

# The Tragedy of the Commons and Economic Growth: Why Does Capital Flow from Poor to Rich Countries?

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We analyze a differential game in which all interest groups have access to a common capital stock. We show that the introduction of a technology that has inferior productivity but enjoys private access may ameliorate the tragedy of the commons. We use this model to analyze capital flight: in many poor countries, property rights are not well defined; since “safe” bank accounts in rich countries (the inferior technology) are available to citizens of these countries, they engage in capital flight. We show that the occurrence of capital flight does not imply that opening the capital account reduces growth and welfare.

## I. Introduction

We present a dynamic model of the tragedy of the commons with two assets. We use it to analyze capital flight and economic growth in countries in which property rights are not well defined.

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A “tragedy of the commons” occurs when property rights over a productive asset are ill defined or cannot be enforced. The classic examples involve cattle grazing in a common pasture or vessels fishing in a lake. Typically, the literature shows that common access leads to overconsumption and underinvestment (see Gordon 1954; Lancaster 1973; Levhari and Mirman 1980; Reinganum and Stokey 1985; Haurie and Pohjola 1987; Benhabib and Radner 1988). This paper shows that introducing a technology with inferior productivity, but private access, into a common-access economy will, under some circumstances, ameliorate the tragedy of the commons and increase welfare. This runs contrary to simple intuition, which might suggest that adding an inferior technology is at best irrelevant, as is the case in a representative agent model.

We present a differential game among interest groups. Each group has an infinite horizon and maximizes lifetime utility derived from consumption. Two technologies can produce the single consumption good: one has common access, meaning that every group can appropriate any share it desires from the common capital stock; the other enjoys private access as in the neoclassical growth model. Both technologies have constant physical rates of return, with the common-access technology having a higher one.

We analyze three symmetric Nash equilibria of the two-asset economy: one interior and two extreme. At the interior equilibrium, the appropriation rate must be such that the *private* rate of return on the common-access technology is equal to the rate of return on the inferior, private-access technology. Otherwise, there would be unexploited arbitrage opportunities.

It follows that the introduction of the inferior technology into a one-asset common-access economy puts a floor on the common-access asset’s rate of return and, thus, a ceiling on the appropriation rate. If this constraint is binding, interest groups will be forced to reduce their appropriation rate. This will increase aggregate capital accumulation, ameliorate the tragedy of the commons, and increase welfare.

This result has strong implications for capital flight and economic growth. Capital flight occurs when productive resources flow from poor to rich countries.<sup>1</sup> This phenomenon apparently contradicts the standard two-factor neoclassical growth model: since poor countries have a lower capital/labor ratio, and thus a higher marginal product of capital, theory suggests that they should experience capital inflows. Most explanations of this paradoxical pattern of capital movement

<sup>1</sup> For the Latin American capital flight experience, see Cuddington (1986), Lessard and Williamson (1987), and Dornbusch and de Pablo (1988). Such flight has often been associated with low domestic investment and slow growth (see Rodrik 1989).

endeavor to show that the relevant rate of return on capital in a poor country is not as high as it might seem at first blush. For instance, Lucas (1990) argues that once external effects and differences in human capital have been accounted for, the return differential between poor and rich countries practically vanishes.

In this paper we offer an alternative explanation. Here, capital flight is a response to the tragedy of the commons, which occurs in poor countries. That is, investing abroad through capital flight represents recourse to the inferior technology of the kind referred to above. Because of a weak system of property rights in poor countries, each interest group has common access to other groups' domestic capital stocks; that is, it can appropriate the fruits of their domestic investment. By contrast, the investments that citizens of poor countries can make abroad—be they in bank deposits or foreign government bonds—enjoy private access. Of course, such investments are not riskless since they can be beset by exchange or interest rate shocks, but their exposure to “political” risk is relatively minimal. At the same time, the relative capital abundance in “rich countries” suggests that such assets must have a lower rate of return than capital in the “poor country.” However, capital flight can emerge as an attempt to place one's wealth beyond the reach of competing interest groups.

In much of the literature, capital outflows are nothing but the optimal response to imprudent macroeconomic policies.<sup>2</sup> By taxing capital too heavily or by following unsustainable monetary and exchange rate policies, governments force agents to protect themselves by holding foreign assets. However, the magnitude and motivation of these policies are exogenous. In this paper, by contrast, we attempt to explain their origin by viewing the government as a clearinghouse for the interests of various groups, with confiscatory or threatening policies emerging as the outcome.<sup>3</sup>

Common access to domestic capital can occur in several ways. The simplest is outright confiscation or banditry. Other, more subtle, mechanisms also exist. Imagine a situation in which each group has the ability to extract from the government any transfers it desires.

<sup>2</sup> There is also a body of work that links underinvestment to uncertainty or lack of credibility of government policies (van Wijnbergen 1985; Rodrik 1989; Tornell 1990). These are primarily theories of why agents choose to invest in short-term liquid assets (domestic and foreign) rather than in fixed capital. They are not necessarily theories of capital flight.

<sup>3</sup> In attempting to understand capital flight as the result of conflict among interest groups, this paper is related to the work of Alesina and Tabellini (1989). In that paper there is conflict between the owners of capital and the owners of labor, each of whom is represented by a political party. Since these parties alternate randomly in power, the possibility of having “the other side” in power in the future can generate international overborrowing and capital flight.

Assume further that the government must balance its budget every period, so that the transfers result in taxes on domestic capital, which is the only asset within the reach of the fiscal authorities. Thus the power to extract transfers gives each group “common access”—via the government budget constraint—to other groups’ capital stocks. This sort of political risk faced by domestic residents is usual in many poor countries. An abundantly analyzed case is that of Argentina.<sup>4</sup>

An economy that suffers from the “tragedy of the commons” and has an open capital account displays some striking behavior. Along the interior equilibrium we find the following: (i) The existence of capital flight does not imply that opening the capital account reduces growth and welfare. The reason is that the disciplinary effect introduced by the option of investing abroad may reduce the appropriation rate. (ii) The *higher* physical productivity of capital, the *lower* economic growth. Under common access, as capital becomes more productive, voracity increases more than proportionally, in order to preserve rate of return equalization. This result may help to explain the bad performance of resource-rich Argentina and the good performance of resource-poor Korea. (iii) Economies with *more* interest groups have *higher* growth rates and lower appropriation rates. (iv) Capital flows occur gradually, despite linear technologies and the absence of adjustment costs associated with the movement of capital.<sup>5</sup> It does not pay to deviate unilaterally and take capital abroad, even at an infinite rate.

The paper is structured as follows. In Section II we present a dynamic, one-asset model of the tragedy of the commons. This case is useful not only as a benchmark but also as a simple setting in which to describe the feedback Nash equilibria of such a game. In Section III, which contains the core of the paper, we introduce the inferior private-access technology and characterize the interior equilibrium of the differential game with two state variables. A comparison between the two regimes—with one or two technologies—is undertaken in Section IV. The implications of the model for capital flight and economic growth are reviewed in Section V. Section VI characterizes the extreme equilibria, and Section VII contains some conclusions.

<sup>4</sup> In reality, many types of distributive conflicts are associated with capital flight, for instance, conflict between producers of traded and nontraded goods or between industry and agriculture (Hirschman 1971, 1981; Mallon and Sourrouille 1975); conflict over the allocation of the fiscal adjustment burden (Alesina and Drazen 1989) or between government agencies (Tabellini 1986); or attempts to use seigniorage to force other groups to finance one’s spending (Aizenman 1989).

<sup>5</sup> This result stands in contrast to the standard capital flight literature (Krugman 1979; Eaton 1987), where discrete capital outflows occur as a result of speculative attacks.

## II. A Dynamic Model of the Tragedy of the Commons

The economy is populated by  $n$  symmetric groups, where  $n$  is an integer no smaller than two, and there exists one asset. Even though each group  $j$  owns a piece of this asset, the other  $n - 1$  groups have “common access” to it. That is, any group can appropriate any share it desires of group  $j$ 's piece. Thus, in practice, there is a “common” stock of the asset. In the literature, this common-access asset is typically identified with a natural resource, such as underground oil or fisheries. One can also identify it—in a country in which property rights are not well defined—with the aggregate stock of domestically held capital.

At each point in time, a group must decide how much of the common stock to appropriate. Since there exists only one asset, a group must consume all that it appropriates. The trade-off faced by each group is that appropriating too little reduces the resources available for own consumption today, but appropriating too much may kill the goose that lays the golden egg, by leading others to increase their consumption. Each group maximizes

$$U_j = \int_0^{\infty} \frac{\sigma}{\sigma - 1} c_j(t)^{(\sigma-1)/\sigma} e^{-\delta t} dt, \quad (1)$$

where  $c$  is consumption,  $\delta$  is the subjective rate of time preference, and  $\sigma (> 0)$  is the elasticity of intertemporal substitution in consumption. Notice that if  $\sigma = 1$ , the instantaneous utility function is given by  $\log(c)$ .

Even though each group nominally owns a share of the asset's stock, the relevant stock, from a group's perspective, is the aggregate because there is common access. Therefore, the budget constraint faced by each group is

$$\dot{k}(t) = ak(t) - d_j(t) - \sum_{i \neq j} d_i(t), \quad (2)$$

where  $a$  is the constant marginal product of the common-access asset,  $k(0) > 0$ , and  $d_j$  is the amount removed from the aggregate stock by group  $j$ . Since in the aggregate there cannot be a short position in the common-access asset, groups are also constrained by

$$k(t) \geq 0 \quad \text{for all } t. \quad (3)$$

### A. Feedback Nash Equilibria

The problem defined by (1)–(3) is a differential game. The solution concept we use is closed-loop feedback Nash. That is, we assume that

groups reoptimize at every instant and that in choosing its actions each group takes as given the rules followed by the other groups. Furthermore, as is typical in this literature, we constrain strategies to depend only on the current value of the state variables (i.e., Markovian strategies). We assume away more complex behavior based on the previous history of the game, such as trigger strategies.<sup>6</sup> We also assume that strategies are symmetric and linear.

At every instant, each group chooses an optimal sequence  $\{c_j(t)\}$  in order to maximize (1), subject to (2), (3), and the strategies of the other  $n - 1$  groups. A major difficulty in solving differential games explicitly is that the first-order conditions involve partial derivatives of the unknown optimal strategies of the other players. In order to obtain a closed-form solution, we assume that  $d_j(t) = \alpha k(t)$  and obtain  $\alpha$  endogenously. Section C of the Appendix shows that in this equilibrium<sup>7</sup>

$$d_j^o(t) = c_j^o(t) = \alpha k^o(t) = \frac{a(1 - \sigma) + \delta\sigma}{n - \sigma(n - 1)} k^o(t), \tag{4a}$$

$$k^o(t) = k(0) \exp \frac{\sigma(a - n\delta)}{n - \sigma(n - 1)} t. \tag{4b}$$

The superscript  $o$  stands for one-asset economy. For the utility index to be bounded, it is necessary to assume

$$[a(1 - \sigma) + \delta\sigma][n - \sigma(n - 1)] > 0. \tag{5}$$

To obtain the indirect utility function, we substitute (4a) in (1):

$$U_j^o = \frac{\sigma}{\sigma - 1} k(0)^{(\sigma-1)/\sigma} \left[ \frac{a(1 - \sigma) + \delta\sigma}{n - \sigma(n - 1)} \right]^{-1/\sigma} \quad \text{for } \sigma \neq 1$$

$$= \frac{\log[k(0)\delta]}{\delta} + \frac{a - n\delta}{\delta^2} \quad \text{for } \sigma = 1. \tag{4c}$$

*B. The First-Best Allocation*

The outcome above is clearly not first-best. The first-best would be attained by maximizing the representative group’s welfare (1), subject

<sup>6</sup> For a formal definition, see Başar and Olsder (1982, chap. 6). For examples of such an equilibrium, see Tabellini (1986) and Haurie and Pohjola (1987).

<sup>7</sup> To get the intuition behind (4a), note first that in a representative agent model in which the rate of return on capital is  $a$ ,  $c = [a(1 - \sigma) + \delta\sigma]k$ . Second, under common access, the private rate of return for group  $j$  is  $a - (n - 1)\alpha_{-j}$ . Thus  $c_j = \{[a - (n - 1)\alpha_{-j}](1 - \sigma) + \delta\sigma\}k$ . Third, the appropriation and the consumption rates are equal in a one-asset economy:  $c_j = \alpha_j k$ . Finally, strategies are symmetric. Hence, equating the coefficients of the last two equations, one gets (4a).

to (2) and (3). This solution is the same as the one that would prevail if (i) each group, enjoying “private access” to the returns of its asset, solved the decentralized problem; or (ii) in spite of the existence of “common access,” groups achieved a cooperative solution. Section C of the Appendix shows that

$$c_j^f(t) = \frac{[a(1 - \sigma) + \delta\sigma]k^f(t)}{n}, \tag{6a}$$

$$k^f(t) = k(0)e^{\sigma(a-\delta)t}, \tag{6b}$$

where the superscript *f* stands for the first-best of a one-asset economy. For the utility index to be bounded, it is necessary to assume

$$a(1 - \sigma) + \delta\sigma > 0. \tag{7}$$

By substituting (6a) in (1) we get

$$U_j^f = \frac{\sigma}{\sigma - 1} \left[ \frac{k(0)}{n} \right]^{(\sigma-1)/\sigma} [a(1 - \sigma) + \delta\sigma]^{-1/\sigma} \quad \text{for } \sigma \neq 1$$

$$= \frac{\log[k(0)\delta/n]}{\delta} + \frac{a - \delta}{\delta^2} \quad \text{for } \sigma = 1. \tag{6c}$$

Now we can give a precise definition of the tragedy of the commons.

**DEFINITION 1.** A sequence *X* of capital stocks exhibits a tragedy of the commons with respect to a sequence *Y* if the sequence *X* is strictly bounded above by sequence *Y*, for *t* > 0.

Since the initial stock of the common-access asset is equal under both regimes, (4b) exhibits a tragedy of the commons if and only if

$$\frac{\dot{k}^f}{k^f} - \frac{\dot{k}^o}{k^o} = \{(a - \delta)[n - \sigma(n - 1)] - (a - n\delta)\}[n - \sigma(n - 1)]^{-1}\sigma$$

$$= (n - 1)[a(1 - \sigma) + \delta\sigma][n - \sigma(n - 1)]^{-1}\sigma$$

is positive. Condition (5) implies that this expression is positive. By comparing (4a) and (6a), one can see that the marginal propensity to consume is higher in the common-access regime than in the first-best solution. This leads to a tragedy of the commons and thus to a lower consumption growth rate and a lower welfare.<sup>8</sup>

<sup>8</sup> The reason is that  $U^f - U^o > 0$  for all  $\sigma$  that satisfy (5) and (7):

$$U^f - U^o = \{n^{1-\sigma} - [n - \sigma(n - 1)]\}^{1/\sigma} \left\{ \left( \frac{\sigma}{\sigma - 1} \right) [a(1 - \sigma) + \delta\sigma]^{-1/\sigma} k(0)^{(\sigma-1)/\sigma} \right\}.$$

Let  $\sigma > 1$ . Condition (7) implies that the second factor in braces is positive. For  $n > 1$ , the first factor in braces is also positive because (i) (5) and (7) imply that the second term is positive, (ii) both terms and their derivatives with respect to *n* are equal for  $n = 1$ , and (iii) both terms are decreasing in *n*, with the first term convex in *n* and the

### III. The Tragedy of the Commons with Two Assets

In this section we introduce an additional technology that has an inferior physical rate of return ( $r$ ) but enjoys "private access." In the context of fisheries, one can think of a big, clean lake to which there is common access and small, private, stagnant lakes, where fish reproduce at a lower rate.

With two assets, the appropriation and the consumption decisions are separate. Now, each group must choose how much of the common-access asset to appropriate and, out of this, how much to consume and how much to invest in the private-access technology. That is, each group maximizes (1) subject to (2), (3),  $d_j(t) = \beta k(t)$ , and<sup>9</sup>

$$\dot{f}_j(t) = rf_j(t) + d_j(t) - c_j(t) \quad (8)$$

and

$$a > r > 0, \quad (9)$$

where  $f_j$  is group  $j$ 's stock of the private-access asset, and  $d_j$  is the amount removed from the common-access asset by group  $j$ . We set the initial stock of the private-access asset equal to zero in order to ease comparisons with other cases.

Since there are no diminishing returns, the first-best would entail taking the shortest possible position in the inferior asset and taking a long position in the common-access asset. This would imply borrowing at a rate  $r$  to invest in an asset that yields  $a$ . However, if all groups have common access to the stocks of others and decisions are decentralized, the equilibria that will emerge will be very different from the first-best. This springs from the fact that capital may "go uphill" where the physical rate of return is lower.

We shall analyze three symmetric Nash equilibria of this game: an "interior" equilibrium in which the  $d_j$ 's are in the interior of the appropriation set and two "extreme" equilibria: a "pessimistic" one in which each group attempts to appropriate as much as it can of the common-access asset and an "optimistic" one in which the  $d_j$ 's are in the lower bound of the appropriation set.

Section A of the Appendix shows that in the interior equilibrium, the optimal strategy for each group is

$$d_j^{\text{in}}(t) = \beta^{\text{in}} k^{\text{in}}(t) = \frac{a - r}{n - 1} k^{\text{in}}(t), \quad (10a)$$

second term linear. The same argument applies to the case in which  $\sigma < 1$ . When  $\sigma = 1$ , we get  $U^f - U^o = [(n - 1) - \log(n)]/\delta$ , which can be shown to be positive using the same argument.

<sup>9</sup> If  $a < r$ , the common-access technology will not be used in equilibrium, and the model will be identical to the neoclassical one.



where “in” stands for interior equilibrium of a two-asset economy. This implies that, despite the absence of adjustment costs, the state variables  $k$  and  $f_j$  do not jump, but evolve as smooth functions of time:

$$k^{\text{in}}(t) = k(0)e^{[(nr-a)/(n-1)]t}, \quad (10b)$$

$$f_j^{\text{in}}(t) = k(0)\{e^{\sigma(r-\delta)t} - e^{[(nr-a)/(n-1)]t}\}, \quad (10c)$$

$$\begin{aligned} c_j^{\text{in}}(t) &= [r(1 - \sigma) + \delta\sigma]k(0)e^{\sigma(r-\delta)t} \\ &= [r(1 - \sigma) + \delta\sigma][k^{\text{in}}(t) + f_j^{\text{in}}(t)]. \end{aligned} \quad (10d)$$

Equations (10a)–(10d) characterize the behavior of the system in the interior feedback Nash equilibrium. For the utility index to be bounded, it is necessary to assume

$$z \equiv r(1 - \sigma) + \delta\sigma > 0. \quad (11)$$

This expression will appear repeatedly throughout the paper. To obtain the indirect utility function, we substitute (10d) in (1):

$$\begin{aligned} U_j^{\text{in}} &= \frac{\sigma}{\sigma - 1} k(0)^{(\sigma-1)/\sigma} z^{-1/\sigma} \quad \text{for } \sigma \neq 1 \\ &= \frac{\log[k(0)\delta]}{\delta} + \frac{r - \delta}{\delta^2} \quad \text{for } \sigma = 1. \end{aligned} \quad (10e)$$

In order to interpret these results, we shall introduce the following definition.

**DEFINITION 2.** The “private rate of return” is the rate that a group realizes on its portion of the common-access asset, after appropriation by other groups.

The first feature of this solution is that each group will appropriate a portion  $\beta^{\text{in}} = (a - r)/(n - 1)$  of the aggregate stock of the common-access asset at each instant of time. This can be understood if we rewrite (10a) to read  $r = a - \beta^{\text{in}}(n - 1)$ , which has a clear interpretation. The left-hand side is the riskless rate of return a group can obtain from the private-access asset. The right-hand side is the private rate of return on the common-access asset. In equilibrium the rates of return on these two assets must be equalized.

This result has two implications. First, since the rate of return on any investment is  $r$ , it follows (as is standard in any optimizing model) that consumption will grow at the rate  $\sigma(r - \delta)$ , which is smaller than in a representative agent model, where the growth rate of consumption would be  $\sigma(a - \delta)$ . Second, both capital stocks evolve gradually. This might seem surprising because with no adjustment costs of moving capital and with linear technologies, one would expect jumps in these stocks. To check that these jumps would not occur even if they

were allowed, consider the case in which a deviant appropriated the entire stock of the common-access asset.<sup>10</sup> In this case, the deviant would invest all the stock in the private-access technology and would solve a standard consumption-savings problem, with an interest rate  $r$ . It follows that its consumption will be given by  $c_j(t) = k(0)Ze^{\sigma(r-\delta)t}$ , which is identical to the consumption it has in the interior equilibrium.<sup>11</sup> Therefore, even in this extreme case, a unilateral deviation does not pay off.

When the deviation from the interior equilibrium is just a change in the appropriation rate (i.e., group  $j$  chooses a  $\beta_j$  different from  $\beta^{in}$  and the remaining  $n - 1$  groups choose  $\beta^{in}$ ), then the welfare attained as a result of the unilateral deviation is (see [A13'])

$$U_j^{dev}(\beta_{-j} = \beta^{in}) = \frac{\sigma}{\sigma - 1} \left[ \frac{\beta_j k(0)}{r - a + (n - 1)\beta^{in} + \beta_j} \right]^{(\sigma-1)/\sigma} z^{-1/\sigma}. \quad (12)$$

Since  $\beta^{in} = (a - r)/(n - 1)$ , this expression collapses to equation (10e), the welfare level that would have been obtained by group  $j$  had it not deviated. Therefore, unilateral deviations are not profitable.<sup>12</sup> We can summarize these results in the following proposition.

**PROPOSITION 1.** In the interior equilibrium of a two-asset economy, the private rate of return on the common-access asset,  $a - (n - 1)\beta$ , is equal to the rate of return on the private-access asset,  $r$ . Therefore, (i) the stocks of both assets evolve gradually, and (ii) consumption is independent of the physical rate of return on the common-access asset,  $a$ .

#### IV. The Effects of Introducing the Additional Technology

In this section we investigate whether the introduction of a technology that has an inferior physical rate of return but enjoys private access ameliorates the tragedy of the commons and raises welfare.

In order to address these issues, we compare the equilibria of Sec-

<sup>10</sup> This type of deviation, which involves stepwise changes in the capital stock, is ruled out by the linear strategy restriction.

<sup>11</sup> In this extreme case, at  $t = T$  the deviant would maximize (1) subject to  $df_j/dt = rf_j - c_j$  and  $f_j(T) = k(T)$ . The first-order conditions are  $(c_j)^{-1/\sigma} = \pi$  and  $d\pi/dt = \pi(\delta - r)$ . Since  $c_j$  has the form  $\gamma f_j$  (see the Appendix), it follows that  $\gamma = Z = r(1 - \sigma) + \delta\sigma$  and  $c_j(t) = Zf_j(t)$  because

$$-\sigma \frac{\dot{\pi}}{\pi} = \frac{\dot{c}_j}{c_j} = \frac{\dot{f}_j}{f_j} = r - \gamma.$$

<sup>12</sup> If a small fixed cost for deviating from the interior equilibrium were introduced, then it would be unprofitable for any group to appropriate a share different from  $\beta^{in}$ . It would also be unprofitable to steal the entire stock of the common-access asset.

tions II and III. In order to do this, we need to make sure that the utility indexes under both regimes are bounded, that is, that (5) and (11) are simultaneously satisfied. Conditions on  $\sigma$  for this to occur are given by (A16) and (A17) in the Appendix.

To determine whether the introduction of the private-access technology induces more investment in the common-access asset, we compare the growth rates under both regimes. By differentiating (4b) and (10b) with respect to time, we get

$$\frac{\dot{k}^{in}}{k^{in}} - \frac{\dot{k}^o}{k^o} = \frac{nr - a}{n - 1} - \frac{\sigma(a - n\delta)}{n - (n - 1)\sigma} = \frac{\beta - z}{n - \sigma(n - 1)}. \tag{13}$$

Surprisingly, the effect on investment is proportional to the difference between the appropriation rate ( $\beta$ ) and the marginal propensity to consume ( $z$ ).

Subtracting (4c) from (10e), we see that the welfare effect of introducing the additional asset has the same sign as (13):<sup>13</sup>

$$\begin{aligned} \text{sgn}\{U^{in} - U^o\} &= \text{sgn}\left\{\frac{a(1 - \sigma) + \delta\sigma}{n - \sigma(n - 1)} - z\right\}(\sigma - 1) \\ &= \text{sgn}\left\{(\beta - z)\left(\sigma - \frac{n}{n - 1}\right)\right\}. \end{aligned} \tag{14}$$

The results above can be summarized as follows.

**PROPOSITION 2.** The introduction of a private-access technology with an inferior rate of return into a common-access economy ameliorates the tragedy of the commons and increases welfare if and only if, in the resulting equilibrium, (i)  $\sigma > n/(n - 1)$  and the appropriation rate is greater than the marginal propensity to consume, or (ii)  $1 \leq \sigma < n/(n - 1)$  and the appropriation rate is lower than the marginal propensity to consume.<sup>14</sup>

In order to get some insight, recall that the possibility of using the private-access technology places a floor to the private rate of return on the common-access asset; that is, it acts as a threat that limits the temptation to overappropriate. If this floor  $r$  is higher than the private rate of return in the one-asset economy,  $a - (n - 1)\alpha$ , then introducing the additional technology will reduce the appropriation

<sup>13</sup> The sign of  $U^{in} - U^o$  is equal to (14) because  $Z, \alpha > 0$  and

$$U^{in} - U^o = \frac{\sigma}{\sigma - 1} k(0)^{(\sigma-1)/\sigma} \left[ \frac{\alpha^{1/\sigma} - Z^{1/\sigma}}{(Z\alpha)^{1/\sigma}} \right].$$

<sup>14</sup> We have excluded  $\sigma < 1$  because it is inconsistent with  $\beta > Z$ . Section D of the Appendix analyzes which combinations of signs of  $\beta - Z$  and  $n - \sigma(n - 1)$  are compatible with (5), (7), (9), and (11). It shows that  $\beta < Z$  is consistent with  $1 \leq \sigma < n/(n - 1)$ , that  $\beta < Z$  is inconsistent with  $\sigma < 1$ , and that  $\beta > Z$  is consistent with  $\sigma > n/(n - 1)$ . Nothing can be said in general about the other cases.

rate (from  $\alpha$  to  $\beta^{\text{in}}$ ) and increase the private rate of return on the common-access asset. To illustrate this, let  $\sigma = 1$  so that  $\alpha = z = \delta$ . In this case, if  $r > a - (n - 1)\delta$ , the appropriation rate falls (i.e.,  $\alpha < \beta^{\text{in}}$ ) because  $\delta < (a - r)/(n - 1)$ . A higher rate of return leads to a higher growth rate of the common-access asset,  $a - n[(a - r)/(n - 1)] > a - n\delta$ . Finally, since in this case the marginal propensity to consume  $z$  is equal to  $\delta$  in both regimes, it follows that welfare increases:  $U^{\text{in}} - U^o = [(r - \delta) - (a - n\delta)]/\delta^2$ .

Up to this point we have not imposed the restriction  $f_j(t) \geq 0$ ,<sup>15</sup> that is, that short positions cannot be taken in the inferior technology. For some technologies, such as fisheries and oil wells, this is a relevant restriction because lakes, for example, cannot have negative stocks. In order to determine the applicability of proposition 2 to this case, we note that

$$\text{sgn} \left\{ \frac{df_j(t)}{dt} \right\} = \text{sgn} \left\{ \sigma(r - \delta) - \frac{nr - a}{n - 1} \right\} = \text{sgn}\{\beta - z\}.$$

Since the sign of the time derivative of  $f_j(t)$  is the same as the condition that appears in proposition 2, it follows that when  $\beta < z$ ,  $1 \leq \sigma < n/(n - 1)$ , and  $f_j(t = 0) = 0$ , welfare increases if and only if the inferior private-access technology is not used in equilibrium.

### V. Capital Flight and Economic Growth

In this section we identify the common-access asset with capital held in a country in which property rights are not well defined—for instance, in which inflation and other taxes serve to subsidize rent-seeking groups—and we identify the inferior private-access technology with bank accounts in foreign countries that offer a safe but low return (relative to the capital held at home). We refer to countries with scarce capital and ill-defined property rights as “poor” and to foreign countries with a low return on capital as “rich.” We start by defining capital flight.

**DEFINITION 3.** Capital flight is the derivative, with respect to time, of the ratio of the private-access asset’s stock to the common-access asset’s stock:<sup>16</sup>

$$KF \equiv \frac{d[f_j(t)/k(t)]}{dt} = \frac{a - r}{n - 1} - [r(1 - \sigma) + \delta\sigma] = \beta - z. \quad (15)$$

<sup>15</sup> Imposing the restriction  $f_j(t) \geq 0$  would not alter the form of solution (10). If  $\beta > Z$ , nothing changes. If  $\beta < Z$ , then since  $f_j(0) = 0$ , we would substitute zero for  $f_j(t)$  in (10c) and (10d).

<sup>16</sup> As shown in Sec. IV, capital flight could also be defined as the derivative, with respect to time, of the private-access asset’s stock (i.e.,  $df_j/dt > 0$ ). Its sign would be the same as that of (15).

Equation (15) states that there is positive capital flight if the appropriation rate ( $\beta$ ) is greater than the marginal propensity to consume ( $z$ ). Intuitively, the reason is that in the interior equilibrium, group  $j$  must appropriate  $\beta^{\text{in}} k(t)$  each instant. However, this group finds it optimal to consume  $zk(t)$  out of the domestic capital stock. If  $\beta^{\text{in}} > z$ , group  $j$  must invest the remainder abroad. On the contrary, if it invested the remainder at home, this would be equivalent to appropriating less than  $\beta^{\text{in}}$ . Hence, this would constitute a deviation from the interior equilibrium.

A puzzling result is that the higher the physical rate of return on domestic capital  $a$ , the higher the extent of capital flight. Intuitively, the higher  $a$ , the more a group can appropriate, while still leaving other groups with a private rate of return equal to  $r$ . On the other hand, since it is independent of  $a$ , consumption remains unchanged. Hence, since capital flight is the difference between the appropriation and the consumption rates, a higher productivity of domestic capital leads to higher capital flight!

Note also that for sufficiently small  $a$  (but  $a > r$ ), capital flight is negative. That is, a country with low productivity of domestic capital will have a low appropriation rate and thus may experience capital inflows.

Next, we consider the evolution of domestic capital. By differentiating (10b) with respect to time, we get

$$\frac{\dot{k}^{\text{in}}}{k^{\text{in}}} = a - n \left( \frac{a - r}{n - 1} \right). \quad (16)$$

Note that the higher the physical productivity of domestic capital, the lower the rate of domestic capital accumulation. Intuitively, as  $a$  goes up, total output ( $ak$ ) increases proportionally. At the same time, the aggregate appropriation rate goes up by a factor of  $n/(n - 1)$  because the appropriation rate of each group increases by a factor of  $1/(n - 1)$ . Hence, total appropriation increases more than total output, with the corresponding decline in growth.

Since gross domestic output is proportional to domestic capital ( $ak$ ), the rate of economic growth is equal to the rate of domestic capital accumulation. Thus we can rephrase the results above as follows.

**PROPOSITION 3.** In an economy with an open capital account and poorly defined property rights, the higher the physical productivity of capital, the lower the rate of economic growth and the more severe the extent of capital flight.

In the representative agent models, the opposite occurs. This contrast illustrates the effects of the strategic element introduced by common access. Under common access, as capital becomes more produc-

tive, voracity increases, leading to a worse economic performance. This result may help to explain the bad performance of Brazil and Argentina and the good performance of Singapore and Korea, the former being resource-rich and the latter resource-poor.

We can also calculate the effects of changes in the foreign interest rate. First, higher  $r$  leads to higher economic growth because if a group has a more attractive opportunity abroad, the other groups will be forced to appropriate less. Second, unlike the effect of increments in  $a$ , the effect of increments in  $r$  on capital flight changes sign for different values of the intertemporal rate of substitution ( $\sigma$ ). For  $\sigma < n/(n - 1)$ , the higher the foreign interest rate, the smaller the extent of capital flight; the opposite occurs if  $\sigma > n/(n - 1)$ . Recall that capital flight is the difference between the appropriation and the consumption rates. When  $r$  increases, appropriation falls by  $[n/(n - 1)]dr$ . However, consumption may or may not fall. If the intertemporal rate of substitution is less than one, consumption goes up by  $n(1 - \sigma)dr$  and capital flight is reduced unambiguously. When  $\sigma > 1$ , consumption goes down. If  $\sigma > n/(n - 1)$ , this latter effect dominates and capital flight increases.

It is commonly argued that a negative aspect of opening the capital account in a poor country is the capital flight that may occur and the consequent reduction in growth. Does this lead to an advocacy of capital controls? Our results suggest that if a country suffers from the tragedy of the commons, then even if capital flight occurs, opening may enhance growth and welfare. In the closed economy, since groups have no alternative but to consume what they appropriate, the tragedy is reflected in a low savings rate. With the opening of the economy, the private-access technology acts as a disciplinary device, which may lead to a decline in the appropriation rate, an increase in the savings rate, and a welfare improvement. In this situation, the occurrence of capital flight would just imply that opening leads to a bigger fall in the marginal propensity to consume than in the appropriation rate (i.e., that  $\sigma > n/[n - 1]$ ).

Next we show that if  $r > \delta$  domestic capital accumulation can be positive when capital flight occurs. To see this, note that (i)  $k(t)$  is increasing if and only if  $[(a - r) - (n - 1)r]/(n - 1) < 0$ , and (ii) since  $r > \delta$ , it follows that

$$KF = \frac{(a - r) - (n - 1)[r(1 - \sigma) + \sigma\delta]}{n - 1} > \frac{(a - r) - (n - 1)r}{n - 1}.$$

Thus (16) can be positive when  $KF > 0$ . For example, if  $\sigma = 1$ ,  $k(t)$  is increasing when capital flight is positive if  $a$  lies in the interval  $(r + (n - 1)\delta, r + (n - 1)r)$ . This result is interesting because it reproduces the experience of some countries such as Argentina.

The welfare implications of opening the capital account follow from proposition 2. Just note that the one-asset economy corresponds to a closed capital account, that introducing the additional asset is equivalent to allowing perfect capital mobility,<sup>17</sup> and that the expression for capital flight in (15) is equal to the expression that determines the sign of  $U^{in} - U^o$  in (14). Thus from proposition 2 we have the following corollary.

**COROLLARY 1.** Opening the capital account is welfare improving if and only if the open economy is characterized by (i) capital flight and  $\sigma > n/(n - 1)$  or (ii) capital inflows and  $1 \leq \sigma < n/(n - 1)$ .

Finally, we address the following issue: Does an open economy with more interest groups grow faster than one with fewer? It depends on the size of the aggregate appropriation rate ( $n\beta$ ). With more groups, the individual appropriation rate must be lower in order to ensure that the domestic private rate of return and the foreign one remain equal ( $a - [n - 1]\beta = r$ ). However, more groups appropriate from the same pie. Since (10a) implies  $\Delta\beta/\beta = -\Delta n/(n - 1)$ , from (10b), (10e), and (15) we have the following proposition.<sup>18</sup>

**PROPOSITION 4.** Consider a set of open economies that have common access to capital held domestically, and let  $n > 1$ . Those economies with *more* interest groups will have a *greater* growth rate and less capital flight. Welfare is the same across the set.

### VI. Extreme Feedback Nash Equilibria

In these equilibria, groups choose their  $d_j$ 's on the boundaries of the appropriation set:  $\bar{\theta}$  and  $\underline{\theta}$ . In the pessimistic equilibrium, they set  $\beta = \bar{\theta}$ , and in the optimistic equilibrium they set  $\beta = \underline{\theta}$ . We impose the following restrictions on these bounds.<sup>19</sup>

$$\frac{a - r}{n} < \underline{\theta} < \frac{a - r}{n - 1} < \bar{\theta} < \infty. \tag{17}$$

Consider first the pessimistic equilibrium. This equilibrium emerges if all groups expect that at least one of the other groups will attempt to appropriate more than  $\beta^{in}$  of the common-access capital

<sup>17</sup> By perfect capital mobility we mean that domestic groups can borrow and lend abroad at the given foreign interest rate; i.e., it is appropriate to allow for negative capital flight. For some less developed countries, the relevant case is the one in which domestic groups cannot borrow abroad, i.e.,  $f_j(t) \geq 0$ .

<sup>18</sup> Note that  $\Delta(n\beta)/n\beta = (\Delta n/n) + (\Delta\beta/\beta) + (\Delta n\Delta\beta/n\beta)$ . Also, if one were to compare two economies, one with  $n = 1$  and another with  $n > 1$ , the opposite would hold.

<sup>19</sup> A bound of  $\bar{\theta} < \infty$  prevents infinite disinvestment rates;  $\underline{\theta} > (a - r)/n$  is necessary for the shadow value of the common- and private-access assets (the costate variables) to be positive. See (A11) and note that  $q(\beta) = \lambda(t)/\Phi(t)$  for all  $t$ . Since  $(a - r)/(n - 1)$  is the appropriation rate in the interior equilibrium,  $\bar{\theta} > (a - r)/(n - 1) > \underline{\theta}$  is just a definition.

stock and invest it in the private-access technology. In this case, it is a best response for every group to try to take as much capital as possible and run. Such an equilibrium resembles that found in Eaton (1987), in which pessimistic expectations lead to a sudden and total capital flight.<sup>20</sup>

In order to formalize the discussion, we denote by  $\beta_j$  and by  $\beta_{-j}$  the appropriation rates of group  $j$  and of the other groups, respectively. Now we ask the question, What is the best response of group  $j$  (denote it as  $\hat{\beta}_j$ ) to  $\beta_{-j} > \beta^{in}$ ? To answer this question, note that when  $\beta_j$  is different from  $\beta_{-j}$ , the utility attained by group  $j$  can be written as (see [A13'])

$$U_j^{dev} = \frac{\sigma}{\sigma - 1} \left[ \frac{\beta_j k(0)}{r - a + (n - 1)\beta_{-j} + \beta_j} \right]^{(\sigma-1)/\sigma} z^{-1/\sigma}. \tag{18}$$

We shall show that if  $\beta_{-j} > \beta^{in}$ , this expression is maximized at  $\beta_j = \bar{\theta}$ , its upper bound. First note that if  $\beta_{-j} > \beta^{in}$ , then  $r - a + (n - 1)\beta_{-j} > 0$ . Thus the derivative of the term in brackets in (18) with respect to  $\beta_j$  is positive. Second, since condition (11) ensures that  $z$  is positive, the result follows. Therefore,  $\hat{\beta}_j(\beta_{-j} > \beta^{in}) = \bar{\theta}$ .<sup>21</sup> With the same argument, it follows that every group will set its  $\beta$  equal to  $\bar{\theta}$ . Thus  $\beta = \bar{\theta}$  is an equilibrium.

Consider now the optimistic equilibrium. This equilibrium occurs when each group expects that others will appropriate less than  $\beta^{in}$ . In this case, the best response of each group is to set  $\beta = \underline{\theta}$ . The discussion above, conducted in terms of the pessimistic outcome, is applicable to this case. Since  $\beta_{-j} < \beta^{in}$ , it follows that  $r - a + (n - 1)\beta_{-j}$  is negative. Thus the derivative of the bracketed term in (18) with respect to  $\beta_j$  is negative. Consequently,  $\hat{\beta}_j(\beta_{-j} < \beta^{in}) = \underline{\theta}$ .

To sum up, in an extreme equilibrium,  $\beta$  is equal to  $\underline{\theta}$  or  $\bar{\theta}$ . There is a continuum of extreme equilibria, corresponding to the values of  $\underline{\theta}$  and  $\bar{\theta}$ . The allocation is given by (see [A8]–[A12])

$$q(\beta) = \frac{\beta}{r - a + n\beta}, \tag{19a}$$

$$k^e(t) = k(0)e^{(a-n\beta)t}, \tag{19b}$$

<sup>20</sup> In that model, capital flight happens when multiple equilibria are possible and a coordination failure lands the economy in the “bad” equilibrium. If a government has a fixed revenue need and can tax only capital, then the tax rate faced by an individual investor will depend on the tax base provided by others’ investments. If they invest at home, then the tax rate is low, the rate of return is high, and all is well. Expectations of capital flight are self-fulfilling: if others are anticipated to invest abroad, the expected tax rate is high, and investing abroad as well is optimal from an individual point of view. Hence, if there is capital flight, all capital leaves instantaneously.

<sup>21</sup> When  $\sigma < 1$ , (18) is negative. However, the exponent on the term in brackets is also negative. Thus the result holds.



$$f_j^e(t) = q(\beta)k(0)[e^{\sigma(r-\delta)t} - e^{(a-n\delta)t}], \tag{19c}$$

$$c_j^e(t) = q(\beta)zk(0)e^{\sigma(r-\delta)t} = Z[q(\beta)k^e(t) + f_j^e(t)]. \tag{19d}$$

The superscript  $e$  stands for extreme equilibrium from the two-asset economy;  $q(\beta)$  is the constant shadow value of the common-access asset in terms of the private-access asset (see n. 22 below). Indirect utility is given by

$$U_j^e = \frac{\sigma}{\sigma - 1} [q(\beta)k(0)]^{(\sigma-1)/\sigma} z^{-1/\sigma} \quad \text{for } \sigma \neq 1$$

$$= \frac{\log[q(\beta)k(0)\delta]}{\delta} + \frac{r - \delta}{\delta^2} \quad \text{for } \sigma = 1. \tag{19e}$$

Since  $q(\beta^{in}) = 1$ ,  $q(\bar{\theta}) < 1$ , and  $q(\underline{\theta}) > 1$ , it follows that welfare in the optimistic equilibrium is greater than under the interior equilibrium, which in turn is greater than in the pessimistic one. Note also that the same ranking can be made with respect to the private rates of return,  $a - (n - 1)\beta$ .

Next, we analyze whether the results of Section IV regarding the introduction of an inferior, private-access technology hold if the resulting equilibrium is extreme. First, from (4b) and (19b), we know that the tragedy of the commons is ameliorated if and only if the appropriation rate in the two-asset equilibrium ( $\beta$ ) is lower than under the one-asset equilibrium ( $\alpha$ ). Second, the sign of the welfare change is given by

$$\text{sgn}\{U^e - U^o\} = \text{sgn}\{\alpha - zq(\beta)^{1-\sigma}\}(\sigma - 1).$$

When  $\underline{\theta}$  tends toward its lower bound  $(a - r)/n$ ,  $q(\beta)$  tends to infinity and  $U^e > U^o$ . As  $\underline{\theta}$  increases, the difference between  $U^e$  and  $U^o$  diminishes.

Thus if the resulting equilibrium is extreme, introducing the inferior technology will, under some circumstances, ameliorate the tragedy of the commons and increase welfare. Note, however, that these two results do not need to hold for the same set of parameter values. Since  $zq(\alpha)^{1-\sigma} = [r(1 - \sigma) + \delta\sigma][(\alpha/r) - a + n\alpha]^{1-\sigma}$  is in general different from  $\alpha$ , it follows that  $\alpha = \beta$  does not imply  $U^e = U^o$ . Hence the sign of  $\alpha - \beta$  need not be equal to the sign of  $U^e - U^o$ , unless  $q(\beta) = 1$ , as is the case in the interior equilibrium.

**VII. Conclusions**

Start from a single-asset economy in which groups have “common access” to this asset. Now introduce a second asset that has an inferior rate of return but that is safe from the voracity of others. Will this additional choice be beneficial, irrelevant, or harmful?

The introduction of a technology with inferior productivity places a floor on the rate of return each group realizes on its holding of the common-access asset. If this floor is sufficiently high, it acts as a discipline device on competing interest groups, limiting the temptation to overappropriate and, hence, reducing the negative welfare implications of common access. On the other hand, if the rate of return on the inferior technology is low, it will be used in equilibrium, generating the wrong portfolio allocation, with the consequent social loss. Thus the introduction of an inferior technology, which under ordinary circumstances (i.e., no common access) would not be used, can actually ameliorate or worsen the tragedy of the commons!

In situations in which there is a tragedy of the commons, the first-best policy is to eliminate the common-access property of the system. The insight to be gleaned from this paper is that, in some circumstances, a second-best policy is to find a technology that has inferior productivity but enjoys private access. Even if this technology is not used, it may reduce appropriation and enhance growth.

The model has strong implications for capital flight and economic growth. When interest groups can appropriate each other's domestic assets, the opening of the capital account may generate capital flight to another country in which the physical rate of return is lower. Note, however, that the occurrence of capital flight does not imply that opening diminishes growth and welfare. The new opportunity acts as a disciplinary device that may reduce overappropriation. Such a result stands in contrast to a representative agent model, where capital flight reduces growth and welfare.

## Appendix

### A. Feedback Nash Equilibria of the Two-Asset Economy

In the differential game we consider, at each instant group  $j$  takes as given the strategies of the other  $n - 1$  groups  $\{d_{-j}(t)\}$  and chooses the sequences  $\{c_j(t)\}$  and  $\{d_j(t)\}$  in order to maximize (1) subject to (2), (3), (8), (17), and  $f_j(0) = 0$ . We assume that best responses depend linearly on the current value of the state variables and that they are symmetric. That is,  $d_j(t) = \beta k(t)$ . Next, we shall obtain endogenously the value of  $\beta$  for each of the Nash equilibria. The Hamiltonian of a representative group is

$$H_j = \frac{\sigma}{\sigma - 1} c_j(t)^{(\sigma-1)/\sigma} + \lambda(t)[ak(t) - d_j(t) - (n - 1)\beta k(t)] + \phi(t)[rf_j(t) + d_j(t) - c_j(t)] + \underline{\mu}(t)[d_j(t) - \theta k(t)] + \bar{\mu}(t)[\bar{\theta}k(t) - d_j(t)].$$

We do not impose explicitly the nonnegativity constraint (3) on  $k$ . We show below that it is satisfied. The first-order conditions for  $j$ 's problem are

$$c_j(t)^{-1/\sigma} = \phi(t), \tag{A1}$$

$$\underline{\mu}(t) - \bar{\mu}(t) = \lambda(t) - \phi(t), \tag{A2}$$

$$\frac{\partial \lambda(t)}{\partial t} = \lambda(t)[\delta - a + (n - 1)\beta] + \underline{\mu}(t)\underline{\theta} - \bar{\mu}(t)\bar{\theta}, \tag{A3}$$

$$\frac{\partial \phi(t)}{\partial t} = \phi(t)(\delta - r), \tag{A4}$$

$$\underline{\mu}(t)[d_j(t) - \underline{\theta}k(t)] = 0, \quad \underline{\mu}(t) \geq 0, \tag{A5}$$

$$\bar{\mu}(t)[\bar{\theta}k(t) - d_j(t)] = 0, \quad \bar{\mu}(t) \geq 0, \tag{A6}$$

and the transversality conditions

$$\lim_{t \rightarrow \infty} \lambda(t)k(t)e^{-\delta t} = 0, \quad \lim_{t \rightarrow \infty} \phi(t)f_j(t)e^{-\delta t} = 0. \tag{A7}$$

It follows from theorems 2 and 10 of Seierstad and Sydsaeter (1977) that conditions (A1)–(A7) are also sufficient for optimality because the control set is convex, and  $\max_{(c_j, d_j)} H_j$  is concave in  $(k, f_j)$ . The first principal minor of the Hessian of  $\max_{(c_j, d_j)} H_j(k, f_j)$  is negative—equal to  $(-1/\sigma)c_j(t)^{-(1+\sigma)/\sigma}$ —and the second is zero. Thus the Hessian is negative semidefinite.

When we solve the differential equation in (A4) and substitute the result in (A1), it follows that in any Nash equilibrium

$$c_j(t) = c_j(0)e^{\sigma(r-\delta)t}. \tag{A8}$$

Equation (A8) implies that consumption grows at the same rate in all Nash equilibria. Only  $c_j(0)$  differs across equilibria. From (2), (8), and (A8), it follows that

$$k(t) = k(0)e^{(a-n\beta)t} \tag{A9}$$

and

$$f_j(t) = \left[ qk(0) - \frac{c_j(0)}{r(1-\sigma) + \delta\sigma} \right] e^{rt} - qk(0)e^{(a-n\beta)t} + \frac{c_j(0)}{r(1-\sigma) + \delta\sigma} e^{\sigma(r-\delta)t}, \tag{A10}$$

where  $q = \lambda(t)/\phi(t)$ <sup>22</sup> is the constant shadow value of the common-access asset in terms of the private-access asset:

<sup>22</sup> To prove this, define  $p_t = \lambda_t/\phi_t$ . We shall show that  $p_t$  is a constant and is equal to  $q(\beta)$  in (A11). Consider the case in which  $\beta = \bar{\theta}$  (the same argument applies to different  $\beta$ 's). In this case,

$$\frac{\dot{p}_t}{p_t} = \frac{\dot{\lambda}_t}{\lambda_t} - \frac{\dot{\phi}_t}{\phi_t} = r - a + (n - 1)\bar{\theta} + \frac{\lambda_t - \phi_t}{\lambda_t}\bar{\theta} = r - a + n\bar{\theta} - \frac{\bar{\theta}}{p_t}.$$

Thus  $p_t = (\bar{\theta}/\alpha) - xe^{\alpha t}$ , where  $\alpha \equiv r - a + n\bar{\theta} > 0$ , and  $x$  is a constant of integration. The positive sign of  $\alpha$  follows from (17). To prove that  $p_t$  is a constant equal to  $q$ , it is sufficient to show that  $x = 0$ . To do this, use the first transversality condition in (A7) together with (A4) and (A9), and note that  $\alpha, k(0)$ , and  $\phi_0$  are positive:

$$\begin{aligned} \lim_{t \rightarrow \infty} \lambda(t)k(t)e^{-\delta t} &= \lim_{t \rightarrow \infty} p(t)\phi(t)k(t)e^{-\delta t} \\ &= \phi(0)k(0) \lim_{t \rightarrow \infty} \left( \frac{\bar{\theta}}{\alpha} e^{-\alpha t} - x \right) = 0 \Leftrightarrow x = 0. \end{aligned}$$

$$q(\beta) \equiv \frac{\beta}{r - a + n\beta} > 0 \quad \forall \beta \in [\underline{\theta}, \bar{\theta}]. \tag{A11}$$

Next, we consider the nonnegativity constraint (3). Since  $k(0) > 0$ , (A9) implies that  $k(t)$  will never be negative.

In order to obtain the value of  $c_j(0)$ , we use the second transversality condition. Substituting (A1), (A8), and (A10) into (A7), we get

$$\lim_{t \rightarrow \infty} \phi(0) \left\{ qk(0) - \frac{c_j(0)}{r(1 - \sigma) + \delta\sigma} - qk(0)e^{-(r-a+n\beta)t} - \frac{c_j(0)}{r(1 - \sigma) + \sigma\delta} e^{-[r(1-\sigma)+\sigma\delta]t} \right\} = 0.$$

Note that as  $t$  tends to infinity, the third term vanishes because we have assumed that  $\beta \geq \underline{\theta} > (a - r)/n$ . Condition (11) implies that the fourth term vanishes. Thus for (A7) to be satisfied, we need

$$c_j(0) = q(\beta)[r(1 - \sigma) + \delta\sigma]k(0). \tag{A12}$$

By substituting (A8) and (A12) in (1), we get the utility level attained by the representative group (when  $\sigma$  is different from one):

$$\begin{aligned} U_j &= \frac{\sigma}{\sigma - 1} [c_j(0)]^{(\sigma-1)/\sigma} \int_0^\infty e^{-[r(1-\sigma)+\delta\sigma]t} dt \\ &= \frac{\sigma}{\sigma - 1} [q(\beta)k(0)]^{(\sigma-1)/\sigma} [r(1 - \sigma) + \delta\sigma]^{-1/\sigma}. \end{aligned} \tag{A13}$$

Condition (11) implies that this integral exists. When  $\sigma = 1$ , the instantaneous utility function is given by  $\log(c)$ . Thus

$$U_j = \int_0^\infty \log[qk(0)\delta] e^{-\delta t} dt + \int_0^\infty (r - \delta)te^{-\delta t} dt = \frac{\log[q(\beta)k(0)\delta]}{\delta} + \frac{r - \delta}{\delta^2}.$$

The only difference among the three Nash equilibria is the value that  $\beta$  and  $q(\beta)$  take. In the interior equilibrium,  $\beta$  is in the interval  $(\underline{\theta}, \bar{\theta})$ . Thus the multipliers  $\underline{\mu}$  and  $\bar{\mu}$  are zero. Therefore, it follows from (A2) that  $\lambda = \phi$ . Hence, by setting (A3) and (A4) equal, we get

$$\beta^{in} = \frac{a - r}{n - 1}, \quad q(\beta^{in}) = 1. \tag{A14}$$

In the extreme equilibria, either  $\beta = \underline{\theta}$  or  $\beta = \bar{\theta}$ . In the first case,  $\bar{\mu} = 0$ , and in the second  $\underline{\mu} = 0$ .

*B. Unilateral Deviations*

If  $n - 1$  groups are following a strategy  $\beta_{-j}$  and group  $j$  chooses  $\beta_j$ , then its problem is to maximize (1) subject to (3), (8), (10), (17), and

$$\dot{k}(t) = a - (n - 1)\beta_{-j} - \beta_j. \tag{A15}$$

Following the same procedure as in section A, we get

$$c_j(t) = \frac{\beta_j[r(1 - \sigma) + \delta\sigma]}{r - a + (n - 1)\beta_{-j} + \beta_j} k(0)e^{\sigma(r-\delta)t}. \tag{A8'}$$

Thus the utility obtained by group  $j$  is

$$U_j^{\text{dev}} = \frac{\sigma}{\sigma - 1} \left[ \frac{\beta_j k(t)}{r - a + (n - 1)\beta_{-j} + \beta_j} \right]^{(\sigma-1)/\sigma} [r(1 - \sigma) + \delta\sigma]^{-1/\sigma}. \tag{A13'}$$

C. *The One-Asset Economy*

In this case each group maximizes (1) subject to (2), (3), and  $d_j(t) = \alpha k(t)$ . Since  $c_j(t) = d_j(t)$ , the Hamiltonian of group  $j$  is

$$H_j = \frac{\sigma}{\sigma - 1} c_j(t)^{(\sigma-1)/\sigma} + \psi(t)[\alpha k(t) - c_j(t) - (n - 1)\alpha k(t)].$$

The first-order conditions are

$$c_j(t)^{-1/\sigma} = \psi(t) \tag{A1'}$$

and

$$\dot{\psi} = \psi(t)[\delta - a + (n - 1)\alpha] \tag{A3'}$$

plus the transversality condition. From  $c_j(t) = \alpha k(t)$  and (A1'),

$$\frac{\dot{k}(t)}{k(t)} = \frac{\dot{c}(t)}{c(t)} = -\sigma \frac{\dot{\psi}(t)}{\psi(t)}.$$

Combining this with (2) and (A3'), we get

$$\alpha = \frac{a(1 - \sigma) + \delta\sigma}{n - \sigma(n - 1)}.$$

Substituting this value of  $\alpha$  in  $c_j(t) = \alpha k(t)$  and in (2), we obtain equations (4a) and (4b). The first-best is obtained by maximizing (1) (i.e., an average of utilities) subject to (2), (3), and  $\dot{k}(t) = \alpha k(t) - nc_j(t)$ . The first-order conditions are

$$c_j(t)^{-1/\sigma} = n\psi(t) \tag{A1''}$$

and

$$\dot{\psi} = \psi(t)(\delta - a). \tag{A3''}$$

Since  $c_j(t)$  has the form  $\eta k(t)$ , it follows that  $\sigma(a - \delta) = a - n\eta$ . Following the same procedures as above, we get (6a) in the text.

D. *Compatibility of the Restrictions on Parameters*

In this section we show that the restrictions we have imposed on the parameters (5), (7), (9), and (11) can be simultaneously satisfied and show also which signs of (15) are consistent with them. First we analyze (5) and (11), that is, whether or not the utility indexes in the one- and two-asset decentralized economies can be simultaneously bounded. We consider two cases.

Case A.1:  $a(1 - \sigma) + \delta\sigma > 0$

For (5) to be satisfied, it is necessary that  $\sigma < n/(n - 1)$ . For (11), it is necessary that  $\sigma < r/(r - \delta)$  if  $r > \sigma$  and  $\sigma > 0 > r/(r - \delta)$  if  $r \leq \sigma$ . Therefore,

$$\begin{aligned} \sigma &< \min \left\{ \frac{n}{n-1}, \frac{r}{r-\delta} \right\} && \text{if } r > \delta, \\ &< \frac{n}{n-1} && \text{if } r \leq \delta. \end{aligned} \tag{A16}$$

Case A.2:  $a(1 - \sigma) + \delta\sigma < 0$

For (5) to be satisfied, it is necessary that  $\sigma > n/(n - 1)$ . For (11), the condition is the same as in case A.1. Therefore, in this case it is necessary that  $r < n\sigma$  and

$$\begin{aligned} \frac{n}{n-1} < \sigma < \frac{r}{r-\delta} &&& \text{if } \delta < r < n\delta, \\ \frac{n}{n-1} < \sigma &&& \text{if } r \leq \delta. \end{aligned} \tag{A17}$$

Next, we check which signs of capital flight (15) are compatible with (5) and (11). We shall parameterize  $\sigma$  as follows:  $\sigma = \epsilon + [n/(n - 1)]$ . We consider three cases.

Case B.1:  $a(1 - \sigma) + \delta\sigma > 0$  and  $1 \leq \sigma < n/(n - 1)$

This is always compatible with a nonpositive sign for (15). No general statement can be given for a positive (15). To prove this, we set  $0 > \epsilon \geq -1/(n - 1)$  and let (15) be nonpositive. Thus we can express (15) and (5) as

$$a \leq (1 - n)r\epsilon + [n + (n - 1)\epsilon]\delta, \quad a < \frac{n + (n - 1)\epsilon}{1 + (n - 1)\epsilon} \delta. \tag{A18}$$

Denote these inequalities by  $a \leq Y(r; \epsilon, n, \delta)$  and  $a < X(\epsilon, n, \delta)$ , respectively. Since  $\epsilon < 0$  and  $a > r$ ,  $Y(r; \epsilon, n, \delta) < Y(a; \epsilon, n, \delta)$ . Since  $\epsilon \geq -1/(n - 1)$ , after some manipulations,  $a < Y(a; \epsilon, n, \delta)$  becomes  $a < X(\epsilon, n, \delta)$ , which is identical to the second inequality in (A18). Hence, a nonpositive (15) is compatible with (5) in this case. If (15) were positive, we would get  $a > Y(r; \epsilon, n, \delta) < Y(a; \epsilon, n, \delta)$ , from which no conclusion can be derived.

Case B.2:  $a(1 - \sigma) + \delta\sigma > 0$  and  $0 < \sigma < 1$

This is always incompatible with a nonpositive (15). No general statement can be given for a positive (15). To prove this, we set  $-n/(n - 1) < \epsilon < -1/(n - 1)$  and let (15) be nonpositive. With the same procedure as above, it follows that  $a \leq Y(r; \epsilon, n, \delta) < Y(a; \epsilon, n, \delta)$  becomes  $a > X(\epsilon, n, \delta)$ , which contradicts (5) as expressed in (A18).

Case B.3:  $a(1 - \sigma) + \delta\sigma < 0$ ,  $n/(n - 1) < \sigma < r/(r - \delta)$ , and  $r < n\delta$

This is always compatible with a positive (15). No general statement can be given for a nonpositive (15). To prove this, let (15) be positive and set  $\epsilon > 0$ . Note that in this case (A18) becomes  $a > Y(r; \epsilon, n, \delta)$  and  $a > X(\epsilon, n, \delta)$ . Since  $\epsilon > 0$ , it follows that  $a > Y(r; \epsilon, n, \delta) > Y(a; \epsilon, n, \delta)$ , which yields (5), that is,  $a > X(\epsilon, n, \delta)$ . For a nonpositive (15), no general conclusions can be derived.

It follows from (A16) and (A17) that the arguments in cases B.1–B.3 are

robust to restriction (11). Case B.1 holds also if  $\sigma < r/(r - \delta) < n/(n - 1)$ , case B.2 holds because  $r/(r - \delta)$  and  $n/(n - 1)$  are greater than one, and case B.3 holds for any  $\sigma > n/(n - 1)$ . Finally, if one restricts (7) to be satisfied simultaneously with (5) and (11), then only cases A.1, B.1, and B.2 are relevant.

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