

**THE TRANSITIONAL DYNAMICS OF FISCAL POLICY:
LONG-RUN CAPITAL ACCUMULATION, AND GROWTH**

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1. Introduction

The effect of fiscal policy on capital accumulation and long-run growth continues to be a fundamental issue in macroeconomics. An initial attraction of the endogenous growth model was that it assigned a key role to fiscal policy as a determinant of long-run economic growth; see Barro (1990), Rebelo (1991), Jones Manuelli and Rossi (1993), Ireland (1994), Turnovsky (1996, 2000). However, these models suffer several shortcomings, leading to a reassessment of their merits. First, endogenous growth obtains only under strict knife-edge conditions on the technology; see Solow (1994). Second, such models are frequently associated with “scale effects”, meaning that the steady-state growth rate increases with the size (scale) of the economy, as indexed by say population. But empirical evidence by Backus, Backus and Kehoe (1992) for the United States and by Jones (1995b) for OECD economies does not support such scale effects. Moreover, Easterly and Rebelo (1993) and Stokey and Rebelo (1995) find at best weak evidence for the effects of fiscal instrument on the long-run rate of growth, although Kneller, Bleaney, and Gemmell (1999) argue that these results are biased because of the incomplete specification of the government budget constraint. The third limitation is that the widely employed one-sector version of the AK endogenous growth model implies instantaneous adjustment to fiscal and other shocks; there are no transitional dynamics. This too contradicts the empirical evidence suggesting that per capita output converges to its steady-state equilibrium rate at around 2-3% per annum; see Barro (1991), Barro and Sala-i-Martin (1992), Mankiw, Romer, and Weil (1992).¹

These considerations have stimulated the development of non-scale growth models; see Jones (1995a, 1995b), Segerstrom (1998) and Young (1998). The advantage of such models is that they are consistent with balanced growth under quite general production structures. Indeed, if the knife-edge restriction that generates traditional endogenous growth models is not imposed, then any

¹ Subsequent studies suggest that the convergence rates are more variable and sensitive to time periods and the set of countries than originally suggested and a wider range of estimates have been obtained. For example, Islam (1995) estimates the rate of convergence to be 4.7% for nonoil countries and 9.7% for OECD economies. Temple (1998) estimates the rate of convergence for OECD countries to be between 1.5% and 3.6% and for non-oil countries to be between 0.3% and 6.7%. Evans (1997) obtains estimates of the convergence rate of around 6% per annum.

stable balanced growth equilibrium is characterized by the absence of scale effects. The non-scale equilibrium is therefore the norm. In this case the long-run equilibrium growth rate is determined by technological parameters and is independent of macro policy instruments.

But the fact that the equilibrium growth rate is independent of fiscal effects does not imply that fiscal policy is unimportant for long-run economic performance. In fact quite the contrary is true. First, fiscal policy has important effects on the *levels* of key economic variables, such as the per capita stock of capital and output. Moreover, the non-scale model typically yields slow asymptotic speeds of convergence, consistent with the empirical evidence of 2-3% per annum; see Eicher and Turnovsky (1999).² This implies that policy changes can affect growth rates for sustained periods of time so that their accumulated effects during the transition from one equilibrium to another may therefore translate to potentially large impacts on steady-state levels. Thus, although the economy grows at the same rate across steady states, the corresponding bases upon which the growth rates compound may be substantially different.

These considerations suggest that attention should be directed to determining the impact of fiscal policy on the transitional dynamics. This is the focus of the present paper. The model we employ is of a one-sector economy in which output depends upon the stocks of both private and public capital, as well as endogenously supplied labor. Public capital introduces a positive externality in production, so that the complete production function is one of overall increasing returns to scale in these three productive factors. In addition to accumulating public capital, the government allocates resources to a utility-enhancing consumption good. These expenditures are financed by taxing capital, labor income, and consumption, or by imposing non-distortionary lump-sum taxation. We set out the dynamic equilibrium of this economy and show how the stable adjustment is characterized by a two dimensional locus in terms of the two stationary variables, referred to as “scale-adjusted” per capita stocks of private and public capital.

Specifying the dynamics in terms of these scale-adjusted variables is important. Being stationary, long-run changes in these quantities reflect the accumulated effects of policy changes

² The short-run speeds of convergence are higher and vary over time.

during the transition. The fact that the transitional paths are two-dimensional introduces flexibility to the dynamics. This contrasts with the standard one-sector neoclassical growth model, or the familiar two-sector Lucas endogenous growth model in which the stable locus is one-dimensional, so that all variables converge at the same constant speed; see Bond, Wang, and Yip (1996), Ortigueira and Santos (1997). Instead, two-dimensional manifolds imply that the convergence speeds will vary through time and across variables, often dramatically, allowing different variables to follow very different transitional paths; see Eicher and Turnovsky (1999). This characteristic is relevant to the empirical evidence of Bernard and Jones (1996a, 1996b) who find that while growth rates of output among OECD countries converge, the growth rates of manufacturing technologies exhibit markedly different time profiles.³

Our analysis focuses on two aspects. First, we characterize the steady state equilibrium and analyze the effects of various fiscal changes on the long-run labor-leisure allocation, the long-run changes in the capital stocks, and output. We compare the long-run effects of the two forms of government expenditure – investment versus consumption – and changes in the alternative tax rates. We show that lump-sum tax-financed expenditure increases of equal magnitudes on the two types of public goods have identical positive effects on long-run employment. These effects are smaller proportionately than are the impacts on capital and output. A higher fraction of output devoted to government consumption leads to identical proportional increases in the long-run stock of private capital, public capital, and output. These effects are all smaller than are the corresponding long-run impacts of an equivalent increase in government investment, which has its greatest impact on public capital. All distortionary taxes are contractionary, and the consumption and wage taxes are equivalent to a reduction in government consumption expenditure. A higher tax on capital income has its most adverse effect on private capital. Distortionary tax-financed increases in either form of expenditure are shown to be amalgams of these effects

³The empirical evidence on the constancy of convergence rates is mixed. Barro and Sala-i-Martin (1995), who abstract from technological change, can reject constancy for Japan, but not for the US or Europe. Nevertheless, all reported rates of convergence (0.4%-3%, 0.4%-6%, and 0.7%-3.4% for Japan, the US, and Europe respectively) are similar in the range that our non-scale model generates.

Most of our attention is devoted to calibrating the model to a benchmark economy and assessing the numerical effects of various types of policy shocks, relative to the benchmark. We consider both the long-run equilibrium responses and the transitional adjustment paths. Particular attention is devoted to the welfare of the representative agent, both the time profile of instantaneous utility and the intertemporal welfare, as represented by the discounted sum of the short-run benefits. The implications for the government's intertemporal budget balance are also addressed. Our numerical analysis yields many interesting insights into the transitional dynamics, and the following are noted. First, the asymptotic speeds of adjustment are remarkably stable across different tax rates, being around 2.7%, consistent with much of the empirical evidence. The effects of fiscal shocks on growth rates are in fact pervasive; indeed most fiscal shocks still exert significant impacts on growth rates over substantial periods of time (say 20 periods). Second, the magnitudes of the long-run responses are often substantial. For example, a 6 percentage point increase in the fraction of output devoted to government investment more than doubles the long-run stock of government capital relative to the benchmark economy. Third, substituting income taxes with a consumption tax leads to a short-run welfare loss, which is more than offset by gains through subsequent growth leading to an overall welfare gain for the consumer. Similarly, while this substitution raises the current deficit through time, the higher transitory growth rates reduces the discounted value, leading to a favorable effect on the government's intertemporal balance. This example, as do others we consider, highlight the importance of the intertemporal aspect of fiscal policy.

There is an extensive recent literature analyzing fiscal policy on capital accumulation and growth. Before proceeding, we wish to indicate the relationship of this paper to that literature. We have already noted the AK models pioneered by Barro (1990) and Rebelo (1991). Government production expenditure in these models is introduced mostly as a flow; see Barro (1990), Ireland (1994), Turnovsky (1996, 2000), Bruce and Turnovsky (1999). But to the extent that productive government expenditure represents infrastructure it is more appropriately modeled as a stock. Futagami, Morita, and Shibata (1993) and Turnovsky (1997) follow this approach in an AK model with fixed labor supply, showing how the transitional dynamics are represented by a one-

dimensional locus. Baxter and King (1993) analyze the dynamics of fiscal policy in a stationary Ramsey model, emphasizing particularly the role of productive government expenditure on the government expenditure multiplier. The present analysis also addresses this in a growing economy, though focusing on somewhat different aspects. Specifically, a more disaggregated set of fiscal shocks is considered and the intertemporal welfare implications emphasized.

The remainder of the paper is structured as follows. Section 2 sets out the model, while its equilibrium dynamics are characterized in Section 3. Section 4 discusses some of the long-run comparative static properties. Section 5 discusses the stationary equilibrium for the first-best optimum, mainly as a benchmark against which the calibrations of Section 6 can be judged. Section 7 concludes and some technical details are provided in the Appendix.

2. The Model

The economy consists of N identical individuals, each of whom has an infinite planning horizon and possesses perfect foresight. Each individual is endowed with a unit of time that can be allocated either to leisure, l_i or to labor, $(1 - l_i)$. Labor is fully employed so that total labor supply, equal to population, N , grows exponentially at the steady rate $\dot{N} = nN$. The representative agent produces output, Y_i , using the Cobb-Douglas production function

$$Y_i = \alpha(1 - l_i)^{1-\sigma} K_i^\sigma K_G^\eta \quad 0 < \sigma < 1, \eta > 0 \quad (1a)$$

where K_i denotes the agent's individual stock of private capital, and K_G is the stock of government capital, such as infrastructure. We assume that the services derived from the latter are not subject to congestion, so that K_G is a pure public good. The producer faces constant returns to scale in the two private factors, and increasing returns to scale, $1 + \eta$, in all three factors of production.

The representative agent's welfare is represented by the intertemporal isoelastic utility function:

$$\int_0^{\infty} (V_\gamma)(C_i^\theta H^\phi)^\gamma e^{-\rho t} dt; \quad (1b)$$

$$\eta > 0, \theta > 0; \quad - < \gamma < 1; \quad 1 > \gamma(1 + \phi); \quad 1 > \gamma(1 + \theta + \phi)$$

where C denotes aggregate consumption, so that per capita consumption of the individual agent at time t is $C/N = C_i$, H denotes the consumption services of a government-provided consumption good, and the parameters θ and ϕ measure the impacts of leisure and public consumption on the agent's welfare.⁴ The remaining constraints on the coefficients appearing in (1b) are imposed in order to ensure that the utility function is concave in the quantities C_i, l_i , and H .

The agent's objective is to maximize (1b) subject to his capital accumulation equation

$$\dot{K}_i = [(1 - \tau_k)r - n - \delta_K]K_i + (1 - \tau_w)w(1 - l_i) - (1 + \tau_c)C_i - T_i \quad (1c)$$

where r = gross return to capital, w = (before-tax) wage rate, τ_k = tax on capital income, τ_w = tax on wage income, τ_c = consumption tax, $T_i = T/N$ = agent's share of lump-sum taxes (transfers). Equation (1c) embodies the assumption that private capital depreciates at the rate δ_K , so that with the growing population the net after-tax private return to capital is $(1 - \tau_k)r - n - \delta_K$.

Performing the optimization, yields:

$$C_i^{\gamma-1} l_i^{\theta\gamma} H^{\phi\gamma} = \lambda_i(1 + \tau_c) \quad (2a)$$

$$\theta C_i^{\gamma} l_i^{\theta\gamma-1} H^{\phi\gamma} = \lambda_i w(1 - \tau_w) \quad (2b)$$

$$r(1 - \tau_k) - n - \delta_K = \rho - \frac{\dot{\lambda}_i}{\lambda_i} \quad (2c)$$

Equation (2a) equates the marginal utility of consumption to the individual's tax-adjusted shadow value of wealth, λ_i , while (2b) equates the marginal utility of leisure to its opportunity cost, the after-tax real wage, valued at the shadow value of wealth.⁵ The third equation is the standard Keynes-Ramsey consumption rule, equating rate of return on consumption to the after-tax rate of

⁴ The parameter γ is related to the intertemporal elasticity of substitution, s say, by $s = 1/(1 - \gamma)$.

⁵ Since all agents are identical each devotes the same fraction of time to leisure, and henceforth we can drop the agent's subscript to l .

return on capital. Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality condition must be imposed:

$$\lim_t \lambda_i K_i e^{-\rho t} = 0 \quad (2d)$$

Aggregating over the individual production functions, (1a), aggregate output, Y , is

$$Y = NY_i = \alpha [(1-l)N]^{1-\sigma} K^\sigma K_G^\eta \quad (3)$$

where $K = NK_i$ denotes aggregate capital. The equilibrium real return to private capital and the real wage are thus respectively:

$$r = \frac{\partial Y}{\partial K} = \frac{\sigma Y}{K} = \frac{\sigma Y_i}{K_i}; \quad w = \frac{\partial Y}{\partial N} = \frac{(1-\sigma)Y}{N(1-l)} = \frac{(1-\sigma)Y_i}{(1-l)} \quad (4)$$

Government capital accumulates in accordance with

$$\dot{K}_G = G - \delta_G K_G \quad (5)$$

where G denotes the gross rate of government investment expenditure, and government capital depreciates at the rate δ_G . The government finances its gross expenditure flows from aggregate tax revenues earned on capital income, labor income, consumption, or lump-sum in accordance with:

$$G + H = \tau_k r K + \tau_w w (1-l)N + \tau_c C + T \quad (6)$$

Aggregating (1c) over the N individuals and combining with (6) leads to the aggregate resource constraint

$$\dot{K} = Y - C - G - H - \delta_K K \quad (7)$$

Finally, we assume that the government sets its current gross expenditures on the consumption good and the investment good as fixed fractions of output, namely:

$$G = gY \quad (8a)$$

$$H = hY \quad (8b)$$

where g, h are fixed policy parameters. Using (8a) and (8b), together with the optimality conditions (4), we may express the government's budget as

$$T = [g + h - \tau_k \sigma - \tau_w (1 - \sigma) - \tau_c (C/Y)]Y \quad (9)$$

Written in this way, (9) expresses the primary budget deficit and therefore the amount of lump-sum taxation (or transfers) necessary to maintain current balance. T is therefore a measure of current fiscal imbalance; Bruce and Turnovsky (1999).⁶ Substituting (8a), (8b) into (7), we may write the growth rate of private capital as:

$$\frac{\dot{K}}{K} = 1 - g - h - \frac{C}{Y} \frac{Y}{K} - \delta_K \quad (10a)$$

Likewise, substituting (8a) into (5), the growth rate of public capital may be written as:

$$\frac{\dot{K}_G}{K_G} = g \frac{Y}{K_G} - \delta_G \quad (10b)$$

3. Equilibrium Dynamics

Our objective is to analyze the dynamics of the aggregate economy about a stationary growth path. Along such an equilibrium path, aggregate output, private capital stock and public capital are assumed to grow at the same constant rate, so that the output-capital ratio and the ratio of public capital to private capital remain constant, while the fraction of time devoted to leisure remains constant. Taking percentage changes of the aggregate production function, the long-run equilibrium growth rate of output, private and public capital, ψ , is:

$$\psi = \frac{1 - \sigma}{1 - \sigma - \eta} n \quad (11)$$

We shall show that one condition for the dynamics to be stable is that $\sigma + \eta < 1$, in which case the long-run equilibrium growth rate $\psi > 0$. As long as government capital is productive, (11) implies that long run per capita growth is positive as well.

⁶Because of Ricardian Equivalence, the lump-sum tax is equivalent to debt.

To analyze the transitional dynamics of the economy about the long-run stationary growth path, we express the system in terms of the following stationary variables: (i) the fraction of time devoted to leisure, l , and (ii) the scale-adjusted per capita quantities⁷

$$k = \frac{K}{N^{(1-\sigma)\gamma(1-\sigma-\eta)}}; \quad k_g = \frac{K_G}{N^{(1-\sigma)\gamma(1-\sigma-\eta)}}; \quad y = \frac{Y}{N^{(1-\sigma)\gamma(1-\sigma-\eta)}} \quad (12)$$

Using this notation, the scale-adjusted output can be written as:

$$y = \alpha(1-l)^{1-\sigma} k^\sigma k_g^\eta \quad (13)$$

In the Appendix we show how the equilibrium dynamics can be expressed as the following system in the redefined stationary variables, l , k , k_g :

$$\begin{aligned} \dot{l} = F(l) &= (1-\tau_k)\sigma - [1-\gamma(1+\phi)]\sigma(1-c-g-h) + \frac{\eta g k}{k_g} \left[\frac{y}{k} - \delta_K(1-\sigma[1-\gamma(1+\phi)]) \right] \\ &+ \delta_G \eta [1-\gamma(1+\phi)] - \left([(1-\sigma)[1-\gamma(1+\phi)] + \gamma n] + \rho \right) \end{aligned} \quad (14a)$$

$$\frac{\dot{k}}{k} = (1-c-g-h) \frac{y}{k} - \delta_K - \psi \quad (14b)$$

$$\frac{\dot{k}_g}{k_g} = g \frac{y}{k_g} - \delta_G - \psi \quad (14c)$$

where⁸

$$c = \frac{C}{Y} = \frac{1-\sigma}{\theta} \frac{l}{1-l} \frac{1-\tau_w}{1+\tau_c} \quad (14d)$$

$$F(l) = \frac{l(1-l)}{(1-\gamma) - (1-\sigma)[1-\gamma(1+\phi)]l - \theta\gamma(1-l)}$$

Equation (14d) is obtained by dividing the optimality conditions (2a) and (2b), while noting (4). It asserts that the marginal rate of substitution between consumption and leisure, which grows with per

⁷ Under constant returns to scale, these expressions reduce to per capita quantities, as in the usual neoclassical model.

⁸ We shall assume that $F(l) > 0$. Sufficient conditions that ensure this is so include: (i) $\gamma < 0$, and (ii) $\sigma > \phi$, both of which are plausible empirically, and imposed in our numerical simulations.

capita consumption, must equal the tax-adjusted wage rate, which grows with per capita income.⁹ With leisure being complementary to consumption in utility, the equilibrium consumption-output ratio thus increases with leisure.

The steady state to this economy, denoted by “ \sim ” can be summarized by:

$$(1 - \tilde{c} - g - h) \frac{\tilde{y}}{k} = \delta_K + \psi \quad (15a)$$

$$g \frac{\tilde{y}}{k_g} + \delta_G + \psi \quad (15b)$$

$$(1 - \tau_k) \sigma \frac{\tilde{y}}{k} = \delta_K + \rho + [1 - \gamma(1 + \phi)]\psi + \gamma n \quad (15c)$$

together with the production function, (13), and (14d). These five equations determine the steady-state equilibrium in the following sequential manner. First, (15c) determines the output-capital ratio so that the long-run net return to private capital equals the rate of return on consumption. Having determined the output-capital ratio, (15a) determines the consumption-output ratio consistent with the growth rate of capital necessary to equip the growing labor force and replace depreciation, while (15b) determines the corresponding equilibrium ratio of public to private capital. Given \tilde{c} , (14d) determines the corresponding allocation of time, l . Having obtained y/k , k_g/k , l , the production function then determines k , with k_g then being obtained from (15b). Moreover, given the restrictions on utility and production this solution is unique, and economically viable in the sense of all quantities being non-negative, and in particular the fractions $0 < \tilde{c} < 1$, $0 < \tilde{l} < 1$, if and only if¹⁰

$$\frac{(1 - g - h)}{\sigma(1 - \tau_k)} > \frac{\delta_K + \psi}{\delta_K + \rho + [1 - \gamma(1 + \phi)]\psi + \gamma n}$$

Linearizing around the steady state denoted by \tilde{l} , \tilde{k} , \tilde{k}_g , the dynamics may be approximated by:

⁹ The marginal rate of substitution between consumption and leisure is $\theta C_l/l$. Equating this to the tax-adjusted real wage. Given in (4), yields (14d).

¹⁰ This condition holds throughout our simulations.

$$\begin{array}{l} \dot{l} \\ \dot{k} \\ \dot{k}_g \end{array} = \begin{array}{cccc} a_{11} & a_{12} & a_{13} & l - \tilde{l} \\ \frac{-y}{1-l} (1-\sigma)(1-c-g-h) - \frac{c}{l} & -(1-\sigma)\frac{y}{k}[1-c-g-h] & \eta\frac{y}{k_g}[1-c-g-h] & k - \tilde{k} \\ -\frac{g(1-\sigma)y}{1-l} & \frac{g\sigma y}{k} & \frac{g(\eta-1)y}{k_g} & k_g - \tilde{k}_g \end{array} \quad (16)$$

where

$$\begin{aligned} a_{11} &= \frac{F(y/k)}{1-l} - G(1-\sigma) + [1-\gamma(1+\phi)]\frac{\sigma c}{l} \\ a_{12} &= -\frac{Fy}{k^2} - G(1-\sigma) + [1-\gamma(1+\phi)]\frac{\eta g k}{k_g} ; \quad a_{13} = \frac{Fy\eta}{kk_g} - G - [1-\gamma(1+\phi)]\frac{\eta g k}{k_g} \\ G &= (1-\tau_k)\sigma - [1-\gamma(1+\phi)]\sigma(1-c-g-h) + \frac{\eta g k}{k_g} \end{aligned}$$

We can readily establish that the determinant of the matrix is proportional to $(1-\sigma-\eta)$, so that provided $\eta < 1-\sigma$ the determinant is positive, which means that there are either 3 positive or 1 positive roots. This condition imposes an upper bound on the positive externality generated by government capital. Unfortunately, due to the complexity of the system we cannot find a simple general condition to rule out the explosive growth case of three positive roots. But one condition that does suffice to do so is if (i) $\gamma = 0$ and (ii) $\tilde{c} > (\delta_K + \psi)(1-\sigma-\eta) + (1-\sigma)\rho$. This latter condition holds in our simulations, and indeed, in all of the many simulations carried out over a wide range of plausible parameter sets, 1 positive and 2 negative roots were always obtained. Thus since the system features two state variables, k and k_g and one jump variable, l , we are confident that the equilibrium is generally characterized by a unique stable saddlepath.¹¹

3.1 Characterization of Transitional Dynamics

¹¹ The fact that our simulations are associated with unique stable saddlepaths does not rule out the possibility of more complex dynamic behavior for other less plausible parameter values. In cases where $\gamma > 0$ (intertemporal elasticity of substitution greater than unity) and for large shares of government expenditure (in excess of 40%) it is possible to obtain complex roots, giving rise to cyclical behavior.

Henceforth we assume that the stability properties are ensured so that we can denote the two stable roots by μ_1, μ_2 , with $\mu_2 < \mu_1 < 0$. The two state variables are scale-adjusted public and private capital. The generic form of the stable solution for these variables is given by:

$$k(t) - \tilde{k} = B_1 e^{\mu_1 t} + B_2 e^{\mu_2 t} \quad (17a)$$

$$k_g(t) - \tilde{k}_g = B_1 v_{21} e^{\mu_1 t} + B_2 v_{22} e^{\mu_2 t} \quad (17b)$$

where B_1, B_2 are constants and the vector $(1 \quad v_{2i} \quad v_{3i}) \quad i=1,2$ (where the prime denotes vector transpose) is the normalized eigenvector associated with the stable eigenvalue, μ_i . The constants, B_1, B_2 , appearing in the solution (17) are determined by initial conditions, and depend upon the specific shocks. Thus suppose that the economy starts out with given initial stocks of private and public capital, k_0, k_{g0} , and through some policy shock converges to \tilde{k}, \tilde{k}_g . Setting $t = 0$ in (17a), (17b) and letting $d\tilde{k} = \tilde{k} - k_0, d\tilde{k}_g = \tilde{k}_g - k_{g0}$, B_1, B_2 are given by:

$$B_1 = \frac{d\tilde{k}_g - v_{22} d\tilde{k}}{v_{22} - v_{21}}; \quad B_2 = \frac{v_{21} d\tilde{k} - d\tilde{k}_g}{v_{22} - v_{21}} \quad (18)$$

Having thus derived B_1, B_2 , the implied time path for leisure is determined by

$$l(t) - \tilde{l} = B_1 v_{31} e^{\mu_1 t} + B_2 v_{32} e^{\mu_2 t} \quad (17c)$$

so that $l(0) = \tilde{l} + B_1 v_{31} + B_2 v_{32}$ is now determined in response to the shock. In studying the dynamics, we are interested in characterizing the slope along the transitional path in $k - k_g$ space. In general, this is given by:

$$\frac{dk_g}{dk} = \frac{B_1 v_{21} \mu_1 e^{\mu_1 t} + B_2 v_{22} \mu_2 e^{\mu_2 t}}{B_1 \mu_1 e^{\mu_1 t} + B_2 \mu_2 e^{\mu_2 t}} \quad (19)$$

and is time varying. Note that since $0 > \mu_1 > \mu_2$, as $t \rightarrow \infty$ this converges to the new steady state along the direction $(dk_g/dk)_t = v_{21}$, for all shocks. The initial direction of motion is obtained by setting $t = 0$ in (12) and depends upon the source of the shock.¹²

One key issue, discussed at length in the recent growth literature, concerns the speed of convergence along the transitional path; see Barro and Sala-i-Martin (1992) and Ortigueira and Santos (1997). In previous growth models, in which all variables moved in proportion to one another, the associated unique stable eigenvalue sufficed to characterize the transition. Equations (17a) and (17b) highlight the fact that with the transitional dynamics being governed by two stable eigenvalues, $0 > \mu_1 > \mu_2$, say, the speeds of adjustment of the two types of capital are neither constant nor equal over time, although asymptotically, all scale-adjusted variables converge to their respective equilibrium at the rate corresponding to the larger negative eigenvalue, $-\mu_1$.

In general, we define the speed of convergence at time t , of a variable x say, as

$$\phi_x(t) = -\frac{\dot{x}(t) - \dot{\tilde{x}}}{x(t) - \tilde{x}} \quad (20)$$

where \tilde{x} is the equilibrium balanced growth path, which may or may not be stationary, depending upon the specific variable.¹³ In the case where \tilde{x} is constant or follows a steady growth path and the stable manifold is one dimensional, this measure equals the magnitude of the unique stable eigenvalue (see Ortigueira and Santos 1997). Most of the discussion focuses on the convergence speeds of per capita quantities, particularly per output and capital, $Y/N, K/N$ which in steady-state equilibrium grow at the rates $\psi - n$. Applying the measure (20) to say K/N , we find:

$$\phi_K(t) = -\frac{\dot{k}(t)}{k(t) - \tilde{k}} - (\psi - n) = \phi_k(t) - (\psi - n) \quad (21)$$

¹² In the Appendix we indicate the formal solutions for temporary shocks. These relations underly the numerical simulations of the temporary shocks undertaken in section 6.C.

¹³ In a converging economy this measure is positive. A negative value implies divergence. Because of the nonlinear stable adjustment path it is possible for $x(t)$ to overshoot its long-run equilibrium, \tilde{x} , during the transition. If that occurs, then at the point of overshooting, the speed of convergence will become infinite (positively or negatively, depending upon the direction of motion). An example of this occurs when government investment is financed by a tax on capital and is illustrated in Fig. 8 below.

with analogous adjustments for the other variables. All per capita variables converge asymptotically to their respective steady-state at the same rate $-\mu_1 - (\psi - n)$, although during the transition the rates of convergence of different variables will deviate markedly, as our simulations below will illustrate.

4. Some Steady State Fiscal Effects

Table 1 summarizes the long-run effects of fiscal changes on key economic variables. By the nature of the non-scale model, the long-run growth rate is unaffected. The responses of the scale-adjusted per capita quantities, k , k_g , and y are, however, very important because they describe the effects on the base levels on which the constant steady-state growth rates compound. They represent the accumulated effects on the growth rates during the transition, and as our numerical results shall indicate, they can turn out to be of very significant magnitudes. Part A of Table 1 considers partial effects, assuming that the changes are accommodated by lump-sum taxes. From these expressions we see the following responses.

An increase in government consumption expenditure h has no effect on either the y/k or the y/k_g ratios (see (15b), (15c)). The two capital stocks and output therefore all change proportionately. Given that y/k remains constant, the increase in h crowds out an equal quantity of private consumption. The reduction in c reduces the utility of leisure leading to a substitution toward more labor. The productivity of capital increases so that k , k_g , and y in fact all increase proportionately.¹⁴ An increase in the rate of government investment, g , has the same impact on consumption and therefore on labor. It also leaves the y/k ratio unchanged, but it reduces the y/k_g ratio. Thus it has a relatively larger impact on public capital than on either private capital or output. By directly influencing the stock of a productive factor, government investment is more expansionary than is an equivalent amount of government consumption expenditure.

Since the wage tax and consumption tax impact through the ratio $(1 - \tau_w)/(1 + \tau_c)$ they have similar effects, which in fact are identical if these taxes are initially zero. Otherwise, the wage tax has a more potent impact and each operates precisely as a negative consumption expenditure shock.

¹⁴ The fact that output changes in proportion to the two capital stocks, despite the less proportionate change in employment, is a consequence of the overall increasing returns to scale of the production function in the three factors.

A higher tax on capital increases the y/k ratio, while leaving the y/k_g ratio unchanged. The reduction in the net return to capital induces a switch toward consumption and leisure. The decline in the labor supply reduces the productivity of both types of capital, and because of the adverse effect on the return to private capital, has a relatively larger (negative) impact on private capital than it does on either public capital or output.

Part B presents expenditure shocks that are financed by changes in distortionary taxes that ensure that the initial government deficit remains unchanged; subsequent deficits brought about through growth, are financed by lump-sum taxes.¹⁵ In the case where government consumption expenditure, h , is financed by a higher consumption tax, the contractionary effects of the latter just offset the expansionary effects of the former; employment and production remain unchanged. In the case where h is financed by a tax on wage income, the y/k and y/k_g ratios remain unchanged, again leading to proportionate adjustments in k , k_g , and y . Whether the contractionary effect of the higher τ_w on employment dominates the positive effect of the higher h depends upon whether $l \gtrless \theta/(1+\theta)$. In the numerical examples in Section 6 this condition is uniformly satisfied, in which case wage tax-financed government consumption expenditure will reduce the long-run stocks of capital and output, all proportionately. By increasing the y/k ratio, a higher government consumption expenditure financed by a higher tax on capital is likely to have a more contractionary effect on labor and therefore on capital stocks and output. Output and public capital decline proportionately, and private capital even more so.

While a consumption tax-financed increase in government investment has no effect on employment, by directly increasing the stock of public capital it enhances the productivity of private capital and therefore output. The impacts of a wage tax-financed increase in g differ from those of an increase in h , by the introduction of a similar direct productive component. This is likely to dominate the other component, so that all effects may be expansionary as in our simulations which introduce differential effects that favor public capital over both output and private capital. In the

¹⁵ Holding T/Y constant, the changes in the tax rates induced by the changes in government expenditure, $i = g, h$ are: $\partial\tau_w/\partial i = 1/\beta$; $\partial\tau_k/\partial i = 1/(1-\beta)$; $\partial\tau_c/\partial i = 1/c - c\partial\tau_c/\partial i$.

final case where the productive expenditure is financed by a tax on capital, we find a clear ranking; private capital declines more than output, which in turn declines more than public capital.

5. Steady-State Equilibrium in the Centrally Planned Economy

As a benchmark for understanding the numerical results, it is useful to set out the steady-state equilibrium for the centrally planned economy in which the planner controls resources directly.¹⁶ The optimality conditions for such an economy consist of equations (13), (15a), (15b), together with:

$$c \frac{C}{Y} = \frac{1-\sigma}{\theta} \frac{l}{1-l} s \quad (14d')$$

$$s\sigma \frac{\tilde{y}}{k} = \delta_K + \rho + [1 - \gamma (1 + \phi)]\psi + \gamma n \quad (15c')$$

$$s = 1 + (q - 1)g + (\phi c - h) \quad (22a)$$

$$s\sigma \frac{\tilde{y}}{k} - \delta_K = s \frac{\eta}{q} \frac{\tilde{y}}{k_g} - \delta_G \quad (22b)$$

where s denotes the shadow price of a marginal unit of output in terms of capital and q denotes the shadow price of public capital in terms of private capital. These equations determine the steady-state solutions for c, y, l, k, k_g, s, q in terms of the arbitrarily set expenditure parameters g and h .

In contrast to the decentralized economy the after-tax prices relevant for marginal rate of substitution condition in (14d') and for the return to capital in (15c') are replaced by the relative price of output in terms of consumption (capital). Equation (22a) determines the relative price of output to capital. In the absence of government expenditure, $s = 1$. Otherwise, the social value of a unit of output deviates from the social value of capital due to the claims of government on output and the value this has for the consumer. Specifically, with the size of government expenditure being tied to aggregate output, an increase in output will divert resources away from private consumption,

¹⁶ The transitional dynamics turn out to be described by a higher (fifth) order system, the extra dynamic variable being the relative shadow values of the two capital stocks. To replicate the transitional dynamics of the centrally planned economy requires the introduction of a time-varying tax rate in the decentralized economy. An example of this for an AK technology (where the dynamics are lower order) is provided by Turnovsky (1997).

leaving $1 - g - h$ available to the agent. But offsetting this, public investment augments the stock of public capital, valued at qg and provides utility benefits equal to ϕc , making the overall value of output to capital as described in (22a). The final equation equates the long-run net social returns to investments in the two types of capital.

Choosing the expenditure shares g and h optimally, implies the first-best optimum is

$$\hat{h} = \phi c; \quad \hat{q} = 1, \text{ and hence } s = 1.$$

The marginal benefit of government consumption expenditure should equal its resource cost, while the shadow values of the two types of capital should be equated. Substituting these conditions into (15b), (15c') and (22b), the optimal share of output devoted to government production expenditure is

$$\hat{g} = \frac{\eta[\delta_G + \psi]}{\delta_G + \rho + \frac{[1 - \gamma(1 + \phi)](1 - \sigma)}{1 - \sigma - \eta} + \gamma n} \quad (23)$$

which provided $\gamma < 0$ implies $\hat{g} < \eta$.¹⁷

Equating (14d') to (14d) and (15c) to (15c'), we see that the decentralized economy can replicate the steady state of the centrally planned economy if and only if the tax rates are set in accordance with

$$1 - \tau_k = \frac{1 - \tau_w}{1 + \tau_c} = s \quad (24)$$

In the case that expenditures are set optimally, (24) simplifies to

$$\hat{\tau}_k = 0, \quad \hat{\tau}_c = -\hat{\tau}_w \quad (24')$$

That is, capital income should remain untaxed, while the tax on consumption must be equal and opposite to that on wage income. Interpreting the tax on wage income as a negative tax on leisure, (19') says that in the absence of any externality, the optimal tax structure requires that the two

¹⁷ This result contrasts with the optimal government expenditure in the Barro (1990) model, where, when government production expenditure impacts output as a flow, $\hat{g} = \eta$, it is consistent with endogenous growth models, in which government production appears as a stock; see Futagami, Murata, and Shibata (1993), Turnovsky (1997).

utility-enhancing goods, consumption and leisure, be taxed uniformly. This result can be viewed as an intertemporal application of the Ramsey principle of optimal taxation; see Deaton (1981), Lucas and Stokey (1983). If the utility function is multiplicatively separable in c and l , as we are assuming here, then the uniform taxation of leisure and consumption is optimal.¹⁸

6. Numerical Analysis of Transitional Paths

Further insights into the effects of fiscal policy can be obtained by carrying out numerical analysis of the model. We begin by characterizing a benchmark economy, calibrating the model using the following parameters representative of the U.S. economy:

<i>Production parameters:</i>	$\alpha = 1, \sigma = 0.35, \eta = 0.20, n = 0.015$
<i>Preference parameters:</i>	$s = 1/(1 - \gamma) = 0.4, \text{ i.e. } \gamma = -1.5, \rho = 0.04, \theta = 1.75, \phi = 0.3$
<i>Fiscal parameters:</i>	$g = 0.08, h = 0.14, \tau_w = 0.28, \tau_k = 0.28, \tau_c = 0$

The elasticity on capital implies that approximately 35% of output accrues to private capital and the rest to labor, which grows at the annual rate of 1.5%. The elasticity $\eta = 0.20$ on public capital implies that public capital generates a significant externality in production. The chosen value is somewhat smaller than the extreme value (0.39) suggested by Aschauer (1989) and lies within the range of the consensus estimates; see Gramlich (1994).

The preference parameters imply an intertemporal elasticity of substitution in consumption of around 0.4. The elasticity of leisure $\theta = 1.75$ accords with the value generally chosen by real business cycle theorists and implies an elasticity of substitution in labor supply of around 1.0 consistent with early estimates obtained; see e.g. Lucas and Rapping (1969).¹⁹ The elasticity of 0.3

¹⁸ Two other points should be noted. First, the optimal tax must also be consistent with the government budget constraint. Given that the constraints on tax rates, $\tau_w < 1, \tau_c > 0$, this may well require the supplementation by lump-sum taxation in order to sustain the equilibrium. In addition, these tax rates replicate only the steady state. To replicate the transitional dynamics of the centrally planned economy requires a time-varying tax rate, as noted.

¹⁹ The evidence on the elasticities of labor substitution are mixed; see Mankiw, Rotemberg, and Summers (1985).

on government consumption implies that the optimal ratio of government consumption to private consumption is 0.3.

Our benchmark setting $\tau_w = 0.28$ reflects the average marginal personal income tax rate in the US. Given the complex nature of capital income taxes, part of which may be taxed at a lower rate than wages, and part of which at a higher rate, we have chosen the common rate $\tau_k = 0.28$ as the benchmark. The benchmark assumes a zero consumption tax. Government expenditure parameters have been chosen so that the total fraction of net national production devoted to government expenditure on goods and services equals 0.22, the historical average in the United States. The breakdown between $g_c = 0.14$ and $g_p = 0.08$ is arbitrary, but plausible. Government investment expenditure is less than 0.08 and our choice of $g_p = 0.08$ is motivated by the fact that a substantial fraction of government consumption expenditure, such as public health services, impact as much on productivity as they do on utility.

These parameters lead to the following plausible benchmark equilibrium, reported in Row 1 of Table 2: fraction of time allocated to leisure: $l = 0.71$, consumption-output ratio = 0.64; the ratio of public to private capital = 0.58.²⁰ The equilibrium levels of scale adjusted private capital, public capital, and output are 0.57, 0.33, and 0.30, respectively. Since these units are arbitrary (depending on α) they have all been normalized to unity. The corresponding quantities in the rows below are all measured relative to the respective benchmark values of unity.²¹ In addition the steady-state growth rate, which by the non-scale nature of the economy is independent of policy, equals 2.17%.

The table also reports the two stable eigenvalues, which for the benchmark economy are approximately -0.034 , and -0.104 . These imply that per capita output and capital converge at the asymptotic rate of approximately 2.7%, consistent with the accepted empirical evidence. An interesting feature of our results is that both stable roots are remarkably stable over all the fiscal

²⁰Estimates of time allocation studies suggest that households allocate somewhat less than one-third of their discretionary time to market activities (labor) and our equilibrium value $l = 0.71$ is generally consistent with that. Direct evidence on the ratio of public to private capital is sparse. Using the following relationships: (i) $\dot{K} = I - \delta_K K$, $\dot{K}_G = G - \delta_G K_G$. Assuming that on the balanced growth that $\dot{K}/K = \dot{K}_G/K_G$ and $\delta_K = \delta_G$, implies the long-run relationship $K_G/K = k_g/k = G/I$. Taking $G = 0.08$, $I = 0.14$ yields the long-run ratio of public to private capital of around 0.57.

²¹ Thus, for example, in Row 3 the new steady-state stocks of private and public capital when h is increased by 0.06 are 0.629.

exercises conducted.²² The unstable root (not reported) is much larger, and much more variable across policies. This implies that the speeds of adjustments are fairly uniform across permanent fiscal changes, though they may vary across temporary policy changes.

Two other measures of economic performance are summarized in Table 1. Economic welfare is the optimized utility of the representative agent:

$$W = \int_0^{\infty} Z(t) dt = \int_0^{\infty} \frac{1}{\gamma} ((C/N)l^\theta H^\phi)^\gamma e^{-\rho t} dt \quad (25)$$

where $Z(t)$ denotes instantaneous utility and C/N , l , H are evaluated along the equilibrium path. The welfare gains reported in the final column are calculated as the percentage change in the flow of base income necessary to maintain the level of welfare unchanged in response to the policy shock. Defining the primary deficit to be $T(t) = [g + h + v - \tau_w(1 - \sigma) - \tau_k\sigma - \tau_c c]Y(t)$, the quantity

$$V = \int_0^{\infty} T(t) e^{-s(1-\tau_k)t} dt \quad (26)$$

where $s(1 - \tau_k) = r(1 - \tau_k) - \delta_K$ is the implied equilibrium rate of interest, measures the present discounted value of the lump-sum taxes necessary to continuously balance the budget. This depends in part upon government transfers, v , which are taken to be 0.12, close to the long-run historical average for the United States. The quantity V is thus a measure of the intertemporal imbalance of the fiscal deficit. Since both V and Y are proportional to α , which has been set arbitrarily to unity, we interpret V as a fraction of initial base income. Evaluating (26) along the balanced growth path we find that in the benchmark case, $V = 0.298$, or nearly 30% of the initial base income. Much of this due to the assumed size of transfers of 12% of current income. In their absence, (26) implies an intertemporal fiscal surplus; in that case tax revenues exceed government expenditures on goods and services by around 6% of current income.²³

Row 2 in Table 2 describes the decentralized economy that would replicate the optimal centrally planned economy. In such an economy leisure would be reduced to 0.59 and the

²² The speed of convergence is much more sensitive to structural changes, such as changes in the productive elasticities.

²³ This accords approximately with recent data on surplus of government account on goods and services.

consumption to output ratio to 0.54. This would lead to private and public capital stocks in excess of three times the stocks in the benchmark economy and to a 240% higher flow of income. Such a steady state would require government consumption and investment to be somewhat larger than in the benchmark economy, namely 16.2% and 10.9% of output, respectively. To sustain this equilibrium the tax on capital should be eliminated and the tax on consumption and leisure equated. The long-run welfare gains from implementing this policy immediately are nearly 16%. They would be even larger if the taxes were time-varying to replicate the transition path as well.

6.1 Uncompensated Fiscal Changes

Rows 3-7 in Table 2A describe various basic policy changes from the benchmark economy. These are uncompensated, meaning that they lead to changes in the government's current deficit, T . In some cases, the corresponding dynamic transition paths are illustrated [Figs. 1-4]. One striking pattern throughout all simulations is that the labor supply responds almost completely upon impact to an unanticipated permanent shock. After the initial jump, the transitional path for labor supply is virtually flat. The reason for this is that for plausible parameter values the elements a_{12}, a_{13} in the transitional matrix in (16) are both small relative to $a_{11} > 0$; there is little feedback from the changing stocks of capital to labor supply. In other words, the dynamics of labor can be approximated by the *unstable* first order system:

$$dl(t)/dt = a_{11}(l(t) - \tilde{l})$$

which for bounded behavior essentially requires that l jump to steady state.

Increase in government consumption expenditure: An increase in h of 0.06 increases long-run private capital, public capital, and output proportionately by 10%, while crowding out the long-run private consumption–output ratio by 0.06 percentage points, with a corresponding reduction in leisure by 0.02. The additional government expenditure leads to a substantial deterioration in the government's long-run balance; V more than doubles to 0.6. The lower private consumption and higher labor supply reduce welfare, but the higher government consumption is welfare increasing.

The accumulation of capital and growth of output over time implies that the fall in absolute consumption is small so that overall welfare rises by 3.13% relative to the benchmark.

The dynamics of this shock are illustrated in the 6 panels of Fig. 1. The immediate effect of the lump-sum tax financed increase in expenditure is to reduce the private agent's wealth, inducing him to supply more labor thereby raising the marginal productivity of both types of capital and raising output.

The phase diagram 1.A indicates that the two capital stocks accumulate approximately proportionately. This is also reflected in their growth rates, which both jump initially to around 2.5% and gradually decline to their steady-state rates of around 2.2%, as both types of capital accumulate and their respective rates of return decline. The convergence rates of all per capita quantities are roughly equal, and close to the asymptotic rate of 2.7% throughout most of the transition. The initial increase in the labor supply (which remains virtually unchanged thereafter), and the associated decline in consumption, is welfare deteriorating, though this is more than offset by the direct benefits of the higher expenditure on the public consumption good. Upon impact, the initial expenditure increase raises instantaneous welfare by 1% and this grows uniformly with the growth in the capital stocks and output, to an asymptotic improvement relative to the benchmark, of over 6%, the present value of which is 3.13%, as noted. Finally, the expenditure increase immediately doubles the current government deficit, which then continues to increase modestly throughout the transition.

Increase in government production expenditure: An increase in g by 0.06 leads to large increases in long-run private capital and output of 41.5%. Long run public capital increases by 147%, raising the ratio of public to private capital from 0.58 to 1.02. The effects on employment and the consumption-output ratio are precisely as for h , as noted in Table 1. The reduction in initial consumption and leisure, illustrated in Fig 2B, with no public expenditure benefits, leads to an initial reduction in welfare of 5% (Fig. 2.E). The initial claim on capital by the government crowds out private investment, so that the growth rate of private capital is reduced below the growth rate of population; the scale-adjusted stock of private capital therefore initially declines. By contrast, public capital

initially grows at over 7% (see Fig. 2.C). As the new public capital is put in place, its productivity raises the growth rate of private capital, and as output and consumption grow, so does welfare, and asymptotically the instantaneous welfare rises 25% above the base level. The present value of this increase, after allowing for the initial loss, is around 4.4%. Finally, public and private capital converge at different rates. The initial decline in scale adjusted private capital means that per capita private capital initially diverges, while public capital converges at the rate of 4%.

Increase in Tax on Capital: Row 5 raises the tax on capital income from 0.28 to 0.40. This has a dramatic long-run effect, reducing private capital by about 30% and public capital and output by around 16%. The higher tax revenues improve the government's long-run balance significantly.²⁴ The dynamics are illustrated in Fig. 3. The higher tax on capital reduces the return to labor leading to an initial substitution toward more leisure and consumption. Initially welfare rises by 4% (Fig. 3.E). Upon impact, the higher tax reduces the growth rate on private capital to almost zero, so that the scale-adjusted per capital stock, k , begins to fall rapidly. The reduction in labor and the reduction in private capital reduces the growth rate of output, and the growth rate of public capital begins to fall as well, so that k and k_g follow the declining paths in Fig. 3.A. As private capital increases in relative scarcity its productivity rises, inducing investment in private capital and thereby restoring its growth rate. This in turn raises the productivity of public capital, the declining growth rate of which is reversed after 15 periods. The initial rapid decline in the growth rate of private capital implies that it initially converges at a relatively fast rate of 6%, while public capital starts out in quite the opposite way. The steady decline in relative consumption implies that after the initial increase, instantaneous welfare declines steadily relative to the benchmark, declining asymptotically by about 12.5%. The present value of this decline is equivalent to a 3.54% reduction in base income.

²⁴ Policy discussions have raised the possibility that in a growing economy *reducing* the income tax, particularly on capital will stimulate growth and thereby improve the long-run government balance. This possibility, known in policy circles as 'dynamic scoring' has been investigated in an AK model by Bruce and Turnovsky (1999), who find that it may hold only if the intertemporal elasticity of substitution exceeds unity. We have investigated this possibility in a number of simulations and never found it to obtain. This is because any positive effects on the growth rate occur only along the transitional path and are therefore only temporary.

Increase in Tax on Wage Income and Consumption: The final two unilateral tax changes are an increase in the wage tax from 0.28 to 0.40 and the introduction of a consumption tax of 10%. They are generally qualitatively similar, and only the former is illustrated (see Fig. 4). In some respects the dynamics of the economy are opposite to that of an increase in h ; in particular public and private capital follow similar contractionary paths. By falling on a broader base, labor, the wage tax generates more revenues and the initial government deficit is transformed to a substantial surplus. The steady decline in consumption, to around 84% of the benchmark economy is mirrored in a steady decline in welfare, relative to the benchmark, despite the increase in leisure and overall welfare declines by 5.4. Under a consumption tax, these contractions and losses are mitigated (cf. Rows 6 and 7).

6.2 Revenue-Neutral Fiscal Changes

Many discussions of fiscal policy focus on revenue neutral policy changes. Table 2.B describes a number of fiscal changes where the change is accompanied by some other accommodating change so that the current government deficit, T , remains unchanged (see footnote 15).

Tax Substitution: Row 1 introduces a 10% consumption tax, which permits income taxes to be reduced to 21.8% to maintain the initial deficit, T , unchanged. This change in the tax structure raises long-run output and public capital by 7.3% and private capital by 16.5%, relative to the benchmark economy. The shift from the wage tax to the consumption tax causes a slight reduction in long-run leisure, with a proportionately slightly larger reduction in the consumption-output ratio. The government's long-run fiscal balance, V , improves slightly and intertemporal welfare improves by 1.3%. The dynamics are illustrated in Fig. 5. The reduction in the tax on capital favors private capital the growth rate of which rises to over 3.2% on impact. This stimulates the productivity of public capital, the growth rate of which also rises in the short run, though more modestly. Private capital thus converges initially much faster than does public capital. The tax switch causes an initial drop in consumption of 2% and an increase in labor supply of 1%, thus leading to an initial

deterioration in welfare. However, the higher output level and resulting higher consumption throughout the transition causes current welfare to improve over time, doing so by 5% asymptotically. Observe, also, that the move from a uniform income tax to a consumption tax increases the current deficit at each instant of time. However, the higher rate of return on capital and resulting higher interest rate, reduces the present value, so that the intertemporal fiscal imbalance actually improves, as noted.

Row 2 describes a more drastic version of the same experiment, whereby the current income tax is eliminated entirely, and the current government deficit financed by a 49.4% consumption tax. This has very favorable effects on the long-run stocks of capital and growth, leading to an overall welfare gain of 4.3%, together with a significant reduction in the fiscal imbalance. Both these cases support the proposition, being increasingly debated in policy circles, of moving away from taxing income to taxing consumption.

Restructuring Government: Row 3 describes the case where the government changes its expenditure priorities, while maintaining the initial deficit. Specifically, it increases its investment expenditure from 8% to 14% of output, and this requires it to reduce its expenditure on consumption from 14% to 8.1% of output. This switch in spending priorities has no effect on long-run employment or the consumption-output ratio. It raises long-run private capital and output by 28.4% and public capital by 125%. Long-run government fiscal balance improves slightly, but intertemporal welfare declines by 3.7%.

The dynamics are illustrated in Fig. 6. The initial switch toward government investment crowds out private investment, the growth of which is initially reduced to around 1%, and thus the private scale-adjusted capital stock declines. Public capital initially grows at over 6% and as its stock accumulates, the productivity of private capital is enhanced and the incentives to invest increase. The decline in the private capital stock is reversed and it begins to accumulate. The reduction in public consumption induces agents to switch initially to more private consumption and more leisure. However, these effects are small, and while they yield a small positive impact on short-run welfare, this is heavily outweighed by the direct welfare loss due to the reduction in

government consumption. On balance, welfare is reduced by 10% in the short run. Over time, as the government investment yields its benefits to production, output and consumption grow relative to the benchmark and the initial welfare losses become welfare gains. However, since these occur in the distant future (after 20 years) they are discounted heavily and on balance intertemporal welfare declines, as noted. As in the case of the tax switch, the expenditure switch increases the current deficit at each instant of time, as output expands. However, the higher rate of return on capital, reduces the present value, so that fiscal imbalance again declines.

One final comment on this case concerns the result that the expenditure switch is associated with a reduction in intertemporal welfare. This is a consequence of the fact that the proposed change [g from 0.08 to 0.14, h from 0.14 to 0.08] is taking the expenditure mix further away from the first best optimum [$g = 0.11$, $h = 0.16$]. If the expenditure switch is reversed we find that the economy experiences a welfare gain of around 3.9%.

Alternatively-Financed Modes of Government Consumption Expenditure: Rows 4 – 6 describe the effects of increasing government consumption expenditure, h , from 0.14 to 0.20, under the three alternatives of (i) consumption tax-financing; (ii) wage tax-financing; and (iii) capital tax-financing, such that the initial deficit remains unchanged. These effects can also be compared with those of lump-sum tax financing summarized in Table 2.A, Row 3. These results confirm the analytical effects reported in Table 1B. Some aspects of the corresponding dynamics are illustrated in Fig. 7.

The interesting aspect of the results is the sharp contrast in the effects of government consumption expenditure between the alternative modes of finance. This is most dramatic in the case of the phase diagram in Fig 7.A. Suppose that the initial equilibrium is at point A. As discussed previously, a lump-sum tax financed increase in government consumption expenditure is expansionary, increasing private and public capital proportionately, along AB. If it is consumption tax-financed, the contractionary effect of the tax just balances the expansionary effect of the expenditure and there is no net change; the economy remains at A. If it is wage tax-financed, private and public capital again move proportionately. However, this time, the contractionary effect dominates the expansionary effect and both decline along the direction AC. Finally, if the

expenditure is financed by a tax on capital, the contractionary effect on capital is more than proportionate and the economy evolves along AD, with a larger contraction in both capital goods.

These responses are reflected in the time paths for welfare illustrated in panel B. As noted before, under lump-sum tax financing there is a uniform steady improvement in current welfare from 1% to 6%, relative to the benchmark, which in present terms is 3.13%. Although the consumption tax has no effect on activity and therefore no induced impact on welfare, the higher government consumption expenditure being financed does have a uniform welfare benefit of just under 1%, the present value of which is 0.7%. Under wage-tax financing, there are small short-run benefits associated with the higher expenditure, but these are more than offset by the long-run losses as the economy contracts, leading to an overall long-run small welfare loss of -0.31% . The intertemporal tradeoff in welfare is more dramatic in the case where the expenditure is capital tax financed. In that case, the reduction in private capital accumulation and employment, together with the switch in private consumption, together with the higher government consumption, leads to short-run welfare gain of about 7% on impact. However, this quickly erodes as the economy contracts, leading to an asymptotic reduction of around 12%, and an overall intertemporal welfare loss in present value terms of nearly 2%.

The final panel in Figure 7 plots the time profiles of the standard government expenditure multipliers. These are given by

$$M_H(t) = \frac{dY(t)}{d[hY(t)]}$$

This represents the change in current output per unit of change in government consumption expenditure induced by the policy change h . It mirrors closely the time profile of the changes in welfare. In the case of lump-sum tax financing M_H starts around 0.7, converging to approximately 1.1. It is zero for consumption tax-financing and it is around -0.4 for wage tax-financing. Surprisingly, M_H is initially very slightly positive in the case of capital tax financing. This is because the tax on capital income has a less adverse effect on labor supply than does the wage tax.

But over time, as the economy contracts, the output multiplier becomes strongly negative, converging to around -5 .

Alternatively-Financed Modes of Government Production Expenditure: Rows 7 – 9 describe the corresponding effects of increasing government investment expenditure from 0.08 to 0.14. These effects can also be compared with those of lump-sum tax financing summarized in Table 2.A, Row 4. And confirm the qualitative effects summarized in Table 1B. Some aspects of the corresponding dynamics are illustrated in Fig. 8.

While the time paths for the two capital stocks exhibit interesting differences across financing modes, these differences are less dramatic than in the previous case of government consumption expenditure. In all cases government capital increases steadily, as does private capital ultimately in the case of lump-sum, consumption, and wage tax financing. An interesting contrast arises in the case of capital tax-financing. Private capital declines over time and actually overshoots its long-run decline after about 8 periods, before being partially restored. At that point its rate of convergence has a singularity, becoming infinite as the point is traversed.

The time profiles for the instantaneous welfare changes contrast sharply with those of government consumption expenditure. In the short run welfare declines by about 5% for the three cases of lump-sum tax, consumption tax, and wage tax financing. This is because the government investment stimulates employment, leading to substitution against consumption, while providing no initial direct benefits. Over time, as the government investment yields its productive payoff and output rises, instantaneous welfare in these three cases rises steadily. With lump sum tax financing it converges to a higher long-run level of 25% above that of the benchmark economy, with a present value of 4.35%; with a consumption tax it converges to 19% higher with a present value of 1.7%; for the wage tax it converges to 15% higher with a present value of 0.5%. The case of capital-tax financing again provides an interesting contrast. The initial reduction in employment leads to an instantaneous welfare gain. As the higher capital tax exerts its bite and private capital declines, there is a welfare loss, lasting through 40 periods. But after that, when the new government capital is in

place, the economy experiences a long run welfare gain. However, The second phase dominates, and overall there is a net welfare loss of around 1.4%.

These results suggests that irrespective of the mode of finance, spending on government investment provides larger intertemporal benefits than does comparable government consumption. But again, it should be borne in mind that this depends upon the level of the particular government expenditure relative to the optimum. If, for example, government consumption expenditure was much further from the social optimum than is the stock of government capital, then the relative desirability of the two forms of public expenditure would be amended.

Figure 8.C plots the time profiles of the corresponding government expenditure multipliers. These are all increasing through time and all are ultimately positive. Thus the lump-sum tax financed multiplier starts out below unity and converges to 3.5, exhibiting a pattern that is entirely consistent with that of Baxter and King (1993). Because of the initial positive employment effects the wage tax and consumption tax financed expenditures are initially slightly negative, but they eventually converge to values in excess of two. While the capital tax-financed expenditure is initially zero and approaches unity.

6.3 Temporary Increases in Expenditure

The formal solutions are presented in the Appendix. A critical aspect of the solution is the behavior of leisure (labor supply) at time T say, the time the temporary fiscal shock ceases. If the termination of the policy is correctly foreseen, the shadow price should adjust continuously at that instant. Whether or not employment moves discontinuously at that instant then depends upon whether the shock impinges directly on the consumer's optimality conditions (2a), (2b). Since government consumption expenditure, h , does impact directly on these conditions, it causes a jump in $l(T)$, which is mirrored in a corresponding jump in both consumption and output at that time. By contrast, since government investment, g , does not influence these conditions, it does not generate any jump in $l(T)$.

Government Consumption Expenditure: Figure 9 illustrates some of the dynamics of an unanticipated increase in government consumption expenditure, which is (correctly) expected to last 25 periods. The initial response is similar to that of a permanent increase; marginal utility of wealth initially rises, labor supply increases relative to the benchmark, and so does output, inducing the accumulation of both private and public capital. However, being temporary the marginal utility of wealth discounts the change so that the initial jumps are dampened. As the economy evolves, the government expenditure is viewed as increasingly temporary. The marginal utility of wealth declines, employment is reduced and the productivity of private capital declines. After around 18 periods its growth rate falls below the equilibrium rate, ψ . The scale-adjusted stock of private capital begins to decline, the accumulation of public capital levels off and scale-adjusted output begins to decline as well. By 25 periods, when the government expenditure is about to cease, the growth rate of private capital has been reduced to 1.5%, and that of public capital to around 2.2%. The output level is approximately 5% above that of the benchmark economy. At that time, when the government expenditure ceases, additional resources are available for private consumption, which jumps up by 4 percentage points, causing employment and therefore output to drop by 5 percentage points and 3 percentage points, respectively. The reduction in output causes the growth rate of public capital to drop to under 2% and for private capital jump to jump up to around 2.2%, and thereafter both converge to the common long-run growth rate ψ .

The initial increase in government consumption crowds out private consumption. While this is welfare-deteriorating, this is more than offset by the direct benefits provided by government consumption, so that on impact welfare improves by 1%. The increase in leisure and increase in consumption during the initial phase (prior to 25) leads to further welfare gains. However, the elimination of government expenditure at time 25 lead to a sharp reduction in welfare at that time, taking it below that of the benchmark economy. The subsequent continuing decline labor supply and increasing consumption implies that welfare continues to rise. Overall the initial gains more than outweigh the subsequent losses and intertemporal welfare improves by around 2.2%. This is a smaller gain than if the shock were permanent and indeed we can show that the gains are proportional to the length of the shock.

Finally, one can see in Fig 9 is that the speed of adjustment throughout the first 25 periods, while the temporary shock is in effect, is much more rapid than during the second phase, when the expansion is permanently removed. In Fig 9A, for example the locus BCD is traversed in 25 periods and is much longer than the return locus DB, which is completed only asymptotically.

Government Production Expenditure: Figure 10 illustrates the case of a 25 period increase in government production expenditure. Again, the economy starts as if the shock is permanent, with output and labor growing relative to the benchmark economy, though much more modestly than if the expansion were permanent. Private capital initially declines slightly, but increases as its productivity is enhanced by the rapidly growing public capital (more than 7% initially). After 20 periods, as the end of the fiscal expansion approaches, the marginal utility of wealth declines and employment and output begin to decline, as does the growth rate of private capital, so that its scale-adjusted stock declines as well. After period 25, when government investment ceases, public capital begins to decline, while the growth rate of private capital increases to over 4%. The fact that this rate of investment is higher than that in the previous case of temporary government consumption expenditure is a consequence of the positive permanent impact of the temporary public investment on the economy's productive capacity. As private capital accumulates over this second phase, its rate of return declines and so does its growth rate. In fact it is reduced below its long-run equilibrium growth, so that scale-adjusted private capital actually declines during the final phase (beyond time 50).

As for the permanent shock, the economy experiences an initial welfare loss of 5% before the benefits of the government investment are realized. However, large temporary gains do eventuate, reaching a maximum of around 13% a few years after the investment is terminated. Taken in conjunction with the initial losses, the present value of welfare increases by 3.4%.

7. Conclusions

This paper has analyzed the effects of fiscal policies in a non-scale growing economy with public and private capital. We have characterized the equilibrium dynamics and have been

concerned with contrasting two types of government expenditure – expenditure on an investment good and expenditure on a consumption good – under different modes of tax financing. Most of our attention has focused on the numerical simulations of a calibrated economy, which we have found to provide helpful intuition. We have obtained many results, of which the following merit noting at this stage.

1. Despite the fact that fiscal policy in such an economy has no effect on the long-run equilibrium growth rate, the slow rate of convergence implies that fiscal policy exerts has a sustained impact on growth rates for substantial periods during the transition. These accumulate to substantial effects on the long-run equilibrium levels of crucial economic variables, including welfare.

2. As examples of the accumulated impacts of policy, an increase in government investment from 0.08 to 0.14 of output raises the long level of output by 40%. Raising the tax on capital income from 0.28 to 0.40 reduces long-run output by 16%.

3. For the calibrated economy allocating a fixed fraction of output to government investment is better than allocating the same resources to government consumption. However, the intertemporal time profiles of the respective benefits are different. The benefits of (lump-sum tax-financed) government consumption are uniformly positive; government investment involves short-run losses, which are more than more than offset over time. These comparisons depend upon the sizes of the two government expenditures, relative to their respective first-best optimal values and could be reversed in other situations.

4. The time paths and growth rates of private and public capital contrast sharply for policies such as g and τ_k , which impact on one or other directly; they move closely for those fiscal shocks -- h, τ_w, τ_c -- which do not impact directly on either form of capital. The most dramatic contrasts in the time paths for the two types of capital occur with respect to an increase in government consumption expenditure, under the four alternative modes of tax financing. Long-run stocks of both *increase proportionately* under lump-sum taxes; *remain unchanged* under consumption tax-financing, *decrease proportionately* under wage tax-financing, and lead to a more than proportional decline in private capital under capital tax-financing.

5. Our numerical simulations suggest the following ranking for the different modes of financing. For either form of expenditure, lump-sum tax financing dominates consumption tax financing, which in turn dominates wage tax financing and finally capital tax financing, in terms of long-run welfare. These rankings are reflected in the long-run output multipliers.

6. The fact that wage tax-financing dominates capital tax-financing, despite the fact that a given increase in the former has a more adverse effect than does a comparable increase in the latter is of interest. It reflects the fact that being levied on a larger base, a smaller rise in the wage tax is required to generate the required revenue to finance the higher expenditure.

7. The analysis highlights the intertemporal welfare tradeoffs involved in policy changes. For example, both the substitution of a consumption tax for a uniform reduction in the income tax and a revenue-neutral switch from government consumption to government investment lead to a short-run welfare losses, which in both cases are more than offset by long-run welfare gains. This is a consequence of the growth generated during the subsequent transition.

One final aspect of the results is that in all cases employment responds virtually instantaneously to permanent shocks. This is because there is little feedback from the slowly evolving capital stocks to the labor-leisure choice along the transitional path. This result, which tends to run against the empirical evidence, is a consequence of the Cobb-Douglas production function and the relatively high degree of substitution between capital and labor in production. This suggests that a useful extension of this analysis would be to generalize the production function to allow for a wider range of substitutability between these two factors of production.

Table 1

Long-run Partial Effects

A. Fiscal Changes
(lump-sum Tax financed)

	g	h	k	w	c
$\frac{dl}{(1-l)}$	$-\frac{l}{c} < 0$	$-\frac{l}{c} < 0$	$\frac{l(1-c-g-h)}{c(1-k)} > 0$	$\frac{l}{1-w} > 0$	$\frac{l}{1+c} > 0$
$\frac{dk}{k}$	$\frac{(1-)\frac{l}{c} + \frac{1}{g}}{1- -} > 0$	$\frac{(1-)\frac{l}{c}}{1- -} > 0$	$-\frac{[c(1-) + (1-)(1-c-g-h)l]}{c(1-k)(1- -)} < 0$	$-\frac{(1-)}{(1- -)} \frac{l}{(1-w)} < 0$	$-\frac{(1-)}{(1- -)} \frac{l}{(1+c)} < 0$
$\frac{dk_g}{k_g}$	$\frac{(1-)\frac{l}{c} + \frac{1}{g}}{1- -} > 0$	$\frac{(1-)\frac{l}{c}}{1- -} > 0$	$-\frac{[c + (1-)(1-c-g-h)l]}{c(1-k)(1- -)} < 0$	$-\frac{(1-)}{(1- -)} \frac{l}{(1-w)} < 0$	$-\frac{(1-)}{(1- -)} \frac{l}{(1+c)} < 0$
$\frac{dy}{y}$	$\frac{(1-)\frac{l}{c} + \frac{1}{g}}{1- -} > 0$	$\frac{(1-)\frac{l}{c}}{1- -} > 0$	$-\frac{[c + (1-)(1-c-g-h)l]}{c(1-k)(1- -)} < 0$	$-\frac{(1-)}{(1- -)} \frac{l}{(1-w)} < 0$	$-\frac{(1-)}{(1- -)} \frac{l}{(1+c)} < 0$

B. Long-run Expenditure Effects

I. Increase in h

	Financed by Adjustments in:		
	k	w	c
$\frac{dl}{(1-l)}$	$-\frac{l}{c} \frac{(1-k)-(1-c-g-h)}{(1-k)+c(1-c-g-h)}$	$-\frac{l}{(1-l)c} [-(1+\lambda)]$	0
$\frac{dk}{k}$	$\frac{-(1-\lambda)(1+c)+\frac{(1-\lambda)l}{c} [(1-k)-(1-c-g-h)]}{[(1-k)+c(1-c-g-h)](1-\lambda)}$	$\frac{(1-\lambda)}{c} [-(1+\lambda)] \frac{l}{1-l}$	0
$\frac{dk_g}{k_g}$	$\frac{-(1+c)+\frac{(1-\lambda)l}{c} [(1-k)-(1-c-g-h)]}{[(1-k)+c(1-c-g-h)](1-\lambda)}$	$\frac{(1-\lambda)}{c} [-(1+\lambda)] \frac{l}{1-l}$	0
$\frac{dy}{y}$	$\frac{-(1+c)+\frac{(1-\lambda)l}{c} [(1-k)-(1-c-g-h)]}{[(1-k)+c(1-c-g-h)](1-\lambda)}$	$\frac{(1-\lambda)}{c} [-(1+\lambda)] \frac{l}{1-l}$	0

II. Increase in g

	Financed by Adjustments in:		
	k	w	c
$\frac{dl}{(1-l)}$	$-\frac{l}{c} \frac{(1-k)-(1-c-g-h)}{(1-k)+c(1-c-g-h)}$	$-\frac{l}{(1-l)c} [-(1+\lambda)]$	0
$\frac{dk}{k}$	$\frac{-(1-\lambda)(1+c)+\frac{(1-\lambda)l}{c} [(1-k)-(1-c-g-h)]}{[(1-k)+c(1-c-g-h)](1-\lambda)} + \frac{(l/g)}{(1-\lambda)}$	$\frac{(1-\lambda)}{c} [-(1+\lambda)] \frac{l}{1-l} + \frac{1}{g}$	$\frac{1}{(1-\lambda)g}$
$\frac{dk_g}{k_g}$	$\frac{-(1+c)+\frac{(1-\lambda)l}{c} [(1-k)-(1-c-g-h)]}{[(1-k)+c(1-c-g-h)](1-\lambda)} + \frac{(1-\lambda)lg}{(1-\lambda)}$	$\frac{(1-\lambda)}{c} [-(1+\lambda)] \frac{l}{1-l} + \frac{1}{g}$	$\frac{(1-\lambda)}{(1-\lambda)g}$
$\frac{dy}{y}$	$\frac{-(1+c)+\frac{(1-\lambda)l}{c} [(1-k)-(1-c-g-h)]}{[(1-k)+c(1-c-g-h)](1-\lambda)} + \frac{l/g}{(1-\lambda)}$	$\frac{(1-\lambda)}{c} [-(1+\lambda)] \frac{l}{1-l} + \frac{1}{g}$	$\frac{1}{(1-\lambda)g}$

Table 2

Long-run Effects of Fiscal Shocks

A. Lump-sum Tax Financed

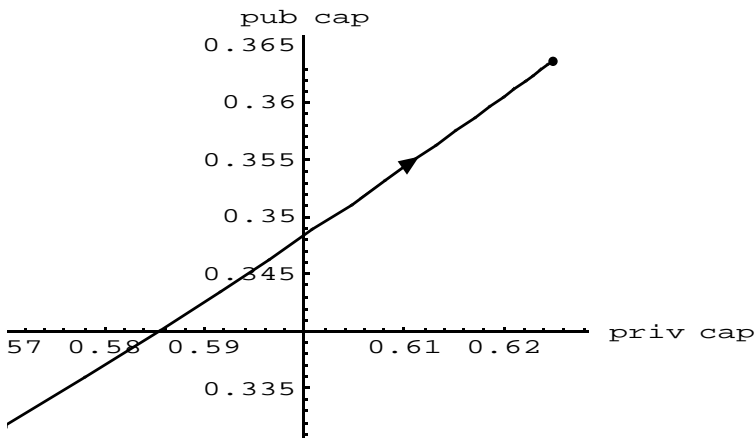
	k	w	c	g	h	l	c/y	μ_1, μ_2	k_g/k	Priv. cap. rel. to base	Pub. cap. rel. to base	Output rel. To base	Long-run govt. bal	Long-run welf. gains
Base Economy	0.28	0.28	0	0.08	0.14	0.706	0.642	-0.03428 -0.1040	0.582	0.570 (=1)	0.332 (=1)	0.297 (=1)	0.2976	--
First-best Optimum	0	w	$-w$	0.11	0.16	0.592	0.538	-0.03423 -0.09612	0.571	3.307	3.244	2.384	--	15.7%
increase in h	0.28	0.28	0	0.08	0.20	0.685	0.583	-0.03428 -0.1025	0.582	1.104	1.104	1.104	0.6009	3.13%
increase in g	0.28	0.28	0	0.14	0.14	0.685	0.583	-0.03428 -0.1025	1.019	1.415	2.477	1.415	0.5783	4.35%
increase in k	0.40	0.28	0	0.08	0.14	0.713	0.665	-0.03426 -0.1062	0.699	0.698	0.837	0.837	0.1058	-3.54%
increase in w	0.28	0.40	0	0.08	0.14	0.743	0.643	-0.03430 -0.1049	0.582	0.826	0.826	0.826	-0.0896	-5.35%
introduce c	0.28	0.28	0.10	0.08	0.14	0.726	0.643	-0.03429 -0.1045	0.582	0.906	0.906	0.906	-0.0213	-2.66%

Long-run Effects of Fiscal Shocks

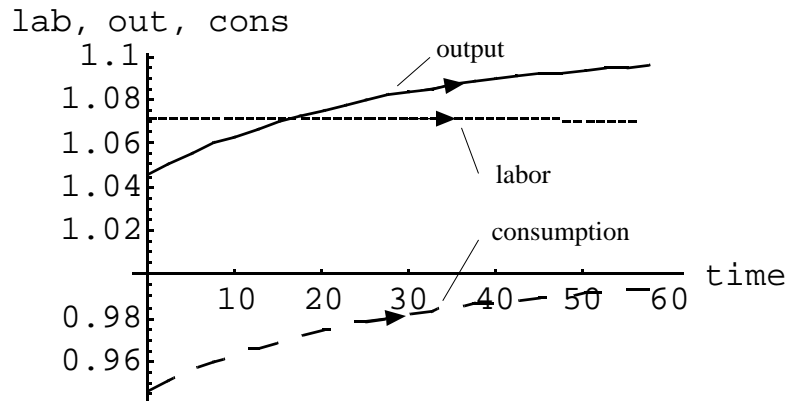
B. Compensated Changes

	k	w	c	g	h	l	c/y	μ_1, μ_2	k_g/k	Priv. cap. rel. to base	Pub. cap. rel. to base	Output rel. to base	Long-run govt bal	Long-run welf. gains
dec in k, w accom.	0.218	0.218	0.10	0.08	0.14	0.705	0.631	-0.03429 -0.1030	0.536	1.165	1.073	1.073	0.2799	1.28%
elim. income tax	0	0	0.494	0.08	0.14	0.703	0.589	-0.03434 -0.0997	0.419	1.819	1.310	1.310	0.2342	4.32%
restructure gov. exp.	0.28	0.28	0	0.14	0.081	0.706	0.642	-0.03428 -0.1040	1.019	1.284	2.246	1.284	0.2834	-3.71%
increase in h accom.	0.28	0.28	0.103	0.08	0.20	0.706	0.583	-0.03429 -0.1030	0.582	1.000	1.000	1.000	0.2976	0.69%
increase in h accom.	0.28	0.371	0	0.14	0.20	0.714	0.583	-0.03430 -0.1032	0.582	0.873	0.873	0.873	0.3019	-0.31%
increase in h accom.	0.452	0.28	0	0.08	0.20	0.697	0.615	-0.03424 -0.1060	0.765	0.584	0.767	0.767	0.3862	-1.90%
increase in g accom.	0.28	0.28	0.099	0.14	0.14	0.705	0.583	-0.03429 -0.1030	1.019	1.287	2.252	1.287	0.2892	1.66%
increase in g accom.	0.28	0.370	0	0.14	0.14	0.713	0.583	-0.03426 -0.1032	1.019	1.238	2.166	1.238	0.2863	0.47%
increase in g accom.	0.451	0.28	0	0.14	0.14	0.697	0.615	-0.03430 -0.1032	1.335	0.829	1.901	1.086	0.3349	-1.90%

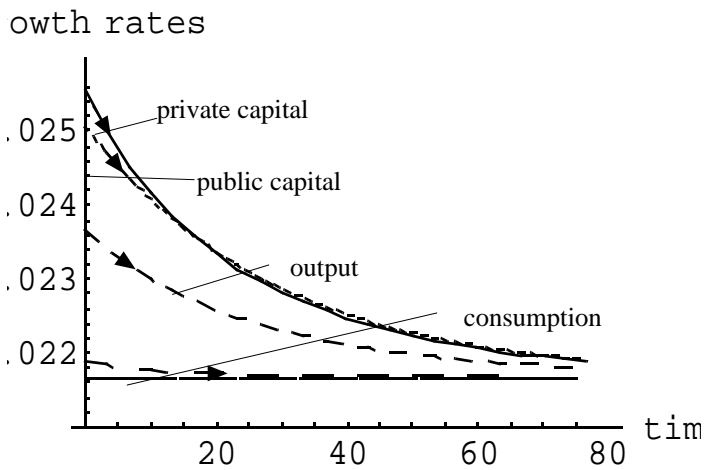
Figure 1: Increase in Government Consumption Expenditure



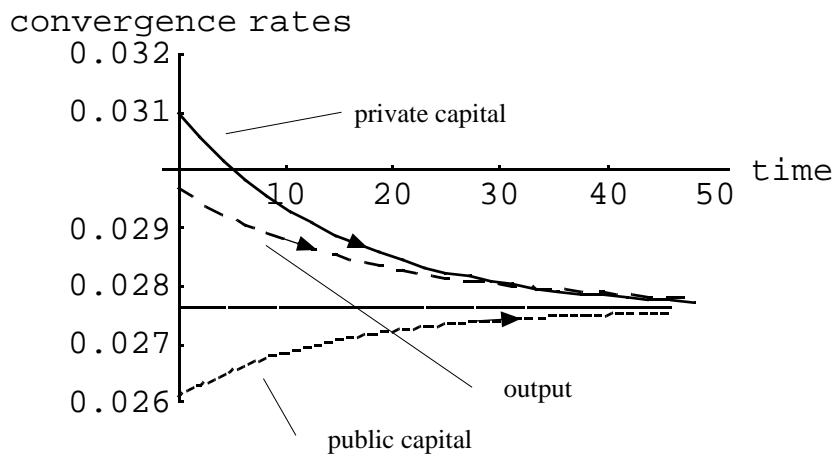
A. Phase Diagram



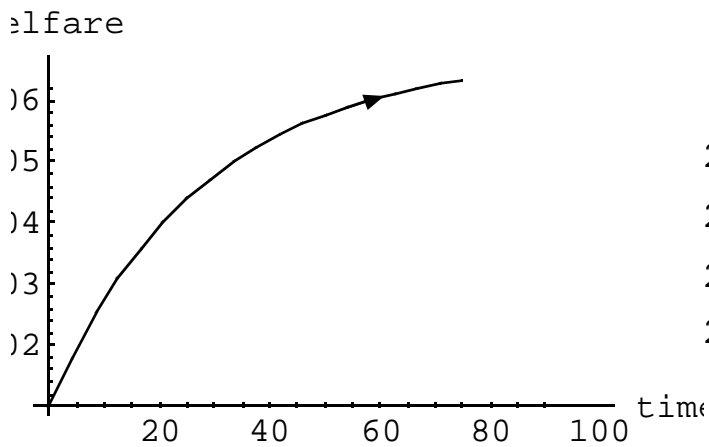
B. Transition Paths for Labor, Output, Consumption Relative to Benchmark Economy



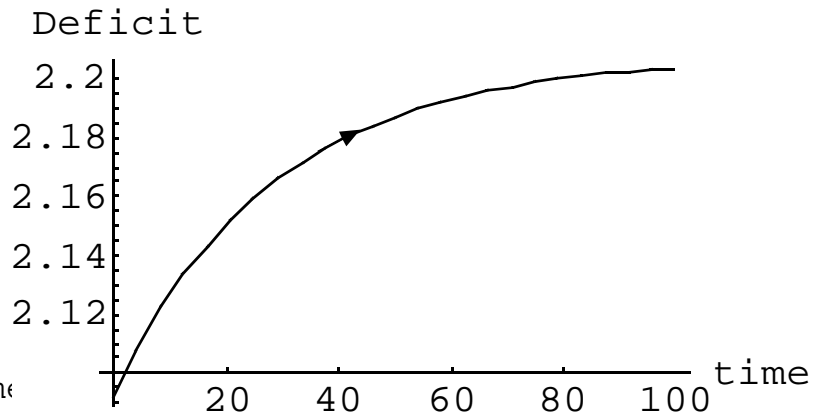
C. Growth Rates



D. Convergence Rates

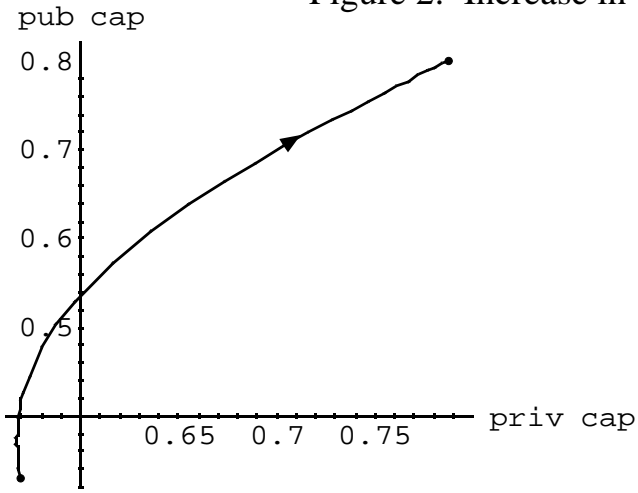


E. Time Path for Instant. Welfare Gains

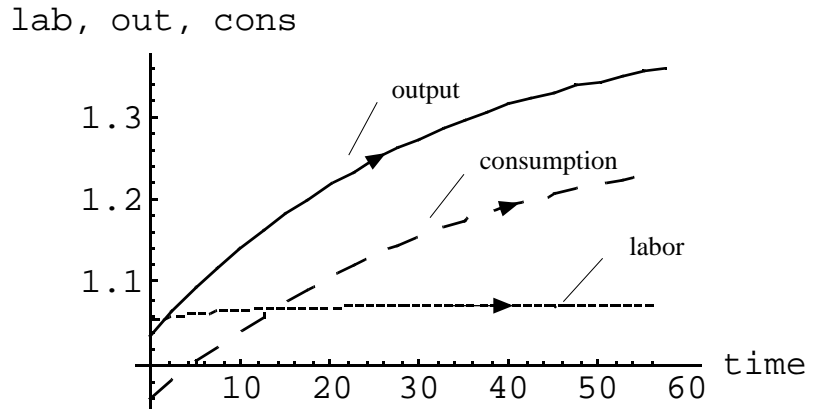


F. Time Path for Current Rel. Deficit

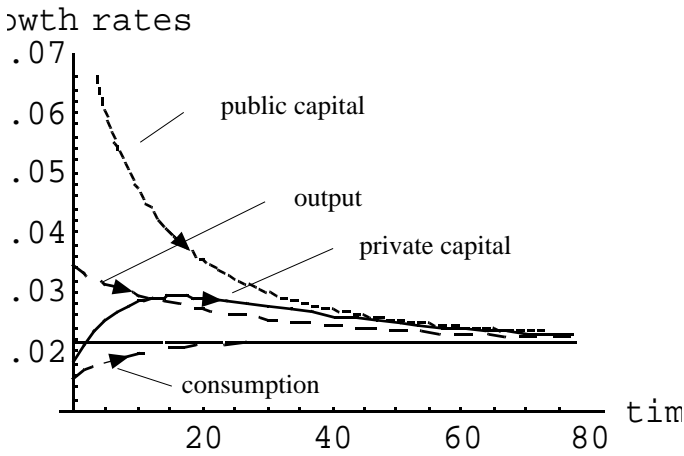
Figure 2: Increase in Government Investment Expenditure



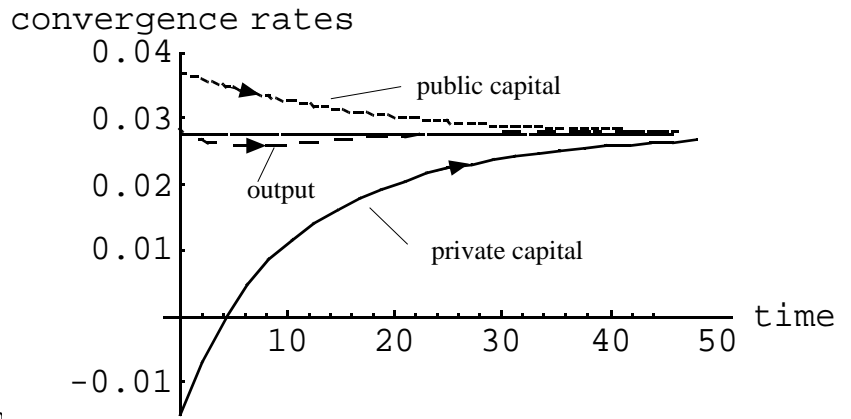
A. Phase Diagram-- Scale Adj. Priv. and Pub. Cap.



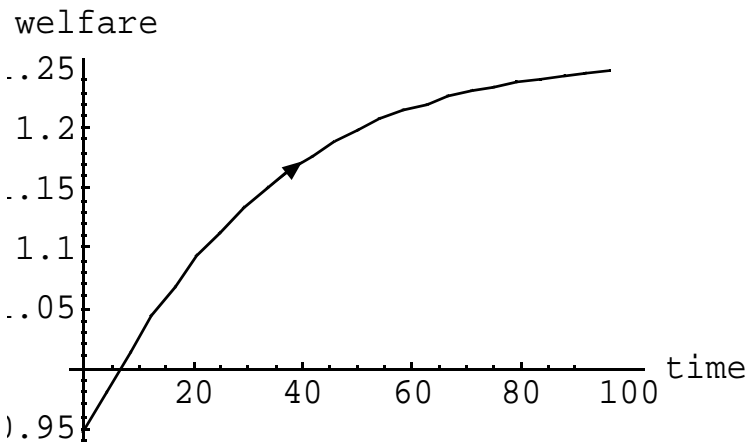
B. Transition Paths for Labor, Output, Consumption Relative to Benchmark Economy



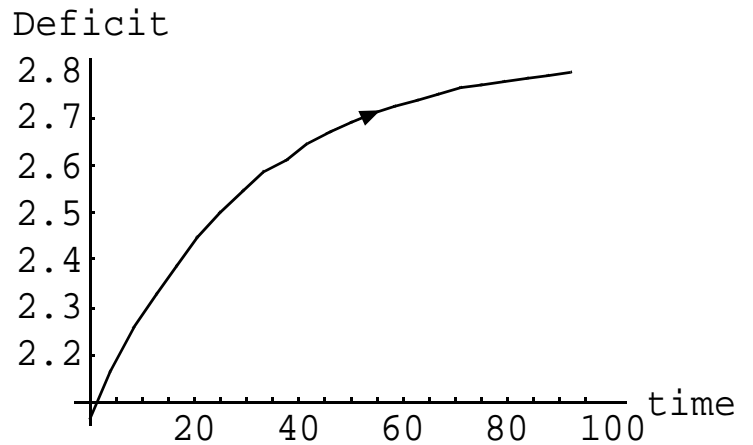
C. Growth Rates



D. Time Paths for Convergence Rates

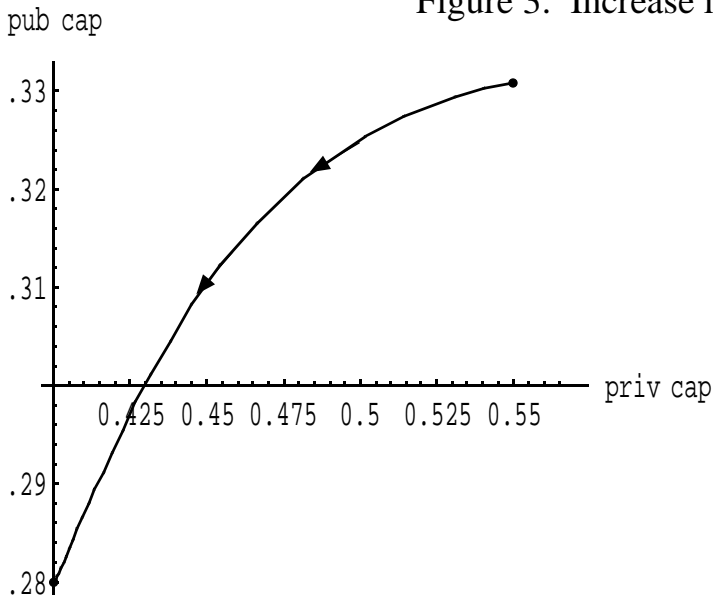


E. Time Path for Instant. Welfare Gains

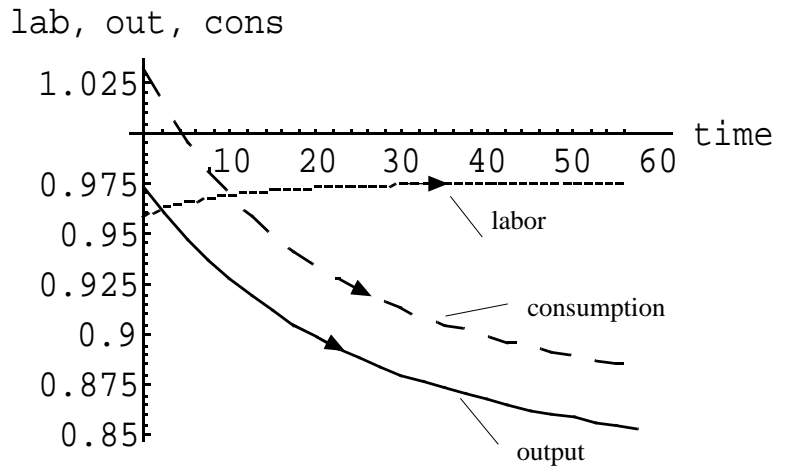


F. Time Path for Current Rel. Deficit

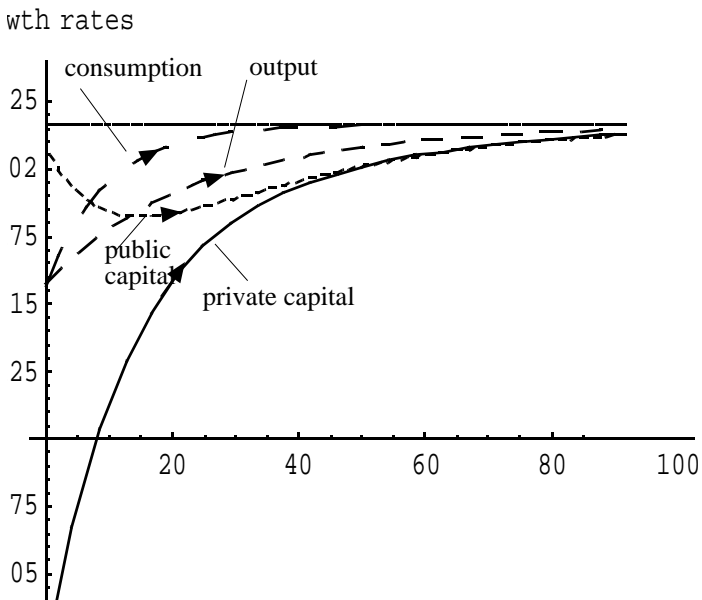
Figure 3: Increase in Tax on Capital



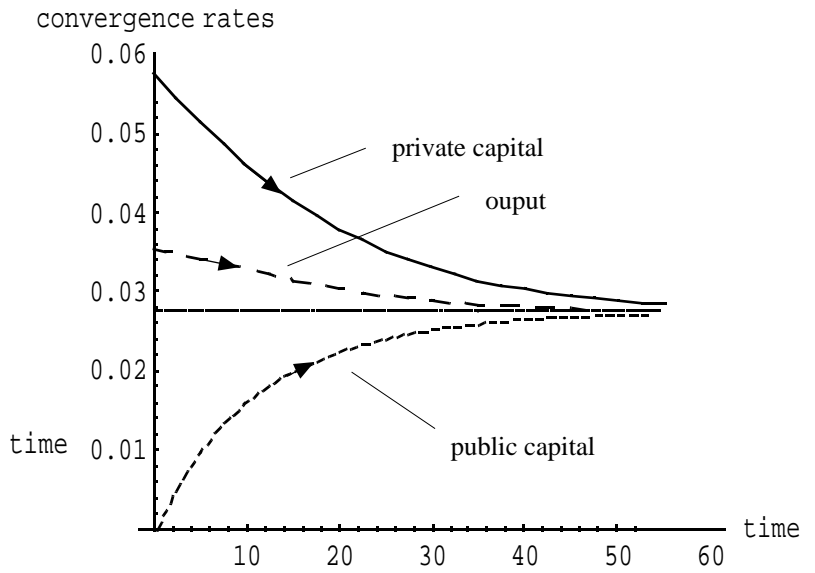
A. Phase Diagram-- Priv. and Pub. Cap.



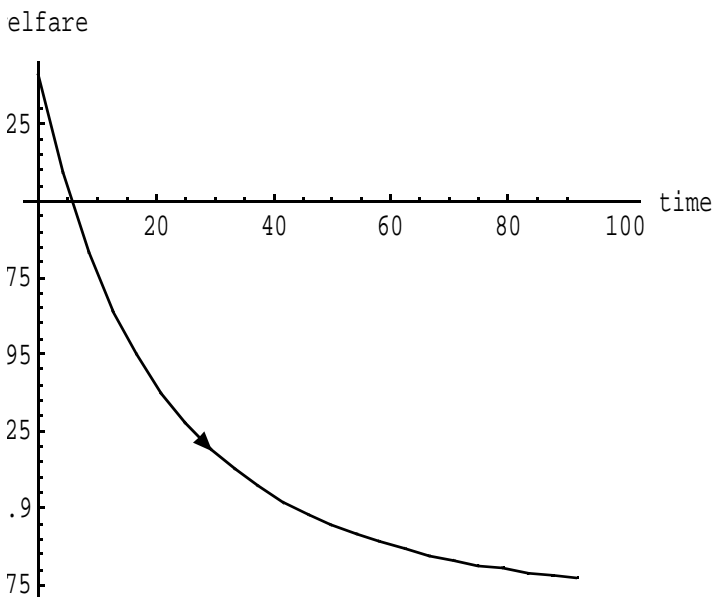
B. Transitory Paths for Labor, Output, Consumption Relative to Benchmark Economy



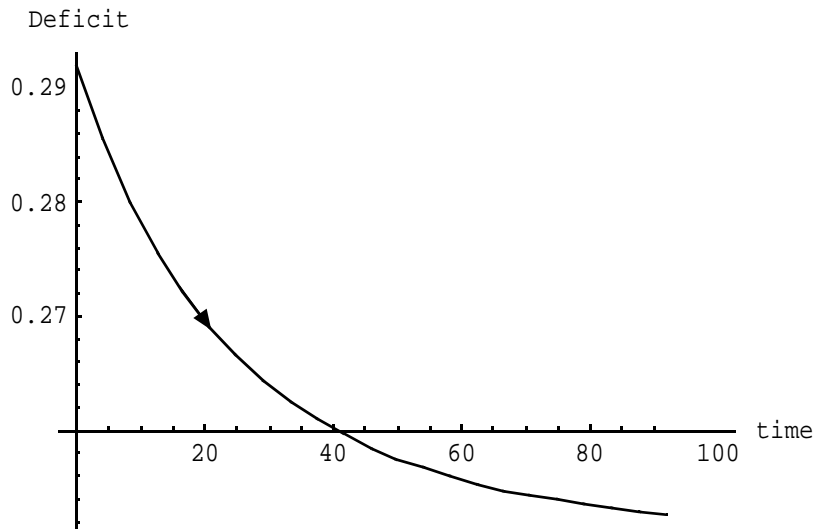
E. Time Paths for Growth Rates



D. Time Paths for Convergence Rates

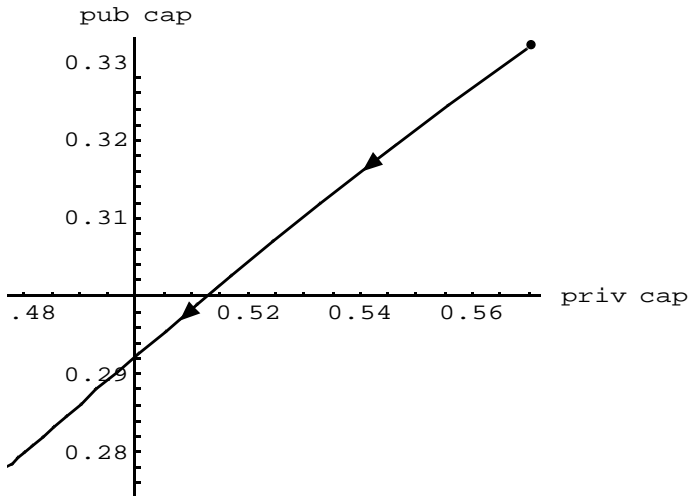


F. Time Path for Instant Welfare Gains

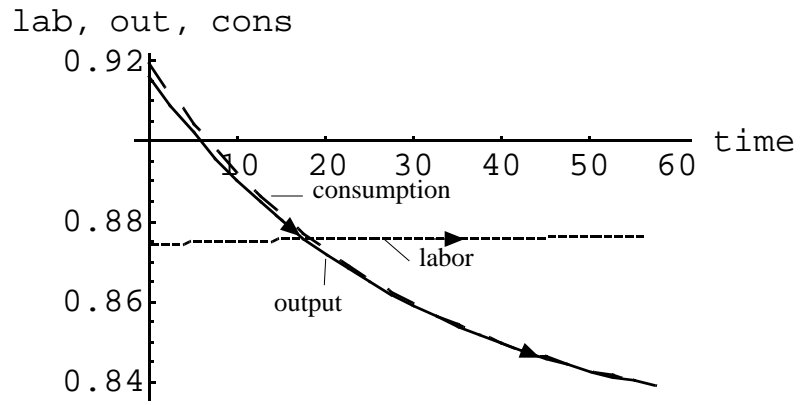


G. Time Path for Current Per. Deficit

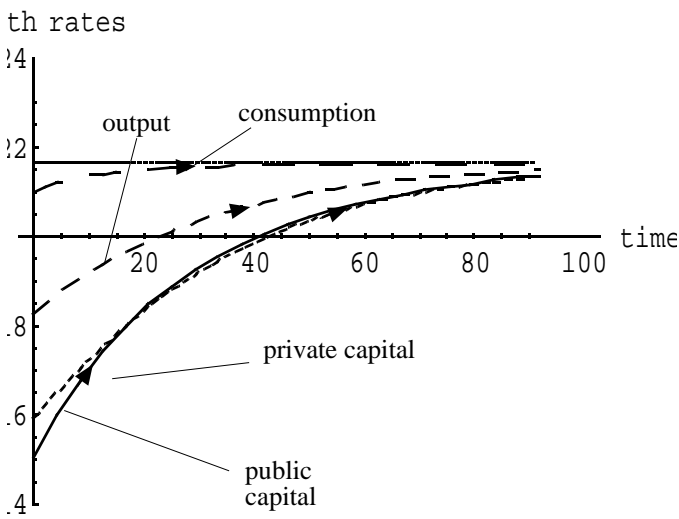
Figure 4: Increase in Tax on Wages



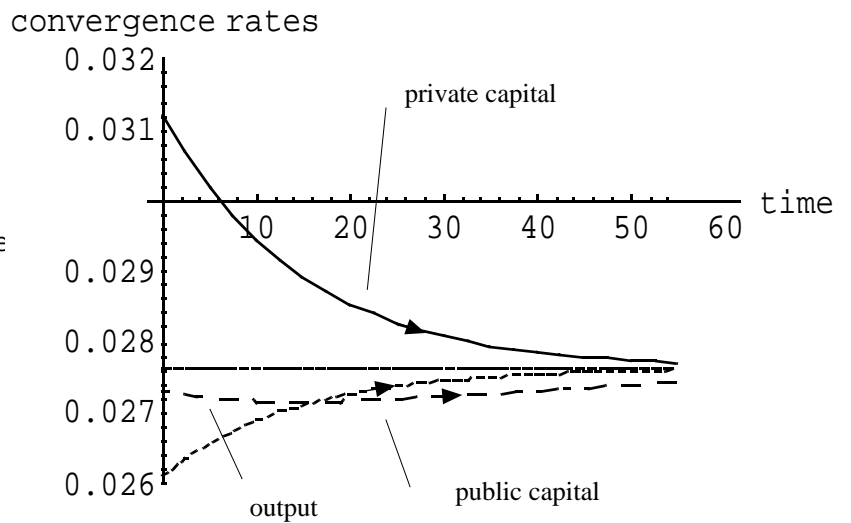
A. Phase Diagram-- Scale Adj. Priv. and Pub. Cap.



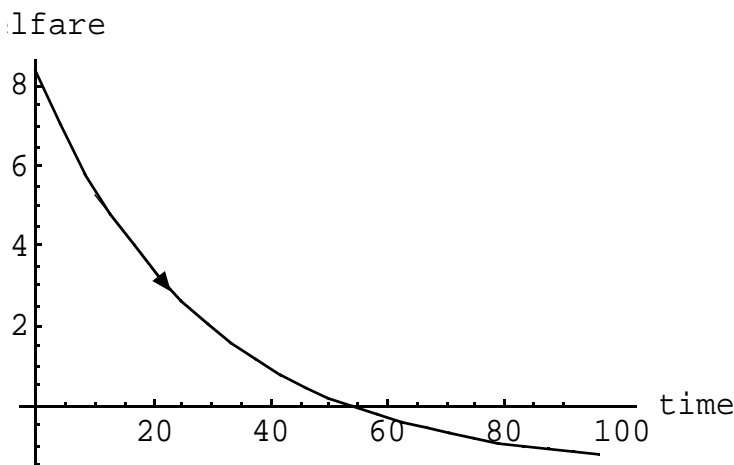
B. Transitional Paths for Labor, Output, Consumption Relative to Benchmark Economy



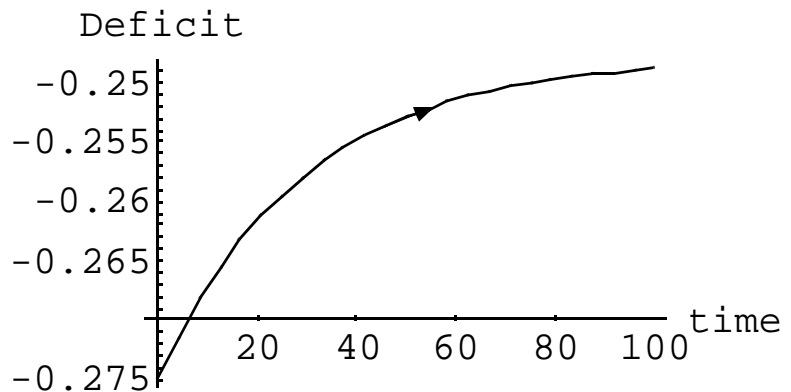
C. Time Paths for Growth Rates



D. Time Paths for Convergence Rates

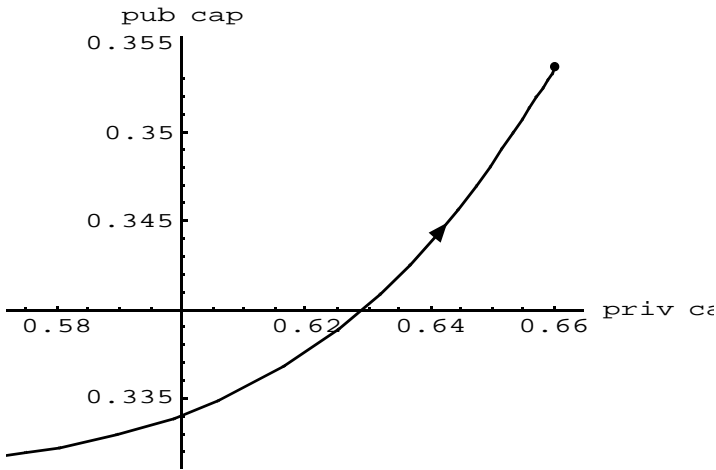


E. Time Path for Instantaneous Welfare Gain

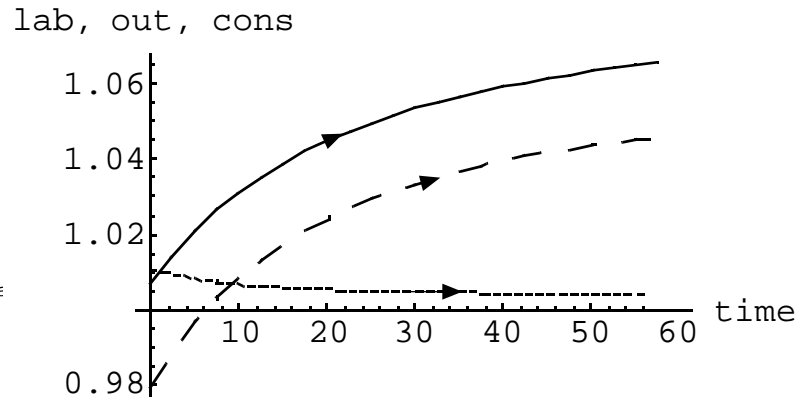


F. Time Path for Current Period Deficit

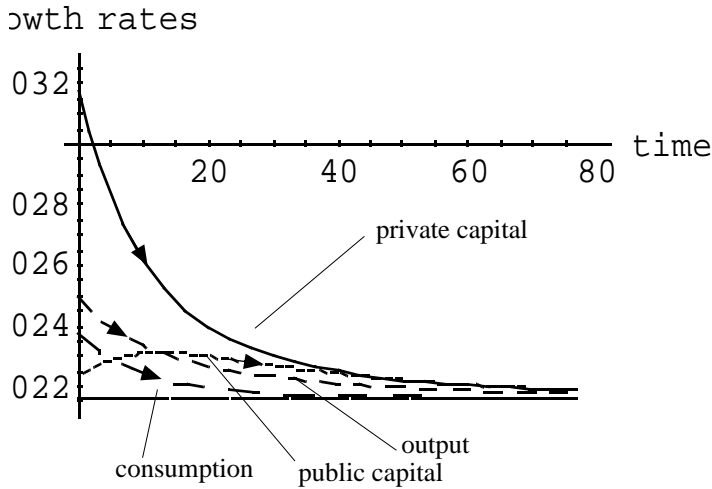
Figure 5: Substitution of Consumption Tax for Income Tax



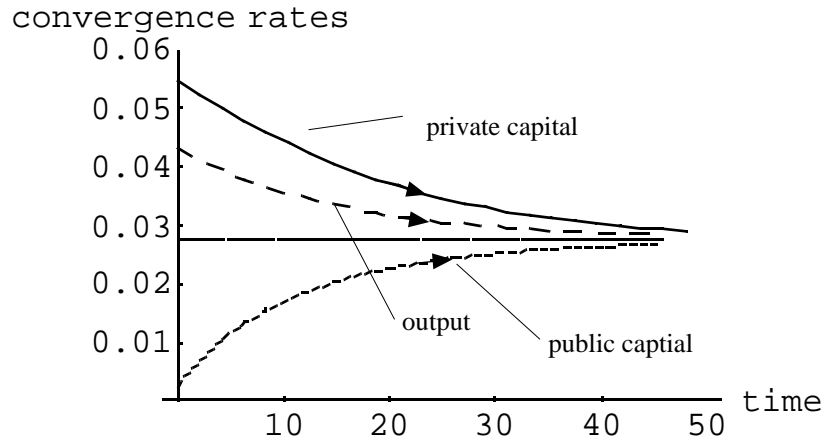
A. Phase Diagram-- Scale Adj. Priv. and Pub. Cap.



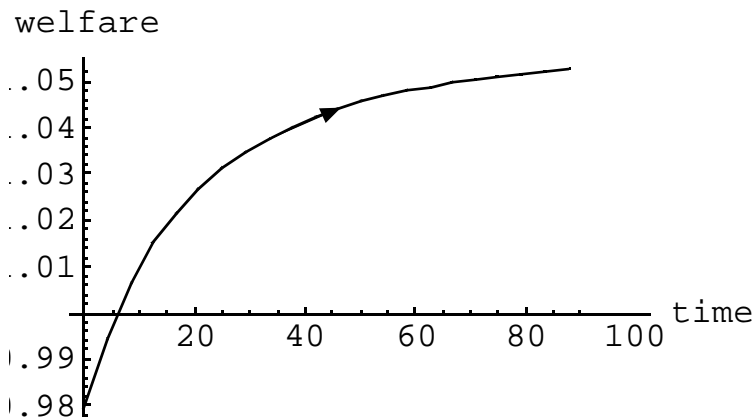
B. Transition Paths for Labor, Output, Consumption Relative to Benchmark Economy



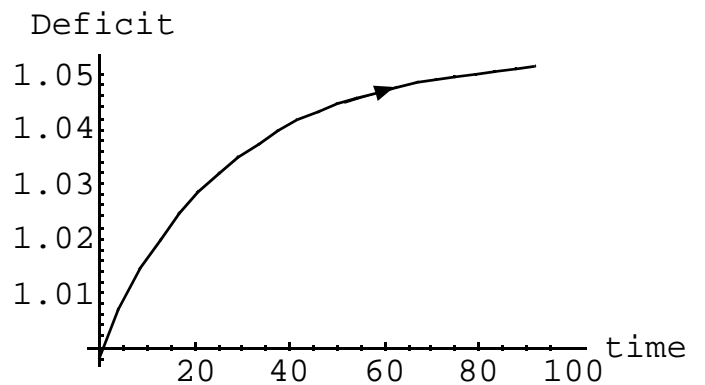
C. Time Paths for Growth Rates



D. Time Paths for Convergence Rates

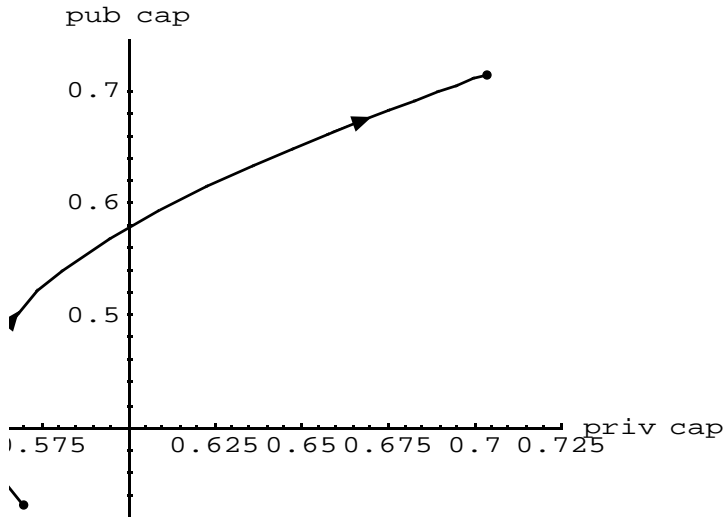


E. Time Path for Instant Welfare Gains

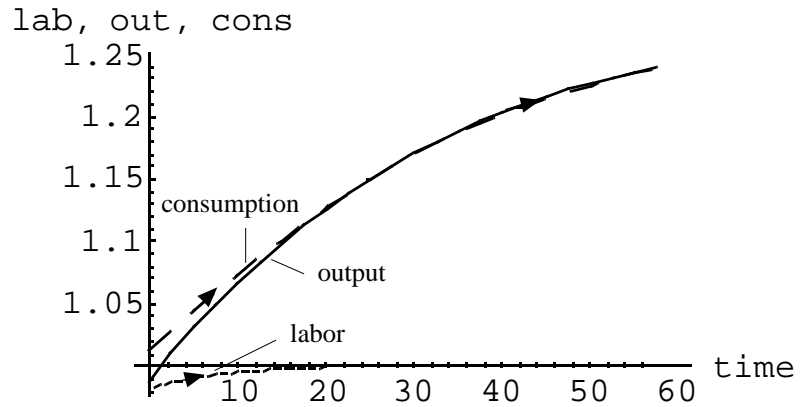


F. Time Path for Current Bal. Deficit

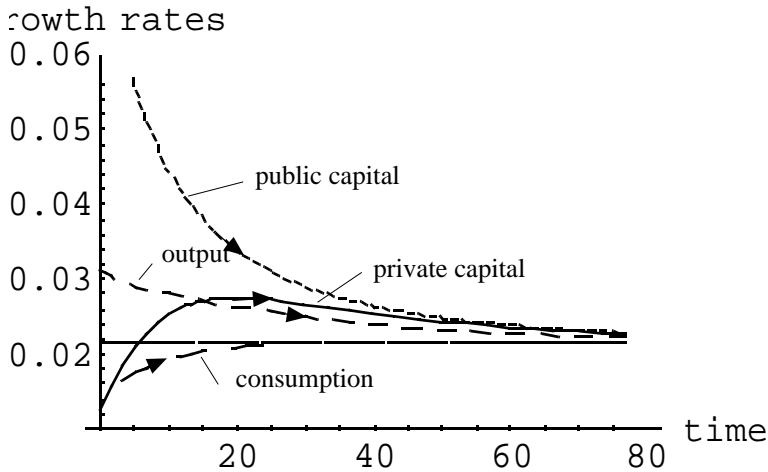
Figure 6: Balanced Expenditure Switch



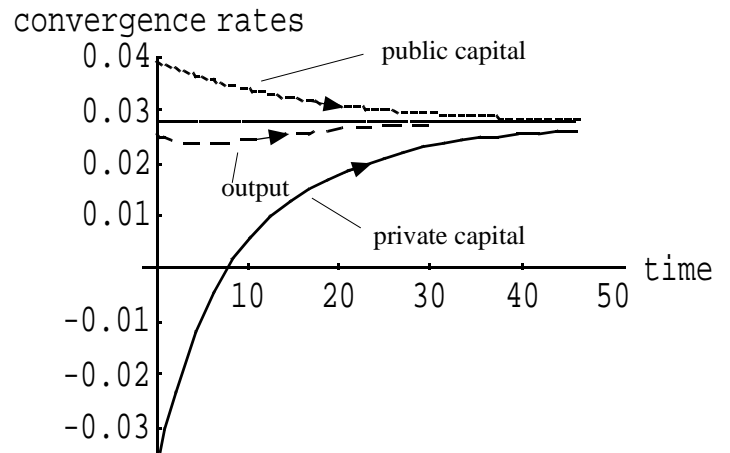
A. Phase Diagram-- Scale Adj. Priv. and Pub. Cap.



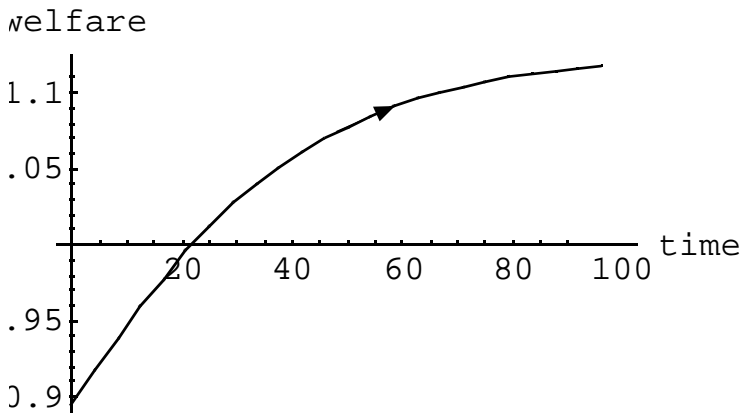
B. Transition Paths for Labor, Output, Consumption Relative to Benchmark Economy



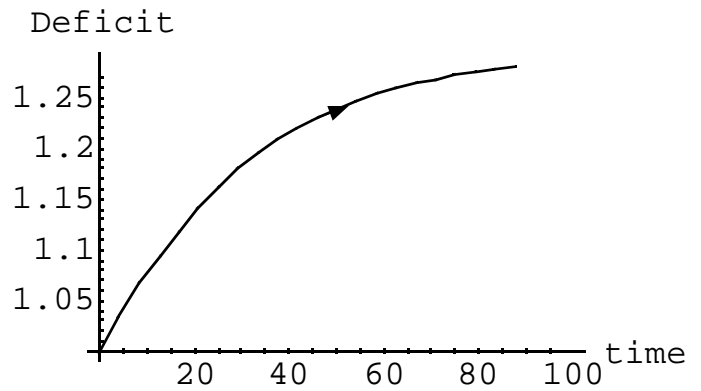
D. Time Paths for Growth Rates



D. Time Paths for Convergence Rates

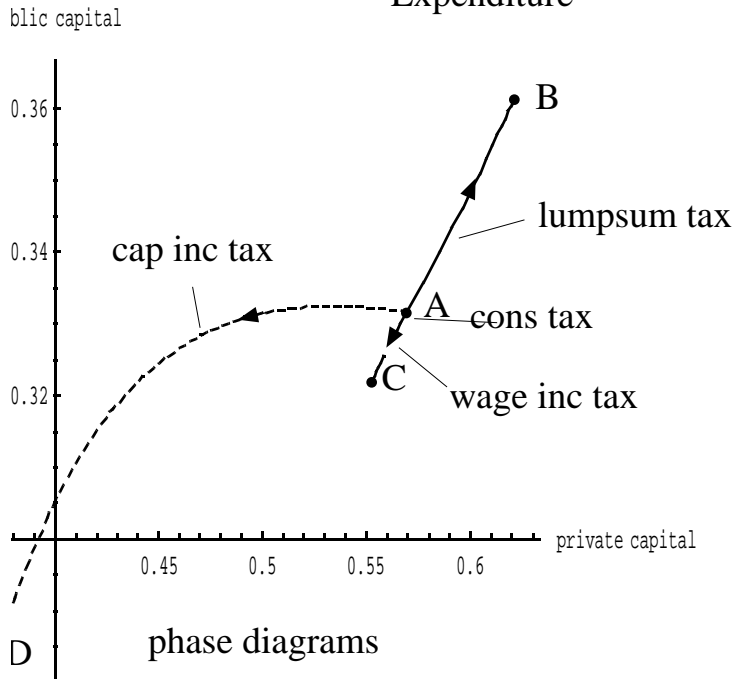


E. Time Path for Instant. Welfare Gains



F. Time Path for Current Rel. Deficit

Figure 7: Alternative Forms of Financing Government Consumption Expenditure



D phase diagrams

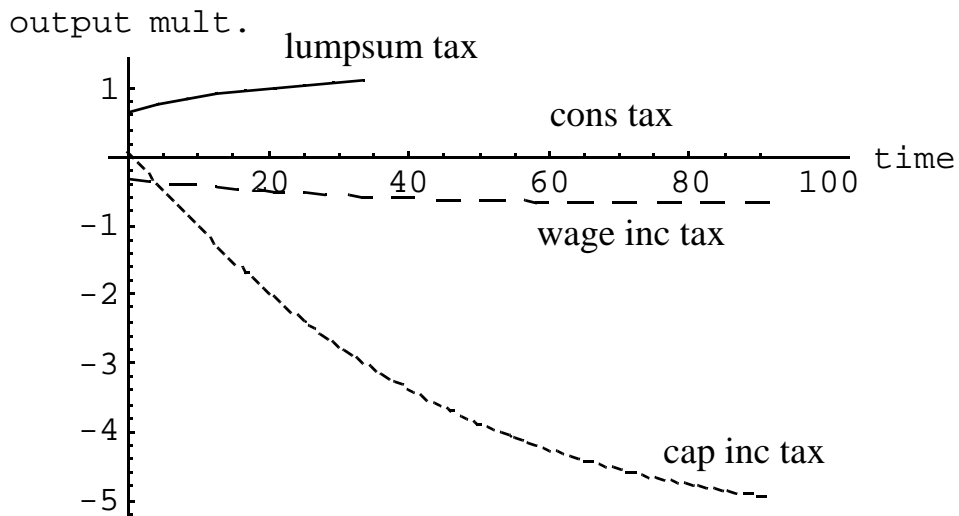
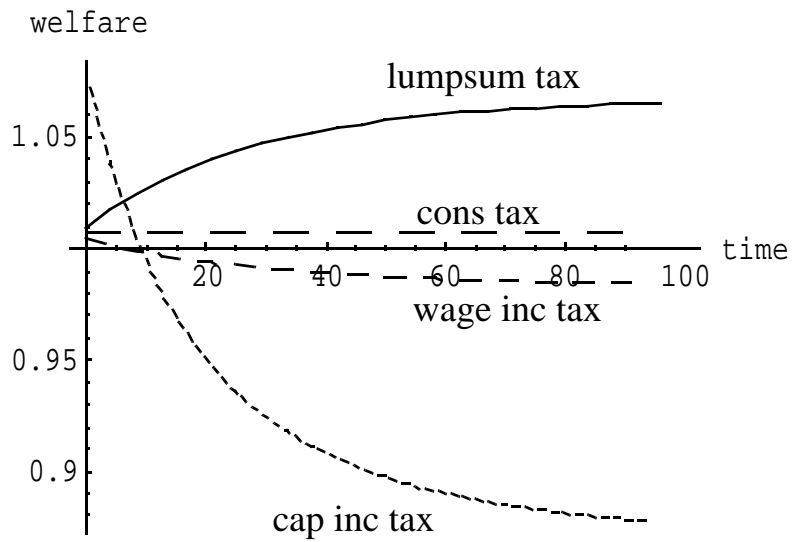


Figure 8: Alternative Forms of Financing Government Investment Expenditure

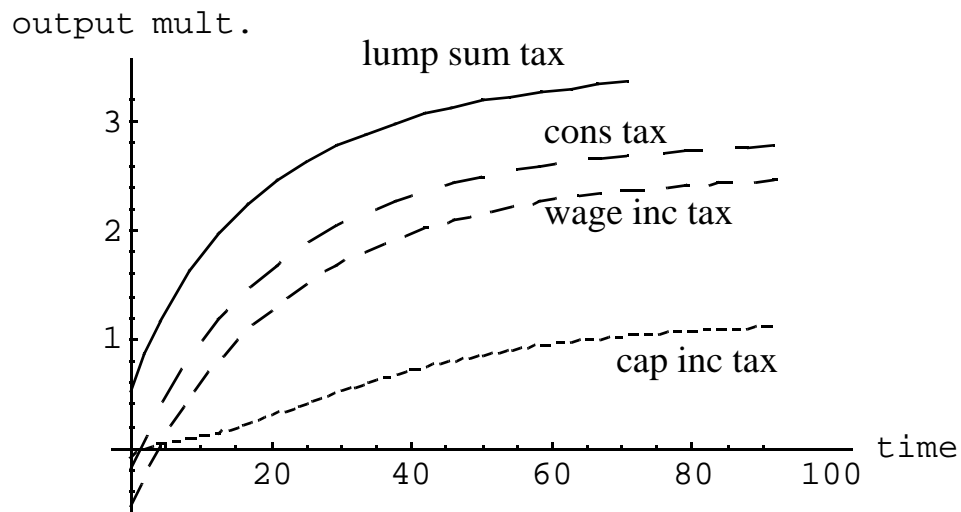
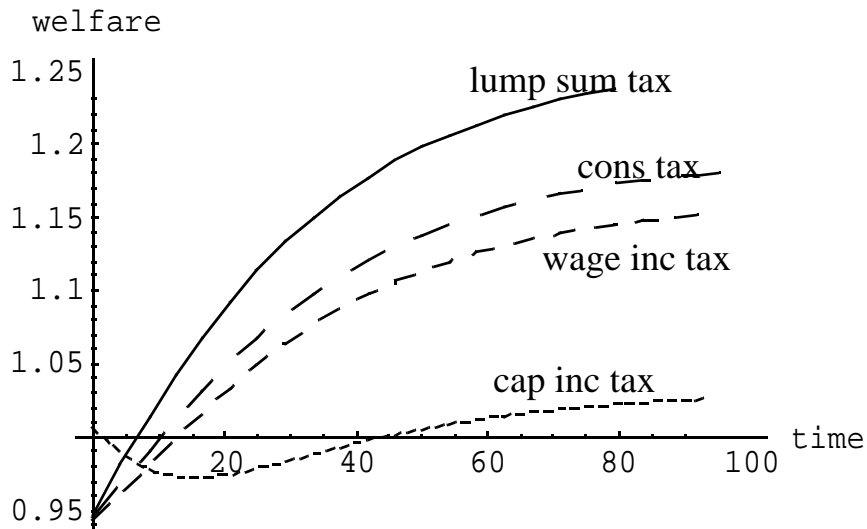
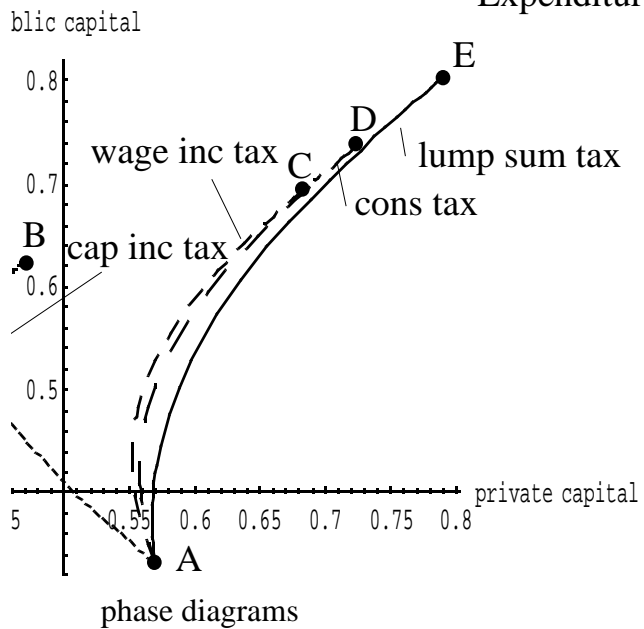
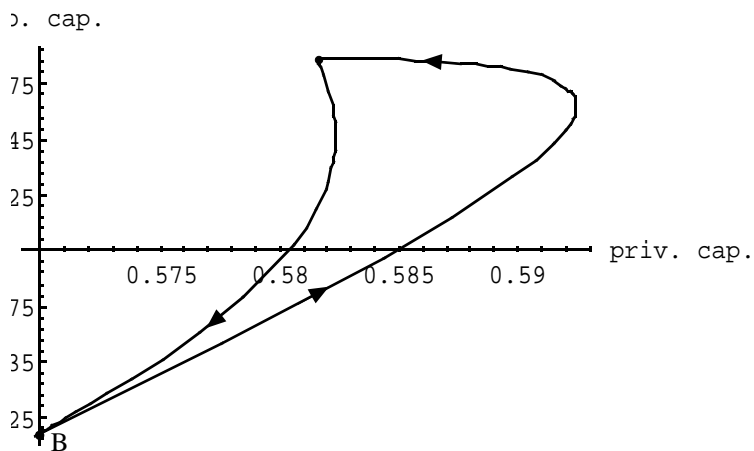
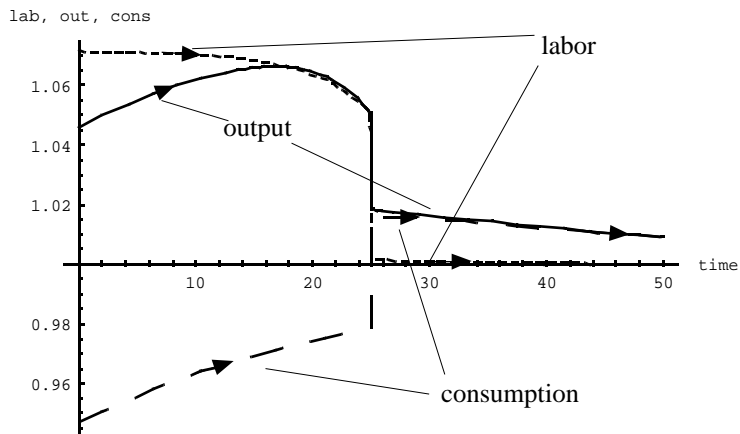


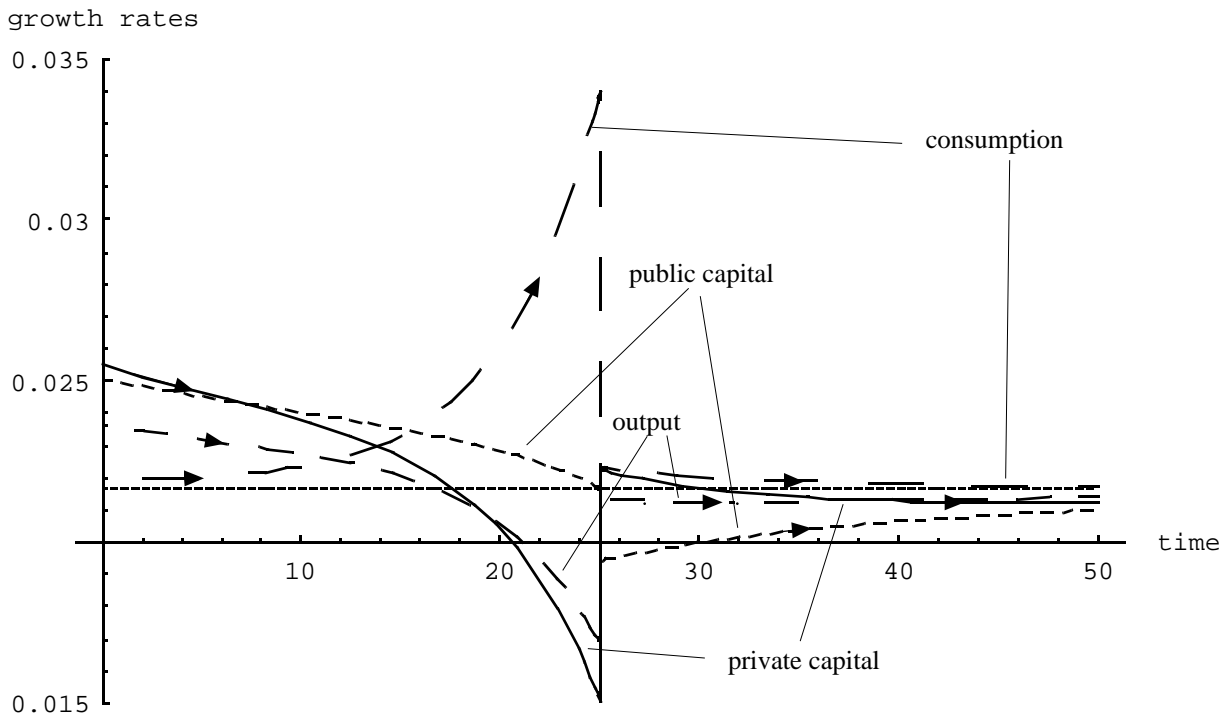
Figure 9: Temporary Increase in Government Consumption Expenditure



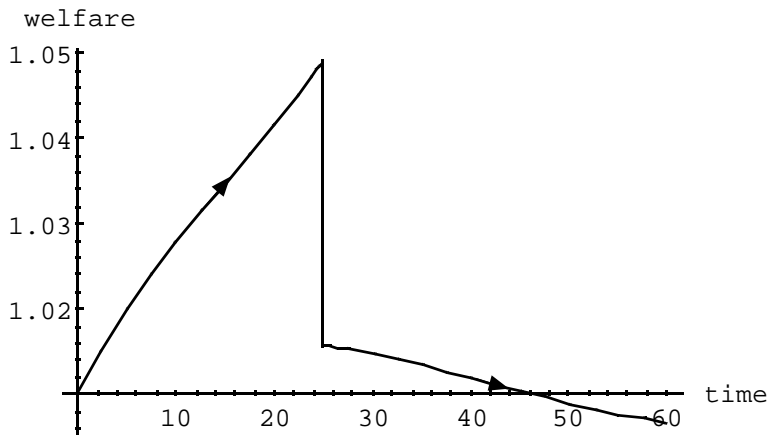
A. Phase diagram



B. Transitional Paths for Labor, Output, Consumption Relative to Benchmark Economy

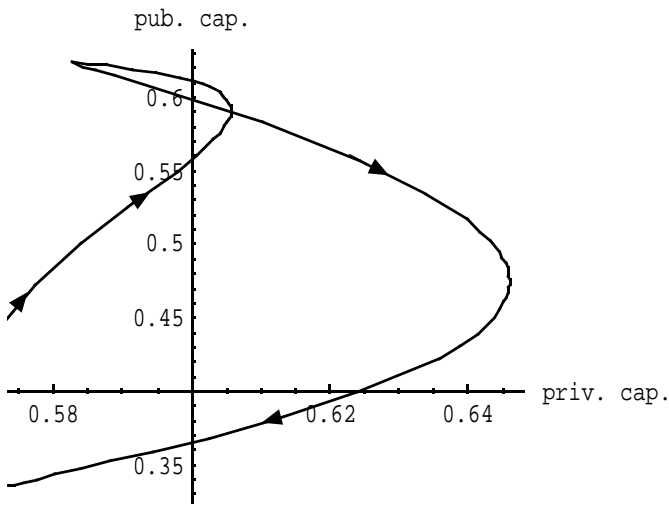


C. Time Paths for Growth Rates

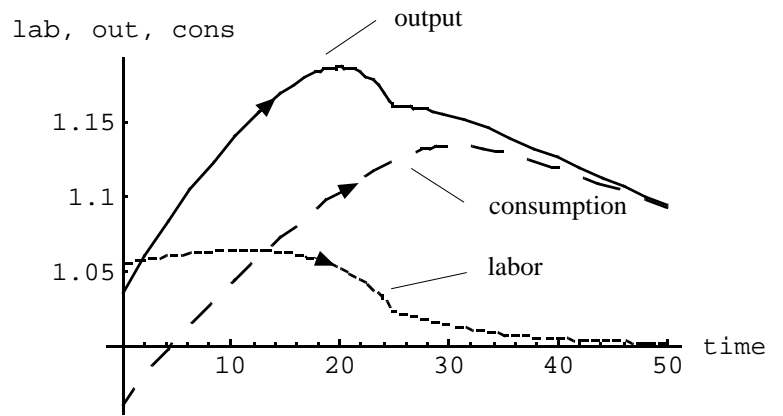


D. Time Path for Instant. Welfare Gains

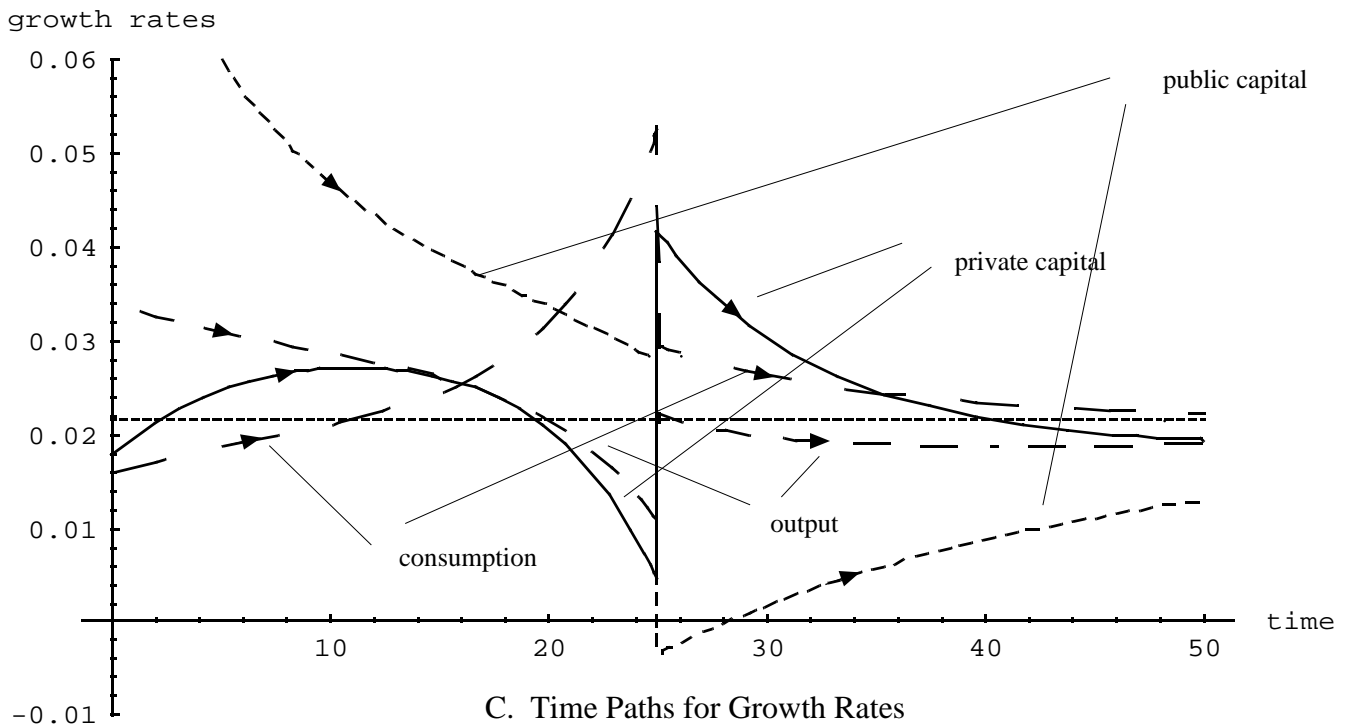
Figure 10: Temporary Increase in Government Investment Expenditure



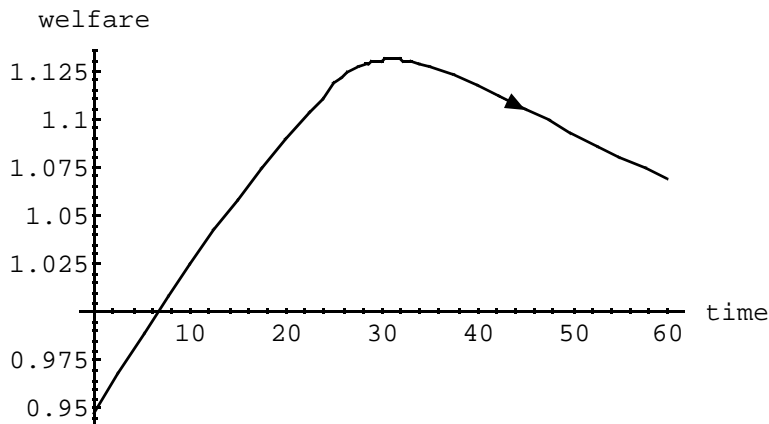
A. Phase Diagram



B. Transitional Paths for Labor, Output, Consumption Relative to Benchmark Economy



C. Time Paths for Growth Rates



Appendix

1. Derivation of Equilibrium System (14)

Differentiating the optimality condition (2a) together with (8b), the aggregate production function (3) the definition of C_i , and the marginal rate of substitution condition (14d) we obtain:

$$\frac{\dot{\lambda}_i}{\lambda_i} = (\gamma - 1) \frac{\dot{C}_i}{C_i} + \theta\gamma \frac{\dot{l}}{l} + \phi\gamma \frac{\dot{Y}}{Y} \quad (\text{A.1a})$$

$$\frac{\dot{Y}}{Y} = -(1 - \sigma) \frac{\dot{l}}{1 - l} + (1 - \sigma)n + \sigma \frac{\dot{K}}{K} + \eta \frac{\dot{K}_G}{K_G} \quad (\text{A.1b})$$

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C} - n \quad (\text{A.1c})$$

$$\frac{\dot{C}}{C} = \frac{\dot{l}}{l(1-l)} + \frac{\dot{Y}}{Y} \quad (\text{A.1d})$$

From these four equations, together with (2c), (10a), and (10b,) we can eliminate the dynamic variables $\dot{\lambda}_i/\lambda_i$, \dot{C}_i/C_i , \dot{C}/C , \dot{Y}/Y , \dot{K}/K , and \dot{K}_G/K_G , reducing them to a single equation in \dot{l} .

Recalling the definition of r given in (4) this can be expressed as (14a) of the text. Taking the time differentials of k, k_g given in (12) and combining with (10a) and (10b) leads to equations (14b) and (14c).

2. Derivation of Dynamics of Temporary Changes

We consider the case where it is announced at time 0 that there is to be temporary a change that is to last until time T , when the shock is removed. We assume that the system starts out from the initial steady states , $k_0, k_{g,0}, l_0$ to which it will return asymptotically. The solutions for k, k_g , and l are of the following general forms:

I. $0 \leq t \leq T$:

$$k(t) - \tilde{k} = B_1 e^{\mu_1 t} + B_2 e^{\mu_2 t} + B_3 e^{\mu_3 t} \quad (\text{A.2a})$$

$$k_g(t) - \tilde{k}_g = B_1 \nu_{21} e^{\mu_1 t} + B_2 \nu_{22} e^{\mu_2 t} + B_3 \nu_{23} e^{\mu_3 t} \quad (\text{A.2b})$$

$$l(t) - \tilde{l} = B_1 \nu_{31} e^{\mu_1 t} + B_2 \nu_{32} e^{\mu_2 t} + B_3 \nu_{33} e^{\mu_3 t} \quad (\text{A.2c})$$

II. $t > T$:

$$k(t) - k_0 = B_1 e^{\mu_1 t} + B_2 e^{\mu_2 t} + B_3 e^{\mu_3 t} \quad (\text{A.2a}')$$

$$k_g(t) - k_{g,0} = B_1 \nu_{21} e^{\mu_1 t} + B_2 \nu_{22} e^{\mu_2 t} + B_3 \nu_{23} e^{\mu_3 t} \quad (\text{A.2b}')$$

$$l(t) - l_0 = B_1 \nu_{31} e^{\mu_1 t} + B_2 \nu_{32} e^{\mu_2 t} + B_3 \nu_{33} e^{\mu_3 t} \quad (\text{A.2c}')$$

As in the text, we let $d\tilde{k} = \tilde{k} - k_0$, $d\tilde{k}_g = \tilde{k}_g - k_{g,0}$, and $d\tilde{l} = \tilde{l} - l_0$. The 6 arbitrary constants, B_i , B_i , $i = 1, 2, 3$ are derived as follows.

(i) The initial conditions on the two capital stocks imply

$$B_1 + B_2 + B_3 = -d\tilde{k} \quad (\text{A.3a})$$

$$B_1 \nu_{21} + B_2 \nu_{22} + B_3 \nu_{23} = -d\tilde{k}_g \quad (\text{A.3b})$$

(ii) The transversality condition implies:

$$B_3 = 0 \quad (\text{A.3c})$$

(iii) The continuous adjustment of the two capital stocks implies that the two solutions (A.2a, A.2a') and (A.2b and (A.2b')) coincide at time T , yielding (with $B_3 = 0$):

$$B_1 e^{\mu_1 T} + B_2 e^{\mu_2 T} + B_3 e^{\mu_3 T} = B_1 e^{\mu_1 T} + B_2 e^{\mu_2 T} + d\tilde{k} \quad (\text{A.4a})$$

$$B_1 \nu_{21} e^{\mu_1 T} + B_2 \nu_{22} e^{\mu_2 T} + B_3 \nu_{23} e^{\mu_3 T} = B_1 \nu_{21} e^{\mu_1 T} + B_2 \nu_{22} e^{\mu_2 T} + d\tilde{k}_g \quad (\text{A.4b})$$

(iv) The continuous adjustment of the shadow value, λ_i , along with that of the capital stocks at that time imposes the following constraint on the behavior of l at time T . Taking the

differentials at of equations (2a, 2b) we can show that at time T , l undergoes a discrete jump, $l(T)$, given by:

$$[1 - \gamma(1 + \theta)] \frac{1}{l(T-)} + [\phi\gamma(1 - \sigma) + \sigma(1 - \gamma)] \frac{1}{1 - l(T-)} dl(T) = \phi\gamma \frac{dh}{h} + (1 - \gamma) \frac{d\tau_w}{1 - \tau_w} - \gamma \frac{d\tau_c}{1 + \tau_c}$$

Note that l undergoes jumps at time T in response to changes in g, τ_w, τ_c , as these impact on its utility at that time. It does not undergo any jump at time T in response to changes in behavior at time T in response to changes in either g or τ_k . The solutions for $l(t)$ given by (A.2c) and (A.2c') must thus satisfy:

$$B_1 \nu_{31} e^{\mu_1 T} + B_2 \nu_{32} e^{\mu_2 T} + B_3 \nu_{33} e^{\mu_3 T} = B_1 \nu_{31} e^{\mu_1 T} + B_2 \nu_{32} e^{\mu_2 T} + d\tilde{l} + dl(T) \quad (\text{A.5})$$

where $l(T)$ is given above. Given the specific shock, equations (A.2), (A.3), (A.4) and (A.5) yield 6 equations that determine the arbitrary constants B_i , B_i , $i = 1, 2, 3$, thereby determining the time paths for the system before and after the shock.

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