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The Timing of Sales

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This paper presents a model of intertemporal price discrimination. A fixed number of sellers produce a homogeneous good. Consumers with different preferences enter the market in each period and leave when they make a purchase. The sellers typically vary their prices over time, charging a high price in most periods, but occasionally cutting the price to sell to a large group of customers with a low reservation price. In some equilibria, all stores lower their price at the same time and to the same level.

1. INTRODUCTION

Retail stores change their price frequently. Periodic price reductions, or sales, occur with sufficient regularity to suggest that they are not entirely due to random variations in supply, demand, or the aggregate price level. Certain types of sale (for example, the January White Sale) are traditional and so well publicized that it is difficult to justify them as devices to separate informed from uninformed consumers. This paper presents a model in which sellers want to reduce prices periodically in order to exploit the differing preferences of their customers. It studies why sales occur, and how they are timed.

The model used extends that of Conlisk, Gerstner, and Sobel (1984). A new cohort of consumers enters the market each period, interested in making a once-and-for-all purchase of a non-resaleable durable, either immediately or after a delay. Production takes place at a constant cost per unit; consumers are well informed; each consumer cohort is the same as the last; but consumers vary within cohort in their tastes for the product. When there is a single seller, Conlisk, Gerstner, and Sobel show that the monopolist typically varies his price over time, periodically cutting the price sharply to sell to a large group of consumers with a low reservation price. This paper shows that if there are many sellers the motivation to hold sales remains. The question of how sales are timed is not given a definitive answer. If sellers use strategies that do not allow today's prices to depend directly on past prices, then symmetric equilibrium behaviour has the following form. Typically, only one store cuts its price at a time, although it is possible for several stores to cut their price—to different levels—on the same day. After this sale, prices remain high for an interval, then another sale occurs. The interval between sales varies, but has a minimum length. The sale price varies, but the expected price decreases as the time from the last sale increases. Each store is equally likely to have a sale. The reason for variation in sale prices and intervals, in spite of the nonstochastic environment, is that stores must use mixed strategies to support the symmetric equilibrium.

On the other hand, many other equilibria exist if stores are allowed to base today's price on yesterday's price. These strategies allow a seller to tell his competitors "if you cut your price today, then I will cut my price by more tomorrow". In general, threats of this form are credible, and can be used to support equilibria that yield profit greater than that obtained from the strategies described earlier. In Section 4 I argue that the most likely equilibrium of this form involves stores cutting their prices simultaneously and to the same level. Thus, perfectly coordinated sales can arise in a noncooperative equilibrium.

There are static models in which sellers pursue mixed strategies. The mixed strategies that arise in Rosenthal (1980), Shilony (1977), and Varian (1980) could represent price variation over time if time periods were thought of as independent repetitions of the game. Prices in these models must be independent of previous prices and independent of the other sellers' prices. In my model prices vary from period to period in a systematic way. Specifically, they always rise after a sale. Without modeling consumers' behaviour explicitly, Rosenthal (1982) presents a dynamic version of Rosenthal (1980). His equilibrium shares features of the mixed-strategy equilibrium described in Section 3. In particular, this model has the property that in a stochastic sense, low prices tend to be followed by high prices.

There are other ways in which temporal price variation might occur despite stationary demand and supply. Salop (1977) presents a model in which a monopolist, faced with well-informed and ill-informed consumers, can separate the ill-informed consumers, who typically pay the high price charged at most retail outlets, from the well-informed, who pay the low price offered by selected outlets. Salop's model is static, but he argues that randomly varying the location of the low prices over time might be part of an appropriate dynamic strategy. Salop and Stiglitz (1982) show that stores might have (unannounced) sales that induce consumers to purchase for future consumption. Doyle (1983) presents a model in which buyers of a non-durable good are uncertain about whether they like the item before they make a purchase. Stores offer low prices periodically to attract new customers and then raise the price to capture surplus from customers who like the good.

2. THE MODEL

Setting. Time is discrete. The product is infinitely durable. Agents are fully informed, risk neutral, and infinite lived.

Supply side. There are a fixed number of sellers, n, where n is greater than one. The ith seller chooses prices to maximize discounted present value, calculated at discount factor ρ_i (with $0<\rho_i<1$), taking the prices of the other sellers as given. Each seller has the same, constant cost per unit, assumed without further loss of generality to be zero. At a given date, no seller can make binding commitments about future prices.

Demand side. N consumers enter the market each period. A fraction α value the product at V_1 dollars; they are assumed to have discount factor zero. The remaining fraction $1-\alpha$ value the product at V_2 dollars; they are assumed to have discount factor β. Here $0<\alpha<1$, $V_1>V_2>0$, and $0<\beta<1$. Also, the convention $N=1$ is adopted; profit can be interpreted as profit per cohort. Once a consumer buys the product, he leaves the market forever. A consumer who has not bought the product stays in the market indefinitely, regardless of the date at which he first entered. No consumer buys more than one unit and there are no resales. All consumers are price takers. If a consumer is otherwise indifferent between buying immediately and buying later (at a lower price), he is assumed to purchase immediately. If more than one seller charges a price that induces the type-two consumers to buy, then each of the sellers attracts the same number of customers. In Section 4, I make this assumption about type-one consumers as well. However, in Section 3, I assume that all the consumers who value the product at V_1 buy at a particular store. That is, there exist non-negative numbers α_i, $i=1,\ldots,n$, summing to α, such that a fraction α_i/α of the entering consumers who value the product at V_1 always buy from seller i.

Throughout the paper I assume that the discount factor of the type-one consumer is zero. Thus, type-one consumers are impatient. In Section 3, they are unwilling or unable to go to more than one store or to wait in order to buy at a lower price. This is meant to capture the idea of an urgent need. In each period some consumers must have the product immediately; provided that they can afford it, these consumers buy at the most convenient store as soon as the need arises. This amounts to assuming that type-one consumers have extremely high search costs both spacially and temporally.

That each store has monopoly power over some type-one consumers is essential for the arguments of Section 3. Otherwise there is nothing to prevent prices from falling to zero. On the other hand, I assume for simplicity that type-one consumers are impatient. I discuss the effect of relaxing this assumption in Section 5.

Type-one consumers are referred to as "high" (and, in Section 3, "loyal") consumers and type-two consumers are referred to as "low" consumers because of their relative reservation prices. A seller who charges a price low enough to sell to low consumers is said to "hold a sale".

3. SIMPLE EQUILIBRIA

In this section, I characterize a class of equilibria. I assume that each store uses a pricing strategy that depends only on the number of high and low consumers in the market; these are called *simple strategies*. The restriction prohibits a seller from threatening to cut his price in the future in response to a sale by one of his competitors. In equilibrium, each seller's strategy maximizes his expected discounted profits given the pricing strategies of the other stores. If all but one seller uses a simple strategy, then the remaining seller can always select a best response that depends only on the number of high and low consumers in the market. Thus an equilibrium in simple strategies is an equilibrium even when more general strategies are allowed. Other equilibrium do exist, however; they are discussed in the next section.

I begin the analysis with some general comments about equilibrium strategies. In each period a seller has the choice of selling to his high customers, or attempting to sell to all of the low customers. The low consumers certainly will not buy at his store if the price is above V_2, the high consumers will buy provided that the price is no higher than V_1. Since future earnings are discounted, the seller will want to sell to his high customers as soon as they enter the market. Thus, the only price above V_2 charged by the seller is V_1. In principle, low consumers may refuse to buy at a price below V_2, choosing instead to wait in hope of a still better price in the future. However, a seller never cuts his price unless he expects to sell to the lows when he charges the lowest price in the market. If seller expects to sell only to his loyal customers when his price is below his competitors' prices, then he should not cut his price. Instead, he should charge V_1. Thus, all low consumers buy on the first sale day at the lowest price available. It follows that the state of the market can be summarized by the number of days, m, since the last sale.

Mixed strategies play an important role in the development that follows. Recall that a seller uses a nondegenerate mixed strategy only if each pure strategy used in the randomization yields the same profit; otherwise he does better by using only strategies yielding the highest profit.

For the remainder of this section, I analyse the symmetric case, in which each store has the same number of loyal customers. Let $a = \alpha/n$; I assume that $\alpha_i = a$ for all i.

Equilibrium behaviour involves periodic sales. The intuition is as follows. Sellers are monopolists with regard to the highs. Thus, they will cut their price only if they expect to make a comparable profit by selling to the lows. Eventually, enough lows will accumulate to make price cuts attractive. However, sellers do compete for the lows; hence, they will cut prices until no one makes a profit from the low consumers.

Theorem 1. *If the sellers are symmetric, then in any simple equilibrium, all sellers expect to earn* aV_1 *in each period.*

Proof. By selling only to its loyal customers, a store makes aV_1 per period. Therefore, I need only show that no store expects to earn more than aV_1 in any period. In order to reach a contradiction, assume that some store expects to earn more than aV_1 in some period. This store must hold a sale with probability one in that period because in equilibrium the store is indifferent between all pure strategies it uses in a mixed strategy. Since the store cannot earn more than aV_1 from a non-sale price, but can earn more than aV_1 by assumption, it always would choose the sale prices that lead to an expected profit greater than aV_1. Thus, a seller only expects to earn more than aV_1 in a period if he always holds a sale.

Let m be the first date at which some seller has a sale with probability one. In order to reach a contradiction, it is enough to show that no seller expects to earn more than aV_1 in this period. If seller 1 expects to earn more than aV_1 in period m, then seller 1 always holds a sale. In fact, every seller must hold a sale, because, for $i \neq 1$, the ith seller can make expected profits greater than aV_1 by charging a price slightly smaller than the lowest price charged by seller 1. Using this strategy, seller i expects to sell to low consumers at least as frequently as seller 1, so that seller i's expected profit can be made arbitrarily close to seller 1's expected profit, which exceeds aV_1 by assumption. Let p be the supremum of the prices charged in period m. Since all sellers hold sales with probability one, $p \leqq V_2$. If p is charged with positive probability by more than one store, then one of the sellers charging p can do better by using a price slightly less than p. If p is charged with positive probability by only one store, then this store sells to low consumers with probability zero. Therefore, this seller makes $ap < aV_1$ using p and would do better to charge V_1. Finally, if p is not charged with positive probability, then prices in a neighbourhood of p attract low customers with arbitrarily low probability. Thus, prices in this neighbourhood earn less than aV_1, and they would not be used in equilibrium. Hence, p cannot be the supremum of the prices charged in period m. This contradiction establishes the theorem. ||

Notice that Theorem 1 depends only on the profit obtainable from the high customers being equal across sellers. This could be the case even if the loyal consumers of store i were willing to pay more than those of store j provided that the relative number of these customers was adjusted appropriately. In this case, the high price of different stores would differ.

Theorem 1 does depend on the assumption of symmetry. If the stores are indexed so that $\alpha_i V_1$ is nondecreasing, then the first seller could make a profit in excess of $\alpha_1 V_1$ per period if and only if $\alpha_2 > \alpha_1$. In general, the store with the fewest loyal customers wants to have sales a bit earlier than the other stores; it can have a sale without competition provided that the revenue from the sale is no greater than what the other stores can obtain by selling to their loyal customers. In an asymmetric situation an equilibrium would consist of periodic sales by the stores with the fewest loyal customers, while the other stores maintain high prices.

The theorem suggests one type of equilibrium: only one store has sales; the other stores always charge the high price. The sale occurs at a price that yields profit aV_1 (so that in particular, low consumers buy) and above which low consumers prefer to wait. Such an equilibrium exists in a continuous-time version of this model, but need not exist in the discrete version since the first price at which low consumers are willing to buy yielding profit at least aV_1 cannot be guaranteed to yield exactly aV_1; however, qualitatively similar equilibria do exist. Nevertheless, symmetric equilibria are suggested when the stores are identical. The rest of this section concentrates on characterizing the symmetric, simple equilibrium.

To begin, I show that no symmetric, pure-strategy, simple equilibrium exists. This follows from two observations. First, note that the only price that can be charged in a symmetric, pure-strategy, simple equilibrium is V_1. To see this, suppose that all of the sellers charge $p \neq V_1$ and argue to a contradiction. Each seller must earn at least aV_1, otherwise he could charge V_1 and increase profit. However, since no one buys if $p > V_1$, to make at least aV_1 it must be that $0 < p < V_1$ and some low consumers buy at p. But then a seller can increase his revenue by setting a price slightly less than p. This price attracts all of the low consumers instead of $1/n$-th of them. Thus, p cannot be an equilibrium price. The argument depends on the loyalty of high consumers, which prevents $p = 0$ from being an equilibrium price, and the disloyalty of low consumers, which allows a small reduction in price to lead to a discontinuous increase in market share.

The second observation is that prices must fall below V_1 eventually. This is because low consumers accumulate so that, ultimately, it will be profitable for a store to cut its price and sell to them. Formally, if a store charges p and sells to its loyal customers and an m-period accumulation of lows, then it earns $p(a+(1-\alpha)m)$. Moreover, if $p = (1-\beta)V_2$ then all of the low consumers in the market will buy at p, because they obtain the surplus $\beta V_2 = V_2 - (1-\beta)V_2$, which is at least as great as the surplus available if they wait. Thus, if

$$\hat{m} > a[V_1 - (1-\beta)V_2]/(1-\alpha),$$

then charging $(1-\beta)V_2$ and earning $(1-\beta)V_2(a+(1-\alpha)\hat{m})$ dominates charging V_1 and earning aV_1. It follows that all stores cannot charge V_1 forever in symmetric, pure-strategy, simple equilibrium, in no more than $\hat{m}$ periods one store will hold a sale. When combined with the observation that stores must always charge V_1 in a symmetric, pure-strategy, simple equilibrium, this shows that no such equilibrium exists.

Mixed strategies arise in this model because having a sale becomes attractive to all stores as soon as there are enough low consumers to make profitable a sale that attracts all of the low customers. Since there will not be enough low customers to support sales by all of the stores at this time, equilibrium behaviour cannot involve all of the stores having a sale with probability one. For a discussion of why mixed-strategy equilibria arise in economic models, and some general existence results, see Dasgupta and Maskin (1981).

Theorem 2. *There exists a symmetric equilibrium in simple strategies. Further, in any symmetric equilibrium in simple strategies there exists an $m^* \geqq 0$ such that sellers charge only high prices for the first m^* periods after a sale, then use mixed strategies, which put positive probability on V_1 and positive density on prices below V_2.*

Proof. The proof proceeds in a series of steps. The first three steps characterize the strategy used when a fixed number of periods, m, have passed since the last sale.

These strategies depend on $\underline{p}_m = aV_1(a+(1-\alpha)m)^{-1}$, the lowest price that a seller can charge and still hope to make a profit of aV_1 and on $\bar{p}_m \leqq V_2$ the highest price that can be charged at which the low consumers buy. Steps 4 and 5 construct the sequence $\{\bar{p}_m\}$ and verify that it has the required properties.

Step 1. If $\bar{p}_m < \underline{p}_m$, then all stores charge V_1 with probability one.

Proof. By definition of $\bar{p}_m$, a seller makes at most $\bar{p}_m(a+(1-\alpha)m) < aV_1$ if he holds a sale. Thus, all stores do better charging V_1 than having a sale.

Step 2. If $\bar{p}_m \geqq \underline{p}_m$ and all but one of the stores use the strategy given by

$$F_m(p) = \begin{cases} 0 & \text{if } p < \underline{p}_m, \\ 1-((aV_1 - ap)((1-\alpha)mp)^{-1})^{1/(n-1)} & \text{if } p \in [\underline{p}_m, \bar{p}_m], \\ 1-((aV_1 - a\bar{p}_m)((1-\alpha)m\bar{p}_m)^{-1})^{1/(n-1)} & \text{if } p \in [\bar{p}_m, V_1), \text{ and} \\ 1 & \text{if } p \geqq V_1, \end{cases}$$

then it is a best response for the other store to use $F_m(p)$ as well.

Proof. The expected profit to a seller charging p when the other stores use $F_m(p)$ is

$$\begin{cases} p(a+(1-\alpha)m) & \text{if } p < \underline{p}_m, \\ p(a+(1-\alpha)m(1-F_m(p))^{n-1}) & \text{if } p \in [\underline{p}_m, \bar{p}_m], \\ ap & \text{if } p \in (\bar{p}_m, V_1], \text{ and} \\ 0 & \text{if } p > V_1. \end{cases} \tag{1}$$

For $p \in [\underline{p}_m, \bar{p}_m]$ sales are made to high consumers, and to the low consumers exactly when p is less than the other $n-1$ prices because no price less than V_1 is charged with positive probability. If $F_m(p)$ gives the distribution of the other stores' prices, then $(1-F_m(p))^{n-1}$ is the probability that p is the lowest price. This explains the second line of (1); the first and third lines follow from the definitions of F_m and $\bar{p}_m$.

It follows that profit is equal to aV_1 if and only if p is in the support of the distribution $F_m(p)$ and less than aV_1 otherwise. This proves Step 2.

Step 3. Given $\bar{p}_m$, symmetric equilibrium strategies must be described by $F_m(p)$.

Proof. If $\bar{p}_m < \underline{p}_m$, then the assertion follows from Step 1. So, suppose $\bar{p}_m \geqq \underline{p}_m$ and, in order to reach a contradiction, that there exists a cumulative distribution function $J_m(p) \neq F_m(p)$ that describes equilibrium strategies for some m. The revenue from charging p if all other stores use $J_m(p)$ is

$$R(p) = \begin{cases} p(a+(1-\alpha)m(1-J_m(p))^{n-1}) & \text{if } p \in [0, \bar{p}_m], \\ ap & \text{if } p \in (\bar{p}_m, V_1], \text{ and} \\ 0 & \text{if } p > V_1. \end{cases}$$

There is a constant R such that $R \geqq R(p)$ with equality if p is in the support of $J_m(p)$. $R \geqq aV_1$, for otherwise a seller could increase his payoff by charging V_1. Now suppose that $J_m(p)$ is continuous for $p < V_1$. Then, $R = aV_1$ implies that $J_m = F_m$. If $R > aV_1$, then let $\bar{p}$ be the supremum of the support of J_m. It follows that $\bar{p} \leqq \bar{p}_m$ and that

$R = R(\bar{p}) = a\bar{p} < aV_1$, a contradiction. Finally, if $J_m(p)$ is discontinuous at some $\hat{p} < V_1$, then either $\hat{p} \leqq \bar{p}_m$, and a seller gains by cutting his price slightly, or $\hat{p} \in (\bar{p}_m, V_1)$ and a seller gains by charging V_1. Both of these cases contradict the assertion that $J_m(p)$ describes an equilibrium strategy. This completes Step 3 of the proof.

Let $G_m(p) = 1-(1-F_m(p))^n$ be the probability that p is the lowest price charged in period m and let

$$E_m(p) = \begin{cases} \displaystyle\int_{\underline{p}_m}^{p} qG'_m(q)\,dq + p(1-G_m(p)) & \text{if } V_1 \geqq p \geqq \underline{p}_m, \\ p & \text{if } 0 \leqq p \leqq \underline{p}_m. \end{cases} \tag{2}$$

Step 4 establishes useful properties related to the difference equation

$$\bar{p}_{m-1} = (1-\beta)V_2 + \beta E_m(\bar{p}_m). \tag{3}$$

Step 4.
(i) The difference equation (3) has a unique solution.
(ii) If $\{p_m\}$ solves (3), then $p_m > p_{m+1}$ for all m.
(iii) If $\{p_m\}$ solves (3), then there exists an m^* such that $p_m > \underline{p}_m$ if and only if $m > m^*$.

The proof of Step 4 is in Appendix A. Facts 1 and 3 of Appendix A combine to prove (i); (ii) follows from Fact 1; and (iii) follows from Fact 2.

Step 5. If $\{\bar{p}_m\}$ satisfies (3) and sellers use the strategies described by the distributions F_m, then low consumers optimize if they purchase at state m if and only if the lowest price is less than or equal to $\bar{p}_m$.

Proof. For $m = 1, 2, \ldots,$ let

$$S_m = (V_2 - \bar{p}_{m-1})/\beta. \tag{4}$$

Since $\bar{p}_{m-1} \geqq \bar{p}_m$ for all m, $S_{m-1} \leqq S_m$ for all m. (2) and (3) imply that

$$S_m = \begin{cases} \displaystyle\int_{\underline{p}_m}^{\bar{p}_m} (V_2 - q)G'_m(q)\,dq + (1-G_m(\bar{p}_m))\beta S_{m+1} & \text{if } \bar{p}_m \geqq \underline{p}_m \\ \beta S_{m+1} & \text{if } \bar{p}_m < \underline{p}_m. \end{cases}$$

Therefore, S_m is the expected surplus of a low consumer if there have been m days since the last sale. Given that all of the other low consumers purchase if and only if the lowest price offered, p, is no greater than $\bar{p}_m$, a low consumer does not purchase at p if

$$V_2 - p < \beta S_{m+1},$$

because the surplus he expects to obtain by waiting is greater than what he receives from buying at p. Therefore, (4) implies that no lows buy at prices greater than $\bar{p}_m$. Moreover, if $p \leqq \bar{p}_m$, then a low consumer expects all of the other lows to buy. Therefore he expects $\beta S_1 \leqq \beta S_{m+1}$ if he waits. Thus, (4) implies that low consumers do best to buy at a sale if and only if $p \leqq \bar{p}_m$. This completes the proof of Step 5.

Steps 1 through 5 combine to prove Theorem 2. ‖

The equilibrium constructed has the following qualitative form. For m^* periods all stores charge the high price. After this point, until there is a sale, there is a positive probability that any store will have a sale. When the price finally falls, all of the low consumers buy and the process repeats. What is observed is a periodic price cut of one of the stores followed by an interval (of length m^* or more) of high prices. Because there are no mass points in the distribution of sale prices, the probability of more than one store charging the same low price on a sale date is zero. However, there is a positive probability that several stores cut their price at the same time. In this case, only the lowest price store attracts the low consumers. The minimum number of periods between sales is greater than one for interesting parameter values; a sufficient condition is $\underline{p}_1 > V_2$ for in that case it is impossible to attract low customers at a price high enough to generate a profit of aV_1. It can be checked that the expected sale price falls as the time between sales increases, and that the expected time between sales is finite.

The following sensitivity results are straightforward to deduce. Changes in V_2 and β leave $\{\underline{p}_m\}$ unchanged. When V_2 is increased or β is decreased, m^* is reduced and $\{\bar{p}_m\}$ is decreased. Higher prices can be charged to low consumers who are willing to pay or are impatient, thus sales are more frequent. Increases in α or V_1 increase $\{\underline{p}_m\}$ and $\{\bar{p}_m\}$: if loyal customers become more profitable, then sale prices increase (in the sense of first-order stochastic dominance). The effect of a change in α or V_1 on $\underline{p}_m^*$ dominates that on $\bar{p}_m^*$, so making high customers more profitable by raising α or V_1 increases m^* and therefore reduces the frequency of sales.

In order to evaluate the effect of a change in the number of sellers on the equilibrium, the nature of this change must be made specific. If the number of sellers increases while the total market remains the same size (so that increases in n reduce the number of loyal customers per seller), then the expected sale price falls and m^* decreases. More sellers lead to lower prices in the sense that the probability that a price below a specific level is charged in a period is nondecreasing in n. The same qualitative features hold if an increase in the number of customers increases in proportion to n (so that increases in n affect neither the number of loyal customers per seller nor the fraction of low customers in the market) because it is the fraction of low customers in the total market that determines the equilibrium strategies. In both cases, the probability of a price greater than or equal to p is $[\alpha(V-p)((1-\alpha)mnp)^{-1}]^{n/(n-1)}$, which is decreasing in n. On the other hand, Rosenthal (1980) considers the case in which each new seller brings with him a share of loyal customers while the total number of low customers remains constant (so that increases in n keep the number of loyal customers per seller and the total number of low customers constant). Interpreted in this way, an increase in n leaves $\underline{p}_m$ unchanged and increases sale prices (in the sense of first-order stochastic dominance). Here the probability of a price greater than or equal to p is $[\alpha(V-p)((1-\alpha)mp)^{-1}]^{n/(n-1)}$, which is increasing in n. The difference arises because an increase in the number of sellers in Rosenthal's sense does not alter the profit available to an individual seller from selling only to loyal customers. Thus, the expected profit from having a sale must not change when the number of firms increases. It follows that the probability that a particular store sets the lowest sale price must be independent of n. In order for this to be true, the probability that a particular seller charges a price below a certain level must decrease with n.

The proof of Theorem 2 shows that the symmetric, simple equilibrium strategies are unique given $\{\bar{p}_m\}$. However, $\{\bar{p}_m\}$ is not uniquely determined. In order for low consumers to buy if and only if the lowest price at state m is $\bar{p}_m$, it is sufficient that

$$\beta S_{m+1} \geqq V_2 - \bar{p}_m \geqq \beta S_1. \tag{5}$$

This is because when all other lows buy if and only if $p \leqq \bar{p}_m$, a low expects tomorrow's state to be 1 if some seller offers a price below $\bar{p}_m$, and to be $m+1$ otherwise. The sequence $\{\bar{p}_m\}$ used in the proof of Theorem 2 satisfies $\bar{p}_m = V_2 - \beta S_{m+1}$. Any sequence that satisfies (5) would work. The qualitative features of equilibrium do not change. Moreover, if $\beta S_{m+1} > V_2 - \bar{p}_m$, a seller could credibly argue to all low buyers that they would be better off if they refused $\bar{p}_m$ and waited. If low consumers believe this type of argument, then the equilibrium values of $\bar{p}_m$ must satisfy $\bar{p}_m = V_2 - \beta S_{m+1}$ and then Theorem 2 characterizes the only symmetric, simple equilibrium.

The mixed-strategy equilibrium characterized in this section describes one form that the competition for low consumers might take; actual randomization is not necessary. First, the analysis makes it clear that equlibria exist in which only one or two stores ever have sales. In fact, these equilibria seem to be the prominent ones in asymmetric models; the stores with fewer loyal customers have the sales. Second, recent work (for example, Milgrom and Weber (1984)) has shown that mixed-strategy equilibria in games with complete information can be identified with pure-strategy equilibria in games with incomplete information. These results suggest that if entry of new customers is random, and stores have different information about the number of consumers in the market, then pure-strategy equilibria qualitatively similar to the one analysed in this section would exist. As a final note about mixed strategies, observe that unlike Shilony (1977) and Varian (1980), randomization itself is not identified with sales behaviour. Rather it is that some price below V_1 must be charged when enough low consumers have accumulated.

I assume that high consumers are loyal to a particular store in this section. In general, some assumption is needed to prevent the equilibrium price from falling to zero. A more complete model could include differential search costs for the consumers. If consumers with high reservation prices had higher search costs—the model of this section is an extreme example, then the incentive to keep the price high in most periods, but to have periodic sales, would persist; symmetric, pure-strategy equilibria would fail to exist.

4. EQUILIBRIA USING PUNISHMENT STRATEGIES

Theorem 1 shows that simple strategies do not allow any store to make a profit from the low customers. This section studies equilibrium strategies that increase the profit of the sellers. It is unnecessary to assume any consumer loyalty to obtain these results; therefore I assume that high consumers buy at the lowest price. If more than one price induces a consumer to buy, then each seller charging that price is assumed to attract the same number of consumers. The punishment strategies studied in this section work roughly as follows. The sellers "agree" to charge high prices. In order to enforce this agreement, defections are punished by a return to "non-cooperative" (simple) strategies. Provided that the gains associated with adhering to the agreement exceed the gains associated with defecting, a non-cooperative equilibrium with relatively high levels of profit results. The idea that collusive behaviour can be a non-cooperative equilibrium in a repeated game is not new (see, for example, Green and Porter (1984), Porter (1983), and Radner (1980)).

Consider a list of strategies consisting of prices $p_i(t)$ to be charged by the ith store in period t, including the time and place of sales, and strategies to be used in the event that $p_i(t)$ is not charged for some i and t or if low consumers refuse to buy at a sale. These strategies determine $\pi_i(t)$, the discounted profit of store i from period t if no defections occur, and $C_i(t)$ the maximum discounted profit to store i associated with a defection in period t. The strategies will be in equilibrium provided that $\pi_i(t) \geqq C_i(t)$ for all i and t, and that low consumers are maximizing their surplus on the day they buy. The first constraint rules out defections. The second constraint puts an upper bound on the price charged during a sale.

If a non-cooperative equilibrium in punishment strategies is expected, then a relevant question is: Which of these equilibria is most profitable? Theorem 3 suggests an answer to that question.

Theorem 3. *If all sellers have the same discount factor, then the level of total profit in any equilibrium can be attained in a symmetric equilibrium.*

As this result is a direct consequence of the symmetry of sellers, I omit a formal proof. Given any equilibrium, a symmetric equilibrium that achieves the same level of total profit can be constructed as follows. In each period, let all stores charge the minimum price offered in the given equilibrium. The strategies that result are symmetric and, because all consumers buy at the lowest price, achieve the same level of total profit. To guarantee that defections are unattractive, assume that stores charge zero forever (starting from the next period) if there is a defection. These penalty strategies are part of a non-cooperative equilibrium since if one store is charging zero, then it is a best response for the other stores to do so as well. Since the symmetric strategies achieve the same level of total profit and do not increase the gains from defecting for any seller, they are in equilibrium.

Theorem 3 says that sales should be coordinated in order to maximize the sum of profits because that is the way to minimize the temptation to cheat on the "agreement" to keep prices positive. The theorem depends on the symmetry assumption. If one seller was relatively impatient, then he would be more willing to defect in order to make a short-term profit. The result is also sensitive to the assumption that stores attract equal shares of the market if they all charge the same price. However, in a model where this assumption was not appropriate, one would expect that a store attracting a relatively small share of the market when all prices were the same would be unable to capture the entire market when prices were slightly different. This would be the case, for example, in a locational model in which stores typically sell to customers who live nearby. Modifying the loyalty assumption, while maintaining symmetry, would not change the result. The only effect would be to change the punishment strategies. If all high consumers were loyal, as in Section 3, then a defecting store would be guaranteed aV_1 in each period after the defection; the threat of cutting prices to zero no longer would be credible, but playing a simple equilibrium strategy that holds profit to aV_1 per period would be credible.

By Theorem 3, there exists a symmetric equilibrium that maximizes joint profits over all equilibria. Next I give conditions under which cycles of length m, in which all sellers charge V_1 for $m-1$ periods and then charge V_2, can be sustained. For $1 \leqq r \leqq m$, let $\pi(m, r)$ be the present value of expected profits to a representative seller if there have been r periods since the last sale and sales occur every m periods. If ρ is the common discount factor of the sellers, then

$$\pi(m, r) = [\alpha V_1(1-\rho)^{-1} + \rho^{m-r}((1-\alpha)mV_2 - \alpha V_1)(1-\rho^m)^{-1}]/n. \tag{5}$$

To derive (5), note that a seller earns $\alpha V_1/n$ in every non-sale period. The first term on the right-hand side of (5) is the discounted profit from earning $\alpha V_1/n$ in every period. The second term is the appropriately discounted increment due to sales. To determine whether a cycle of length m can be supported as an equilibrium, I need to compute the gains from defecting. Provided that a defection leads all stores to charge zero forever, the gain from defecting r periods after the last sale is

$$\begin{aligned} C(m, r) &= \max\{\alpha V_1, (\alpha + r(1-\alpha))(1-\beta)V_2\} \quad \text{if } r \neq m \\ &= \max\{\alpha V_2, (\alpha + m(1-\alpha))(1-\beta)V_2\} \quad \text{if } r = m. \end{aligned} \tag{6}$$

There are two types of defection to consider. First, a seller could attract all of the high consumers. This means undercutting V_1 on a non-sale day and undercutting V_2 on a sale day; this type of defection yields a profit arbitrarily close to αV_1 if $r \neq m$ and to αV_2 if $r = m$. Alternatively, the seller could attract all of the low consumers. To do this, he must charge no more than $(1-\beta)V_2$. Otherwise the lows, noting the defection, wait until the next period, when the price falls to zero. This type of detection earns $(\alpha + r(1-\alpha))(1-\beta)V_2$ when there have been r periods since the last sale.

Therefore, there exists an equilibrium with cycle length m if d only if

$$\pi(m, r) \geqq C(m, r) \quad \text{for } r = 1, \ldots, m. \tag{7}$$

From (5) and (6), it follows that (7) is easier to satisfy if n is small, so that each firm earns a greater fraction of the total profit, or if β is near one, so that it is less profitable to cheat on a sale day. Moreover, for fixed m and n, (7) is satisfied for all ρ sufficiently close to one. This is because $C(m, r)$ does not depend on ρ, while $\pi(m, r)$ goes to infinity as ρ approaches one. Theorem 4 establishes a stronger result.

Theorem 4. *If all sellers have the same discount factor, ρ, then for fixed n there is a $\rho^* < 1$ such that if $\rho > \rho^*$, then there exists an equilibrium in which total profit equals monopoly profit.*

Proof. Let $m^*(\rho)$ maximize

$$\rho^m((1-\alpha)mV_2 - \alpha V_1)(1-\rho_m)^{-1}.$$

It follows from (5) that m^* maximizes $\pi(m, r)$ for all r, and that sellers earn monopoly profit if all stores cut their price to V_2 every m^* periods and charge V_1 otherwise. Thus, it is sufficient to show that for large enough ρ, (7) is satisfied when $m = m^*(\rho)$. In Appendix B, I show that

$$((1-\alpha)m^*V_2 - \alpha V_1)(1-\rho^{m^*})^{-1} \geqq \rho(1-\alpha)V_2(1-\rho)^{-1}. \tag{8}$$

Combining (8) with (5) yields

$$\pi(m^*, r) \geqq (1-\rho)^{-1}(\alpha V_1 + \rho^{m^*}(1-\alpha)V_2)/n \quad \text{for all } r = 1, \ldots, m^*. \tag{9}$$

Also, from (6),

$$(\alpha + m(1-\alpha))V_1 > C(m, r) \quad \text{for all } r = 1, \ldots, m. \tag{10}$$

Inequalitites (9) and (10) guarantee that $m = m^*(\rho)$ satisfies (7) whenever

$$\alpha V_1 + \rho^{m^*}(1-\alpha)V_2 \geqq (1-\rho)n(\alpha + m^*(1-\alpha))V_1. \tag{11}$$

In Appendix B, I show that

$$\lim_{\rho \to 1}(1-\rho)m^*(\rho) = 0.$$

It follows that the right-hand side of (11) converges to zero as ρ approaches one. Thus, there exists a $\rho^* < 1$ such that for $\rho > \rho^*$ (11) holds. This completes the proof. ||

The message of this section is that, by using punishment strategies, sellers are able to increase their profit above the level attainable by simple strategies; to increase joint profits to their highest sustainable (in a non-cooperative equilibrium) level the stores should have simultaneous sales. The punishment for defecting from the collusive agreement, zero prices forever, seems very extreme. Part of the reason for this is that the punishments are never invoked in equilibrium; this is due to the complete information assumption. However, as shown by Green and Porter (1984), if sellers had imperfect information about the state of demand and the prices charged by the other sellers, then equilibrium strategies involving intervals of zero prices would exist. A seller would cut his price to zero if he felt that there was a sufficiently high probability that one of the other sellers was cheating. When there is incomplete information, punishment phases will be entered with positive probability even if no one cheats. This suggests that the optimal length of a penalty phase is finite, so that intervals of zero prices caused by imperfect monitoring of demand would be less damaging. Nevertheless, Porter (1983) has shown that the optimal punishment interval is infinite in many situations; it is often better to have a low probability of long penalty intervals than to have more frequent, less severe penalty phases. Besides, the reason that the simultaneous sales strategy does better than other strategies is that it provides less incentive for cheating. This would be the case even if a class of "extreme" punishments were not feasible.

Other reasons to expect simultaneous sales do not appear specifically in the model. First, the strategy is easily described. All stores make it clear to their customers that they will match the price of competitors. Sales are then coordinated automatically. Also, if symmetry is required, but there is incomplete information about demand, it would be more difficult to maintain equal profit across stores with staggered sales. The interval between sales would be determined by how many low consumers have accumulated and would therefore be stochastic. If stores were identical, then simultaneous sales at regular, non-stochastic intervals would generate equal profit levels and would make defections readily identifiable.

5. CONCLUSION

This paper presents an oligopoly model in which sales are held periodically and tries to find conditions which imply that all sellers have their sale at the same time. Prices fall occasionally as a means of price discrimination: In most periods the price is high and only people with a high reservation price make a purchase, but periodically it is attractive to lower the price and sell to a large group of consumers with low reservation prices. Sales become attractive to all stores at the same time, but whether all sellers actually cut their price simultaneously depends on the equilibrium concept.

I use several devices to induce price variation. In Section 3, stores could maintain high prices because they had monopoly power over their loyal customers. While the assumptions made regarding loyalty were extreme, search costs or product differentiation typically give sellers of durable goods some monopoly power. In Section 4, I show that stores are able to maintain high prices even without monopoly power. This is possible if punishment strategies are used. This type of strategy seems most likely to be important when there are relatively few sellers and each store is able to monitor the pricing decisions of its competitors.

The crucial assumptions of the model appear to be these. First, there must be sufficient consumer heterogeneity to create a profit opportunity through price discrimination. Second, there must be a continual influx of new consumers so that, when one price cycle ends, another will begin. Third, the assumption that there is no new entry of sellers plays a key role. This assumption could be justified by modifying the specification of production costs. The simplest way to do this would be to add a fixed set-up cost. If the marginal cost of production was not constant, then the incentives to have sales would change. In addition, holding inventories might become attractive. Fourth, the assumption that the low consumers are completely informed and fully rational guarantees that only the lowest sale price attracts the low consumers. It is reasonable to assume that the low consumers do not have complete information about prices; presumably it would be in the interest of the sellers to tell consumers about sales, to that advertising should develop.

To keep the analysis simple, several restrictive assumptions were made. Some of these do not appear to be essential. Consumers with high reservaton prices could be willing to wait before making a purchase. Provided that the high consumers were not considerably more patient than the lows, high consumers would still buy in the period that they enter the market, while low consumers will wait for a sale. However, in order to induce the high customers to buy, the price charged to them must fall as the expected sale date approaches. This case was analysed in Conlisk, Gerstner, and Sobel (1984) when there is only one seller. There could be more than two consumer types. In general, the incentive to hold periodic sales would persist if the fraction of consumers above the lowest type was large enough and their willingness to pay great enough. If the product were of limited durability, and the "new" consumers entering in a period could include old consumers returning to the market, then the cycle-producing incentives would remain. Similarly, the model should be a good approximation to the case in which new entry is stochastic.

APPENDIX A

Appendix A establishes three facts about solutions to the difference equation

$$p_{m-1}=(1-\beta)V_2+\beta E_m(p_m) \quad \text{for } m>1 \tag{A1}$$

where

$$E_m(p)=\begin{cases}\displaystyle\int_{\underline{p}_m}^{p} qG'_m(q)\,dq+p(1-G_m(p)) & \text{if } V_1\geqq p\geqq \underline{p}_m,\\ p & \text{if } 0\leqq p\leqq \underline{p}_m.\end{cases}$$

Fact 1. *There exists a solution $\{\bar{p}_m\}$ to* (A1) *that satisfies $V_2\geqq \bar{p}_m\geqq \bar{p}_{m+1}\geqq(1-\beta)V_2$ for all m.*

Proof. It is useful to analyse the function

$$H_m(p)=(1-\beta)V_2+\beta E_m(p).$$

For each m, there is a unique $\hat{p}_m\in[(1-\beta)V_2,\ V_2]$ such that

$$\begin{gathered} p<H_m(p)<\hat{p}_m \quad \text{for } p<\hat{p}_m,\\ H_m(\hat{p}_m)=\hat{p}_m,\end{gathered} \tag{A2}$$

and

$$p>H_m(p)>\hat{p}_m \quad \text{for } p>\hat{p}_m.$$

To verify (A2), first note that, since $0<E_m(p)<p$ for $\underline{p}_m<p$, it follows that

$$H_m(V_2)<V_2 \quad \text{and} \quad H_m((1-\beta)V_2)>(1-\beta)V_2 \quad \text{whenever } V_2>\underline{p}_m. \tag{A3}$$

Moreover, since $H_m(p)=(1-\beta)V_2+\beta p$ for $p\leqq \underline{p}_m$,

$$H_m(p)>p \quad \text{for } p\leqq \underline{p}_m \quad \text{whenever } V_2>\underline{p}_m. \tag{A4}$$

Also,

$$H'_m(p) = \beta E'_m(p) = \beta(1 - G_m(p)) < 1 \quad \text{for } \underline{p}_m < p < V_1. \tag{A5}$$

Provided that $V_2 > \underline{p}_m$, (A3) implies the existence of $\hat{p}_m \in [(1-\beta)V_2, V_2]$ that satisfies $H_m(\hat{p}_m) = \hat{p}_m$, while (A4) and (A5) guarantee that (A2) holds. If $V_2 \leqq \underline{p}_m$, then $\hat{p}_m = V_2$ satisfies (A2). Therefore, (A2) holds for all m. Finally, note that $H_m(p)$ is non-increasing in m; it follows that $\{\hat{p}_m\}$ is non-increasing.

To construct a solution to (A1), for each i let $\{p^i_m\}$ satisfy (A1) for $m \leqq i$ (so that $H_m(p^i_m) = p^i_{m-1}$ for $m \leqq i$) and let $p^i_m = (1-\beta)V_2$ for $m \geqq i$. Then, for all i and m,

$$V_2 \geqq p^i_{m-1} \geqq p^i_m \geqq (1-\beta)V_2 \tag{A6}$$

and

$$p^{i+1}_m \geqq p^i_m. \tag{A7}$$

To show (A6), notice that for all i, $p^i_{i+1} = (1-\beta)V_2$ so that (A2) implies

$$(1-\beta)V_2 = p^i_{i+1} < H_{i+1}(p^i_{i+1}) = p^i_i < \hat{p}_i \leqq V_2 \tag{A8}$$

so that

$$(1-\beta)V_2 < p^i_i < \hat{p}_i. \tag{A9}$$

Similarly, (A2) implies that, whenever $m \leqq i$ and $p^i_m < \hat{p}_m$,

$$p^i_m < H_m(p^i_m) = p^i_{m-1} < \hat{p}_m \leqq \hat{p}_{m-1} \leqq V_2. \tag{A10}$$

Together (A9) and (A10) establish (A6) for $m \leqq i$ and for $m \geqq i+1$ (A6) follows because

$$(1-\beta)V_2 = p^i_{m-1} = p^i_m.$$

(A7) follows because (A4) implies that

$$p^i_i = (1-\beta)V_2 = p^{i+1}_{i+1} \leqq H_{i+1}(p^{i+1}_{i+1}) = p^{i+1}_i$$

(the first two equations follow from the definition of p^i_m) so that $p^i_m \leqq p^{i+1}_m$ for $m \leqq i$ since $H_m(p)$ is increasing in p and $p^i_m = p^{i+1}_m = (1-\beta)V_2$ for $m > i$ by definition.

By (A6) and (A7) $\lim_{i\to\infty} p^i_m$, call it $\bar{p}_m$, exists for each m. By construction, it follows that $\{\bar{p}_m\}$ satisfies (A1) and

$$V_2 \geqq \bar{p}_m \geqq \bar{p}_{m+1} \geqq (1-\beta)V_2. \quad \|$$

Fact 2. *If $\{p_m\}$ is a solution to (A1), then there exists an m^* such that $p_m \geqq \underline{p}_m$ if and only if $m > m^*$.*

Proof. Let $\{p_m\}$ satisfy (A1). Since $\lim_{m\to\infty} \underline{p}_m = 0$ and $p_m \geqq (1-\beta)V_2$ for all m, there exists k such that $p_m \geqq \underline{p}_m$ for $m \geqq k$. Let m^* be the largest value of m that satisfies $p_m < \underline{p}_m$ ($m^* = 0$ if $p_m \geqq \underline{p}_m$ for all m). I will show that

$$p_m < \underline{p}_m \quad \text{for all } m \leqq m^*. \tag{A11}$$

(A11), together with the definition of m^*, suffices to prove the fact.

To verify (A11) note that $p_{m^*} < \underline{p}_{m^*}$ and $p_{m^*+1} \geqq \underline{p}_{m^*+1}$, so that

$$\underline{p}_{m^*} > p_{m^*} = (1-\beta)V_2 + \beta E_{m^*+1}(p_{m^*+1}) \geqq (1-\beta)V_2 + \beta \underline{p}_{m^*+1} \tag{A12}$$

where the equation follows from (A1) and the second inequality follows because $E_{m^*+1}(p)$

is increasing in p, $p_{m^*+1} \geqq \underline{p}_{m^*+1}$ by assumption, and $E_{m^*+1}(\underline{p}_{m^*+1}) = \underline{p}_{m^*+1}$. Since

$$\underline{p}_{i-1} - \beta \underline{p}_i = aV_1(a+(1-\alpha)(i-1))^{-1} - \beta a V_1(a+(1-\alpha)i)^{-1}$$

is decreasing in i, (A12) implies that

$$\underline{p}_{i-1} > (1-\beta)V_2 + \beta \underline{p}_i \quad \text{for } i \leqq m^*. \tag{A13}$$

However, if $p_i < \underline{p}_i$, then $E_i(p_i) = p_i$ so that

$$p_{i-1} = (1-\beta)V_2 + \beta p_i \quad \text{if } p_i < \underline{p}_i. \tag{A14}$$

Subtracting (A14) from (A13) yields

$$\underline{p}_{i-1} - p_{i-1} > \beta(\underline{p}_i - p_i) \quad \text{for any } i \leqq m^* \tag{A15}$$

provided that $p_i < \underline{p}_i$.

Since $p_{m^*} < \underline{p}_{m^*}$, repeated applications of (A15) prove (A11) and establish the fact. ‖

Fact 3. *(A1) has a unique solution.*

Proof. Let $\{p_m\}$ and $\{q_m\}$ satisfy (A1). I will show that $p_m = q_m$ for all m. Fact 2 shows that there is an m^* such that p_m and $q_m \geqq \underline{p}_m$ for $m > m^*$. Hence, for $m > m^*$,

$$\begin{aligned} |p_{m-1} - q_{m-1}| &= \beta |E_m(p_m) - E_m(q_m)| \\ &\leqq \beta \max_{p \in [\underline{p}_m, V_1]} |E'_m(p)(p_m - q_m)| \\ &\leqq \beta |p_m - q_m| \end{aligned} \tag{A16}$$

because, for $p \in [\underline{p}_m, V_1]$, $E'_m(p) = 1 - G_m(p)$ and $0 \leqq G_m(p) \leqq 1$. Since

$$\lim_{m\to\infty} p_m = \lim_{m\to\infty} q_m = (1-\beta)V_2,$$

(A16) implies that $p_m = q_m$ for all $m \geqq m^*$. That $p_m = q_m$ for all m now follows from (A1). ‖

APPENDIX B

Appendix B establishes equation (8) and shows that $\lim_{\rho\to 1}(1-\rho)m^*(\rho) = 0$.

Recall that $m^*(\rho)$ maximizes

$$f(m) = \rho^m((1-\alpha)mV_2 - \alpha V_1)(1-\rho^m)^{-1}.$$

Since

$$\begin{aligned} f(m+1) - f(m) = \rho^m(1-\rho^m)^{-1}(1-\rho^{m+1})^{-1}[(1-\alpha)V_2\rho(1-\rho^m) \\ -((1-\alpha)mV_2 - \alpha V_1)(1-\rho)], \end{aligned}$$

it follows that $m^*(\rho)$ satisfies

$$\begin{aligned} [(1-\alpha)(m^*-1)V_2 - \alpha V_1](1-\rho^{m^*-1})^{-1} &\leqq \rho(1-\alpha)V_2(1-\rho)^{-1} \\ &\leqq [(1-\alpha)m^*V_2 - \alpha V_1](1-p^{m^*})^{-1}. \end{aligned} \tag{A17}$$

This establishes (8).

Now let $g(\rho) = (1-\rho)m^*(\rho)$. The first inequality in (A17) implies that for all $\rho < 1$,

$$(1-\alpha)V_2[g(\rho) - \rho + \rho^{m^*}] \leqq (1-\rho)[(1-\alpha)V_2 + \alpha V_1]. \tag{A18}$$

Let $\bar{g} = \limsup_{\rho \to 1} g(\rho)$. From (A18) and L'Hopital's Rule it follows that

$$\bar{g} - 1 + e^{-\bar{g}} \leqq 0.$$

Therefore $\bar{g} = 0$ and, since $g(\rho) > 0$, $\lim_{\rho \to 1} g(\rho) = 0$.

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