THE TIME VARYING VOLATILITY OF MACROECONOMIC FLUCTUATIONS

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Abstract. This paper deals with the estimation of DSGE models when structural innovations have volatilities that are allowed to vary over time. In particular, we develop an efficient algorithm for jointly inferring the model parameters, underlying shocks and time varying volatilities. We apply our estimation strategy to a large-scale model of the US business cycle and identify the main determinants of the important shifts in the volatility of macroeconomic variables that has characterized the postwar period. We find that investment specific technology shocks account for most of the sharp decline in volatility of the last two decades.

1. Introduction

It has been well documented that the volatility of output, inflation, interest rates and many other macroeconomic variables of the U.S. economy has exhibited a very high degree of time variation over the last fifty years (see, for instance, Sims and Zha (2004) or Stock and Watson (2003a)). Perhaps, the most notorious episode of substantial fluctuation of volatilities in recent U.S. economic history is the "Great Moderation,"[1] which corresponds to the sharp decline in the volatility of output as well as other macroeconomic and financial variables since the mid 1980s. While substantial efforts have been devoted to determine the timing of these volatility changes (see, among others, Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Stock and Watson (2002), Chauvet and Potter (2001), Herrera and Pesavento (2005)), there have been surprisingly few studies attempting to identify the structural disturbances responsible for all these volatility shifts.

In this paper we fill this gap, by estimating a DSGE model in which the volatility of the structural innovations is allowed to change over time. First, we describe an

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[1] The name Great Moderation is due to Stock and Watson (2002), although the phenomenon was first noted by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000).

algorithm that allows for simultaneous inference on both the parameters and the stochastic volatilities. Then, we apply our modeling and estimation strategy to a large-scale business cycle model of the US economy. The theoretical framework we adopt follows along the lines of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003) and has been shown to fit the U.S. data fairly well. The model exhibits a number of real and nominal frictions, and various shocks with a precise (although sometimes debatable) microeconomic interpretation. The novelty of our set-up is that all of these shocks have a variance which is allowed to change over time.

We believe that this is an interesting innovation because it allows us to identify the sources of the changes in the volatility of the main macro variables in the postwar period. Thereafter, we are able to shed light on the nature of the underlying disturbances responsible for the Great Moderation and other fluctuations in the volatility of the U.S. business cycle.

The main conclusions we reach in this study are the following. First, the exogenous structural disturbances to the US economy display a very large degree of stochastic volatility. Nonetheless, the degree of variation in variances differs quite substantially across shocks, being more pronounced for technology shocks and, particularly, monetary policy shocks. Consequently, while stochastic volatility is present in all the endogenous observed variables of the model, different series exhibit contrasting patterns of fluctuations in their variances.

Second, although disturbances have different prominence in accounting for the Great Moderation across series, the decline in the volatility of output, investment and consumption is largely driven by investment specific technology shocks. So our results suggest that a fall in the variance of the relative price of consumption to investment goods has played a prominent role in the lower volatility of the U.S. business cycle of the last two decades.

From the methodological standpoint, this paper is related to the statistics literature on stochastic volatility models (for an overview, see Kim, Shephard, and Chib (1998)) and, more generally, on partial non-Gaussian state-space models (Shephard (1994)). Drawing from this literature, we develop an efficient algorithm, based on Bayesian Markov chain Monte Carlo (MCMC) methods, for the numerical evaluation of the posterior of the parameters of interest. Methodologically, the paper closest to ours is Laforte (2005), although in Laforte (2005) the time varying variances are modeled as Markov switching processes as opposed to smoother processes, as we do in this paper.

From the point of view of the application, this paper is related to the large literature using estimated micro-founded models to understand the main sources of U.S. business cycle fluctuations (see, for instance, Rotemberg and Woodford (1997), Ireland (2004), Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003), Altig, Christiano, Eichenbaum, and Linde (2005)). However, we depart from previous work in this area by allowing for time variation in the volatility of the structural disturbances. Our approach is related to the fairly large literature dealing with the estimation of vector autoregressions with heteroskedastic shocks (see, for example, Bernanke and Mihov (1998), Cogley and Sargent (2003), Sims and Zha (2004), Primiceri (2005) or Canova, Gambetti, and Pappa (2005)). In contrast to this strand of work, one advantage of our approach is that a fully-fledged model provides an easy interpretation for the structural disturbances hitting the economy.

The paper is organized as follows. Section 2 presents the class of models we will deal with and outlines some methodological issues (the details are in the appendix). Section 3 illustrates our application to the model of the US business cycle. Section 4 and 5 discuss the estimation results and address the causes of the Great Moderation. Section 6 concludes with some final remark and priorities for future research.

2. Stochastic Volatility in DSGE Models

The general class of models we will work with is summarized by the following system of equations:

$$E_t \left[f \left(y_{t+1}, y_t, y_{t-1}, \eta_t, \theta \right) \right] = 0, \tag{2.1}$$

where y_t is a $k \times 1$ vector of states and endogenous variables, η_t is an $n \times 1$ vector of exogenous disturbances, θ is a $p \times 1$ vector of structural parameters and E_t denotes the mathematical expectation operator, conditional on the information available at time t. For example, (2.1) can be thought as a collection of constraints and first order conditions derived from a micro-founded model of consumers and/or firms behavior. The novelty here is that the standard deviation of the elements of η_t is allowed to change over time. In particular, we make the assumption that

$$\begin{aligned} \log \eta_t &\equiv \hat{\eta}_t = \Sigma_t \varepsilon_t \\ \varepsilon_t &\sim N(0, I_n), \end{aligned}$$

where N indicates the normal distribution, I_n denotes an $n \times n$ identity matrix and Σ_t is a diagonal matrix with the $n \times 1$ vector σ_t of time varying standard deviations on the main diagonal. Following the stochastic volatility literature (see, for instance, Kim, Shephard, and Chib (1998)), we assume that each element of σ_t evolves (independently) according to the following stochastic processes:

$$\begin{aligned} \log \sigma_{i,t} &= (1-\rho_{\sigma_i}) \log \sigma_i + \rho_{\sigma_i} \log \sigma_{i,t-1} + \nu_{i,t} \\ \nu_{i,t} &\sim N(0, s_i^2) \qquad\qquad i = 1, ..., n. \end{aligned} \tag{2.2}$$

Observe that modeling the logarithm of σ_t, as opposed to the σ_t itself, ensures that the standard deviation of the shocks remains positive at every point in time.

Our objective is characterizing the posterior distribution of the model structural parameters (θ) and the time varying volatility of the shocks ($\{\sigma_t\}_{t=1}^{T}$). Observe that the model described by (2.1) is in general nonlinear and does not have a closed-form solution. Therefore, the solution must be approximated. Notice also that commonly used log-linearization methods would not serve our purposes, since the time varying standard deviations would disappear in the log-linearized version of the model. Moreover, log-linear methods would be accurate in this set-up only if the variability of the standard deviations is small. Higher order approximations would instead preserve the interaction term $\Sigma_t \varepsilon_t$. However, they would also generate many additional nonlinear terms, considerably complicating the estimation. For these reasons, we develop what we call a *partially nonlinear approximation* of the model, which combines the appeal of both log-linearization and higher order approximations. In particular, we approximate the solution of the model by the partially nonlinear function

$$\hat{y}_t = A\hat{y}_{t-1} + B\hat{\eta}_t = A\hat{y}_{t-1} + B\Sigma_t \varepsilon_t, \tag{2.3}$$

where $\hat{y}_t$ denotes log deviations from the non-stochastic steady state of the variable y. (2.3) is a perfectly valid approximation of the model's solution. Moreover, when the standard deviations of the model disturbances are actually time varying, (2.3) approximates the model's solution better than standard log-linearization methods. These results are proven in appendix A.

3. The Model

We apply our method to a relatively large-scale model of the U.S. business cycle, which has been shown to fit the data nearly as well as Bayesian vector autoregressions (Smets and Wouters (2003)). The model is based on work by Christiano, Eichenbaum, and Evans (2005) and we refer to them for some of the details. Our brief illustration of the model follows closely Del Negro, Schorfheide, Smets, and Wouters (2004).

3.1. **Final goods producers.**

At every point in time t, perfectly competitive firms produce the final consumption good Y_t, using the intermediate goods $Y_t(i)$, $i \in [0,1]$ and the production technology

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} di \right]^{1+\lambda_{p,t}}.$$

$\lambda_{p,t}$ follows the exogenous stochastic process

$$\log \lambda_{p,t} = (1-\rho_p) \log \lambda_p + \rho_p \log \lambda_{p,t-1} + \sigma_{p,t} \varepsilon_{p,t},$$

where $\varepsilon_{p,t}$ is $i.i.d. N(0,1)$ and $\sigma_{p,t}$ evolves as in (2.2).
Profit maximization and zero profit condition for the final goods producers imply the following relation between the price of the final good (P_t) and the prices of the intermediate goods ($P_t(i)$, $i \in [0,1]$)

$$P_t = \left[\int_0^1 P_t(i)^{\frac{1}{\lambda_{p,t}}} di \right]^{\lambda_{p,t}},$$

and the following demand function for the intermediate good i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t.$$

3.2. **Intermediate goods producers.**

A monopolist firm produces the intermediate good i using the following production function:

$$Y_t(i) = \max \left\{ A_t^{1-\alpha} K_t(i)^{\alpha} L_t(i)^{1-\alpha} - A_t F; 0 \right\},$$

where, as usual, $K_t(i)$ and $L_t(i)$ denote respectively the capital and labor input for the production of good i, F represents a fixed cost of production and A_t is an exogenous stochastic process capturing the effects of technology. In particular, we model A_t as a unit root process, with a growth rate ($z_t \equiv \log \frac{A_t}{A_{t-1}}$) that follows the exogenous process

$$z_t = (1-\rho_z)\gamma + \rho_p z_{t-1} + \sigma_{z,t} \varepsilon_{z,t},$$

with the usual assumption about the properties of $\sigma_{z,t}$ and $\varepsilon_{z,t}$. As in Calvo (1983), a fraction ξ_p of firms cannot re-optimize their prices and, therefore, set their prices following the indexation rule

$$P_t(i) = P_{t-1}(i)\pi_{t-1}^{\iota_p}\pi^{1-\iota_p},$$

where π_t is defined as $\frac{P_t}{P_{t-1}}$ and π denotes the steady state value of π_t. Subject to the usual cost minimization condition, re-optimizing firms choose their price $(\tilde{P}_t(i))$ by maximizing the present value of future profits

$$E_t\sum_{s=0}^{\infty}\xi_p^s\beta^s\lambda_{t+s}\left\{\left[\tilde{P}_t(i)\left(\Pi_{j=0}^{s}\pi_{t-1+j}^{\iota_p}\pi^{1-\iota_p}\right)\right]Y_{t+s}(t) - P_{t+s}\left[W_tL_t(i) + R_t^kK_t(i)\right]\right\},$$

where λ_{t+s} is the marginal utility of consumption, W_t and R_t^k denote respectively the wage and the rental cost of capital.

3.3. **Households.** The firms are owned by a continuum of households, indexed by $j \in [0,1]$. As in Erceg, Henderson, and Levin (2000), while each household is a monopolistic supplier of specialized labor $(L_t(j))$, a number of 'employment agencies' combines households' specialized labor into labor services available to the intermediate firms

$$L_t = \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_w}}dj\right]^{1+\lambda_w}.$$

Profit maximization and zero profit condition for the perfectly competitive employment agencies imply the following relation between the wage paid by the intermediate firms and the wage received by the supplier of specialized labor $L_t(j)$

$$W_t = \left[\int_0^1 W_t(j)^{\frac{1}{\lambda_w}}dj\right]^{\lambda_w},$$

and the following labor demand function for labor type j:

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}}L_t.$$

Each household maximizes the utility function[2]

$$E_t\sum_{s=0}^{\infty}\beta^s b_{t+s}\left[\log\left(C_{t+s}(j) - hC_{t+s-1}(j)\right) - \varphi_{t+s}\frac{L_{t+s}(j)^{1+\nu}}{1+\nu}\right],$$

where $C_t(j)$ is consumption, h is the "degree" of habit formation, φ_t is a preference shock that affects the marginal disutility of labor and b_t is a "discount factor" shock

[2]We assume a cashless limit economy as described in Woodford (2003).

affecting both the marginal utility of consumption and the marginal disutility of labor. These two shocks follow the stochastic processes

$$\begin{aligned}\log b_t &= \rho_b \log b_{t-1} + \sigma_{b,t}\varepsilon_{b,t}\\ \log \varphi_t &= (1-\rho_\varphi)\log\varphi + \rho_\varphi \log\varphi_{t-1} + \sigma_{\varphi,t}\varepsilon_{\varphi,t}.\end{aligned}$$

The household budget constraint is given by

$$\begin{aligned}P_{t+s}C_{t+s}(j) + P_{t+s}I_{t+s}(j) + B_{t+s}(j) \quad &\leq \quad R_{t+s-1}B_{t+s-1}(j) + Q_{t+s-1}(j) + \Pi_{t+s} + \\ &+W_{t+s}(j)L_{t+s}(j) + R^k_{t+s}(j)u_{t+s}(j)\bar{K}_{t+s-1}(j) - P_{t+s}a(u_{t+s}(j))\bar{K}_{t+s-1}(j),\end{aligned}$$

where $I_t(j)$ is investment, $B_t(j)$ is holding of government bonds, R_t is the gross nominal interest rate, $Q_t(j)$ is the net cash flow from participating in state contingent securities, Π_t is the per-capita profit that households get from owning the firms. Households own capital and choose the capital utilization rate which transform physical capital ($\bar{K}_t(j)$) in effective capital

$$K_t(j) = u_t(j)\bar{K}_{t-1}(j),$$

which is rented to firms at the rate $R^k_t(j)$. The cost of capital utilization is $a(u_{t+s}(j))$ per unit of physical capital. As in Altig, Christiano, Eichenbaum, and Linde (2005), we assume that $u_t = 1$ and $a(u_t) = 0$ in steady state. The usual physical capital accumulation equation is described by

$$\bar{K}_t(j) = (1-\delta)\bar{K}_{t-1}(j) + \mu_t\left(1 - S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right)I_t(j),$$

where δ denotes the depreciation rate and, as in Christiano, Eichenbaum, and Evans (2005) and Altig, Christiano, Eichenbaum, and Linde (2005), the function S captures the presence of adjustment costs in investment, with $S' = 0$ and $S'' > 0$. μ_t is a random shock to the price of investment relative to consumption and follows the exogenous process

$$\log\mu_t = \rho_\mu \log\mu_{t-1} + \sigma_{\mu,t}\varepsilon_{\mu,t}.$$

As in Erceg, Henderson, and Levin (2000), a fraction ξ_w of households cannot re-optimize their wages and, therefore, set their wages following the indexation rule

$$W_t(j) = W_{t-1}(j)\left(\pi_{t-1}e^{z_{t-1}}\right)^{\iota_w}\left(\pi e^{\gamma}\right)^{1-\iota_w}.$$

The remaining fraction of re-optimizing households set their wages by maximizing

$$E_t\sum_{s=0}^{\infty}\xi_w^s\beta^s b_{t+s}\left\{-\varphi_{t+s}\frac{L_{t+s}(j)^{1+\nu}}{1+\nu}\right\},$$

subject to the labor demand function.

3.4. **Monetary and Government Policies.** Monetary policy sets short term nominal interest rates following a Taylor type rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{A_t}\right)^{\phi_Y}\right]^{1-\rho_R} e^{\sigma_{R,t}\varepsilon_{R,t}},$$

where R is the steady state for the nominal interest rate and $\varepsilon_{R,t}$ is an $i.i.d.N(0,1)$ monetary policy shock.

Fiscal policy is assumed to be fully Ricardian and public spending is given by

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t,$$

where g_t is an exogenous disturbance following the stochastic process

$$\log g_t = (1-\rho_g)\log g + \rho_g \log g_{t-1} + \sigma_{g,t}\varepsilon_{g,t}.$$

3.5. **Market Clearing.** The resource constraint is given by

$$C_t + I_t + G_t + a(u_t)\bar{K}_{t-1} = Y_t,$$

3.6. **Steady State and Model Solution.** Since the technology process A_t is assumed to have a unit root, consumption, investment, capital, real wages and output evolve along a stochastic growth path. Once the model is rewritten in terms of detrended variables, we can compute the non-stochastic steady state and employ the partially nonlinear method illustrated in section 2 and appendix A, to approximate the model around the steady state. This delivers a partial non-linear state space model of the kind described in Shephard (1994).

We conclude the discussion of the model by specifying the vector of observables, completing the state space representation of our model:

$$[\Delta\log Y_t, \Delta\log C_t, \Delta\log I_t, \log L_t, \Delta\log\frac{W_t}{P_t}, \pi_t, R_t], \tag{3.1}$$

where $\Delta\log X_t$ denotes $\log X_t - \log X_{t-1}$.

4. Inference

4.1. **The Data.** We estimate the model using seven series of U.S. quarterly data, as in Levin, Onatski, Williams, and Williams (2005) and Del Negro, Schorfheide, Smets, and Wouters (2004). These series correspond to the ones reported in the vector of observable variables of our model (3.1). Our dataset spans a sample from 1954QIII to 2004Q4. All data are extracted from Haver Analytics database (series mnemonics in parenthesis). Following Del Negro, Schorfheide, Smets, and Wouters (2004), we construct real GDP by diving the nominal series (GDP) by population (LF and LH) and the GDP Deflator (JGDP). Real series for consumption and investment are obtained in the same manner, although consumption corresponds only to personal consumption expenditures of non-durables (CN) and services (CS), while investment is the sum of personal consumption expenditures of durables (CD) and gross private domestic investment (I). Real wages corresponds to nominal compensation per hour in the non-farm business sector (LXNFC) divided by the GDP deflator. Our measure of labor is given by the log of hours of all persons in non-farm business sector (HNFBN) divided by population. The quarterly log difference in the GDP deflator constitutes our measure of inflation, while for nominal interest rates we use the effective Federal Funds rate. We do not demean or detrend any series.

4.2. **Bayesian Inference.** Bayesian Markov chain Monte Carlo (MCMC) methods are used to characterize the posterior distribution of the model's structural parameters (θ), the time varying volatility of the shocks ($\{\sigma_t\}_{t=1}^T$) and the coefficients of the volatility processes ($\left[\sigma, \rho_\sigma, s^2\right]$). Observe that, dealing with unobservable components, where the distinction between parameters and shocks is less clear than in other situations, a Bayesian approach is the natural one. Moreover, Bayesian methods deal efficiently with the high dimension of the parameter space and the nonlinearities of the model, splitting the original estimation problem in smaller and simpler ones. In particular, MCMC is carried out in three steps. First, a Metropolis step is used to draw from the posterior of the structural coefficients θ. Drawing the sequence of time varying volatilities σ^T (conditional on θ, σ, ρ_σ and s^2) is instead more involved and relies mostly on the method presented in Kim, Shephard, and Chib (1998). It consists of transforming a nonlinear and non-Gaussian state space form in a linear and approximately Gaussian one, which allows the use of simulation smoothers like Carter and Kohn (1994) or Durbin and Koopman (2002). Simulating the conditional posterior of $\left[\sigma, \rho_\sigma, s^2\right]$ is standard, since it is the product of independent normal-inverse-Gamma distributions. The details are left to appendix B.

4.3. **Priors.** We fix a small number of the model parameters to values that are very common in the existing literature. In particular, we set the steady state share of capital income (α) to 0.3, the quarterly depreciation rate of capital (δ) to 0.025

and the steady state government spending to GDP ratio $(1 - 1/g)$ to 0.22, which corresponds to the average value of G_t/Y_t in our sample. Moreover, in order to reduce the number of free coefficients, we set all the ρ_σ's to 1, which reflects the assumption that the volatilities follow geometric random walk processes.

The first three columns of table 1 report our priors for the remaining parameters of the model. While most of these priors are relatively disperse and reflect previous results in the literature, a few of them deserve some further discussion. For all but one persistence parameters we use a Beta prior, with mean 0.5 and standard deviation 0.15. The exception is the persistence of the mark-up shock and the reason is the weak identification problem between this parameter and the one capturing the degree of price indexation. We decided to impose a tighter prior on ρ_p to avoid convergence problems in our MCMC algorithm.

The priors on the standard deviations (s) of the innovations to the log-volatility processes deserve some comment as well, as these coefficients are new in the DSGE literature. We chose to use an inverse-Gamma prior with mean equal to 0.01^2 for several reasons. First, assuming for simplicity that the log-volatilities behave as random walks, this number implies an average variation of about 15 percent over our sample of forty years. We regard this as a conservative estimate. Second, in the context of time varying vector autoregressions, Primiceri (2005) has experimented and tested several values and concluded that this value attained the highest marginal likelihood. We assessed the sensitivity of the estimates to alternative specifications of the prior (especially for the variance of the innovation to the log-volatilities) and found that these modifications had no important influence on the results.

5. Estimation Results

5.1. **Parameter estimates.** The last three columns of table 1 summarize the posterior distribution of the model coefficients, reporting posterior medians, standard deviations and 5th and 95th percentiles computed with the draws. All coefficients are quite precisely estimated and their estimate seem plausible. For example, the estimates of the Calvo stickiness parameters for prices (ξ_p) is approximately equal to $\frac{2}{3}$, which is a value slightly below to the ones obtained by the previous literature estimating DSGE models.[3] This number seems still higher than what obtained

[3]See Altig, Christiano, Eichenbaum, and Linde (2005) for an example of a model generating lower estimates of the price stickiness parameter.

by micro studies (see, for instance, Bils and Klenow (2004)), but the presence of indexation mechanisms (which assures that prices are actually changed in every period) makes the results potentially consistent with the micro evidence on the high frequency of price changes.

For comparison, table 1 also reports posterior medians, standard deviations, 5th and 95th percentiles of a model estimated with time invariant volatilities. Notice that, as should be expected, most of the coefficient estimates are similar to the time varying model, although there are some important exceptions. Most notably, two of the coefficients related to the labor market part of our framework change importantly when stochastic volatility is allowed for. Specifically, the Calvo wage stickiness parameter (ξ_w) drops from 0.71 to 0.38 and the inverse Frisch elasticity of labor supply (ν) declines as well from 3.8 to 2.5. The higher elasticity of labor supply and the more volatile behavior of wages are compensated by a smoother pattern of the intra-temporal preference shock (φ_t), whose autocorrelation coefficient is estimated much higher in the stochastic volatility model.[4] This suggests that not accounting for stochastic might introduce some bias in the estimation.

5.2. **Volatility estimates.** Figure 1 presents the plots of the time varying standard deviations of the seven shocks of our models. Notice that the degree of stochastic volatility varies substantially across shocks. Three of the disturbances seem to have relatively constant standard deviations. This is the case for the price mark-up shock ($\lambda_{p,t}$), and the two taste shocks (b_t and φ_t). The evidence is very different for the volatility of the remaining four shocks, which exhibit a very important amount of time variation. The exogenous disturbance showing the highest degree of time varying volatility is the monetary policy shock (ε_t^{MP}, figure 1a), for which the difference between the lowest and the highest levels of the standard deviation is about 500 percent. Observe that the "Volcker episode"[5] is perfectly captured in

[4]It is worth pointing out that the posterior distribution of the time invariant model is bimodal. The values reported in table 1 are relative to the global maximum. However, there exists a local maximum for which the level of the log-posterior is only slightly lower (just by one point) and the values of the coefficients are much closer to the estimates of the time varying model. Notice also that the time varying model does not exhibit the problem of the two modes, suggesting that accounting for stochastic volatility might help solving some of the identification problems, which are common in thie class of models.

[5]The "Volcker episode" refers to the high volatility of interest rates in the 1979-1983 period, due to the monetary targeting regime initiated by chaiman Paul Volcker in response to the dramatic rise in U.S. inflation in the 1970s.

our estimates, as well as the reduction in the volatility of monetary policy shocks during the Greenspan period.

Monetary policy shocks are not the only ones exhibiting time varying volatilities. The standard deviation of technology shocks (z_t, figure 1b) seems to decrease by almost 50 percent in the second part of the sample. This is potentially consistent with the observed reduction in the volatility of GDP in the last two decades. A similar pattern is followed by the volatility of the investment specific technology shock (μ_t, figure 1d) and the government spending shock (g_t, figure 1c), although the fall in volatility at the beginning of the 1980s seems more dramatic in the case of the investment specific technology shock.

One important component of our analysis (which will lead us to the next subsection, addressing the causes of the Great Moderation) is the variance decomposition. We perform the variance decomposition exercise in the following way: given the estimated variances of the exogenous disturbances, we construct the implied variances of the (endogenous) observable variables. Then, we re-compute the variances of the observable variables, setting to zero the variance of each disturbance, one-at-a-time. In this way we are able to investigate the contribution of each shock to the variance of the endogenous variables.[6] Observe that, since our variances are changing over time, our variance decomposition is a time varying "object". In order to save space, we do not present the graph of the variance decomposition for all of the observables. Instead, we have chosen to give a complete characterization only of the variance decomposition for GDP. For the remaining series we only report the time varying share of the variance explained by selected shocks.

Figure 2 presents the time varying shares of the variance of GDP growth due to each exogenous disturbances. Consistently with Greenwood, Hercowitz, and Krusell (1997) and Fisher (2005), the most important shock in explaining the variability of GDP seems to be the *investment specific technology shock* (figure 5d), which, at least in the first part of the sample, explains about 40 percent of the variance of GDP growth. However, the importance of the investment specific technology shock declines over time. On average, *neutral technology shocks* and labor preference shocks explain 20 percent of the variance of GDP each. This share seems to remain relatively stable over time for the neutral technology shock (figure 2b), while it increases in the last two decades for the labor preference shock (figure 2f). Other

[6]Of course there are alternative ways of doing a variance decomposition exercise and we are busy trying these alternatives.

shocks are less central. The limited importance of the monetary policy shock (figure 2a) is striking and probably related to the observation that, in this class of models, flexible price output seems to track quite closely actual output (Walsh (2005)), which represents a problem for current sticky price models.

Figure 3 presents the graphs of the time varying shares of the variance of the other observable variables, explained by selected shocks. A major portion of the variance of consumption is explained by the intertemporal shock to the discount factor (figure 3a). Although not fundamental for output, monetary policy, mark-up shocks are quite important for the volatility of interest rates (figure 3b) and inflation (figure 3c). Moreover, as one would expect, the investment specific technology shock and the labor preference shock explain most of the variability of investment (figure 3d) and hours (figure 3*e*) respectively, while the neutral technology shock accounts for about 40 percent of the variance of real wages (figure 3f).[7]

6. The Great Moderation

6.1. **The Great Moderation and the role of investment specific technology shocks.** We now apply our method and our results to the Great Moderation episode. In two very influential papers, Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) drew the attention on the dramatic reduction in the volatility of U.S. GDP, which has characterized the last two decades with respect to the pre-1980s period.[8] The change seems to be more abrupt than gradual (Kim and Nelson (1999) and Stock and Watson (2002)) and the break date is estimated to approximately correspond to 1984. In our sample, the standard deviation of GDP growth over the 1984-2004 period is almost one half of the standard deviation computed over the 1955-1983 sample. The literature has labeled this phenomenon as the Great Moderation.

Clearly, a number of explanations have been provided and exhaustive reviews can be found in Blanchard and Simon (2001), Stock and Watson (2002) and Stock and Watson (2003a). Explanations of this phenomenon can be broadly bunched as corresponding to simply good luck, technological progress (particularly in managing inventories) or improvements in the conduct of monetary policy under the

[7]The complete set of variance decomposition graphs is available upon request.

[8]Stock and Watson (2003b) show a similar pattern for other G7 countries.

Volcker and Greenspan chaimanships. However, the evidence in favor of the technological progress and improved monetary policy hypotheses is rather tenuous (see, for instance, Stock and Watson (2002) or Maccini and Pagan (2003)).

Therefore, the starting point of the analysis of the Great Moderation undertaken in this paper is the very robust finding of Stock and Watson (2002), Stock and Watson (2003a) and Ahmed, Levin, and Wilson (2004), who conclude that "this reduction in volatility is associated with an increase in the precision of forecasts of output growth" (Stock and Watson (2002), p 42). Notice that our framework is a natural candidate to understand the structural causes of the reduction in forecast errors. In fact, our methodology allows for time varying volatilities and is based on a fully-fledged model, which provides an easy interpretation for the structural disturbances hitting the economy.

To assess the role played by each shock in the Great Moderation, we rely on counterfactual simulations exercises. Our approach consists of using our model to simulate the volatility of GDP growth under alternative paths for the volatility of each structural disturbance. This counterfactual simulations can be interpreted as the hypothetical pattern of the volatility of GDP growth in the period 1981-2004, had the standard deviation of that particular structural shock only not changed with respect to the 1981 level.

Figure 4 plots the volatility of GDP growth implied by our model. There are at least two things to notice. First, although the evolution of the standard deviation of GDP growth is very similar to the one obtained from an unrestricted, reduced form, univariate AR(4) with stochastic volatility, we notice that the DSGE model somewhat overpredicts the level of the volatility. This problem is common to the time invariant version of the model and is therefore indicative of difficulties in simultaneously matching the levels of persistence, comovements and volatilities observed in the data, even with state of the art DSGE models (Del Negro, Schorfheide, Smets, and Wouters (2004)). Second, nonetheless, the model captures remarkably well the timing and the size of the Great Moderation, despite the abrupt nature of the decline in volatility.

Figure 5 presents our counterfactual simulations. Our approach gives a very strong conclusion about the causes of the Great Moderation. As apparent from figure 5d, the main explanation for the Great Moderation seems to be the sharp reduction in the volatility of investment specific technology shocks. That is, had the volatility of investment specific technology shocks remained at its 1981 level, then the standard deviation of GDP growth would have been much higher than the realized one in the 1981-2004 period.

Observe that an alternative (isomorphic) interpretation of the investment specific technology shocks is of *shocks to the inverse price of efficient units of investment in terms of consumption goods*.[9] Although this variable is not used in our estimation, data on the price of investment relative to consumption may serve as a proxy for investment specific technology shocks. In order to verify that the reduction in the volatility of investment specific technology shocks is not somewhat spurious and specific to our model, we analyze the volatility of this relative price. In particular, we construct the chain-weighted deflators for our components of consumption (non-durables and services) and investment (durables and total private investment) and estimate an AR(4) process in the growth rate of this relative price, allowing for stochastic volatility. Figure 6 plots the time varying standard deviation of the AR(4) innovation and makes clear that this volatility has sharply decreased in the second part of the postwar sample. We regard the fact that our model provides a very similar insight (without using any data on the relative price of investment) as a remarkable result.

6.2. **Robustness issues.** In this subsection we perform robustness checks on our important finding that the Great Moderation seems to be driven by the decline in volatility of investment specific technology shocks. In particular, we perform some simple experiments in the context of our model which suggest that the lower variability of U.S. output is difficult to explain even when considering changes in the systematic part of monetary policy or when using series that abstract from the role of inventories. We believe that these observations echo findings elsewhere in the literature using other methods and models. More importantly, for our purposes, when these variants are considered in all cases the decline in the variability of GDP growth is once again largely attributed to a reduction in the volatility of investment specific technology shocks.

Our strategy to address the importance of the changes in the systematic part of monetary policy consists of estimating our model (without stochastic volatility) on two separate subsamples, 1953-1979:II and 1983-2004:III. Notice that, following Hanson (2003), we exclude the 1979:III-1982:IV period from the estimation since monetary policy during this period may not be correctly characterized by a Taylor

[9] See Fisher (2005) for an explanation of this alternative interpretation.

rule.[10] Table 2 presents posterior modes and standard deviations of the coefficients estimated over these two periods. Notice that there are some differences in the coefficient estimates, particularly, consistently with evidence presented earlier, in the standard deviation of the shocks. In addition, table 3 highlights that, in line with the reduction in stochastic volatility, the unconditional standard deviation of output growth in the second subsample relative to the first is 0.52.

In the context of our model this discrepancy can potentially be explained by three different sets of parameters: the monetary policy coefficients, the remaining structural parameters and the variances of the shocks. With regard to the first possible explanation, notice that, as expected, monetary policy in the second subsample seems to have been more responsive to inflation. To assess the role of these change in the systematic part of policy on the volatility of output, table 3 presents the relative standard deviation of GDP growth when the coefficients of the Taylor rule estimated in the second subsample replace the correspondent coefficients in the first subsample. In order to gauge the second possible explanation, table 3 also presents the relative volatility of GDP growth when a similar counterfactual exercise is performed by replacing all coefficients (other than the volatilities) from the second sub-sample. Table 3 makes clear that neither changes in the systematic part of monetary policy nor the remaining structural coefficients of the model seem to account for the decline in volatility of output. This result is in line with Sims and Zha (2004), Hanson (2003), Leduc and Sill (2003), Primiceri (2005), Ahmed, Levin, and Wilson (2004).

With regard to the role of inventories in explaining the Great Moderation, we proceed with the rather simple exercise of constructing series of investment and, therefore, output that abstract from inventories. This approach corresponds with the measurement of investment series in some business cycle quantitative studies (for example, Fisher (2005)). Furthermore the rationale for subtracting inventories from output, as opposed to working with the series for inventories themselves, arises from the ambiguity of whether inventories buffer or rather amplify the economic fluctuations.[11] That is, it remains unclear whether an increase in the volatility of

[10] To further check the robustness of this experiment, we have also estimated the model from 1979:III to 2004:III allowing for stochastic volatility in order to capture the large degree of variability of interest rates during this period. Our results are unaffected by this modification.

[11] Kahn and McConnell (2002), for instance, argue that while in theory inventories should buffer production from fluctuations in sales, in practice the opposite occurs as inventories and sales comove in the same direction.

inventories would map into a rise or a decline of the volatility of output. Our aim here is simply to check whether when removing inventories the Great Moderation is still evident and, furthermore, whether our conclusions regarding the importance of investment specific technology shocks in accounting for this episode still hold.

Using this new series for investment and output (obtained by chain weighting) we re-estimate the model with stochastic volatility over the full sample. Figure 7 presents counterfactual exercises constructed in a similar way to section 6.1, using the new coefficient and volatility estimates. In this case, the moderation in the variability of output growth is still evident although the decline is somewhat more prolonged as opposed to abrupt. Moreover, monetary policy shocks seem to be slightly more important than in the baseline case. Notice, however, that the role of investment specific technology shocks in accounting for the decline in the volatility of output growth remains for the most part unaltered. We conclude from this exercise that, while inventories may be important for understanding the sharp drop in the variability of output growth, it would seem that additional explanations are needed to address the Great Moderation (see also Stock and Watson (2002) and Maccini and Pagan (2003)). In this respect, and consistently once again with our previous findings, our results point to the predominant role of investment specific technology shocks.

7. Concluding Remarks

In this paper we have estimated a large scale DSGE model of the US business cycle, allowing for the volatility of the structural innovations to change over time. We have found that the volatility of several shocks have changed dramatically in the postwar period. However, the sharp reduction in the standard deviation of GDP growth that has characterized the last twenty years can be explained mostly due to the decline in the variability of a single disturbance: the investment specific technology shock. This crucial disturbance has the equivalent interpretation of shock to the inverse price of efficient units of investment in terms of consumption goods and, indeed, this series has exhibited a substantial moderation in its variability, in accordance with the predictions of our the model.

Our results provide guidance for future research on the volatility of the U.S. business cycle, suggesting that a fruitful avenue would be to model the variability of disturbances affecting the relative price of investment goods. Particularly important seems to be offering a structural interpretation for the sharp reduction in the volatility of these shocks. Although beyond the scope of this paper, we just note that explanations based on increased access to credit markets (Campbell and Hercowitz (2004)) and a decline in investment financial frictions (like the ones modeled in Bernanke, Gertler, and Gilchrist (1999) or Iacoviello (2005)) are potentially consistent with the decline in the volatility of the relative price of investment.

Appendix A. Partially non-linear approximation

This is a sketch of the proof that the partially nonlinear approximation is a valid approximation of the model's solution. To simplify the notation, here we work with linearizations (as opposed to log-linearizations). Consider the general class of models described by the following system of rational expectation equations:

$$\begin{aligned} E_t \left[F\left(y_{t+1}, y_t, \varepsilon_t, \sigma_t\right)\right] &= 0, \\ \varepsilon_t &\sim N(0, I) \end{aligned} \tag{A.1}$$

where we have the assumption that each element of σ_t evolves as in (2.2). Let's define

$$\eta_t \equiv \sigma_t \circ \varepsilon_t, \tag{A.2}$$

with '$\circ$' denoting the element-by-element product between two vectors. The proof involves two steps: first, we show that the first order approximation of the solution (policy rule) as a function of η_t (as opposed to ε_t) does not include σ_t. Second, we show that the first order approximation of the solution as a function of η_t is not worse (might be better) than the (usual) first order approximation of the solution as a function of ε_t.

A.1. **Step 1: The first order approximation of the solution (policy rule) as a function of η_t (as opposed to ε_t) does not include σ_t.** Given (A.2), we can reparameterize (A.1) as a function of η_t and σ_t as follows:

$$E_t \left[f\left(y_{t+1}, y_t, \eta_t, \sigma_t\right)\right] = 0, \tag{A.3}$$

where it is easy to see that the functions $f(\cdot)$ and $F(\cdot)$ are the same. Suppose that (A.3) admits a unique solution in the neighborhood of the non-stochastic steady state (described by $f(y, y, 0, 0) = 0$). The solution has the general form

$$y_t = g(\eta_t, \sigma_t), \tag{A.4}$$

Let's now characterize the first order approximation of (A.4). Plug (A.4) into (A.3), obtaining

$$E_t\left[f\left(g(\eta_{t+1},\sigma_{t+1}),g(\eta_t,\sigma_t),\eta_t,\sigma_t\right)\right]=0 \tag{A.5}$$

Take a first order Taylor expansion of f in (A.5)

$$E_t\left[f_1g_1\eta_{t+1}+f_1g_2\sigma_{t+1}+f_2g_1\eta_t+f_2g_2\sigma_t+f_3\eta_t+f_4\sigma_t\right]\approx 0,$$

where f_j denotes the partial derivative of f with respect to its jth argument. Since ε_t and σ_t never enter (A.1) separately, $f_4=0$. This implies that $g_2=0$, which proves our result. This implies that a first order expansion of (A.4) would be equal to

$$y_t=g_1\eta_t+o(||\eta_t,\sigma_t||).$$

It is also easy to see that the first order approximation of the solution as a function of η_t has the same form as the (usual) first order approximation of the solution as a function of ε_t. In fact, if we plug $y_t=h(\varepsilon_t,\sigma_t)$ into (A.1) and take a Taylor expansion, we obtain

$$E_t\left[f_1h_1\varepsilon_{t+1}+f_1h_2\sigma_{t+1}+f_2h_1\varepsilon_t+f_2h_2\sigma_t+f_3\varepsilon_t+f_4\sigma_t\right]\approx 0.$$

Notice that $f_4=0$ also in this case. This implies that $h_2=0$ and h_1 must solve $f_2h_1+f_3=0$, exactly like g_1 has to solve $f_2g_1+f_3=0$ (so g_1 must equal h_1).

A.2. **Step 2: The first order approximation of the solution as a function of η_t is not worse (might be better) that the (usual) first order approximation of the solution as a function of ε_t.** We aim at approximating the locally unique solution in the neighborhood of the non-stochastic steady state. Consider again the system of equations given by the solution (A.4) and the law of motion for the elements of σ_t, given by (2.2). Rewrite (2.2) as

$$\begin{aligned}\log\tilde{\sigma}_{i,t} &= \rho_{\sigma_i}\log\tilde{\sigma}_{i,t-1}+\nu_{i,t}\\ \tilde{\sigma}_{i,t} &= \frac{\sigma_{i,t}}{\sigma_i}.\end{aligned}$$

Therefore, (A.4) becomes

$$y_t=g(\underbrace{\sigma\underbrace{\tilde{\sigma}_t\circ\varepsilon_t}_{\xi_t}}_{\eta_t},\sigma\tilde{\sigma}_t) \tag{A.6}$$

Following Kim et al. (2004), we can take a first order expansion of (A.6) with respect to the first and second arguments, obtaining

$$y_t = g_1 \sigma \xi_t + o_p(||\sigma||),$$

which states that the first order approximation is valid as long as the average standard deviation of the shocks is not too large.

For comparison, suppose to approximate the solution in the usual way (as a function of ε_t). In this case we would obtain

$$y_t = g_1 \sigma \varepsilon_t + o_p(||\sigma, s||),$$

which states that for the approximation to be valid not only we need the average standard deviation of the shocks to be small, but also the volatility of the time varying standard deviations to be small.

Appendix B. The estimation algorithm

B.1. **The Standard Case: Homoskedastic Disturbances.** For the model without stochastic volatility, the estimation algorithm is simply a random walk Metropolis MCMC procedure, as suggested originally by Schorfheide (2000). To initialize the chains we compute the posterior ordinate for $5,000$ draws from the priors, select the ten points attaining the highest posterior density and use a maximization algorithm (Chris Sims' csminwel) to find the posterior mode. Having observed that all chains lead to the same mode, the inverse Hessian at the peak is used as the variance of a proposal density for generating draws with the random walk metropolis. We initialize multiple chains by scaling the inverse Hessian upwards and drawing randomly from a normal centered at the mode. The variance-covariance matrix of the proposal density is adjusted to attain an acceptance rate close to 0.25, as it is usually suggested. Trace plots, kernel estimates as well as the variants of the potential scale-reduction factors proposed by Brooks and Gelman (1998) are used to gauge the convergence of the algorithm.

B.2. **Stochastic Volatility.** When the structural shocks exhibit stochastic volatility, this algorithm must be modified to account for inference on the unobserved stochastic volatilities. A Metropolis within Gibbs MCMC algorithm allows us to iteratively draw from the posterior densities of the DSGE model's parameters, stochastic volatilities and associated innovation variances. As discussed below, generating a draw for the stochastic volatilities entails using a normal mixture approximation and sampling a set of latent indicators for the components of this mixture.

To illustrate the steps involved in sampling from the different blocks, let the vector θ collect all parameters of the DSGE model (other than the standard deviations of the structural disturbances of the time invariant model) and notice that the solution of the linearized DSGE model leads to a state-space representation of the form

$$x_t = Dy_t \tag{B.1}$$

$$y_t = A(\theta)y_{t-1} + B(\theta)\eta_t \tag{B.2}$$

where x_t and y_t represent the observable variables and the endogenous / state variables respectively. (B.2) is the same equation as in (2.3), but we have dropped the "hats" to simplify the notation. As discussed in section A the novelty of our framework is that the vector of structural innovations η_t (dimension $n \times 1$) is allowed to have a time varying variance covariance matrix. Indexing each structural shock by i, the stochastic volatilities for each shocks are modelled as

$$\eta_{i,t} = \sigma_{i,t}\varepsilon_{i,t} \tag{B.3}$$

$$\log \sigma_{i,t} = (1-\rho_{\sigma_i})\log \sigma_i + \rho_{\sigma_i} \log \sigma_{i,t-1} + \nu_{i,t} \tag{B.4}$$

$$\begin{aligned} \varepsilon_{i,t} &\sim N(0,1) \\ \nu_{i,t} &\sim N(0, s_i^2) \qquad i = 1, ..., n. \end{aligned} \tag{B.5}$$

Let the vector h_t, with entry i given by $h_{i,t} = \log \sigma_{i,t}$, collect the log volatilities for all shocks at time t and stack the whole sample of stochastic volatilities into the matrix $H^T = [h_1, h_2, ..., h_t, ..., h_T]'$. Finally, we denote the sample of structural shocks as $\eta^T = [\eta_{1,}\eta_2, ..., \eta_t, ..., \eta_T]'$.

Suppose that the MCMC algorithm has completed iteration g (> 0), producing samples $\theta^{(g)}$, $H^{T,(g)}$ and $V^{(g)}$.of the parameters of interest (individual elements of a vector are indexed by i while (g) indicates the current state of the chain). In iteration $g+1$, the following five steps are used to generate a set of new draws.

B.2.1. *Step 1: Draw the structural shocks $\eta^{T,(g+1)}$.* In order to generate a new sample of the stochastic volatilities we must first obtain a new draw of the structural shocks. This can be done easily using the efficient simulation smoother for disturbances developed by Durbin and Koopman (2002). The simulation smoother is applied to the state space representation given by (B.1) and (B.2).

B.2.2. ***Step 3: Draw the stochastic volatilities*** $H^{T,(g+1)}$. With a draw of η^T in hand the system of nonlinear measurement equations in (B.3) for each structural shock, can be easily converted in a linear one, by squaring and taking logarithms of every element. Due to the fact that the squared shocks $\eta_{i,t}^2$ can be very small, an offset constant is used to make the estimation procedure more robust. Dropping the iteration indicators momentarily for ease of notation, this leads to the following approximating state space form:

$$\tilde{\eta}_{i,t} = 2h_{i,t} + e_{i,t} \tag{B.6}$$

$$h_{i,t} = h_{i,t-1} + \nu_{i,t}. \tag{B.7}$$

where $\tilde{\eta}_{i,t} = \log[(\eta_{i,t})^2 + \bar{c}]$; $\bar{c}$ is the offset constant (set to 0.001); $e_{i,t} = \log(\varepsilon_{i,t}^2)$. Observe that the e's and the ν's are not correlated. The resulting system has a linear, but non-Gaussian state space form, because the innovations in the measurement equations are distributed as a $\log \chi^2(1)$. In order to further transform the system in a Gaussian one, a mixture of normals approximation of the log χ^2 distribution is used, as described in Kim, Shephard, and Chib (1998). Under the assumption of orthogonality across the ε's (recall the variance covariance matrix of the ε's is the identity matrix) this implies that the variance covariance matrix of the v's is also diagonal, allowing to use the same (independent) mixture of normals approximation for any for each innovation:

$$f(e_{i,t}) = \sum_{k=1}^{K} q_k f_N(e_{i,t}|s_{i,t} = k), \qquad i = 1, ..., n$$

where $s_{i,t}$ is the indicator variable selecting which member of the mixture of normals has to be used at time t for the innovation i, $q_k = \Pr(s_{i,t} = k)$ and $f_N(\cdot)$ denotes the pdf of a normal distribution. Kim, Shephard, and Chib (1998) select a mixture of 7 normal densities ($K = 7$) with component probabilities q_k, means $m_k - 1.2704$, and variances r_k^2, $j = 1, ..., 7$, chosen to match a number of moments of the $\log \chi^2(1)$ distribution. For completeness the constants are reported below $\{q_j, m_j, r_j^2\}$ below.[12]

ω	$q_j = \Pr(\omega = j)$	m_j	r_j^2
1	0.00730	-10.12999	5.79596
2	0.10556	-3.97281	2.61369
3	0.00002	-8.56686	5.17950
4	0.04395	2.77786	0.16735
5	0.34001	0.61942	0.64009
6	0.24566	1.79518	0.34023
7	0.25750	-1.08819	1.26261

Source: Kim, Shephard and Chib (1998).

Conditional on $S^{T,(g)}$, the system has an approximate linear and Gaussian state space form. Therefore a new draw for the complete history of the volatility $H^{T,(g+1)}$can be obtained recursively with the standard Gibbs sampling for state space forms using, for instance, the forward-backward recursion of Carter and Kohn (1994).

B.2.3. *Step 3: Draw the indicators of the mixture approximation $s^{T,(g+1)}$.* A new sample of the indicators, $s_t^{i,(g+1)}$, for the mixture is obtained conditional on $\eta^{T,(g+1)}$ and $H^{T,(g+1)}$ by independently sampling each from the discrete density defined by

$$\Pr(s_{i,t}^{(g+1)} = j \mid \tilde{\eta}_{i,t}^{(g+1)}, h_{i,t}^{(g+1)}) \propto q_j f_N(\tilde{\eta}_{i,t}^{(g+1)} | 2h_{i,t}^{(g+1)} + m_j - 1.2704, r_j^2), \quad j = 1, ..., 7$$

Consistent with notation above, collect the indicators for which component of the mixture of the normal approximation to use for each structural shock and time period into a stacked matrix $s^{T,(g+1)} = [s_1^{(g+1)}, s_2^{(g+1)}, ..., s_t^{(g+1)}, ..., s_T^{(g+1)}]'$

B.2.4. ***Step 4: Draw the coefficients of the stochastic volatility processes.*** Having generated a sample $H^{T,(g+1)}$, the vector $\left[\sigma_i^{(g+1)}, \rho_{\sigma_i}^{(g+1)}, s_i^{2\ (g+1)}\right]$, $i = 1, ..., n$, can be generated easily from the usual Normal inverse-Gamma distribution.

B.2.5. ***Step 5: Draw the DSGE parameters $\theta^{(g+1)}$.*** As in the time invariant algorithm, a new candidate parameter θ^* is drawn from a proposal density. However, in this case, the computation of the likelihood used to construct the probability

[12] We neglect the reweighting procedure used in Kim, Shephard, and Chib (1998) to correct the minor approximation error.

of acceptance depends on $H^{T,(g+1)}$. More formally the candidate draw is accepted with probability

$$a = \min\left\{1; \frac{\mathcal{L}(Y^T|\theta^*, H^{T,(g+1)})\pi(\theta^*)}{\mathcal{L}(Y^T|\theta^{(g)}, H^{T,(g+1)})\pi(\theta^{(g)})}\right\},$$

where $\mathcal{L}(\cdot)$ and $\pi(\cdot)$ denote the likelihood and the prior distribution respectively.

These five steps are repeated N times, across multiple chains. As in the case of the time invariant model, we apply a battery of diagnostics to gauge the convergence of the chains.

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Table 1: Prior densities and posterior estimates with and without stochastic volatility

	Prior			Posterior 2/			Posterior with Stochastic Volatility /3		
Coefficient	Prior Density 1/	Mean	Std	Median	Std	[5,95] Prob	Median	Std	[5,95] Prob
ι_p	B	0.5	0.15	0.149	0.059	[0.072 , 0.263]	0.288	0.091	[0.158 , 0.4529]
ι_w	B	0.5	0.15	0.11	0.031	[0.064 , 0.164]	0.134	0.035	[0.081 , 0.197]
γ	N	0.5	0.025	0.425	0.024	[0.386 , 0.464]	0.434	0.023	[0.396 , 0.472]
h	B	0.5	0.1	0.809	0.03	[0.759 , 0.864]	0.748	0.027	[0.705 , 0.793]
λ_p	N	0.15	0.05	0.243	0.038	[0.18 , 0.304]	0.218	0.034	[0.161 , 0.275]
λ_w	N	0.15	0.05	0.127	0.046	[0.054 , 0.207]	0.171	0.038	[0.114 , 0.241]
L^{ss}	N	396.83	0.5	397.218	0.451	[396.507 , 397.986]	396.799	0.479	[396.045 , 397.6244]
P^{ss}	N	0.5	0.1	0.578	0.098	[0.415 , 0.735]	0.712	0.056	[0.625 , 0.81]
R^{ss}	N	0.5	0.1	0.997	0.073	[0.876 , 1.117]	0.996	0.067	[0.89 , 1.114]
ν	G	2	0.75	3.856	0.914	[2.598 , 5.618]	2.423	0.759	[1.57 , 4.036]
ξ_p	B	0.75	0.1	0.784	0.023	[0.745 , 0.821]	0.681	0.027	[0.637 , 0.725]
ξ_w	B	0.75	0.1	0.718	0.051	[0.619 , 0.784]	0.379	0.051	[0.302 , 0.472]
χ	G	5	1	7.136	1.104	[5.552 , 9.133]	7.328	1.050	[5.773 , 9.222]
S	N	4	1.5	2.051	0.586	[1.331 , 3.244]	2.447	0.490	[1.702 , 3.338]
φ_p	N	1.7	0.3	2.014	0.144	[1.804 , 2.275]	2.503	0.184	[2.225 , 2.832]
φ_y	G	0.125	0.1	0.072	0.014	[0.05 , 0.097]	0.024	0.011	[0.008 , 0.045]

Table 1: Prior densities and posterior estimates with and without stochastic volatility

	Prior			Posterior 2/			Posterior with Stochastic Volatility /3		
Coefficient	**Prior Density** 1/	**Mean**	**Std**	**Median**	**Std**	**[5,95] Prob**	**Median**	**Std**	**[5,95] Prob**
ρ_r	B	0.5	0.15	0.806	0.022	[0.768 , 0.84]	0.843	0.017	[0.812 , 0.87]
ρ_z	B	0.5	0.15	0.295	0.055	[0.201 , 0.385]	0.275	0.051	[0.19 , 0.357]
ρ_g	B	0.5	0.15	0.984	0.006	[0.972 , 0.993]	0.985	0.006	[0.974 , 0.993]
ρ_μ	B	0.5	0.15	0.909	0.028	[0.859 , 0.949]	0.831	0.036	[0.772 , 0.888]
ρ_λ	B	0.8	0.1	0.876	0.036	[0.81 , 0.925]	0.908	0.029	[0.854 , 0.95]
ρ_ψ	B	0.5	0.15	0.489	0.07	[0.379 , 0.609]	0.924	0.033	[0.857 , 0.964]
ρ_b	B	0.5	0.15	0.822	0.051	[0.716 , 0.884]	0.822	0.042	[0.745 , 0.883]
σ_r	I	0.15	0.15	0.254	0.014	0.232 , 0.28]			
σ_z	I	0.15	0.15	1.097	0.057	1.006 , 1.192]			
σ_g	I	0.15	0.15	0.553	0.035	0.501 , 0.615]			
σ_μ	I	0.1	0.1	0.136	0.021	0.108 , 0.178]			
σ_λ	I	0.15	0.15	0.096	0.011	0.08 , 0.115]			
σ_ψ	I	0.15	0.15	0.988	0.283	0.587 , 1.52]			
σ_b	I	0.15	0.15	0.55	0.202	0.387 , 1.083]			

Calibrated coefficients: α at 0.3, δ is 0.025 and gss 0.22

1/ N stands for Normal, B Beta, G Gamma and I Inverted-Gamma1 distribution

2/ Median, standard deviations and posterior percentiles of 110,000 draws from the Random Walk metropolis algorithm for the model without stochastic volatility. We disrcard the initial 50,000 draws.

3/ Median, standard deviations and posterior percentiles of 120,000 draws from the Random Walk metropolis within Gibbs algorithm for the model with stochastic volatility. We discard the initial 50,000 draws.

Table 2: Posterior estimates on Split Sample (without stochastic volatility)

	Posterior Sample I: 1954q3 - 1979q3		Posterior Sample II: 1980q1 - 2004q4	
Coefficient	**Mode**	**Std**	**Mode**	**Std**
ι_p	0.218	0.087	0.178	0.068
ι_w	0.094	0.032	0.41	0.077
γ	0.460	0.024	0.476	0.024
h	0.817	0.018	0.689	0.042
λ_p	0.170	0.035	0.122	0.044
λ_w	0.141	0.042	0.18	0.011
L^{ss}	397.081	0.449	396.94	0.492
P^{ss}	0.577	0.093	0.646	0.062
R^{ss}	0.733	0.078	0.732	0.068
ν	2.266	0.692	1.726	0.564
ξ_p	0.706	0.029	0.721	0.035
ξ_w	0.621	0.062	0.351	0.057
χ	6.026	1.023	6.181	1.034
S	0.800	0.176	1.872	0.524
φ_p	1.639	0.114	2.387	0.194
φ_y	0.058	0.001	0.031	0.012

Table 2: Posterior estimates on Split Sample (without stochastic volatility)

	Posterior Sample I: 1954q3 - 1979q3		Posterior Sample II: 1980q1 - 2004q4	
Coefficient	**Mode**	**Std**	**Mode**	**Std**
ρ_r	0.786	0.029	0.815	0.024
ρ_z	0.309	0.063	0.321	0.069
ρ_g	0.922	0.025	0.965	0.012
ρ_μ	0.912	0.028	0.847	0.046
ρ_λ	0.891	0.036	0.902	0.035
ρ_ψ	0.511	0.070	0.919	0.029
ρ_b	0.504	0.077	0.833	0.057
σ_r	0.235	0.018	0.153	0.013
σ_z	1.193	0.084	0.727	0.060
σ_g	0.700	0.054	0.438	0.036
σ_μ	0.085	0.005	0.061	0.011
σ_λ	0.143	0.017	0.102	0.019
σ_ψ	0.680	0.191	0.247	0.024
σ_b	0.883	0.182	0.203	0.035

Calibrated coefficients: α at 0.3, δ is 0.025 and gss 0.22

1/ Prior distributions are identical to those reported in Table 1.

Table 3: Model implied relative standard deviations for output, split-sample estimates and counterfactual

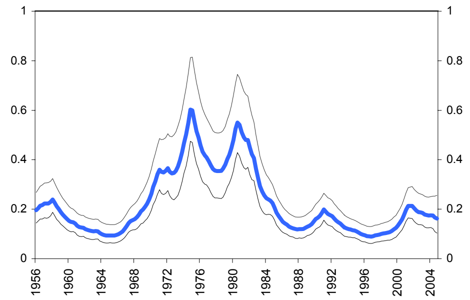
	A	B	C	D
	Sample I *: 1954q3-1979q3*	***Sample II*** *: 1984q1 - 2004q4*	*Sample I; monetary policy coefficients Sample II* 1/	*Std only Sample I; all other coefficients Sample II* 2/
Std of output (relative to std sample I)	1	0.526	1.026	1.114

1/ All coefficients and standard deviations at median estimates for sample I, but replace coefficients of the Taylor rule and the mean of inflation with median estimates for sample II.

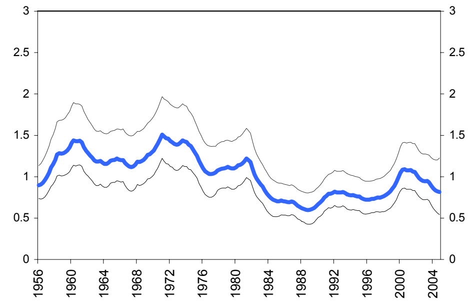
2/ Standard deviations at median estimates for sample I, while all other coefficients are at their median estimates for sample II.

Figure 1: Stochastic Volatiliy of the DSGE Model Shocks

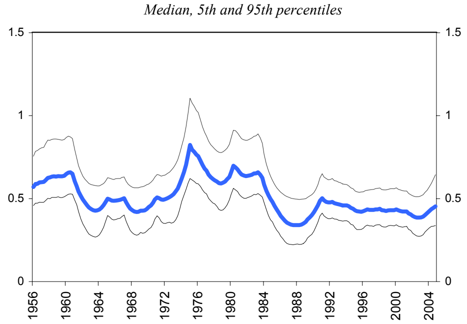
A. Monetary Policy Shock
Median, 5th and 95th percentiles



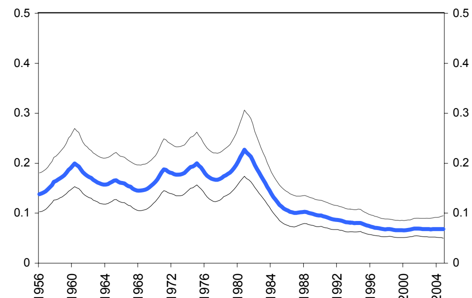
B. Technology Shock
Median, 5th and 95th percentiles



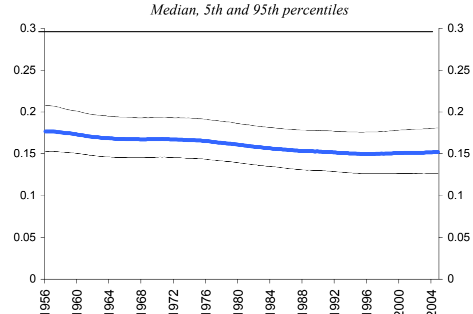
C. G Shock
Median, 5th and 95th percentiles



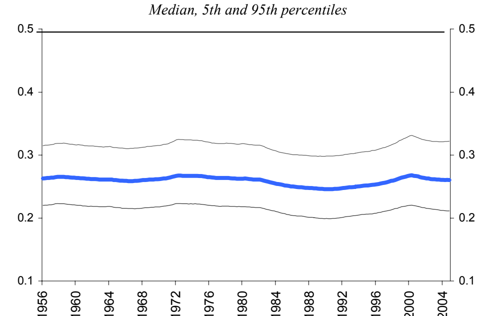
D. μ Shock
Median, 5th and 95th percentiles



E. λ Shock
Median, 5th and 95th percentiles



F. ψ Shock
Median, 5th and 95th percentiles



G. β Shock
Median, 5th and 95th percentiles

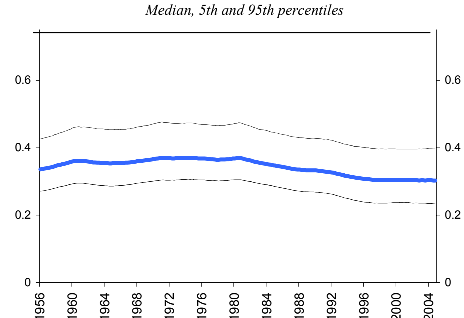
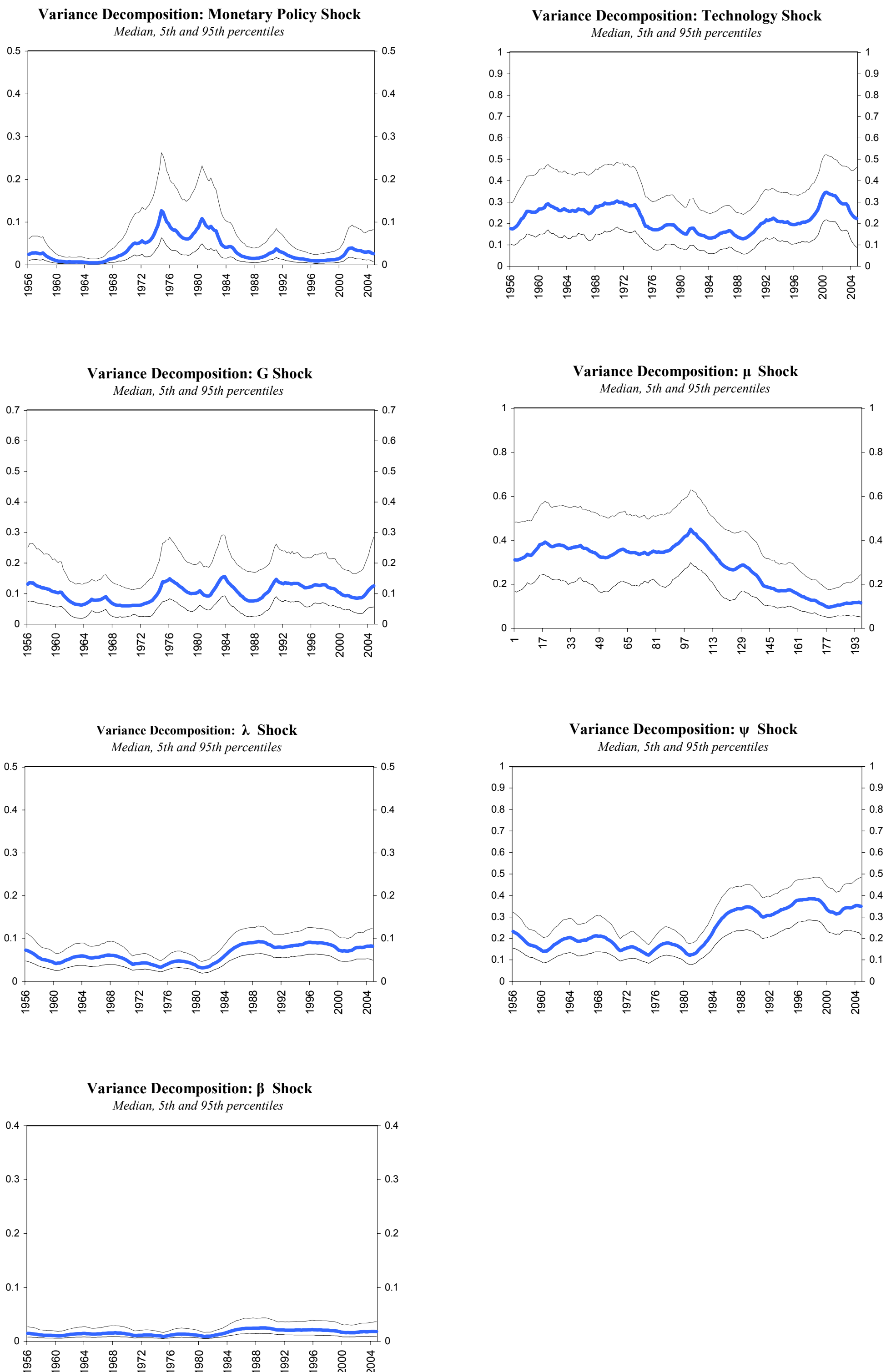


Figure 2: Variance Decomposition for Output Growth 1/



1/ For variance decompositions, medians need not add up to exactly one, but means do

Figure 3: Selected Variance Decomposition (VD) for Other Series

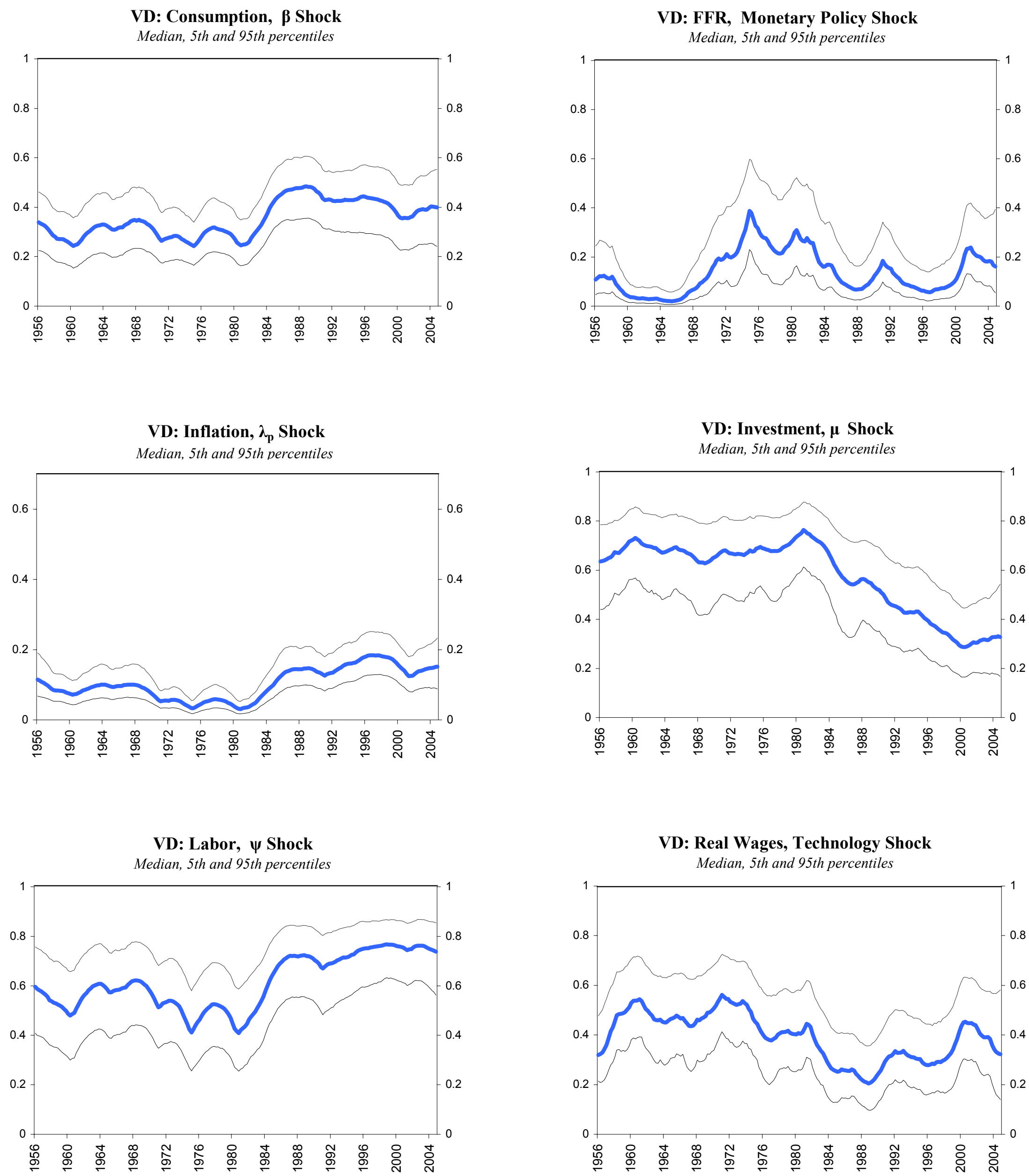


Figure 4: DSGE Time-Varying Standard Deviation of Output Growth

Median and 5th and 95th percentiles

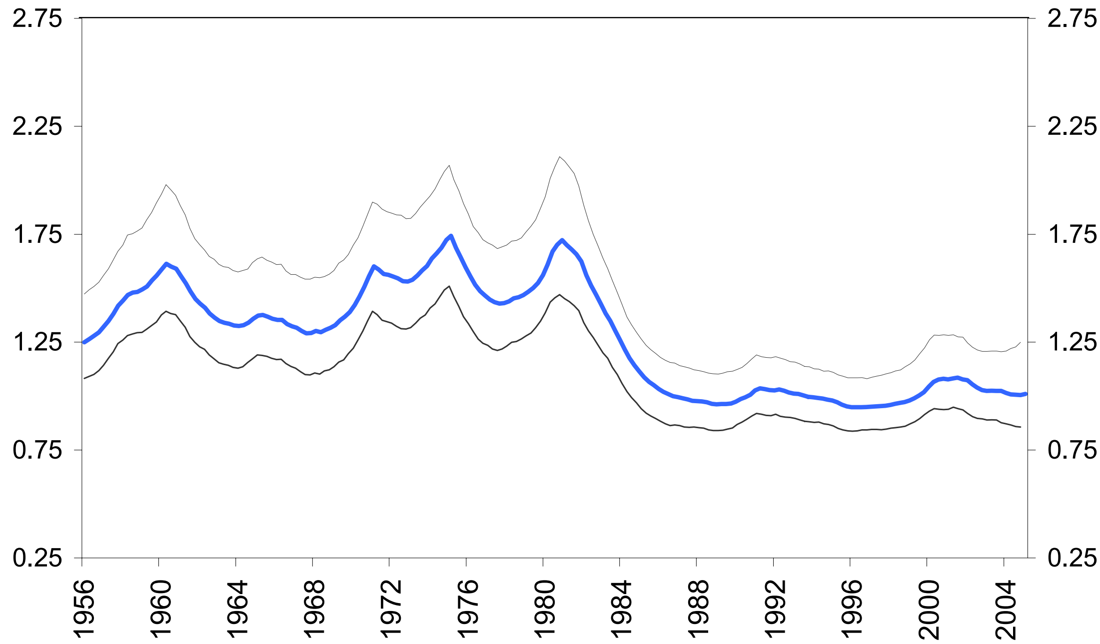


Figure 5: Actual and counterfactual standard deviation (std) for output growth

Counterfactual std obtained by fixing for the remainder of the sample the std of each shock, one at a time, to the average level of the time variying standard deviations in 1980 (MA4)

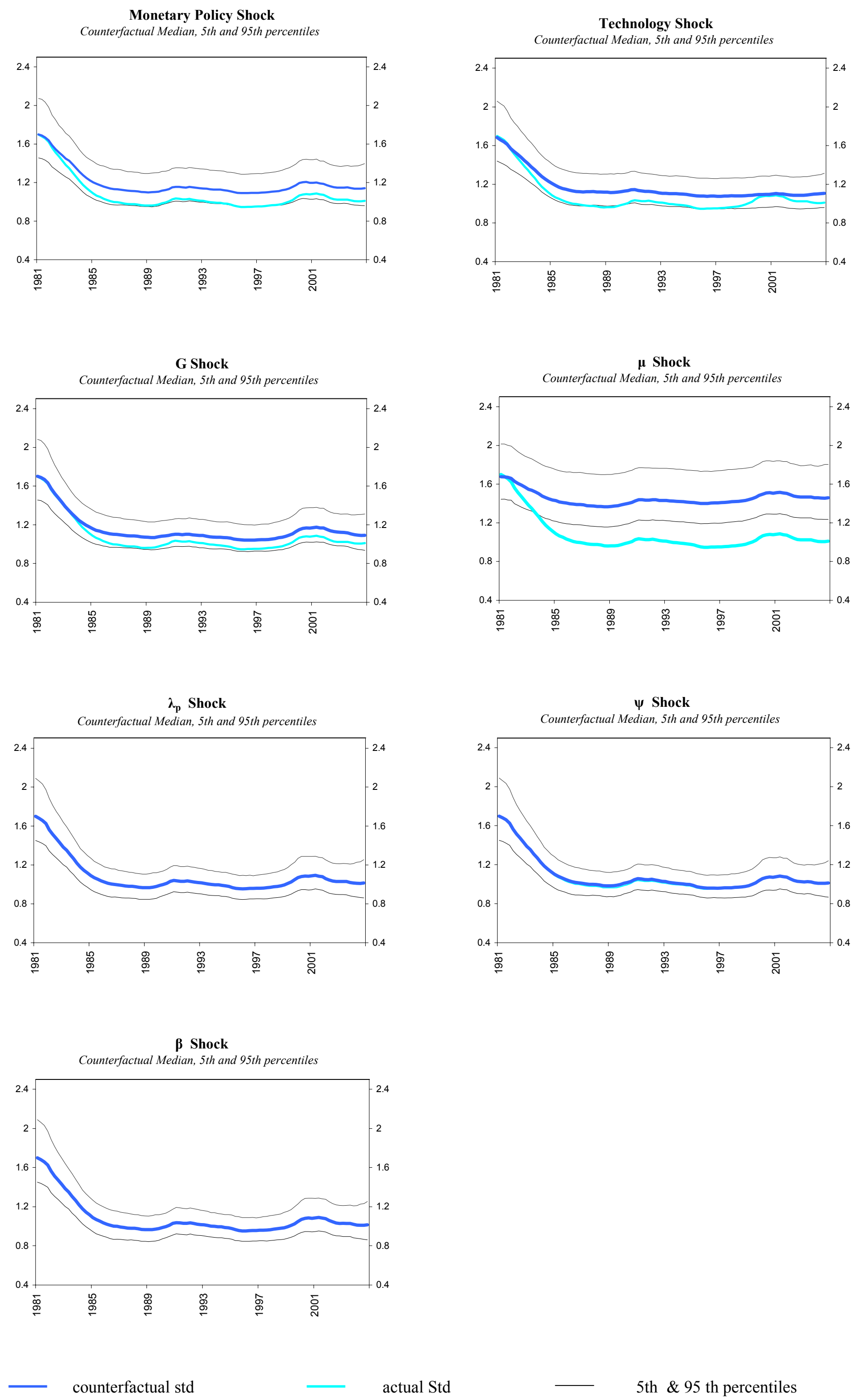


Figure 6: Time-Varying Standard Deviation Relative Price of Investment to Consumption

Obtained using an AR(4) with stochastic volatility

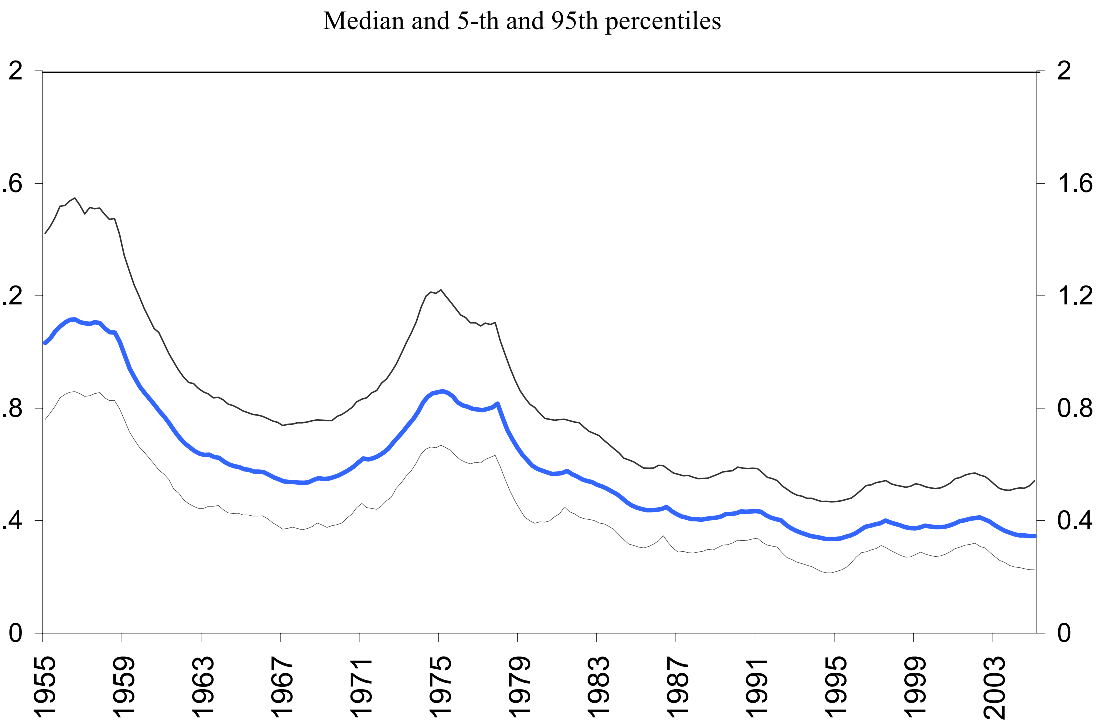


Figure 7: Actual and counterfactual standard deviation (std) for output growth exlcuding stocks

Counterfactual std obtained by fixing for the remainder of the sample the std of each shock, one at a time, to the average level of the time variying standard deviations in 1980 (MA4)