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## THE TIMING OF TRANSACTIONS IN GENERAL EQUILIBRIUM

by Peter Howitt

Until recently one of the main weaknesses in the foundations of monetary theory has been the lack of an acceptable theory of the timing of an individual's exchange transactions. Such a theory is now being developed by authors who are extending the basic Baumol-Tobin theory of the timing of bond transactions, to deal with transactions of all sorts.<sup>1</sup> However this theory as it has been developed is limited in scope to individual experiments rather than market experiments. The purpose of the present paper is to suggest a way of extending the theory to deal with market experiments by describing a class of models that analyze the timing of transactions in general equilibrium.

In its simplest form the received theory begins by supposing that the individual is in a stationary state with fixed rates of production and consumption of various stock-flow commodities. The individual faces a market environment in which unlimited quantities of these commodities can be bought or sold at announced, constant, prices, and at dates that may be chosen by the individual. The individual incurs two types of trade-related costs. First there are transactions costs, assumed to be of the set-up variety; that is, costs incurred on each transaction date that are independent of the size of the transaction. The existence of set-up costs induces the individual to trade at discrete points rather than continuously in time. The second type of cost is the cost of holding inventories, which can take the form of storage cost, interest opportunity cost, or depreciation. Generally speaking, the greater the frequency of transactions the lower will be the individual's average inventory

holdings and hence the lower will be the amount of holding costs incurred per unit of time. On the other hand, the greater the frequency of transactions the greater will be the transaction costs incurred per unit of time. Thus the individual will choose a frequency of transactions in each good so as to minimize the sum of transaction costs and holding costs.

This simple theory has been extended in various directions,<sup>2</sup> but in all cases a basic problem remains that stands in the way of generalizing the results of the theory to deal with market experiments. The problem is that, whereas the activity of exchange is by its nature a cooperative activity involving at least two individuals, the present theory assumes that trading decisions can be made unilaterally. This begs the question of how the trading decisions of separate individuals are coordinated; what ensures that a trader who decides to buy every Wednesday at 3:16 p.m. will find a seller at every such date? This problem of coordination is not solved simply by assuming that all markets clear in the usual sense. Even if for each commodity the rate of planned production equals the sum of planned consumption and planned accumulation, there is still the problem of coordinating the individuals' specific transaction dates. In Perlman's (1971) terminology there is a problem not only of the double coincidence of wants but also of the double coincidence of timing.

Our problem is thus part of the larger one of analyzing the coordination of economic activities. It is instructive to note that this problem does not appear in the neo-Walrasian models of money in general equilibrium, in which only the fully coordinated states of general equilibrium are examined, even though these models do contain explicit costs of transacting.<sup>3</sup> One reason for this is that in many of these models the transaction costs are not of the set-up

variety that induce lumpy transactions; rather they are describable by a convex market set which, in a perfectly stationary economy, would result in all agents transacting continuously and hence simultaneously. But even when these models are generalized to include set-up costs of transacting the problem is avoided by the heroic assumptions that all agents can predict future prices with perfect accuracy, and that these future prices are such that desired sales and purchases of each commodity are equated at each instant.

Once the unrealistic neo-Walrasian assumption that trading activities may be coordinated costlessly is given up, it is apparent that an incentive exists for some agents to accumulate inventories and set themselves up as specialist middleman agents. Such middlemen would stand open to do business continuously at announced prices, and allow their inventories to fluctuate in order to accommodate the timing decisions of the nonspecialist traders. By doing so, they would provide services of "availability" to the nonspecialist traders, the price of which would be a spread between their buying and selling prices. The size of the spread would be determined by competition between the middlemen, and by their costs. The approach of the present paper is to develop a class of models in which this coordination problem is handled by such middlemen.<sup>4</sup>

Since our primary concern is not to analyze the question of price formation, and since that question poses difficult analytical problems of its own, the present paper takes prices as given at their equilibrium values. Nevertheless, the class of models described herein is intended as a prototype for a more general class in which all of

the activities that the auctioneer undertakes costlessly in neo-Walrasian models are undertaken by profit-maximizing middlemen in competition with one another. The inventory holdings of the middlemen thus serve the purpose of coordinating not only the primary agents' timing decisions but also their quantity decisions. If and when a systematic trend becomes apparent to the middlemen in their inventory fluctuations they may suspect a fundamental excess supply or demand at the going prices, and be induced to change their prices. Thus the approach may prove to be useful in resolving one of the outstanding problems of economic theory--how to characterize the process of price-adjustment. The likely usefulness of this approach is suggested by the following observation. The general equilibrium approach suggests that when markets are not clearing some traders will be rationed. Given the unlikeliness of ever observing an economy in a general equilibrium one might then ask why we do not observe the widespread rationing that is predicted by this approach. The obvious answer to that question is that specialist middleman traders in the form of shopkeepers, brokers, wholesalers distributors, retailers, and so forth allow their inventories to fluctuate in the face of excess demands and supplies, thus allowing the nonspecialist primary traders to realize their trading plans even when markets are not clearing.

In the absence of any obvious excess demands or supplies the middlemen have a strong incentive to maintain constant prices in the face of

what may prove to be transitory fluctuations in their inventories, for by doing so they reduce the primary agents' costs of uncertainty (Alchian, 1970). This saving in costs of uncertainty is another of the services of availability rendered by the middleman. Thus, the present paper may be interpreted as dealing exclusively with situations in which none of the middlemen perceive enough of a fundamental excess demand or supply to change any price.

One insight that emerges from this approach is that the activity of exchange involves a fundamental externality. In the absence of middleman coordinators this externality takes the form of the necessity for a double coincidence of timing. Thus my trading plan will have an effect on the welfare of other traders, independent of its effects on any market prices. If I decide to trade when others are planning to trade that confers a benefit upon them. With middleman traders the problems associated with this externality are reduced but not entirely eliminated. The primary traders determine their own transaction dates but to the middlemen the timing of their sales is a random variable. Thus they will hold precautionary inventories against the possibility of running out. One of the costs of doing business as a middleman will be the cost of holding these inventories. The amounts of these inventories that the middlemen choose to hold will depend among other things upon the variance of their daily sales, which in turn will depend upon the frequency decisions of the primary traders. Thus the externality arises from the direct dependency of the middlemen's costs upon the primary traders' trading decisions.



Externalities of this sort have been mentioned previously in the literature. They are hinted at in the common notion of money as a public good.<sup>5</sup> My money holdings will confer direct benefits upon my trading partners, if they provide a buffer against shortfalls in liquidity which would otherwise induce me to present notes for immediate payment to my debtors. In the present context they will also confer benefits upon the middlemen if when my money holdings are increased that is associated with a reduction in my holdings of commodity inventories which in turn is associated with more frequent purchases of a smaller size. Given the expected daily rate of sales to the middlemen such an increase in frequency and reduction in purchase size will reduce the variance of their daily sales.

The presence of this externality casts doubt upon conventional answers to questions of monetary efficiency. In particular, in the presence of these externalities the triangle measured under the demand for money function will not capture all of the welfare costs of inflation, even of perfectly anticipated inflation in a stationary economy.<sup>6</sup> The height of the money demand function will measure the marginal private benefits of money whereas in the presence of positive externalities the marginal social benefits will exceed this. Furthermore, the optimum quantity of money in the presence of this externality can generally not be achieved by the payment of competitive interest on money. The standard argument is that without competitive interest being paid on money individuals see the private costs of holding money as being greater than the social costs; thus competitive interest is a way of setting private and social costs equal to each other.<sup>7</sup> However, with the

externality individuals also see the private benefits as being less than the social benefits. Thus to achieve optimality the private marginal cost of holding money must actually be set less than the marginal social cost. Another purpose of the present paper is to explore these issues of monetary efficiency.

In Section 1 a simple model is laid out involving households, producers and middlemen. The model has been kept drastically simple in order to present an easily understood prototype of the class of models implied by this approach and in order to illustrate several implications of the approach. Section 2 explores the comparative statics implications of the model. Section 3 addresses the issues of monetary efficiency. Section 4 presents the conclusions and suggestions for further research.

#### 1. The Model

The model portrays a monetary economy consisting of three groups of agents: households, producers, and middlemen; and three groups of tradeable commodities: money, factor services and goods. Within each group each agent is identical in all essential respects. Households sell factor services to producers and middlemen for money, producers sell goods to middlemen for money, and middlemen sell goods to households for money. In order to establish a basis for monetary exchange we must assume that each producer specializes in the production of only some of the goods and that each household specializes in selling its factor services to only some of the producers or middlemen. But in order to avoid the inessential complications in analysis and notation that arise from the diversity of goods we also assume that all goods are produced by identical production functions and that all goods enter symmetrically into the typical household's utility function.

In the interest of simplicity we also restrict our attention to the examination of stationary-state equilibrium positions in which the time path of each agent's inventories repeats itself at regular intervals and all markets are clearing. This last condition, combined with our symmetry assumptions, implies that all goods have a constant relative price equal to one.

(a) Households. Consider the situation of a household in a stationary state who is consuming at the rate  $c$ , with

$$(1) \quad c = y/n$$

where  $y$  represents total real income in the economy and  $n$  represents the number of identical households.<sup>8</sup> Each household is able to purchase in one shopping trip to one of the middlemen all of the goods that it consumes, and chooses a constant interval of length  $h$  between each successive shopping trip. Thus under the assumption that the household maintains no precautionary inventories its average holdings of commodity inventories will be given by

$$(2) \quad \bar{c} = ch/2 = C/2$$

where  $C$  is the size of each purchase. Each household is paid by a producer or middleman at fixed, regular intervals of length  $\theta$ , where  $\theta > h$ .<sup>9</sup> The household also maintains inventories of money. At the beginning of each payment interval total inventory holdings will consist entirely of the amount  $c\theta$  of real money balances, where here and throughout the paper the price deflator is the level of buying prices faced by the households,  $P_b$ .

At the end of the interval no inventories are held.<sup>10</sup> During the interval the sum of real money balances and goods inventories are run down continuously at the constant rate  $c$ . Thus average inventory holdings over the period will be  $c\theta/2$ , and average real money balances will be

$$(3) \quad \bar{M}_h = c\theta/2 - ch/2 .$$

The household will choose  $h$  so as to minimize the sum of all trade-related costs. Thus its decision problem will be

$$(4) \quad \text{Min}_{\{h\}} \rho c\theta/2 + (\alpha + \pi)(c\theta/2 - ch/2) + \beta ch/2 + a/h .$$

The first term in (4) represents total waiting costs, assumed to be proportional to the average holdings of all inventories, where  $\rho$  is the household's subjective rate of time preference. The second term represents the storage costs on average money holdings, where  $\alpha$  is the physical storage cost coefficient and  $\pi$  is the expected rate of inflation.<sup>11</sup> The third term represents storage costs on average holdings of goods inventories where  $\beta$  is the storage cost coefficient. The fourth term represents the average transaction cost per unit of time, where  $a$  is the set-up cost per transaction and  $1/h$  is the frequency of transactions. The solution to this decision problem is given by the familiar square root formula

$$(5) \quad h = \sqrt{\frac{2a}{(\beta - \alpha - \pi)c}}$$

which implies average real money balances of

$$(6) \quad \bar{M}_h = c\theta/2 - \sqrt{\frac{ac}{2(\beta - \alpha - \pi)}} .$$

(b) Middlemen. Suppose that all shopping trips are evenly spaced throughout any time interval. Thus over a time interval of unit length the total number of shopping trips made by all individual households is given by

$$(7) \quad N = n/h .$$

Suppose that there are  $m$  identical middlemen in the economy. In order to make explicit the source of random variation in each middleman's sales, suppose that each household engages in search activities by randomly selecting to which of the middlemen each shopping trip is made. Of course the probability that any shopping trip will be made to a particular middleman ought to depend upon the posted prices of that middleman relative to those of the other middlemen. However, since these relative prices are all fixed at unity, we assume that at each of the  $N$  purchase dates the probability that the purchase will be made from any one middleman equals  $1/m$ .

Suppose that the typical middleman places a regular order for goods with a frequency of one regular order per period. The middleman's problem is to decide the quantity of goods with which to begin each period immediately after the regular order. Large quantities will imply large holding costs, but small quantities will imply a large probability of running out during the period. In the event of a run out it is assumed that the middleman places a special order.<sup>12</sup> Each time a special order is placed a set-up cost is incurred. Thus the middleman will try to choose the beginning-of-period quantity that minimizes the sum of expected holding costs and expected set-up costs.

Consider the time period between regular orders. Let the number of sales by a middleman during this period be  $x$ . The total volume of sales during the period,  $xC$ , will be governed by the binomial distribution:

$$(8) \quad f(xC) = \binom{N}{x} \left(\frac{1}{m}\right)^x \left(1 - \frac{1}{m}\right)^{N-x}; \quad x=0,1,\dots,N$$

The mean of this distribution is

$$(9) \quad \mu = \frac{NC}{m} = y/m .$$

The standard deviation is

$$(10) \quad \sigma = C \sqrt{N \left(\frac{1}{m}\right) \left(1 - \frac{1}{m}\right)}$$

Using (1), (2), (5), (7), and (10) the standard deviation can be written as

$$(11) \quad \sigma = y^{3/4} \left(\frac{2a}{\beta - \alpha - \pi}\right)^{1/4} z$$

where  $z$  is a constant given by

$$(12) \quad z = \sqrt{\left(\frac{1}{m}\right) \left(1 - \frac{1}{m}\right) / n^{1/4}}$$

The middleman's value added, and thus its factor payments, come entirely from the spread between the buying price  $P_b$  and the selling price  $P_s$ . Assume that with each sale the middleman retains the value added in a special payment fund, from which it makes factor payments at regular intervals of length  $\theta$ . In order to simplify the analysis of the middlemen it is assumed that  $\theta$  is an integer. Whenever the middleman runs out and places a special order, the size of the special order is just equal to the middleman's current money balances minus the factor payment fund. This quantity is referred to as the middleman's active money balances. Likewise, since the middleman will choose the size of each regular order so as to begin each subsequent regular

order period with a constant amount of goods inventories, the size of each regular order must also be equal to the middleman's current active money balances.

Over any payment interval of length  $\theta$  the average real money balances of the typical middleman will be equal to the random variable:

$$(13) \quad M_m = M_a + M_\pi$$

where  $M_a$  denotes average active balances and  $M_\pi$  the average size of the payment fund.

It is easily seen that the expected value of  $M_\pi$  is

$$(14) \quad E(M_\pi) = \frac{1}{2} E \left[ \left( \frac{P_b - P_s}{P_b} \right) \theta Cx \right] = \left( \frac{P_b - P_s}{P_b} \right) \frac{\theta y}{2m}$$

Furthermore the Appendix proves that, provided that the cost of running out is high enough, so that the middleman chooses a low enough probability of running out, the expected value of  $M_a$  can be approximated by the formula:

$$(15) \quad E(M_a) = E(P_s \times C/2 P_b) = P_s y/2 P_b m.$$

Thus the expected value of the middleman's average real balances over any such interval is:

$$(16) \quad \bar{M}_m = E(M_\pi) + E(M_a) = \left[ \theta - \frac{P_s}{P_b}(\theta - 1) \right] \frac{y}{2m}$$

Suppose that the middleman plans to begin each period with commodity inventories equal in amount to  $S$ . By our stationarity assumption, the sum of the two kinds of inventories will be

$$(17) \quad \bar{M}_m + \bar{S}_m = S$$

where  $\bar{S}_m$  is the expected value of goods inventories. Using (16) and (17) we see that expected goods inventories will be

$$(18) \quad \bar{S}_m = S - g\mu$$

where  $g = \frac{1}{2} \left[ \theta - \frac{P_s}{P_b} (\theta - 1) \right]$ , so that  $0 < g < \frac{\theta}{2}$ .

The middleman will attempt to minimize the expected value of trade-related costs. Thus, its decision problem will be:

$$(19) \quad \text{Min}_{\{S\}} \rho S + (\alpha + \pi) g\mu + \beta(S - g\mu) + b(1 - F(S))$$

The first term in (19) represents waiting costs where once again  $\rho$  is the subjective rate of time discount. The second and third terms represent the storage costs on expected money and goods inventories, respectively. The fourth term represents the expected set-up cost of special orders, where  $b$  is the cost per special order, and  $F(S)$  is the cumulative density of daily sales evaluated at  $S$ , so that  $1 - F(S)$  is the probability of running out. The solution to this problem is given by

$$(20) \quad f(S) = \frac{\rho + \beta}{b}$$

which can conveniently be rewritten as

$$(21) \quad S = \mu + k \left( \frac{\rho + \beta}{b} \right) \sigma. \quad 13$$

(c) Producers. The producers will generally face a problem that is similar in nature to that of the middlemen. Just as the middlemen cannot predict with perfect accuracy the purchasing plans of the individual households, nor can the producers predict with perfect accuracy the purchasing plans of the middlemen. However, in order to simplify the analysis, suppose that producers are very few in number relative to middlemen, so that from the point of view of the typical producer orders come in almost continuously at a constant rate of  $y/l$ , where  $l$  is the number of identical producers. The producers thus need to hold



no goods inventories. Each producer holds no money at the beginning of each payment interval and accumulates real money balances steadily at the rate  $\frac{P_s}{P_b} y/l$ . Thus, average real money balances of the typical producer will be:

$$(22) \quad \bar{M}_f = \left(\frac{P_s}{P_b}\right) y \theta / 2l. \quad 14$$

## 2. Comparative Statics

We can now derive an expression for the aggregate demand for money:

$$(23a) \quad \bar{M} = n\bar{M}_h + m\bar{M}_m + l\bar{M}_f = y\left(\theta + \frac{P_s}{2P_b}\right) + \sqrt{\frac{a n y}{2(\beta - \alpha - \pi)}}$$

The aggregate demand for goods inventories will be

$$(23b) \quad \bar{G} = n\bar{C} + m\bar{S}_m = y(1 - g) + \sqrt{\frac{a n y}{2(\beta - \alpha - \pi)}} + m z k \left(\frac{\rho + \beta}{b}\right) \left(\frac{2a}{\beta - \alpha - \pi}\right)^{1/4} y^{3/4}$$

Total demand for inventories will be

$$(24) \quad \bar{M} + \bar{G} = y \left[ \frac{\theta}{2} \left(1 + \frac{P_s}{P_b}\right) + 1 \right] + m z k \left(\frac{\rho + \beta}{b}\right) \left(\frac{2a}{\beta - \alpha - \pi}\right)^{1/4} y^{3/4}$$

Comparative statics analysis of these aggregate demands is quite straightforward, and the results are summarized in Table 1.

|                                   | a | b | $\alpha + \pi$ | $\rho$ | $\beta$ | y |
|-----------------------------------|---|---|----------------|--------|---------|---|
| $\bar{M}$                         | - | 0 | -              | 0      | +       | + |
| $\bar{G}$                         | + | + | +              | -      | -       | ? |
| $\bar{M} + \bar{G}$               | + | + | +              | -      | -       | + |
| $\bar{M} + \bar{G}(\bar{\sigma})$ | 0 | + | 0              | -      | -       | + |

Table 1

The fourth line of this table gives the results of adding together the partial effects of parameter changes on the inventories of producers, households, and middlemen when the interdependency of middleman decisions upon household decisions coming through the standard deviations faced by the middlemen is ignored, by keeping the standard deviation constant. While this table of results contains no great surprises, four observations are in order.

First, the ambiguity of the income elasticity of  $\bar{G}$  disappears if  $\frac{P_s}{P_b}$  is close enough to unity that  $g < 1$ . Second, the externality referred to in the introduction does have some effect upon the qualitative behavior of this model, as can be seen in the comparison of lines 3 and 4. For instance, in the event that the storage cost coefficient on money were to rise, conventional analysis which focused only upon the household behavior would suggest that goods inventories would rise, money holdings would fall, and the sum of money and goods inventories would remain constant, as indicated by column 3, line 4. However, when the interdependency of trading decisions is taken into account we see in column 3, line 3 that following such a parameter change total inventory holdings will actually rise. The reason is that when households reduce their money holdings and increase their holdings of goods inventories they also make fewer transactions of larger size. This has the effect of raising the middlemen's standard deviation and inducing them to increase their goods inventories.

Third, the present analysis has implications regarding economies of scale in the holding of trade inventories. From (24) it is easily seen that the income elasticity of total inventory holdings will lie somewhere between 1 and 3/4. Thus, economies of scale do exist, but not nearly to the extent that would be predicted by focussing only upon households. Along the same lines if

transaction costs depend proportionately upon real wages, and if real wages are in turn proportional to real income, then the economies of scale will be consistent with measured income elasticities of greater than unity.<sup>15</sup> An x percent increase in y will be associated with an x percent increase also in a and b which can be seen to produce more than an x percent increase in  $\bar{M} + \bar{G}$ . While the implications of the deterministic inventory models regarding these income elasticities is well known, the implications of the stochastic models have been problematical ever since Edgeworth's seminal contribution. The problem is that these stochastic models rather than including an explicit income variable depend upon the variance of the flow of transactions, which is difficult to relate unambiguously to the level of income or even the level of transactions. When income changes the effect on the variance will depend upon the extent to which this change in income takes the form of an increased number of transactions each of the same size, or an increased size of transactions occurring at the same frequency, or some combination of increased frequency and increased size.<sup>16</sup> One of the advantages of the present approach is that it utilizes the implications of the deterministic models to solve this problem. Thus the square root formulae imply that a doubling of income will take the form of a 50 percent rise in the size of transactions and a 50 percent rise in the frequency, so that the standard deviation of the middleman's sales can be related unambiguously to the level of income by the formula in (11).

Fourth, it should be noted that many of these results are quite sensitive to the particular specification of the model. For example, suppose

that the households are paid more frequently than they purchase.<sup>17</sup> Then the average money holdings and goods inventories of the typical household will be

$$(25) \quad \bar{M}_h = \frac{c(h-\theta)}{2} ; \quad \bar{G}_h = c/2 = ch/2$$

and the household minimization problem analogous to (4) will produce the solution

$$(26) \quad h = \sqrt{\frac{2a}{c(2\rho + \alpha + \pi + \beta)}}$$

so that the aggregate demand for money and goods inventories will be

$$(27a) \quad \bar{M} = \sqrt{\frac{a n y}{2(\alpha + \pi + \beta + 2\rho)}} + \frac{y P_s}{2 P_b}$$

$$(27b) \quad \bar{G} = y(1-g) + \sqrt{\frac{a n y}{2(\alpha + \pi + \beta + 2\rho)}} + m z k \left(\frac{\rho + \beta}{b}\right) \left(\frac{2a}{2\rho + \pi + \alpha + \beta}\right)^{1/4} y^{3/4}$$

and the total demand for inventories will be:

$$(28) \quad \bar{M} + \bar{G} = \sqrt{\frac{2 a n y}{\alpha + \beta + \pi + 2\rho}} + y \left[ 1 - \frac{\theta}{2} \left( 1 - \frac{P_s}{P_b} \right) \right] + m z k \left(\frac{\rho + \beta}{b}\right) \left(\frac{2a}{2\rho + \pi + \alpha + \beta}\right)^{1/4} y^{3/4}$$

This produces the comparative statics results summarized in Table 2.

|                                   | a | b | $\alpha + \pi$ | $\rho$ | $\beta$ | y |
|-----------------------------------|---|---|----------------|--------|---------|---|
| $\bar{M}$                         | + | 0 | -              | -      | -       | + |
| $\bar{G}$                         | + | + | -              | -      | -       | ? |
| $\bar{M} + \bar{G}$               | + | + | -              | -      | -       | ? |
| $\bar{M} + \bar{G}(\bar{\sigma})$ | + | + | -              | -      | -       | ? |

Table 2

The ambiguities disappear once again if  $\frac{P_s}{P_b}$  is close enough to unity. The only parameters whose effects are the same in both cases are  $b$ , the middleman's cost of special orders, and  $y$ , the level of income. The reversal of many of the other effects occurs for the same reason as spelled out by Grossman and Policano (1976).<sup>18</sup> Notice that in this case an increase in the storage cost coefficient on money has a different feedback on the middlemen's holdings of goods inventories, because it will lead households in this case to make smaller, but more frequent, transactions thus reducing the standard deviation of daily sales to the middleman and inducing them to hold fewer commodity inventories.

One special feature of both of these models is that the frequency decisions of households have no feedback effects on the money balances of the middlemen. Indeed these balances are determined just by the level of income. In general, however, one would expect them to be affected positively by the standard deviation of daily sales, which can have interesting comparative statics implications. For example, an increase in the storage cost coefficient on money will lead households to change their frequency of purchases, which will alter this standard deviation. In the case where  $h < \theta$ , it might even be the case that these indirect effects of a change in the storage cost coefficient on money override the direct effects, with the result that an increase in the storage cost coefficient on money actually produces a rise in the aggregate demand for money.

### 3. Monetary Efficiency

As mentioned earlier, the present approach has implications regarding the optimum quantity of money, and the measurement of the welfare costs of inflation. In order to analyze these questions, suppose that we reformulate the household decision problem as

$$(29) \quad \underset{\{\bar{M}_h\}}{\text{Max}} \quad B(\bar{M}_h) - (\rho + \pi)\bar{M}_h$$

where  $B(\bar{M}_h)$  represents the private benefits to the typical household of holding the average quantity of money ( $\bar{M}_h$ ). The solution is given by the first-order condition

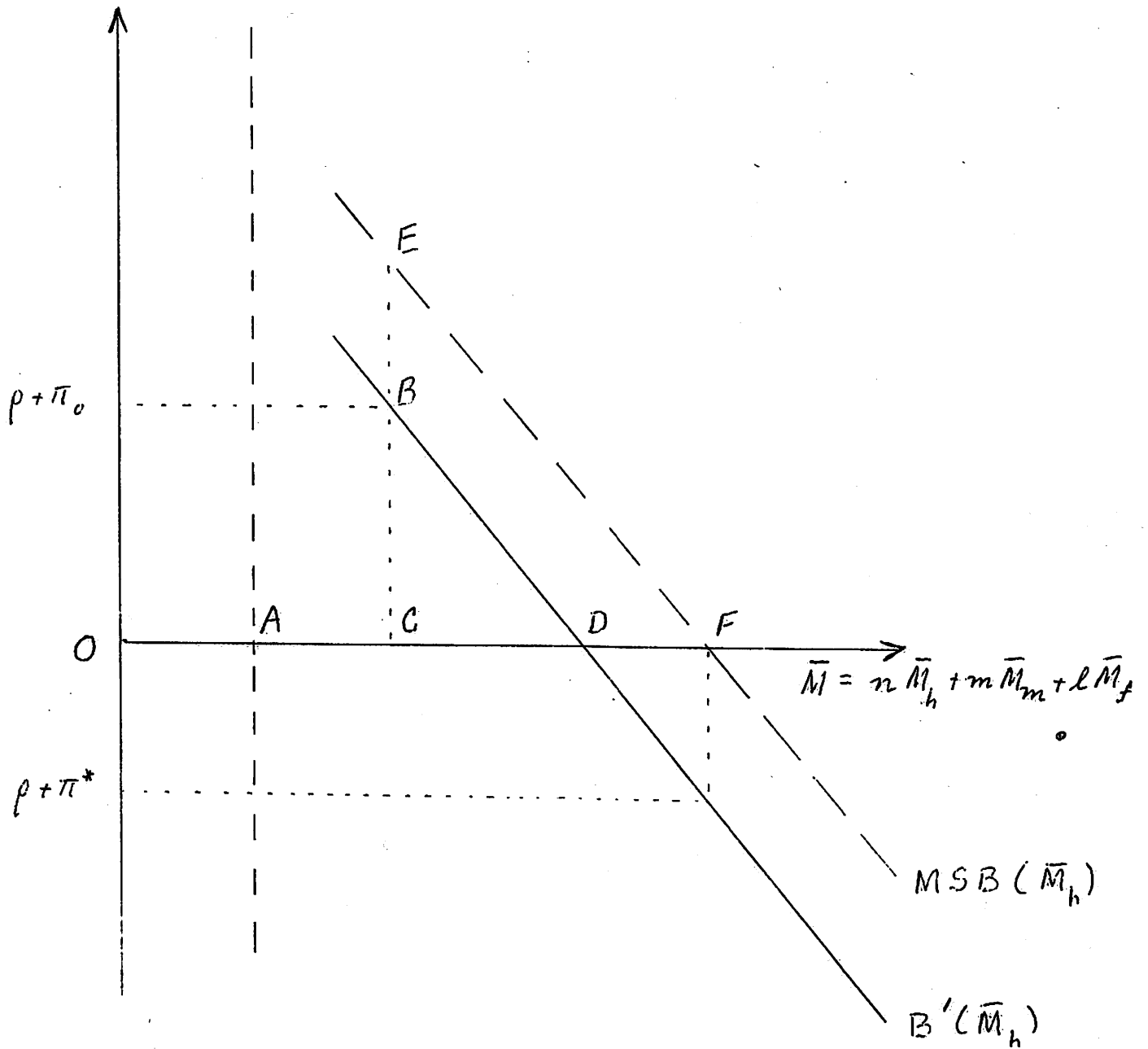
$$(30) \quad B'(\bar{M}_h) = \rho + \pi$$

This decision problem is the same as (4) where the benefits of money holdings are

$$(31) \quad B(\bar{M}_h) = -(\rho + \beta)y^{\theta/2} + (\beta + \rho - \alpha)\bar{M}_h - na/(\theta - 2\bar{M}_h/y)$$

The aggregate demand for money function is represented in Figure 1 where the opportunity cost of money holdings is measured on the vertical axis, and the aggregate demand for money is measured on the horizontal axis. The horizontal distance OA represents the total demand for money by producers and middlemen, and the distance from the vertical line A to the demand function going through B and D represents the demand for money by households. According to equation (30), the height of the demand function at any point represents the marginal private benefit to households from holding the quantity of money ( $\bar{M}_h$ ).

Figure 1



A consequence of the externality involved in this approach is that social benefits from household money holdings exceed private benefits. The social benefits per household may be expressed as

$$(32) \quad SB(\bar{M}_h) = B(\bar{M}_h) + \frac{mb}{n} (F(s; \sigma(\bar{M}_h)) - 1)$$

where  $\sigma(\bar{M}_h)$  represents the functional dependency of the standard deviation on household decisions, through equations (2), (3), and (10).<sup>19</sup> Thus, the marginal social benefit from the typical household's money holdings is

$$(33) \quad MSB(\bar{M}_h) = B'(\bar{M}_h) + \frac{mb}{n} \frac{\partial F(s; \sigma)}{\partial \sigma} \cdot \frac{d\sigma}{d\bar{M}_h}$$

It is easy to establish that

$$(34) \quad \frac{\partial F(s; \sigma)}{\partial \sigma} < 0$$

$$(35) \quad \frac{d\sigma}{d\bar{M}_h} = \frac{d\sigma}{dh} \cdot \frac{dh}{d\bar{M}_h} = - \frac{\sigma}{hy} < 0$$

Thus we may express the marginal social benefit as

$$(36) \quad MSB(\bar{M}_h) = B'(\bar{M}_h) + X(\bar{M}_h)$$

where the marginal external benefit is  $X(\bar{M}_h) > 0$ .

The marginal social benefit of money holdings is represented in Figure 1 by the dashed line above the demand function. It follows immediately that in this case the conventional measure of the welfare costs of inflation; i.e., the area of the triangle BCD understates the social costs of inflation at the rate  $\pi_0$ , which should be the area of the triangle ECF. This will always be the case when positive externalities of this sort exist regardless of their source. However, as indicated in Section 2, the externalities do not necessarily go all in this direction. For example, if  $h > \theta$ , then the partial effect of household money holdings on the standard deviation instead of



being negative as indicated in (35), would be positive, the marginal external benefit would be negative, and the marginal social benefit curve would lie to the left of the demand function.<sup>20</sup> In this case the conventional triangle would overstate the welfare costs of inflation. In summary, the present approach casts doubt upon the validity of conventional measures of the welfare costs of inflation, even under the idealized circumstances of perfectly anticipated inflation in a stationary state; however, the direction of the bias of the conventional measure cannot be determined a priori.

The other issue on which the present analysis has some bearing is that of the optimum quantity of money. Suppose that the marginal social cost of money holdings is zero. Then social optimality requires that

$$(37) \quad 0 = \text{MSB}(\bar{M}_h)$$

It can be seen from (30) and (37) that this condition will be satisfied in equilibrium only if

$$(38) \quad \pi = -\rho - X(M_h^*)$$

where  $M_h^*$  is the solution to (37). The conventional approach implies that monetary optimality can be achieved by setting a rate of inflation equal to the negative of the common rate of time discount in the economy. Equation (38) implies that in the case of positive externalities the rate of deflation should be pushed beyond this point.<sup>21</sup> This is illustrated in Figure 1 where  $\pi^*$  is the socially optimal rate of inflation. Once again, however, while the present analysis casts doubt upon previous findings, the direction of bias in those findings is not clear. In the case where  $h > \theta$  and the marginal external benefit is negative the optimal rate of deflation will be less than the rate

of time discount.

In a more general model we would expect the producers' and middlemen's demand for money to depend upon  $\rho + \pi$ , but the above conclusions would still be valid in a qualitative sense. In the case where  $h < \theta$  ( $h > \theta$ ), monetary optimality would require a rate of deflation greater (less) than the rate of time preference and the standard triangle measure of the welfare costs of inflation is an underestimate (overestimate) of those costs. However, the results of establishing the optimal rate of deflation will be a second-best optimum. The kind of externality that is involved here is one that cannot generally be corrected completely by changing the common prices paid by all agents. Rather, there is a presumption in the literature on externalities and public goods that the achievement of Pareto efficiency requires different individuals to pay different prices depending upon the particular nature of the externalities involved.<sup>22</sup> In the present case the households should see the costs of holding money as being  $\rho + \pi^*$ . But at this rate of deflation the middlemen, of whose money holdings the private and social benefits are equal, will see the private costs as being different from social costs. Hence full Pareto optimality could only be achieved if each agent had different expectations of inflation. Otherwise the optimal rate of (expected) inflation will clearly lie somewhere between  $-\rho$  and  $-(\rho + E(\bar{M}_h^*))$ .

#### 4. Conclusion

In summary, this paper has attempted to show that, first, it is possible to generalize the theory of the timing of individual transactions to deal with market experiments by means of the class of models illustrated in this paper; second, there is at least in principle an interconnectedness between the money holding and transaction

timing decisions of one economic agent and those of another, which must be taken into account in order to derive accurate comparative statics predictions; third, this same interconnectedness has implications on questions of monetary efficiency.

The empirical significance of this interconnectedness is a question that remains to be explored. Clearly the externality is to some extent internalized in the form of long-term contracts between suppliers and their customers that minimize the variance of the suppliers' sales. But just as clearly it remains external in cases such as the retailing of mass-produced consumer goods. In any event, future development of this approach ought to be more fruitful in providing a microeconomic foundation for short-run questions of monetary dynamics than for questions of welfare economics. Particularly valuable extensions of the present class of models would be to allow for trade credit and temporary bond holdings by the various agents, search behavior by the households, price adjustment by the middlemen, and speculative behavior on the part of all agents.<sup>23</sup>

Also worth pursuing is the question of how many middlemen will exist in a competitive situation. On the grounds of risk-pooling economies it would appear that only one middleman would survive competition, in which case the randomness of the present model would disappear, and along with it the externality. However, spatial considerations and limitational factors such as entrepreneurship<sup>24</sup> should serve to limit the size of each middleman's operations; indeed, if the overhead costs of middlemen's operations were large enough, one would expect the firms each to undertake their own retailing operations. Clearly this question must be addressed before a satisfactory analysis of the efficiency problems raised in this paper can be undertaken, and before the approach can be satisfactorily developed as a micro-foundations for other questions of monetary economics.

Footnotes

<sup>1</sup>See Clower (1970), Feige and Parkin (1971), Barro and Santomero (1976). Similar probabalistic models have been developed by Miller and Orr (1966) and Patinkin (1965). Patinkin's model is closest to the present approach, as it focuses upon proper market experiments; however it takes the timing of payments as given (although probabilistic), and analyzes the demand for money as a result of this random payments process, rather than analyzing how the timing decisions themselves are made.

<sup>2</sup>See Clower and Howitt (1976), Howitt (1977).

<sup>3</sup>See Hahn (1973), Kurz (1974). This literature is usefully summarized by Ulph and Ulph (1975).

<sup>4</sup>This class of models has been proposed by Clower (1975), Clower and Leijonhufvud (1975), Clower and Howitt (1976) and Howitt (1976).

<sup>5</sup>This was the subject of the paper by Weldon (1973). The notion is also implicit in Friedman's discussion of the non-optimality of the holdings of non-interest bearing money (1969, p. 15). The following discussion borrows heavily from Laidler (1975).

<sup>6</sup>For the standard theory of this triangle measurement, see Bailey (1956) and Marty (1967).

<sup>7</sup>See Samuelson (1968) and Friedman (1969).

<sup>8</sup>We are assuming that all trade-related costs are subjective so that income and consumption are equal for the typical household. Likewise the real capital losses to households' real balances are continuously offset by lump-sum transfer payments of new money.

<sup>9</sup>The assumption that  $\theta$  is exogenous could easily be relaxed along the lines of Barro and Santomero (1976), and is retained here merely for simplicity.

<sup>10</sup>We are assuming that  $\theta$  is an integer multiple of  $h$ , although later on we shall treat  $h$  as continuously variable. In a simple model of this form, as Barro (1976) has shown, this analytical approximation is reasonably accurate. However, in more complicated models, as Clower and Howitt (1976) have shown, the approximation may give rise to qualitatively invalid comparative statics predictions.

<sup>11</sup>Note that in order to have inflation we must assume that the middlemen's prices are continuously indexed and thus are fixed in real, not nominal terms.

<sup>12</sup>The alternative of assuming that run-outs imply lost sales seems just as acceptable but would involve the complications of (a) households having to take the probability of run-outs into account in their decisions, and (b) the spillover effects on each middleman's sales of households leaving other middlemen who have just run out.

<sup>13</sup>Note that we must assume that an interior solution exists, with total costs no greater than  $(\alpha + \pi)g\mu - \beta g\mu + b$ , or else the solution will be:  $S = 0$ . Also, formula (21) assumes that, using the central limit theorem,

we may treat daily sales as being distributed normally with mean  $\mu$  and standard deviation  $\sigma$ . This formula is similar to the one originally derived by Edgeworth (1888), who treated  $k$  as fixed. It also appears in Laidler (1976).

<sup>14</sup>We assume that some set-up costs exist in the making of factor payments that discourage the firm from economizing on its money holdings by staggering the payment dates to its various factors. If such staggering did occur it would not materially affect our results as long as there were enough set-up costs that factor payments were not made continuously through time.

<sup>15</sup>This set of assumptions has been made by Fried (1973) in the context of a different model, in which the measured income elasticity of the demand for money was exactly equal to unity. The empirical evidence on this elasticity is summarized by Laidler (1976).

<sup>16</sup>See Patinkin (1965, 86-88), Miller and Orr (1966, 425-426).

<sup>17</sup>It can be seen by comparing indirect cost functions that the household will indeed choose  $h < \theta$  if  $\theta > \sqrt{\frac{2a}{(\delta-\gamma)}}$ , will choose  $h > \theta$  if  $\theta < \sqrt{\frac{2a}{(\delta+\gamma)}}$ , and will choose  $h=\theta$  otherwise; where  $\delta \equiv (\rho+\beta)c$ , and  $\gamma \equiv (\rho+\alpha+\pi)c$ .

<sup>18</sup>Grossman and Policano consider a somewhat more complicated model of household behavior in which there are two consumption goods; one with a purchase interval less than the payments interval and the other with a purchase interval greater than the payments interval.

<sup>19</sup>This social benefit function suppresses all of the notation referring to costs that are independent of  $\bar{M}_h$ .

<sup>20</sup>Actually the relative sizes of  $h$  and  $\theta$  depends upon the rate of inflation, as in fn. 17 above. If  $\theta$  is fixed, then  $h$  approaches  $\theta$  as  $\pi$  increases, and will equal  $\theta$  for large enough values of  $\pi$ . This is true regardless of whether, for small  $\pi$ ,  $h$  is greater or less than  $\theta$ . An increase in  $\pi$  cannot, however, change  $h > \theta$  to  $h < \theta$  or vice versa. Further complications would obviously also arise if  $\theta$  were made endogenous.

<sup>21</sup>Of course a negative money rate of interest would be implied, which would require sufficiently large physical storage costs on money for such an equilibrium to be attainable as a stationary state.

<sup>22</sup>The case of pure public goods is analyzed by Samuelson (1964).

<sup>23</sup>For an analysis of price adjustment and speculative behavior by middlemen in a similar context, see Howitt (1976).

<sup>24</sup>See Scitovsky (1943).

## APPENDIX

This appendix shows that formula (15) in the text is approximately valid if the probability of running out is small enough. Define

$\phi = \frac{P_s}{P_b} \cdot C$ . The average active balances of the typical middleman will be:

$$(A.1) \quad M_a = \frac{1}{N} \sum_{t=1}^N M(t)$$

where  $M(t)$  is the average active money holding over the period from purchase date  $t-1$  to purchase date  $t$ , which, if no run-out occurs, equals:

$$(A.2) \quad \begin{aligned} M(1) &= \phi x_1 / 2 \\ M(2) &= \phi(x_1 + x_2) / 2 \\ &\vdots \\ M(N) &= \phi(x_1 + x_2 + \dots + x_{N-1} + x_N) / 2 \end{aligned}$$

Therefore:

$$(A.3) \quad M_a = (\phi/N) \sum_{t=1}^N (N + \frac{1}{2} - t)x_t.$$

Define  $T$  as the event that no run-out occurs and the total value of sales (minus value added) over the "purchase period" equals  $T$ . Then the expected sale per purchase period conditional upon  $T$  is  $E(\phi x_t / T) = \frac{T}{N}$ . Thus

$$(A.4) \quad E(M_a | T) = \frac{T}{2}.$$



Extending the definition of event T to include events in which the middleman runs out, we get:

$$\begin{aligned}
 \text{(A.5)} \quad E(M_a) &= \sum_{T=0}^{NC} E(M_a | T) \cdot f(T) \\
 &= \sum_{T=0}^{NC} \left(\frac{T}{2}\right) \cdot f(T) + \sum_{T=S+1}^{NC} (E(M_a | T) - \frac{T}{2})f(T)
 \end{aligned}$$

If the probability of running out is small the last term will be small, so that (A.5) can be approximated by (15) in the text.

References

- Alchian, A. A., "Information Costs, Pricing, and Resource Unemployment," in Microeconomic Foundations of Employment and Inflation Theory, edited by E. S. Phelps, Norton, New York, 1970.
- Bailey, M. J., "The Welfare Cost of Inflationary Finance," Journal of Political Economy 64, April 1956, 93-110.
- Barro, R. J., "Integral Constraints and Aggregation in an Inventory Model of Money Demand," Journal of Finance 31, March 1976, 77-87.
- Barro, R. J. and Santomero, A. M., "Output and Employment in a Macro Model With Discrete Transaction Costs," Journal of Monetary Economics 2, July 1976, 297-310.
- Clower, R. W., "Is there an Optimal Money Supply?," Journal of Finance 25, May 1970, 425-433.
- Clower, R. W., "Reflections on the Keynesian Perplex," Zeitschrift für Nationalökonomie, July 1975, 1-24.
- Clower, R. W. and Leijonhufvud, A., "The Coordination of Economic Activities: A Keynesian Perspective," American Economic Review 65, May 1975, 182-188.
- Clower, R. W. and Howitt, P. W., "Money, Credit, and the Timing of Transactions," Discussion Paper No. 72, Department of Economics, UCLA, June 1976.
- Edgeworth, F. Y., "The Mathematical Theory of Banking," Journal of the Royal Statistical Society 51, 1888, 113-127.
- Feige, E. L. and Parkin, M., "The Optimal Quantity of Money, Bonds, Commodity Inventories and Capital," American Economic Review 61, June 1971, 335-349.
- Fried, J. S., "Money, Exchange and Growth," Western Economic Journal 11, September 1973, 285-301.
- Friedman, M., "The Optimum Quantity of Money," in The Optimum Quantity of Money and Other Essays by M. Friedman, Chicago: Aldine, 1969.

- Grossman, H. I. and Policano, A. J., "Money Balances, Commodity Inventories, and Inflationary Expectations," Journal of Political Economy 83, December 1975, 1093-1112.
- Hahn, F. H., "Foundations of Monetary Theory," Essays on Modern Economics, edited by M. Parkin, Longmans, London, 1973, 230-242.
- Howitt, P. W., "The Role of Speculation in Competitive Price Dynamics," Research Report 7612, Department of Economics, University of Western Ontario, May 1976.
- Howitt, P. W., "Intertemporal Utility Maximization and the Timing of Transactions," American Economic Review, forthcoming 1977.
- Kurz, M., "Equilibrium with Transaction Cost and Money in a Single Market Exchange Economy," Journal of Economic Theory 7, April 1974, 418-452.
- Laidler, D. E. W., "The Welfare Costs of Inflation in Neoclassical Theory--Some Unsettled Problems." Paper presented to the I.E.A. Conference on Inflation and Anti-Inflation Policy, Saltsjöbaden, Sweden, August 28 - September 3, 1975.
- Laidler, D. E. W., The Demand for Money: Theories and Evidence, 2nd Edition, Scranton: International Textbook, 1976.
- Marty, A. L., "Growth and the Welfare Cost of Inflationary Finance," Journal of Political Economy 75, February 1967, 71-76.
- Miller, M. M., and Orr, D., "A Model of the Demand for Money by Firms," Quarterly Journal of Economics 80 (August 1966), 413-435.
- Patinkin, D., Money, Interest, and Prices, 2nd edition, New York: Harper and Row, 1965.

Perlman, M., "The Roles of Money in an Economy and the Optimum Quantity of Money,"  
Economica 38, August 1971, 233-252.

Samuelson, P. A., "The Pure Theory of Public Expenditure," Review of Economics  
and Statistics, November 1964, 387-389.

Samuelson, P. A., "What Classical and Neo-Classical Monetary Theory Really Was,"  
Canadian Journal of Economics 1, February 1968, 1-15.

Scitofsky, T., "A Note on Profit Maximisation and its Implications," Review  
of Economic Studies 11, 1943, 57-60.

Ulph, A. M. and Ulph D.T., "Transaction Costs in General Equilibrium Theory -  
A Survey," Economica, November 1975, 355-372.

Weldon, J. C., "On Money as a Public Good." Paper presented to Meetings of  
the Canadian Economics Association, Kingston, Ontario, June 1973.