THE TIME-VARYING BEHAVIOR OF REAL INTEREST RATES: A RE-EVALUATION OF THE RECENT EVIDENCE

Basma Bekdache[§]
Department of Economics
Wayne State University
Detroit, MI, 48202
Phone: (313) 577-3231
Fax: (313) 577-0149
Email: BBEKDAC@ECON.WAYNE.EDU

Prepared for the
Second International Conference of the
Society for Computational Economics
Geneva, Switzerland
26-28 June 1996

(Preliminary, please do not quote)

[§]I am grateful to Christopher Baum for many helpful comments and discussions on earlier drafts of this paper. I also wish to thank Chang-Jin Kim for providing me with his Gauss computer programs. The usual disclaimer applies.

ABSTRACT

Several recent studies have documented structural shifts in the stochastic process of the ex post (or realized) real interest rate (see for example Huizinga and Mishkin (1986) (HM hereafter), Antoncic (1986), Garcia and Perron (1994) (GP hereafter), and Evans and Lewis (1995)). Given its central role in economics, the changing behavior of real interest rates over the past three decades has attracted a good deal of attention in the economics literature, especially in relation to recent monetary and fiscal policy regime changes.

This paper investigates sources of shifts in real rates by incorporating systematic time variation in the parameters and variance shifts in the equation specified by HM to predict the ex ante real rate. HM use a general specification in which the ex ante real rate depends on the nominal interest rate, the inflation rate and a supply shock variable. This approach permits us to simultaneously model two types of shifts in the stochastic process of real rates: 1) shifts in the coefficients of the relationship between the ex ante real rate and its determinants and 2) unconditional shifts in the variance of the stochastic process. The model is estimated using Kim's (1993,1994) methodology combining dynamic linear models with Markov switching heteroscedasticity.

Using monthly data for the period between January 1961 and January 1991, I find considerable support for a general time-varying parameter model. The results are broadly consistent with the GP study in which the ex ante real rate is characterized as a three-state Markov switching model. However, the results from our longer sample indicate that the mean and variability of the ex ante real rate change again after 1986 where the GP sample ends. This highlights the importance of modeling continual change in the ex ante real rate in terms of other economic variables rather than relying on a statistical characterization that only permits a limited number of discrete jumps in the mean of the process.

I examine the contribution of each explanatory variable to the mean of the ex ante real rate using the estimated coefficients at each time period. I find that the nominal interest rate has the largest effect on the mean of the ex ante real rate over most of the sample period and that this relationship becomes stronger after 1981. The overall predictive ability of the variables is examined using a period-by-period Bayes factor that compares the general model to the random walk model. Interestingly, I find that the predictability of the ex ante real rate diminishes considerably after 1986. Finally, the estimates of the model are used to infer a time-series for expected inflation. The estimates indicate that the Fisher effect is weak in the post-1981 period and that this period is characterized by unusually high nominal rates and low expected inflation.

1. Introduction

The unusual increase in real interest rates in the late 1980's spawned a great deal of interest in the impact of the so-called Federal Reserve monetarist experiment of October 1979 (when the Federal reserve announced a change in operating procedures from partially targeting interest rates to targeting nonborrowed reserves) on the time series behavior of real interest rates. The empirical evidence on whether a shift in the stochastic process of real interest rates occurred at the time of the change in Fed operating procedures is somewhat mixed. Antoncic (1986) estimates the real interest rate using an unobserved component model in which the real rate is assumed to follow a random walk process. Using data that covers the period from January 1965 to December 1984, the author rejects the hypothesis that the variance of the real rate process is the same before and after October 1979. However, given the finding that the most likely break time occurs in April 1980, the study concludes that the Fed regime change could not be directly associated with the rise in the level and volatility of real rates. Using a more general specification in which ex ante real rates are modeled as a function of a set of economic variables, Huizinga and Mishkin (1986) (HM hereafter) conclude that significant shifts in the stochastic process of real rates coincide with the Fed regime switch dates.

In a more recent study, Garcia and Perron (1994) (GP hereafter) apply Hamilton's (1988,1989) state dependent Markov-switching model to ex post real interest rates and inflation. They find that, for the 1961-1986 period, the real rate process is best characterized by a model in which the mean and variance vary over three different regimes. Essentially, they "..indicate the presence of three segments with different means.." (Garcia and Perron (1994, p 3.) and variances in the time series of ex post real rates. Similar to the results in Antoncic (1986), regime shifts occur at the beginning of 1973 and the middle of 1981, corresponding more closely to the timing of the rise in oil prices and the federal budget deficit rather than to the change in the Fed operating procedures.

The GP study improves on previous work by using econometric methods that are designed to detect structural change in the stochastic process governing real interest rates. However, its focus is limited to the statistical properties of the real rate as derived solely from its past history and is thus uninformative about potential economic determinants of the documented shifts in the real rate process. Alternatively, the general approach taken in the HM study offers a framework suitable for such an investigation. HM estimate the real interest rate by specifying an equation relating the ex post real interest rate to the nominal interest rate, the inflation rate (lagged one and two months) and a supply shock variable (lagged one month)[1]. Although their regression results indicate that these variables are good predictors of the ex post real rate, they reject the hypothesis of coefficient stability during their study periods. Similarly to Antoncic (1986), they use Quandt's (1958) procedure to find the timing of the break points that maximize the likelihood function of the real rate process. Their findings are that the regime shifts occur in October 1979 and October 1982, the dates corresponding to the Fed regime switch.

This paper reconsiders and extends the evidence in the HM study by modeling the real rate process using Kim's (1993,1994) methodology of combining a time-varying parameter model with Markov switching conditional heteroscedasticity. Given the previous findings of coefficient instability and changes in volatility, this approach systematically accounts for changing regression coefficients as well as for shifts in the variance of the real rate process which are documented in previous studies. This model differs from the GP study mainly by allowing the mean of the ex ante real rate to depend on other economic variables rather than just on its past history. Another difference is that the mean continually changes over time rather than undergoing a discrete number of shifts. As compared to the random walk model used in Antoncic (1986), this approach is more general since, in addition to modeling continuous time variation in the mean, it permits additional economic factors to potentially predict movements in the real rate while simultaneously modeling changes in the variance of the real rate process. In this framework, the estimated coefficients can be used to track the effect of each explanatory variable on the conditional mean of the real rate as it varies over time and can thus be very informative about the underlying sources of the documented shifts in the mean of the process. Further, the importance of the variables in explaining the real rate can be assessed on a period by period basis by comparing the predictive ability of the dynamic HM model to the random walk model used in Antoncic (1986) using a ratio of likelihoods computed at each time period.

The rest of the paper is organized as follows. Section 2 introduces the equations to be estimated and briefly reviews the approaches taken in previous studies. The data and estimation methodology used in this study are presented at the end of section 2 . Section 3 presents the empirical results from estimating the HM model in a dynamic framework and compares it to the random walk model used in Antoncic (1986). This section also investigates the relevance of each variable to the shifts in the mean of the real rate process. The predictive power of the explanatory variables is assessed using a period-by-period Bayes factor that compares the dynamic HM model to the random walk

[1]HM arrive at this specification by eliminating insignificant variables from a general distributed lag specification for the ex post real rate. A more complete discussion of the HM study is deferred to section 2.3.

model. A brief discussion on derived estimates of expected inflation concludes section 3. Summary remarks and extensions for future research are presented in section 4.

2. Methodology

The ex ante real interest rate on one-period nominal bond is given by

$$rr_t = i_t - \pi_t^e, \tag{1}$$

where rr_t and i_t denote the ex ante real interest rate and the nominal interest rate on the one-period bond respectively. π_t^e is expected inflation from time t to $t+1$. Given that we do not observe inflationary expectations, the ex ante real rate is generally unobservable. The assumption of rational expectations is often used to replace the expectation term by actual inflation and a random forecast error. We can rewrite equation (1) by substituting for π_t^e using $\pi_t = \pi_t^e + e_t$, where e_t is the inflation forecast error assumed to be unforecastable using information available at time t:

$$rr_t = i_t - \pi_t + e_t. \tag{2}$$

In equation (2), $i_t - \pi_t$ is the observed realized real return from time t to $t+1$ and is usually referred to as the *ex post* real interest rate. Using this definition we have

$$eprr_t = rr_t - e_t. \tag{3}$$

Since the forecast error e_t is expected to equal zero under the assumption of rational expectations, equation (3) shows that analyzing the time series behavior of the ex post real rate is equivalent to analyzing the ex ante real rate. Next, I present a brief review of previous studies' estimates of Equation (3).

2.1 The Ex ante real rate as a random walk process

In Antoncic (1986), the ex ante real interest rate is assumed to follow a random walk process. Adding this assumption to equation (3), the model can be expressed in the state-space form as follows

$$\begin{aligned} eprr_t &= rr_t - e_t, \quad \text{(Observation Equation)} \\ rr_t &= rr_{t-1} + v_t, \quad \text{(Transition Equation)} \\ e_t &\sim N(0, \sigma_e^2), \end{aligned} \tag{4}$$

$$v_t \sim N(0, \sigma_v^2).$$

In this model, rr_t is an unobserved state variable which is assumed to evolve according to the transition (or system) equation in (4) with v_t representing the stochastic element in the evolution of rr_t. Under the stated distributional assumptions, the Kalman filter recursions are used to obtain maximum likelihood estimates of the time series for rr_t. To account for potential structural change in the variance of ex ante real rates, Antoncic (1986) relaxes the assumption of constant variance for the transition equation error, v_t. The author estimates σ_v^2 for two subsamples before and after October 1979 and is able to reject the hypothesis that the variances in the two subintervals are equal. However, using Quandt's (1958) maximum likelihood procedure[2], she finds that the most likely time for the shift in the variance occurs in April 1980 rather than in October 1979 when the Fed changed its operating procedures.

2.2 A Markov switching model of the ex ante real rate

Garcia and Perron (1994) model the ex post real interest rate using the following autoregressive specification.

$$\begin{aligned} eprr_t - \mu(S_t) &= \phi_1[eprr_{t-1} - \mu(S_{t-1})] + \phi_2[eprr_{t-2} - \mu(S_{t-2})], + \sigma(S_t)\epsilon_t, \\ \mu(S_t) &= \alpha_0 + \alpha_1 S_{1t} + \alpha_2 S_{2t}, \\ \sigma(S_t) &= \omega_0 + \omega_1 S_{1t} + \omega_2 S_{2t}, \end{aligned} \tag{5}$$

where μ and σ are the regime dependent mean and variance of the process respectively. The variable S_t can take the value 0, 1 or 2 to index three potential regimes[3]. Specifically, $S_{it} = 1$ when $S_t = i$ and 0 otherwise. Following Hamilton (1988, 1989, 1994), GP model the evolution of S_t as a first-order Markov process with transition probabilities given by

$$\begin{aligned} & p_{ij} = Pr[S_t = j | S_{t-1} = i], \\ & \sum_{j=0}^{2} p_{ij} = 1. \qquad i, j = 0, 1, 2 \end{aligned} \tag{6}$$

[2] The procedure consists of choosing a set of breakpoints that maximizes a likelihood ratio statistic that compares the maximized likelihood function assuming there are no breaks to the maximized likelihood function for a given set of breakpoints.

[3] GP arrive at this specification after a thorough selection procedure and a number of sensitivity analysis tests. The reader is referred to Garcia and Perron (1994) for more details.

The α's, ω's , ϕ's and p_{ij}'s are model parameters to be estimated using an algorithm proposed by Hamilton (1989). Essentially, the idea is to estimate the parameters and transition probabilities by maximizing the likelihood function conditional on a specified number of states[4]. The algorithm is also useful for making inferences about the various regimes using the "filter" probabilities of each state, which are the probabilities of being in state 0, 1 or 2 at time t, given the information set at time t.

2.3 A more general model of the ex ante real rate

The specification used in Huizinga and Mishkin (1986) (HM hereafter) starts from the assumption that the ex ante real rate can be described as a linear function of a set of variables contained in the information set available at time t so that we can write rr_t as follows:

$$rr_t = X_t\beta + u_t, \tag{7}$$

where X_t is a matrix of variables used to predict rr_t, β is a vector of estimable coefficients, and u_t is the residual from the linear projection of rr_t onto X_t and is by definition orthogonal to X_t . Substituting (7) into (3), we get

$$eprr_t = X_t\beta + u_t - e_t = X_t\beta + \epsilon_t. \tag{8}$$

Recall that e_t is the forecast error for inflation, which under the assumption of rational expectations is uncorrelated with any information available at time t. Given that the variables in X_t are a subset of the information set used to predict inflation, e_t and the composite error term $\epsilon_t = u_t - e_t$ are also uncorrelated with X_t. Therefore, the coefficients in β can be consistently estimated using an ordinary least squares (OLS) regression of the ex post real rate, $eprr_t$, on X_t. The fitted values from this regression are used as estimates of the ex ante real rate,

$$\hat{r}r_t = X_t\hat{\beta}, \tag{9}$$

where $\hat{\beta}$ denotes the OLS estimate of β[5].

[4] For more details on Markov processes and Markov switching models, the reader is referred to Hamilton (1994, pp 677-703).

[5] HM note that although we cannot directly assess the accuracy of $\hat{r}r_t$ from the variance of u_t (since it is not identifiable), we can examine the composite fitted residuals $\hat{\epsilon}_t$ to see if they are white noise as a diagnostic check on the specification of X_t. To see this note that since e_t is assumed to be serially uncorrelated under rational expectations, serial correlation in $\hat{\epsilon}_t$ would occur if u_t is serially correlated and its variance is large enough to dominate the variance of e_t. Alternatively, given the autocovariances of e_t and u_t, $\hat{\epsilon}_t$ may be uncorrelated if the variance of u_t is small relative to that of e_t. Thus, the model specification is tested jointly with the rational expectations assumption

HM build their model starting from a specification for X_t that includes a distributed lag of ex post real rates, inflation rates and other variables such as industrial production, in addition to economic variables known at time t, such as the current nominal interest rate and a measure of supply shocks. Since the nominal rate is comprised of the ex ante real rate and expected inflation, its inclusion in X_t is meant to capture the relationship between expected inflation and the ex ante real rate as well as ".. other, hard to measure, influences on the ex ante real rate." (HM, p 238.). After eliminating insignificant variables, HM's final specification for X_t consists of a constant term, the nominal interest rate (i_t), the inflation rate lagged one and two months (π_{t-1} and π_{t-2}), and a supply shock variable lagged one month ($supply_{t-1}$)[6]. The supply shock variable is calculated using the relative price of fuel and related products to the overall producer price index.

HM test for the constancy of the real interest rate using the standard Chow test (F test) for coefficient stability. They find that two significant shifts in the real rate process (as indicated by shifts in the coefficients in the real rate equation) occur in October 1979 and October 1982. In addition, they find that the coefficients after October 1982, when the Fed returned to its pre-October 1979 procedures, are statistically different from those obtained in the pre-October 1979 period.

The HM results of coefficient instability suggest that the model should be estimated in a time-varying parameter setting where the coefficients can systematically vary over time. As HM explain in the context of parameter instability, movements in the coefficients of equation (8) potentially reflect a change in the way information is used to predict rr_t which would occur if agents update their beliefs about the relationships between economic variables. The next section introduces the estimation methodology proposed in this study.

2.4 The TVP model with conditional heteroskedasticity

To account for instability in the regression coefficients of equation (8) as well for changes in the variance of the real rate process documented in Antoncic (1986), I propose to employ the following

[6] Variations on equation (8) have been used in previous studies to test for the constancy of the real rate and to examine the correlations of real rates with nominal rates and expected inflation (e.g. Fama (1975), Mishkin (1981,1988) and others). In a recent study, Mishkin (1995) estimates the following two equations :

$$\begin{aligned} eprr_t &= \alpha + \beta i_t + u_t \\ eprr_t &= \alpha + \beta \pi_t^e + u_t \end{aligned}$$

In the second equation, the nominal rate i_t, and two lags of the inflation rate are used as instruments for the expected inflation term, π_t^e. Tests for the constancy of the real rate consist of testing whether $\beta = 0$. In the first equation this is equivalent to testing whether the coefficient on the nominal rate is equal to 1 in an equation of the Fisher " type" given by: $\pi_t = -\alpha + (1 - \beta) i_t - u_t$.

model developed by Kim (1993,1994):

$$
\begin{aligned}
&eprr_t = X_t\beta_t + \epsilon_t, \qquad t = 1, 2, \ldots, T \\
&\beta_t = \beta_{t-1} + v_t, \\
&\epsilon_t \sim N(0, h_t), \\
&v_t \sim N(0, Q), \\
&h_t = \sigma_0^2 + (\sigma_1^2 - \sigma_0^2)S_t,
\end{aligned}
\tag{10}
$$

where $X_t = (1, i_t, \pi_{t-1}, \pi_{t-2}, Supply_{t-1})$ is the vector of explanatory variables as previously defined, and $\beta_t = (\beta_{0t}, \beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t})$ is a vector of time-varying parameters relating the ex post real rate to the explanatory variables. Q is the variance-covariance matrix of the errors in the coefficients' transition equation. The potential for shifts between high and low variance states in the error of the real rate equation (σ_1^2 and σ_0^2 respectively) is modeled using a two-state Markov switching process[7]. The unconditional variance of the forecast errors, h_t, is governed by the unobserved discrete-valued state variable S_t which takes on a value of 1 in one regime (high variance) and 0 in the other (low variance)[8]. Following Hamilton (1989), S_t, evolves according to a Markov chain with the following transition probabilities:

$$
\begin{aligned}
&Pr[S_t = 1|S_{t-1} = 1] = p_{11}, \\
&Pr[S_t = 0|S_{t-1} = 1] = 1 - p_{11}, \\
&Pr[S_t = 1|S_{t-1} = 0] = 1 - p_{00}, \\
&Pr[S_t = 0|S_{t-1} = 0] = p_{00}
\end{aligned}
$$

The model's unknown parameters, σ_0^2, σ_1^2, p_{11}, p_{00}, and the elements of Q are estimated via maximum likelihood using an approximation developed in Kim (1993)[9]. The advantage of using this approach to model ex ante real rates is that we can simultaneously analyze two types of shifts in the real rate process. The first type of shift arises due to changes in the coefficients (β's) in the

[7]Although GP use a three-state model for the mean and the variance, their results show that the variance is virtually the same in two of the three states. This, coupled with the Antoncic (1986) findings supporting two variance regimes suggest that the simple two-state specification is a reasonable starting point.

[8]According to Kim (1994), Markov switching heteroscedasticity can be viewed as an alternative to ARCH type models. However he notes that an important difference between the two is that ARCH models assume that the unconditional variance is constant whereas it is subject to structural change in the Markov switching model.

[9]The reader is referred to Kim (1993a,1993b) for details on the estimation technique.

equation used to predict the real rate while changes in the variance of the process (h_t) constitute another type of shift. This approach resembles the GP paper in modeling the variance with Markov switching heteroscedasticity. However as previously noted, GP use a univariate specification with discrete shifts in the mean of the real rate process while this study models continuous time variation in the mean of ex ante real rates as a function of a number of economics variables. This provides for a more general framework for studying the underlying sources of shifts in the real rate process. I now turn attention to a description of the data and empirical results.

3. Empirical Results

3.1 Data and OLS results

The nominal yield, i_t, is a one-month zero coupon interest rate, expressed in percent per month, obtained from the term structure data set provided by McCulloch and Kwon in their 1993 working paper. The spot (or zero coupon) yields in this data set are derived from a tax-adjusted cubic spline discount function applied to government coupon bonds. The one month inflation rate is calculated from a consumer price index (CPI) series that adjusts for housing costs on a rental equivalence basis which is available from the Bureau of Labor Statistics (BLS). Following HM, the supply shock variable is computed as the log of the relative price and related products in the producer price index, also available from the BLS. The data consist of monthly observations for the period from January 1961 to January 1991 for a total of 373 observations.

Tables 1 presents the OLS regression results from estimating equation (8) over the whole sample period (1/60-1/91) as well as over the three subsamples corresponding to the periods before, during and after the change in Federal Reserve policy in October 1979 Through October 1982. Starting with the whole sample results, we can see that, consistent with HM, all of the variables in the regression are significant at the 1% confidence level. The subperiod regressions reveal that significant positive correlation between the nominal interest rate, i_t, and the ex post real rate is only present in the post-November 1982 period of the sample. This implies that the correlation between inflation and the nominal rate becomes weaker in the last subsample[10]. Consistent with HM's results, the coefficient on the supply shock variable is negative in the pre-October 1979 period but is positive for the other subsamples[11]. Overall, the results in Table 1 support the relevance of these variables

[10]To see this, note that equation (8) can be written as follows :

$$\pi_t = -\beta_0 + (1 - \beta_1)i_t - \beta_2\pi_{t-1} - \beta_3\pi_{t-2} - \beta_4 supply_{t-1} - \epsilon_t$$

so that a small coefficient on i_t (β_1) implies a high correlation between inflation and the nominal interest rate.

[11]Wilcox (1983) introduced the supply variable to tests of the Fisher relationship and hypothesized that a rise

to explaining the ex post real rate. The different results over the subsamples suggests the need for a dynamic framework that allows for structural change and permits the predictive ability of the explanatory variables to vary over time. The results from estimating such a model are presented in the next section.

3.2 TVP model estimates

Table 2 presents maximum likelihood estimates of the time-varying parameter (TVP) model shown in (10). We can see from the table that the variance in the high volatility state ($\sigma_1^2 = 0.139$) is six times larger than the variance during low volatility periods ($\sigma_1^2 = 0.0238$). Figure 1 shows a plot of the estimated conditional variance of the errors as given by $(h_t|I_{t-1}) = \sigma_0^2 + (\sigma_1^2 - \sigma_0^2)Pr[S_t = 1||I_{t-1}]$, with I_{t-1} denoting the information set up to time $t-1$. Periods of high volatility occur in August 1973, February 1977, July 1980 and June 1982. Volatility is also high around March 1986 and January 1990. The episodes of high volatility are not as persistent as those of low volatility. This is indicated by a probability of remaining in state 1 (p_{11}) of only about 60% as compared to a greater than 90% probability of remaining in state 0 (p_{00}). As a check on the specification of the model, we examine the standardized residuals and the square of the standardized forecast errors for serial correlation. The Q-statistics indicate that the model is able to capture most of the dynamics of the ex post real interest rate process ($Q-$statistics are: $Q(12) = 16.9, Q(24) = 32, Q(36) = 55.58$ for the standardized residuals and $Q(12) = 10.09, Q(24) = 17.8, Q(36) = 23.6$) although there does appear to be some serial correlation in the standardized residuals at lag 36. The variance of the process appears to be adequately modeled since there is no serial correlation in the squared standardized residuals.

The estimated (predicted) values of the ex ante real interest rate , i.e., $\hat{r}r_t = X_t\hat{\beta}_t$, are plotted and compared to the ex post real rate in figures 2. It is readily apparent that the mean and the volatility of ex ante real interest rates have changed dramatically over the sample period. Consistent with previous studies, the estimates indicate that ex ante real rates are positive and relatively low until the beginning of 1974 when they turn negative and remain negative until November 1980. They steadily increase after 1981 and reach their highest level of an annualized 9.78% in March 1982. Although ex ante real rates remain positive until the end of the sample, they decrease considerably in May 1986 after which they do not exceed 4.3%. Average values for the ex ante and

in the relative price of energy reduces investment demand and accordingly reduces ex ante real interest rates. The author finds evidence supportive of this negative relationship by estimating an equation relating the nominal interest rate to expected inflation and the supply shock variable among others for a sample ending in 1979. HM note that the positive coefficient in the post-November period could signal that the supply variable is proxying for other factors that may affect real rates ex post real rates over these distinguishable subperiods are shown in Table 3. These results are consistent with GP who find that, over their sample period which ends in 1986, the ex post real rate can be characterized by three different means. However, given that the mean changes again in the last four years of our sample, it is questionable whether the GP three-state model would still be valid for a longer sample[12]. The mean values for each subperiod should be interpreted with caution since the ex ante real rate fluctuates widely within each subperiod as is apparent in the figures. For instance, ex ante real rates become as low as -5.86% in the February 1974 to November 1980 period.

The time-varying coefficients of each of the explanatory variables in equation (10) are shown in Figures 3a-3d. As anticipated from the OLS results, all of the coefficients vary a great deal over the whole sample period. The real rate is positively correlated with the nominal interest rate for most of the sample period with the exception of the 1973 to 1980 period. Interestingly and perhaps not surprisingly, the coefficient on the lagged supply shock variable and its variability decrease considerably after 1975. Although the OLS results indicate that the variable is significant in all subperiods, its contribution to the mean of the ex ante real rate becomes very small after 1975. The same pattern is true of the coefficient on the constant which captures autonomous changes in the mean of the ex ante real interest rate.

We can focus on the contribution of each variable to the mean of the ex ante real interest rate by looking at the estimated coefficient at each t multiplied by the value of the variable at time period t. Decomposing the mean of the ex ante real rate into its components is useful for analyzing the role each of the explanatory variables potentially plays in accounting for the various structural shifts which occurred over the sample period. Figures 4a through 4c show plots of the effects from inflation and the nominal interest rate on the ex ante real rate. It is obvious from the graphs that the nominal rate can account for a great deal of the movements in the ex ante real rate throughout the sample although much more so after 1980. The nominal rate closely follows the movement in the ex ante real rate even in the early part of the 1973-1980 period when they become negatively correlated. Note that the role of inflation in explaining the negative ex ante real rate becomes more pronounced only towards the end of 1975 and the beginning of 1976. In figure 4c we can see that, after 1980, the high level of ex ante real rates is mainly due to high levels of nominal rates. This finding is, as expected from the post-1982 OLS results, reflective of a weakening of the relationship between inflation and the nominal interest rate as seen by the increase in the coefficient on the nominal interest rate in figure 3b. Recall that a large coefficient on the nominal rate in equation

[12] GP acknowledge this limitation of their model in the paper.

(10) implies that the coefficient on the nominal rate in a regression of inflation on the nominal rate and the rest of the variables is small and less than one. Thus, a 1% rise in the nominal rate is associated with a smaller increase in inflation (or equivalently a 1% increase in inflation is matched by a larger than 1% increase in the nominal rate) which leads to the increase in the real rate. The effect from inflation is very small but can account for the pattern of variability in ex ante real rates especially around September 1982 and June 1986. This strong relationship between the ex ante real rate and the nominal rate seems to contradict the conclusion from GP that the nominal rate is a poor predictor of the ex ante real rate over the whole sample (GP, p. 8)[13]

The results in this section reaffirm conclusions from previous studies that the stochastic process of ex ante real rates has undergone important changes over time. We also found that the impact of the explanatory variables on the mean of the ex ante real rates varies over the sample. The next section further examines the role of these variables in predicting real interest rates by performing a period-by-period comparison between the model in (10) and the random walk model used in Antoncic (1986). The idea is to test whether the model's ability to predict ex ante real rates changes as a result of the various shifts that occurred in the stochastic process of real interest rates.

3.3 Predictability of the ex ante real rate

Recall from section 2.1 that Antoncic (1986) estimates the random walk model in (4) by separating the sample into subintervals in order to allow for the variance of the real rate to vary overtime. However, we can systematically model changes in the variance of the unobserved component in (4) using Kim (1993)'s methodology which generalizes unobserved component models to incorporate Markov-switching heteroscedasticity[14]. With this modification, we can rewrite the model in (4) as follows:

$$
\begin{aligned}
eprr_t &= rr_t - e_t, && \text{(Observation Equation)} \\
rr_t &= rr_{t-1} + v_t, && \text{(Transition Equation)} \\
e_t &\sim N(0, \sigma_e^2), && \\
v_t &\sim N(0, q_t), &&
\end{aligned}
\tag{11}
$$

[13]Evans and Watchel (1992) arrive at similar conclusions using a generalized Fisher equation which they derive from a Consumption CAPM model with taste shocks. They find that monthly changes in the nominal rate are mostly indicative of changes in real rates rather than expected inflation especially in the mid to late 1980's. Their finding that the increase in the nominal interest rate in late 1970 early 1981 was due to a very large increase in real rates is consistent with the ex ante real rate estimates shown in figure 2. These results have the policy implication that short-term rises in the nominal rate may signal changes in liquidity rather than inflationary expectations.

[14]The reader is referred to Kim (1993) for details on the estimation methodology.

$$q_t = \sigma_{v0}^2 + (\sigma_{v1}^2 - \sigma_{v0}^2)S_t.$$

Equation (11) augments the model in (4) by specifying the movement in the unconditional variance of the real rate process as given by q_t. Similarly to the model presented in section 2.4, the variance switches between the high and low states following the unobserved indicator variable S_t which evolves according to a first-order Markov process[15]. In this model, several variance shifts may occur at any time during the sample period, whereas estimating σ_v^2 from (4) over two subsamples effectively implies that the variance changes only once at the time of the breakpoint.

The estimated ex ante real rate is plotted in figure 5a and is compared to the ex post real rate and the ex ante real rate from the TVP model in figure 5b. The random walk model produces an overall smoother estimate of the ex ante real rate but appears to be more sensitive to large changes in the ex post real rate[16]. This is most apparent during the high volatility periods occurring in 1973-4, mid-1981 and 1986. Those periods are identified by the model as high variance states as shown by the estimated state probabilities plotted in figure 6. Consistent with Antoncic (1986), ex ante real rates increase to abnormal levels towards late 1980 early 1981.

All of the results presented thus far indicate that the relationship between the ex ante real rate and the explanatory variables in equation (10) is time-varying. In order to answer the question of whether the predictability of the ex ante real rate is affected by the regime shifts that occurred over the sample period, we conduct a period-by-period analysis using a Bayes Factor (BF) involving the ratio of model likelihoods between the augmented random walk model in (11)and the TVP model in (10). Suppose that we consider two models for a variable y_t which differ only in the explanatory variables contained in the matrix X_t. Denoting the predictive density of the first model by $p_1(y_t|I_{t-1})$ and the second model by $p_2(y_t|I_{t-1})$, then the relative likelihood of model 1 versus model 2 based on the observation y_t, at time t is simply the ratio

$$H_t = p_1(y_t|I_{t-1})/p_2(y_t|I_{t-1}) \tag{12}$$

This likelihood ratio, alternatively called *Bayes' Factor*, provides a basic measure of predictive performance of model 1 relative to model 2. More generally, this ratio can be formed to compare any two alternative hypotheses H_1 and H_2. Multiplying (12) by the prior odds $Pr(H_1|I_0)/Pr(H_2|I_0)$ yields the familiar posterior odds ratio in favor of model 1. Given equal prior probabilities on the two hypotheses, the data is more in accord with H_1 than H_2 when H_t exceeds unity. On the log scale, a positive value of the log of H_t is supportive of model 1 whereas a negative value indicates

[15]The reader is referred back to sections 2.2 and 2.4 for a more complete exposition of this material.
[16]Estimates of this model are available upon request.

support for model 2. The ratio in (12) provides evidence for or against a given hypothesis in a given time period t based on the observation y_t. More generally, the overall likelihood of the observations $y_t, y_{t-1}, \ldots, y_k$,($k = 1, \ldots, t$) is given by:

$$\begin{aligned} H_t(t-k+1) &= \prod_{r=k}^{t} H_r \\ &= p_1(y_t, y_{t-1}, \ldots, y_k | I_{k-1}) / p_2(y_t, y_{t-1}, \ldots, y_k | I_{k-1}) \end{aligned} \quad (13)$$

A Cumulative Log Bayes Factor (CLBF) can be obtained by taking the log of expression (13). The traditional interpretation for the CLBF introduced by Jeffreys (1961) is that evidence in favor of model 1 is indicated by a value of 1, with a value of 2 or more indicating strong evidence. Alternatively, a value of -1 indicates evidence in favor of model 2 with a value of -2 or more indicating strong evidence.

Thus, the hypothesis that the nominal rate, the lagged inflation rate and the supply shock variable help predict the ex ante real rate (model 1=TVP/HM) is compared to the alternative hypothesis that these variables do not contain information useful for forecasting the ex ante real rate (model 2= augmented random walk) using the LBF and CLBF measures given by the log of expressions (12) and (13) respectively. The CLBF shown in figure 7 indicates that there is very strong evidence in favor of the TVP model over most of the sample period . Recall that a value of 2 or more indicates strong evidence in favor of model 1. By looking at the LBF in figure 8, we can identify a great deal of variability in the period after 1973. Specifically, although the random walk model outperforms the TVP model in certain periods (i.e. the LBF is negative), on average, evidence from model 2 is stronger until around the end of 1982 when the CLBF attains a value of less than than 2. Not surprisingly, this evidence suggests that the ex ante real rate becomes less predictable in periods of high volatility. Interestingly, the performance of the TVP model deteriorates considerably after 1985 as shown by the large and negative CLBF value.

3.3 Expected inflation

Based on the specification used to model the ex ante real rate, we can derive estimates of expected inflation ($\hat{\pi}_t$) by subtracting the estimated ex ante real rate ($\hat{r}r$) from the nominal interest rate (i_t)[17]. A Plot of expected inflation along with actual inflation (π_t) is shown in figure 9. The model appears to capture the behavior of actual inflation reasonably well throughout the sample with the exception of the large forecast errors which occur at the time of the oil shocks in the early 1970's as well as in the early 1980's. We can also see from figure 9 that inflation is consistently

[17]This static forecast of inflation is essentially given by: $\hat{\pi}_t = -\hat{\beta}_{0t} + (1 - \hat{\beta}_{1t})i_t - \hat{\beta}_{2t}\pi_{t-1} - \hat{\beta}_{3t}\pi_{t-2} - \hat{\beta}_{4t}supply_{t-1}$.

underpredicted in the post-1982 period. However, with the exception of the unusual episodes noted earlier, the model appears to perform equally well over the whole sample. The relationship between expected inflation and the nominal interest rate can be casually inspected using the plot shown in figure 10. The graph shows that the nominal rate moves very closely with expected inflation for most of the sample period until late 1980, early 1981. After this period, expected inflation is below the nominal rate leading to high levels of ex ante real rates for the post 1980 period[18] A crude measure of the changing strength of the Fisher effect (i.e., the positive relationship between expected inflation and the nominal rate) is given by an examination of the simple correlation between the nominal rate and expected inflation over the various subsamples. Consistent with HM and the OLS results from section 3.1, there is a strong Fisher effect in the pre-1979 period of the sample with a correlation coefficient of 0.92. Although this relationship weakens considerably in the October 1979-November 1982 subperiod where the correlation decreases to 0.5, it does not become virtually null as in the HM study. In addition, HM find that the Fisher effect is strong again between November 1982 and December 1984 (the end of their sample period) whereas here the correlation is lowest (0.285) between November 1982 and the end of the sample in January 1991. Therefore, the high correlation reported in HM after 1982 does not persist in the longer sample. These results compound the evidence that a structural change in real rates occurred in the 1981-1982 period rather than at the time the Fed changed its operating procedure.

4. Conclusions and direction for future research

The ex ante real rate is an important variable on which economic agents base many important economic decisions. It also constitutes a basic component of many theoretical models that are used to predict relationships among economic variables. Thus, it is not surprising that its changing behavior over time has attracted a good deal of attention in the economics literature, especially in relation to recent monetary and fiscal policy regime changes. The issue of whether shifts in the stochastic process of the ex ante real rate are associated with identifiable policy regime shifts such as the switch in Federal reserve policy in October 1979 is somewhat mixed depending on the model which is used to explain the ex ante real rate.

This study has extended the general model used by Huizinga and Mishkin (1986) by employing a time-varying parameter model with Markov switching heteroscedasticity to examine the relationship between the ex ante real rate and the nominal rate, the lagged inflation rate and a supply shock variable which are used to predict the ex ante real rate. This approach systematically accounts

[18]In the context of the Volcker anti-inflationary policies of this period, the high nominal rates may reflect a lack of credibility in the long term efficacy of these policies.

for coefficient instability and changes in volatility which are documented by the authors and other previous studies.

Consistent with previous results, this study finds that the behavior of the ex ante real rate has changed dramatically over the period between January 1961 and January 1991. However, the evidence does not support the conclusion from HM that structural shifts occurred in October 1979 and November 1982 at the time the Fed changed its operating procedures. Rather the results reinforce the finding by Antoncic (1986) and Garcia and Perron (1994) that the level of real rates rose to abnormal levels in late 1980 and early 1981. The results differ from those found in the GP study in two main respects. First, the expected value of ex ante real rates changes again after 1986 confirming GP's doubts about the continued validity of the three-state statistical characterization of the ex ante real rate. Second, I find that the nominal interest rate comprises the largest component of the mean of the ex ante real rate over most of the sample period and that this relationship becomes stronger after 1981 consistent with a weaker Fisher effect. This is inconsistent with GP's conclusion that overall, the nominal rate contains little useful information for predicting real rates. An important policy implication of these findings is that short term movements in nominal interest rates are more closely related to changes in real borrowing costs than they are to changes in inflationary expectations.

The predictive power of the variables in the general model is assessed using a period-by-period Bayes Factor that compares the TVP model to a random-walk model. The evidence is in favor of the general model over most of the sample but decreases considerably in periods of high volatility and is totally reversed after 1986. Finally, a time series for expected inflation is derived from the estimated ex ante real rate and the nominal rate. The estimates confirm a weakening of the Fisher effect after 1981 and indicate that the high ex ante real rates of this period are a result of abnormally high nominal interest rates coupled with lower expectations of inflation.

Overall, the methodology used in this paper is useful for analyzing the time-varying behavior of real interest rates. Although the real rate puzzle (namely the high real rates of the 1980's) is still unsolved, I believe that the approach taken in this study is a step in the right direction since it focuses on combining current techniques needed to model structural change with an economic model of the ex ante real rate. I plan to extend this work to incorporate additional fiscal and monetary variables into the model . Another interesting extension relates to specifying a more general model for the variance of the real rate process as a function of other economic variables. Following Kim (1993), this model can also be used to construct measures of uncertainty about real interest rates by decomposing the forecast error variance into two components: one due to parameter uncertainty and the other due to heteroscedasticity in the error term.

REFERENCES

Antoncic, M., 1986, High and volatile real interest rates: Where does the Fed fit in?, *Journal of Money Credit and Banking* 18 (1), 18-27.

Evans, M.D., and K.K. Lewis, 1995, Do expected shifts in inflation affect estimates of the long-run Fisher relationship?, *Journal of Finance* vol? , 225-251.

Evans, M.D., and P. Watchel, 1992, Interpreting the movements in short-term interest rates, *Journal of Business* 65 (3), 395-429.

Fama, E.F., 1975, Short-term Interest Rates as predictors of inflation, *American Economic Review* 65, 269-282.

Garcia, R. and P. Perron, 1994, An analysis of the real rate under regime shifts, working paper, Centre de recherches et development en economique.

Hamilton, J.D., 1988, Rational expectations econometric analysis of changes in regime: An investigation of the term structure of interest rates, *Journal of Economic Dynamics and Control* 12, 385-423.

______, 1989, A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica* 57, 357-384.

______, 1994, Time series analysis, Princeton: Princeton University Press.

Huizinga, J., and Mishkin F.S., 1986a, Monetary policy regime shifts and the unusual behavior of real interest rates, *Carnegie-Rochester Conference Series on Public Policy*, 24, 231-274.

______, 1986b, How robust are the results? A reply, *Carnegie-Rochester Conference Series on Public Policy* 24, 289-302.

Jeffreys, H., 1961, *Theory of Probability* (3rd edn.) (Oxford University Press, London).

Kim, C.J, 1993, Unobserved-component time series models with Markov-switching heteroscedasticity: Changes in regime and the link between inflation rates and inflation uncertainty, *Journal of Business and Economic Statistics* 11 (3), 341-349.

______, 1994, Dynamic linear models with Markov-switching, *Journal of Econometrics*, 60, 1-22.

McCulloch J.H. and H. Kwon, 1993, U.S. term structure data, 1947-1991, Ohio State University Working Paper nb 93-6.

Mishkin, F.S., (1981), The real rate of interest: An empirical investigation, *The cost and consequences of inflation, Carnegie-Rochester Conference Series on Public Policy*, 15, 151-200.

______, 1988, Understanding real interest rates, *American Journal of Agricultural Economics*, 70, 1064-1072.

______, 1995, Nonstationarity of regressors and tests of real-interest-rate behavior, *Journal of Business and Economic Statistics*, 13 (1), 47-51.

Quandt, R.E., 1958, The estimation of the parameters of a linear regression system obeying two separate regimes, *Journal of the American Statistical Association* 53, 873-80.

Wilcox, J.A., 1983, Why real rates were so low in the 1970's, *American Economic Review* 73 (1), 44-53.

Table 1: OLS regressions of the ex post real rate †				
	Dependent variable: $eprr_t$ (% per month)			
Period	1/60-1/91	1/60-10/79	11/79-10/82	11/82-1/91
Constant	0.00187	-0.00037	0.00269	0.00105
	(0.00085)	(0.00100)	(0.00229)	(0.00126)
i_t	0.49731	-0.19859	0.20298	0.63204
	(0. 12163)	(0.21736)	(0.27892)	(0.16603)
π_{t-1}	-0.35034	-0.02350	-0.19923	-0.36177
	(0.07748)	(0.10103)	(0.21395)	(0.12613)
π_{t-2}	-0.22408	-0.12632	0.12212	0.10252
	(0.06137)	(0.05653)	(0.15938)	(0.09035)
$Supply_{t-1}$	0.00191	-0.00253	0.03833	0.00441
	(0.00076)	(0.00084)	(0.00985)	(0.00140)
R^2	0.329	0.210	0.516	0.369
$D.W.$	2.01	1.97	2.02	1.880
$S.E.E.$	0.0023	0.0020	0.0026	0.0019
Q‡	96.98 (36)	45.78 (36)	4.97 (9)	30.78 (25)

† Consistent standard errors are in parentheses.
‡ Degrees of freedom are in parentheses .

Table 2: Estimates from the TVP model

$$eprr_t = X_t\beta_t + \epsilon_t,$$
$$\beta_t = \beta_{t-1} + v_t,$$
$$e_t \sim N(0, h_t),$$
$$v_t \sim N(0, Q),$$
$$h_t = \sigma_0^2 + (\sigma_1^2 - \sigma_0^2)S_t,$$
$$Pr[S_t = 1|S_{t-1} = 1] = p_{11},$$
$$Pr[S_t = 0|S_{t-1} = 0] = p_{00}$$

Parameter	Estimate (standard error)
σ_0	0.154232 (0.019886)
σ_1	0.372703 (0.119518)
Q_{11}	0.000010 (0.000002)
Q_{22}	0.037048 (0.019238)
Q_{33}	0.010835 (0.008867)
Q_{44}	0.018101 (0.015324)
Q_{55}	0.011145 (0.008847)
p_{11}	0.587815 (0.237749)
p_{00}	0.928426 (0.062514)
Log Likelihood	52.8989

Table 3: Average values for Ex post and Ex ante real rates

Period	Ex post (% per year)	Ex ante (% per year)
Jan '61 -Jan '74	0.82	1.01
Feb '74 - Nov '80	-1.55	-1.85
Dec '80 - Apr '86	5.29	4.82
May '86 -Jan '91	1.99	2.28