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THE TIMING OF INTERGENERATIONAL TRANSFERS, TAX POLICY, AND AGGREGATE SAVINGS

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ABSTRACT

We analyze the interest rate and savings effects of fiscal policy in an overlapping generations framework that accommodates two observations: (1) The interest rate on consumption loans exceeds the rate of return to household savings. (2) Private intergenerational transfers are widespread and primarily occur early in the lifecycle of recipients. The wedge between borrowing and lending rates in our model arises from the asymmetric tax treatment of interest income and interest payments. Intergenerational transfers are altruistically motivated.

Under the assumption that altruistic transfers occur in at least some family lines and other plausible conditions, we prove the invarience of capital's steady-state marginal product to government expenditures, government debt, the labor income tax schedule, and the tax rate on capital income. In contrast, we find that the tax treatment of interest payments has powerful effects on captial's marginal product and aggregate savings in life-cycle and, especially, altruistic linkage models.

Our theoretical analysis also generates new testable implications for empirical work on how tax policy effects aggregate savings and on the connection between the age distribution of resources and the age distribution of consumption. Simulations of our model suggest that the 1986 Tax Reform Act's elimination of interest deductibility on consumer loan repayments will significantly increase per capita savings.

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1. Introduction

The interest rate on consumption loans greatly exceeds the rate of return to household savings. As documented in Table 1, during selected years over the past two decades the after-tax nominal interest rate on unsecured personal loans averaged 12.4% per year, while the after-tax nominal rate of return on government securities averaged only 6.5%. The after-tax wedge between household borrowing and lending rates averaged 5.7 percentage points. This wedge increases to a full eight percentage points, if we use the credit card rate as the measure of household borrowing rates. A wedge of six to eight percentage points is too large to explain away by a simple adjustment for positive default rates on unsecured consumer loans. Thus, households face a kink in their intertemporal budget constraint. We take this simple empirical observation as one stepping-off point for our analysis of how tax and debt policy affect aggregate savings and interest rates.

We develop our analysis in the context of an overlapping generations framework that encompasses a wedge between borrowing and lending rates. We model the source of this wedge as the asymmetric tax treatment of interest income and interest payments on consumption loans. We focus on this source of the wedge for three reasons: (i) this component of the wedge is directly manipulable by tax policy; (ii) as the positive entries in row (9) of Table 1 indicate, asymmetries in the tax code make the wedge larger; and (iii) many past and proposed reforms of the U.S. tax code imply nontrivial changes in the wedge.

As an example of tax policy's impact on the size of the wedge between borrowing and lending rates, consider the Tax Reform Act of 1986. Comparing the 1984 and post-reform entries in Table 1 indicates that a direct effect of the Tax Reform Act is to increase the size of the wedge by three percentage points. We calculate this figure by applying the 1984 tax rates to the post-reform, pre-tax interest rates.[1]

While tax code asymmetries contribute to the wedge between borrowing and lending

[1] The figures in row (5) of Table 1 are not adjusted for provisions in the tax code governing tax-sheltered savings, i.e., IRAs. Since the Tax Reform Act of 1986 greatly restricted the availability of IRAs, Table 1 understates the Act's impact on the wedge. Our attempts to adjust the measure of ρ for IRAs suggests that the 1986 Act increased the average after-tax wedge by more than three and one-half percentage points.

TABLE 1

	Household Borrowing and Savings Rates, Selected Years					
	1972	1980	1983	1984	Post 1986 Tax Reform	Avg.
(1) Average rate on two-year personal loans (credit cards)[a]	.127 (.172)	.155 (.173)	.165 (.188)	.165 (.188)	.147 (.178)	.152 (.180)
(2) Average Marginal Subsidy Rate to Borrowing, δ^b	.181	.247	.224	.249	0	.187
(3) After-tax Borrowing Rate, $(1-\delta)$ times (1)	.104 (.141)	.117 (.130)	.128 (.146)	.124 (.141)	.147 (.178)	.124 (.147)
(4) Rate on three-year U.S. Treasury Securities[c]	.057	.116	.105	.119	.083	.092
(5) Average Marginal Tax Rate on Interest Income, ρ^d	.313	.346	.302	.292	.217	.296
(6) After-tax Rate of Return to Savings, $(1-\rho)$ times (4)	.039	.076	.073	.084	.065	.065
(7) Pre-tax Wedge Between Borrowing and Saving Rates, (1) minus (4)	.07 (.115)	.039 (.057)	.060 (.083)	.046 (.069)	.064 (.095)	.060 (.084)
(8) After-tax Wedge Between Borrowing and Saving Rates, (3) minus (6)	.065 (.102)	.041 (.054)	.055 (.073)	.040 (.057)	.082 (.113)	.057 .080
(9) Tax Wedge, $(\rho-\delta)$	.132	.099	.078	.043	.217	.109

Table 1, continued

Notes:

a. Source: *Federal Reserve Bulletin*, various issues. 1972 is the first year that these series are reported in the *Bulletin*.

b. Values for 1972, 1980, 1983, and 1984 were calculated by the authors as the borrowings-weighted average of marginal subsidy rates on unsecured personal loans. The calculations are based on data in *Statistics of Income*, published by the Internal Revenue Service. An appendix detailing the calculations is available from the authors upon request. The Post-1986 Tax Reform rate is based on the fully phased-in consumer loan provisions in the Tax Reform Act of 1986.

c. Source: *Federal Reserve Bulletin*, various issues.

d. Values for 1972 and 1980 are savings-weighted averages of marginal tax rates on interest income from Estrella and Fuhrer (1983). The values for 1983 and 1984 were calculated by the authors using the same procedure as Estrella and Fuhrer. The post-Reform value is from Hausman and Poterba (1987).

rates, Table 1 also indicates that other features of the economy account for the bulk of the wedge. In this connection, we remark that our framework accomodates (with minor modifications) any capital market imperfection that amounts to a proportional transactions cost in the consumption-loans market.

As a second stepping-off point for our analysis, we note the prevalence and magnitude of intergenerational transfers. Based on a representative cross-section of U.S. households, Cox and Raines (1985) report high incidence rates for the receipt of private transfers over the first eight months of 1979, especially among family units headed by a person less than twenty-five years old. Cox and Raines also provide evidence that most private transfers are intergenerational, that the overwhelming bulk of intergenerational transfers are from older to younger generations, and that most intergenerational transfers occur inter vivos. Using the same data set as Cox and Raines, Kurz (1984) estimates that private intergenerational transfers amounted to $63 billion in 1979, excluding inheritances.[2]

We do not integrate a full range of transfer motives into our analytical framework. Instead, we focus on intergenerational altruism as a transfer motive and explore its implications in economies with a wedge between borrowing and lending rates. It seems to us that a complete explanation for the magnitude and prevalence of intergenerational transfers is likely to involve an important role for intergenerational altruism. In any case, several of our chief results require only that altruism motivates some intergenerational transfers, not that it motivate all or even most intergenerational transfers.

Our results provide answers to four questions. First, how does the existence of a wedge between borrowing and lending rates affect the life-cycle timing of altruistically motivated intergenerational transfers? Second, in economies that contain a wedge in the loan market and at least some altruistic family lines, what are the long-run effects of

[2]Other empirical approaches bear out the importance of intergenerational transfers. Kotlikoff and Summers (1981) construct age-earnings and age-consumption profiles to compute life-cycle wealth (savings for retirement) for various age cohorts in the United States. By comparing their computation for life-cycle wealth to aggregate wealth, they conclude that intergenerational transfers account for the bulk of aggregate savings. See, also, Kotlikoff (1988) and Modigliani (1988) and references therein. Our analysis does not address the aggregate savings puzzle identified by these studies. As we show below, intergenerational transfers in our framework occur inter vivos and are used to finance consumption.

government expenditures, government debt, unfunded social security, and labor income taxation on aggregate savings and capital's marginal product? Third, how do tax policy changes that alter the size of the wedge affect aggregate savings and capital's marginal product? Fourth, what does the existence of a wedge between borrowing and lending rates imply about the relationship of overlapping generations models with altruistic family lines to models with infinitely-lived representative agents?

With respect to the first question, the existence of a wedge between borrowing and lending rates pins down the optimal timing of intergenerational transfers. Altruistically motivated intergenerational transfers occur early in the life cycle when borrowing rates exceed lending rates. This timing result implies that the wedge destroys the fully interconnected set of budget constraints that undergirds standard Ricardian neutrality results. We show, for example, that an increase in the scale of an unfunded social security program causes a short-run reduction in aggregate savings. This outcome occurs in a model in which each generation is linked to its successor generation by altruistic transfers early in the life cycle.

With respect to the second question, we derive a powerful long-run neutrality result relating changes in government expenditures, government debt, the scale of social security programs, and the labor income tax schedule to capital's marginal product: If at least *some* family lines are characterized by (a) an operative transfer motive and (b) young persons who are at an interior solution with respect to their borrowing or saving decision, then capital's steady-state marginal product is invariant to each of these interventions.

Unlike neutrality results in the tradition of Barro (1974), Becker (1984), and Bernheim and Bagwell (1988), the proof of our neutrality result does not rest upon a network of interconnected budget constraints. Thus, our neutrality result is more robust and less comprehensive than the Ricardian Equivalence Theorem. It is more robust in three senses: it applies to a wider class of interventions, it does not require perfect capital markets, and it does not rest upon pervasive intergenerational altruism. It is less comprehensive in the sense that it applies only to the steady-state marginal product of capital.

With respect to tax policy interventions that affect the size of the wedge, we show the following. First, if the household borrowing rate exceeds the rate of return to saving (as in Table 1), and if the young borrow and condition (a) holds in at least some family lines, then changes in the proportional tax rate on capital income have no long-run effect on capital's marginal product. It follows that for a plausible elasticity of aggregate labor supply, aggregate savings is highly inelastic with respect to changes in the tax rate on capital income. Second, under the same conditions, capital's long-run marginal product is highly sensitive to changes in the proportional subsidy rate on interest payments. It follows that aggregate savings is highly elastic with respect to changes in the subsidy rate on interest payments, regardless of whether labor supply is elastic. Thus, our analysis identifies empirically plausible conditions under which the tax treatment of household borrowing provides a much more potent tool for influencing aggregate savings than the tax treatment of capital income.

Finally, with respect to the fourth question, our analysis highlights the sharp distinctions between overlapping generations models with altruistic linkages and representative agent models. Since even a small wedge between borrowing and lending rates pins down the optimal timing of intergenerational transfers, altruistic linkage models are quite generally *not* isomorphic to representative agent models. The distinct fiscal policy implications of these two models, and the life-cycle model, emerge clearly in some numerical simulations summarized in section 6. The simulations focus on the long-run response of aggregate savings to changes in the tax rate on capital income and changes in the subsidy rate on interest payments.

We turn now to a description of our analytical framework.

2. An Overlapping Generations Framework with Capital Income Taxation

Consider an overlapping generations production economy populated by persons who live three periods. Each member of generation t supplies homogeneous labor services (L_{1t}, L_{2t}, L_{3t}) over the life cycle according to a lifetime productivity profile $(\alpha_1, \alpha_2, \alpha_3)$ and a labor-leisure choice spelled out below. Aggregate period-t labor supply is given by

$$(1+n)^t L_t = \left[\alpha_1 L_{1t} + \frac{\alpha_2 L_{2,t-1}}{1+n} + \frac{\alpha_3 L_{3,t-2}}{(1+n)^2}\right](1+n)^t, \tag{1}$$

where n is the population growth rate, and we have normalized so that generation 0 has one member.

Defining $k = \frac{K}{L}$ as the capital-labor ratio, we write the aggregate production function as

$$Y_t = F[K_t, (1+n)^t L_t] \equiv (1+n)^t L_t f(k_t) \tag{2}$$

where $f'(\cdot) > 0$, $f''(\cdot) < 0$, $\lim_{k\to 0} f(k) = \infty$, and $\lim_{k\to\infty} f'(k) = 0$. The representative firm's competitive profit maximization conditions are

$$W_t = f(k_t) - k_t f'(k_t) \tag{3}$$

and

$$r_t = f'(k_t) \tag{4}$$

where W_t is the period-t wage in units of the produced good and r_t is the rate of return on physical capital held from time $t-1$ to time t.

The representative member of generation t chooses a sequence over consumption, labor supply, and intergenerational transfers to maximize:

$$U_t = \sum_{i=1}^{3} \beta^{i-1} u(C_{it}) + \sum_{i=1}^{3} \beta^{i-1} v(L_{it}) + \beta\gamma U_{t+1}^{*} \tag{5}$$

where

C_{it} = consumption by a member of generation t in the ith period of life,

L_{it} = labor supply by a member of generation t in the ith period of life,

β = intertemporal discount factor, $0 < \beta < 1$,

γ = interpersonal discount factor, $0 \leq \gamma < (1+n)/\beta$ (insures a positive steady-state interest rate when transfer motives operate and capital markets are perfect),

$u(\cdot)$ = period utility function (over consumption), satisfying $u'(\cdot) > 0$, $u''(\cdot) < 0$, $\lim_{C\to 0} u'(C) = \infty$, and $\lim_{C\to\infty} u'(C) = 0$,

$v(\cdot)$ = period utility function (over labor supply), satisfying $v'(\cdot) < 0$, $v''(\cdot) < 0$, $\lim_{L\to 0} v'(L) = 0$, and $\lim_{L\to\overline{L}} v'(L) = -\infty$, where $\overline{L}$ is a positive upper bound on labor supply,

U_{t+1}^{*} = maximum utility attainable by a generation $t+1$ agent as a function of intergenerational transfers received.

The specification of altruistic preferences in (5) mirrors the specification in Barro (1974) and many other analyses. We allow for operative and inoperative transfer motives, so that (5) also encompasses pure life-cycle economies.

Turning to the household budget constraints, we consider lifetime productivity profiles such that the middle-aged choose to save and the young potentially save or borrow. A key feature of our model is a wedge between household borrowing and lending rates. We explicitly model the source of this wedge as distortionary taxation of interest income that is not (fully) matched by the subsidy applied to interest payments on consumption loans. Alternatively, we could interpret the wedge as arising from any capital market imperfection that amounts to a proportional transaction cost in the consumption-loans market. (This alternative interpretation would lead to a slight modification of some results.)

It is worthwhile to observe that, for a sufficiently large wedge between borrowing and savings rates, young households may choose a corner position at which they neither save nor borrow. A wedge economy with a corner outcome is (locally) equivalent to an economy with binding borrowing constraints that stem from the absence of ex post enforcement mechanisms in the consumption-loans market or any other capital market imperfection severe enough to shut down the consumption-loans market. Thus, our overlapping generations framework encompasses capital market imperfections that take the form of borrowing constraints. In this paper, we focus primarily on equilibria in which the young are at an interior solution with respect to either their savings or their borrowing decision. Corner outcomes are considered in some of our numerical exercises. For a complete analysis of corner equilibria, we refer the reader to Altig and Davis (1989, 1991).

With these remarks in mind, we write the budget equations for a representative member of generation t as

$$C_{1t} + a_{1t} + T_{1t} = \alpha_1 L_{1t} W_t + b_{1t} + x_t \tag{6}$$

$$C_{2t} + (1+n) b_{1,t+1} + \psi_{t+1} x_t + a_{2t} + d_{t+1} + T_{2t} = \phi_{t+1} a_{1t} + \alpha_2 L_{2t} W_{t+1} + b_{2t} \tag{7}$$

$$C_{3t} + (1+n) b_{3,t+1} + T_{3t} = \phi_{t+2}(a_{2t} + b_{3t} + d_{t+1}) + \alpha_3 L_{3t} W_{t+2} \tag{8}$$

where

x_t = borrowings by generation t when young,

a_{1t} = savings (claims to capital) by generation t when young,

a_{2t} = savings (in the form of claims to capital or repayment of consumption loans) by generation t when middle- aged,

$b_{i,t+1}$ = transfers made by a generation-t parent to each $(1 + n)$ offspring in the children's ith period of life (an *inter vivos* transfer for $i = 1, 2$, a bequest for $i = 3$),

T_{it} = lump-sum taxes (subsidies if negative) levied on a member of generation t during the ith period of life,

d_{t+1} = government debt issued at time $t+1$ per middle-aged person,

r_t = the pre-tax rate of return from $t-1$ to t on claims to physical capital, government debt, and the repayment of consumption loans,

$\phi_t = 1 + r_t(1-\rho)$ where ρ = proportional tax rate on interest income, and

$\psi_t = 1+r_t(1-\delta)$ where δ = the proportional subsidy rate applied to interest payments on consumption loans.

For simplicity, and without loss, the budget constraints incorporate the assumption that all government debt is purchased by the middle-aged.

The representative consumer maximizes (5) subject to (6)-(8) and the non-negativity constraints on consumption, labor supply, transfers, savings, and borrowings. Assuming nonpositive savings by the young ($a_{1t} = 0$), the consumer's intertemporal first-order conditions can be written

$$u'(C_{1t}) \leq \beta(1 + r_{t+1}(1-\delta))u'(C_{2t}) \quad \text{and,} \tag{9}$$

$$u'(C_{2t}) = \beta(1 + r_{t+2}(1-\rho))u'(C_{3t}). \tag{10}$$

Equation (9) holds as an equality when the loan market is active; it holds as an inequality when the loan market is inactive and the young are at a corner.

Using the envelope theorem, the first-order conditions governing intergenerational transfers are

$$u'(C_{2t}) \geq \frac{\gamma}{1+n}u'(C_{1,t+1}) \quad \text{with equality if } b_{1,t+1} > 0 \tag{11}$$

$$u'(C_{3t}) \geq \frac{\gamma}{1+n}u'(C_{2,t+1}) \quad \text{with equality if } b_{2,t+1} > 0 \tag{12}$$

for *inter vivos* transfers, and

$$u'(C_{3t}) \geq \frac{\gamma\beta}{1+n}(1 + r_{t+2}(1-\rho))u'(C_{3,t+1}) \quad \text{with equality if } b_{3,t+1} > 0 \tag{13}$$

for bequests. Equations (11)-(12) state that, when an *inter vivos* transfer motive operates, the discounted marginal rate of substitution of parents' consumption for children's consumption equals the (population growth) deflated interpersonal discount factor. Equation (13) has a similar interpretation.

The static first-order conditions characterizing the labor-leisure tradeoff for a member of generation t are given by

$$v'(L_{it}) = -\alpha_i W_{t+i-1} u'(C_{it}), \quad \text{for i=1,2,3.} \tag{14}$$

To complete the framework, we specify the government budget constraint, the goods market-clearing condition, and the capital market-clearing condition:

$$g_t + \frac{(1+r_t)}{1+n} d_{t-1} = (1+n)\Gamma_{1t} + \Gamma_{2,t-1} + \frac{\Gamma_{3,t-2}}{1+n} + d_t \tag{15}$$

$$(1+n)L_{t+1}k_{t+1} - L_t k_t + C_{1t} + \frac{C_{2,t-1}}{1+n} + \frac{C_{3,t-2}}{(1+n)^2} + g_t = L_t f(k_t) \tag{16}$$

$$L_t k_t + (1+n)x_{t-1} = (1+n)a_{1,t-1} + a_{2,t-2} + b_{3,t-2} \tag{17}$$

where

g_t = government expenditures on goods and services at time t per middle-aged person,

$\Gamma_{1t} = T_{1t}$,

$\Gamma_{2,t-1} = T_{2,t-1} - \delta r_t x_{t-1} + \rho r_t a_{1,t-1}$,

$\Gamma_{3,t-2} = T_{3,t-2} + \rho r_t (a_{2,t-2} + b_{3,t-2} + d_{t-1})$.

We assume that, on the margin, government expenditures are unproductive and do not substitute for private consumption. For our purposes, nothing essential is altered by relaxing these assumptions.

For economies that fit within this framework, an equilibrium is a sequence $\{C_{1t}, C_{2,t-1}, C_{3,t-2}, L_{1t}, L_{2,t-1}, L_{3,t-2}, x_t, a_{1t}, a_{2,t-1}, b_{1t}, b_{2,t-1}, b_{3,t-2}, W_t, r_{t+1}, k_t, g_t, d_t, T_{1t}, T_{2,t-1}, T_{3,t-2}\}_{t=0}^{\infty}$ that satisfies (3)-(14), the non-negativity constraints, the market-clearing conditions and the government budget constraint for all t, given the initial condition $(x_{-1}, a_{1,-1}, a_{2,-2}, k_0, d_0)$.

3. The Optimal Timing of Altruistic Intergenerational Transfers

In Barro's (1974) Ricardian environment the optimal timing of altruistic intergenerational transfers is indeterminate. Since capital markets are perfect, children and parents care only about the present value of intergenerational transfers and not their exact timing. This timing indeterminacy supports an extensive set of intergenerational linkages, and these linkages, in turn, play a key role in neutralizing certain fiscal policies. A straightforward, but central, result that emerges from our framework is the knife-edge character of this timing indeterminacy.

The slightest friction in the consumption-loans market in the form of a wedge between borrowing and lending rates–or a strong friction like binding borrowing constraints–pins down the optimal timing of altruistically motivated intergenerational transfers. Once the timing of intergenerational transfers is pinned down, the extensive set of intergenerational linkages in Ricardian environments breaks down. Despite this general observation, the fiscal policy implications of pinning down the timing of intergenerational transfers depend very much on whether capital market imperfections drive potential borrowers to a corner solution, whether capital market imperfections arise from transaction costs or tax considerations, and on the elasticity of labor supply.

We now state two propositions that characterize the optimal timing of altruistically motivated transfers. The first proposition applies when borrowing rates exceed lending rates in an active consumption-loans market or when the wedge between borrowing and lending rates is large enough to drive the young to a corner with respect to their borrowing decision. The second proposition applies when lending rates exceed borrowing rates.

Proposition 1: Assume that borrowing rates exceed lending rates ($\rho > \delta$) in the consumption-loans market and that the nonnegativity constraint binds on a_1. Then, if intergenerational transfers are positive, $b_1 > 0$ and $b_2 = b_3 = 0$.

Proof:

Interior solution for x:

Suppose that $b_2 > 0$, so that (12) holds with equality. Combining (12) and (10) yields

$$r = \frac{1+n-\beta\gamma}{\beta\gamma(1-\rho)}. \tag{18}$$

Substituting into (9) yields

$$u'(C_1) = \beta[1 + (\frac{1+n}{\gamma\beta} - 1)(\frac{1-\delta}{1-\rho})]u'(C_2)$$

Equation (11) requires that $u'(C_1) \leq (\frac{1+n}{\gamma})u'(C_2)$. This condition holds if and only if

$$\beta[1 + (\frac{1+n}{\gamma\beta} - 1)(\frac{1-\delta}{1-\rho})]u'(C_2) \leq \frac{1+n}{\gamma}u'(C_2)$$

$$\Rightarrow (\frac{1+n}{\gamma\beta} - 1)(\frac{1-\delta}{1-\rho}) \leq \frac{1+n}{\gamma\beta} - 1$$

$$\Rightarrow 1-\delta \leq 1-\rho,$$

which implies $\delta \geq \rho$, violating the hypothesis (a). Thus, b_2 cannot be positive.

Now suppose $b_3 > 0$. Then (13) leads to (18), and we obtain a contradiction in the same way as before. Thus, b_3 cannot be positive.

Finally, when $b_1 > 0$, (9) and (11) imply

$$r = \frac{1+n-\beta\gamma}{\beta\gamma(1-\delta)}. \tag{19}$$

It is straightforward to verify that equations (12) and (13) are consistent with (19) when $b_2 = b_3 = 0$. Thus, if intergenerational transfers are positive, only $b_1 > 0$.

Corner solution for x:

As before, suppose $b_2 > 0$ or $b_3 > 0$. Then (12) or (13) in combination with (10) yields (18). Since the young are at a corner with respect to their savings decision, and making use of (18),

$$u'(C_1) > \beta[1 + (\frac{1+n}{\gamma\beta} - 1)(\frac{1-\rho}{1-\rho})]u'(C_2) \quad \Rightarrow u'(C_1) > \frac{1+n}{\gamma}u'(C_2),$$

which contradicts (11). Thus, $b_2 = b_3 = 0$. Furthermore, $b_1 > 0$ is consistent with (9)-(13).

Q.E.D.

Following the same line of argument as in the preceding proof, we have

Proposition 2: Assume that lending rates exceed borrowing rates in the consumption loans market and that the nonnegativity constraint binds on a_1. Then, if intergenerational transfers are positive, $b_2 > 0$ or $b_3 > 0$, or both, and $b_1 = 0$.

The intuition behind these timing propositions is straightforward. Parents choose the timing of intergenerational transfers to exploit the wedge between the after-tax borrowing rate faced by the child and the after-tax rate of return on own savings. More generally, in the cases covered by Proposition 1 (2), the marginal rate of substitution of current for future consumption is higher (lower) for children than for parents. Thus, transfers early (late) in the life cycle dominate transfers late (early) in the life cycle. As we show below, this timing result has important implications for fiscal policy.

4. Equilibrium Linkage Regimes

Our analytical framework admits several possible patterns of capital market participation by the young and altruistic linkages between generations. We refer to a feasible equilibrium pattern as a linkage regime. While our framework implies a unique equilibrium linkage regime for any particular parametrization, different parametrizations correspond to different linkage regimes. In this section we identify the set of equilibrium linkage regimes, and we briefly analyze the factors that determine the prevailing regime. We focus on the empirically relevant case where borrowing rates exceed the rate of return to savings.

Given that borrowing rates exceed lending rates, Proposition 1 rules out linkage regimes with altruistic transfers late in the life cycle. Given that the middle-aged always save in our framework, there remain six candidates for equilibrium linkage regimes:

Regime A: No transfers, young borrow.

Regime B: No transfers, young at a corner with respect to borrowing/saving decision.

Regime C: No transfers, young save.

Regime D: Middle-aged make transfers, young borrow.

Regime E: Middle-aged make transfers, young at a corner.

Regime F: Parents make transfers, young save.

Note that Regime C is the standard life-cycle model with distortionary capital income taxation, and Regime F is Barro's dynastic model with distortionary capital income taxation.

It turns out that all six candidates emerge as the equilibrium linkage regime in some plausible region of the parameter space. We illustrate this result in Figures 1-3. Each figure indicates the prevailing regimes in a two-dimensional slice of the parameter space. These figures are constructed assuming Cobb-Douglas production with capital's share θ equal to .25, zero population growth, $\beta = (.99)^{25}$, inelastic labor supply, $\rho = .22$, no government purchases, and the return of all distortionary taxes and subsidies to the impacted generation via lump-sum transfers. Unless otherwise indicated, the figures assume a value of zero for δ and a value of .33 for σ_c, the consumption intertemporal substitution elasticity. The lifetime productivity profile is set at $(\alpha_1, \alpha_2, \alpha_3) = (1.5, 6, 2.5)$ in Figures

Figure 1.
Boundary Loci: Inelastic labor supply, rho=.22, beta = .778, sigma_C=.33, theta=.25

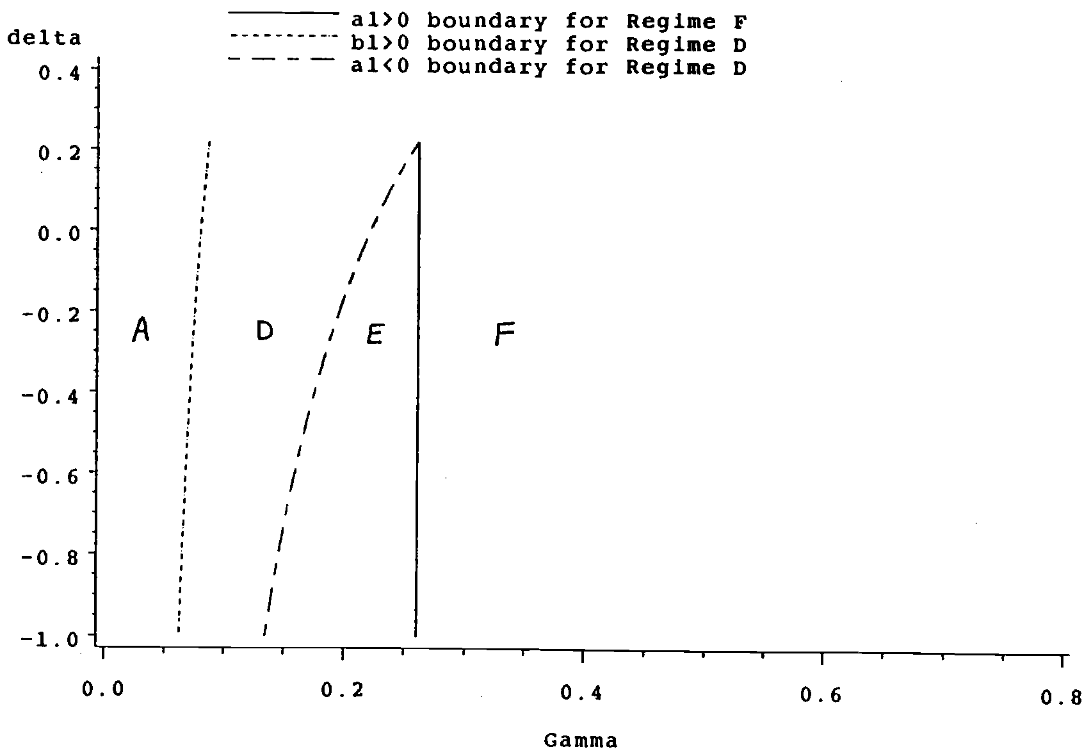


Figure 2.
Boundary Loci: Inelastic labor, rho=.22, delta=0, sigma_c=.33, beta=.778, theta=.25

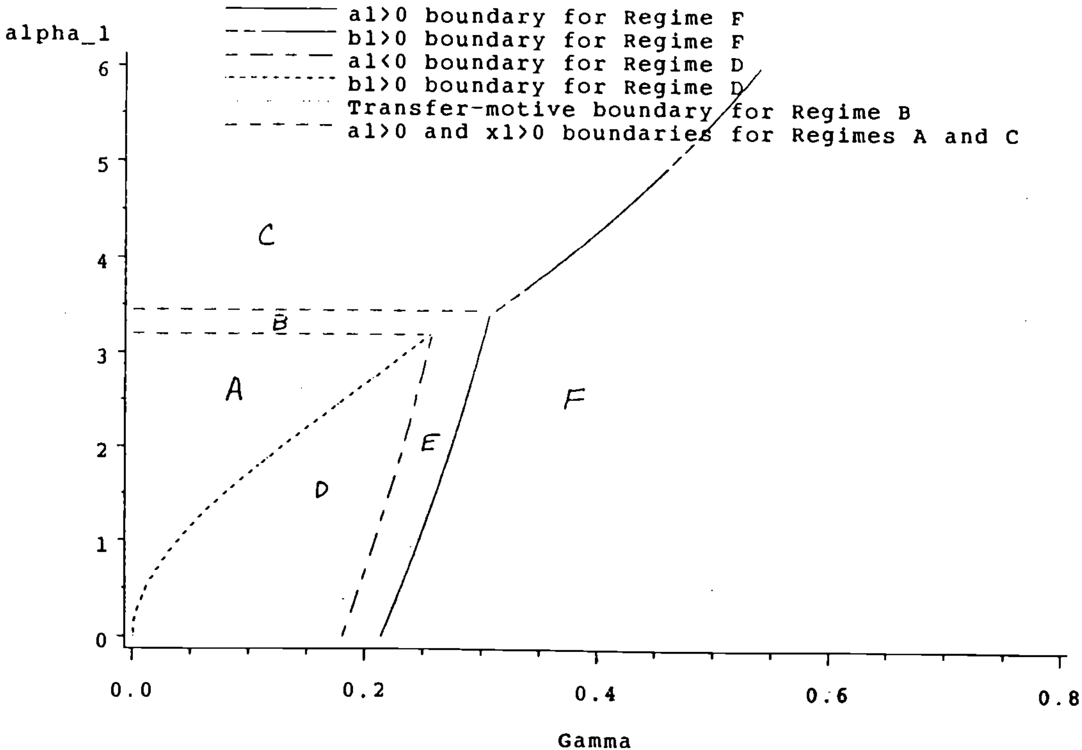
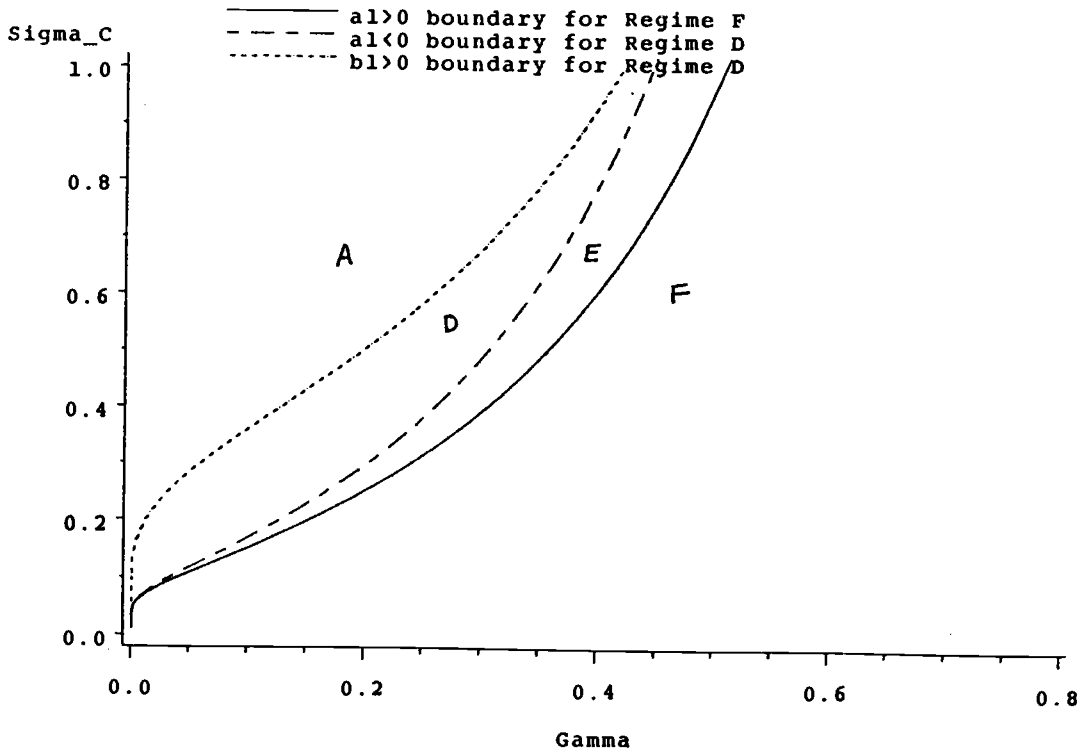


Figure 3.
Boundary Loci: Inelastic labor supply, rho=.22, delta=0, beta=.778, theta=.25



1 and 2. Figure 3 uses identical values for α_2 and α_3, but allows α_1 to vary from zero to six.[3]

Figure 1 shows boundary loci in $\delta - \gamma$ space. Negative values for δ can be interpreted as either a tax on borrowing or as a proportional transaction cost in the consumption-loans market.[4] As the figure indicates, at high levels of parental altruism intergenerational transfers are large enough to obviate the kink in the intertemporal budget constraint – consumption behavior satisfies $u'(C_1) = \beta[1 + r(1 - \rho)]u'(C_2)$, and the economy lies in Regime F. At more modest levels of parental altruism, transfers become smaller and young persons choose the kink point on their intertemporal budget constraint – the economy lies in Regime E. At yet more modest levels of parental altruism, transfers are sufficiently small that young persons choose to borrow – consumption behavior satisfies $u'(C_1) = \beta[1 + r(1 - \delta)]u'(C_2)$, and the economy lies in Regime D. The greatest scope for Regime-D equilibria, in which the young both borrow and receive transfers, involves a modest wedge between borrowing and lending rates. Finally, for sufficiently small levels of parental altruism, no transfers occur but the young continue to borrow – the economy lies in Regime A.[5]

Figure 2 shows boundary loci in $\alpha_1 - \gamma$ space. All six linkage regimes emerge as the equilibrium outcome for some combinations of α_1 and γ. Given α_2 and α_3, larger values for α_1 correspond to a smaller slope in the lifetime productivity profile over the first two periods of life. Hence, the figure shows that a more steeply-sloped lifetime productivity profile tends to increase the incidence of borrowing and increase the incidence of transfers.

[3]An appendix, available upon request from the authors, discusses the choice of parameters and the techniques used to determine the boundaries between the linkage regimes. The appendix also displays additional slices of the parameter space involving capital's share, the time discount factor, and the labor supply intertemporal substitution elasticity.

[4]The figure is constructed under the tax interpretation of the wedge between borrowing and lending rates. The transaction cost interpretation requires a modified goods market-clearing condition and leads to slightly different boundary loci separating Regimes A,D and E.

[5]Although not shown in Figure 1, for low values of γ and sufficiently large taxes on borrowing, the economy lies in Regime B. Given the other parameter settings underlying Figure 1, there are no points in $\delta - \gamma$ space such that the economy lies in Regime C.

Lastly, Figure 3 shows boundary loci in $\sigma_c - \gamma$ space. This figure highlights the interaction between the intertemporal substitution elasticity and the strength of the altruism motive in determining the prevailing linkage regime. When consumption is relatively elastic, operative transfers, as well as positive savings by the young, require stronger altruism. Depending on the consumption intertermporal substitution elasticity, each of the regimes appearing in Figure 3 is consistent with a wide range of values for the altruism parameter.[6]

Recapitulating the central message of Figures 1-3, we see that simple overlapping generations models with intergenerational altruism are consistent with a variety of equilibrium linkage regimes. Research on fiscal policy and savings behavior in overlapping generations constructs typically focuses on outcomes in a single regime. Thus, dynastic models – implicitly, Ramsey-type models as well – focus on outcomes in Regime F. Traditional life-cycle models focus on outcomes in Regime C. While the contrasting implications of the dynastic and traditional life-cycle models are well understood, fiscal policy consequences and savings behavior also differ, often sharply, across the remaining regimes. In addition, fiscal policy consequences and savings behavior in the remaining regimes often differ sharply from outcomes in either dynastic or life-cycle models.

We develop this key point about the importance of the prevailing linkage regime in the subsequent sections of the paper. We shall focus our analysis on the properties of Regime D and, to a lesser extent, Regime A. We have analyzed the properties of Regimes B and E in Altig and Davis (1989, 1991).[7]

5. Lump-Sum Fiscal Policy in the Altruistic Linkage Model

We turn now to an analysis of lump-sum fiscal policy in economies with altruistic intergenerational linkages and a wedge between borrowing and lending rates. We prove two results under the assumption of an active loan market. First, all lump-sum social security and government debt interventions are fully neutral in their effect on steady-state equilibrium. Second, we show by way of a simple example that these same fiscal policies

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[6]Regimes B and C also emerge as equilibrium linkage patterns for a wide range of values for the altruism parameter, given sufficiently large σ_c.

[7]In these papers, we consider overlapping generations models with a nonnegativity constraint on each person's holdings of nonhuman wealth. Under this type of nonnegativity constraint, Regimes B and E expand to fully encompass Regimes A and D.

are typically nonneutral in their short-run impact.

A. Long-Run Neutrality

Proposition 3: If

(a) the consumption-loans market is active,

(b) the altruistic transfer motive operates, and

(c) the level of government expenditures is constant,

then all fiscal policies that redistribute resources between generations in a lump-sum manner have no effect on steady-state values of interest rates, the capital stock, and the lifetime consumption profile.

Proof:

Case (i): $\rho > \delta$*:*

By hypothesis (a),

$$u'(C_1) = \beta\big[1 + r(1-\delta)\big]u'(C_2).$$

By hypothesis (b), $\rho > \delta$, and applying Proposition 1,

$$u'(C_2) = \frac{\gamma}{1+n}u'(C_1).$$

Combining these two equations yields (19). The parameters on the r.h.s. of (19) are independent of lump-sum fiscal policies. Thus, the capital-labor ratio is also independent of lump-sum fiscal policies.

Next, use the first-order conditions (9) and (10) to rewrite the goods market-clearing condition as

$$G(C_2, k, \delta, \rho) = L\big[f(k) - nk\big] - g, \tag{21}$$

where $\frac{\partial G(\cdot)}{\partial C_2} > 0$. By (19), the term in square brackets is a constant.

Now suppose that the capital stock rises following the fiscal intervention. k and g constant $\Rightarrow L$ rises $\Rightarrow C_2$ rises. But an increase in C_2 implies L falls by (14), a contradiction. We also obtain a contradiction when we suppose the capital stock falls. Thus, the capital stock does not change.

It follows that L, W, and aggregate consumption are also unchanged. Finally, since aggregate consumption and the interest rates are unchanged, it follows from (9) and (10) that the lifetime consumption profile is unchanged.

Case (ii), $\rho \leq \delta$:

The proof proceeds along lines parallel to case (i). Note that the steady-state interest rate is now given by (18).

Q.E.D.

The main distinguishing feature of Proposition 3 is the line of proof. To develop this point, consider the logic behind the neutrality results that appear in the literature. Fiscal policy neutrality results in the tradition of Barro (1974), Becker (1974), and Bernheim and Bagwell (1988) exploit the interconnectedness of budget constraints implied by operative altruistic transfers. (Bernheim and Bagwell refer to the interconnectedness of budget constraints as the linkage hypothesis.) Neutrality theorems in this tradition basically state that a government-imposed transfer between two persons or generations who are linked by altruistic transfers (before and after the government action) is neutral in its effects on consumption patterns and prices. Rather than resting upon the interconnectedness of budget constraints for the parties involved in the government-imposed transfer, our proof combines an intertemporal first-order condition with the transfer-motive first-order condition to pin down the interest rate in terms of preference, growth rate, and tax parameters. The remainder of the proof then follows from the intertemporal first-order condtions and the goods market-clearing condition.

The difference between our proof and the traditional logic can be easily seen in the following example. Suppose the economy lies in Regime D, and consider a government intervention that engages in lump-sum transfers from the middle-aged to the old. We have already established that the middle-aged and old are not linked by altruistic transfers in Regime D – i.e., their budget constraints are not interconnected. Thus, the traditional logic suggests that this government intervention will be nonneutral. In contrast, Proposition 3 informs us that the intervention is neutral in its steady-state effects.

The substance of Proposition 3 differs in two respects from the Ricardian Equivalence Theorem as proved by Barro (1974) and as reformulated many times in the subsequent literature. First, the neutrality result in Proposition 3 holds despite distortionary capital income taxation and, more generally, the asymmetric tax treatment of interest income and interest payments on consumption loans. Second, Proposition 3 applies only to the steady-state effects of debt and social security interventions. When borrowing and lending rates differ ($\rho \neq \delta$), lump-sum interventions typically imply nonneutralities along the transition path.

B. Short-Run Nonneutrality

We now demonstrate that a wedge between borrowing and lending rates implies the short-run nonneutrality of lump-sum fiscal policies in the altruistic linkage model. Our discussion focuses on the impact effects of a surprise increase in lump-sum payments to the old, financed by an increase in lump-sum taxes on the middle-aged. Thus, the experiment represents a surprise increase in the scale of an unfunded social security system.

To make the argument transparent, we adopt several simplifying assumptions: no population growth, inelastic labor supply, no labor supply by the old, no government expenditures, and the redistribution of all distortionary taxes to the affected generations via lump-sum transfers. We further assume that the economy is in a steady-state equilibrium at time t, prior to the intervention at time $t+1$.

Let T_{2t} denote the additional lump-sum tax levied on middle-aged persons at time $t+1$. Normalizing so that $\alpha_1 + \alpha_2 = 1$, write the goods market-clearing condition as

$$f(k_{t+1}) + k_{t+1} - C_{3,t-1} = C_{1,t+1} + C_{2t} + k_{t+2}.$$

Given $\rho > \delta$, Proposition 1 informs us that the old's marginal utility of consumption exceeds the γ-discounted marginal utility of their middle-aged children's consumption. Hence, the old at time $t+1$ will choose to increase $C_{3,t-1}$ by the full amount of a small, surprise increase in social security payments. This is the key observation.

Now use the budget constraint (8) and the government budget constraint to rewrite the goods market-clearing condition as

$$f(k_{t+1}) + k_{t+1} - (1 + r_{t+1})a_{2,t-1} - T_{2t} = C_{1,t+1} + C_{2t} + k_{t+2}. \tag{20}$$

Except for T_{2t}, every term on the l.h.s. of (20) is predetermined at $t+1$. It follows from the key observation in the preceding paragraph that the social security payment to the old translates dollar-for-dollar as reductions in the sum of consumption by the young, consumption by the middle-aged, and aggregate savings. The impact effect is nonneutral.

Consumption-smoothing incentives (both between persons and over time) imply that part of the decline takes the form of a reduction in aggregate savings. Thus, the capital stock falls and the interest rate rises. Since (9) holds with equality, consumption falls for both the young and middle-aged. If we allow for elastic labor supply, the impact effects also include increased aggregate output and a reduction in the wage. Since the middle-aged reduce savings by more if they anticipate higher future social security benefits, the impact effects on the capital stock are smaller for a transitory increase in old-age benefits than for an increase expected to persist for two or more periods. By the same token, the impact effects on labor supply, output, the wage, and consumption by the middle-aged and young are larger in response to a transitory increase in old-age benefits.

These remarks show that altruistic linkage models lead to short-run nonneutrality and long-run neutrality in response to (small) lump-sum interventions. The wedge between borrowing and lending rates is essential for this dichotomy between long-run and short-run responses. If borrowing rates equal lending rates, then adjacent generations are connected at the margin by intergenerational transfers at all stages of the life cycle. In this case, arguments based on the interconnectedness of budget constraints apply and full neutrality prevails.

6. Long-Run Interest Rate Neutrality in the Altruistic Linkage Model

We now turn our attention to the long-run effects of the tax policy parameters, ρ and δ, on interest rates and aggregate savings in the altruistic linkage model. We first build on our earlier analysis to obtain a surprising neutrality result. We then show that the proportional subsidy rate on interest payments has powerful effects on aggregate savings when borrowing rates exceed lending rates.

A. Interest-Rate Neutrality

Consider a version of the altruistic linkage model in which borrowing rates exceed lending rates in an active consumption-loans market (Regime D). Retracing the first part of the proof to Proposition 3 yields equation (19), reproduced here for convenience:

$$r = \frac{1+n-\beta\gamma}{\beta\gamma(1-\delta)}. \tag{19}$$

Equation (19) implies that the steady-state pre-tax interest rate (i.e., capital's marginal product) is unaffected by changes in the tax rate on income from investments in physical capital or consumption loans.[8]

[8]Of course, this neutrality result fails for a change in ρ that is large enough to push the

This interest rate neutrality result is even stronger than it appears. Since the derivation of (19) does not rest upon complete interconnectedness of budget constraints, it does not require pervasive altruistic preferences. Provided there exist at least *some* family lines characterized by (a) an operative altruistic transfer motive and (b) young members who are at an interior solution with respect to their borrowing (or saving) decision, then equation (19) (or (18)) holds at a steady-state equilibrium. Hence, this interest-rate neutrality result is consistent with the following observations: some family lines behave as pure life-cycle consumers; some intergenerational transfers are motivated by exchange considerations rather than altruism; and many persons are at a corner with respect to their borrowing and saving decisions.[9]

We make three other straightforward observations about this neutrality result. First, if $\rho < \delta$, then a similar line of argument establishes that equation (18) holds in the steady-state equilibrium, provided that at least some family lines have an operative altruistic transfer motive. Second, when conditions (a) and (b) hold for at least some family lines, neither government expenditures, government debt or labor income taxes affect capital's steady-state marginal product. Finally, equation (4) implies that interest-rate neutrality is equivalent to aggregate-savings neutrality when aggregate labor supply is inelastic.

We summarize these results for the case of $\rho > \delta$ in

Proposition 4: If borrowing rates exceed lending rates, and at least some family lines are characterized by

(a) positive intergenerational transfers motivated by a preference specification of the form (5)

(b) young persons who are at an interior solution with respect to their borrowing decision,

then (i) changes in government expenditures, (ii) fiscal policies that redistribute resources between generations or over time in a lump-sum manner, (iii) changes in taxes on labor income, and (iv) changes in the tax rate on interest income have no effect on capital's steady-state marginal product. If aggregate labor supply is inelastic, then these interventions also have no effect on steady-state aggregate savings.

We are aware of two previous analyses that use a line of proof similar to the one

economy out of Regime D.

[9]We demonstrate these claims formally in the appendix, which is available upon request from the authors.

underlying Proposition 4. Altig and Davis (1991) prove an interest-rate neutrality result in the context of a model with borrowing constraints and child-to-parent altruistic gift motives. They also discuss the role played by the separability assumptions embedded in the preference specification (5) in this line of proof. Summers (1982) derives an interest-rate neutrality result in an overlapping generations model with capital income taxation but no wedge between borrowing and lending rates. Summers stresses the infinite elasticity of savings with respect to the after-tax rate of return implied by the neutrality result in his setting. In sharp contrast, depending on the elasticity of labor supply, we obtain a zero long-run elasticity of savings with respect to the after-tax rate of return on savings. The difference between our results and those of Summers reflect the wedge between borrowing and lending rates in our framework as compared to the absence of a wedge in his framework.

B. The Long-Run Effect of the Subsidy on Interest Payments

In contrast to the neutrality of capital's marginal product with respect to the proportional tax rate on capital income, capital's marginal product is highly sensitive to changes in the proportional subsidy rate on interest payments. This result, too, follows directly from equation (19). Thus, we have,

Proposition 5: Under the hypotheses of Proposition 4, the steady-state marginal product of capital, given by equation (19), is an increasing function of the proportional subsidy rate applied to interest payments on consumption loans.

Consider a simple numerical example in which $n = .641$ and $\beta = .778$. Interpreting a period as twenty-five years, these values correspond to an annual population growth rate of 2% and an annual time discount factor of .99. Assume that parents weight each child's utility one-half as heavily as own utility. Now consider the impact of a reduction in δ from .25 to 0, which corresponds closely to the estimated effect of the 1986 tax reform in Table 1. From equation (19), this reduction in the subsidy rate on interest payments implies a reduction in the steady-state value of r from 4.29 to 3.22. In annualized terms, this change corresponds to a reduction in the pre-tax rate of return on capital from 6.89% to 5.92%. Thus, the recent tax policy change governing the proportional subsidy rate on interest payments implies a 14% decline in the steady-state marginal product of capital in this partial parametrization of the altruistic linkage model. This sizable reduction in the marginal product of capital implies that the elimination of interest payment deductibility causes a sizable increase in the steady-state capital stock, even if aggregate labor supply is inelastic in the long-run.

7. Tax Policy and Aggregate Savings: Experiments in Three Models

With respect to the effects of tax policy on aggregate savings, two basic points emerge from the analysis in section 6. First, in the altruistic linkage model with $\rho > \delta$ and an active loan market, aggregate savings is considerably more sensitive to changes in the subsidy rate on interest payments (δ) than to changes in the tax rate on interest income (ρ). Second, the aggregate savings response to changes in δ or ρ in the altruistic linkage model differ from the response in life-cycle and dynastic/representative agent models.

In this section, we more fully develop these points by quantifying the long-run aggregate savings response to tax policy changes in the three models. The three models we consider are the altruistic linkage (AL) model with operative transfers and differential borrowing and lending rates, the life-cycle (LC) model with no transfers but differential borrowing and lending rates, and the dynastic/representative agent (DRA) model. These three models correspond to linkage regimes D, A and F, respectively, of the general model specified in section 2. For each of these models, we calculate the percentage change in the steady-state capital stock associated with permanent changes in the tax policy parameters.

A. Parametrization

In conducting our simulations, we interpret a period as twenty-five years and use the following parametrization:

Technology:

$y_t = k_t^\theta, \ \theta = .25$

Productivity Profile:

$(\alpha_1, \alpha_2, \alpha_3) = (1.5, 6.0, 2.5)$

Population Growth:

$n' = .01, \ n = (1+n')^{25} - 1$

Time Preference:

$\beta' = .99, \ \beta = (\beta')^{25}$

Interpersonal Discount Factor:

$\gamma = 0$, .25, or .75, depending on the model

Period Utility (over consumption):

$$u(C_{it}) = \frac{C_{it}^{1-\frac{1}{\sigma_C}}}{1-\frac{1}{\sigma_C}}, \qquad \sigma_C = .33$$

Period Utility (over labor supply):

$$v(L_{it}) = \frac{-L_{it}^{1-\frac{1}{\sigma_N}}}{1-\frac{1}{\sigma_N}}, \qquad \sigma_N \in \{0, .15, .3, 1\}$$

In the interests of brevity, our discussion of these parameter choices is relegated to an appendix available from the authors.

All of our tax policy experiments maintain a balanced budget condition for the government by adjusting lump-sum taxes and subsidies. In the AL and LC models the generational incidence of lump-sum taxes matters. For simplicity, we assume that all distortionary taxes are returned to the impacted generation via lump-sum subsidies, and we treat distortionary subsidies analogously.

We report the results of two types of experiments:

Experiment 1: The subsidy rate, δ, is fixed and the marginal tax rate on interest income, ρ, is varied.

Experiment 2: ρ is fixed and δ is varied.

In our simulations, we measure the capital stock response relative to a benchmark tax structure with $\delta = 0$ and $\rho = .22$. These values closely reflect the fully phased-in provisions of the Tax Reform Act of 1986.[10],[11]

B. The Savings Response to Changes in the Tax Rate on Interest Income

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[10]Interest expense on pure consumption loans will no longer be deductible as of 1991. The effect of eliminating deductions of interest payments on non-mortgage consumer debt may be muted for many households by the availability of home-equity lines of credit. In fact, lending in the form of home equity lines of credit has expanded dramatically since 1986. The extent to which this type of debt instrument will eventually substitute for traditional, non-tax-favored forms of consumption loans is not yet clear. See Canner and Luckett (1989).

[11]The benchmark value of ρ is from Hausman and Poterba (1987) who estimate the marginal tax rate on interest income in 1988 to be 21.7%, based on the NBER's TAXSIM model.

Table 2 summarizes the results of our simulation experiments in the LC, AL, and DRA models. Each panel reports, for a particular model, the percentage change in the steady-state capital stock under experiments 1 and 2 relative to the benchmark specification of the tax policy parameters. Column headings indicate the value of ρ or δ in the new steady-state equilibrium.

The first panel of Table 2 shows that changes in the marginal tax rate on interest income have significant effects on the steady-state capital stock in the LC model. For example, assuming $\sigma_N = .3$, an increase in ρ from .22 to .33 causes the capital stock to decline by 6.7%. Elimination of interest income taxation causes the capital stock to rise by 12.6%. Similar results hold for other values σ_N.

Turning to the second panel, changes in ρ have even larger effects on the steady-state capital stock in the DRA model. Assuming $\sigma_N = .3$, an increase in ρ from .22 to .33 causes the capital stock to decline by 17%. Elimination of interest income taxation causes the capital stock to rise by 36%. Thus, simulations in both the LC and DRA models indicate that long-run aggregate savings show significant sensitivity to the tax rate on interest income. These results are similar to previous results in the literature; see Summers (1982).

The effect of changes in ρ differ sharply for the AL model. We know from Proposition 4 that changes in ρ have zero effect on the steady-state capital stock when labor supply is inelastic. The third panel of Table 2 reveals qualitatively similar responses when labor supply is elastic. The effects of changes in ρ in the AL model are roughly an order of magnitude smaller than in the LC and DRA models. The contrast between the two models with altruistic transfers – the AL and DRA models – is especially striking. Assuming $\sigma_N = .3$, elimination of interest income taxation causes the steady-state capital stock to rise by a mere 1% in the AL model, as compared to 36% in the DRA model.

C. The Savings Response to Changes in the Subsidy Rate on Interest Expense

In the LC model, changes in ρ and δ have roughly symmetric effects on the steady-state capital stock. For example, again focusing on $\sigma_N = .3$, an increase in δ from 0 to .11 causes the capital stock to fall by 9.7%. An increase in δ from 0 to .22 causes the capital stock to fall by 19.5%. Thus, aggregate savings shows significant sensitivity to the subsidy rate on interest expenses in the LC model.

In the AL mode., the aggregate savings effects of changes in δ are even larger. For

TABLE 2

The Effects of Tax Policy on the Capital Stock in Three Models

Tax Policy Parameters in the Initial Steady State: $\rho = .22$ and $\delta = 0$

		Percent Change in the Steady State Capital Stock as a Result of Changing One of the Tax Policy Parameters to:				
		$\rho = .33$	$\rho = .11$	$\rho = 0$	$\delta = .11$	$\delta = .22$
LC Model	Inelastic	-8.95	8.65	17.01	-9.53	-19.19
	$\sigma_N = .15$	-7.63	7.31	14.37	-9.55	-19.25
(Regime A,	$\sigma_N = .3$	-6.74	6.41	12.57	-9.65	-19.47
$\gamma = 0$)	$\sigma_N = 1$	-4.46	4.17	8.11	-10.26	-20.06
DRA Model	Inelastic	-18.35	18.73	39.18	0.00	0.00
	$\sigma_N = .15$	-17.48	18.18	36.88	0.00	0.00
(Regime F,	$\sigma_N = .3$	-17.02	17.57	35.64	0.00	0.00
$\gamma = .75$)	$\sigma_N = 1$	-16.24	16.62	33.58	0.00	0.00
AL Model	Inelastic	0.00	0.00	0.00	-14.39	-28.20
	$\sigma_N = .15$	-0.33	0.31	0.60	-13.50	-26.58
(Regime D,	$\sigma_N = .3$	-0.58	0.54	1.05	-12.98	-25.62
$\gamma = .25$)	$\sigma_N = 1$	-1.18	1.10	2.12	-11.96	-23.74

Note: In the AL model with inelastic labor supply, γ is set to .15.

example, when $\sigma_N = .3$, an increase in δ from 0 to .11 causes the capital stock to fall by 13%, and an increase in δ from 0 to .22 causes the capital stock to fall by 25.6%. Again, the contrast between the AL and DRA models is striking: since intergenerational transfers are large enough in the DRA model to obviate the kink in the intertemporal budget constraint, changes in δ have no effect on capital's marginal product in this model.

To summarize, the simulations point to powerful long-run savings effects of the borrowing subsidy in the two models with an active consumption-loans market – the AL and LC models. With respect to the 1986 Tax Reform Act's elimination of interest expense deductibility (on consumer loans), the simulations suggest that this reform will lead to an eventual 10-25% increase in the capital stock.

8. Concluding Remarks

This paper analyzes an overlapping generations framework that accommodates simple capital market imperfections like the asymmetric tax treatment of interest income and interest payments. Both the traditional life-cycle model and Barro's dynastic model represent feasible equilibrium linkage regimes within our framework, but these standard models correspond to only two of several feasible equilibrium linkage regimes. A central theme of our analysis is that the consequences of various fiscal policy interventions hinge critically on the prevailing linkage regime.

Our results do not conform neatly to any of the prominent positions in the vigorous debate over the aggregate savings effects of fiscal policy. On the one hand, we prove the invariance of capital's steady-state marginal product to government debt and social security policies, to the labor income tax schedule, and to the tax rate on capital income under plausible conditions. For reasonable parametrizations of the labor supply elasticity, the effects of these government interventions on the steady-state capital stock are also small. Notably, our long-run invariance theorem does not rest upon an extensive network of interconnected budget constraints, either within family lines or across family lines. Nor does it rest upon the assumed absence of binding borrowing constraints or otherwise perfect capital markets. Thus, our invariance theorem is immune to the most fequently invoked arguments against the Ricardian position.

On the other hand, the scope of our invariance theorem is narrower than the Ricardian Equivalence Theorem in many respects. The invariance of capital's steady-state marginal product (and the approximate invariance of steady-state aggregate savings) in our altruistic linkage model is consistent with important short-run effects of lump-sum government debt and social security policies and distortionary factor income taxation on capital's marginal product and aggregate savings. Our invariance theorem is also fully consistent with the view that these fiscal policies have important long-run and short-run consequences for the distribution of consumption across age cohorts and among heterogeneous individuals within age cohorts.

Furthermore, our analysis points to powerful long-run effects of certain types of tax policy on aggregate savings, regardless of whether intergenerational altruism plays an important role. For example, our simulations suggest that the elimination of interest expense deductibility by the Tax Reform Act of 1986 will lead to significant long-run increase in aggregate savings.

Most of our novel results follow from Proposition 1, which describes the optimal timing of altruistically motivated intergenerational transfers when borrowing rates exceed lending rates. While we doubt that our simple altruistic linkage model–and the optimal timing proposition, in particular–completely characterizes real-world savings and transfer behavior, we are willing to entertain the hypothesis that the model captures an element of truth for a significant fraction of the population. This hypothesis suggests two interesting and testable implications that we plan to pursue in future empirical work.

The first testable implication follows directly from the optimal timing proposition and involves the connection between the age distribution of resources and the age distribution of consumption. (See Boskin and Kotlikoff (1985), Abel and Kotlikoff (1988), and Altonji, Hayashi, and Kotlikoff (1989) for related empirical work.) According to Proposition 1, shocks that redistribute income between middle-aged and young persons imply no change in the age distribution of consumption, whereas shocks that redistribute income from middle-aged (or young) persons to old persons lead to increased consumption by the old. This strict testable implication follows when all family lines exhibit non-strategic altruistic behavior. More plausibly in our view, when some family lines operate as pure life cyclers and other family lines operate as altruists, the testable implication becomes: a one dollar redistribution of resources from the middle-aged to the old leads, on average, to a larger decline in consumption by the middle-aged than a one dollar redistribution of income from the middle-aged to the young. This implication can be tested with panel data on consumption, income (or wealth), and familial relationships.

A second testable implication follows from Propositions 4 and 5, which describe the long-run aggregate savings response to the tax treatment of interest income and interest expense in the altruistic linkage model. If our analysis captures an important element of real-world behavior, then cross-country differences in the tax treatment of consumer loan interest expenses will help to explain differences in aggregate savings rates. At a minimum, the subsidy rate on interest payments will have more explanatory power than the tax rate on interest income.

To close, we briefly remark upon two natural directions for further theoretical research. First, we have abstracted from individual uncertainty about lifetime earnings and longevity. Coupled with less than perfect insurance and annuity markets, these factors imply incentives for altruistic parents to defer transfers to children, as they await the resolution of uncertainty. Hence, uncertainity about earnings and longevity mitigates against the optimal timing result in Proposition 1. Modifying our framework to incorporate longevity or earnings uncertainty is likely to yield a richer and more nuanced set of implications about the timing of altruistically motivated intergenerational transfers. Whether the tax policy parameters δ and ρ continue to have sharply asymmetric effects in an altruistic linkage model with individual uncertainty is an open question.

Second, parents and children could potentially arbitrage the difference between their after-tax lending and borrowing rates in our framework. For example, parents might make "gifts" to their young children with the understanding that the children will reciprocate when they become middle-aged. This reciprocal gift-giving behavior would amount to a family-based consumption loan that circumvents the tax wedge and, possibly, other transaction costs associated with market-based loans. To be viable, family-based consumption loans require some reliable extra-market enforcement mechanism to insure that children will reciprocate in the appropriate amount when they become middle-aged. Intergenerational altruism, even two-sided altruism, will not by itself provide the enforcement mechanism. Thus, our analysis can be rationalized by the assumption that the requisite extra-market enforcement mechanisms are unavailable. However, these observations about the requirements for viable family-based loans do not deny their existence in the real world. The tax policy implications of family-based loans, and the extra-market enforcement mechanisms that support these loans, are issues that we leave for future research.

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Appendix A: Calculating Equilibrium Outcomes

This appendix explains how to numerically calculate the steady-state equilibrium in Regimes A-F, given isoelastic preferences and Cobb-Douglas production. To simplify the exposition, we assume zero population growth, no government purchases, and the redistribution of all distortionary taxes (and subsidies) to the impacted generations via lump-sum transfers. (The analysis is easily modified when one or more of these simplifying assumptions is altered.) Under these assumptions, we show that the calculation of equilibrium can be reduced to solving a system of one, two or three nonlinear simultaneous equations, depending on the linkage regime. The resulting equation system can be solved numerically using Newton-Raphson or other appropriate methods.

From (14), the static first-order conditions governing the consumption-labor supply tradeoff satisfy

$$L_i = \left[\frac{C_i^{1/\sigma_c}}{\alpha_i W}\right]^{-\sigma_n}, \qquad i = 1,2,3, \tag{A.1}$$

where σ_c and σ_n denote the intertemporal substitution elasticities in consumption and labor supply, respectively. From (3) and (4), factor prices satisfy

$$r = \theta k^{\theta-1}, \quad \text{and} \tag{A.2}$$

$$W = (1-\theta)k^{\theta}. \tag{A.3}$$

Aggregate labor supply is defined as

$$L = \alpha_1 L_1 + \alpha_2 L_2 + \alpha_3 L_3, \tag{A.4}$$

and the goods market-clearing condition is given by

$$C_1 + C_2 + C_3 = Lk^{\theta}. \tag{A.5}$$

Most of the remaining conditions involved in computing the equilibrium vary across linkage regimes, so we treat each regime separately.

Regime A: No Transfers, Young Borrow

Let

$$A_1 = \left\{\beta[1 + r(1-\delta)]\right\}^{\sigma_c}, \quad \text{and}$$

$$A_2 = \{\beta[1 + r(1-\rho)]\}^{\sigma_c}.$$

In Regime A, the intertemporal first-order conditions governing the evolution of consumption are

$$C_1 = C_2/A_1, \quad \text{and} \tag{A.6}$$

$$C_3 = A_2 C_2. \tag{A.7}$$

Substituting these conditions into the goods market-clearing condition yields

$$\left[\frac{1}{A_1} + 1 + A_2\right] C_2 = Lk^{\theta}, \tag{A.8}$$

where A_1, A_2, and L can be written as explicit functions of C_2 and k using the preceding expressions.

The capital market-clearing condition in Regime A is

$$Lk = a_2 - x_1. \tag{A.9}$$

Using the budget constraints and intertemporal first-order conditions, we obtain

$$a_2 = \frac{A_2 C_2 - \alpha_3 L_3 W}{1+r}, \quad \text{and} \tag{A.10}$$

$$x_1 = \frac{(C_2/A_1) - \alpha_1 L_1 W}{1+r}. \tag{A.11}$$

Now, using these expressions to eliminate a_2 and x_1 from $(A.9)$, we obtain

$$(1+r)Lk = \left(A_2 - \frac{1}{A_1}\right) C_2 - \alpha L_3 W + \alpha_1 L_1 W, \tag{A.12}$$

where r, A_1, A_2, L, L_1 and L_3 can be written as explicit functions of C_2 and k. Thus, $(A.8)$ and $(A.12)$ is a system of two nonlinear equations in the unknowns C_2 and k.

To verify that a solution to $(A.8)$ and $(A.12)$ constitutes a Regime-A equilibrium, we must check the following conditions:

$$x_1 > 0, \qquad \text{positive borrowing by young,}$$

$$C_2^{-1/\sigma_c} > \gamma C_1^{-1/\sigma_c}, \qquad \text{inoperative transfer motive.}$$

Regime B: No Transfers, Young at Corner

In this regime, labor supply and consumption by the young are solutions to a static utility maximization problem:

$$C_1 = \alpha_1 L_1 W, \quad \text{and} \tag{A.13}$$

$$L_1^{-1/\sigma_n} = -\alpha_1 W\left[\alpha_1 L_1 W\right]^{-1/\sigma_c}, \tag{A.14}$$

where W is a function of k.

Using $(A.7)$, we can write the goods market-clearing condition as

$$C_1 + C_2(1 + A_2) = Lk^\theta, \tag{A.15}$$

where C_1 is a function of L_1 and k by $(A.13)$.

Next, using the budget constraint for the middle-aged, we can write the capital market-clearing condition as

$$Lk = \alpha_2 L_2 W - C_2, \tag{A.16}$$

which can be written as a function of C_2, k and L_1. Thus, $(A.14)$ - $(A.16)$ constitute a system of three nonlinear equations in the unknowns C_2, k and L_1.

To verify that a solution to $(A.14)$ - $(A.16)$ constitutes a Regime-B equilibrium, we must check the following conditions:

$$C_1^{-1/\sigma_c} < A_1 C_2^{-1/\sigma_c}, \qquad \text{no desire to borrow,}$$

$$C_1^{-1/\sigma_c} > A_2 C_2^{-1/\sigma_c}, \qquad \text{no desire to save,}$$

$$C_2^{-1/\sigma_c} > \gamma C_1^{-1/\sigma_c}, \qquad \text{inoperative transfer motive.}$$

Regime C: No Transfers, Young Save

Consumption behavior satisfies the intertemporal first-order conditions $(A.7)$ and

$$C_2 = A_2 C_1. \tag{A.17}$$

Substituting $(A.7)$ and $(A.17)$ into the goods market-clearing condition yields

$$\left[\frac{1}{A_2} + 1 + A_2\right] C_2 = Lk^\theta, \tag{A.18}$$

where A_2 and L can be written as explicit functions of C_2 and k. Using the budget constraints and intertemporal first-order conditions to eliminate a_1 and a_2 from the capital market-clearing condition, we obtain

$$(1+r)Lk = \left(A_2 - \frac{1}{A_2}\right)C_2 - \alpha L_3 W + \alpha_1 L_1 W, \tag{A.19}$$

where r, L, A_2, L_3, L_1 and W can be written as explicit functions of C_2 and k. Thus, $(A.18)$ and $(A.19)$ constitute two nonlinear equations in the unknowns C_2 and k.

To verify that a solution to $(A.18)$ and $(A.19)$ constitutes a Regime-C equilibrium, we must check the following conditions:

$$a_1 > 0, \qquad \text{positive saving by young,}$$

$$C_2^{-1/\sigma_c} > \gamma C_1^{-1/\sigma_c}, \qquad \text{inoperative transfer motive.}$$

Regime D: Positive Transfers, Young Borrow

Equation (18) in the text informs us that

$$r = \frac{1-\beta\gamma}{\beta\gamma(1-\delta)}. \tag{18}$$

From $(A.2)$ we have

$$k = (r/\theta)^{1/(\theta-1)}. \tag{A.20}$$

Consumption satisfies the intertemporal first-order conditions $(A.6)$ and $(A.7)$ and the transfer-motive first-order condition

$$C_1 = BC_2, \tag{A.21}$$

where $B = \gamma^{\sigma_c}$.

Substituting $(A.7)$ and $(A.21)$ into the goods market-clearing condition, we obtain

$$\left[B + 1 + A_2\right]C_2 = Lk^{\theta}, \tag{A.22}$$

where A_2, L and k can be written as explicit functions of C_2 using (18), $(A.1)$ - $(A.4)$, $(A.7)$, $(A.20)$ and $(A.21)$. Thus, $(A.22)$ can be written as a single nonlinear equation in the unknown C_2.

To verify that a solution to $(A.22)$ constitutes a Regime-D equilibrium, we must check the following conditions:

$$x_1 > 0, \qquad \text{positive borrowing by young,}$$

$$b_1 > 0, \qquad \text{positive transfers.}$$

Regime E: Positive Transfers, Young at a Corner

Consumption behavior satisfies $(A.7)$ and $(A.21)$. Substituting these conditions into the goods market-clearing condition, we obtain $(A.22)$, where all variables can be written as explicit functions of C_2 and k. Next, use the budget constraint of the old and $(A.7)$ to write the capital market-clearing condition as

$$A_2C_2 = (1+r)Lk + \alpha_3 L_3 W, \tag{A.23}$$

where A_2, r, L, L_3 and W can be written as explicit functions of C_2 and k. Thus, $(A.22)$ and $(A.23)$ constitute two nonlinear equations in the two unknowns C_2 and k.

To verify that a solution to $(A.22)$ and $(A.23)$ constitutes a Regime-E equilibrium, we must check the following conditions:

$$C_1^{-1/\sigma_c} < A_1 C_2^{-1/\sigma_c}, \qquad \text{no desire to borrow,}$$

$$C_1^{-1/\sigma_c} > A_2 C_2^{-1/\sigma_c}, \qquad \text{no desire to save,}$$

$$b_1 > 0, \qquad \text{positive transfers.}$$

Regime F: Positive Transfers, Young Save

Computation of the equilibrium proceeds as in Regime D, except that the interest rate is now given by

$$r = \frac{1-\beta\gamma}{\beta\gamma(1-\rho)}. \tag{19}$$

To verify that a solution to $(A.22)$ constitutes a Regime-F equilibrium, we must check the following conditions:

$$a_1 > 0, \qquad \text{positive saving by young,}$$

$$b_1 > 0, \qquad \text{positive transfers.}$$

Appendix B: Determining the Equilibrium Linkage Regime

This appendix describes our techniques for determining the prevailing linkage regime in various regions of the parameter space. These techniques underlie the construction of Figures 1-3 in the text and the additional figures presented below.

Drawing on the results of Appendix A, a brute-force approach to determining the equilibrium linkage regime is available as follows. For each point in a grid on the parameter space, first solve the model in all six linkage regimes. Second, check the verification conditions in Appendix A to determine which linkage regime is the equilibrium outcome at each point in the grid. Third, isolate the boundaries of each linkage regime and plot these boundaries in various two-dimensional slices of the parameter space. While this aproach is conceptually staightforward, it is cumbersome in practice. For the case of inelastic labor supply, we have developed a simpler method that allows us to directly calculate the regime boundaries in the parameter space. We describe this method presently.

To lie in Regime F, a point in the parameter space must satisfy $b_1 > 0$ and $a_1 > 0$. Using the budget constraints, the intertemporal first-order conditions $(A.6)$ and $(A.7)$, the goods market-clearing condition and the capital market-clearing condtion, $b_1 > 0$ implies

$$\left(\frac{1+r-A_2^2}{1+A_2+A_2^2}\right)Lk^{\theta} + L(1+r)k + \left[\alpha_3 - \alpha_1(1+r)\right]W > 0, \qquad (B.1)$$

where r, k and W satisfy (18), $(A.2)$ and $(A.3)$. Now, fix all parameter values except, say, γ, and solve for the value of γ such that $(B.1)$ holds with equality. The solution yields the infimum value of γ such that $b_1 > 0$ holds. Looping over the value of some other parameter, say σ_c, and repeatedly solving $(B.1)$ yields the $b_1 > 0$ boundary in σ_c - γ space.

Similarly, $a_1 > 0$ leads to the boundary condition

$$Lk(1+r) + \alpha_3 W - \frac{Lk^{\theta}A_2^2}{1+A_2+A_2^2} > 0. \qquad (B.2)$$

Proceeding as before, $(B.2)$ can be used to trace out the $a_1 > 0$ boundary locus in some two-dimensional slice of the parameter space, say σ_c - γ space.

Recall that a parameter vector must satisfy both $(B.1)$ and $(B.2)$ to lie in Regime F. Thus, the only relevant segment of the $b_1 > 0$ boundary locus occurs where $a_1 > 0$ also holds, and vice versa. Similar remarks apply to all of the boundary loci described below. In Figures 1-3 and below, we plot only the relevant segments of the various boundary loci.

The calculation of the other boundary loci proceeds in a similar manner. One general remark is in order, however. Each boundary locus can be computed from two different directions, but the computations are typically much easier from one direction than the other. We exploit this fact when choosing how to compute each boundary locus.

To lie in Regime D, a point in the parameter space must satisfy $b_1 > 0$ and $x_1 > 0$. These restrictions imply the boundary conditions

$$\left(\frac{1+r-A_1A_2}{1+A_2+A_1A_2}\right)Lk^{\theta} + L(1+r)k + \left[\alpha_3 - \alpha_1(1+r)\right]W > 0, \qquad (B.3)$$

and

$$Lk(1+r) + \alpha_3 W - \frac{Lk^{\theta}A_1A_2}{1+A_2+A_1A_2} > 0. \qquad (B.4)$$

where r, k and W satisfy (18), $(A.2)$ and $(A.3)$.

To find the $x_1 > 0$ boundary for Regime A, we first compute the equilibrium. In this regime, the equilibrium value of k solves

$$(1+r)Lk = \left[\frac{A_1A_2-1}{1+A_1+A_1A_2}\right]Lk^{\theta} + (\alpha_1 - \alpha_3)W, \qquad (B.5)$$

where r and W are explicit functions of k from $(A.2)$ and $(A.3)$. After solving the non-linear equation $(B.5)$, we compute the implied level of borrowings by the young for the given parameter configuration. Iterating over this procedure, we search for the value of a particular parameter, say σ_c, such that $x_1 = 0$ holds. This value of σ_c determines the $x_1 = 0$ locus in σ_c - γ space, since the Regime-A equilibrium is independent of γ.

The $a_1 > 0$ boundary for Regime C is found in the same way as the $x_1 > 0$ boundary for Regime A. In Regime C, the equilibrium value of k solves

$$(1+r)Lk = \left[\frac{A_2^2-1}{1+A_2+A_2^2}\right]Lk^{\theta} + (\alpha_1 - \alpha_3)W, \qquad (B.6)$$

Finally, to find the transfer-motive boundary separating Regimes B and E, we first compute the Regime-B equilibrium. With inelastic labor supply, the Regime-B value of k solves

$$\alpha_1 W A_2 + (1+A_2)\left[(1+r)Lk + \alpha_3 W\right] = Lk^{\theta}A_2, \qquad (B.7)$$

where r and W satisfy $(A.2)$ and $(A.3)$. Given the values of C_1 and C_2 implied by a solution to $(B.7)$, a Regime-B equilibrium satisfies $C_1 > BC_2$, where $B = \gamma^{\sigma_c}$. This inequality implies the boundary value for γ that separates Regimes B and E:

$$\gamma = \left(\frac{C_1}{C_2}\right)^{1/\sigma_c}.$$

Summarizing, conditions $(B.1)$ - $(B.7)$ generate seven boundary loci that can be used to partition a two-dimensional slice of the parameter space into six or fewer regions. Each region corresponds to an equilibrium linkage regime. Figures 1-3 in the text and Figures B.1 and B.2 below were constructed directly from the boundary conditions. Figure B.3, which involves elastic labor supply, was constructed using the brute-force method outlined in the second paragraph of this appendix.

Figure B.1
Boundary Loci: Inelastic labor supply, rho=.22, delta=0, sigma_C=.33, theta=.25

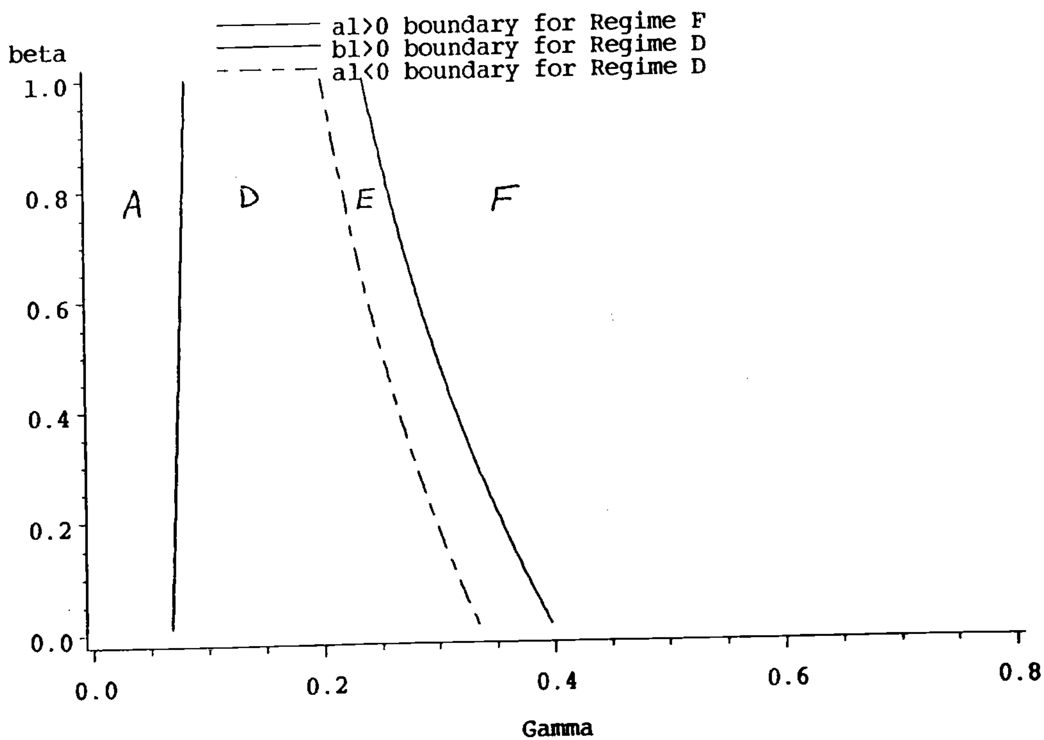


Figure B.2

Boundary Loci: Inelastic labor supply, rho=.22, delta=0, sigma_c=.33, beta=.778,

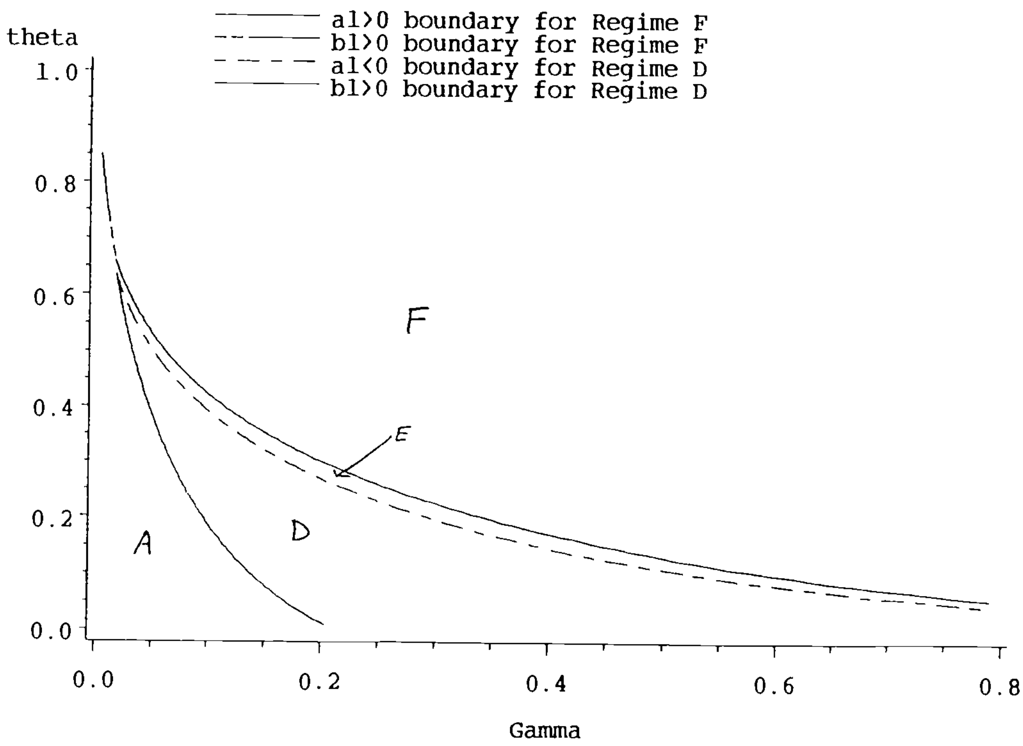
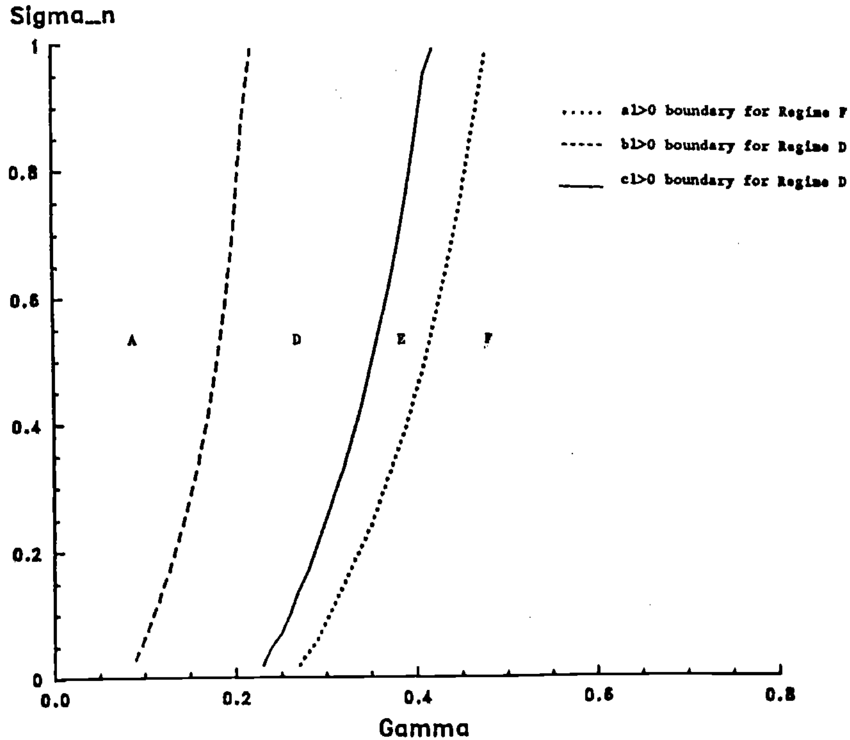


Figure B.3

Boundary Loci: Elastic labor supply, rho=.22, delta=0, sigma_c=.33



Appendix C: Equilibrium with Two Agent Types

In section 6.A of the text, we claim that the interest rate neutrality result contained in Proposition 4 holds provided that *at least some family lines* are characterized by

(a) positive intergenerational transfers motivated by a preference specification of the form (5), and

(b) young persons who are at an interior solution with respect to their borrowing decision.

We now demonstrate this claim formally.

If $\rho > \delta$ and (a) and (b) above hold, then the derivation of (19) in a heterogeneous agent economy proceeds exactly as in the proof to Proposition 3. Thus, to establish our claim, we need only show that there exist equilibria in which (a) and (b) are satisfied by only some family lines. To show existence, we construct a simple example involving a mixture of altruistic family lines and pure life-cyclers.

Let ϵ denote the fraction of family lines that are altruistic. As before, assume Cobb-Douglas production and isoelastic preferences. To simplify the algebra and notation, assume no population growth, inelastic labor supply, and the redistribution of all distortionary taxes and subsidies to the directly impacted agents via lump-sum instruments. To enrich the range of examples we can consider, we allow for differences in the lifetime productivity profile between the two agent types.

When conditions (a) and (b) hold, budget constraints for the altruists are given by

$$C_1^a = \alpha_1^a W + b_1 + x^a,$$

$$C_2^a + b_1 + a_2^a + (1+r)x^a = \alpha_2^a W, \quad \text{and}$$

$$C_3^a = a_2^a(1+r) + \alpha_3^a W.$$

The budget constraints for the life-cyclers depend on whether their young save, borrow, or choose a corner. For example, if they borrow, the life-cyclers' budget constraints become

$$C_1^n = \alpha_1^n W + x^n,$$

$$C_2^n + a_2^n + (1+r)x^n = \alpha_2^n W, \quad \text{and}$$

$$C_3^n = a_2^n(1+r) + \alpha_3^n W.$$

With two types of agents, aggregate labor supply becomes

$$L = \epsilon(\alpha_1^a + \alpha_2^a + \alpha_3^a) + (1-\epsilon)(\alpha_1^n + \alpha_2^n + \alpha_3^n). \qquad (C.1)$$

The goods and capital market-clearing conditions are also modified in the obvious manner.

We can now calculate and verify an equilibrium as follows: First, conjecture an equilibrium satisfying (a) and (b) above. Second, use (19), $(A.2)$ and $(A.3)$ to calculate the wage and the interest rate in the conjectured equilibrium. Third, given W and r, solve the (partial equilibrium) problem confronting the life-cyclers. In the case where the young life-cylcers borrow, we obtain

$$C_2^n = \frac{\left[(1+r)\alpha_1^n + \alpha_2^n + \alpha_3^n(1+r)^{-1}\right]W}{\left[(1+r)/A_1 + 1 + A_2(1+r)^{-1}\right]}. \tag{C.2}$$

C_1^n and C_3^n follow from the intertemporal first-order conditions $(A.6)$ and $(A.7)$, and a_2^n and x_n can be recovered from the budget constraints.

Fourth, use the intertemporal first-order conditions $(A.6)$ and $(A.7)$ to write the goods market-clearing condition as

$$C_2^a = \frac{Lk^\theta}{\epsilon[1+(1/A_1)+A_2]} - \left(\frac{1-\epsilon}{\epsilon}\right)C_2^n. \tag{C.3}$$

Substituting $(C.1)$ and $(C.2)$ into $(C.3)$ and rearranging yields a solution for C_2^a. The intertemporal first-order conditions and budget constraints then deliver solutions for C_1^a, C_3^a, x^a, b, and a_2^a.

Finally, to verify the conjectured equilibrium, we must check that $b > 0$, $x^a > 0$, and $x^n > 0$.

For example, consider the benchmark parameter configuration used in the text: $\sigma_c = .33$, $\beta = .99^{25}$, $\theta = .25$, $\rho = .22$, $\delta = 0$, and $(\alpha_1, \alpha_2, \alpha_3) = (1.5, 6.0, 2.5)$ (for both types). Suppose that one-half the agents are pure life-cyclers, while the other half are weak altruists with $\gamma = .10$. Then the following is a two-type equilibrium that satisfies the hypotheses of Proposition 4:

$$r = 11.856, \qquad k = .005825, \qquad w = .2072,$$

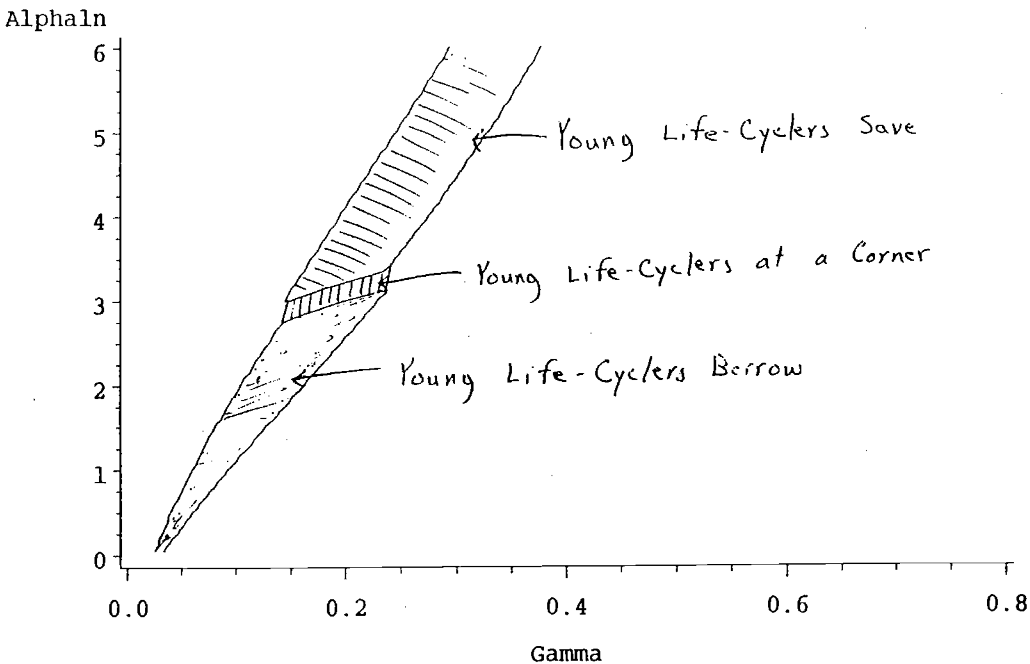
$$C_1^a = .4043, \quad C_2^a = .8643, \quad C_3^a = 1.7146, \quad x^a = .0162, \quad a_2^a = .0931, \quad b = .0772$$

$$C_1^n = .3445, \quad C_2^n = .7365, \quad C_3^n = 1.4612, \quad x^n = .0337, \quad a_2^n = .0734.$$

This example completes the proof of our claim that Proposition 4 extends to economies in which only some family lines satisfy conditions (a) and (b).

Conditions (a) and (b) hold over a wide range of values for the altruism parameter in two-type economies. This point is illustrated in Figure C.1, which modifies the previous example by varying α_1^n and γ. (Except for α_1^n and γ, the parameter settings underlying the construction of Figure C.1 are identical to the preceding example.) Both conditions (a) and (b) hold in the shaded regions of the figure. Thus, we see that conditions (a) and (b) are consistent with the observation that some family lines behave as pure life-cyclers and the observation that many persons are at a corner with respect to borrowing and saving decisions. With a suitable modification of preferences and constraints, we could easily introduce an additional motive for intergenerational transfers along the lines of Cox (1987), while preserving Proposition 4.

Figure C.1
Parameter Values that Lead to Equilibria Satisfying the Hypotheses of Proposition 4 When the Half the Agents are Altruists
Different lifetime productivity profiles for Altruists and Life-Cyclers



Appendix D. Extensions to Distortionary Labor Income Taxation and Nominal Capital Income Taxation

This appendix extends the results in the text regarding the long-run neutral impact of various fiscal policy interventions on capital's marginal product. We briefly consider the implications of distortionary labor income taxes and the distortionary effects of inflation when the capital income tax base involves nominal variables.

A. Distortionary Labor Income Taxes

Provided that there exist at least some family lines characterized by an operative altruistic transfer motive and young persons who choose an interior solution with respect to borrowing or saving, arbitrary labor income tax schedules have no effect on the steady-state marginal product of capital. Under these circumstances, equation (19) describes the marginal product of capital when the after-tax borrowing rate exceeds the after-tax lending rate. (Alternatively, if the lending rate exceeds the borrowing rate or the young altruists are savers, then equation (18) describes the marginal product of capital.) As before, this result follows directly by combining the intertemporal consumption first-order condition for the young with the transfer-motive first-order condition for the middle-aged.[1] Hence, the results stated in Propositions 3-5 carry over without alteration to economies with distortionary labor income taxation. In addition to the long-run neutrality results in these propositions, we add

Proposition 6: Under the hypotheses of Proposition 4, the steady-state marginal product of capital is invariant to arbitrary changes in the labor income tax schedule.

B. Inflation and Nominal Taxation

We model inflation by introducing an exogenously determined unit of account. This device enables us to capture the distortion arising from the interaction between inflation and the tax structure without explicitly modelling the inflationary mechanism. We continue to assume a proportional tax rate on interest income and a proportional subsidy rate on interest payments. In contrast to our previous analysis, however, we assume that tax calculations are based on nominal interest rates.

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[1]The steady-state invariance of capital's marginal product with respect to the labor income tax schedule does not require separability between consumption and leisure in the utility function. This observation is easily verified by relaxing the intra-period separability assumption embodied in (5) and retracing the derivation of (18) and (19).

Denote the rate of inflation–i.e., the growth rate of the unit of account–from time t to $t+1$, as π_{t+1}. Approximating the nominal interest rate as the sum of the real rate of return to capital and the rate of inflation, the first-order conditions (9) and (10) become

$$u'(C_{1t}) \geq \beta(1 + r_{t+1}(1-\delta) - \delta\pi_{t+1})u'(C_{2t}) \tag{D.1}$$

$$u'(C_{2t}) = \beta(1 + r_{t+2}(1-\rho) - \rho\pi_{t+1})u'(C_{3t}) \tag{D.2}$$

Using equations (D.1) and (D.2) to argue along familiar lines, we have

Proposition 7: Assume that interest income taxes and interest payment subsidies are calculated on nominal rates. Then

(i) If after-tax borrowing rates exceed after-tax lending rates and conditions (a) and (b) of Proposition 4 hold for at least some family lines, the steady-state marginal product of capital is given by

$$r = \frac{1 + n + \gamma\beta\delta\pi - \gamma\beta}{\gamma\beta(1-\delta)}. \tag{D.3}$$

(ii) If after-tax lending rates exceed after-tax borrowing rates, and condition (a) of Proposition 4 holds for at least some family lines, then the steady-state marginal product of capital is given by

$$r = \frac{1 + n + \gamma\beta\rho\pi - \gamma\beta}{\gamma\beta(1-\rho)}. \tag{D.4}$$

Three interesting results follow directly from Proposition 7. First, for a fixed inflation rate, the neutrality results in Propositions 3-6 extend to economies with nominal interest income taxation.[2] Second, the long-run sensitivity of capital's marginal product to the tax

[2]In an explicit monetary model, the government's budget constraint implies a relationship between the growth rate of the money supply and fiscal policy instruments. A higher level of government debt, for example, would be associated with a higher inflation rate, if the interest payments on government debt were financed by money creation. In this scenario, and under the assumptions of Proposition 7, changes in the steady-state level of government debt would be associated with changes in the marginal product of capital. Alternatively, if interest payments on the higher level of government debt were financed by an increase in labor income taxes, the steady-state marginal product of capital would be unaffected.

parameters, ρ or δ, is an increasing function of the inflation rate. To see this point when, for example, borrowing rates exceed lending rates, differentiate equation $(D.3)$ to obtain

$$\frac{\mathrm{d}r}{\mathrm{d}\delta} = \frac{\pi + r}{1 - \delta}.$$

Third, when borrowing rates exceed lending rates, the effect of inflation on capital's steady-state marginal product hinges crucially on the interest payment subsidy rate, δ, and is independent of the interest income tax rate, ρ. From $(D.3)$

$$\frac{\mathrm{d}r}{\mathrm{d}\pi} = \frac{\delta\pi}{1 - \delta}.$$

Thus, the inflation effect on capital's marginal product is an increasing function of the proportional subsidy rate on interest payments. Furthermore, eliminating the subsidy to interest payments eliminates the effect of inflation on capital's marginal product.

The implications of these observations for aggregate savings can be summarized as follows. When borrowing rates exceed lending rates in the altruistic linkage model, the magnitude of any inflation-induced decline in aggregate savings is much more sensitive to the subsidy rate on nominal interest payments than to the tax rate on nominal interest income. If aggregate labor supply is inelastic, then the long-run response of aggregate savings to inflation is independent of the tax rate on nominal interest income.

Appendix E: Choosing Parameter Values for the Numerical Exercises

This appendix explains our choice of parameter values for the numerical simulation exercises reported in the paper, particularly Table 2.

A priori, the magnitude of the aggregate savings response to changes in the tax policy parameters seems likely to be sensitive to the intertemporal substitution elasticities, σ_C and σ_N, as the following remarks suggest. First, it is well known that the intertemporal elasticity of substitution in consumption strongly influences the savings response to changes in the after-tax interest rate in the life-cycle (LC) and dynastic/representative agent (DRA) models. Second, in altruistic linkage (AL) models, Altig and Davis (1989) show that small changes in the willingness to substitute consumption intertemporally have powerful effects on the magnitude of intergenerational transfers and the scale of activity in the consumption-loans market. Finally, the analysis in section 6 of the text shows that, at least for the AL model, the aggregate savings response to changes in the marginal tax rate on interest income depends critically on the elasticity of labor supply.

MaCurdy's (1981) study of men's labor supply behavior suggests values of σ_N in the range $(.1, .45)$, a finding which is largely confirmed in related studies (see Pencavel (1986)). Despite much greater disparity in the estimates of the labor supply elasticity of women, there is broad agreement among labor economists that the elasticity is higher for women than men (see Killingsworth and Heckman (1986)). Thus, evidence on the labor supply behavior of women points to a larger value for the aggregate labor supply elasticity. In addition, Hansen (1985) shows that indivisibilities in labor supply behavior can lead to an aggregate intertemporal substitution elasticity much larger than the elasticity of individuals. These considerations lead us to consider unit elastiticity as an upper value for σ_N.

Turning to the intertemporal elasticity of substitution in consumption, Hall's (1988) empirical study suggests a value of σ_C near .1. Hall's estimates of σ_C (as well as most other estimates in the literature) are based on short-run consumption growth responses to anticipated movements in real returns on financial assets. However, given the three-period-lived agents in our analytical framework and our focus on the long-run response to tax policy changes, it is more appropriate to parametrize the model in terms of the willingness to substitute consumption over broad epochs of life. We are unaware of formal econometric attempts to estimate this notion of an intertemporal substitution elasticity, although descriptive work suggests that the elasticity is large. For example, Carroll and Summers (1989) show that the shape of the lifetime consumption profile differs greatly across education and occupation groups, and that the shape of group average consumption profiles closely mirrors the shape of group average income profiles. Aside from pointing to important departures from perfect capital markets, these patterns suggest that consumers exhibit considerable willingness to substitute consumption intertemporally over broad epochs of life.

These observations prompted us to simulate the long-run response to tax policy interventions under several sets of values for the intertemporal substitution elasticities. We considered values of σ_N in the set $\{0, .1, .3, 1\}$ and values of σ_C in the set $\{.33, .5, 1\}$. Table 2 in the text reports results for $\sigma_C = .33$. An earlier version of the paper finds similar results for $\sigma_C = .5$ or 1.

Our values for capital's share of output (25%), the population growth rate (1% per year), and the annual time discount factor (.99) are unlikely to be controversial. In any case, some experimentation indicates that reasonable variation in these parameter values does not greatly affect our results.

The primary effect of changes in γ, the interpersonal discount factor, involves its impact on the prevailing linkage regime. Within a particular linkage regime, changes in γ do not greatly affect the results reported in Table 2. Thus, the effect of γ on the response to changes in the tax policy parameters can be seen by comparing across panels in Table 2.

In choosing a lifetime productivity profile, we sought to provide a reasonably strong motive for the young to borrow. This motive can stem from either an upwardly-sloped lifetime income profile during the early years of working life or, in a slightly more general model, age-specific utility function shifters that increase the demand for consumption expenditures early in the life cycle.

Finally, we chose benchmark values for the tax policy parameters, ρ and δ, to reflect the fully phased-in provisions of the Tax Reform Act of 1986.

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