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Beta Working Paper series 413

BETA publicatie	WP 413 (working paper)
ISBN	
ISSN	
NUR	804
Eindhoven	March 2013

The Time-Dependent Vehicle Routing Problem with Soft Time Windows and Stochastic Travel Times

Duygu Taş*, Nico Dellaert, Tom van Woensel, Ton de Kok

School of Industrial Engineering and Innovation Sciences, Eindhoven University of Technology, P.O. Box 513, 5600 MB, Eindhoven, The Netherlands

Abstract

This paper studies a vehicle routing problem with time-dependent and stochastic travel times. In our problem setting, customers have soft time windows. A mathematical model is used in which both efficiency for service as well as reliability for customers are taken into account. Depending on whether service times are included or not, we consider two versions of this problem. Two metaheuristics are built: a Tabu Search and an Adaptive Large Neighborhood Search. We carry out our experiments for well-known problem instances and perform comprehensive analyses on the numerical results in terms of the computational time and the solution quality. Experiments confirm that the proposed procedure is effective to obtain very good solutions to be performed in real-life environment.

Keywords:
Vehicle routing, Time-dependency, Stochastic travel times, Time windows

1. Introduction

In real-life applications, travel times on individual arcs are stochastic. In addition, vehicle routes are operated in a traffic network which has different levels of congestion depending on the time of the day. Routing customers with respect to a deterministic and static schedule is a strong assumption for real-life environment, leading to inefficient operations. To consider more realistic

* *Corresponding author*
Email address: `d.tas@tue.nl` (Duygu Taş)

representations of real-life problems, several adapted versions of the classical Vehicle Routing Problem (VRP) have been addressed, such as stochastic routing problems (see, e.g., Gendreau et al., 1996) and time-dependent routing problems (see, e.g., Ichoua et al., 2003; Van Woensel et al., 2008).

In the VRP, each capacitated vehicle is assigned to a closed route with a sequence of customer locations, such that the total cost is minimized, each customer is visited once and all demands are fulfilled (see, e.g., Laporte, 2007; Toth and Vigo, 2002). Formally, this problem can be defined on a connected digraph $G = (N, A)$ where $N = \{0, 1, ..., n\}$ is the set of nodes and $A = \{(i, j)|i, j \in N, i \neq j\}$ is the set of arcs. Each vehicle route originates and ends at node 0. The latter (depot) is the central location with a fixed size fleet of identical vehicles given in set V, each with a capacity Q. The other nodes in N (nodes 1 to n) represent customers. For each customer i, a non-negative demand q_i and a non-negative service time s_i are given. The cost of visiting node j immediately after node i is denoted by d_{ij}, that usually corresponds to the distance of the arc (i, j). The classical Vehicle Routing Problem with Time Windows (VRPTW) is an extension of the VRP where the service at each customer must take place within a given time interval (hard time window). The latter is often relaxed in practice (leading to soft time window) which enables early and late servicing with some penalty costs. A soft time window with non-negative boundaries $[l_i, u_i]$ is then defined for each node i. The time window at the depot $[l_0, u_0]$ can be thought of as the scheduling horizon of the problem. The interested reader is referred to Bräysy and Gendreau (2005a,b) for reviews on the VRPTW.

In this paper, we focus on the VRP with time-dependent and stochastic travel times, including soft time windows. We use the mathematical model proposed in Taş et al. (2013). These authors focus on a VRP with stochastic (but time-independent) travel times and soft time windows, and develop a new solution procedure based on Tabu Search (TS). For the problem in this paper, we adapt both the model and the solution approach given in Taş et al. (2013) with respect to time-dependency. In addition, we implement an Adaptive Large Neighborhood Search (ALNS) procedure based on Ropke and Pisinger (2006). Our objective is to minimize the total weighted cost which includes two components, transportation costs and service costs. The distributions of the arrival times need to be obtained for the calculations of both cost components. We first exclude the service times which enables us to derive the exact distributions (the exact values of the mean and the variance) of the arrival times. We then consider the case with the original service times in which approximations of the arrival time distributions are needed. We conduct extensive analyses on numerical results: (i) solutions of two heuristics are compared with respect to each other, (ii) solutions of two heuristics are compared with the ones obtained in Taş et al. (2013) for a VRP with soft time windows and stochastic travel times (time-independent), (iii) solutions of two heuristics are compared with the optimal/best-known solutions obtained for the classical VRPTW with hard time windows and deterministic travel times (time-independent).

The main contributions of this paper are threefold:

1. We model the travel times with respect to time-dependency and stochasticity.
2. We solve a rather complex mathematical model with two separate metaheuristic procedures, where we have complicated arrival time distributions.
3. We perform a number of computational experiments and analyze the results comprehensively.

The remainder of this paper is organized as follows. The relevant literature on time-dependent and stochastic VRP is presented in Section 2. We describe the mathematical model applied to our problem in Section 3, and the properties of the arrival times in Section 4. In the latter section, the calculations of the expected values employed in the model are also given. We explain the methods used to solve the model in Section 5. Results and analyses obtained on Solomon's problem instances Solomon (1987) are presented in Section 6. Finally, the main findings and conclusions are highlighted in Section 7.

2. Literature Review

We focus on a time-dependent and stochastic VRP with soft time windows. Traveling Salesman Problem (TSP) is one of the special cases of the VRP where all customers are served by one vehicle with infinite capacity. Malandraki and Dial (1996) present a dynamic programming algorithm for solving a TSP where travel times are time-dependent. Discrete step functions are used to have different travel times in different time periods. Jula et al. (2006) study a TSP with Time Windows (TSPTW) where travel times are time-dependent and stochastic, and service times are stochastic. An approximate procedure is developed to estimate the arrival times at each customer.

Chang et al. (2009) develop a heuristic algorithm where the applied techniques are based on *n-path* relaxation method given by Houck et al. (1980) for the classical TSP, and the convolution-propagation approach given by Chang et al. (2005). In the latter study, the proposed approach approximates travel times where networks are both time-dependent and stochastic. For the time-dependent and stochastic networks, the interested reader is referred to Gao and Huang (2012) in which the routing decisions are taken with respect to real-time information.

To generate applicable models for the VRP and the VRPTW in real-life environments, most literature centers merely on time-dependent travel times. For the VRP with time-dependent travel times, Hill and Benton (1992) provide a model and a number of procedures to estimate the related parameters. The drawback of the proposed model is that it does not satisfy the First-In-First-Out (FIFO) property. Donati et al. (2008) develop a model that optimizes the total travel time along with the number of routes. A solution procedure which combines the ant colony system with a local search method is developed. Van Woensel et al. (2008) study time-dependent travel times resulted from a stochastic process due to the traffic congestion. They develop queueing models for this problem and propose a Tabu Search (TS) method to solve these models. Recently, Jabali et al. (2009) focus on time-dependent travel times where vehicles have unexpected delays at the customers' docking stations. A solution procedure based on TS is proposed to solve large-sized problem instances. Time-dependency can also be used to model other components in routing problems. The interested reader is referred to Tagmouti et al. (2011) for an arc routing problem in which the service cost on each arc is a function of the time of beginning service.

For the VRPTW with time-dependent travel times, the FIFO property is satisfied by Ichoua et al. (2003) by means of modeling travel times as continuous piecewise linear functions. The authors solve their model by a procedure based on a parallel TS method. Fleischmann et al. (2004) describe modern traffic information systems and time-dependent travel times obtained from these systems. A general framework is provided to implement time-dependent travel times in different algorithms modeled for the VRPTW. They report that using constant average travel times causes approximately a 10% underestimation of the total travel time. Haghani and Jung (2005) consider heteregoneus fleet of vehicles and solve their model by a solution procedure based on the Genetic Algorithm. Potvin et al. (2006) propose a model for the dynamic VRPTW by considering real-time customer demands and time-dependent travel times. A number of dispatching algorithms are defined to construct initial routes which are further improved by a local procedure.

Lecluyse et al. (2009) consider both time-dependent and stochastic travel times for VRP applications. The objective is to minimize the weighted sum of the mean and the standard deviation of the total travel time. They focus on a TS method to solve their model effectively. Nahum and Hadas (2009) develop a model based on a chance constrained programming. Small-sized problem instances are tested under several conditions such as considering deterministic environment, considering time-dependent and stochastic environment, and so on. Results are then compared to the optimal solutions which can easily be obtained by means of the small number of customers.

In this paper, we study a VRP where we have both time-dependent and stochastic travel times, and soft time windows. The aim is to minimize the sum of transportation costs and service costs. Transportation costs comprise three main elements which are the total distance traveled, the number of vehicles used and the total expected overtime of the drivers. Service costs are the penalty costs paid by the carrier company due to early and late servicing. To our knowledge, no research has addressed the problem considered in this paper, which is difficult both to model and to solve. Furthermore, we deal with a rather complex model and apply two separate solution approaches (TS and ALNS) to both cases (no service times and with service times), each with complicated integration calculations.

3. Model Description and Formulation

We first describe the notations employed in the mathematical model. The assignments of vehicles and the sequences of customers in these assignments are represented by the vector $\mathbf{x}$, where $\mathbf{x} = \{x_{ijv} | i, j \in N, v \in V\}$. In this vector, the decision variable x_{ijv} takes the value 1 if vehicle v visits node j immediately after node i and 0, otherwise. The *expected* delay and the *expected* earliness at node j served by vehicle v are denoted by $D_{jv}(\mathbf{x})$ and $E_{jv}(\mathbf{x})$, respectively. Associated with each vehicle v used for servicing, $O_v(\mathbf{x})$ is the *expected* overtime of the driver working on the route of that vehicle. The details about these expected values, which are calculated with respect to the time-dependent and stochastic travel times, are described in Sections 4.1.1 and 4.2.1.

One unit of delay and one unit of earliness are penalized by c_d and c_e, respectively. Each vehicle activated for the service brings a fixed cost c_f. Moreover, the costs paid by the company for one unit of distance and one unit of overtime are denoted by c_t and c_o, respectively. The model considering time-dependency and stochasticity is stated as follows:

$$\begin{aligned}\min \quad & \rho\frac{1}{C_1}\left(c_d\sum_{j\in N}\sum_{v\in V}D_{jv}(\mathbf{x})+c_e\sum_{j\in N}\sum_{v\in V}E_{jv}(\mathbf{x})\right)\\ & +(1-\rho)\frac{1}{C_2}\left(c_t\sum_{i\in N}\sum_{j\in N}\sum_{v\in V}d_{ij}x_{ijv}+c_f\sum_{j\in N\setminus\{0\}}\sum_{v\in V}x_{0jv}+c_o\sum_{v\in V}O_v(\mathbf{x})\right)\end{aligned} \tag{1}$$

$$\text{subject to} \sum_{j\in N}\sum_{v\in V}x_{ijv}=1, \qquad i\in N\setminus\{0\}, \tag{2}$$

$$\sum_{i\in N}x_{ihv}-\sum_{j\in N}x_{hjv}=0, \qquad h\in N\setminus\{0\}, v\in V, \tag{3}$$

$$\sum_{j\in N}x_{0jv}=1, \qquad v\in V, \tag{4}$$

$$\sum_{i\in N}x_{i0v}=1, \qquad v\in V, \tag{5}$$

$$\sum_{i\in N\setminus\{0\}}q_i\sum_{j\in N}x_{ijv}\le Q, \qquad v\in V, \tag{6}$$

$$\sum_{i\in S}\sum_{j\in S}x_{ijv}\le |S|-1, \qquad S\subseteq N\setminus\{0\}, v\in V, \tag{7}$$

$$x_{ijv}\in\{0,1\}, \qquad i\in N, j\in N, v\in V. \tag{8}$$

The objective (1) is to minimize the total weighted cost which comprises service costs incurred due to late and early servicing, and transportation costs resulting from distance, vehicles used for the service, and overtime. Constraints (2) and (3) ensure that each customer is served by exactly one vehicle and the vehicle leaves that customer location after serving. Constraints (4), (5) and (6) state that each vehicle starts and ends its route at the depot, and it delivers a quantity which does not exceed the vehicle capacity. Constraints (7) satisfy the subtour elimination and constraints (8) indicate that partial servicing is not allowed. Parameter ρ yields diverse combinations of the service cost and transportation cost components where their values are calibrated by using parameters C_1 and C_2. The interested reader is referred to Taş et al. (2013) for the details about calculations of these scaling parameters.

4. Properties of the Arrival Times

Assume that the scheduling horizon is divided into k intervals, each with a multiplier c_k. These multipliers are used to specify that we have different travel speeds for different time intervals, where a larger multiplier indicates that more time is needed for traveling. The following calculations related to arrival times are carried out with respect to the values given in Table 1. We refer to the time domain represented by the intervals given in this table as the b-domain.

Time Interval	**Multiplier**
$[0, b_1)$	c_1
$[b_1, b_2)$	c_2
$[b_2, b_3)$	c_3
$[b_3, b_4)$	c_4
$[b_4, \infty)$	c_5

Table 1: Travel time multipliers given for each arc (b-domain)

The speed of the vehicle changes along an arc in case the vehicle traverses from one time interval to the next time interval (transition parts) in the b-domain. To satisfy the FIFO property with these transition parts, the boundaries of the time intervals are defined as follows:

$$t_n = \frac{(b_n - b_{n-1})}{c_n} + t_{n-1}, \quad n = \{1, ..., k-1\}, \tag{9}$$

where $t_0 = b_0 = 0$, and $t_5 = \infty$. We refer to this latter time domain translated from the b-domain, as the t-domain.

The expected values mentioned in Section 3 are derived from the arrival time distributions. The exact values of the mean and the variance of the arrival times are obtained by means of assuming no service times at the customers. When including the original service times, we estimate the arrival time distributions by approximating the mean and the variance. In the next two sections, we provide the details of the arrival time distributions and the calculations of the expected values both for the case assuming no service times and for the case with service times.

4.1. Without Service Times

Arrival times defined in Taş et al. (2013) (time-independent and stochastic travel times) resulted from a travel time distribution, where $c_m = 1$, $\forall m \in K$ and K is the set of time intervals. We translate these time-independent arrival times into factual arrival times by using different multipliers for different time intervals. Assume that T, which is the random travel time spent for one unit of distance, is Gamma distributed with shape parameter α and scale parameter λ. This approach enables us to derive Gamma distributed arc traversal times by scaling T with respect to the distance of the corresponding arc. T_{ij}, which is the time spent on arc (i, j) for traveling from node i to node j, then follows a Gamma distribution with parameters αd_{ij} and λ. As vehicles do not wait at customer locations, the arrival time of vehicle v at node j, Y_{jv}, is defined in the time-independent case as follows:

$$Y_{jv} = \sum_{(p,r)\in A_{jv}} T_{pr}, \tag{10}$$

where A_{jv} is the set of arcs traversed by vehicle v until node j. Gamma distributed arrival times are obtained by means of Equation (10), where shape and scale parameters of Y_{jv} are calculated as follows:

$$\alpha_{jv} = \alpha \sum_{(p,r)\in A_{jv}} d_{pr}, \tag{11}$$

$$\lambda_{jv} = \lambda. \tag{12}$$

The mean and the variance of the arrival time of vehicle v at node j, which is visited immediately after node i, are calculated by:

$$E[Y_{jv}] = E[Y_{iv}] + E[T_{ij}] \text{ and,} \tag{13}$$

$$\mathrm{Var}(Y_{jv}) = \mathrm{Var}(Y_{iv}) + \mathrm{Var}(T_{ij}), \tag{14}$$

respectively.

As we have different multipliers for different time intervals, the mean and variance of the factual arrival time R_{jv} for each node need to be obtained. To derive these values, we need to compute $E[R_{jv}]$ and $E[{R_{jv}}^2]$, where $\mathrm{Var}(R_{jv}) = E[{R_{jv}}^2]$ - $(E[R_{jv}])^2$. Related formulations are then given as follows:

$$
\begin{aligned}
E[R_{jv}] &= \int_{t_0}^{t_1} (b_0 + c_1(z - t_0))dF_{jv} + \int_{t_1}^{t_2} (b_1 + c_2(z - t_1))dF_{jv} \\
&+ \int_{t_2}^{t_3} (b_2 + c_3(z - t_2))dF_{jv} + \int_{t_3}^{t_4} (b_3 + c_4(z - t_3))dF_{jv} \\
&+ \int_{t_4}^{t_5} (b_4 + c_5(z - t_4))dF_{jv},
\end{aligned}
\tag{15}
$$

and

$$
\begin{aligned}
E[R_{jv}^2] &= \int_{t_0}^{t_1} (b_0 + c_1(z - t_0))^2 dF_{jv} + \int_{t_1}^{t_2} (b_1 + c_2(z - t_1))^2 dF_{jv} \\
&+ \int_{t_2}^{t_3} (b_2 + c_3(z - t_2))^2 dF_{jv} + \int_{t_3}^{t_4} (b_3 + c_4(z - t_3))^2 dF_{jv} \\
&+ \int_{t_4}^{t_5} (b_4 + c_5(z - t_4))^2 dF_{jv},
\end{aligned}
\tag{16}
$$

where $dF_{jv} = \dfrac{(e^{-z/\lambda_{jv}})(z)^{\alpha_{jv}-1}}{\Gamma(\alpha_{jv}){\lambda_{jv}}^{\alpha_{jv}}}dz$ and $\Gamma(\alpha_{jv}) = \int_0^\infty e^{-r} r^{\alpha_{jv}-1} dr$.

The interested reader is referred to Appendix A for the calculations of $E[R_{jv}]$ and $E[R_{jv}^2]$ in detail. General representations of $E[R_{jv}]$ and $E[R_{jv}^2]$ are given as follows:

$$
\begin{aligned}
E[R_{jv}] &= \alpha_{jv}\lambda_{jv} \sum_{n=0}^{k-1} c_{n+1}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_{n+1}) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_n)] \\
&+ \sum_{n=0}^{k-1} (b_n - c_{n+1}t_n)[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_{n+1}) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_n)], \text{ and}
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
E[R_{jv}^2] &= \alpha_{jv}(\alpha_{jv}+1)\lambda_{jv}^2 \sum_{n=0}^{k-1} c_{n+1}^2 [\Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_{n+1}) - \Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_n)] \\
&+ \sum_{n=0}^{k-1} (b_n - c_{n+1}t_n)^2 [\Gamma_{\alpha_{jv},\lambda_{jv}}(t_{n+1}) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_n)] \\
&+ 2\alpha_{jv}\lambda_{jv} \sum_{n=0}^{k-1} (b_n - c_{n+1}t_n) c_{n+1} [\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_{n+1}) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_n)].
\end{aligned}
\tag{18}
$$

4.1.1. Calculations of Expected Delay, Earliness and Overtime

The mean and the variance of the factual arrival time at customer j can be calculated by means of Equations (17) and (18). We then calculate the shape and the scale parameters of the factual arrival time distribution ($\overline{\alpha_{jv}}$ and $\overline{\lambda_{jv}}$) to estimate the expected delay, $D_{jv}(\mathbf{x})$ and the expected earliness, $E_{jv}(\mathbf{x})$ at each customer j, and the expected overtime, $O_v(\mathbf{x})$ for drivers working on the routes of each vehicle v allocated.

$D_{jv}(\mathbf{x})$ and $E_{jv}(\mathbf{x})$ are calculated as follows by using the shape and the scale parameters of the factual arrival time distribution:

$$D_{jv}(\mathbf{x}) = \overline{\alpha_{jv}}\,\overline{\lambda_{jv}}(1 - \Gamma_{\overline{\alpha_{jv}}+1,\overline{\lambda_{jv}}}(u_j)) - u_j(1 - \Gamma_{\overline{\alpha_{jv}},\overline{\lambda_{jv}}}(u_j)) \text{ and,} \tag{19}$$

$$E_{jv}(\mathbf{x}) = l_j \Gamma_{\overline{\alpha_{jv}},\overline{\lambda_{jv}}}(l_j) - \overline{\alpha_{jv}}\,\overline{\lambda_{jv}} \Gamma_{\overline{\alpha_{jv}}+1,\overline{\lambda_{jv}}}(l_j). \tag{20}$$

$O_v(\mathbf{x})$ is calculated with respect to the distribution of the factual arrival time at the (ending) depot as follows:

$$O_v(\mathbf{x}) = \overline{\alpha_{0v}}\,\overline{\lambda_{0v}}(1 - \Gamma_{\overline{\alpha_{0v}}+1,\overline{\lambda_{0v}}}(w)) - w(1 - \Gamma_{\overline{\alpha_{0v}},\overline{\lambda_{0v}}}(w)), \tag{21}$$

where w is the labor shift time agreed by the company.

4.2. With Service Times

Suppose that the stochastic process $G_{ij}(t)$ represents the travel time on arc (i,j) in case the vehicle enters that arc at time t. Note that t denotes the departure time in the t-domain. The departure time of vehicle v from node i, denoted by Z_{iv}, is computed as follows:

$$Z_{iv} = Y_{iv} + s_i, \tag{22}$$

where Y_{iv} is the arrival time of vehicle v at node i. The arrival time of that vehicle at node j, which is visited immediately after node i, is then described as follows:

$$Y_{jv} = Z_{iv} + T_{ij}, \tag{23}$$

where the random variable T_{ij} is the travel time needed for traveling from node i to node j by traversing the arc (i, j) given the departure time from node i, $T_{ij} = G_{ij}(Z_{iv})$.

The mean and the variance of the stochastic process $G_{ij}(t)$ are represented by $\mu_{ij}(t)$ and $\sigma^2_{ij}(t)$, respectively. $\mu_{ij}(t)$ is calculated as follows:

$$\begin{aligned}
\mu_{ij}(t) = E[G_{ij}(t)] &= \int_{t_0-t}^{t_1-t} (b_0 - b + c_1(z - (t_0 - t)))dF_{ij} \\
&+ \int_{t_1-t}^{t_2-t} (b_1 - b + c_2(z - (t_1 - t)))dF_{ij} \\
&+ \int_{t_2-t}^{t_3-t} (b_2 - b + c_3(z - (t_2 - t)))dF_{ij} \\
&+ \int_{t_3-t}^{t_4-t} (b_3 - b + c_4(z - (t_3 - t)))dF_{ij} \\
&+ \int_{t_4-t}^{t_5-t} (b_4 - b + c_5(z - (t_4 - t)))dF_{ij},
\end{aligned} \tag{24}$$

where $dF_{ij} = \dfrac{(e^{-z/\lambda})(z)^{\alpha d_{ij}-1}}{\Gamma(\alpha d_{ij})\lambda^{\alpha d_{ij}}}dz$, $\Gamma(\alpha d_{ij}) = \int_0^\infty e^{-r} r^{\alpha d_{ij}-1}dr$, and b is the corresponding departure time in the b-domain. Note that $(t_l - t)$ takes the value of 0 if $t_l \leq t$, $\forall l \in K$. A general representation of $\mu_{ij}(t)$ can be stated as follows (see Appendix B for the calculations of $\mu_{ij}(t)$ in detail):

$$\begin{aligned}
\mu_{ij}(t) &= \alpha d_{ij}\lambda \sum_{n=0}^{k-1} c_{n+1}[\Gamma_{\alpha d_{ij}+1,\lambda}(t_{n+1} - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_n - t)] \\
&+ \sum_{n=0}^{k-1}(b_n - b - c_{n+1}(t_n - t))[\Gamma_{\alpha d_{ij},\lambda}(t_{n+1} - t) - \Gamma_{\alpha d_{ij},\lambda}(t_n - t)].
\end{aligned} \tag{25}$$

$\sigma^2_{ij}(t)$ is calculated by:

$$\sigma^2_{ij}(t) = E[G^2_{ij}(t)] - (E[G_{ij}(t)])^2, \tag{26}$$

where

$$
\begin{aligned}
E[G_{ij}^2(t)] &= \int_{t_0-t}^{t_1-t} (b_0 - b + c_1(z-(t_0-t)))^2 dF_{ij} \\
&+ \int_{t_1-t}^{t_2-t} (b_1 - b + c_2(z-(t_1-t)))^2 dF_{ij} \\
&+ \int_{t_2-t}^{t_3-t} (b_2 - b + c_3(z-(t_2-t)))^2 dF_{ij} \\
&+ \int_{t_3-t}^{t_4-t} (b_3 - b + c_4(z-(t_3-t)))^2 dF_{ij} \\
&+ \int_{t_4-t}^{t_5-t} (b_4 - b + c_5(z-(t_4-t)))^2 dF_{ij}, \qquad (27)
\end{aligned}
$$

and $(t_l - t)$ takes the value of 0 if $t_l \leq t$, $\forall l \in K$. A general representation of $E[G_{ij}^2(t)]$ can be stated as follows (see Appendix B for the calculations of $E[G_{ij}^2(t)]$ in detail):

$$
\begin{aligned}
E[G_{ij}^2(t)] &= \alpha d_{ij}(\alpha d_{ij}+1)\lambda^2 \sum_{n=0}^{k-1} c_{n+1}^2 [\Gamma_{\alpha d_{ij}+2,\lambda}(t_{n+1}-t) - \Gamma_{\alpha d_{ij}+2,\lambda}(t_n - t)] \\
&+ \sum_{n=0}^{k-1} (b_n - b - c_{n+1}(t_n - t))^2 [\Gamma_{\alpha d_{ij},\lambda}(t_{n+1}-t) - \Gamma_{\alpha d_{ij},\lambda}(t_n - t)] \\
&+ 2\alpha d_{ij}\lambda \sum_{n=0}^{k-1} (b_n - b - c_{n+1}(t_n - t)) c_{n+1} [\Gamma_{\alpha d_{ij}+1,\lambda}(t_{n+1}-t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_n - t)]. \qquad (28)
\end{aligned}
$$

The mean arrival time of vehicle v at node j is calculated as follows with respect to Equations (22) and (23):

$$
\begin{aligned}
E[Y_{jv}] &= E[Z_{iv}] + E[E[T_{ij}|Z_{iv} = z_{iv}]] \\
&= E[Y_{iv}] + s_i + E[E[T_{ij}(Z_{iv})]] \\
&= E[Y_{iv}] + s_i + E[\mu_{ij}(Z_{iv})], \qquad (29)
\end{aligned}
$$

where

$$
E[\mu_{ij}(Z_{iv})] = \int_0^\infty \mu_{ij}(z_{iv}) f_{Z_{iv}}(z_{iv}) dz_{iv}. \qquad (30)
$$

The first-order approximation of Equation (30) is obtained as follows in a similar way given by Fu and Rilett (1998), and Jula et al. (2006) by assuming that $\mu_{ij}(t)$ is differentiable at point $t = E[Z_{iv}]$, and by using the Taylor's series expansion:

$$E[\mu_{ij}(Z_{iv})] \cong \mu_{ij}(E[Z_{iv}]). \tag{31}$$

By substituting Equation (31) in Equation (29), we calculate $E[Y_{jv}]$ by:

$$E[Y_{jv}] \cong E[Y_{iv}] + s_i + \mu_{ij}(E[Z_{iv}]). \tag{32}$$

The variance of the arrival time of vehicle v at node j is obtained as follows:

$$\mathrm{Var}(Y_{jv}) \cong \mathrm{Var}(Y_{iv}) + \sigma_{ij}^2(E[Y_{iv}] + s_i). \tag{33}$$

4.2.1. Calculations of Expected Delay, Earliness and Overtime

The mean and the variance of the arrival time of vehicle v at customer j can be calculated by means of Equations (32) and (33) by assuming that we have approximated Gamma distributed arrival times. We then calculate the shape and the scale parameters of the arrival time distribution ($\overline{\alpha_{jv}}$ and $\overline{\lambda_{jv}}$) to estimate the expected delay, $D_{jv}(\mathbf{x})$ and the expected earliness, $E_{jv}(\mathbf{x})$ at each customer j, and the expected overtime, $O_v(\mathbf{x})$ for drivers working on the routes of each vehicle v allocated.

To have better approximations of $D_{jv}(\mathbf{x})$ and $E_{jv}(\mathbf{x})$, we shift the time window at node j to the left in the time horizon by the total service time spent by that vehicle until it serves node j (s_{jv}). The adjusted time window is denoted by $[l'_j, u'_j]$ where $l'_j = l_j - s_{jv}$ and $u'_j = u_j - s_{jv}$. The two expected values are then calculated by using the adjusted limits, and the shape and the scale parameters of the factual arrival time distribution as follows:

$$D_{jv}(\mathbf{x}) = \begin{cases} \overline{\alpha_{jv}}\,\overline{\lambda_{jv}}(1 - \Gamma_{\overline{\alpha_{jv}}+1,\overline{\lambda_{jv}}}(u'_j)) - u'_j(1 - \Gamma_{\overline{\alpha_{jv}},\overline{\lambda_{jv}}}(u'_j)), & \text{if } u_j > s_{jv} \\ E[Y_{jv}] - u_j, & \text{otherwise} \end{cases} \tag{34}$$

$$E_{jv}(\mathbf{x}) = \begin{cases} l'_j\Gamma_{\overline{\alpha_{jv}},\overline{\lambda_{jv}}}(l'_j) - \overline{\alpha_{jv}}\,\overline{\lambda_{jv}}\Gamma_{\overline{\alpha_{jv}}+1,\overline{\lambda_{jv}}}(l'_j), & \text{if } l_j > s_{jv} \\ 0, & \text{otherwise} \end{cases} \tag{35}$$

To have a better approximation of $O_v(\mathbf{x})$, we adjust the labor shift time by the amount of the total service time spent by vehicle v along its route (s_{ov}). The adjusted labor shift time is denoted by w' where $w' = w - s_{0v}$. $O_v(\mathbf{x})$ is then calculated by using w' and the distribution of the factual arrival time at the (ending) depot as follows:

$$O_v(\mathbf{x}) = \begin{cases} \overline{\alpha_{0v}}\,\overline{\lambda_{0v}}(1 - \Gamma_{\overline{\alpha_{0v}}+1,\overline{\lambda_{0v}}}(w')) - w'(1 - \Gamma_{\overline{\alpha_{0v}},\overline{\lambda_{0v}}}(w')), & \text{if } w > s_{0v} \\ E[Y_{0v}] - w, & \text{otherwise} \end{cases} \tag{36}$$

5. Solution Methodology

In our solution approach, we apply both a TS and an ALNS metaheuristic. We start the optimization with a solution obtained using the initialization algorithm proposed in Taş et al. (2013). In the latter algorithm, the insertion heuristic I1 given in Solomon (1987) is extended by considering the expected violations of the time windows where travel times are stochastic. The solution constructed by this heuristic is further improved with respect to the total transportation cost by applying a TS procedure, leading to the initial feasible solution. We modify the initialization algorithm developed in Taş et al. (2013) by calculating the expected values with respect to the time-dependent and stochastic travel times. Note that y_{init} denotes the initial feasible solution generated by the algorithm adapted. Solutions obtained by the metaheuristics are then further processed by a post-optimization method to deal with the detailed schedule of the vehicle routes in the corresponding solution. The overview of our solution procedure is given in Algorithm 1.

1. Construct an initial feasible solution with respect to the time-dependent and stochastic travel times
2. Improve the solution generated in Step 1 by using a TS method with respect to the total weighted cost
3. Improve the solution generated in Step 1 by using an ALNS method with respect to the total weighted cost
4. Apply a post-optimization method to the solutions obtained in Step 2 and in Step 3

Algorithm 1: Overview of the solution procedure

5.1. Tabu Search

The TS procedure applied in this paper is based on the algorithm given by Taş et al. (2013). In this algorithm, the total cost of the (current) solution y is denoted by $z(y)$. The neighborhood of y is constructed by two operators. One operator changes the location of the customer within the route while the other one removes the customer from a route and inserts it into another route.

Each solution y' constructed by the neighborhood operators are evaluated with respect to a measure which is $c(y') = z(y') + \nu q(y')$. In this measure, $q(y')$ is equal to $max\{0, ((\sum_{v \in V} \sum_{i \in N \setminus \{0\}} q_i \sum_{j \in N} \widetilde{x}_{ijv}) - Q)\}$ where $\widetilde{x}_{ijv}$ takes the value 1 if arc (i, j) is traversed by vehicle v in solution y' and 0, otherwise. The coefficient ν is the cost paid for one unit of demand violation, and its value is updated at each iteration with respect to the solution accepted. ν takes the value $\nu/(1+\varphi)$ if the solution accepted does not violate the vehicle capacity and $\nu(1 + \varphi)$, otherwise. In case $c(y') \geq c(y)$, a diversification cost is added to $c(y')$ to expand the search in the neighborhood. Note that the measure, which calculates the additional cost, employs a parameter Ω to adjust the intensity of the search expanded.

The TS method accepts the first solution y' in the neighborhood which is non-tabu and $c(y') < c(y)$, or satisfies the aspiration criterion. In case the method cannot find any such solution, the best non-tabu solution in the neighborhood is accepted. Note that the non-tabu solutions are identified by means of a list with size ϑ which stores the customers prohibited to relocate. Moreover, the search in the neighborhood is intensified by a medium-term memory mechanism. This mechanism focuses on the promising regions of the neighborhood obtained by the best feasible solution (y^*). The interested reader is referred to Taş et al. (2013) for the details of the intensification technique and the pseudo-code in which the framework of the algorithm is described.

Two stopping criteria are used in the procedure explained above. The primary criterion is the maximum number of iterations (θ). A threshold number of iterations is employed as the secondary criterion where the algorithm terminates in case the best feasible solution cannot be updated for τ iterations. As we compare the routes in the best feasible solution generated by the TS procedure to those obtained by the ALNS method, these two metrics are also used as the stopping criteria in the ALNS.

5.2. Adaptive Large Neighborhood Search

The scheme of the ALNS method is introduced by Pisinger and Ropke (2005, 2007), and Ropke and Pisinger (2006) to handle various types of the routing problems. These authors extend the Large Neighborhood Search (LNS), which is developed by Shaw (1997), in terms of the operators implemented and the neighborhood considered. In the next sections, we present the selection procedure of heuristics and the acceptance of solutions generated. We also explain the details of the removal and insertion heuristics. The overall ALNS procedure is then explained in pseudo-code as Algorithm 2.

5.2.1. Adaptive Weight Adjustment Procedure and Acceptance of Solutions

In the ALNS, the removal and insertion heuristics to be employed in the current iteration are selected by a roulette-wheel technique. Suppose that each heuristic i has a weight, ω_i. At the beginning of the ALNS method, a probability which is equal to $\frac{\omega_i}{\sum_{j=1}^{h} \omega_j}$ is calculated for each heuristic i. Note that the selection of heuristics depends on the independent probabilities which are calculated over the set of removal heuristics, and the set of insertion heuristics.

The weight adjustment procedure needs to separate out the total number of iterations of the ALNS into the segments denoted by θ_ω. Throughout the ALNS method, weight values to be employed in segment $j+1$ are updated with respect to segment j as $\omega_{i,j+1} = \omega_{i,j}(1-r_\omega)+r_\omega \frac{\Phi_i}{\xi_i}$. In the latter, r_ω is the control parameter, Φ_i is the score of heuristic i and ξ_i is the number of times that heuristic i is employed in segment j. Note that scores of all heuristics are set to 0 at the beginning of each segment. If the solution obtained in the current iteration improves the best feasible solution y^*, the scores of both the removal and the insertion heuristics applied in that iteration are increased by Θ_1. If the cost of the new solution is better than the cost of the current solution, the scores are increased by Θ_2. If the cost of the new solution is worse than the current solution but it has been accepted, the scores are updated by Θ_3.

The solutions obtained in the ALNS iterations are evaluated with respect to a criterion based on the simulated annealing framework. The new solution y_{new} with the total cost $z(y_{new})$ is always accepted if $z(y_{new})$ is better than $z(y)$. If $z(y_{new})$ is worse than $z(y)$, y_{new} is accepted with a probability $e^{-(y_{new}-y)/H}$ where H denotes the temperature. The initial value of H is set to $z(y_{init})c_{init}$ in which c_{init} is the coefficient used to initialize the temperature. H is updated at each iteration by c_H, which is the coefficient employed to decrease the temperature, as $c_H H$.

5.2.2. Removal Heuristics

We apply three removal heuristics similar to ones given by Ropke and Pisinger (2006). Each heuristic starts with a solution y, and removes one customer from the current solution at each iteration. The heuristic is repeated until it reaches the maximum number of given iterations (ϕ).

Shaw Removal Heuristic

This heuristic, which is introduced by Shaw (1997), aims to remove similar customers where similarity is defined according to a measure. The algorithm employs a removal list that includes a randomly selected customer at the initialization step. At each iteration, a customer i from that list is randomly chosen and the relatedness value of each customer j in y (L_{ij}) is calculated. Suppose that in solution y, customers i and j are visited by vehicles v_1 and v_2, respectively. L_{ij} is then equal to $(\gamma d_{ij} + \chi|E[Y_{iv_1}] - E[Y_{jv_2}]| + \delta|q_i - q_j|)$ where d_{ij}, $E[Y_{iv_1}]$, $E[Y_{jv_2}]$, q_i and q_j are normalized in the range of [0,1]. In case these two customers are served by the same vehicle, ψ is added to L_{ij}. Customers in y are then ranked with respect to their L_{ij} values in increasing order. The algorithm randomly selects a customer in this order to remove it from y and insert into the removal list. Note that the degree of randomness in the selection part is arranged by the parameter ζ.

Random Removal Heuristic

This heuristic randomly removes ϕ customers from the given solution and inserts them into the removal list. In our ALNS procedure, this method may be viewed as a special case of Shaw removal heuristic where ζ is set to 1.

Worst Removal Heuristic

This heuristic repeatedly removes customers with respect to a measure calculated for each customer j in the solution y. The measure is the change in the total cost of the solution y by removing customer j (M_j). Customers in y are then ranked with respect to their M_j values in decreasing order. The algorithm randomly selects a customer in this order to remove it from y and insert into the removal list. Note that the degree of randomness in the selection part is arranged by the parameter ζ.

5.2.3. Insertion Heuristics

We apply two insertion heuristics similar to ones given by Ropke and Pisinger (2006). Each heuristic starts with a solution y_{temp} and a removal list that are obtained by the removal procedure. At each iteration, the heuristic removes one customer from that list and inserts it into the current solution until the list becomes empty.

Greedy Heuristic

At each iteration, this heuristic determines the best possible customer to be inserted into a route of solution y_{temp} at its best possible place in that route. For each unrouted customer j, we calculate the minimum change in the total cost of the solution y_{temp} by inserting that customer into y_{temp} (M'_j). This measure yields the best possible place for customer j in terms of the route and the location in that route. The algorithm then selects the customer $j^* = \text{argmin}_{j \in C}\{M'_j\}$ where C denotes the customers given by the removal list (ones not yet served by any route).

Regret Heuristic

This heuristic inserts customers with respect to a measure calculated for each customer j in the removal list. The measure is the difference between the total cost of solution y_{temp} in case of inserting customer j at its best place and that in case of inserting customer j at its second-best place (M^*_j). The algorithm then selects the customer $j^* = \text{argmax}_{j \in C}\{M^*_j\}$ where C denotes the unrouted customers given by the removal list. Recall that the greedy heuristic delays the insertion of the customers with high M'_j values to the last iterations where few places are available for these customers. The regret heuristic copes with this problem by trying to insert the customers which may not have several available places as the algorithm proceeds.

5.3. Post-optimization Method

The vehicle routes obtained by the TS and the ALNS depart from the depot at time 0. We repeatedly shift the starting time of each vehicle route from the depot by 10 minutes until the total cost of that route is not improved. This method yields a better balance between the expected delay and the expected earliness. Note that shifting the departure time also affects the expected overtime, which is not seen in time-independent and stochastic case.

```
Construct initial feasible solution, y_init
Set y := y_init, y* := y_init and z(y*) := z(y_init)
Calculate the initial probabilities for each removal heuristic and for
each insertion heuristic
Initialize H with respect to y_init
Set κ := 1, stop := 0
while κ ≤ θ and stop = 0 do
    Choose a removal heuristic with respect to removal probabilities
    Apply this heuristic to y, leading to solution y_temp
    Choose an insertion heuristic with respect to insertion probabilities
    Apply this heuristic to y_temp, leading to solution y_new
    if z(y_new) < z(y) then
        Set y := y_new and z(y) := z(y_new)
    end
    else
        Generate a random number ε
        if ε < e^{-(y_new - y)/H} then
            Set y := y_new and z(y) := z(y_new)
        end
    end
    if z(y) < z(y*) then
        Set y* := y and z(y*) := z(y)
    end
    if y* is not updated for τ iterations then
        Set stop := 1
    end
    Set H = c_H H
    Update the scores and weights with respect to weight adjustment
    procedure, and calculate the probabilities of removal and insertion
    heuristics
    Set κ := κ + 1
end
return y*
```

Algorithm 2: The general framework of the ALNS procedure

6. Numerical Results

Our computational experiments are conducted on the well-known data sets given by Solomon (1987). In these sets, we have three types of geographic distribution: (i) Clustered (C), (ii) Random (R), and (iii) Randomly Clustered (RC). Moreover, we have two types of instances with respect to the time windows: (i) instances with tight time windows (C1, R1 and RC1), and (ii) instances with large time windows (C2, R2 and RC2). Each problem instance has 100 customers and one depot. The travel time spent for one unit of distance is Gamma distributed where (α, λ) are equal to (1.0, 1.0). The latter leads to a Coefficient of Variation (CV) taking the value 1.0. We set w to 480 and ρ to 0.50. Cost coefficients (c_d, c_e, c_t, c_f, c_o) are equal to (1.0, 0.1, 1.0, 400, 5/6) and travel-time multipliers (c_1, c_2, c_3, c_4, c_5) are equal to (1.0, 1.2, 1.0, 1.4, 1.0), respectively. The scheduling horizon in the b-domain is designed with respect to w for C1, R1 and RC1 sets. In C2, R2 and RC2 sets, we potentially have more customers in a route due to the large time windows. Thus, we use the upper bound of the time window at the depot (u_0) to arrange the boundaries of the b-domain. (θ,τ) are set to (500,100) in the improvement procedure employed by the initialization algorithm, and to (2000, 500) in the TS and in the ALNS. Moreover, ν is initially equal to 1. We implement our algorithms in JAVA, and experiments are performed on an Intel Core Duo with 2.93 GHz and 4 GB of RAM.

We first conduct a number of preliminary tests to set the parameters to be used in the TS and in the ALNS. In these tests, results are obtained by employing the different values of a parameter on an interval where the other parameters are kept unchanged. We implement a similar procedure to that given by Cordeau et al. (1997) and Taş et al. (2013) for the TS, and a similar procedure to that given by Ropke and Pisinger (2006) for the ALNS. Ω, φ and ϑ are tested over the intervals [0.005,0.025], [$5\log_{10}|N|$,$15\log_{10}|N|$] and [0.25,1.25], respectively. These tests lead to three main sets of results where we accordingly set Ω, φ and ϑ to 0.015, 0.25 and $5\log_{10}|N|$, respectively. According to preliminary tests performed for the ALNS, (ω, θ_ω, r_ω, Θ_1, Θ_2, Θ_3, c_{init}, c_H, ϕ, γ, χ, δ, ψ, ζ) are set to (2, 25, 0.10, 5, 3, 2, 10, 0.99, 6, 10, 10, 6, 8, 4), respectively.

6.1. Solutions obtained by the TS and the ALNS

This section discusses the different aspects of the results obtained by the two metaheuristics employed in the paper. Table 2 provides the details of the Initial Feasible Solutions (IFS) which are generated both for the case assuming no service times, and for the case with the original service times. Note that the total expected delay (Del.), total distance (Dist.), total expected earliness (Earl.), total expected overtime (Over.), number of vehicles used for the service (Veh.), and the CPU times in seconds represent the average values obtained over all problem instances in the corresponding data set. We do not report the objective function values (Obj.) of the starting solutions since they are equal to 1 by means of the scaling parameters, C_1 and C_2. Solutions in this table show that when the original service times are included, the total expected delay is increased and the total expected earliness is decreased in all problem sets. Moreover, the procedure given for the case with service times provides the solutions in less CPU time with respect to the procedure given for the case assuming no service times.

Tables 3 and 4 present the solutions obtained by the TS for the case assuming no service times and for the case with original service times, respectively. The post-optimization method leads to the final solutions with the adjusted departure times from the depot. Solutions in Table 3 show that in all problem sets, the TS method decreases the total expected earliness with respect to the initial solutions. Solutions in Table 4 show that the TS method decreases both the total expected delay and the total expected earliness in all problem sets with respect to the initial solutions. These reductions mostly bring an increase in the total distance and in the total expected overtime for both cases. The post-optimization method provides an improvement in all solutions obtained by the TS for the case assuming no service times. This method cannot improve the solutions for the case with service times due to the long travel times along each vehicle route.

Tables 5 and 6 present the solutions obtained by the ALNS for the case assuming no service times and for the case with the original service times, respectively. Solutions in Table 5 show that in all problem sets except C2, the ALNS method decreases the total expected earliness with respect to the initial solutions. Solutions in Table 6 show that the ALNS method decreases both the total expected delay and the total expected earliness in all problem sets with respect to the initial solutions. These reductions mostly bring an increase in the total distance and in the total expected overtime for both cases. The post-optimization method provides an improvement in all solutions obtained by the ALNS for the case assuming no service times. This method can slightly improve the solutions of C1 for the case with service times; however, it cannot further improve the other solutions due to the long travel times along each vehicle route.

According to the objective function values given in Tables 3 and 5, the TS method provides better final results for C1, R1, RC1 and C2 problem sets, and the ALNS method provides better final results for R2 and RC2 problem sets. According to the objective function values given in Tables 4 and 6, the TS method yields better final results for C1 problem set while the ALNS method provides better final results for RC1, R2 and RC2 problem sets. The detailed average values given for R1 and C2 problem sets show that the TS method provides better final results for these sets with respect to the ALNS.

6.2. Solutions obtained for the time-independent and stochastic travel times

In this section, we aim to analyze the benefits gained by the time-dependent travel times compared to the time-independent travel times. The solutions obtained by Taş et al. (2013) with respect to only stochastic travel times are evaluated under the time-dependent and stochastic travel times. The original service times are included in this evaluation since they are considered to generate the solutions in Taş et al. (2013). Note that for a fair comparison, each travel-time multiplier is set to 1.12 which is the average of five multipliers employed to obtain solutions in Section 6.1. Moreover, each vehicle starts its route from the depot at time 0 to adequately assess the solutions in terms of the performance of the TS and the ALNS. The average values obtained by the evaluation for each set are presented in Table 7 (on the left part). According to the values given in Tables 4 and 6, for C1, RC1, C2 and R2 sets, the solutions originally generated with respect to time-dependency and stochasticity perform better than the solutions generated with respect to only stochasticity. The stochastic and time-independent procedure provides better solutions for R1 and RC2 sets for the time-dependent environment where the increase in the delay due to the longer travel times is well compensated with a decrease in the earliness.

6.3. Optimal solutions obtained for the classical VRPTW

In this section, we aim to analyze the benefits gained by the time-dependent and stochastic travel times compared to the time-independent and deterministic travel times. The optimal/best known solutions obtained for the classical VRPTW (see Desaulniers et al., 2008; Baldacci et al., 2011) are evaluated under the time-dependent and stochastic travel times by considering soft time windows. Note that these solutions are originally generated for a set of customers with hard time windows where the travel times are time-independent and deterministic. In the evaluation, each travel-time multiplier is set to 1.12 which is the average of five multipliers employed to obtain solutions in Section 6.1. Moreover, each vehicle starts its route from the depot at time 0 to adequately assess the solutions in terms of the performance of the TS and the ALNS. The average values obtained by the evaluation for each set are presented in Table 7 (on the right part). According to the values given in Tables 4 and 6, for R1, RC1, R2 and RC2 sets, the solutions originally generated with respect to time-dependency and stochasticity perform better than the solutions generated with respect to time-independent and deterministic environment. For C1 and C2 sets, the deterministic procedure provides better solutions for the time-dependent and stochastic environment due to the network structure of these sets (see Solomon, 1987). The time windows in C1 and C2 sets have been arranged with respect to the arrival times of the vehicles at the customers, leading to very reasonable delay and earliness values even if the travel times are time-dependent and stochastic. Moreover, we have low transportation costs due to the objective of the classical VRPTW (minimizing the total distance) and the distribution of the customers in these sets (clustered).

6.4. Quality of the Arrival Time Distributions

We assess the quality of the arrival time distributions by running a simulation procedure for each final solution obtained by the TS and by the ALNS. In this method, we employ 1000 iterations. The average values obtained by the simulation for each set are presented in Tables 3, 4, 5 and 6 (on the right-most part). We evaluate the difference in the total weighted cost of the final solutions calculated by the formulations proposed in this paper and by the simulation. This difference is at most 3.27% for the solutions obtained by the TS for the case assuming no service times, at most 1.56% for the solutions obtained by the ALNS for the case assuming no service times, and at most 1.66% for the solutions obtained by the ALNS for the case with the original service times. These results confirm that the distributions of the arrival times and the expected values used to calculate the total weighted cost are reliably estimated in this paper.

Table 2: Details of initial feasible solutions

	IFS for the case assuming no service times						IFS for the case with original service times					
Set	Del.	Dist.	Earl.	Over.	Veh.	CPU	Del.	Dist.	Earl.	Over.	Veh.	CPU
C1	0.36	882.49	28095.74	0.00	10.00	538	11332.42	835.52	9452.03	84.35	10.00	349
R1	182.42	895.54	2453.84	0.00	8.00	577	2783.07	895.54	1224.15	0.00	8.00	432
RC1	52.59	1014.41	2557.96	0.00	9.00	455	1394.69	1014.41	1253.70	0.00	9.00	352
C2	0.00	689.98	102854.84	0.00	3.00	2039	32985.51	689.98	24103.02	138.55	3.00	1850
R2	180.51	693.78	13465.09	0.02	2.00	3236	7842.11	698.83	7144.00	12.55	2.09	2712
RC2	169.20	727.94	14499.33	0.01	2.25	2362	3839.44	727.35	8690.68	10.16	2.50	1975

Table 3: Details of solutions obtained by the TS for the case assuming no service times

	Solutions of TS							Final Solutions				Simulation			
Set	Del.	Dist.	Earl.	Over.	Veh.	Obj.	CPU	Del.	Earl.	Over.	Obj.	Del.	Earl.	Over.	Obj.
C1	1.60	2217.11	13367.10	1.97	10.00	0.86	1473	32.17	9299.73	4.66	0.82	38.65	9330.25	4.64	0.82
R1	11.93	1241.55	450.55	0.00	8.00	0.60	1420	12.54	441.49	0.00	0.60	19.68	481.21	0.00	0.61
RC1	9.38	1516.41	359.28	0.00	9.00	0.62	1187	11.57	319.44	0.00	0.61	17.19	350.65	0.00	0.63
C2	3.96	662.82	102206.85	0.00	3.00	0.99	3281	131.88	87750.30	0.22	0.93	130.12	87788.36	0.19	0.93
R2	9.33	922.27	4951.41	1.94	2.00	0.75	4665	27.52	4351.50	2.35	0.74	22.21	4384.05	2.29	0.74
RC2	19.57	946.73	6794.47	1.32	2.00	0.76	3929	37.40	6357.23	1.68	0.75	32.65	6395.61	1.62	0.75

Table 4: Details of solutions obtained by the TS for the case with original service times

Set	Solutions of TS							Final Solutions				Simulation			
	Del.	Dist.	Earl.	Over.	Veh.	Obj.	CPU	Del.	Earl.	Over.	Obj.	Del.	Earl.	Over.	Obj.
C1	80.86	970.00	166.52	86.64	10.00	0.52	413	80.86	166.52	86.64	0.52	82.47	167.15	86.58	0.52
R1	1048.63	952.89	809.37	0.00	8.00	0.69	941	1048.63	809.37	0.00	0.69	1055.85	818.07	0.00	0.69
RC1	530.89	1147.57	684.55	0.00	9.00	0.70	1179	530.89	684.55	0.00	0.70	549.66	690.14	0.00	0.70
C2	201.22	798.49	1069.15	140.62	3.00	0.52	1068	201.22	1069.15	140.62	0.52	192.18	1071.99	140.55	0.52
R2	1015.67	802.83	3811.56	14.82	2.09	0.62	1509	1015.67	3811.56	14.82	0.62	1002.24	3823.09	14.77	0.62
RC2	707.52	899.80	5262.41	13.19	2.50	0.70	1704	707.52	5262.41	13.19	0.70	695.42	5281.51	13.14	0.70

Table 5: Details of solutions obtained by the ALNS for the case assuming no service times

Set	Solutions of ALNS							Final Solutions				Simulation			
	Del.	Dist.	Earl.	Over.	Veh.	Obj.	CPU	Del.	Earl.	Over.	Obj.	Del.	Earl.	Over.	Obj.
C1	4.46	2900.79	12993.10	1.08	10.00	0.93	177	31.00	9849.67	3.43	0.89	38.73	9884.04	3.45	0.90
R1	21.63	1349.91	508.71	0.00	8.25	0.64	335	21.82	505.32	0.00	0.64	31.58	543.02	0.00	0.65
RC1	9.66	1681.22	306.42	0.00	10.00	0.67	312	11.22	280.85	0.00	0.67	16.91	311.44	0.00	0.68
C2	0.00	689.98	102854.84	0.00	3.00	1.00	254	25.24	88789.58	0.31	0.94	24.69	88825.13	0.26	0.94
R2	6.74	952.60	4086.18	2.77	2.00	0.75	1693	19.41	3500.21	3.19	0.74	14.25	3528.38	3.14	0.74
RC2	12.43	968.70	5886.27	1.35	2.00	0.74	1638	29.41	5384.37	1.84	0.74	24.49	5412.85	1.78	0.74

Table 6: Details of solutions obtained by the ALNS for the case with original service times

	Solutions of ALNS							Final Solutions				Simulation			
Set	Del.	Dist.	Earl.	Over.	Veh.	Obj.	CPU	Del.	Earl.	Over.	Obj.	Del.	Earl.	Over.	Obj.
C1	1066.54	1614.60	1698.17	93.55	10.56	0.62	104	1066.55	1696.63	93.56	0.62	1073.83	1703.08	93.64	0.63
R1	817.17	1019.31	898.37	0.00	8.75	0.69	268	817.17	898.37	0.00	0.69	832.49	906.38	0.00	0.70
RC1	273.01	1225.32	777.99	0.00	9.88	0.69	242	273.01	777.99	0.00	0.69	288.78	787.11	0.00	0.69
C2	214.54	849.36	1095.07	141.62	3.00	0.52	549	214.54	1095.07	141.62	0.52	203.72	1099.12	141.55	0.52
R2	880.84	777.09	4151.72	14.35	2.09	0.61	628	880.84	4151.72	14.35	0.61	866.03	4168.39	14.28	0.60
RC2	651.12	904.02	5098.30	13.31	2.50	0.69	677	651.12	5098.30	13.31	0.69	636.26	5116.78	13.25	0.69

Table 7: Details of solutions evaluated under time-dependency and stochasticity with original service times

	Stochastic and time-independent solutions						Deterministic and time-independent solutions					
Set	Del.	Dist.	Earl.	Over.	Veh.	Obj.	Del.	Dist.	Earl.	Over.	Veh.	Obj.
C1	22.77	1204.59	1181.31	77.52	11.89	0.55	14.92	828.38	111.04	85.46	10.00	0.50
R1	584.77	1068.37	633.38	0.00	9.17	0.69	94.84	1178.46	879.42	0.00	13.25	0.82
RC1	423.74	1244.95	458.08	0.00	9.75	0.73	110.50	1338.13	558.29	0.00	12.50	0.75
C2	10.07	825.97	1956.11	128.94	4.63	0.52	52.52	589.86	20.48	137.01	3.00	0.49
R2	53.01	914.07	2629.37	8.56	3.27	0.63	3.63	876.92	11066.82	1.24	5.45	0.79
RC2	56.65	1004.60	4750.30	7.55	3.50	0.68	0.31	1003.98	12577.14	0.66	6.25	0.92

7. Conclusions

In this paper, we consider a VRP with soft time windows in which the travel times are modeled with respect to time-dependency and stochasticity. The aim is to construct both reliable and efficient routes by a solution procedure with three main phases. The initial solution constructed in the first phase is improved in the routing phase which is handled by a TS and an ALNS. In the third phase, a post-optimization method is applied to the solutions obtained by the metaheuristics. We formulate the arrival time distributions both exactly (no service times) and approximately (with service times). We conduct our computational experiments on well-known problem instances, and perform comprehensive analyses. Results indicate that the formulations proposed in this paper reliably estimate the distributions and the expected values employed in the mathematical model. Even though this model is rather complex and the arrival time distributions are rather complicated, we have an effective solution procedure which provides very good final solutions in a reasonable amount of time.

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Appendix A. $\boldsymbol{E[R_{jv}]}$ and $\boldsymbol{E[R_{jv}^2]}$

$E[R_{jv}]$ is calculated as follows according to given time periods:

$$\begin{aligned}
E[R_{jv}] &= (b_0 - c_1 t_0)[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_1) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_0)] + c_1\alpha_{jv}\lambda_{jv}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_1) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_0)] \\
&+ (b_1 - c_2 t_1)[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_2) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_1)] + c_2\alpha_{jv}\lambda_{jv}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_2) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_1)] \\
&+ (b_2 - c_3 t_2)[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_3) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_2)] + c_3\alpha_{jv}\lambda_{jv}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_3) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_2)] \\
&+ (b_3 - c_4 t_3)[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_4) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_3)] + c_4\alpha_{jv}\lambda_{jv}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_4) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_3)] \\
&+ (b_4 - c_5 t_4)[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_5) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_4)] + c_5\alpha_{jv}\lambda_{jv}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_5) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_4)]. \qquad \text{(A.1)}
\end{aligned}$$

$E[R_{jv}^2]$ is calculated as follows according to given time periods:

$$\begin{aligned}
E[R_{jv}^2] &= (b_0 - c_1 t_0)^2[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_1) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_0)] + 2(b_0 - c_1 t_0)c_1\alpha_{jv}\lambda_{jv}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_1) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_0)] \\
&+ c_1^2\alpha_{jv}(\alpha_{jv}+1)\lambda_{jv}^2[\Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_1) - \Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_0)] \\
&+ (b_1 - c_2 t_1)^2[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_2) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_1)] + 2(b_1 - c_2 t_1)c_2\alpha_{jv}\lambda_{jv}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_2) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_1)] \\
&+ c_2^2\alpha_{jv}(\alpha_{jv}+1)\lambda_{jv}^2[\Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_2) - \Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_1)] \\
&+ (b_2 - c_3 t_2)^2[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_3) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_2)] + 2(b_2 - c_3 t_2)c_3\alpha_{jv}\lambda_{jv}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_3) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_2)] \\
&+ c_3^2\alpha_{jv}(\alpha_{jv}+1)\lambda_{jv}^2[\Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_3) - \Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_2)] \\
&+ (b_3 - c_4 t_3)^2[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_4) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_3)] + 2(b_3 - c_4 t_3)c_4\alpha_{jv}\lambda_{jv}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_4) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_3)] \\
&+ c_4^2\alpha_{jv}(\alpha_{jv}+1)\lambda_{jv}^2[\Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_4) - \Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_3)] \\
&+ (b_4 - c_5 t_4)^2[\Gamma_{\alpha_{jv},\lambda_{jv}}(t_5) - \Gamma_{\alpha_{jv},\lambda_{jv}}(t_4)] + 2(b_4 - c_5 t_4)c_5\alpha_{jv}\lambda_{jv}[\Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_5) - \Gamma_{\alpha_{jv}+1,\lambda_{jv}}(t_4)] \\
&+ c_5^2\alpha_{jv}(\alpha_{jv}+1)\lambda_{jv}^2[\Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_5) - \Gamma_{\alpha_{jv}+2,\lambda_{jv}}(t_4)]. \qquad \text{(A.2)}
\end{aligned}$$

Appendix B. $\mu_{ij}(t)$ and $E[G_{ij}^2(t)]$

$\mu_{ij}(t)$ is calculated as follows according to given time periods:

$$\begin{aligned}
\mu_{ij}(t) &= (b_0 - b - c_1(t_0 - t))[\Gamma_{\alpha d_{ij},\lambda}(t_1 - t) - \Gamma_{\alpha d_{ij},\lambda}(t_0 - t)] \\
&+ c_1 \alpha d_{ij} \lambda [\Gamma_{\alpha d_{ij}+1,\lambda}(t_1 - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_0 - t)] \\
&+ (b_1 - b - c_2(t_1 - t))[\Gamma_{\alpha d_{ij},\lambda}(t_2 - t) - \Gamma_{\alpha d_{ij},\lambda}(t_1 - t)] \\
&+ c_2 \alpha d_{ij} \lambda [\Gamma_{\alpha d_{ij}+1,\lambda}(t_2 - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_1 - t)] \\
&+ (b_2 - b - c_3(t_2 - t))[\Gamma_{\alpha d_{ij},\lambda}(t_3 - t) - \Gamma_{\alpha d_{ij},\lambda}(t_2 - t)] \\
&+ c_3 \alpha d_{ij} \lambda [\Gamma_{\alpha d_{ij}+1,\lambda}(t_3 - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_2 - t)] \\
&+ (b_3 - b - c_4(t_3 - t))[\Gamma_{\alpha d_{ij},\lambda}(t_4 - t) - \Gamma_{\alpha d_{ij},\lambda}(t_3 - t)] \\
&+ c_4 \alpha d_{ij} \lambda [\Gamma_{\alpha d_{ij}+1,\lambda}(t_4 - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_3 - t)] \\
&+ (b_4 - b - c_5(t_4 - t))[\Gamma_{\alpha d_{ij},\lambda}(t_5 - t) - \Gamma_{\alpha d_{ij},\lambda}(t_4 - t)] \\
&+ c_5 \alpha d_{ij} \lambda [\Gamma_{\alpha d_{ij}+1,\lambda}(t_5 - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_4 - t)]
\end{aligned} \tag{B.1}$$

$E[G_{ij}^2(t)]$ is calculated as follows according to given time periods:

$$\begin{aligned}
E[G_{ij}^2(t)] &= (b_0 - b - c_1(t_0 - t))^2[\Gamma_{\alpha d_{ij},\lambda}(t_1 - t) - \Gamma_{\alpha d_{ij},\lambda}(t_0 - t)] \\
&+ 2(b_0 - b - c_1(t_0 - t))c_1 \alpha d_{ij} \lambda [\Gamma_{\alpha d_{ij}+1,\lambda}(t_1 - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_0 - t)] \\
&+ c_1^2 \alpha d_{ij}(\alpha d_{ij} + 1)\lambda^2 [\Gamma_{\alpha d_{ij}+2,\lambda}(t_1 - t) - \Gamma_{\alpha d_{ij}+2,\lambda}(t_0 - t)] \\
&+ (b_1 - b - c_2(t_1 - t))^2[\Gamma_{\alpha d_{ij},\lambda}(t_2 - t) - \Gamma_{\alpha d_{ij},\lambda}(t_1 - t)] \\
&+ 2(b_1 - b - c_2(t_1 - t))c_2 \alpha d_{ij} \lambda [\Gamma_{\alpha d_{ij}+1,\lambda}(t_2 - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_1 - t)] \\
&+ c_2^2 \alpha d_{ij}(\alpha d_{ij} + 1)\lambda^2 [\Gamma_{\alpha d_{ij}+2,\lambda}(t_2 - t) - \Gamma_{\alpha d_{ij}+2,\lambda}(t_1 - t)] \\
&+ (b_2 - b - c_3(t_2 - t))^2[\Gamma_{\alpha d_{ij},\lambda}(t_3 - t) - \Gamma_{\alpha d_{ij},\lambda}(t_2 - t)] \\
&+ 2(b_2 - b - c_3(t_2 - t))c_3 \alpha d_{ij} \lambda [\Gamma_{\alpha d_{ij}+1,\lambda}(t_3 - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_2 - t)] \\
&+ c_3^2 \alpha d_{ij}(\alpha d_{ij} + 1)\lambda^2 [\Gamma_{\alpha d_{ij}+2,\lambda}(t_3 - t) - \Gamma_{\alpha d_{ij}+2,\lambda}(t_2 - t)] \\
&+ (b_3 - b - c_4(t_3 - t))^2[\Gamma_{\alpha d_{ij},\lambda}(t_4 - t) - \Gamma_{\alpha d_{ij},\lambda}(t_3 - t)] \\
&+ 2(b_3 - b - c_4(t_3 - t))c_4 \alpha d_{ij} \lambda [\Gamma_{\alpha d_{ij}+1,\lambda}(t_4 - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_3 - t)] \\
&+ c_4^2 \alpha d_{ij}(\alpha d_{ij} + 1)\lambda^2 [\Gamma_{\alpha d_{ij}+2,\lambda}(t_4 - t) - \Gamma_{\alpha d_{ij}+2,\lambda}(t_3 - t)] \\
&+ (b_4 - b - c_5(t_4 - t))^2[\Gamma_{\alpha d_{ij},\lambda}(t_5 - t) - \Gamma_{\alpha d_{ij},\lambda}(t_4 - t)] \\
&+ 2(b_4 - b - c_5(t_4 - t))c_5 \alpha d_{ij} \lambda [\Gamma_{\alpha d_{ij}+1,\lambda}(t_5 - t) - \Gamma_{\alpha d_{ij}+1,\lambda}(t_4 - t)] \\
&+ c_5^2 \alpha d_{ij}(\alpha d_{ij} + 1)\lambda^2 [\Gamma_{\alpha d_{ij}+2,\lambda}(t_5 - t) - \Gamma_{\alpha d_{ij}+2,\lambda}(t_4 - t)]
\end{aligned} \tag{B.2}$$

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