to a parabola, but is in fact much more nearly an exponential curve.

7. Limiting Condition when Signal is Indefinitely Strong.

The case of a very strong signal (much more than one volt) is met with in relay reception; but it is of interest more especially with reference to ordinary heterodyne reception (discussed in Section 12). Even if it were feasible to determine accurately the higher differential coefficients, and so calculate the rectified current from the expansion in Section 5, there would not be much practical interest in doing so here. For with very large alternating grid potential the rectification tends to become "perfect"; that is to say, that whereas during the positive half-cycle large currents flow through the detector, during the negative half the currents are negligible in comparison.

If the maximum value of the alternating P.D. applied to the detector, now assumed to be sinusoidal, is E, the mean value during a half-wave is $(2/\pi)E = (2\sqrt{2}/\pi)\mathcal{E}$, where $\mathcal{E}$ is the R.M.S. value. If E is so large that we can take the characteristic as sensibly a straight line over the great majority of the positive half-period, then the mean current during that half-period tends towards $(2\sqrt{2}/\pi)\mathcal{E} . di/dv$, and the mean current over the whole period (or any number of whole periods) is approximately $(\sqrt{2}/\pi)\mathcal{E} . di_a/dv_g = 0{\cdot}45\mathcal{E} . di_a/dv_g$.

Complete rectified current curve for anode rectification.—We have seen that the rectified current curve starts as the parabola $I_r = 16\mathcal{E}^2$ micro-amperes (Section 6), and must, if saturation is not approached, finally become asymptotic to the line

$$I_r = 0{\cdot}45\mathcal{E} . di_a/dv_g$$

Fig. 9 shows the observed curve for an "R" triode in which $di_a/dv_g = 200$ micro-amperes per volt, together with the straight line $I_r = 0{\cdot}45\mathcal{E} . di_a/dv_g = 90\mathcal{E}$ micro-amperes, which the observed curve approaches as $\mathcal{E}$ increases.

The same argument applies to grid rectification with large applied signal. But such rectification cannot become apparent in the anode circuit without the insertion into the grid circuit of a condenser or resistance, or both. Large rectified grid currents with fixed mean

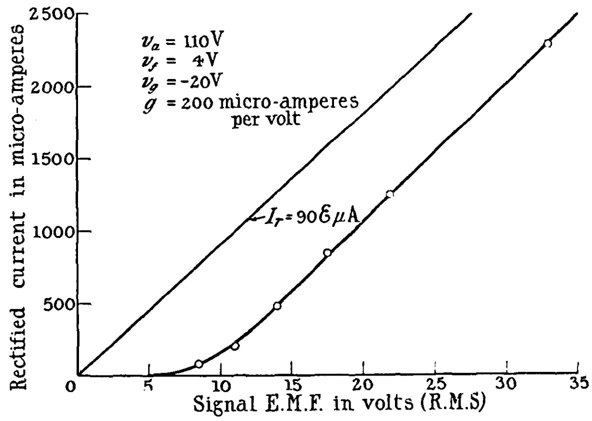

Fig. 9.—Anode rectification.

grid potential are consequently without practical interest; and no experimental curves have been taken for this case.

8. Cumulative Grid Rectification.

In the method which goes under this name the grid is partially insulated by the insertion of a "leaky" condenser, as in Fig. 10. The signal current is now the change of mean anode current accompanying change

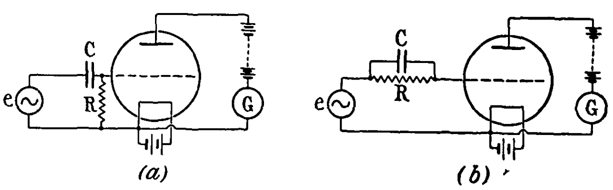

Fig. 10.

of mean grid potential due to rectification in the grid circuit. The method is well known, but its quantitative theory has not been clearly set out. It is best approached graphically, as the significance of the necessary approximations made is then more clearly appreciated. The two methods of connection shown in the figure are electrically equivalent, and in each case the steady non-signal grid potential is found from the simultaneous solution of the equations

$$i_g = f(v_g) \text{ and } i_g = -v_g/R$$

The shape of the grid current curve $f(v_g)$ is such that its analytical treatment is impracticable, and the solution can best be found graphically as shown in Fig. 11. The steady grid potential is given by OA.

the abscissa of the point of intersection B of the grid and resistance characteristics.

The function of the condenser in the two methods of connection is slightly different. In Fig. 10 (*a*) its function is to interrupt the circuit RC*e* so far as steady currents are concerned; and C is supposed to be so large that the alternating current flowing through it produces no sensible alternating P.D. across it. In other words the full signal E.M.F. may be considered

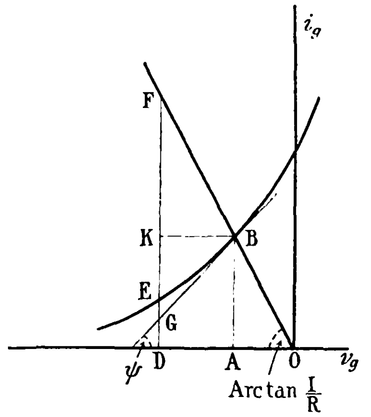

FIG. 11.—Cumulative grid rectification.

to be applied between grid and filament. In Fig. 10 (*b*) the function of C is to provide a high-frequency shunt across R, so that again the full signal E.M.F. may be considered to be applied between grid and filament. The alternating potential of the grid makes the mean grid current increase; and since, in either method of connection, this increased current must flow through R, the mean grid potential is thereby lowered. The

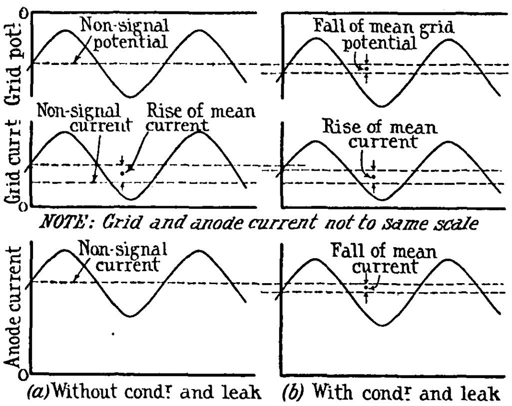

FIG. 12.

accompanying reduction of mean anode current is utilized to operate an indicating instrument.

The effect in the anode circuit of the events in the grid circuit are well exhibited in Fig. 12. Fig. 12 (*a*) depicts grid potential, grid current and anode current in the absence of the condenser and resistance: the signal produces no change of mean grid potential or anode current. Fig. 21 (*b*) depicts the same quantities, when the steady state has been reached, with the cumulative grid method of connection: there is now a decrease of mean grid potential; still an increase in mean grid current [though the increase is less than in Fig. 12 (*a*)]; and a decrease in mean anode current. All these changes in mean values can easily be found graphically from Fig. 11 as follows. Suppose that the E.M.F. applied to the grid produces a rectified current I_r. Find a point F on the line OB such that the intercept FE of the ordinate FD between the resistance line and the grid curve is equal to I_r on the scale of the diagram. The new mean grid current is DF, which may be analysed into two parts, FE called the rectified current, and FD the steady grid current associated with the new mean grid potential OD. The fall of mean grid potential is evidently DA, and hence the fall of mean anode current is DA $\times\, di_a/dv_g \equiv g\,.\,$DA. The increase in mean grid current shown in Fig. 12 (*a*) is FE, and in Fig. 12 (*b*) is the smaller amount FK.

The above graphical method is applicable with signals of any strength, and may be used for calculating by trial and error the signal current for any given signal. With weak signals, on the other hand, an approximate formula for the signal current will now be derived. We have found theoretically, and verified experimentally, that with constant mean grid potential the grid rectified current is very closely $\frac{1}{2}\mathcal{E}^2\,.\,d^2i_g/dv_g^2$ provided that $\mathcal{E}$ does not exceed about 0·5 volt. Hence if DA (Fig. 11) is small so that the value of d^2i_g/dv_g^2 at E does not much differ from its value at B, we can say that FE $= \frac{1}{2}\mathcal{E}^2\,.\,d^2i_g/dv_g^2$, where d^2i_g/dv_g^2 is the value of the second derived function at B, i.e. at the state point, determined by R, before the signal arrived. At B draw a tangent to the grid curve, and let it cut FD at G and make an angle ψ with the horizontal. Suppose that DA is so small that the tangent departs very little from the curve between A and D and we can take GF as sensibly equal to EF. Then we have

$$\begin{aligned}\mathrm{DG} &= \mathrm{DF} - \mathrm{FG} \\ &= \frac{1}{R}\,.\,\mathrm{DO} - \mathrm{EF} \\ &= \frac{1}{R}\,.\,\mathrm{DO} - \frac{\mathcal{E}^2}{2}\frac{d^2i_g}{dv_g^2}\end{aligned}$$

Now $$\mathrm{DA} = (\mathrm{BA} - \mathrm{DG})\cot\psi$$

$$\therefore\ \mathrm{DA} = \left(\frac{1}{R}\,.\,\mathrm{AO} - \frac{1}{R}\,.\,\mathrm{DO} + \frac{\mathcal{E}^2}{2}\frac{d^2i_g}{dv_g^2}\right)\cot\psi$$

$$= \left(-\frac{1}{R}\,.\,\mathrm{DA} + \frac{\mathcal{E}^2}{2}\frac{d^2i_g}{dv_g^2}\right)\cot\psi$$

that is, $$\mathrm{DA}\left(\frac{1}{R} + \tan\psi\right) = \frac{\mathcal{E}^2}{2}\frac{d^2i_g}{dv_g^2}$$

$$\therefore\ \mathrm{DA} = \frac{\dfrac{\mathcal{E}^2}{2}\dfrac{d^2i_g}{dv_g^2}}{\dfrac{1}{R} + \dfrac{di_g}{dv_g}}$$

This gives signal current

$$S = g\,.\,\mathrm{DA} = \frac{\dfrac{\mathcal{E}^2}{2}\dfrac{d^2i_g}{dv_g^2}}{\dfrac{1}{R} + \dfrac{di_g}{dv_g}}\cdot\frac{di_a}{dv_g} \quad . \quad . \quad . \quad (2)$$

This formula is not of much service in calculating the signal current from any except extremely weak signals, because we do not know what value to attach to d^2i_g/dv_g^2 until we know the rectified current. The appropriate value is the value at the undetermined point E, not at the non-signal state point B. With very weak signals disregard of any difference between these two values is no doubt a second-order inaccuracy; but with a signal of 0·5 volt (R.M.S.), when $R = 7\cdot2$ megohms the observed signal current (Fig. 15) indicates that the grid potential has fallen by 0·25 volt; and reference to Figs. 8 and 2 shows that this has reduced d^2i_g/dv_g^2 by about 40 per cent.

If we suppose that $\mathcal{E}$ is sufficiently small to permit us to use the non-signal state point value of d^2i_g/dv_g^2, the error introduced by our approximation of ignoring EG in FG = FE + EG can be reduced by putting

$$\mathrm{EG} = \tfrac{1}{2}(\mathrm{DA}) \times \frac{d^2i_g}{dv_g^2}$$

This changes formula (2) into the more accurate formula

$$S = \frac{\dfrac{\mathcal{E}^2}{2}\dfrac{d^2i_g}{dv_g^2}}{\dfrac{1}{R}+\dfrac{di_g}{dv_g}} \cdot \frac{1}{2}\left\{1 + \frac{\dfrac{\mathcal{E}^2}{2}\left(\dfrac{d^2i_g}{dv_g^2}\right)^2}{\left(\dfrac{1}{R}+\dfrac{di_g}{dv_g}\right)^2}\right\} \cdot \frac{di_a}{dv_g} \quad . \quad (3)$$

9. Experimental Verification of Section 8.

As an example of the graphical method of predicting the signal current, let us take the following numerical case. When using a grid leak of 0·43 megohm, a signal E.M.F. of 0·38 volt (R.M.S.) was observed to produce a signal current of 21 μA. Fig. 2 shows that with this value of grid leak the non-signal grid potential is 0·47 volt. The experiments from which Fig. 8 is produced showed that if $v_g = -0\cdot47$ volt an applied E.M.F. of 0·38 volt (R.M.S.) produces a rectified current of 0·5 μA. Fig. 2, again, shows that this increase of grid current brings the grid potential to $-0\cdot57$ volt, a reduction of 0·1 volt from the non-signal value. In this triode with the anode potential in use $di_a/dv_g = 290$ micro-amperes per volt, so that the signal current caused by a change of grid potential of 0·1 volt is 29 μA. This is some 30 per cent above the observed value, 21 μA. If, however, proceeding by trial and error, we now guess that the grid potential would be reduced to (say) $-0\cdot55$ volt instead of $-0\cdot57$, the rectified current shown by Fig. 8 would be only 0·45 μA instead of 0·51 μA. Fig. 2 shows that this increase of 0·45 μA in grid current would lower the grid potential almost exactly to $-0\cdot55$ volt, a decrease due to the signal of 0·8 volt; and the consequent signal current would be 23 μA. The agreement between this and the observed 21 μA is no doubt sufficiently close to verify the above theory of the action of the rectifier with fairly strong signals.

Using the balance method of Section 4 with a highly sensitive galvanometer capable of measuring $\frac{1}{50}$ μA, it is possible to observe the signal current in the anode circuit produced by a signal on the grid of only 0·01 volt.* Figs. 13 and 14 show the observed signal current with signal E.M.F.'s up to 0·12 volt (R.M.S.); and the dotted curves in the same figure show the values calculated from formula (2) above by using the values of d^2i_g/dv_g^2, etc., found graphically from the triode characteristics. The agreement is very close, and shows that the expressions (2) and (3) are sensibly accurate as long as E does not exceed 0·1 volt.

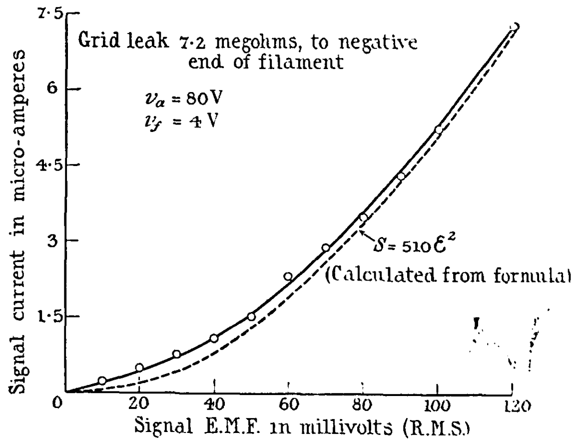

Fig. 13.—Cumulative grid rectification.

The curve of Fig. 15 shows signal current plotted against applied E.M.F. for a grid leak of 7·2 megohms connected to the negative end of the filament. The applied signals are fairly strong, so that the curves do not agree with those given by either of the foregoing formulæ (2) and (3), for the reasons already stated. It will also be noticed that the signal current increases less rapidly than as the square of the applied E.M.F.

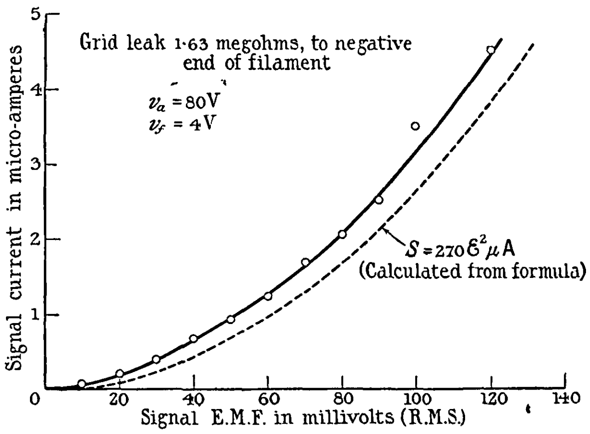

Fig. 14.—Cumulative grid rectification.

Choice of grid leak and state point on grid characteristic.—With any value of grid leak resistance any desired state point may be attained by connecting the outer end of the grid leak to a point at suitable potential, positive or negative, with respect to the

* It may be remarked in passing that we are then observing changes in anode current of one part in 20 or 30 thousand—a testimony to the convenience of the balance arrangement employed.

negative end of the filament. For any assigned state point, the expression $\frac{d^2i_g/dv_g^2}{1/R + di_g/dv_g}$, to which we have seen the signal current is proportional, can be increased

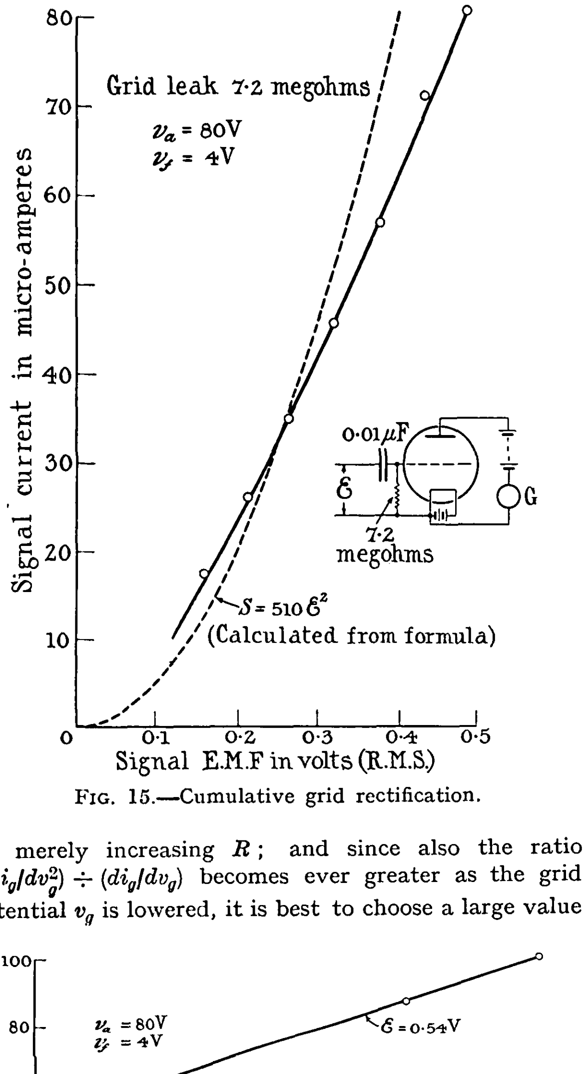

FIG. 15.—Cumulative grid rectification.

by merely increasing R; and since also the ratio $(d^2i_g/dv_g^2) \div (di_g/dv_g)$ becomes ever greater as the grid potential v_g is lowered, it is best to choose a large value

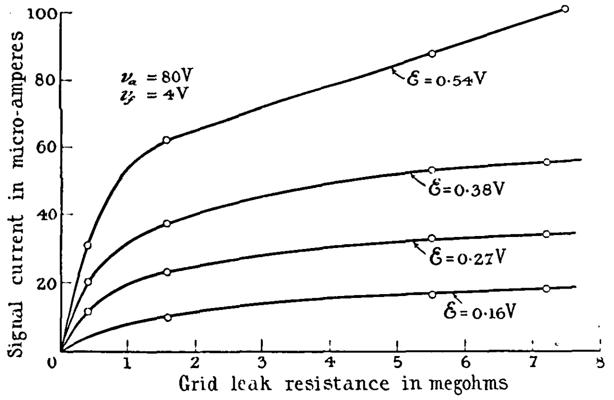

FIG. 16.—Cumulative grid rectification. Effect of size of grid leak on signal current.

for R, and to connect the outer end of the leak to the negative in preference to the positive end of the filament.

Fig. 16 shows the observed relation between signal current and grid leak resistance R for four strengths of constant applied signal: the importance of keeping R not less than (say) 5 megohms is clearly shown. Fig. 17 shows the relation, with a large value of R and for a constant applied signal, between signal current and the non-signal grid potential. In this case it may be seen that if the leak is connected to a point at a potential even as high as + 20 volts, the signal current is only some 25 per cent less than when it is connected to the negative end of the filament. Thus the important point is to make R large rather than select it to arrive

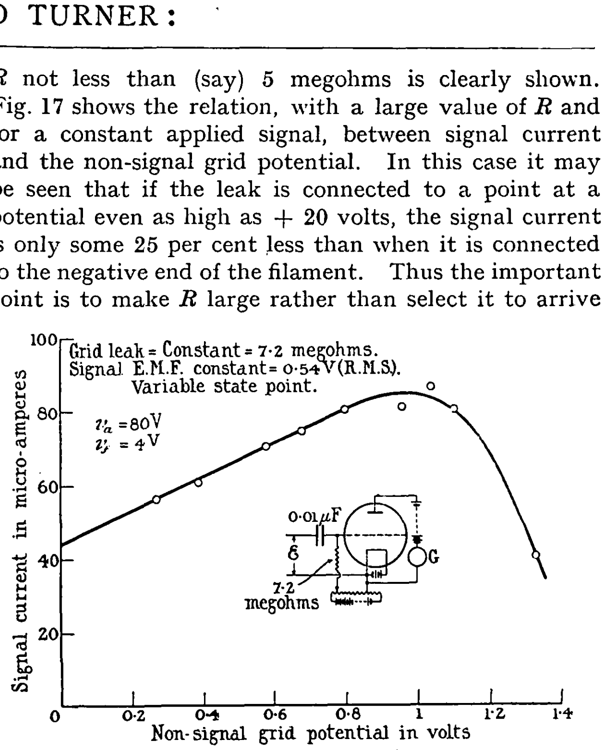

FIG. 17.—Cumulative grid rectification.

at any particular state point. In fact it matters little whether the leak is connected to the positive or negative end of the filament, provided only that R is large, say at least 5 megohms.

Choice of grid condenser.—It has already been pointed out that the function of the condenser is that of a high-frequency shunt. In order to keep the time-constant of the circuit as low as possible, the capacity should be kept as small as is compatible with its efficient

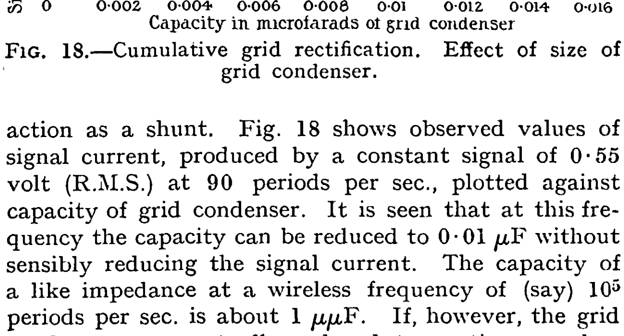

FIG. 18.—Cumulative grid rectification. Effect of size of grid condenser.

action as a shunt. Fig. 18 shows observed values of signal current, produced by a constant signal of 0·55 volt (R.M.S.) at 90 periods per sec., plotted against capacity of grid condenser. It is seen that at this frequency the capacity can be reduced to 0·01 μF without sensibly reducing the signal current. The capacity of a like impedance at a wireless frequency of (say) 10^5 periods per sec. is about 1 $\mu\mu$F. If, however, the grid condenser were actually reduced to so tiny a value, it would be less than the internal capacity of the triode; and Fig. 18 refers only to the case of a grid condenser great in comparison with the internal capacity. No doubt the condenser may safely be reduced to (say) 50 $\mu\mu$F ; and as the equivalent resistance of grid and leak in parallel is about 1·5 megohms, the time-constant of the circuit can be reduced to about 75 micro-seconds. With these values, if signals of 3 000 metres wave-length are being received, the signal current will attain the value given by formula (2) in about 40 periods.

In the circuit of Fig. 10 (*b*) it is possible to omit the condenser altogether; and formula (2) may be used to calculate the signal current in this case. A signal $\mathcal{E}$, applied between the outer end of R and the filament, produces a potential difference $\mathcal{E}'$ between

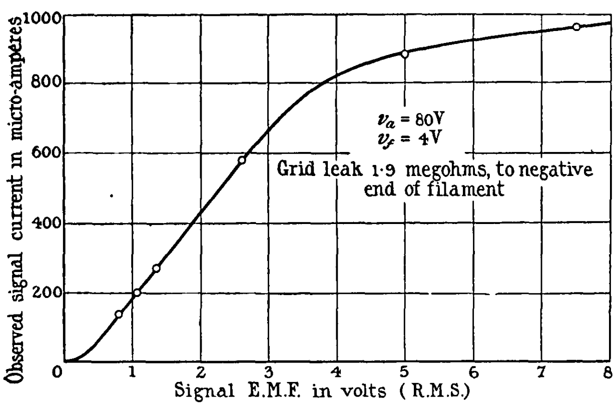

FIG. 19.—Cumulative grid rectification.

grid and filament, which, if we can neglect the internal capacity of the grid, is

$$\mathcal{E}' = \mathcal{E} \cdot \frac{r}{R + r} \quad \text{where} \quad r \equiv 1 \Big/ \frac{di_g}{dv_g} \quad .$$

The signal current is then

$$S = \frac{\mathcal{E}'^2}{2} \cdot \frac{R\,r}{R + r} \cdot \frac{d^2 i_g}{dv_g{}^2} \cdot g$$
$$= \frac{\mathcal{E}^2}{2} \cdot \frac{Rr^2}{(R + r)^3} \cdot \frac{d^2 i_g}{dv_g^2} \cdot g$$

This is easily shown to be a maximum for choice of R when $R = \frac{1}{2}r$, giving an optimum

$$S = \frac{4}{27} \frac{\mathcal{E}^2}{2} \frac{d^2 i_g}{dv_g^2} \Big/ \frac{di_g}{dv_g} \cdot g \, .$$

Taking 2·5 as the appropriate value of $(d^2 i_g/dv_g^2) \div (di_g/dv_g)$ in the " R " triode, we get

$S = 55\mathcal{E}^2$ micro-amperes if $g = 300$ micro-amperes per volt.

Cumulative grid rectification with very strong signals.—We have seen that the rectified current curve is a parabola for signals not exceeding 0·1 volt. Fig. 19 shows that when the signal lies between 0·25 volt and 3 volts the curve is found to be approximately the straight line $S = 210\mathcal{E}$ micro-amperes; and above 3 volts the curve finally becomes asymptotic to the line $S = 1\,000$ micro-amperes. The straight portion is easily accounted for as due to " perfect " rectification; and the final convex portion is explained by the two facts (i) that as the mean grid potential becomes more and more negative, the grid current flows for a decreasing part instead of the whole of the positive half-cycle, and (ii) increasing negative grid potentials tend towards the lower curved portion of the anode current characteristic and so again lessen the signal current.

10. Comparison between Anode and Cumulative Grid Rectification.

The signal currents produced by a sustained signal $\mathcal{E}$ have been found to be given, according to the value of $\mathcal{E}$, approximately by the several formulæ collected in Table 2.

TABLE 2.

Strength of signal $\mathcal{E}$ volts (R.M.S.)	Signal current in micro-amperes, with		Ratio
	Anode rectification	Cum. grid rectn.	
$<0{\cdot}1$	$16\mathcal{E}^2$	$500\mathcal{E}^2$	30
0·25 to 2	$16\mathcal{E}^2$	$210\mathcal{E}$	$6\frac{1}{2}$ when $\mathcal{E}=2$
>10	$100\mathcal{E} - 800$	1 000	1 when $\mathcal{E}=20$

Thus cumulative grid rectification is for weak signals some 30 times, for moderate signals some $6\frac{1}{2}$ times, and for strong signals about once, as sensitive as anode rectification. Such marked superiority as 20 or 30 times is likely to be questioned by those who have used both rectifying arrangements in wireless receivers, and some explanation of this apparent discrepancy between theory and practice is offered below.

First, it is to be remembered that the comparison we have drawn is for continuous-wave working without heterodyne; so that although with weak signals the signal current may indeed be some 20 or 30 times as great with cumulative grid as with anode rectification, there is no means of making this signal current declare itself. The current is too weak to operate a relay, and cannot affect a telephone in the absence of a heterodyne. With stronger signals, or with any signals in the presence of a heterodyne (as will be seen in Section 14), the superiority is reduced from 30 to a much smaller figure.

Secondly, Table 2 is not directly applicable to spark signals; for the cumulative grid rectifier, as its name implies, has a time-constant, whereas the anode rectifier has none. Unless the time-constant CR of the grid condenser and its leak is negligibly small in comparison with the duration of the signal train—and previous remarks on this time-constant show that with short waves it is not—the grid cannot reach as low a potential as it would finally reach if the P.D. were sustained in amplitude: and it is this final value which is given in Table 2. It is shown in Section 13 that this effect may not be very considerable with heterodyne reception of continuous-wave signals; but its importance in the case of spark signals is not easily estimated. In whatever degree this occurs, however, the effect must cause the superiority to fall below the ratios shown in Table 2.

It has been seen that the grid rectifier obtained by omitting the grid condenser, and therefore without time-constant, can with optimum adjustment give a signal current $S = 55\mathcal{E}^2$ micro-amperes, as compared with the $S = 16\mathcal{E}^2$ micro-amperes given by an anode rectifier—a superiority of $3\frac{1}{2}$ times. Perhaps these can be taken as approximating to the conditions obtaining in the cumulative grid rectifier with highly damped short waves when the grid condenser is much too large. This leads to $3\frac{1}{2}$ as a lower limit of cumulative grid superiority with weak spark signals. Rough comparison tests have been made with spark signals

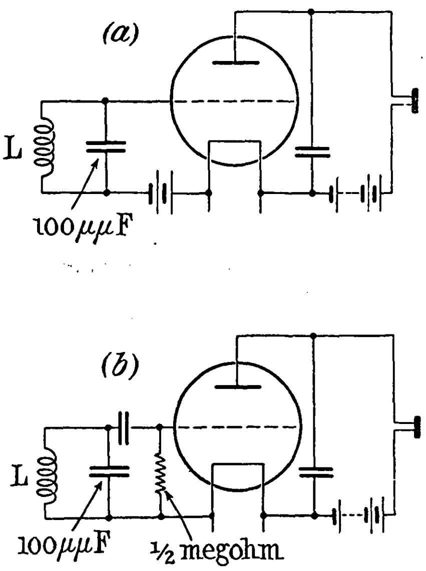

FIG. 20.

from the Eiffel Tower, the two rectifier arrangements being interchangeable by press keys. It was observed that the cumulative grid rectifier was quite definitely superior to the anode rectifier, though not very markedly.

A third reason why the ratios of Table 2 are not substantiated in wireless practice lies in the damping which is necessarily produced in the cumulative grid but not in the anode rectifier arrangement. Owing to this damping, any given signal E.M.F. in the receiving antenna cannot be made to produce as large a signal P.D. on the grid in the former as in the latter arrangement. The extent to which this fact is felt depends on various factors, but that it can be large is easily seen by considering a reasonable numerical example. We will assume that a certain power, say 10^{-10} watt, is fed into the oscillatory circuit with each arrangement;* that the wave-length is 3 000 metres; that the decrement of the oscillatory circuit alone is 0·02; and that for practical reasons the capacity of the condenser cannot be reduced below $100\,\mu\mu$F and is given that value in both cases. With anode rectification [Fig. 20 (*a*)], with the grid at sufficiently low a potential as to produce negligible damping, the signal P.D. across the coil L would be 0·016 volt. With cumulative grid rectification [Fig. 20 (*b*)], if the total leak resistance is (say) $\frac{1}{2}$ megohm, the decrement of the circuit rises to 0·12, and the same signal power fed in produces across L a P.D. of only 0·0064 volt. The ratio of signal currents in the telephone would therefore be not the 30 of Table 2, but $30 \times (0{\cdot}0064/0{\cdot}016)^2 \doteqdot 5$. It is to be noticed that this disability of cumulative grid rectification due to damping introduced by the rectifier is obviated if the rectifier is connected across the resistance in the anode circuit of an amplifying triode inserted between the oscillatory circuit and the rectifier.

* In so far as this assumption is untrue, it unduly favours cumulative grid rectification.

11. Anode Rectification with Heterodyne of Moderate Strength.

It has been shown (Section 6) that the foot of the anode-current/grid-potential characteristic may be expressed by the equation

$$i_a = \alpha + \beta v_g + \gamma v_g^2$$

the rate of change of slope, $d^2 i_a/dv_g^2$, being approximately a constant, 2γ. With the signal potential difference $e = a \sin pt$ and the superposed heterodyne potential difference $b \sin qt$ we have

$$v = a \sin pt + b \sin qt$$

Hence

$$\begin{aligned} i_a &= \alpha + \beta(a \sin pt + b \sin qt) \\ &\qquad + \gamma(a^2 \sin^2 pt + b^2 \sin^2 qt + 2ab \sin pt \sin qt) \\ &= \alpha + \beta(a \sin pt + b \sin qt) \\ &\qquad + \frac{\gamma a^2}{2}(1 - \cos 2pt) + \frac{\gamma b^2}{2}(1 - \cos 2qt) \\ &\qquad\qquad + \gamma ab\{\cos (p - q)t - \cos (p + q)t\} \end{aligned}$$

The effects on the anode circuit of the signal *e* accordingly are:

(i) To increase the mean current by $\frac{1}{2}\gamma b^2$;
(ii) To introduce high-frequency currents of frequencies corresponding to $2q$ and $(p + q)$; and
(iii) To introduce a low-frequency current

$$\gamma ab \cos (p - q)t$$

Item (iii) is responsible for the acoustic effect in the telephone inserted in the anode circuit, and is accordingly here termed the signal current. The total range, $2\gamma ab$, of this acoustic fluctuation is called the magnitude of the signal current.

The signal current, then, is of magnitude

$$2\gamma ab = ab \frac{d^2 i_a}{dv_g^2} = \sqrt{2}\mathcal{E} b \frac{d^2 i_a}{dv_g^2}$$

The same result may be reached as follows. The amplitude of P.D. across the rectifier varies between $(b + a)$ and $(b - a)$ at an acoustic frequency small compared with the wireless frequency $p/2\pi$. Hence

(Section 5) the high-frequency mean of the anode current ranges from

$$\tfrac{1}{4}(b+a)^2 \,.\, d^2i_a/dv_g^2 \quad \text{to} \quad \tfrac{1}{4}(b-a)^2 \,.\, d^2i_a/dv_g^2$$

The signal current is therefore

$$\frac{1}{4}\frac{d^2i_a}{dv_g^2}\left\{(b+a)^2-(b-a)^2\right\} = ab\,\frac{d^2i_a}{dv_g^2} \text{ as before.}$$

But the formulæ

$$\text{mean } i_a = \tfrac{1}{4}(b \pm a)^2 \,.\, d^2i_a/dv_g^2$$

are applicable only for values of $\mathcal{E} = (b+a)/\sqrt{2}$ up to about 2 volts, and over this range for an " R " triode $d^2i_a/dv_g^2 = \text{constant} \doteqdot 32$ micro-amperes per volt per volt. Hence the best signal current with a weak signal (namely, when $b + a \doteqdot b$ is adjusted to be about $2\sqrt{2}$ volt) is

$$S = 128\mathcal{E} \text{ micro-amperes.}$$

12. Anode Rectification with Very Strong Heterodyne.

In practice it is entirely feasible to employ a heterodyne of any strength; and by using a high anode

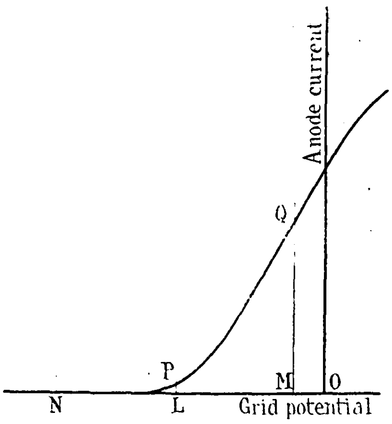

Fig. 21.

potential (e.g., 100–200 volts) the grid can be maintained permanently negative in potential, so avoiding any inconvenience from grid damping despite the large fluctuation of grid potential. Suppose the mean grid potential OL (Fig. 21) is chosen to bring the point P to the foot of the characteristic; and the heterodyne is strong enough to sweep over the curve between N and Q, NP being nearly all horizontal and PQ being nearly all straight at the slope $di_a/dv_g \equiv g$. The rectification then becomes nearly " perfect," and the rectified current as found in Section 7 approaches $0{\cdot}45g$ LM$/\sqrt{2}$. The amplitude LM fluctuates at the acoustic rate between $(b+a)$ and $(b-a)$, so that the signal current tends towards the value $0{\cdot}45g \times \sqrt{2}a = 0{\cdot}90g\mathcal{E}$ as the heterodyne strength is increased, provided that the filament emission is adequate to prevent Q from reaching the convex upper bend of the curve.

In most " R " triodes g is about 300 micro-amperes per volt per volt; so that with a very strong heterodyne we have signal current

$$\begin{aligned} S &= 0{\cdot}90 \times 300\mathcal{E} \text{ micro-amperes} \\ &= 270\mathcal{E} \text{ micro-amperes} \end{aligned}$$

This value is to be compared with the value $128\mathcal{E}$ found in the last section for a heterodyne amplitude of 2 volts. It is thus clearly worth while to use a heterodyne of adequate strength. Referring to Fig. 9, with heterodyne reception it is the slope and not the ordinate of the curve which should be made as large as possible; so that a heterodyne strength of some 15 volts would be suitable, and would give a sensitivity slightly exceeding the above $270\mathcal{E}$ micro-amperes.

13. Cumulative Grid Rectification with Heterodyne.

Owing both to uncertainty as to the equation of the i_g/v_g characteristic and to difficulties of the integration involved in the cumulative action, it is not possible to write down an expression for the instantaneous anode current, as has been done in the case of anode rectification. We have seen in Section 9 that the time-constant of the circuit can be reduced to about 75 micro-seconds, and as the acoustic variation of high-frequency mean anode current is relatively slow (with period, e.g. of 1 000 micro-seconds), it is probably sufficiently accurate to neglect the time effect of the condenser and take the high-frequency mean of grid potential at any instant as sensibly equal to the final value it would assume under continued application of the signal amplitude obtaining at that instant.

With this assumption, the high-frequency mean of anode current fluctuates between

$$\frac{\dfrac{d^2i_g}{dv_g^2}}{\dfrac{1}{R}+\dfrac{di_g}{dv_g}} \cdot \frac{di_a}{dv_g} \cdot \frac{(b+a)^2}{4} \equiv K\frac{(b+a)^2}{4}, \text{ say,}$$

and

$$K\frac{(b-a)^2}{4}$$

Hence signal current is $S = Kab = \sqrt{2}Kb\mathcal{E}$.

The value of K already found (Section 9) for a small potential difference on the grid is, for an " R " triode, 1 000; and if we take $b = 0{\cdot}14$ volt, which is the highest value for which the formula is applicable, we get

$$\begin{aligned} S &= \sqrt{2} \times 1\,000 \times 0{\cdot}14\mathcal{E} \text{ micro-amperes} \\ &= 200\mathcal{E} \text{ micro-amperes} \end{aligned}$$

If, on the other hand, b lies between 1 volt and 5 volts, the mean anode current (from Fig. 19) fluctuates between $210(b+a)/\sqrt{2}$ and $210(b-a)/\sqrt{2}$. Hence

$$\begin{aligned} S &= 210\sqrt{(2)}a \text{ micro-amperes} \\ &= 420\mathcal{E} \text{ micro-amperes} \end{aligned}$$

14. Comparison between Anode and Cumulative Grid Rectification with Heterodyne.

The signal currents produced by a sustained signal $\mathcal{E}$ have been found to be given, according to the strength of the heterodyne P.D., approximately by the several formulæ collected in Table 3.

TABLE 3.

Strength of heterodyne (volts, R.M.S.)	Signal current in micro-amperes, with Anode rectification	Cum. grid. rectn.	Ratio
0·1	$6\mathcal{E}$	$200\mathcal{E}$	30
2	$128\mathcal{E}$	$420\mathcal{E}$	3
> 15	$270\mathcal{E}$	Very small	Very small

Hence it can be seen that with suitable values of the heterodyne strength, and without allowing for the damping introduced in the cumulative grid rectifier, this would be distinctly superior to the anode rectifier. But in circuits where the damping must be taken into account, there is probably little to choose as regards sensitiveness.

15. Anode Rectification with Damped Waves.

Let the incoming signal applied to the detector be now

$$e = a\epsilon^{-mt}\sin pt$$

As before, we take the anode characteristic to be

$$i_a = \alpha + \beta v_g + \gamma v_g^2$$

and we shall suppose that the damping exponent m is very small compared with p. Then the anode current at any instant is

$$\begin{aligned} i_a &= \alpha + \beta a\epsilon^{-mt}\sin pt + \gamma a^2\epsilon^{-2mt}\sin^2 pt \\ &= \alpha + \beta a\epsilon^{-mt}\sin pt + \tfrac{1}{2}\gamma a^2\epsilon^{-2mt}(1 - \cos 2pt) \end{aligned}$$

Integrating this expression over the number N of complete periods per spark, we obtain

$$\begin{aligned} \int_0^{\frac{2\pi N}{p}} i_a dt &= \beta a\left[\frac{-\epsilon^{-mt}\cos pt}{p}\right]_0^{\frac{2\pi N}{p}} + \frac{\gamma a^2}{2}\left[\frac{-\epsilon^{-2mt}}{2m}\right]_0^{\frac{2\pi N}{p}} + \frac{\gamma a^2}{2}\left[\frac{-\epsilon^{-2mt}\sin 2pt}{2p}\right]_0^{\frac{2\pi N}{p}} \\ &= -\frac{\beta a}{p}\left(\epsilon^{-\frac{2\pi Nm}{p}} - 1\right) - \frac{\gamma a^2}{4m}\left(\epsilon^{-\frac{4\pi Nm}{p}} - 1\right) \\ &= \frac{\beta a}{2\pi n}\left(1 - \epsilon^{-\frac{n\delta}{x}}\right) - \frac{\gamma a^2}{4n\delta}\left(\epsilon^{-\frac{2n\delta}{x}} - 1\right) \end{aligned}$$

where n is the frequency, δ the decrement, and x the number of sparks per second.

The first of these two terms is the net quantity of electricity which would be passed through a conductor of conductance β which is independent of the current (i.e. following Ohm's law); it is only not zero on account of the damping. The second term expresses the rectification due to the curvature of characteristic, and with practical signal strengths vastly exceeds the first, which may therefore be ignored. Hence

$$\begin{aligned} \text{mean } i_a &= \frac{\gamma a^2}{4}\left(1 - \epsilon^{-\frac{2n\delta}{x}}\right)\frac{x}{n\delta} \text{ approximately} \\ &= \frac{a^2}{4}\frac{d^2 i_a}{dv_g^2}\left(1 - \epsilon^{-\frac{2n\delta}{x}}\right)\frac{x}{2n\delta} \end{aligned}$$

Now suppose $n = 10^5$ periods per sec., $\delta = 0 \cdot 1$ and $x = 500$; then $n\delta/x = 20$, and

$$\begin{aligned} \text{mean } i_a &= \frac{a^2}{4}\frac{d^2 i_a}{dv_g^2}(1 - \epsilon^{-40}) \times \frac{1}{40} \\ &= \frac{1}{40}\left(\frac{a^2}{4}\frac{d^2 i_a}{dv_g^2}\right) \text{ approximately} \end{aligned}$$

Hence the signal current is only one-fortieth of what it would have been had the wave been undamped and of maximum value equal to the first maximum of the damped train.

Substituting the value of $d^2 i_a/dv_g^2$ in the "R" triode, namely, 32 micro-amperes per volt per volt, we have signal current $S = a^2/5$ micro-amperes. If the wave had been undamped, however, and a suitable heterodyne had been used, it follows from Section 12 that $S = 270\mathcal{E} \doteqdot 200a$. Hence the ratio of signal current when working with heterodyne to signal current when receiving a damped train of the frequency and decrement already stated is $(200a) \div a^2/5 = 1\,000/a$. If, for example, a has the value $0 \cdot 1$ volt, this ratio is 10 000.

Probably a fairer comparison is between undamped waves and spark trains having the same R.M.S. value, rather than the same initial amplitude. The R.M.S. value over one spark period of our damped train $e = a\epsilon^{-mt}\sin pt$ is

$$\begin{aligned} \mathcal{E} &= \tfrac{1}{2}a\sqrt{(x/n\delta)} \\ &= \tfrac{1}{2}a\sqrt{(1/20)} \text{ in our numerical instance} \\ &= 0 \cdot 11a \end{aligned}$$

The signal current with a heterodyned undamped signal of this strength is therefore

$$\begin{aligned} S &= 270\mathcal{E} \\ &= 270 \times 0 \cdot 11a = 30a \end{aligned}$$

The ratio $1\,000/a$ found for equal initial amplitudes thus becomes, for equal R.M.S. values,

$$30a/(a^2/5) = 150/a$$

With $a = 0 \cdot 1$ volt as before, this ratio becomes 1 500.

In the largeness of this figure, it is suggested, lies the chief explanation of the much superior ranges of continuous-wave transmitters as compared with spark transmitters of equal radiated power.

16. Effect of Impedance in Anode Circuit.

In all the foregoing analysis it has been assumed that the anode potential is unchanged by the advent of the signal. In any well-designed receiver this cannot be the case; for the impedance to the signal current offered by the telephone (or other indicating instrument) must be given an appropriate value, which is not negligible. The effect of this impedance, which has hitherto been tacitly ignored, is to reduce the values of the signal currents from the values calculated and observed in the foregoing sections. We proceed to find the correction which must be applied to the signal current S when there is no appreciable impedance of the indicating instrument T (Fig. 22) in order to arrive at the actual signal current S' when there is an impedance X.

It is to be noticed, first, that it is not an impedance to high-frequency currents with which we are concerned, for this is in practice reduced sensibly to zero by the shunting condenser C; it is the impedance (X) of T and

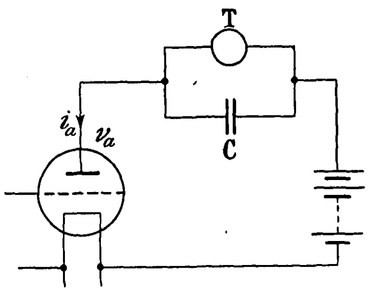

FIG. 22.

C in parallel to the acoustic fluctuations constituting the signal currents. In general terms, we have

$$S' = S - XS'\frac{di_a}{dv_a}$$

$$\therefore S' = \frac{S}{1 + X \cdot \frac{di_a}{dv_a}}$$

The correcting factor $1/(1 + X \cdot di_a/dv_a)$ is therefore to be applied to any of the foregoing calculated or observed signal currents in order to allow for the effect of the impedance X of the particular indicating instrument for the particular signal current, and with the particular anode-anode slope conductance di_a/dv_a of the triode in use.

The magnitude of X is, of course, under the control of the designer. It should usually be chosen to make S'^2X a maximum, in which case $X = 1\bigg/\frac{di_a}{dv_a}$; that is, the impedance should be equal in magnitude to the anode-anode slope resistance of the triode. With this optimum condition, we have

$$S'_{opt.} = \tfrac{1}{2}S$$

In the case of cumulative grid rectification, the appropriate value of di_a/dv_a is usually that obtaining over the straight region of the characteristics—about 30 micro-amperes per volt per volt in the " R " triode; but in the case of anode rectification, where we are necessarily working on the curved region, some lower value depending on the non-signal state point must be taken. This introduces no complication where weak signals are dealt with; but where a large range of grid potential is swept over, some average value of di_a/dv_a must be taken. Since the choice of the precise optimum impedance X is not a matter of much practical moment, it has been thought inadvisable to complicate the investigations in this respect.

DISCUSSION BEFORE THE WIRELESS SECTION, 1 MARCH, 1922.

Dr. W. H. Eccles: First of all I ought to thank the authors for venturing into a somewhat chaotic region and enforcing order therein. A great many papers have been written on this subject, but as far as I can remember there has been no paper covering the same ground in a systematic manner combining experiment and theory so elegantly. The procedure adopted is the ideal one of checking every calculation by a measurement. All the theory is based on the static characteristics, and in particular on those characteristics in which the anode potential is maintained constant. If we take that starting point, there is another way of tackling the theory of rectification which I think is rather simpler and more practical than the use of Taylor's theorem as adopted by the authors. Taylor's theorem expands the function in a series of which the coefficients are differential coefficients at the origin; but instead of using Taylor's theorem we may simply write down that the anode current is of the form:

$$i_a = \alpha + \beta e_g + \gamma e_g^2 + \delta e_g^3 + \dots$$

where i_a is the anode current, e_g is the grid voltage, and the other symbols are functions of the anode voltage, being constants if this is constant.

By taking four or five points these constants can be determined numerically. The grid voltage is of the form

$$e_g = v + E_g \cos \omega t$$

where v is the steady adjusting voltage applied between grid and filament. The undisturbed anode current is obtained by putting $e_g = v$, and if this is subtracted from the general value we obtain

$$\beta E_g \cos \omega t + \gamma(2vE_g \cos \omega t + E_g^2 \cos^2 \omega t) + \delta(3v^2E_g \cos \omega t + 3vE_g^2 \cos^2 \omega t + E_g^3 \cos^3 \omega t) + \dots$$

On integrating over a second of time we get the quantity of rectified electricity flowing round the circuit. The result is

$$\tfrac{1}{2}\gamma E_g^2 + 3(\tfrac{1}{2}\delta v + \epsilon v^2)E_g^2 + \dots$$

This expression agrees in form with the authors' results. Plotting the expression with values of v as abscissæ we can compare with the curves given by the authors. If we neglect third- and fourth-degree terms in E_g we obtain a horizontal straight line. If we then take in the third-degree terms we get another straight line, but sloping. If we then take in the fourth-degree terms, we get a parabola. The fact is that none of these fits perfectly the authors' results. Theirs is really a straight line in the middle—straighter than a parabola, but it is bent at the ends. The authors have taken the extreme case of anode voltage constant; the other extreme case, namely, anode current constant, has not been touched. The case of anode current will arise when a saturated diode is used in series with a triode adjusted for rectifying at either the low-current or high-current rectifying