The Trader's Dilemma: Trading Strategies and Endogenous Pricing in an Illiquid Market

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Abstract

We investigate a large trader's trading strategies in a decentralized market, in which all traders are subject to type switching. The large trader has pressure to liquidate her position by the end of the horizon to avoid extra holding costs. She faces a trade-off: if she trades quickly, she moves the price too much; if she trades slowly, she may not be able to find counterparties in the market in later periods. We derive subgame perfect equilibria under three different spot market structures. The structures are chosen to show various degrees of competitive bargaining. We show that in each equilibrium the large trader chooses the optimal trading strategy to take into account both the price impact effect and liquidity uncertainty. Thus asset prices are generated endogenously through a dynamic bargaining and trading process and reflect the impact of the large trader's trades. Small traders, who possess little market power, cannot be ignored because their reactions to the large trader's trading strategy jointly determines market liquidity. We show that limiting competitive pricing occurs when there are enough small traders, or there are many trading periods. Illiquidity is generated by the thin market for buyers, and their limited capacity to buy the asset sold by the large trader.

1. Introduction

There has been increasing interest in the impact of large traders' trading strategies on asset prices. The fact that a large trader's actions can be significant enough in an asset market to move prices is a concern to large institutional investors. This price impact of trading has been verified by many empirical studies and exists in almost all kinds of markets.[1] Financial stress can occur when investors find themselves in desperate need to liquidate their long positions and market liquidity dries up. A recent well-known occurrence was the LTCM crisis in 1998. Studies of this crisis show that, in addition to poor risk management, it is suspected that LTCM became a victim of predatory trading. Studying the trading behaviour of market makers during the crisis, using a unique dataset of audit trail transactions, Cai (2003) infers that market makers exploited their informational advantage on customers' order-flows (LTCM needed to cover its short position in the treasury bond future market) and front run their customers' trades.

The 1998 turmoil would not have happened if the market had been perfectly liquid. This aspect of asset market illiquidity, which arises from imperfect competition [Basak (1997), Kihlstrom (2000), Pritsker (2004)], along with the other aspects of illiquidity driven by exogenous transaction costs, either deterministic [(Amihud and Mendelson (1986), Constantinides (1986), Vayanos (1998), Vayanos and Vila (1999), Huang (2003), Duffie, Garleanu and Pedersen (2004a, 2004b)], or stochastic [Acharya and Pedersen (2004)] and asymmetric information [Kyle (1985, 1989), Vayanos

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[1] For example, Holthausen, Leftwich and Mayers (1990) examine price effects associated with block trades by investigating the largest 50 trades for 109 firms traded on the NYSE in 1983 and find that most of the price effects are permanent and related to block size. They report a price impact of around 1 percent. Keim and Madhavan (1996) report an even larger price impact (8 percent) in an up-stairs market. Harris and Piwowar (2004) study transaction costs and trading volumes in the U.S. municipal bond market and find that municipal bond trades are substantially more expensive than similar sized equity trades due to the lack of price transparency.

(1999, 2001)], has long been studied in the market microstructure literature but could not completely explain the 1998 illiquidity.

Market liquidity anomalies have aroused a lot of interest, but the models have not given convincing explanations. For example, Longstaff (2001) defined an illiquid market as one in which traders were unable to initiate or unwind a position, and studied a trader's optimal portfolio selection problem. The illiquid market could be regarded as an exogenous trading constraint faced by market participants. However, his model doesn't provide an explanation how such an extreme situation comes into being, or why market participants retreated from trading under such circumstances.

Other issues arise from studying the performance of "large" traders, such as hedge and mutual funds. If large traders have superior analytical technological skills and information, how could they not consistently "beat the market"? Contrary to popular belief, analyses by Braas and Bralver (2003) on the trading profits of more than 40 large trading rooms throughout the world conclude that speculative positioning cannot be the major source of trading revenues. More often than not, this practice loses money rather than makes money. They imply that the profits are obtained through strategic trading.

We may then infer that a large trader, whose trades impact prices, can either benefit or suffer from her own market power, conditioning on market conditions. To better understand the impact of large traders' trading activities on market illiquidity, we need to introduce a large trader into a model with a "thin market" and examine how her behaviour reacts to market conditions.

In this paper, we study a large trader's trading strategy in a scenario of distressed sales and show how her trading strategy impacts on the intertemporal equilibrium price process. We adapt the basic structure of the model by Duffie, Garleanu and

Pedersen (2004a; henceforth DGP). Whereas they study limiting behaviour with large numbers of traders in a stationary stochastic environment, our model assumes limited number of traders and short-run strategic trading and the impact on the price process. We assume symmetric information so that there are no incentives to signal or act in a manner to exploit informational monopoly power. Because we wish to model "thin" markets, we model asset sales in each period by a decentralised bargaining process, where traders bargain over the transaction price. (Examples of such markets are over-the-counter markets for small stocks, or corporate bonds.) Traders have heterogeneous initial endowments (i.e., one large trader with two shares vs. two small traders who have a maximum capacity to own one share each) and differential bargaining power. They also differ by their intrinsic types (high-type versus low-type) in the sense that they have different asset valuations. In addition to different valuations on assets, a low-type asset owner, who places a low valuation on the asset she holds, also incurs a holding cost. We assume that their intrinsic types are subject to random change over time: this generates uncertainty over their future types, and in turn, induces randomness over the future number of small liquidity providers. In this sense there is uncertainty over future market liquidity. There are even scenarios where the large trader cannot find any one to trade with profitably in future periods. (This framework allows us to rationalise Longstaff's (2001) idea of the market "drying up".) We are able to study a dilemma often faced by large traders: trade fast and you move the market too much against you; wait to trade and the market moves around you. The large trader chooses trading strategies that trades-off these two effects.

Our formulation is sufficiently flexible that we can study traders' behaviour under different multilateral bargaining game structures. These bargaining games take place at each date and are contingent on the types and their asset holdings. In particular, we explore three bargaining games. The first bargaining game models the large trader to be in a privileged position in trading with the small traders who cannot communicate. The second game models a situation where all traders are equal in bargaining, but negotiation is a one-shot game at each trading date; and the third game assumes all traders can renegotiate repeatedly to mimic a semi-competitive situation. Using each of the component bargaining games in turn, we analyse the dynamic game to deduce the trading strategies and price process. As a general result we show that the large trader chooses optimal selling strategies, trading off the initial price impact of the large trader's monopolistic market power with the uncertainty of market liquidity in the future periods. The extent of price impact also varies with the constituent bargaining game structure. For example, in the first type of bargaining game, where the large trader faces the two small traders, who cannot trade between themselves, the large trader's first period trade incurs a large price impact. That is, she obtains a lower price if she sells two shares in one period as opposed to spreading the sale over two periods. But with the other bargaining games, where the small traders are less constrained in their bargaining, mimicking a more competitive outcome, the price impact becomes less evident. Her monopoly power weakens when the market becomes more transparent (in terms of the bargaining process) and competitive. She may have to choose to spread the trades over two periods, because the cost to induce small traders to buy in the first period is just too high. For this reason, the large trader may benefit from the improvement of market transparency to some extent that small traders have higher expected payoffs for better trading opportunities. Bargaining with small traders, the large trader gains more by giving up some immediate monopoly advantage, especially when her relative bargaining power to small traders is very high.

The introduction of a large trader distinguishes our model from the competitive search model of DGP (extended by Vayanos and Wang (2003) and Weill (2003)). They assume identical small traders and focus on steady-state equilibria, without considering the impact of time-varying liquidity risk. In contrast, we introduce a large trader into the model. We solve the model by characterizing the subgame perfect equilibrium of a dynamic bargaining game, finding the large trader's optimal trading strategy and associated prices. The model generates a number of different results. Firstly, the impact of a large trader's type-switching is different from a small trader's type-switching in that upon the large trader's type-shifting, a significant change occurs to the security's demand or supply. Secondly, a large trader is able to choose trading strategies to maximize the liquidation value, which in turn influences future market liquidity. Lastly, when choosing trading strategies the large trader takes into consideration both the price impact and liquidity uncertainty, which endogenizes illiquidity cost and price impact.

In reality, large traders sometimes hold dominant market power relative to their dealers or other traders and extract more value from the bargaining. Braas and Bralver's (2003) analyses on trading profits of large intermediations demonstrate that trading profits from market making and from customer business are a function of the relative power of the two trading parties. Green, Hollifield and Schurhoff (2004), estimating a structural bargaining model using transaction data of the U.S. municipal bond market, attribute their finding of decreasing profits on trade sizes to the dealers' relative market power.

Our results contribute to the market microstructure literature in several ways. For example, we show that even without asymmetric information [Kyle (1985)] or the need to share risk [Vayanos (1999, 2001)] large traders trade strategically when the market is illiquid. Moreover, our study shows that the price impact effect could be magnified by market illiquidity; whilst monopoly power has less of an effect in a more liquid market.

Our model and method also contribute to a recent literature trying to incorporate liquidity risk into asset pricing by endogenizing illiquidity cost into asset prices. For example, Pritsker (2004) studies a general equilibrium model in which the competitive fringe takes price as given, whereas large investors face prices as a function of their own orderflows. Illiquidity in this model stems from imperfect competition. He is able to derive a multi-factor asset pricing formula, capturing the imperfect risk sharing by temporary factors[2] in addition to the market risk factor. Acharya and Pedersen (2004), on the other hand, assume a stochastic illiquidity cost and develop a liquidity adjusted CAPM model. Since the stochastic transaction cost is exogenous, the net-of-transaction-cost returns should satisfy the CAPM in a frictionless economy. Using this insight they are able to derive asset prices in an overlapping generation model. They show that in the liquidity-adjusted CAPM, the expected return of an asset has a four-factor structure with a non-zero constant term representing the expected illiquidity cost. Vayanos (2004) complements Acharya and Pedersen (2004) by introducing a link between the liquidity and the volatility. Instead of a time-varying transaction cost assumed by Acharya and Pedersen, he assumes a constant transaction cost, but time-varying horizon, which depends on the volatility of market return. By modeling investors as fund managers subject to performance-based withdrawals, he shows that assets in equilibrium can be priced by a conditional two-factor CAPM adjusted for the transaction cost, with two factors being the market risk and the volatility.

We do not assume a deterministic or stochastic illiquidity cost. We model illiquidity as arising from both imperfect competition and liquidity uncertainty, so that trading and price impacts are endogenized in the process of bargaining and trading. In addition, the existence of a large trader and a few small traders, alters the bargaining situation from bilateral bargaining to multilateral bargaining. This introduces more complexity into the model by requiring us to analyse a large number of contingent trading strategies, but it does allow us to better examine the dynamics of trading strategies and how market competition influences prices.

Lastly, our model provides a theoretical base to Longstaff's (2001) interpretation of an illiquid market. Our results show that there is some probability that there may be no counterparty on the other side of the market, either because of the sudden co-switching of traders or because traders are not willing to trade due to the high uncertainty of liquidity. In either case, markets "disappear" temporarily.

Our work is also related to the literature on market manipulation. For example, Jarrow (1992) investigates market manipulation trading strategies by large traders when their trades move prices. He studies the conditions on the price process, under which large traders generate profits at no risk. Subramanian and Jarrow (2001) study the liquidity cost when a trader's trades have a price impact and there are execution lags in trading. The differences between their models and ours are as follows: first in their model the price process and price impact function are assumed, while in our model prices are produced endogenously and price impact exists as a result of imperfect competition and liquidity uncertainty. Second, they study the large trader's trading behaviour in a partial equilibrium model while we study the large trader's

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[2] These risk factors are temporary in that it is the deviations from Pareto optimal asset holdings by large investors that affect asset prices and these deviations will eventually disappear when the investors' risky asset holdings converge to the competitive levels as time goes to infinity.

trading behaviour in a dynamic game. Finally, there is no liquidity uncertainty in their model.

The rest of our paper is organized as follows. Section 2 describes the basic model. The security market resembles an over-the-counter market, in which traders contact potential trading counterparties and bargain over prices. Section 3 analyses the model and describes the optimal trading strategy for the large trader under different bargaining game structures. Section 4 explores the model in a situation where the low type non-owners must exit the market so that the large trader may not be able to find any trading counterparties on the market. This variation introduces an extreme situation of illiquidity. Section 5 discusses briefly the case of a monopolistic buyer: we show that we can apply our earlier results to obtain symmetric results for the case of a large buyer. Section 6 extends our model to n small traders and t trading periods: we show how an infinite number of small traders and trading periods affect market liquidity and pricing, making them more competitive and reducing the price impact. Conclusions and further implications are discussed in Section 7. Calculations and proofs can be found in Appendices.

2. The Basic Model

This is a three-date model. People trade at t_1 and t_2. No trade takes place at the last date t_3. Investors can either invest in a perfectly liquid risk-free money market account with a return of r or an illiquid security in an over-the-counter market, paying a dividend $\bar{D} > r$ at date t_3. The security can only be traded upon the encounter of two traders.

There are three traders in the market: one big trader (B) with an initial endowment of two shares of the illiquid security;[3] two small traders (S) with either one share of this security or M dollars, $\bar{D} < M < 2\bar{D}$, as initial endowment. Borrowing or short selling is not allowed. Investors are risk neutral. They are heterogeneous in their intrinsic types: high (h) or low (l). We assume that when a low-type investor owns a share of the illiquid asset, she incurs a cost of ε such that $\bar{D} - \varepsilon < r$; while a high-type owner does not incur this cost. This captures the incentive of liquidation for low-type owner. In addition, investors' intrinsic types are subject to changes. The switching rate from high-type to low-type is ρ_d per unit of time, and the opposite switching rate from low to high is ρ_u per unit of time. $\rho_{u/d} \in (0,1)$. The investor types and changes capture the effects of several situations. For instance, a) a liquidity shock, i.e., the need for cash; b) a risk management requirement, e.g., to meet the VaR restriction or hedging needs; c) low utilities for an asset, e.g., a low expectation of future dividend flow.

Therefore an investor's type is drawn from the set {*high-type owner, high-type non-owner, low-type owner, low-type non-owner*}, which is denoted as $I = \{ho, hn, lo, ln\}$.

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[3] Note that the number of shares held by an agent doesn't necessarily mean the exact number of shares. It can represent any number of shares. What the numbers try to capture here is that a large trader owns significantly more shares than a small trader.

When the intrinsic type of an investor switches from *high* to *low*, the investor's valuation of the asset becomes lower and he wants to liquidate the asset. Similarly when an investor's type switches from *low* to *high*, she may want to buy the illiquid asset and consume the dividend at the end. Therefore the asset transfers between hands with different expected payoffs, e.g., from a low-type owner to a high-type non-owner.

The market, however, is decentralized in the sense that buyers and sellers are separated. Although any two agents are free to trade the security whenever they meet, they have to contact a potential trading counterparty and bargain over the price. The timing of the model is as follows.

At the beginning of each date, each trader recognizes her type and endowment and decides whether to trade in this period or not. The agent who decides to trade contacts some other agents. Once two agents meet, they immediately reveal their types and enter into a bilateral pre-trade bargaining over the transaction price. Engaging in a bargaining, however, doesn't guarantee a deal. An agent, who may bargain with more than one trader, will trade at the most advantageous price. If the two parties of a bargaining reach an agreement, transaction occurs. If negotiation breaks, they have to wait till the next trading date to resume trading. From the time after transaction to next date they can trade again, their intrinsic types are subject to changes. By the next date, they learn their new types and trade if necessary. The timing is clearly demonstrated in Figure 1.

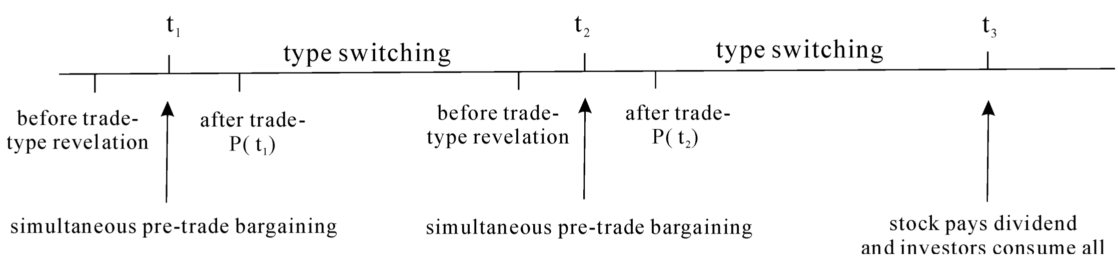
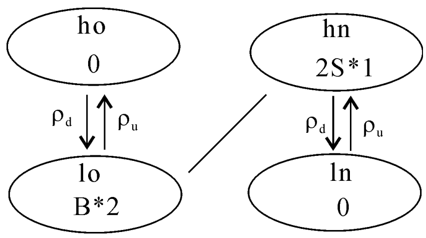


Figure 1. Timing of type switching and trade

Firstly, there are several things about investors that should be emphasized. The investors are heterogeneous in two ways, initial endowments and bargaining powers. We assume small traders' relative bargaining power is *q*, $0 \leq q < 1/2$ and the large trader's bargaining power is 1-*q*. Two small traders are identical and their relative bargaining powers to each other are equal to 1/2. The bargaining power partly captures the idea of the "market power", which translates the market power of an agent into the control of bargains. In this model, the "market power" is also reflected in how many shares a trader owns. We assume a trader can meet and bargain with more than one trader on the market at the same time. Since the large trader has more than one share, she can choose the trading strategy to her best interests by spreading the sales, for example in this case, selling one share in each period, or concentrating the sales, i.e., selling two shares in one period, t_1 or t_2.

Secondly, we need to specify the game structure in more detail. The game starts by nature choosing the types of three traders as if they are randomly drawn from the type set {*ho, hn, lo, ln*}. Any combination of the three traders constitutes a game. The following figure demonstrates an example of the mass configuration of a game at the beginning.



In the game above, the large trader, B, is a *lo*- (low-type owner) trader with two shares, i.e., B*2, and two small traders are both *hn*- (high-type non-owner) traders with endowments to buy one share each (2S*1). There is no trader in other two categories: *ho* (high-type owner) and *ln* (low-type non-owner). In this case, the big *lo* trader wants to sell and the small *hn* traders want to buy. This is the scenario we would like to study in this paper. Of course variations in any trader's type give rise to other combinations, some of which bring on trades, some do not and are thus trivial. To avoid repetitions, we will not study every single case in detail.

In order to study a particular case of depressed sale by the large trader, we assume that the large owner's type, once jumped down to low type, cannot switch back to high-type unless she unwinds her long position.[4] Small traders, however, are not subject to this restriction. In other words, if the large trader, being a low type owner at the beginning, sells one share or doesn't sell in the first period, she will still be a low type owner in the second period and cannot switch to high type. However, if she sells off two shares at t_1 and becomes a low type non-owner after trade, she is then freely subject to type switching and hence symmetric to small traders. With this assumption, the distressed large trader, who suffers a liquidity shock suddenly, cannot take her chances by doing nothing and hoping the situation will improve itself.

Note that this assumption also gives a small low type non-owner incentive to buy from a depressed large seller (*lo* type), but not from a small *lo*-trader. Let's take trades in the last period for example. The expected payoff at t_2 to a low type non-owner buying one share is greater than that to a low type owner who, if unable to unwind her position, has no chance to switch to high type in the last period (i.e., she incurs the holding cost ε at the last date with probability 1). Thus trading is beneficial to both traders. However, trades cannot take place between a low type non-owner and a low type owner, both of whom are subject to type switching in the last period and their expected payoffs of holding one share are the same. Therefore in an economy with all traders being identical, only low type owners and high type non-owners are

[4] This can be justified by situations such as a) a large trader facing margin calls from her broker; b) a fund manager facing sudden withdrawals from fund holders; and c) a risk manager facing the binding constraint, e.g., VaR constraint. In all these cases she has to liquidate at least part of her position to meet the cash need. She cannot wait for the situation to improve by itself.

involved in trading in equilibrium (as in DGP (2004)). In our setting with a depressed large trader, low type non-owners may also participate in trading as long as it is profitable.

We later assume that all low type non-owners must exit the market and cannot come back until their type switches back to high type. This assumption puts the large depressed trader in a harsher market that she may not be able to find anyone to trade with in the second period.

The large depressed trader then contacts two small traders, the potential buyers, for selling two shares. She then has three choices: doesn't sell; sell one or sell two shares in the first period. The pre-trade bargaining is modeled by the Nash bargaining solution. That is, in any pairwise negotiation the buyer and the seller split the joint surplus of trading according to their bargaining power. The price is given by

$$P(t) = q_{hn}\Delta V_l(t) + q_{lo}\Delta V_h(t) \tag{1}$$

where q_{lo} and q_{hn} are the bargaining powers of low-type owner and high-type non-owner respectively, and ΔV_l and ΔV_h refer to the reservation values of *lo* and *hn* traders respectively.

If in period t a seller and a buyer fail to reach an agreement, they remain in their own categories until period t+1, when they participate in the market again given that their types keep unchanged.

In order to study how market structure affects traders' trading strategies and asset prices, we will study the game with different structures. We first assume that small traders are "geographically separate" such that they are unable to contact each other. In other words, they can only be reached by the large trader. This can be thought of as an example of a monopolist in a very opaque market. We then remove this assumption so that the three traders are allowed to contact each other. However, their pre-trade negotiation is one-round. That is, a trader cannot re-open a negotiation with another trader if they fail to reach an agreement at a trading date. If two traders separate, they have to wait till the next trading date to contact and negotiate with each other all over again. Lastly, we allow traders to negotiate iteratively infinite times at each trading date. They can strategically delay and decline in a negotiation until an agreement is made. The bargaining solution depends more on a trader's market power in terms of the market supply and demand at that time.

Lastly, we assume the information is symmetric in that no one can hide her identity when she enters a negotiation. This distinguishes our model from many market microstructure models, such as Kyle (1985) and others, in which the information asymmetry is the major incentive for some market participants to trade strategically.

Next we give the definition for a subgame perfect equilibrium of the model and solve the model by finding the optimal trading strategy and associated prices.

3. Depressed Sale and Asset Prices

In this section we study the distressed large trader's trading behavior under three different game structures. We only study in detail the case that at t_1 the large trader is the only low-type owner with two shares to sell while the other two small traders are both high-type non-owners, who each would like to buy one share.[5] Figure 2 describes the dynamics of population structure, which evolves according to trades and type switching.

Before we define the subgame perfect equilibrium of this dynamic game, we would like to explain the structure of Figures 2 and 3 in more detail. In these two figures, a cluster of ovals represents a mass configuration at some specific time. Since the large trader is the only *lo* trader who has two shares, she can choose to trade one share, two shares or not to trade at all. This is shown in Figure 2 as three branches leading to three mass distributions after trading at t_1. Before the next trading date arrives, traders' types are subject to change. This is represented in the figure as dotted lines leading to possible mass distributions at t_2, following by associated probabilities. On arriving t_2 they find themselves in one cluster of ovals, which develops to a subgame numbered from (i) to (xiii). For instance, the mass configuration highlighted in the rectangle evolves to the subgame demonstrated in Figure 3. Even with two periods the structure of the game can become quite complicated.

At the beginning of each period the large trader chooses the optimal strategy to maximize her expected payoff at the last date. Since she has two shares to sell and she can sell only one share to a buyer upon one encounter, her major concern is "when" to sell and "how many shares" to sell at each period. Selling quickly, for example, two shares in the first period, she may not get a very good price, but then the pressure of liquidation is gone and the payoff is guaranteed. Employing the strategy of smoothing the sales across two periods may be conducive to a higher transaction price by exploiting the large trader's monopolistic power, but the uncertainty of not being able to trade in a later period increases (due to the type switching of *hn* traders). Thus the tradeoff faced by the large trader is between the price impact of trading and the possibility of market deterioration.

Next we define the equilibrium outcome to this game.

Definition: An outcome profile consists of a trading strategy profile and associated transaction prices $\left(\psi(t), P(t)\right)$. An *equilibrium outcome profile* $\left(\psi(t)^*, P(t)^*\right)$ is an outcome profile such that for a trader configuration at each time, given split-the-difference negotiations, the large trader cannot improve her expected payoff by adopting any other strategy profile $\left(\psi(t)', P(t)'\right)$, and no small trader can improve his expected payoff in a pairwise negotiation with the large trader.

Note that there are several differences between this model and the search models in DGP and in Vayanos and Wang (2003). Firstly, this model studies a dynamic process of the liquidation and associated asset prices while their models study a search market in the steady state; Secondly, agents in our model are heterogeneous not only in their intrinsic types but in their initial endowments, while in DGP model agents are identical except their types and in Vayanos and Wang (2003) agents are also heterogeneous in trading horizons, i.e., different preferences to liquidity; Thirdly, they assume a continuum of traders but we do not assume that because otherwise the large trader can always find counterparties and liquidate her position. We study in Section 6 extensions of the model to *n* small traders and *t* periods such that *n* and *t* are allowed to go to infinity. The results verify our claims here that the market is always

[5] We examine a similar case of the large trader being the only buyer (*hn*) at t_1 in a later section.

liquid in the sense that the large trader can liquidate her position at any speed or at any time she wants. This fact is also critical in that since a single trader's activity cannot be ignored the market is not perfectly competitive, and the price is therefore affected by traders' strategic activities.

Without the convenient properties of the steady state, we have to start by analyzing payoffs to different strategies in the last period and work backward to find an equilibrium path. We briefly summarize the approach as follows.

In this two-period game, traders have two opportunities to trade, at t_1 and t_2. Each trader seeks to maximize her value function at t_3. She solves the dynamic program problem

$$\max_{a_{t_1}, a_{t_2}} \quad EV_{t_3}$$

by choosing trading strategies a_{t_1}, a_{t_2} at each date. $a_{t_1}, a_{t_2} \in \{no\ trade, sell\ 1, sell\ 2\}$. In equilibrium, trading activity chosen by a trader must be the best response to the other traders' trading actions.

The second period: We first determine each trader's payoff at the last date, $X^i(t_3)$. Then for a subgame at t_2 we compute the large trader's value function, $V_{t_2}^B(\Gamma(\bullet, t_2))$

$$V_{t_2}^B(\Gamma(\bullet, t_2)) = \max_{a_{t_2}} E\left[X_{t_3}^B(\Gamma(\bullet, t_3) | a_{t_2}) \right]$$

by comparing her expected payoffs across different trading strategies, a_{t_2}, in the game $\Gamma(\bullet, t_2)$, where the dot describes the mass distribution of this game.

The first period (Figure 2): Back in the first period, we determine the large trader's optimal trading strategy by comparing her value functions of adopting different strategies. B's value function of taking an action a at t_1 in the game $\Gamma(\bullet, t_1)$ is given by

$$V_{t_1}^B\left(a_{t_1},\Gamma\left(\bullet,t_1\right)\right)=E_{\omega}\left[V_{t_2}^B\left(\Gamma\left(\bullet,t_2\right)\middle|a_{t_1}\right)\right]$$

$$=\frac{1}{r}\sum\omega\left(\Gamma\left(\bullet,t_2\right)\right)V_{t_2}^B\left(\Gamma\left(\bullet,t_2\right)\right)$$

which is her expected utility over all possible outcomes of subgames resulted from the action a_{t_1}. $\omega\left(\Gamma\left(\bullet,t_2\right)\right)$ is the associated probability of subgame $\Gamma\left(\bullet,t_2\right)$. The optimal trading strategy of the large trader is the strategy which maximizes her expected utility at t_1. $\left(a_{t_1}^*,a_{t_2}^*\right)$ constitute the large trader's optimal trading strategy profile.

Next we solve the model under three different designs of market structures. We first study the game in which small traders are "geographically separate". We then allow small traders to contact each other. But none of them can re-open a negotiation over transaction price at a trading date once a pairwise negotiation is closed between any two. We last relax all the above restrictions so that traders are free to contact anyone and allowed for the opportunity to engage in renegotiations with any traders.

3.1 Geographically Separated Small Traders

3.1.1 Subgames in the Second Period

Subgames started at t_2 are numbered from (i) to (xiii) in Figure 2. To demonstrate how a trader makes a trading decision, it is useful to first compute the expected payoffs to owning one share for traders of all four types. For a high type trader, a large trader or a small trader, the expected payoff to owning one share at t_2 is

$$X^h\left(t_2\right)=\frac{\bar{D}-\varepsilon}{r}\left(\rho_d\Delta t\right)+\frac{\bar{D}}{r}\left(1-\rho_d\Delta t\right)=\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r} \qquad (2)$$

However for low type traders, a large low-type trader's expected payoff is different from that of a small low-type trader.

$$X^{S_{lo}}(t_2) = X^{S_{ln}}(t_2) = \frac{\bar{D}-\varepsilon}{r}(1-\rho_u\Delta t) + \frac{\bar{D}}{r}(\rho_u\Delta t) = \frac{\bar{D}-(1-\rho_u\Delta t)\varepsilon}{r} \tag{3}$$

$$X^{B_{lo}}(t_2) = \frac{\bar{D}-\varepsilon}{r} \tag{4}$$

We assume $1-\rho_u\Delta t-\rho_d\Delta t>0$ to insure that a high type trader is willing to buy from a low type trader (i.e., $X^h(t_2)>X^l(t_2)$). Also note that $X^{S_{ln}}$ is greater than $X^{B_{lo}}$, because the large low-type owner cannot switch up to high type in the second period but a small low-type trader can. Thus trades may take place between a S_{ln} and the B_{lo}, but not between two small traders.

Thus the large trader can always sell to small non-owners as long as she can find some one. But the prices she sells to a small high type non-owner and a small low type non-owner are different. The bargaining price $P_{t_2}^{B_{lo}-S_{hn}}$ between B_{lo} and S_{hn} is determined by

$$q\left[P_{t_2}^{B_{lo}-S_{hn}} - X^{B_{lo}}(t_2)\right] = (1-q)\left[X^{S_{hn}}(t_2) - P_{t_2}^{B_{lo}-S_{hn}}\right] \tag{5}$$

$$\Rightarrow \quad P_{t_2}^{B_{lo}-S_{hn}} = q\left[\frac{1}{r}(\bar{D}-\varepsilon)\right] + (1-q)\left[\frac{1}{r}(\bar{D}-\rho_d\Delta t\varepsilon)\right] \tag{6}$$

Similarly, the bargaining price between B_{lo} and S_{ln} is given by

$$P_{t_2}^{B_{lo}-S_{ln}} = q\left[\frac{1}{r}(\bar{D}-\varepsilon)\right] + (1-q)\left[\frac{1}{r}(\bar{D}-(1-\rho_u\Delta t)\varepsilon)\right] \tag{7}$$

Thus in subgame (i), the large seller simultaneously contacts both small high type non-owners and sells one share to each trader at $P_{t_2}^{B_{lo}-S_{hn}}$. In subgame (ii), B sells one share each to S_{hn} and S_{ln} at $P_{t_2}^{B_{lo}-S_{hn}}$ and $P_{t_2}^{B_{lo}-S_{ln}}$ respectively. In subgame (iii), where both of the small high type non-owners switch to low type between t_1 and t_2, B sells two shares to two S_{ln}'s at $P_{t_2}^{B_{lo}-S_{ln}}$. Therefore, the value function of the large trader adopting the "no trade" strategy in the first period, is her expected payoff to this strategy at t_1.

$$V_{t_1}^{B_{lo}}\left(no\ trade, \Gamma\left(B_{lo}, 2S_{hn}, t_1\right)\right) = \frac{1}{r}\sum \omega\left(\Gamma\left(\bullet, t_2\right)\middle| no\ trade\right) V_{t_2}^{B}\left(\Gamma\left(\bullet, t_2\right)\middle| no\ trade\right)$$

$$= \frac{1}{r}\Bigg[\left(1-\rho_d\Delta t\right)^2 \frac{2\bar{D}-2\varepsilon\left[q+\left(1-q\right)\rho_d\Delta t\right]}{r}$$
$$+2\rho_d\Delta t\left(1-\rho_d\Delta t\right)\frac{2\bar{D}-\varepsilon\left[2q+\left(1-q\right)\left(1-\rho_u\Delta t+\rho_d\Delta t\right)\right]}{r}$$
$$+\left(\rho_d\Delta t\right)^2 \frac{2\bar{D}-2\varepsilon\left[q+\left(1-q\right)\left(1-\rho_u\Delta t\right)\right]}{r}\Bigg]$$

$$= \frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\left[q+2\left(1-q\right)\rho_d\Delta t-\left(1-q\right)\left(\rho_d\Delta t\right)^2-\left(1-q\right)\rho_d\rho_u\Delta t^2\right] \tag{8}$$

Sugames (iv) to (vii) describe all possible trader configurations at t_2 if the large trader sells one share at t_1. If there is no type switching between two trading dates, i.e., subgame (iv), B contacts S_{hn} and sells one share left at $P_{t_2}^{B_{lo}-S_{hn}}$. Thus the large trader's value function for this subgame is her proceeds from selling two shares at two dates, i.e.,

$$V_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo}, S_{hn}, S_{ho}, t_2\right)\right) = rP_{t_1}^{B_{lo}-S_{hn}}\left(1\right)+P_{t_2}^{B_{lo}-S_{hn}} \tag{9}$$

If the large trader finds herself in subgame (v) or (vii), where she only finds one small low-type owner, she sells the share to S_{ln} at $P_{t_2}^{B_{lo}-S_{ln}}$. Her value functions are the same for both subgame (v) and (vii).

$$V_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo}, S_{ln}, S_{ho}, t_2\right)\right) = V_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo}, S_{ln}, S_{lo}, t_2\right)\right) = rP_{t_1}^{B_{lo}-S_{hn}}\left(1\right)+P_{t_2}^{B_{lo}-S_{ln}} \tag{10}$$

Subgame (vi) is the game that different game structures come into effect. There is only one buyer, S_{hn}, in this subgame, facing two heterogeneous low type owners, S_{lo} and B_{lo}. Since the expected payoffs to holding one share at t_2 are different for S_{lo} and B_{lo}, the bilateral bargaining prices between S_{hn} and B_{lo} and between S_{hn} and S_{lo} must be different as well. If S_{hn} can contact both S_{lo} and B_{lo}, from whom she would buy depends on which price is more advantageous to her. The assumption that two small traders are "geographically separate", however, simplifies the analysis here by eliminating the possible trade between S_{hn} and S_{lo}. Thus under this game structure, S_{hn} buys from the large trader B_{lo} at the bilateral bargaining price between them, i.e., $P_{t_2}^{B_{lo}-S_{hn}}$.

We then can compute the large trader's value function of selling one share at t_1.

$$V_{t_1}^{B_{lo}}\left(sell\,1,\Gamma\left(B_{lo},S_{hn},S_{ho},t_1\right)\right)=\frac{1}{r}\sum\omega\left(\Gamma\left(\bullet,t_2\right)\middle|sell\,1\right)V_{t_2}^{B}\left(\Gamma\left(\bullet,t_2\right)\middle|sell\,1\right)$$

$$=P_{t_1}^{B_{lo}-S_{hn}}\left(1\right)+\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\left[q+2\left(1-q\right)\rho_d\Delta t-\left(1-q\right)\left(\rho_d\Delta t\right)^2-\left(1-q\right)\rho_d\rho_u\Delta t^2\right] \quad (11)$$

If the large trader sells two shares in the first period, she becomes a low-type non-owner after trade and is then subject to type switching. Trades will occur if the large trader switches up to high type and at the same time, one or both high type small traders switch down to low type, which are subgames (xii) and (xiii). The large trader then becomes the large buyer in both cases, which are symmetric to subgame (i) and (ii). She will buy back as many share as possible in these two subgames at the bilateral bargaining price $P_{t_2}^{B_{hn}-S_{lo}}$.

$$P_{t_2}^{B_{hn}-S_{lo}}=\left(1-q\right)\left[\frac{1}{r}\left(\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon\right)\right]+q\left[\frac{1}{r}\left(\bar{D}-\rho_d\Delta t\varepsilon\right)\right] \quad (12)$$

Her value functions for subgame (xii) and (xiii) are

$$V_{t_2}^{B_{hn}}\left(\Gamma\left(B_{hn},S_{ho},S_{lo},t_2\right)\right)=\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}+2rP_{t_1}^{B_{lo}-S_{hn}}\left(2\right)-P_{t_2}^{B_{hn}-S_{lo}} \quad (13)$$

$$=2rP_{t_1}^{B_{lo}-S_{hn}}\left(2\right)+\frac{\varepsilon}{r}\left(1-q\right)\left(1-\rho_u\Delta t-\rho_d\Delta t\right)$$

$$V_{t_2}^{B_{hn}}\left(\Gamma\left(B_{hn},2S_{lo},t_2\right)\right)=2\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}+2rP_{t_1}^{B_{lo}-S_{hn}}\left(2\right)-2P_{t_2}^{B_{hn}-S_{lo}} \quad (14)$$

$$= 2rP_{t_1}^{B_{lo}-S_{hn}}\left(2\right)+2\frac{\varepsilon}{r}\left(1-q\right)\left(1-\rho_u\Delta t-\rho_d\Delta t\right)$$

There is no trade in other subgames, in which the large trader's value functions are the same and equal to $2rP_{t_1}^{B_{lo}-S_{hn}}\left(2\right)$, the proceeds of selling two shares at t_1.

The large trader's value function of selling two shares at t_1 is

$$V_{t_1}^{B_{ln}}\left(sell\ 2,\Gamma\left(B_{ln},2S_{ho},t_1\right)\right)=\frac{1}{r}\sum\omega\left(\Gamma\left(\bullet,t_2\right)\middle|sell\ 2\right)V_{t_2}^{B}\left(\Gamma\left(\bullet,t_2\right)\middle|sell\ 2\right)$$

$$= 2P_{t_1}^{B_{lo}-S_{hn}}\left(2\right)+\frac{2\varepsilon}{r^2}\left(1-q\right)\rho_d\rho_u\Delta t^2\left(1-\rho_u\Delta t-\rho_d\Delta t\right) \qquad (15)$$

3.1.2 Subgame in the First Period

To determine what the large trader's optimal trading strategy is in the first period, we compare the large trader's value functions for all strategies. But before the comparison, we need to determine the transaction price at t_1.

At t_1, the large seller may contact and negotiate with one or two small buyers simultaneously. How does she choose one trading strategy over another?

Consider the outcome if the large seller sells only one share. The Nash bargaining outcome requires that the large seller and a small buyer split the joint surplus of trading according to their bargaining power, thereby satisfying

$$q\left[V_{t_1}^{B_{lo}}\left(sell\,1,\Gamma\left(t_1\right)\right)-V_{t_1}^{B_{lo}}\left(no\ trade,\Gamma\left(t_1\right)\right)\right]=\left(1-q\right)\left[V_{t_1}^{S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\,1\right)-P_{t_1}^{B_{lo}-S_{hn}}\left(1\right)\right] \qquad (16)$$

where $V_{t_1}^{S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\,1\right)$ is the expected payoff of the small trader who buys one share given the large trader sells one share at t_1.

$$V_{t_1}^{S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\,1\right)=\frac{1}{r}\left[\left(1-\rho_d\Delta t\right)\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}+\rho_d\Delta t\frac{\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon}{r}\right]$$

$$=\frac{1}{r^2}\left[\bar{D}-\left(2-\rho_d\Delta t-\rho_u\Delta t\right)\rho_d\Delta t\varepsilon\right] \qquad (17)$$

Substituting $V_{t_1}^{B_{lo}}\left(sell\ 1, \Gamma\left(t_1\right)\right)$, $V_{t_1}^{B_{lo}}\left(no\ trade, \Gamma\left(t_1\right)\right)$ and $V_{t_1}^{S_{ho}}\left(1, \Gamma\left(t_1\right) \middle| sell\ 1\right)$ into (17), we derive $P_{t_1}^{B_{lo}-S_{hn}}\left(1\right)$ as

$$P_{t_1}^{B_{lo}-S_{hn}}\left(1\right) = \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\left[q^2 + 2\left(1-q^2\right)\rho_d \Delta t - \left(1-q^2\right)\rho_d^2 \Delta t^2 - \left(1-q^2\right)\rho_d \rho_u \Delta t^2\right] \quad (18)$$

The price is equal to the present value of the dividend paid at t_3 minus a discount, which is a function of the relative bargaining power and the rate of type switching.

Substituting the price back into $V_{t_1}^{B_{lo}}\left(sell\ 1, \Gamma\left(t_1\right)\right)$, we have the value function for the large trader selling one share

$$\begin{aligned} V_{t_1}^{B_{lo}}\left(sell\ 1, \Gamma\left(t_1\right)\right) = \frac{2\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\Big[& q + q^2 + 2\left(1-q\right)\left(2+q\right)\rho_d \Delta t \\ & -\left(1-q\right)\left(2+q\right)\left(\rho_d \Delta t\right)^2 - \left(1-q\right)\left(2+q\right)\rho_d \rho_u \Delta t^2 \Big] \end{aligned} \quad (19)$$

Similarly, if selling two shares is the optimal strategy in the first period, then the following equation should be satisfied.

$$q\left[V_{t_1}^{B_{ln}}\left(sell\ 2, \Gamma\left(t_1\right)\right) - V_{t_1}^{B_{lo}}\left(sell\ 1, \Gamma\left(t_1\right)\right)\right] = \left(1-q\right)\left[V_{t_1}^{S_{ho}}\left(1, \Gamma\left(t_1\right) \middle| sell\ 2\right) - P_{t_1}^{B_{lo}-S_{hn}}\left(2\right)\right] \quad (20)$$

This equation states that the large trader keeps selling until the portion of the profit from selling the second share given up to the second buyer is equal to the portion of the gain that can be claimed from this buyer. The second buyer's expected payoff to buying one share at t_1 is

$$\begin{aligned} V_{t_1}^{S_{ho}}\left(1, \Gamma\left(t_1\right) \middle| sell\ 2\right) = \frac{1}{r}\Bigg\{ & \left(1-\rho_d \Delta t\right)\frac{\bar{D} - \rho_d \Delta t \varepsilon}{r} + \rho_d \Delta t\left(1-\rho_u \Delta t\right)\frac{\bar{D} - \left(1-\rho_u \Delta t\right)\varepsilon}{r} \\ & + \rho_d \Delta t \rho_u \Delta t \frac{\bar{D} - \left[\left(1-q\right)\left(1-\rho_u \Delta t\right) + q\rho_d \Delta t\right]\varepsilon}{r}\Bigg\} \end{aligned} \quad (21)$$

The price at which the large trader sells one share to each S_{hn} is then given by

$$P_{t_1}^{B_{lo}-S_{hn}}(2) = \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2(1+q)}\Big[q^2(1+q) + 2(1-q)(1+q)^2\rho_d\Delta t - (1-q)(1+q)^2\rho_d^2\Delta t^2$$
$$-(1-q)(1+q+2q^2+q^3)\rho_d\rho_u\Delta t^2 - q(1-q)\rho_d^2\rho_u\Delta t^3 - q(1-q)\rho_d\rho_u^2\Delta t^3\Big] \tag{22}$$

And the value function of selling two shares for the large trader is

$$V_{t_1}^{B_{ln}}\left(sell\ 2,\Gamma(t_1)\right) = \frac{2\bar{D}}{r^2} - \frac{2\varepsilon}{r^2}\Big[q^2 + 2(1-q^2)\rho_d\Delta t - (1-q^2)\rho_d^2\Delta t^2$$
$$-\frac{(1-q)\left[q(1+q)^2+2+q\right]}{1+q}\rho_d\rho_u\Delta t^2 + \frac{1-q}{1+q}\rho_d^2\rho_u\Delta t^3 + \frac{1-q}{1+q}\rho_d\rho_u^2\Delta t^3\Big] \tag{23}$$

Of particular interest is equation (20), which can be rewritten as the following by substituting $V_{t_1}^{B_{ln}}\left(sell\ 2,\Gamma(t_1)\right)$ and $V_{t_1}^{B_{lo}}\left(sell\ 1,\Gamma(t_1)\right)$ from (15) and (11).

$$q\left\{\left[P_{t_1}^{B_{lo}-S_{hn}}(2) - h(\bar{D},\varepsilon)\right] + \left[P_{t_1}^{B_{lo}-S_{hn}}(2) - P_{t_1}^{B_{lo}-S_{hn}}(1)\right]\right\} = (1-q)\left(\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r} - P(2,t_2)\right) \tag{24}$$

where,

$$h(\bar{D},\varepsilon) = \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\Big[q + 2(1-q)\rho_d\Delta t - (1-q)\rho_d^2\Delta t^2$$
$$+(1-q)\rho_d\rho_u\Delta t - 2(1-q)\rho_d^2\rho_u\Delta t^3 - 2(1-q)\rho_d\rho_u^2\Delta t^3\Big]$$

The LHS shows that by selling an additional share the large seller not only gains $P_{t_1}^{B_{lo}-S_{hn}}(2) - h(\bar{D},\varepsilon)$ at the margin, but also incurs the *price impact* captured by $P_{t_1}^{B_{lo}-S_{hn}}(2) - P_{t_1}^{B_{lo}-S_{hn}}(1)$. This secondary price effect is crucial in our analysis that when the large trader sells an additional share she has to take into consideration the effect on the price of her own trading. The marginal costs of selling one share and selling two shares are different due to an additional uncertainty of being unable to trade with a high-type non-owner in a later period.

The large trader chooses the optimal strategy by comparing the payoffs of the three trading strategies. Simple algebra gives us the following proposition describing her optimal strategy.

Proposition 1 (geographically separate small traders).

(i) *When* $0 \le q < 1/2$, $1-\rho_u\Delta t-\rho_d\Delta t>0$ *and* $\rho_{u/d}\in(0,1)$, *there exists a unique subgame perfect equilibrium in this game, in which the large trader chooses to sell two shares in the first period.*

(ii) *The large trader's trading incurs price impact, i.e., she receives different prices from selling one share and selling two shares. More specifically,* $P_{t_1}^{B_{lo}-S_{hn}}(1)$ > $P_{t_1}^{B_{lo}-S_{hn}}(2)$ *when* $\frac{1-\rho_d\Delta t-\rho_u\Delta t}{1+q}-q>0$.

3.2 One Round Multilateral Negotiation

In this sub-section, we relax the assumption in the previous sub-section that small traders are unable to approach each other. Instead we allow the three traders to freely contact each other. At each trading date, however, they have to make quick decisions since negotiations between any two traders are one-shot. That is, a trader cannot re-open a negotiation with another trader once she departs without reaching an agreement.

Analysis of the game with this structure follows the same logic as the "separate-small-trader" case. Only subgame (vi) at t_2 displays differences between these two structures. Thus we analyze subgame (vi) here and leave the rest of derivations to the appendix.

Subgame (vi) describes the situation in which one small buyer (*hn*-type) faces two sellers (*lo*-type), one small and one large, each of whom has one share to sell. The small buyer then contacts both sellers and enters into bargaining with both. The bilateral bargaining price between S_{hn} and S_{lo} is

$$P_{t_2}^{S_{hn}-S_{lo}} = \frac{1}{2}\left[\frac{1}{r}\left(\bar{D}-\left(1-\rho_u \Delta t\right)\varepsilon\right)\right]+\frac{1}{2}\left[\frac{1}{r}\left(\bar{D}-\rho_d \Delta t \varepsilon\right)\right] \tag{25}$$

and the bargaining price between S_{hn} and B_{lo} is

$$P_{t_2}^{S_{hn}-B_{lo}} = q\left[\frac{1}{r}\left(\bar{D}-\varepsilon\right)\right]+\left(1-q\right)\left[\frac{1}{r}\left(\bar{D}-\rho_d \Delta t \varepsilon\right)\right] \tag{26}$$

Since a trader can only enter into a bargain with another trader once at each trading date, she cannot strategically delay or decline to provoke competition between other traders. What the buyer, S_{hn}, does is to compare two prices and trade at the most advantageous one.

Therefore, the small buyer buys from the large seller (B_{lo}) if $P_{t_2}^{S_{hn}-B_{lo}} \le P_{t_2}^{S_{hn}-S_{lo}}$ and buys from the small seller (S_{lo}) otherwise.[6] Comparing two prices $P_{t_2}^{S_{hn}-B_{lo}}$ and $P_{t_2}^{S_{hn}-S_{lo}}$, we find that $P_{t_2}^{S_{hn}-B_{lo}} \le P_{t_2}^{S_{hn}-S_{lo}}$ if and only if $2q \ge \frac{1-\rho_d \Delta t-\rho_u \Delta t}{1-\rho_d \Delta t}$. The large seller's value function of this game is thus

$$V_{t_2}^{MN,B_{lo}}\left(\Gamma\left(B_{lo},S_{lo},S_{hn},t_2\right)\right)=\begin{cases} rP_{t_1}^{B_{lo}-S_{hn}}\left(1\right)+P_{t_2}^{B_{lo}-S_{hn}} & \text{when} \quad 2q \ge \dfrac{1-\rho_d \Delta t-\rho_u \Delta t}{1-\rho_d \Delta t} \\ rP_{t_1}^{B_{lo}-S_{hn}}\left(1\right)+\dfrac{\bar{D}-\varepsilon}{r} & \text{when} \quad 2q < \dfrac{1-\rho_d \Delta t-\rho_u \Delta t}{1-\rho_d \Delta t} \end{cases} \tag{27}$$

where $V_{t_2}^{MN,B_{lo}}$ denotes the value function of B_{lo} at t_2 under the game structure "one-round ***M***ultilateral ***N***egotiation".

Continuing backward induction, we derive the optimal outcome of the game as stated in the following proposition.

Proposition 2 (one round multilateral negotiation).

[6] We simply assume here when $P_{t_2}^{S_{hn}-B_{lo}} = P_{t_2}^{S_{hn}-S_{lo}}$, the small buyer buys from the large seller.

(i) *When* $2q \geq \frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}$, *the equilibrium outcome profile is the same as that with "geographically separate" small trades.*

(ii) *When* $2q < \frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}$, $q \in [0,1/2)$, $1-\rho_u\Delta t-\rho_d\Delta t>0$ *and* $\rho_{u/d} \in (0,1)$, *there exists a unique subgame perfect equilibrium in this game, in which the large trader chooses to sell one share or two shares in the first period depending on the relationship between q,* $\rho_d\Delta t$ *and* $\rho_u\Delta t$.

Therefore, the equilibrium outcome of the game is the same as that under the structure of "geographically separate small traders" when $2q \geq \frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}$. When $2q < \frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}$, the large trader's optimal strategy in the first period can either be "sell one share" or "sell two shares" in the first period, depending on the relationship between her relative bargaining power q and type switching probabilities $\rho_d\Delta t$ and $\rho_u\Delta t$. We provide examples to illustrate the sensitivity of the large trader's equilibrium strategy to parameters in Appendix C.

3.3 Iterative Limiting Case

We next consider a setting where three traders are free to contact each other and engage in an arbitrary number of pairwise negotiations prior to trade in which any trader can commit to how many shares to trade and re-open negotiations over his or her transaction price. Again, we only analyze subgames whose outcomes are affected by this game structure.

When traders are allowed to iteratively bargain with others, they can strategically delay or decline in a bargaining without worrying about the consequence of breakdown. Hence the outcome rests more with the role a trader plays on the market. Let's first look at the subgame (i) in which the large trader (*lo*) faces two small traders (both *hn's*)[7]. The large trader has three strategies to choose from in the first period. The "no trade" strategy is ruled out immediately because a) this is the last period; and b) she always gains from trading. Since she is the only seller, she can either exploit her monopoly power by committing to sell only one share and thus drive two small traders into competition and hence the price up to their expected payoffs of owning one share, i.e., $\left(\bar{D}-\rho_d \Delta t\varepsilon\right)/r$; or commit to sell two shares at the bilateral bargaining price $P_{t_2}^{B_{lo}-S_{hn}} = q\left[\frac{1}{r}\left(\bar{D}-\varepsilon\right)\right]+\left(1-q\right)\left[\frac{1}{r}\left(\bar{D}-\rho_d \Delta t\varepsilon\right)\right]$, given in equation (6). Her decision in this game thus depends on which strategy has a higher payoff.

$$V_{t_2}^{IN,B_{lo}}\left(sell\ 1,\Gamma\left(B_{lo},2S_{hn},t_2\right)\right)=\frac{\bar{D}-\varepsilon}{r}+\frac{\bar{D}-\rho_d \Delta t\varepsilon}{r}$$

$$V_{t_2}^{IN,B_{lo}}\left(sell\ 2,\Gamma\left(B_{lo},2S_{hn},t_2\right)\right)=2P_{t_2}^{B_{lo}-S_{hn}}=\frac{2\bar{D}-2\varepsilon\left[q+\left(1-q\right)\rho_d \Delta t\right]}{r}$$

Comparing two value functions, it is easy to see that she will sell two shares as long as $q<1/2$, which is satisfied by our assumption that the large trader's relative bargaining power is greater than the small traders'. Therefore, the large trader's value function for this subgame is $V_{t_2}^{IN,B_{lo}}\left(\Gamma\left(B_{lo},2S_{hn},t_2\right)\right)=\frac{2\bar{D}-2\varepsilon\left[q+\left(1-q\right)\rho_d \Delta t\right]}{r}$.

Another subgame we would like to look at is subgame (vi), in which the small high type non-owner is the only buyer and two sellers, B_{lo} and S_{lo}, differ in their expected payoffs of holding one share at t_2. If there is only one seller on the market, B_{lo} or S_{lo}, the bilateral bargaining price between S_{ho} and B_{lo} (or S_{lo}) is $P_{t_2}^{S_{hn}-B_{lo}}$ (or $P_{t_2}^{S_{hn}-S_{lo}}$) given

[7] Subgame (xiii) is the case where B_{lo} is the monopoly buyer. It is symmetric to subgame (i) and we do not analyze it in detail here.

by equation (26) (or 25) in the previous sub-section. Of course the monopoly buyer S_{ho} would like to buy at the lower price. But now she can do more than that. She keeps negotiating repeatedly with two sellers until the price is driven down to the level at which one seller has no gain from trading and drops out the competition. Who will drop out first?

We know already that the expected payoff of "no trade" to S_{lo}, i.e., $\frac{\bar{D}-(1-\rho_u \Delta t)\varepsilon}{r}$, is higher than that to B_{lo}, $\frac{\bar{D}-\varepsilon}{r}$, which are their "disagreement payoffs" in bargains with S_{ho}. In the competition with S_{lo}, B_{lo} would still benefit from trading at the price $\frac{\bar{D}-(1-\rho_u \Delta t)\varepsilon}{r}$, while S_{lo} is indifferent between selling and holding her share at this level. If the bilateral bargaining price $P_{t_2}^{S_{hn}-B_{lo}}$ is lower than $\left[\bar{D}-(1-\rho_u \Delta t)\varepsilon / r\right]$, the small buyer S_{ho} buys from the large seller at $P_{t_2}^{S_{hn}-B_{lo}}$ immediately, because the small seller S_{lo} will not compete with B_{lo}. However, if $P_{t_2}^{S_{hn}-B_{lo}}$ is greater than $\left[\bar{D}-(1-\rho_u \Delta t)\varepsilon / r\right]$, the competition between S_{lo} and B_{lo} will bring the price down to $\left[\bar{D}-(1-\rho_u \Delta t)\varepsilon / r\right]$. By asking a price a little bit lower than that, the large trader wins the competition (we assume that at $\left[\bar{D}-(1-\rho_u \Delta t)\varepsilon / r\right]$, S_{lo} would rather hold her share than sell it). Therefore, it is always the big seller, B_{lo}, who sells to S_{hn}, but at a different price such as

$$\left[\bar{D}-(1-\rho_u \Delta t)\varepsilon / r\right] \text{ when } P_{t_2}^{S_{hn}-B_{lo}} \geq \left[\bar{D}-(1-\rho_u \Delta t)\varepsilon / r\right] \Rightarrow q \leq \frac{1-\rho_d \Delta t-\rho_u \Delta t}{1-\rho_d \Delta t};$$

$$\text{or } P_{t_2}^{S_{hn}-B_{lo}} \text{ when } P_{t_2}^{S_{hn}-B_{lo}} < \left[\bar{D}-(1-\rho_u \Delta t)\varepsilon / r\right] \Rightarrow q > \frac{1-\rho_d \Delta t-\rho_u \Delta t}{1-\rho_d \Delta t}.$$

Therefore, the large trader's value function of this subgame becomes

$$V_{t_2}^{IN,B_{lo}}\left(\Gamma\left(B_{lo},S_{lo},S_{hn},t_2\right)\right)=\begin{cases} rP_{t_1}^{B_{lo}-S_{hn}}\left(1\right)+\dfrac{\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon}{r} & \text{when} \quad q\le\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \\ rP_{t_1}^{B_{lo}-S_{hn}}\left(1\right)+P_{t_2}^{B_{lo}-S_{hn}} & \text{when} \quad q>\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \end{cases} \tag{28}$$

where $V_{t_2}^{IN,B_{lo}}$ is B_{lo}'s value function at t_2 under the game structure "***I***terative ***N***egotiation". The small buyer S_{hn}'s expected payoff in this subgame then becomes

$$V_{t_2}^{IN,S_{hn}}\left(\Gamma\left(B_{lo},S_{lo},S_{hn},t_2\right)\right)=\begin{cases} \dfrac{\bar{D}-\rho_d\Delta t\varepsilon}{r}-\dfrac{\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon}{r} & \text{when } q\le\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \\ \dfrac{\bar{D}-\rho_d\Delta t\varepsilon}{r}-\dfrac{\bar{D}-\varepsilon\left[q+\left(1-q\right)\rho_d\Delta t\right]}{r} & \text{when } q>\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \end{cases} \tag{29}$$

Outcomes of other subgames at t_2 are the same as in the previous subsections. Having obtained the large traders' value functions for three strategies, we go back to the first period to decide her optimal strategy at t_1.

At the beginning of the first period, the large trader faces the same decision as in subgame (i) at t_2, in which two small buyers intend to buy one share each. If she commits to sell one share only, the competition between two small buyers will drive the price up to such a level that a small high type non-owner is indifferent between acquiring a share now and waiting till the next trading date. That is

$$V_{t_1}^{IN,S_{ho}}\left(1,\Gamma\left(t_1\right)\right)-P_{t_1}^{IN,B_{lo}-S_{hn}}\left(1\right)=V_{t_1}^{IN,S_{hn}}\left(0,\Gamma\left(t_1\right)\right) \tag{30}$$

where $V_{t_1}^{IN,S_{ho}}\left(1,\Gamma\left(t_1\right)\right)$ and $V_{t_1}^{IN,S_{hn}}\left(0,\Gamma\left(t_1\right)\right)$ are value functions of a small trader "buying one share" and "not buying" at the first period when B_{lo} offers to sell one share respectively.

If the large trader commits to sell two shares, she contacts both of the small traders and bargains with them. We need the following lemma to determine small traders' trading strategies and value functions. Small traders must either trade or not trade with the large trader at the same time. This is because if the large trader reaches an agreement with one small trader, this small trader must obtain a higher utility trading than not trading. Since both small traders are identical, the other small trader would be better off by re-opening a negotiation with the large trader and mimicking the first small trader. By symmetry, the situation where one trades and one does not, cannot arise in an equilibrium. Small traders, simultaneously contacted by the large trader, either both trade or none of them trade with the large trader at the same time.

Lemma 1: *When the large trader commits to sell two shares and both small traders are able to re-negotiate with the large trader over their transaction prices, small traders would either trade or not trade with the large trader at the same time.*

Note that small traders take the same action not because they collaborate but this is the equilibrium outcome that none of them want to deviate from. This implies that they bargain over the price until both small traders are indifferent between buying and waiting. The large trader's value function, conditional on both small traders trading, is $V_{t_1}^{IN,B_{ln}}\left(sell\ 2,\Gamma\left(t_1\right)\right)$, and her value function, conditional on neither small traders trading, is $V_{t_1}^{IN,B_{lo}}\left(no\ trade,\Gamma\left(t_1\right)\right)$. For a small trader, the gain from trading is $V_{t_1}^{IN,S_{ho}}\left(1,\Gamma\left(t_1\right)\right)-P_{t_1}^{IN,B_{lo}-S_{hn}}\left(2\right)$. Bargaining with both gives us the price

$$q\left[V_{t_1}^{IN,B_{ln}}\left(sell\ 2,\Gamma\left(t_1\right)\right)-V_{t_1}^{IN,B_{lo}}\left(no\ trade,\Gamma\left(t_1\right)\right)\right]=2\left(1-q\right)\left[V_{t_1}^{IN,S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\ 2\right)-P_{t_1}^{IN,B_{lo}-S_{hn}}\left(2\right)\right] \tag{31}$$

From (30) and (31) we get prices at which the large trader sells one share and sells two shares. Substituting prices back to the large trader's value functions, we obtain the large trader's value functions for three strategies.

The large trader chooses the optimal strategy at t_1 by comparing her expected value functions of three strategies.

Proposition 3 (iterative limiting case):

(i) *When* $q \le \frac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$, $q \in [0,1/2)$, $\rho_{u/d} \in (0,1)$ *and* $\rho_d \Delta t + \rho_u \Delta t < 1$, *there exists a unique subgame perfect equilibrium in this game, in which the large trader chooses to sell either one share or two shares in the first period depending on parameter values.*

(ii) *When* $q > \frac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$, $q \in [0,1/2)$, $\rho_{u/d} \in (0,1)$ *and* $\rho_d \Delta t + \rho_u \Delta t < 1$, *there exist a unique subgame perfect equilibrium in this game, in which the large trader dumps two shares in the first period.*

Similar to the games with “geographically separated small traders” and “one round negotiation”, “no trade” is never the optimal strategy in this game when all traders are able to iteratively bargain with each other. When $q \le \frac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$, the large trader’s trading strategies in the first period may be to sell one share or two shares, depending on valuations of model parameters *q*, ρ_d , ρ_u and Δt . When $q > \frac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$, the payoffs to strategies “sell one share” and “no trade” are the same. Thus both strategies are dominated by the strategy of “sell two shares”. Examples in Appendix C show how the large trader’s strategies are affected by parameters in this game.

3.4 Effects of Different Market Structures on Trading Decision and Prices

The three game structures discussed above represent different market structures. For example, the model with “geographically separated” small traders can be considered as an opaque over-the-counter market, where only the big “dealer” (B_{lo}) can locate other traders. In such a market, the large trader faces no competition from other traders and hence has an absolute control on when to contact small traders. Analysis shows that she always finds it optimal to trade quickly in the first period to fully exploit her monopoly power.

When the market becomes more transparent in the sense that all traders are able to contact and negotiate with each other, the large trader's status as a monopolist is challenged in some situations, e.g., subgame (vi) at t_2, in which her trading with a high type small buyer is no longer guaranteed. As a result, selling quickly in the first period is not always optimal in any circumstance. In contrast, she may either spread the sales over two periods or dump two shares in the first period, depending on whether the price impact effect or the liquidity uncertainty effect is dominant.

Lastly, the market becomes even more transparent and competitive when renegotiations become possible. This can be thought of as a setting in which both small traders participate in a batch auction, which takes place at t_1 and t_2. In these auctions, the distressed trader acts as a dealer trading for her own account and receives orders from small traders. With this structure, not only can the large trader behave strategically, but so do small traders. Again, the large trader may have to spread the sales over two periods to maintain her monopoly position.

The following table summarizes and compares the large trader's optimal strategies at t_1 under different market structures.

Game structures	$\frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}<q$	$q<\frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\le 2q$	$\frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}>2q$
Geographically separate small traders (GS)	Sell two shares	Sell two shares	Sell two shares
One-round multilateral negotiation (MN)	Sell two shares	Sell two shares	Sell one share or two shares
Iterative negotiation (IN)	Sell two shares	Sell one share or two shares	Sell one share or two shares

Table 1. The large trader's optimal strategies t_1 and game structures

According to Proposition 1-3 in the previous sections, the large trader's optimal strategy depends on the relationship between her relative bargaining power and the type switching probabilities. Let $q' = \frac{1-\rho_d\Delta t - \rho_u\Delta t}{1-\rho_d\Delta t}$. Given the level of liquidity uncertainty (i.e., the values of $\rho_d\Delta t$ and $\rho_u\Delta t$), when $q > q'$, the large trader's optimal strategies are the same under three different market structures, which is to sell two shares. When q decreases to a level such that $q < q' \leq 2q$ (or equivalently, $q'/2 \leq q < q'$), the large trader's optimal strategy is to sell two shares under the structure of "separate small traders" and the structure of "one round multilateral negotiation", but to sell one or two shares under the structure of "iterative negotiation". As q decreases further to such a level that $q' > 2q$ (or $q < q'/2$), "sell two shares" is the optimal strategy only under the structure of "separate small traders", while "sell one or two shares" becomes the optimal strategy under the structures "one round multilateral negotiation" and "iterative negotiation". Hence for the later two market structures, "one round multilateral negotiation" and "iterative negotiation", given the values of $\rho_d\Delta t$ and $\rho_u\Delta t$, the large trader tends to trade more slowly as her relative bargaining power, 1- q, increases. She also tends to trade more slowly as the market becomes more transparent, other things being equal, to mitigate the price impact of her trades, because such advantages as the privilege to trade in an opaque market diminish in a more transparent market.

However, the large trader's expected value is not monotonically decreasing in market transparency. For the same values of model parameters q, $\rho_d\Delta t$ and $\rho_u\Delta t$, the large trader may better off in a more transparent market. This is especially so when her relative bargaining power (1-q) is extremely high. For example, compare the large trader's value functions under different market structures for the same set of parameter values, e.g., $\rho_d = \rho_u = 0.2$, $\Delta t = 1$ and q = 0.1. The large trader has the highest expected value by selling two shares at t_1 in the market with "one round multilateral negotiation". However, when q is increased to 0.4, other parameters being equal, the large trader is better off in the most transparent market of the three, i.e., the market with "iterative negotiations". Increasing q further to 0.45, we find that the large trader prefers to trade in the market with "geographically separate small traders", the most opaque market among the three. On the one hand, improved trading opportunities in a more transparent market increases the small traders' expected payoffs, which in consequence increases the large trader's expected value directly from bilateral bargaining. This benefit may be offset by the loss of her monopoly payoff in a more transparent and competitive market. Thus the large trader may have some incentive to improve the market transparency under certain levels of liquidity uncertainty and bargaining power.

Note that no matter how low the price is, the large trader can always liquidate her position within two periods, because even in the worst scenario in which she can only find a small trader as low type non-owner, she still can unwind her position by selling to low type non-owners. We then ask the question how the large trader's trading strategy is affected if trading with low type non-owners is forbidden, for example, low type non-owners' fund are held for collateral or for the purpose of risk management. In the next section, we study the model with a further assumption that low type non-owners must exit the market and will not be able to trade until their type switches to the high type. We repeat the above analyses under the three game structures and study the large trader's trading behavior in a more stringent market.

4. Depressed Sales and Temporary Market Disappearance

In this section, we study the large trader's trading strategy and resulting price function when she may not be able to find trading counterparties in the second period. In contrast to the analyses in section 3, we assume that low type non-owners must exit the economy and cannot come back until their types switch to a high type. With this assumption, the depressed large seller (B_{lo}) cannot trade with a low type non-owner as in the previous section. Only low type owners and high type non-owners participate in trading. Hence there will be no trade in some subgames at t_2 if B_{lo} cannot find any high type non-owner, such as in subgames (iii), (v) and (vii).

We briefly analyze the subgames that are affected by this assumption. In subgame (ii), B_{lo} finds only one S_{hn} (the other small trader occurs a type switching and exits the market) so that she can only sell one share to this small trader at the bilateral bargaining price. In subgames (iii), (v), (vii), there is no high type non-owner in the market so that the large depressed trader, B_{lo}, cannot trade at all. However, if the large trader sells two shares in the first period, she is not affected by this assumption. Having liquidated the entire position, she becomes a low type non-owner and exits the market.

We state the outcomes of the game for different market structures in the following propositions.

Proposition 4 (geographically-separate-small-trades case with low type non-owners exiting the market):

(i) *There exists a unique subgame perfect equilibrium of this game. When* $0 \le q < 1/2$, $1-\rho_u \Delta t - \rho_d \Delta t > 0$ *and* $\rho_{u/d} \in (0,1)$, *the large trader sells two shares in the first period.*

(ii)*The large trader's trading incurs price impact, i.e.* $\tilde{P}_{t_1}^{B_{lo}-S_{hn}}(2) < \tilde{P}_{t_1}^{B_{lo}-S_{hn}}(1)$.

This result provides evidence that when a large trader faces a potential risk that she may not be able to trade later, she accelerates trading even though she has to trade at a more disadvantageous price than when she spreads the trades.

Proposition 5 (one-round-multilateral-negotiation case with low type non-owners exiting the market):

(i) *When* $2q \geq \frac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$, *the equilibrium outcome profile is the same as that with "geographically separated" small traders: that is, the large trader sells two shares in the first period.*

(ii) *When* $2q < \frac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$, $q \in [0, 1/2)$, $1-\rho_u \Delta t - \rho_d \Delta t > 0$ *and* $\rho_{u/d} \in (0,1)$, *there exists a unique subgame perfect equilibrium in this game, in which the large trader chooses to sell either one share or two shares in the first period depending on the relationship between q,* $\rho_d \Delta t$ *and* $\rho_u \Delta t$.

When small traders can contact each other, the large trader may lose the competition to a small trader in subgame (vi) because the small trader is willing to sell at a lower price. This possible outcome affects the large trader's trading decision in the first period such that she may choose to spread the sale over two periods or sell quickly in the first period, depending on the relationship between her relative bargaining power and type switching rates. Afraid of being unable to trade in the second period (such as subgames (iii), (v), (vii)), the large trader never leaves all the trades to the second period.

Proposition 6 (iterative-limiting case with low type non-owners exiting the market):

There exists a unique subgame perfect equilibrium of this game. When $0 \le q < 1/2, 1-\rho_u \Delta t - \rho_d \Delta t > 0$ *and* $\rho_{u/d} \in (0,1)$*, the large trader sells two shares in the first period.*

Lastly, the outcome of the iterative limiting case when low type non-owners must exit the market is different to that with low type non-owners remaining in the market. In equilibrium, the large trader responds to a harsher market by dumping her entire position quickly in the first period. Intuitively, when the market becomes more transparent, the large trader makes less profit because her monopoly power is weakened. Therefore, she tends to trade more lowly to avoid too much competition with small traders. This is the case when she can always trade in the second period, with low-type non-owners. However, when low type non-owners exit the market, the possibility that the large depressed trader may not be able to trade at all in the second period becomes a concern. Weighing her chances to trade in the second period, and the proportion of gain that has to be given to small traders for trading in the first period, the large trader finds it optimal to sell two shares in the first period in all circumstances.

In sum, when low type non-owners are unable to trade, the large trader loses some trading opportunities in the second period, which thereby decreases her expected payoff of trading in the second period and consequently accelerates the speed of liquidation.

5. A Symmetric Case: A Large Buyer vs. Two Small Sellers

It is natural to ask whether these arguments apply in a symmetric case, where a large buyer (*hn*-type) seeks to buy two shares in two periods and two small traders (*lo*-type) hold one share each and hence are eager to sell before t_3. Similar to the scenario of a distressed sale, the large buyer has to buy two shares at the end of the second period to avoid a penalty of δ per share, where $\delta > \bar{D}$. This assumption can be interpreted as a situation where, for example, a large trader faces margin calls and is forced to cover a short position; or a fund manager faces unexpected withdrawals and has to liquidate her short position. In those cases the large trader will have to trade aggressively to avoid the penalty. What happens to a large seller can happen to a large buyer: she may be forced to liquidate her "short" position rather than a "long" position.

The large buyer faces the same dilemma: buying aggressively, she may push up the price; waiting to buy at a better price, she could miss the last opportunity to purchase. On the other side of the market, small sellers are balancing between the payoff of selling a unit today and the expected payoff of keeping it until the next period. Given the same bargaining and trading procedure, we can find the subgame perfect equilibrium for the "large buyer" game which is symmetric to the "large seller" case. As in the above game, we expect that prices in the first period are functions of the relative bargaining power, type switching rates, ρ_u and ρ_d, the holding cost ε for the low-type owner and the penalty δ for the large high-type non-owner. The large trader's trading strategies should depend on the current market liquidity and the expected future market liquidity. Therefore, conditional on the current market condition, there should exist some condition under which the large trader would rather bargain harder now than wait and vice versa.

6. Extensions and Further Interpretations of the Model

When talking about a liquid (or illiquid) market, there is no unanimous definition or measurement. Among efforts to achieve a definition, Kyle (1985) provides a thorough characterization of "market liquidity", which is widely accepted by academics and practitioners. He describes market liquidity in three aspects: tightness, depth and resiliency.

In this paper we also try to provide some insights into the definition of market liquidity. "Market liquidity" has two levels of meaning in our model. It refers to current and future liquidity levels, i.e., the liquidity providers available in our model. To model these two aspects of market liquidity, we assume a limited number of small traders and type switching rates which introduces uncertainty to the future liquidity level. We have shown that such a market, from the large trader's perspective, is neither infinitely tight, i.e., she cannot turn over a position costlessly in two periods even if she can perfectly discriminate across small traders, nor deep enough to avoid a price impact.

This, however, is not the case if there are a large number of small traders in the market or many trading periods. To show this, we now extend our model to n small traders, each being either a high-type non-owner or a low-type non-owner with a probability of p_h and $1-p_h$. In any period, the probability of there being at least two high-type non-owners is $1-p_h\left(1-p_h\right)^{n-2}$, which is asymptotically equal to one for large n. This implies that the large trader is almost sure she can always find somebody on the market even when type switching probabilities are significantly greater than zero. Therefore the market is perfectly liquid in the sense that she can sell whatever number of shares whenever she wants.

Next we consider a market with limited number of small traders and a fixed horizon but t trading periods. Suppose that the large trader only knows that at time τ_1 the probability of a small trader being a high-type is p. Then after τ periods the probability of a small trader being a high-type is $p-\left[p\rho_d\Delta t-(1-p)\rho_u\Delta t\right]\left(\frac{1-(1-\rho_d\Delta t-\rho_u\Delta t)^\tau}{\rho_d\Delta t+\rho_u\Delta t}\right)$, which is $\frac{\rho_u}{\rho_u+\rho_d}$ in the limit when τ goes to infinity.[8] It is easy to see that when two switching rates are equivalent the probability that the large trader will find at least one small trader to trade with is approximately ½. The probability will be less than ½ when the downward switching rate ρ_d is greater than the upward switching rate ρ_u, and be greater than ½ when ρ_d is less than ρ_u. But whatever rate dominates, the probability of the availability of at least one high-type small trader is a constant in the limit and is significantly greater

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[8] We provide a brief derivation here. At time τ_1, the probability that a small trader being either a high-type is p, i.e., $p_h(\tau_1)=p$, $p_l(\tau_1)=1-p$.

After one period, $p_h(\tau_1+\Delta t)=p(1-\rho_d\Delta t)+(1-p)\rho_u\Delta t=p-\left[p\rho_d\Delta t-(1-p)\rho_u\Delta t\right]=p-\gamma$ and $p_l(\tau_1+\Delta t)=1-p+\gamma$, where $\gamma=p\rho_d\Delta t-(1-p)\rho_u\Delta t$.

After two periods,

$$\begin{aligned}p_h(\tau_1+2\Delta t)&=p_h(\tau_1+\Delta t)(1-\rho_d\Delta t)+p_l(\tau_1+\Delta t)\rho_u\Delta t\\&=p-\gamma(1+1-\rho_d\Delta t-\rho_u\Delta t)\\&=p-\gamma(1+x)\end{aligned}$$

where $x=1-\rho_d\Delta t-\rho_u\Delta t$. $p_l(\tau_1+2\Delta t)=1-p+\gamma(1+x)$.

Following the same method, we have $p_h(\tau_1+3\Delta t)=p-\gamma(1+x+x^2)$.

We can show by induction that after τ periods,

$$\begin{aligned}p_h(\tau_1+\tau\Delta t)&=p-\gamma\left(1+x+x^2+\cdots+x^{\tau-1}\right)\\&=p-\gamma\left(\frac{1-x^\tau}{1-x}\right)\end{aligned}$$

because $x\in(0,1)$. Therefore, $\lim_{\tau\to\infty}p_h(\tau_1+\tau\Delta t)=p-\frac{p\rho_d\Delta t-(1-p)\rho_u\Delta t}{\rho_d\Delta t+\rho_u\Delta t}=\frac{\rho_u}{\rho_d+\rho_u}$.

than zero. Therefore if the large trader is allowed to trade frequently enough, she can always liquidate her position without disturbing the price or worrying about illiquidity.

If the horizon is finite, then the probability that there is no high-type small trader is non-zero in some period. This could even last for several periods, which to the large trader would seem as if the market had disappeared.

The above two extensions show that limited traders and limited trading opportunities are both crucial to market illiquidity. Our model also provides a theoretical base for the definition of illiquidity in Longstaff (2001), in which a trader is unable to trade because the market has just disappeared. Our model shows that it is indeed possible for this effect to occur.

7. Conclusions and Future Research

The purpose of this simplified three-date model was to demonstrate the impact of trading strategies on prices under different spot market structures. We demonstrated that with three traders (one large trader and two small traders) the transaction price of the security is determined by the future dividend flow, the trader's type-switching rates and bargaining power.

By studying the large trader's trading strategy, we showed how asset prices are jointly affected by the market conditions for trading and by the large trader's own trading strategy. The risk neutrality of all market participants ensures that the liquidity effect is purely a consideration of future market liquidity. We would conjecture that drastic price changes during a large trader's depressed sale would dramatically increase the asset's volatility, which may also drive risk-averse small traders from the market. In addition, similar to the propagation mechanism of financial contagion described in Allen and Gale (2000), liquidity risk may be contagious across assets and markets through portfolio adjustment or other constraints. This suggests two further questions: will liquidity risk affect market risk? Second, is liquidity risk systemic and how should it be hedged?

Keeping these questions in mind, we could extend this simple multi-period model in several ways. (1) Extend the game to multiple large traders and study how the existence of other large traders would affect individual large trader's trading strategies. In particular it is interesting to explore the activities of other large traders when one of the large traders is in financial distress. Is it possible to obtain front-running as part of a strategic response to a large trader's distressed selling?[9] This extension is reported

[9] Predatory trading or front-run like behaviour are also studied in Brunnermeier and Pedersen (2004), Attari, Mello and Ruckes (2002) and Pritsker (2004).

in Liang (2005). (2) We could generalise the model by letting bargaining power be a function of the shares held, and study how this impacts the distressed large trader's trading strategy and asset prices. (3) We could add into the large trader's portfolio a derivative and study how hedging strategies change due to imperfect competition and liquidity risks on the underlying asset market.

Appendix

A: Games with *ln*-traders staying on the market

Proposition 2: (One round multilateral negotiation)

This game structure doesn't affect the large trader's value functions when she doesn't sell or sells two shares in the first period. That is

$$V_{t_1}^{MN,B_{lo}}\left(no\ trade,\Gamma\left(B_{lo},2S_{hn},t_1\right)\right)=V_{t_1}^{B_{lo}}\left(no\ trade,\Gamma\left(B_{lo},2S_{hn},t_1\right)\right)$$

$$V_{t_1}^{MN,B_{lo}}\left(sell\ 2,\Gamma\left(B_{ln},2S_{ho},t_1\right)\right)=V_{t_1}^{B_{lo}}\left(sell\ 2,\Gamma\left(B_{ln},2S_{ho},t_1\right)\right)$$

given by (8) and (15) respectively. The superscript *MN* in the value function indicates the game structure of "*M*ultilateral *N*egotiation".

The large trader's value function of trading one share in the first period is thus different from that under the assumption of "separate small traders".

$$V_{t_1}^{MN,B_{lo}}\left(sell\,1,\Gamma\left(B_{lo},S_{hn},S_{ho},t_1\right)\right)=\begin{cases}V_{t_1}^{B_{lo}}\left(sell\,1,\Gamma\left(B_{lo},S_{hn},S_{ho},t_1\right)\right) & \text{when}\quad 2q\geq\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\\ \sum\omega\left(\Gamma\left(\bullet,t_2\right)\middle|sell\,1\right)V_{t_2}^{MN,B}\left(\Gamma\left(\bullet,t_2\right)\middle|sell\,1\right) & \\ & \text{when}\quad 2q<\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\end{cases}$$

$$=\begin{cases}P_{t_1}^{MN,B_{lo}-S_{hn}}\left(1\right)+\dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[q+2\left(1-q\right)\rho_d\Delta t-\left(1-q\right)\rho_d^2\Delta t^2 & \\ \qquad -\left(1-q\right)\rho_d\rho_u\Delta t^2\Big] & \text{when}\quad 2q\geq\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\\ P_{t_1}^{MN,B_{lo}-S_{hn}}\left(1\right)+\dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[q+3\left(1-q\right)\rho_d\Delta t-3\left(1-q\right)\rho_d^2\Delta t^2 & \\ \qquad +\left(1-q\right)\rho_d^3\Delta t^3-\left(1-q\right)\rho_d\rho_u\Delta t^2\Big] & \text{when}\quad 2q<\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\end{cases}$$

(A.1)

In the first period, if the large trader determines to sell one share, then the following equation is satisfied.

$$q\left[V_{t_1}^{MN,B_{lo}}\left(sell\,1,\Gamma\left(t_1\right)\right)-V_{t_1}^{MN,B_{lo}}\left(no\ trade,\Gamma\left(t_1\right)\right)\right]=\left(1-q\right)\left[V_{t_1}^{MN,S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\,1\right)-P_{t_1}^{MN,B_{lo}-S_{hn}}\left(1\right)\right] \tag{A.2}$$

where

$$V_{t_1}^{MN,S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\,1\right)=\begin{cases}V_{t_1}^{S_{lo}}\left(1,\Gamma\left(t_1\right)\right) & \text{when}\quad 2q\geq\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\\[2ex] \dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\left[\dfrac{3}{2}\rho_d\Delta t-\rho_d^2\Delta t^2+\dfrac{1}{2}\rho_d^3\Delta t^3\right. & \\ \qquad\left.-\dfrac{1}{2}\rho_d\rho_u\Delta t^2-\dfrac{1}{2}\rho_d^2\rho_u\Delta t^3\right] & \text{when}\quad 2q<\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\end{cases} \tag{A.3}$$

This is because, when $2q\geq\frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}$, this small trader cannot sell in subgame (vi), in which her type switches to low type at t_2; but when $2q<\frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}$, she is able to sell one share in this subgame.

Substituting $V_{t_1}^{MN,B_{lo}}\left(sell\,1,\Gamma\left(t_1\right)\right)$, $V_{t_1}^{MN,B_{lo}}\left(no\ trade,\Gamma\left(t_1\right)\right)$ and $V_{t_1}^{MN,S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\,1\right)$ back to (A.2), we get the price as

$$P_{t_1}^{MN,B_{lo}-S_{hn}}\left(1\right)=\begin{cases}P_{t_1}^{B_{lo}-S_{hn}}\left(1\right) & \text{when}\quad 2q\geq\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\\[2ex] \dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\left[q^2+\left(1-q\right)\left(q+\dfrac{3}{2}\right)\rho_d\Delta t-\left(1-q\right)^2\rho_d^2\Delta t^2+\left(1-q\right)\left(\dfrac{1}{2}-q\right)\rho_d^3\Delta t^3\right. & \\ \left.-\left(1-q\right)\left(\dfrac{1}{2}+q\right)\rho_d\rho_u\Delta t^2-\dfrac{1}{2}\left(1-q\right)\rho_d^2\rho_u\Delta t^3\right] & \text{when}\quad 2q<\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\end{cases} \tag{A.4}$$

$$V_{t_1}^{MN,B_{lo}}\left(sell\,1,\Gamma\left(t_1\right)\right)=\begin{cases}V_{t_1}^{B_{lo}}\left(sell\,1,\Gamma\left(t_1\right)\right) & \text{when}\quad 2q\geq\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\\[2ex] \dfrac{2\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\left[q+q^2+\left(1-q\right)\left(q+\dfrac{9}{2}\right)\rho_d\Delta t+\left(q-4\right)\left(1-q\right)\rho_d^2\Delta t^2+\left(1-q\right)\left(\dfrac{3}{2}-q\right)\rho_d^3\Delta t^3\right. & \\ \left.-\left(1-q\right)\left(\dfrac{3}{2}+q\right)\rho_d\rho_u\Delta t^2-\dfrac{1}{2}\left(1-q\right)\rho_d^2\rho_u\Delta t^3\right] & \text{when}\quad 2q<\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\end{cases} \tag{A.5}$$

Similarly, when the large trader decides to sell two shares at t_1, then

$$q\left[V_{t_1}^{MN,B_{lo}}\left(sell\ 2,\Gamma\left(t_1\right)\right)-V_{t_1}^{MN,B_{lo}}\left(sell\ 1,\Gamma\left(t_1\right)\right)\right]=\left(1-q\right)\left[V_{t_1}^{MN,S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\ 2\right)-P_{t_1}^{MN,B_{lo}-S_{hn}}\left(2\right)\right] \tag{A.6}$$

where $V_{t_1}^{MN,S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\ 2\right)=V_{t_1}^{S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\ 2\right)$, given by (22).

$$P_{t_1}^{MN,B_{lo}-S_{hn}}\left(2\right)=\begin{cases} P_{t_1}^{B_{lo}-S_{hn}}\left(2\right) & \text{when} \quad 2q\geq\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \\ \dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{(1+q)r^2}\Bigg[q^2\left(1+q\right)+\left(1-q\right)\left(q^2+\dfrac{9}{2}q+2\right)\rho_d\Delta t+\left(1-q\right)\left(q^2-4q-1\right)\rho_d^2\Delta t^2 & \\ +q\left(1-q\right)\left(\dfrac{3}{2}-q\right)\rho_d^3\Delta t^3-\left(1-q\right)\left(q^2+\dfrac{1}{2}q+1\right)\rho_d\rho_u\Delta t^2 & \\ -\dfrac{3}{2}q\left(1-q\right)\rho_d^2\rho_u\Delta t^3-q\left(1-q\right)\rho_d\rho_u^2\Delta t^3\Bigg] & \text{when} \quad 2q<\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \end{cases} \tag{A.7}$$

$$V_{t_1}^{MN,B_{lo}}\left(sell\ 2,\Gamma\left(t_1\right)\right)=\begin{cases} V_{t_1}^{B_{lo}}\left(sell\ 2,\Gamma\left(t_1\right)\right) & \text{when} \quad 2q\geq\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \\ \dfrac{2\bar{D}}{r^2}-\dfrac{2\varepsilon}{(1+q)r^2}\Bigg[q^2\left(1+q\right)+\left(1-q\right)\left(q^2+\dfrac{9}{2}q+2\right)\rho_d\Delta t+\left(1-q\right)\left(q^2-4q-1\right)\rho_d^2\Delta t^2 & \\ +q\left(1-q\right)\left(\dfrac{3}{2}-q\right)\rho_d^3\Delta t^3-\left(1-q\right)\left(q^2+\dfrac{3}{2}q+2\right)\rho_d\rho_u\Delta t^2 & \\ -\dfrac{3}{2}q\left(1-q\right)\rho_d^2\rho_u\Delta t^2-q\left(1-q\right)\rho_d\rho_u^2\Delta t^3\Bigg] & \text{when} \quad 2q<\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \end{cases} \tag{A.8}$$

Comparing value functions of three choices at t_1, the large trader chooses her optimal strategy as a function of q, ρ_d , ρ_u and Δt . For example,

$$\begin{aligned} &V_{t_1}^{MN,B_{lo}}\left(no\ trade,\Gamma\left(t_1\right)\right)-V_{t_1}^{MN,B_{lo}}\left(sell\ 2,\Gamma\left(t_1\right)\right) \\ &=\frac{2\left(1-q\right)q}{\left(1+q\right)r^2}\varepsilon\Bigg[-\left(1+q\right)+\left(q+\frac{5}{2}\right)\rho_d\Delta t+\left(q-3\right)\rho_d^2\Delta t^2+\left(\frac{3}{2}-q\right)\rho_d^3\Delta t^3 \\ &-\left(q+\frac{1}{2}+\frac{1}{q}\right)\rho_d\rho_u\Delta t^2-\frac{3}{2}\rho_d^2\rho_u\Delta t^2-\rho_d\rho_u^2\Delta t^3\Bigg] \end{aligned}$$

Let $K\left(q,\rho_d,\rho_u,\Delta t\right)=-\left(1+q\right)+\left(q+\dfrac{5}{2}\right)\rho_d\Delta t+\left(q-3\right)\rho_d^2\Delta t^2+\left(\dfrac{3}{2}-q\right)\rho_d^3\Delta t^3$

$$\frac{\partial K}{\partial q} = -1 + \rho_d \Delta t + \left(\rho_d \Delta t\right)^2 + \left(\rho_d \Delta t\right)^3$$
$$= \left(1 - \rho_d \Delta t\right)\left[\left(\rho_d \Delta t\right)^2 - 1\right] \le 0$$

Since $q \in \left[0, 1/2\right)$, q=0 maximizes *K*, i.e., $K\left(q=0\right) = -1 + \frac{5}{2}\rho_d \Delta t - 3\rho_d^2 \Delta t^2 + \frac{3}{2}\rho_d^3 \Delta t^3$.

$\frac{\partial K\left(q=0\right)}{\partial\left(\rho_d \Delta t\right)} = \frac{5}{2} - 6\rho_d \Delta t + \frac{9}{2}\rho_d^2 \Delta t^2 > 0$ for $\rho_d \Delta t \in \left[0,1\right]$. Therefore, *K* is maximized at $q = 0$ and $\rho_d \Delta t = 1$ and hence $K \le \overline{K}\left(q = 0, \rho_d \Delta t = 1\right) = 0$. It is then easy to see that $V_{t_1}^{MN, B_{lo}}\left(no\ trade, \Gamma\left(t_1\right)\right) - V_{t_1}^{MN, B_{lo}}\left(sell\ 2, \Gamma\left(t_1\right)\right) < 0$.

Therefore, "no trade" is a strictly dominated strategy. The large trader would employ this strategy only when she has all the power in bargains with small traders, i.e., $q = 0$.

□

Iterative limiting case:

We first calculate the large traders' value functions for trading one share, trading two shares or no trade at t_1.

$$V_{t_1}^{IN, B_{lo}}\left(no\ trade, \Gamma\left(B_{lo}, 2S_{hn}, t_1\right)\right) = V_{t_1}^{B_{lo}}\left(no\ trade, \Gamma\left(B_{lo}, 2S_{hn}, t_1\right)\right)$$
$$= \frac{2\overline{D}}{r^2} - \frac{2\varepsilon}{r^2}\left[q + 2\left(1-q\right)\rho_d \Delta t - \left(1-q\right)\rho_d^2 \Delta t^2 - \left(1-q\right)\rho_d \rho_u \Delta t^2\right] \tag{A.9}$$

$$V_{t_1}^{IN, B_{lo}}\left(sell\ 2, \Gamma\left(B_{ln}, 2S_{ho}, t_1\right)\right) = V_{t_1}^{B_{ln}}\left(sell\ 2, \Gamma\left(B_{ln}, 2S_{ho}, t_1\right)\right)$$
$$= 2P_{t_1}^{IN, B_{lo} - S_{hn}}\left(2\right) + \frac{2\varepsilon}{r^2}\left(1-q\right)\left(\rho_d \rho_u \Delta t^2 - \rho_d^2 \rho_u \Delta t^3 - \rho_d \rho_u^2 \Delta t^3\right) \tag{A.10}$$

$$V_{t_1}^{IN,B_{lo}}\left(sell\ 1,\Gamma\left(t_1\right)\right)=\frac{1}{r}\sum\omega\left(\Gamma\left(\bullet,t_2\right)\middle|sell\ 1\right)V_{t_2}^{IN,B}\left(\Gamma\left(\bullet,t_2\right)\middle|sell\ 1\right)$$

$$=\begin{cases} P_{t_1}^{IN,B_{lo}-S_{hn}}\left(1\right)+\dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[q+2\left(1-q\right)\rho_d\Delta t-2\left(1-q\right)\rho_d^2\Delta t^2 \\ \quad +\left(1-q\right)\rho_d^3\Delta t^3-\rho_d\rho_u\Delta t^2+\rho_d^2\rho_u\Delta t^3\Big] \qquad \text{when} \quad q\le\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \\ P_{t_1}^{IN,B_{lo}-S_{hn}}\left(1\right)+\dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[q+2\left(1-q\right)\rho_d\Delta t-\left(1-q\right)\rho_d^2\Delta t^2 \\ \quad -\left(1-q\right)\rho_d\rho_u\Delta t^2\Big] \qquad \text{when} \quad q>\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \end{cases} \quad \text{(A.11)}$$

In the first period, the large trader will sell one share at a price such that a small trader is indifferent between acquiring a share now and waiting till the next trading date. That is

$$V_{t_1}^{IN,S_{ho}}\left(1,\Gamma\left(t_1\right)\right)-P_{t_1}^{IN,B_{lo}-S_{hn}}\left(1\right)=V_{t_1}^{IN,S_{hn}}\left(0,\Gamma\left(t_1\right)\right) \quad \text{(A.12)}$$

If a S_{hn} buys one share at t_1, her expected payoff would be

$$V_{t_1}^{IN,S_{ho}}\left(1,\Gamma\left(t_1\right)\right)=\frac{1}{r}\left[\left(1-\rho_d\Delta t\right)\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}+\rho_d\Delta t\frac{\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon}{r}\right] \quad \text{(A.13)}$$

If she doesn't buy, her expected payoff of being a high type non-owner at t_1 would be

$$\begin{aligned} V_{t_1}^{IN,S_{hn}}\left(0,\Gamma\left(t_1\right)\right)=\frac{1}{r}\Bigg[&\left(1-\rho_d\Delta t\right)^2\left(\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}-\frac{\bar{D}-\varepsilon\left[q+\left(1-q\right)\rho_d\Delta t\right]}{r}\right) \\ &+\rho_d\Delta t\left(\frac{\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon}{r}-\frac{\bar{D}-\varepsilon\left[q+\left(1-q\right)\left(1-\rho_u\Delta t\right)\right]}{r}\right) \\ &+\rho_d\Delta t\left(1-\rho_d\Delta t\right)\left(\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}-\begin{cases}\dfrac{\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon}{r} & \text{when } q\le\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \\ \dfrac{\bar{D}-\varepsilon\left[q+\left(1-q\right)\rho_d\Delta t\right]}{r} & \text{when } q>\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\end{cases}\right)\Bigg] \end{aligned}$$

(A.14)

According to (A.12), the price at which the large trader to sell one share is

$$P_{t_1}^{IN,B_{lo}-S_{hn}}(1)=\begin{cases}\dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[q+3(1-q)\rho_d\Delta t-3(1-q)\rho_d^2\Delta t^2+(1-q)\rho_d^3\Delta t^3 \\ \quad +(q-2)\rho_d\rho_u\Delta t^2+\rho_d^2\rho_u\Delta t^3\Big] & \text{when } q\le\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \\ \dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[q+2(1-q)\rho_d\Delta t-(1-q)\rho_d^2\Delta t^2 \\ \quad +(q-1)\rho_d\rho_u\Delta t^2\Big] & \text{when } q>\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\end{cases} \tag{A.15}$$

Thereby,

$$V_{t_1}^{IN,B_{lo}}\left(sell\ 1,\Gamma(t_1)\right)=\begin{cases}\dfrac{2\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[2q+5(1-q)\rho_d\Delta t-5(1-q)\rho_d^2\Delta t^2+2(1-q)\rho_d^3\Delta t^3 \\ \quad +(q-3)\rho_d\rho_u\Delta t^2+2\rho_d^2\rho_u\Delta t^3\Big] & \text{when } q\le\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \\ \dfrac{2\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[2q+4(1-q)\rho_d\Delta t-2(1-q)\rho_d^2\Delta t^2 \\ \quad +2(q-1)\rho_d\rho_u\Delta t^2\Big] & \text{when } q>\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}\end{cases} \tag{A.16}$$

If the large trader chooses to trade two shares in the first period, she contacts both small buyers simultaneously and bargains with each of them until both of them are indifferent between buying and not buying. We need the following lemma to determine the bargaining outcome.

Lemma 1: *When the large trader commits to sell two shares and both small traders are able to re-negotiate with the large trader over their transaction prices, small traders would either trade or not trade with the large trader at the same time.*

Proof: This is so because if the large trader only reaches an agreement with one small trader, this small trader must obtain a higher utility by trading than waiting. Since both small traders are identical, the other small trader would be better off by re-opening a negotiation with the large trader and mimicking the first small trader. By symmetry, such situation as, one trades while one does not, cannot arise in an equilibrium. Small traders, simultaneously contacted by the large trader, either both of them trade or none of them trade with the large trader at the same time.

□

Since the large trader will trade with either both small traders or none, the solution to this bargaining game is that they split the joint surplus (from three parties) such as in a bilateral bargaining situation with one seller and one buyer.

$$q\left[V_{t_1}^{IN,B_{ln}}\left(sell\ 2,\Gamma\left(t_1\right)\right)-V_{t_1}^{IN,B_{lo}}\left(no\ trade,\Gamma\left(t_1\right)\right)\right]=2\left(1-q\right)\left[V_{t_1}^{IN,S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\ 2\right)-P_{t_1}^{IN,B_{lo}-S_{hn}}\left(2\right)\right] \tag{A.17}$$

where $V_{t_1}^{IN,S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\ 2\right)=V_{t_1}^{S_{ho}}\left(1,\Gamma\left(t_1\right)\middle|sell\ 2\right)$, which is given by (21).

$$P_{t_1}^{IN,B_{lo}-S_{hn}}\left(2\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\left[q^2+2\left(1-q^2\right)\rho_d\Delta t-\left(1-q^2\right)\rho_d^2\Delta t^2-\left(1-q^2\right)\rho_d\rho_u\Delta t^2\right] \tag{A.18}$$

$$\begin{aligned}V_{t_1}^{IN,B_{ln}}\left(sell\ 2,\Gamma\left(t_1\right)\right)=\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\Big[&q^2+2\left(1-q^2\right)\rho_d\Delta t-\left(1-q^2\right)\rho_d^2\Delta t^2\\&-\left(1-q\right)\left(2+q\right)\rho_d\rho_u\Delta t^2+\left(1-q\right)\rho_d^2\rho_u\Delta t^3+\left(1-q\right)\rho_d\rho_u^2\Delta t^3\Big]\end{aligned} \tag{A.19}$$

We can easily show that when $q\in\left[0,1/2\right)$, $\rho_{u/d}\in\left(0,1\right)$ and $\rho_d\Delta t+\rho_u\Delta t<1$, the strategy of "no trade" is strictly dominated by the strategy of "sell two shares".

$$\begin{aligned}&V_{t_1}^{IN,B_{ln}}\left(sell\ 2,\Gamma\left(t_1\right)\right)-V_{t_1}^{IN,B_{ln}}\left(no\ trade,\Gamma\left(t_1\right)\right)\\&=\frac{2\left(1-q\right)\varepsilon}{r^2}\left[q\left(1-\rho_d\Delta t\right)^2+\rho_d\rho_u\Delta t^2\left(1+q-\rho_d\Delta t-\rho_u\Delta t\right)\right]>0\end{aligned}$$

Thus B_{lo} will never choose to sell two shares in this game.

Whether the large trader would choose to sell one share or two shares in the first period depends on the relationship between q, ρ_d and ρ_u. Examples in Appendix C show that either strategy can be optimal.

B. Games when *ln*-traders must exit the market

With the assumption that low type non-owners exit the market, the large trader loses some trading opportunities in the second period, which makes trading in the first period more desirable. Subgames at t_2 that will be affected by this assumption include subgame (ii), (iii), (v) and (vii).

In subgame (ii), B_{lo} can only trade with S_{hn} but not S_{ln}. B_{lo}'s value function for this subgame is

$$\tilde{V}_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo},S_{hn},S_{ln},t_2\right)\middle| no\ trade\right)=\frac{2\bar{D}-\varepsilon\left[1+q+\left(1-q\right)\rho_d\Delta t\right]}{r} \tag{B.1}$$

Even worse, in subgames (iii), (v) and (vii), B_{lo} cannot find anyone to trade with.

$$\tilde{V}_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo},2S_{ln},t_2\right)\middle| no\ trade\right)=\frac{2\bar{D}-2\varepsilon}{r} \tag{B.2}$$

$$\tilde{V}_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo},S_{ho},S_{ln},t_2\right)\middle| sell\ 1\right)=rP_{t_1}^{B_{lo}-S_{hn}}\left(1\right)+\frac{\bar{D}-\varepsilon}{r} \tag{B.3}$$

$$\tilde{V}_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo},S_{lo},S_{ln},t_2\right)\middle| sell\ 1\right)=rP_{t_1}^{B_{lo}-S_{hn}}\left(1\right)+\frac{\bar{D}-\varepsilon}{r} \tag{B.4}$$

These subgames are not affected by different game structures. Therefore, replacing traders' value functions of these subgames in the previous analyses with their value functions under this assumption, we get all the results.

C. Numerical Examples

The large trader's optimal strategy in the first period is sensitive to parameters q, ρ_d , ρ_u and Δt . We give examples in this appendix showing how the large trader's equilibrium strategy changes as parameters change under different specifications of game structures.

(i) One round multilateral negotiation

As shown in Proposition 2, when $2q \geq \frac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$, the large trader's equilibrium strategy in the first period is to sell two shares and the value functions are the same as those under the market structure "geographically separate small traders". On the contrary, when $2q < \frac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$, she may choose to sell either one share or two shares in the first period.

For example, let $\Delta t = 1$, $\rho_d = 0.3$, $\rho_u = 0.3$, $q = 0.3 > \frac{1-\rho_d \Delta t - \rho_u \Delta t}{2(1-\rho_d \Delta t)} \approx 0.2857$.

$$V_{t_1}^{MN,B_{lo}}\left(no\ trade, \Gamma\left(t_1\right)\right) = \frac{2\bar{D}}{r^2} - \frac{\varepsilon}{r^2}(1.188)$$

$$V_{t_1}^{MN,B_{lo}}\left(sell\ 1, \Gamma\left(t_1\right)\right) = \frac{2\bar{D}}{r^2} - \frac{\varepsilon}{r^2}(1.0662)$$

$$V_{t_1}^{MN,B_{lo}}\left(sell\ 2, \Gamma\left(t_1\right)\right) = \frac{2\bar{D}}{r^2} - \frac{\varepsilon}{r^2}(0.8943)$$

Obviously, $V_{t_1}^{MN,B_{lo}}\left(sell\ 2, \Gamma\left(t_1\right)\right) > V_{t_1}^{MN,B_{lo}}\left(sell\ 1, \Gamma\left(t_1\right)\right) > V_{t_1}^{MN,B_{lo}}\left(no\ trade, \Gamma\left(t_1\right)\right)$ and the large trader's optimal trading strategy in the first period is to sell two shares.

Let's again set $\Delta t = 1$, $\rho_d = 0.3$, $\rho_u = 0.3$, but $q = 0.2 < \frac{1-\rho_d \Delta t - \rho_u \Delta t}{2(1-\rho_d \Delta t)} \approx 0.2857$. $V_{t_1}^{MN,B_{lo}}\left(sell\ 2, \Gamma\left(t_1\right)\right)$ is still the largest among the three value functions.

Let $\Delta t = 1$, $\rho_d = 0.38$, $\rho_u = 0.01$ and $q = 0.002$, which is much less than $\frac{1-\rho_d \Delta t - \rho_u \Delta t}{2(1-\rho_d \Delta t)} \approx 0.4919$. In this case, $V_{t_1}^{MN,B_{lo}}\left(sell\ 1, \Gamma\left(t_1\right)\right) > V_{t_1}^{MN,B_{lo}}\left(sell\ 2, \Gamma\left(t_1\right)\right) > V_{t_1}^{MN,B_{lo}}\left(no\ trade, \Gamma\left(t_1\right)\right)$. But when q increases, e.g., $q = 0.05$, then $V_{t_1}^{MN,B_{lo}}\left(sell\ 2, \Gamma\left(t_1\right)\right) > V_{t_1}^{MN,B_{lo}}\left(sell\ 1, \Gamma\left(t_1\right)\right) > V_{t_1}^{MN,B_{lo}}\left(no\ trade, \Gamma\left(t_1\right)\right)$.

(ii) Iterative limiting case

Similarly, the large trader's optimal strategy in the first period under this game structure depends on the relationship between q, ρ_d , ρ_u and Δt .

when $q \le \dfrac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$, the large trader may choose to sell one share or two shares in the first period. For example, let $\Delta t = 1$, $\rho_d = 0.6$, $\rho_u = 0.05$ and $q = 0.05$. $V_{t_1}^{IN,B_{lo}}\left(sell\ 1, \Gamma\left(t_1\right)\right) > V_{t_1}^{IN,B_{lo}}\left(sell\ 2, \Gamma\left(t_1\right)\right) > V_{t_1}^{IN,B_{lo}}\left(no\ trade, \Gamma\left(t_1\right)\right)$. But when ρ_u increase to 0.3, while Δt and q are kept unchanged, the large trader's optimal strategy is to sell two shares in the first period.

When $q > \dfrac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$, $V_{t_1}^{IN,B_{lo}}\left(no\ trade, \Gamma\left(t_1\right)\right) = V_{t_1}^{IN,B_{lo}}\left(sell\ 1, \Gamma\left(t_1\right)\right)$ and both are less than $V_{t_1}^{IN,B_{lo}}\left(sell\ 2, \Gamma\left(t_1\right)\right)$. Thus the large trader's optimal strategy is to sell quickly, i.e., two shares, when $q > \dfrac{1-\rho_d \Delta t - \rho_u \Delta t}{1-\rho_d \Delta t}$.

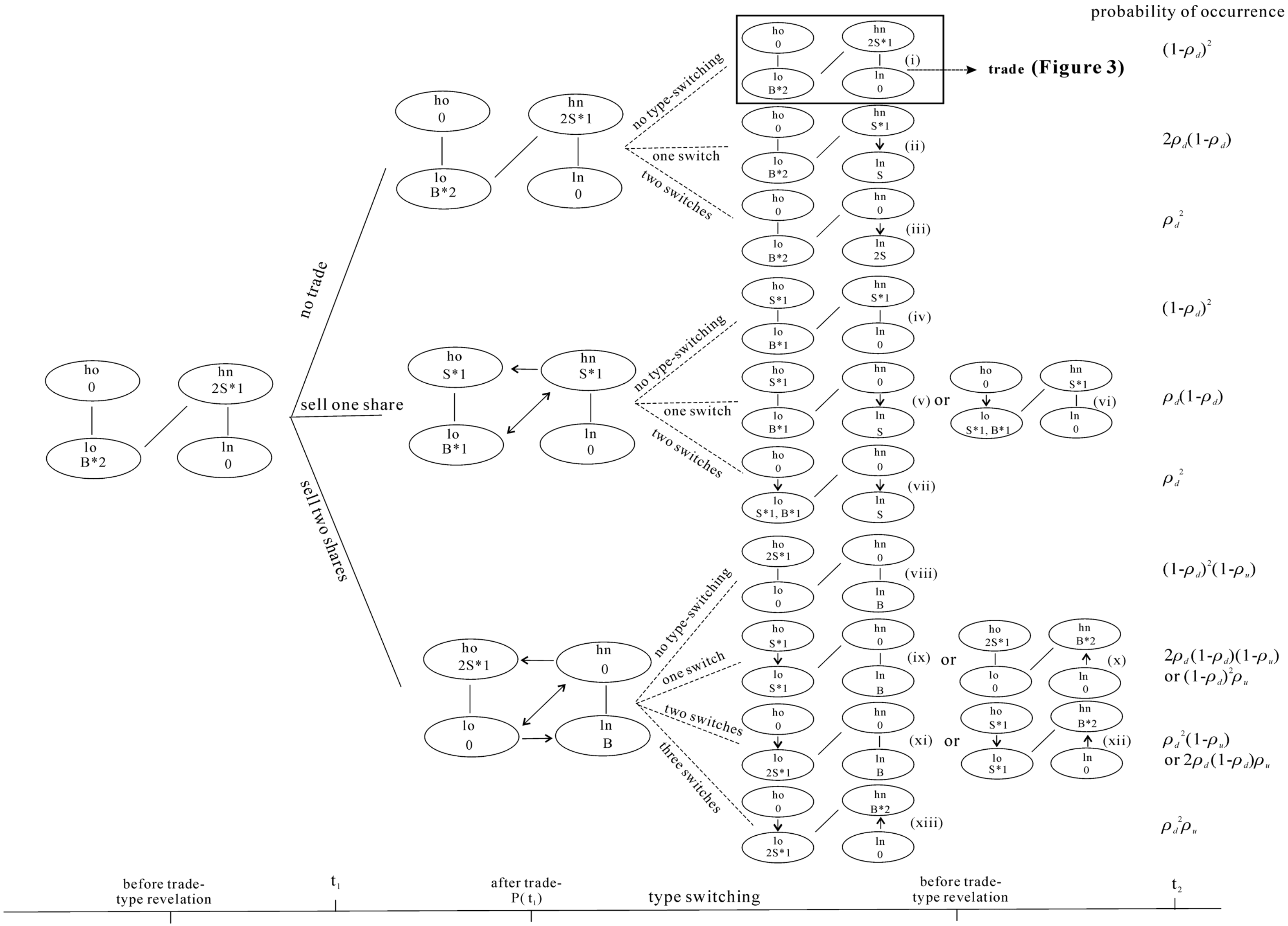

Figure 2. The dynamics of the trading and type evolution

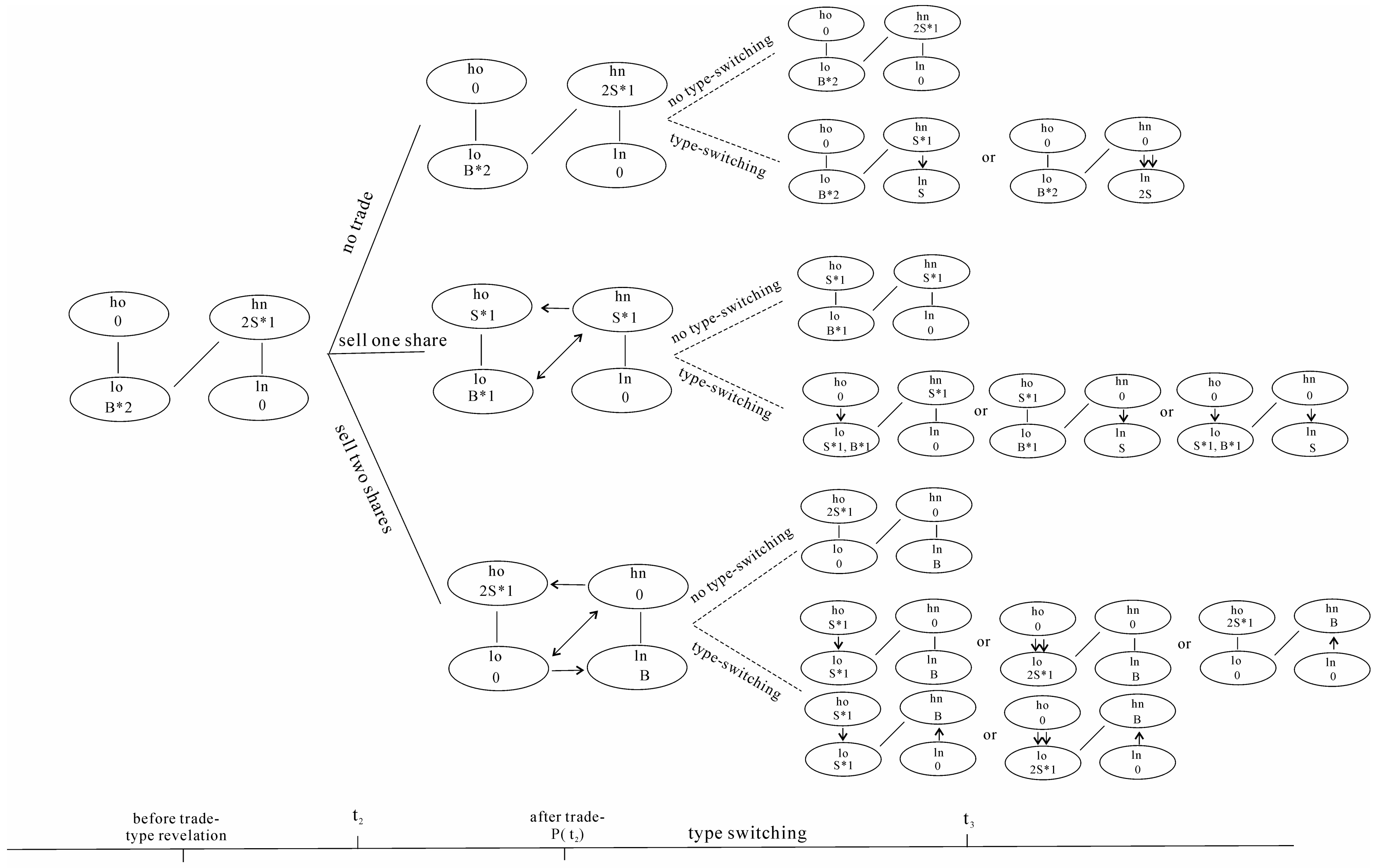

Figure 3. The evolution of a subgame begins at t_2

References:

Acharya, Viral V., and Lasse Heje Pedersen, 2004, Asset pricing with liquidity risk, *Journal of Financial Economics (forthcoming).*

Allen, Franklin, and Douglas Gale, 2000, Financial contagion, *Journal of Political Economy* 108, 1-33.

Amihud, Yakov, and H Mendelson, 1986, Asset pricing and the bid-ask spread, *Jounal of Financial Economics* 17, 223-249.

Attari, Mukarram, Antonio S. Mello, and Martin Ruckes, 2002, Arbitraging arbitrageurs, working paper, University of Wisconsin-Madison.

Basak, Suleyman, 1997, Consumption choice and asset pricing with a non-price-taking agent, *Economic Theory* 10, 437-462.

Braas, Alberic, and Charles Bralver, 2003, An analysis of trading profits: how most trading rooms really make money, in Joel M. Stern, and Donald H. Chew, Jr., eds.: *The revolution in corporate finance* (Blackwell Publishing Ltd.).

Brunnermeier, Markus K., and Lasse Heje Pedersen, 2004, Predatory trading, *Journal of Finance (forthcoming)*.

Cai, Fang, 2003, Was there front running during the LTCM crisis?, International Finance Discussion Papers 758, The Federal Reserve Board.

Constantinides, George M., 1986, Capital market equilibrium with transaction costs, *Journal of Political Economy* 94, 842-862.

Duffie, Darrell, Nicolae Garleanu, and Lasse Heje Pedersen, 2004a, Over-the-counter markets, *Econometrica (forthcoming).*

Duffie, Darrell, Nicolae Garleanu, and Lasse Heje Pedersen, 2004b, Valuation in over-the-counter markets, working paper, Graduate School of Business, Stanford University.

Green, Richard, Burton Hollifield, and Norman Schurhoff, 2004, Financial Intermediation and the cost of trading in an opaque market, *Working paper, Carnegie Mellon University*.

Harris, Lawrence E., and Michael S. Piwowar, 2004, Municipal bond liquidity, *Working paper, Securities and Exchange Commission*.

Holthausen, Robert, Richard W. Leftwich, and David Mayers, 1990, Large-block transactions, the speed of response, and temporary and permanent stock-price effects, *Journal of Financial Economics* 26, 71-95.

Huang, Ming, 2003, Liquidity shocks and equilibrium liquidity premia, *Journal of Economic Theory* 109, 104-129.

Jarrow, Robert, 1992, Market manipulation, bubbles, corners, and short squeezes, *Journal of financial and Quantitative Analysis* 27, 311-336.

Keim, Donald B. , and Ananth Madhavan, 1996, The upstairs market for large-block transactions: analysis and measurement of price effects., *Review of Financial Studies* 9, 1-36.

Kihlstrom, Richard, 2000, Monopoly power in dynamic securities markets, working paper, The Wharton School, University of Pennsylvania.

Kyle, Albert, 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315-1336.

Kyle, Albert, 1989, Informed speculation with imperfect competition, *Review of Economic Studies* 56.

Liang, Dan, 2005, Distressed sales and financial arbitrageurs: front-running in illiquid makrets, Working Paper, Queen's University.

Longstaff, Francis A., 2001, Optimal portfolio choice and the valuation of illiquid securities, *Review of financial studies* 14, 407-431.

Pritsker, Matthew, 2004, Large investors: implications for equilibrium asset returns, shock absorption, and liquidity, mimeo, Federal Reserve Board.

Subramanian, Ajay, and Robert Jarrow, 2001, The liquidity discount, *Mathematical Finance* 11, 447-474.

Vayanos, Dimitri, 1998, Transaction costs and asset prices: a dynamic equilibrium model, *Review of Financial Studies* 11, 1-58.

Vayanos, Dimitri, 1999, Strategic trading and welfare in a dynamic market, *Review of Economic Studies* 66, 219-254.

Vayanos, Dimitri, 2001, Strategic trading in a dynamic noisy market, *Journal of Finance* LVI, 131-171.

Vayanos, Dimitri, 2004, Flight to quality, flight to liquidity, and the pricing of risk, working paper, Sloarn School of Management, MIT.

Vayanos, Dimitri, and J. L. Vila, 1999, Equilibrium interest rate and liquidity premium with transaction costs, *Economic Theory* 13, 509-539.

Vayanos, Dimitri, and Tan Wang, 2003, Search and endogenous concentration of liquidity in asset markets, working paper, Sloan School of Business, MIT.

Weill, Pierre-Olivier, 2003, Liquidity premia in dynamic bargaining markets, working paper, Department of Economics, Stanford University.