THE TRANSPOSITION OF ELECTRICAL CONDUCTORS.

BY FRANK F. FOWLE.

The transmission of electromagnetic energy through the ether, by means of conducting wires whose function is that of directing and concentrating the energy flow, is accompanied by mutual interferences between transmission systems which employ in common any considerable portion of the ether. The extent to which the ether is energized, in a direction transverse to the direction of energy flow, is controlled by the design of the transmission circuit and the rate of energy flow. The energy storage-capacity of a transmission circuit is very small in comparison with the transmitted energy, but the great velocity of transmission renders possible the transmission of large energies. The conformations of the electric and the magnetic fields about a line circuit depend, in alternating-current systems, on the number of phases and on the number, size, and separation of the wires. The field at a given point due to a single-phase circuit is constant in direction but variable in intensity, from instant to instant throughout a cycle; the field due to a polyphase circuit varies both in direction and intensity.

Alternating-current transmission systems resolve themselves into two great classes, one for the transmission of power and the other for the transmission of intelligence. Power systems are characterized by: high pressure; large currents; low or moderate frequencies; only one fundamental frequency; relatively great reaction of terminal apparatus compared with line reactions; magnitude of inductive disturbances in the line usually small and important only as affecting regulation; line length less than a wave length. and the flow of energy usually

in a given direction. Telephonic systems are characterized by; low pressure; small currents; high frequencies, variable over a wide range; relatively small reaction of terminal apparatus compared with line reactions; magnitude of inductive disturbances in the line usually large and rarely negligible; line lengths usually greater than the wave lengths and often equal to many wave lengths, and the flow of energy alternating in direction and source.

The general character of the fields set up about the line is the same, whatever the system, and a consideration of
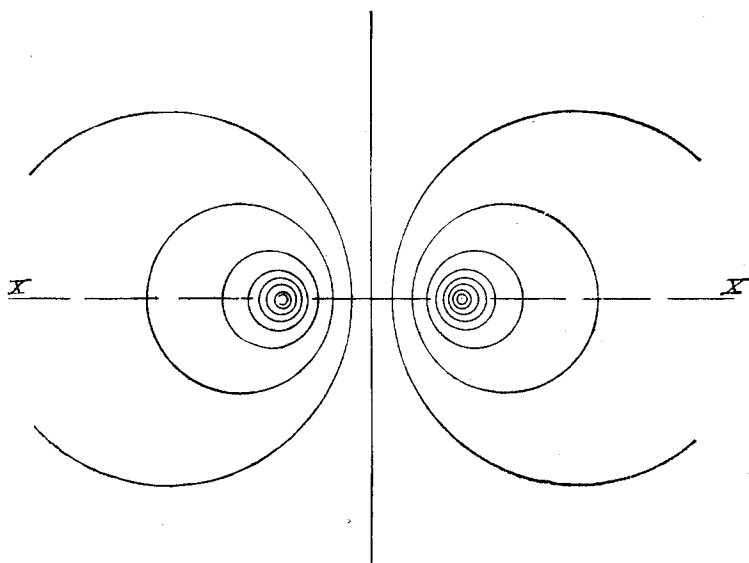

FIG. 1.

this will lead to the theory of transposition. Fig. 1 shows the magnetic field set up about a two-wire circuit; Fig. 2 shows the electric field. These two fields at all points of intersection are at right angles to each other; that is, the electric and the magnetic forces at any given point are perpendicular with respect to each other. Also, the electric pressure is constant in magnitude at all points on any magnetic flux line; or every magnetic flux line is a line of constant electric pressure. The flux density and the intensity of the electric pressure both diminish rapidly as the distance from the wires increases. An instance of this is shown in Fig. 3, which gives, in rectangular

coördinates, the flux density along the X axis of Fig. 1; Fig. 4 shows similarly the electric pressure along the X axis in Fig. 2.

For a single wire having an earth return, the solution above may be applied by considering the surface of the earth coincident with the Y axis· the wire beneath the earth's surface
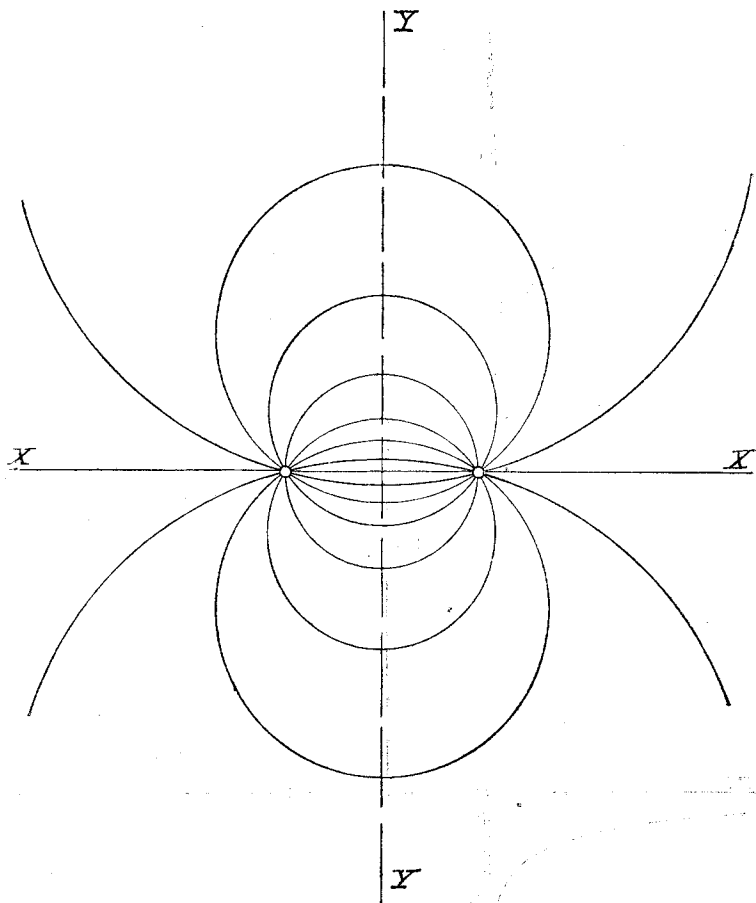

FIG. 2.

will be the electrical image of the real wire above. It will be seen that the fields due to a ground-return circuit in practice extend through a far greater volume of dielectric than in the case of a metallic circuit.

The theory upon which the above curves are based is as follows: Consider a single straight wire in free space, of radius r,

conveying a current I. The work done in carrying a unit magnetic pole circumferentially around the wire, at a radial distance a from it is
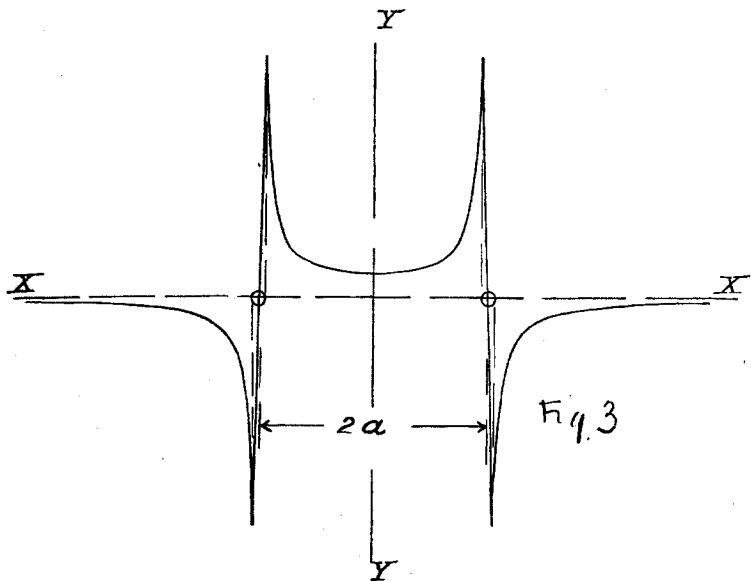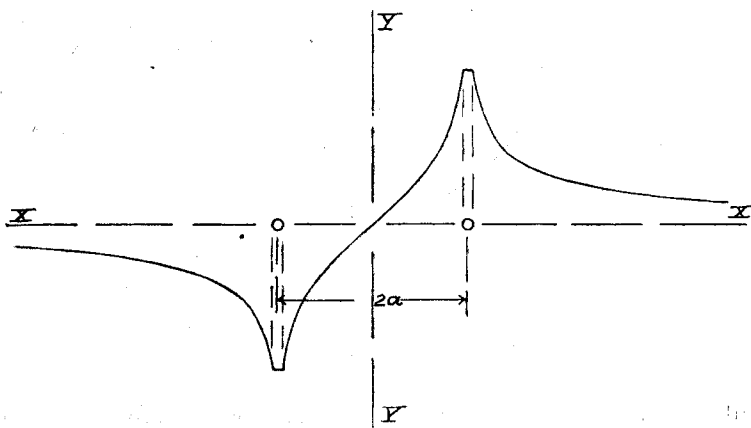
$$W = 4 \pi I = 2 \pi a F \tag{1}$$

FIG. 3.

FIG. 4.

where F is the magnetic force at a. The force F acts through the point a perpendicularly to the plane containing the wire and the point a.

Therefore

$$F = \frac{2\,I}{a} = H \tag{2}$$

where H is the intensity of the magnetic field.   In a unit length of wire the total number of flux lines encircling the wire, multiplied by the current is

$$N_1 = \int_{r}^{\infty} \frac{2\,\mu\,I^2}{x}\,d\,x \tag{3}$$

where x is the radial distance from the wire and μ the magnetic permeability of the medium.

Assuming a uniform distribution of the current I throughout the cross-section of the wire, we have for the field intensity at a point radially distant b from the centre of the wire,

$$\frac{\pi\,b^2}{\pi\,r^2}\left(\frac{2\,I}{b}\right) = \frac{2\,I\,b}{r^2} \tag{4}$$

and this intensity is due to the current enclosed within the circumference $2\,\pi\,b$.   The product of the current into the flux, integrated between the limits $b = o$ and $b = r$ is

$$N_2 = \int_{0}^{r} \frac{2\,\mu_1\,I^2\,x^3}{r^4}\,d\,x \tag{5}$$

where μ_1 is the magnetic permeability of the wire.   The total number of linkages of the flux with the current, multiplied by the current, is

$$N = N_1 + N_2 = \int_{0}^{r} \frac{2\,\mu_1\,I^2\,x^3}{r^4}\,dx + \int_{r}^{\infty} \frac{2\,\mu\,I^2}{x}\,d\,x \tag{6}$$

The inductance* is therefore

$$L = \frac{1}{2}\,\mu_1 + \int_{r}^{\infty} \frac{2\,\mu}{x}\,d\,x \tag{7}$$

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* This demonstration is due to Jackson; *Alternating Currents and Alternating Current Machinery*, p. 140.

If there is a parallel wire within the field of the first wire, distant r_{12} from it, the portion of the total flux which is linked with it, is

$$M = \int_{r_{12}}^{\infty} \frac{2\,\mu}{x}\, d\,x \tag{8}$$

and this is the mutual inductance of the two circuits.

For a two-wire metallic circuit the inductance will be double the difference between expressions (7) and (8), or

$$L = \mu_1 + 2 \int_{r}^{r_{12}} \frac{2\,\mu}{x}\, d\,x$$

$$= \mu_1 + 4\,\mu \log \frac{r_{12}}{r} \tag{9}$$

For a single wire at height h above the earth, the inductance is

$$L = \frac{1}{2}\,\mu_1 + 2\,\mu \log \frac{2\,h}{r} \tag{10}$$

These expressions in henries per mile are respectively

$$L = \left(0.1609 + 1.482 \log_{10} \frac{d}{r}\right) 10^{-3} \tag{11}$$

$$L = \left(0.08047 + 0.7411 \log_{10} \frac{2\,h}{r}\right) 10^{-3} \tag{12}$$

where μ and μ_1 are unity.

The field of force without a two-wire metallic circuit is, from (2)

$$F = 2\,I \left(\frac{1}{r_{1x}} - \frac{1}{r_{2x}}\right) \tag{13}$$

where the point x is perpendicularly distant r_{1x} from one wire and r_{2x} from the other. From this, the field intensity along the X axis in Fig. 1 is proportional to

$$y = \frac{1}{a+x} + \frac{1}{a-x} \tag{14}$$

for points without the wires and

$$y = \frac{2\,x}{r^2} \tag{15}$$

for points within the wire.   Fig. 3 is plotted from (14) and (15).

Considering the case of two two-wire metallic circuits, as shown in cross-section in Fig. 5, the expression for their mutual inductance follows at once from expression (8):

$$M = \int_{r_{13}}^{r_{14}} \frac{2\,\mu}{x}\, d\,x - \int_{r_{23}}^{r_{24}} \frac{2\,\mu}{x}\, d\,x \tag{16}$$
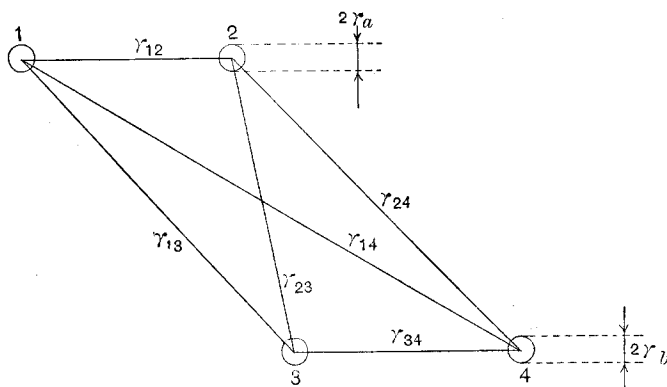
FIG. 5.

$$M = 2\,\mu \log \frac{r_{14}\,r_{23}}{r_{13}\,r_{24}} \tag{17}$$

In henries per mile this is

$$M = \left(0.7411 \ \log_{10} \frac{r_{14}\,r_{23}}{r_{13}\,r_{24}} \right)\ 10^{-3} \tag{18}$$

The foregoing expressions for self and mutual inductance may be established from the standpoint of the kinetic energy of an electromagnetic system, as shown by Maxwell[*] and Heaviside.[†]

The introduction of another circuit within the field of a given circuit does not alter the expression for the self-inductance

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[*] Maxwell; vol. II., art. 685.

[†] Heaviside; Elec. Papers, vol. I., p. 100.

of either circuit. The mutual inductance between the circuits is taken care of in the general equations for the energy of the whole system.

The capacity constants are not quite so simple, because the addition of a circuit within the field of a given circuit changes the constant for the given circuit by itself; and when a third circuit is considered the constants for the two previous circuits are again altered and their mutual constant is also altered and so on, for each circuit added to the group.

Take a system of n conductors with respective charges q_1, q_2, q_3 . . . q_n, and pressures v_1, v_2, v_3 . . . v_n. The pressure of each conductor is a homogeneous linear function of the n charges and the total energy of the system is half the sum of the products of the pressure of each conductor into its charge.

$$W = \tfrac{1}{2} u_{11} q_1{}^2 + u_{21} q_1 q_2 + u_{31} q_1 q_3 + \ . \ . \ .$$
$$+ \tfrac{1}{2} u_{22} q_2{}^2 + u_{32} q_2 q_3 + \ . \ . \ .$$
$$+ \tfrac{1}{2} u_{33} q_3{}^2 + \ . \ . \ . \tag{19}$$

Differentiating W with respect to the charge on any conductor we get the pressure of that conductor, for the pressure is the work done in increasing W by adding a unit charge. This gives equations of the form.

$$\left.\begin{array}{l} V_1 = u_{11} q_1 + u_{21} q_2 + u_{31} q_3 + \ . \ . \ . \ + u_{n1} q_u \\ V_2 = u_{12} q_1 + u_{22} q_2 + u_{32} q_3 + \ . \ . \ . \ + u_{n2} q_n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ V_n = u_{1n} q_1 + u_{2n} q_2 + u_{3n} q_3 + \ . \ . \ . + u_{nn} q_n \end{array}\right\} \tag{20}$$

The coefficient u_{xx} expresses the pressure of the conductor x when its charge is unity and that of all other conductors zero. The coefficient u_{xy} expresses the pressure of the conductor y when the charge on x is unity, the charges on the other conductors being zero, and $u_{xy} = u_{yx}$. Solving (20) for the charges,

$$\left.\begin{array}{l} q_1 = C_{11} V_1 + C_{21} V_2 + C_{31} V_3 + \ . \ . \ + C_{n1} V_n \\ q_2 = C_{12} V_1 + C_{22} V_2 + C_{32} V_3 + \ . \ . \ + C_{n2} V_n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ q_n = C_{1n} V_1 + C_{2n} V_2 + C_{3n} V_3 + \ . \ . \ + C_{nn} V_n \end{array}\right\} \tag{21}$$

where C_{xx} is a coefficient of capacity and C_{xy} is a coeffiicent of induction or mutual capacity. The coefficients C are functions of the coefficients u. Maxwell[*] shows that the coefficients of pressure are all positive, but none of the coefficients u_{xy} are

* Maxwell; vol. I., chap. III.

greater than u_{xx}; none of the coefficients of induction (mutual capacity) are positive and the sum of all those belonging to one conductor is not numerically greater than the coefficient of capacity of that conductor, the latter being always positive; when there is only one conductor in the field, its coefficient of pressure with respect to itself is the reciprocal of its capacity; if a new conductor is brought into the field, the coefficient of pressure of any other conductor with respect to itself is diminished, the coefficients of capacity are increased and the coefficients of induction (mutual capacity) are diminished. To find the expression for the coefficients u, consider a single wire in free space, with a charge q per unit length.
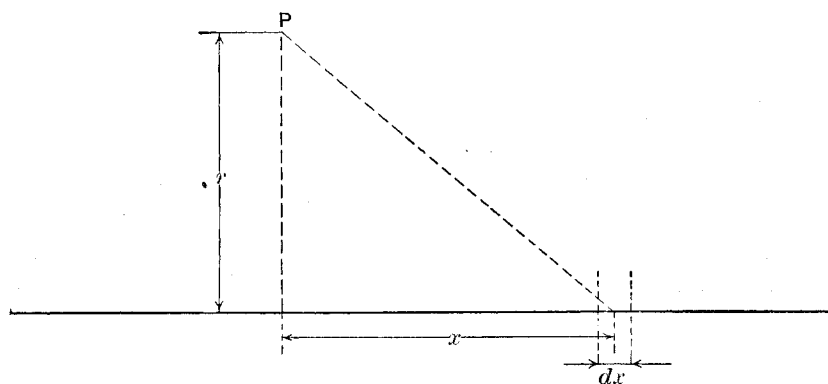
FIG. 6.

The force at P due to the charge $q\,d\,x$ is

$$d\,F_1 = \frac{q\,d\,x}{x^2 + r^2} \tag{22}$$

and the force normal to the wire (the longitudinal component vanishes when the whole wire is considered) is

$$d\,F = \frac{q\,r\,d\,x}{(x^2 + r^2)^{\frac{3}{2}}} \tag{23}$$

$$F = \int_{-\infty}^{+\infty} \frac{q\,r}{(x^2 + r^2)^{\frac{3}{2}}}\,d\,x = \frac{2\,q}{r} \tag{24}$$

But the force is the negative of the partial derivative of the pressure with respect to the distance, or

$$F = -\frac{\partial}{\partial r} (V) \tag{25}$$

whence
$$V = 2q \log \frac{a}{r} \tag{26}$$

a being a constant.

In practice there is always the return conductor of opposite pressure to consider; or if the earth be the return the electrical image must be considered. The pressure at the point P, distant r' from a second parallel wire, with a charge $-q$ per unit length is

$$V^1 = -2q \log \frac{a'}{r'} \tag{27}$$

The total pressure is

$$V_0 = 2q \log \frac{r'}{r} \tag{28}$$

At an infinite distance r and r' approach unity as a ratio and since $\log 1 = 0$, the constants a and a' must be equal.

Considering a two-wire metallic circuit whose wires have a separation d and radius r,

$$V_1 = 2q \log \frac{d}{r} \tag{29}$$

$$V_2 = -2q \log \frac{d}{r} \tag{30}$$

$$V_1 - V_2 = 2q \log \frac{d^2}{r^2} = 4q \log \frac{d}{r} \tag{31}$$

$$C = \frac{q}{V_1 - V_2} = \frac{1}{4 \log \frac{d}{r}} \tag{32}$$

The pressure coefficient in the above case is

$$u = 2 \log \frac{r'}{r}. \tag{33}$$

The pressure of the electric field along the X axis in Fig. 2 is proportional to

$$y = \log \frac{a + x}{a - x} \tag{34}$$

for points without the wires, and is constant within them. The curve in Fig. 4 is plotted from (34).

If the wires are insulated the capacity is slightly increased by the greater specific inductive capacity of the insulation. If the wires are of radius r_1 and the insulating coverings of radius r_2, (32) becomes

$$C = \frac{1}{4 \log \dfrac{d}{r_2} + \dfrac{4}{k} \log \dfrac{r_2}{r_1}} \tag{35}$$

where k is the specific inductive capacity of the insulation. This increase in capacity may amount to several per cent. The above equations are general and apply to a system of any number of wires; a discussion of the subject is given by Heaviside,* with examples worked out for ground-return circuits.

The case of two two-wire metallic circuits in Fig. 5 may be derived as follows:

$$\left.\begin{aligned}
v_1 &= u_{11}\, q_1 + u_{21}\, q_2 + u_{31}\, q_3 + u_{41}\, q_4 \\
v_2 &= u_{12}\, q_1 + u_{22}\, q_2 + u_{32}\, q_3 + u_{42}\, q_4 \\
v_3 &= u_{13}\, q_1 + u_{23}\, q_2 + u_{33}\, q_3 + u_{43}\, q_4 \\
v_4 &= u_{14}\, q_1 + u_{24}\, q_2 + u_{34}\, q_3 + u_{44}\, q_4
\end{aligned}\right\} \tag{36}$$

The circuit conditions are:

$$\left.\begin{aligned}
V_1 - V_2 &= V_a \\
V_3 - V_4 &= V_b \\
q_1 + q_2 &= 0 \\
q_3 + q_4 &= 0
\end{aligned}\right\} \tag{36a}$$

A coefficient of the form u_{xy} equals the coefficient u_{yx}. Then we have

$$\left.\begin{aligned}
V_a &= (u_{11} - 2\,u_{12} + u_{22})\, q_1 + (u_{13} - u_{14} - u_{23} + u_{24})\, q_3 \\
V_b &= (u_{13} - u_{23} - u_{14} + u_{24})\, q_1 + (u_{33} - 2\,u_{34} + u_{44})\, q_3
\end{aligned}\right\} \tag{37}$$

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* Heaviside; Elec. Papers, vol. I., p. 42, p 140; vol. II., p 303, p. 329.

But

$$
(u_{11} - 2\,u_{12} + u_{22}) = 4\log\frac{r_{12}}{r_a}
$$

$$
(u_{33} - 2\,u_{34} + u_{44}) = 4\log\frac{r_{34}}{r_b}
\tag{38}
$$

$$
(u_{13} - u_{14} - u_{23} + u_{24}) = 2\log\frac{r_{14}\,r_{23}}{r_{13}\,r_{24}}
$$

$$
\left.\begin{array}{l}
q_1 = C_a\,V_a + C_{ba}\,V_b \\
q_3 = C_{ab}\,V_a + C_b\,V_b
\end{array}\right\}
\tag{39}
$$

From which it follows that

$$
C_a = \frac{4\log\dfrac{r_{34}}{r_b}}{\left[\,16\log\dfrac{r_{12}}{r_a}.\ \log\dfrac{r_{34}}{r_b} - \left(2\log\dfrac{r_{14}\,r_{23}}{r_{13}\,r_{24}}\right)^2\,\right]}
\tag{40}
$$

$$
C_b = \frac{4\log\dfrac{r_{12}}{r_a}}{\left[\,\phantom{16\log\dfrac{r_{12}}{r_a}}\right]}
\tag{41}
$$

$$
- C_{ab} = \frac{2\log\dfrac{r_{14}\,r_{23}}{r_{13}\,r_{24}}}{\left[\,16\log\dfrac{r_{12}}{r_a}.\ \log\dfrac{r_{34}}{r_b} - \left(2\log\dfrac{r_{14}\,r_{23}}{r_{13}\,r_{24}}\right)^2\,\right]}
\tag{42}
$$

The above formulas for capacity may be expressed in farads per mile by multiplying by 0.07768×10^{-6}.

The first important conclusion from the expressions for mutual inductance (17) and mutual capacity (42) is that these two constants vanish simultaneously when

$$
r_{14}\,r_{23} = r_{13}\,r_{24}
\tag{43}
$$

Let

$$
\frac{r_{23}}{r_{13}} = \frac{r_{24}}{r_{14}} = p
\tag{44}
$$

Let the coördinates of one circuit be fixed, and be $(+a, o)$ and $(-a, o)$, as in Figs. 3 and 4.   If the two wires of the other

circuit are on the locus of the point, (x, y), for any particular value of p, (43) will be satisfied. The conversion of (44) to rectangular coördinates is

$$p^2 [(x+a)^2 + y^2] = [(x-a)^2 + y^2] \qquad (45)$$

or

$$\left[x + a\frac{p^2 + 1}{p^2 - 1} \right]^2 + y^2 = \left(a\frac{p^2 + 1}{p^2 - 1} \right)^2 - a^2 \qquad (46)$$

This is a system of eccentric circles, there being one circle for each value of p. The plot of this system of circles, from (46), gives the magnetic field of Fig. 1. That is, the mutual
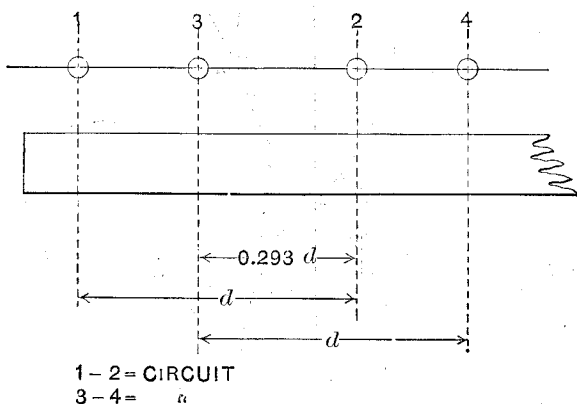
1 − 2 = CIRCUIT
3 − 4 =   "

Fig. 7.

interferences vanish when the wires of one circuit are on a single flux line of the field of the other circuit, and therefore enclose none of the magnetic flux of that circuit and are at points of equal electric pressure due to that circuit.

It is seldom in practice that the conditions permit the use of the relations given in (43) and (46). A particular solution is applicable to two two-wire circuits, as independent circuits or as the respective phases of a four-wire, two-phase line. This is shown in Fig. 7.

To show how the constants of mutual inductance and capacity depend on the relative wire positions, the curves in Figs. 8 and 9 have been plotted from

$$\frac{M}{2} = \log \frac{r_{14}\,r_{23}}{r_{13}\,r_{24}} \qquad (47)$$

If the circuit 3-4 always maintains a horizontal position, in Fig. 8, and the coördinates of P are (x, y), P being constrained to move in a circle, (47) becomes

$$\frac{M}{2} = \frac{1}{2} \log \frac{[(a+b)^2 + c^2]^2 - [2(a+b)x]^2}{[(a-b)^2 + c^2] - [2(a-b)x]^2} \tag{48}$$

$a = b, \quad c = 4a.$

SCALE, $Log_{10} 10 = 30.''$

FIG. 8.

where C is given by the locus of P,

$$x^2 + y^2 = C^2 \tag{49}$$

If the circuit 3-4 maintains a vertical position, in Fig. 9, and the locus of P is again the circle of equation (49),

$$\frac{M}{2} = \frac{1}{2} \log \frac{(x^2 - a^2)^2 + (x+a)^2(y+b)^2 + (x-a)^2(y-b)^2 + (y^2 - b^2)^2}{(x^2 - a^2)^2 + (x-a)^2(y+b)^2 + (x+a)^2(y-b)^2 + (y^2 - b^2)^2} \tag{50}$$

The curves in Figs. 8 and 9 are polar diagrams, giving the relative mutual disturbances as P describes a circle. There are four points in a revolution at which M is zero. It is possible to introduce a third circuit and so locate it that M is zero for any two, and hence for all.

$a = b.$ $c = 4a.$

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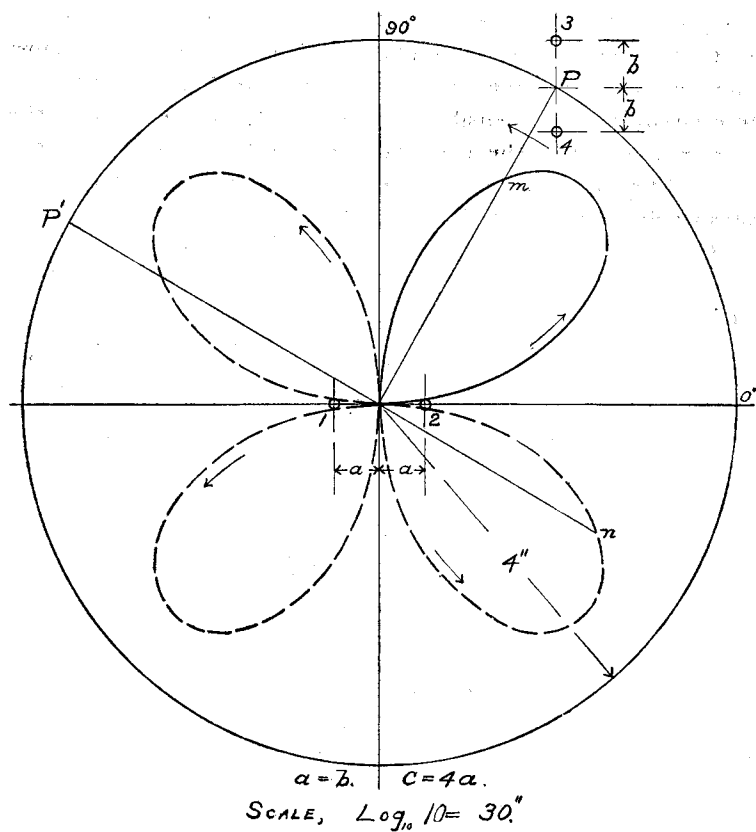
FIG. 9.

When the number exceeds three it becomes, in a practical sense, impossible to prevent mutual interferences. It is evident from expressions (17) and (42) for the mutual inductance and the mutual capacity that the interchange of wires 1 and 2 or wires 3 and 4 will change the signs of these formulas without changing their magnitude. It is evident also that interchanging both wires 1 and 2 and wires 3 and 4 will not alter the formulas in any way; that is the transposition of one of two parallel

circuits will reverse the signs of the mutually induced currents and pressures.

Consider the circuits in Fig. 10.

The additional conditions which are necessary for the disturbances to vanish are as follows. The current in the disturbing circuit must be constant in magnitude and in phase throughout the distance l, and the pressure must likewise be constant in magnitude and in phase. That is, 1 must be so short a portion of the wave length in the disturbing circuit that these conditions are sensibly fulfilled. At low frequencies the wave length is not the primary consideration, but rather the drop in line pressure and the decrease of the line current, due respectively to wire impedance and to the leakage and the static charge.
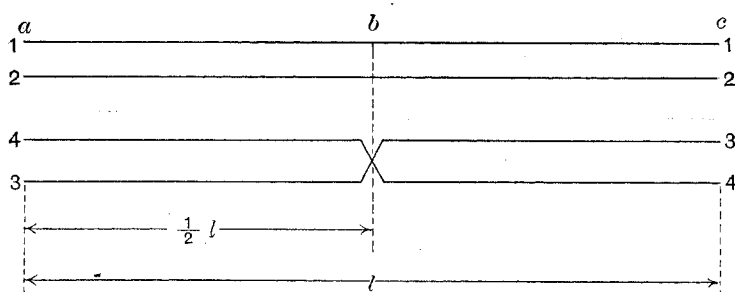
FIG. 10.

As shown by Mr. J. J. Carty, in a paper before the INSTITUTE, on " Inductive Disturbances in Telephone Circuits," in April, 1891, the induced currents due to electric induction cannot be made to vanish entirely, as can those due to magnetic induction.

The treatment of induction between telephone circuits has been by empirical rule rather than theory. It has been determined by experiment, with receivers of a given sensitiveness and transmitters of a given power, how frequently two adjacent circuits should be transposed in order to eliminate the cross-talk. Using two-mile sections, transposed at the centre, the cross-talk is distinguishable with transmission sufficiently powerful for 1000-mile service. Half- or quarter-mile sections, transposed at the centre, are satisfactory; this results in a minimum transposition spacing of one-quarter mile. The existence of cable at each end of the line, in any considerable length, will

reduce the cross-talk. It is necessary to devise different types of transposed circuits, no two transposed alike, in order to treat the cases occurring in practice. The manner of doing this is shown in Fig. 11.

The " exposure," as it is termed, of circuit 1 to circuit 2 is $\frac{1}{4}$; of 1 to 3 is $\frac{1}{4}$; of 2 to 3 is $\frac{1}{2}$; because a transposition at the junction of two sections, each transposed at its centre, has almost no beneficial effect. The exposure of 1 to 5 is $\frac{1}{8}$; of 2 to 6 and 3 to 7, $\frac{1}{8}$; of 2 to 8 and 2 to 9, $\frac{1}{16}$; and so on. The tabulated exposures are given in Fig 12, in terms of the length l of a
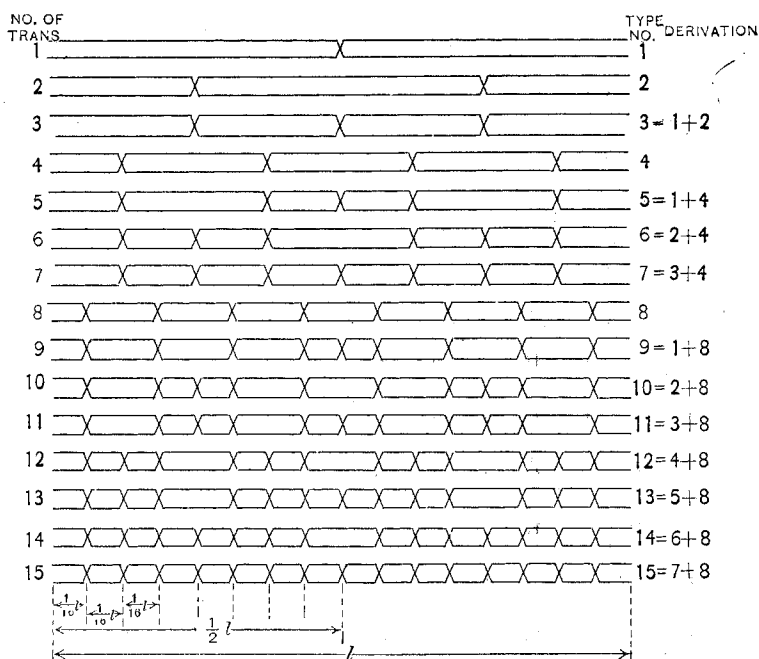
Fig. 11.

transposition section. The derivation of the types is simple; the first three are obvious. The fourth is obtained by doubling the number of divisions of the transposition section from two to four. The fifth is obtained by superposing the first type on the fourth type, the sixth by superposing the second on the fourth, etc. The composition of the complicated types is shown in Fig. 11. So far as the writer is aware this synthetical system of deriving types of dissimilarly transposed circuits is due to Mr. John A. Barrett. The method may be extended as far as desired, but 15 types are usually sufficient.

The choice of a convenient length for l is an important matter. An eight-mile section has been extensively used, but is probably too cumbersome. A much more convenient section would be one four miles in length.

If there are only a few circuits, a two-mile section may be

To	Exposure of Type No.												
---	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	$\frac{1}{4}$													
3	$\frac{1}{4}$	$\frac{1}{2}$												
4	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$											
5	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$										
6	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$									
7	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$								
8	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$							
9	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{2}$						
10	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{1}{4}$					
11	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$				
12	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$			
13	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$		
14	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	
15	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

Fig. 12.

used. A case of this sort is a ten-wire line, on a single-cross-arm, as shown in Fig. 13.

The following table gives the theoretically permissible exposure between any two circuits. The basis on which this table is computed is as follows; calculate $\frac{M}{2}$ from (47) for

each combination of two circuits; if the standard of exposure between two adjacent circuits is, say, one-fourth of a mile, then the exposure between any other two circuits will be inversely proportional to the calculated $\dfrac{M}{2}$ for the two latter circuits.
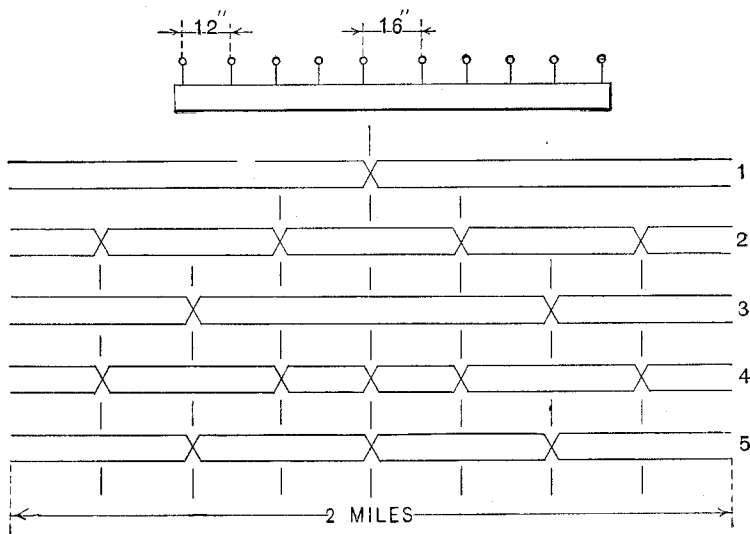

FIG. 13.

	Circuit No.				
To	1	2	3	4	5
1					
2	0.25				
3	0.90	0.21			
4	2.85	1.31	0.21		
5	4.95	2.85	0.90	0.25	

The tabular values are in miles.*

The procedure in applying the sections to a long line is as follows: beginning at the first pole, of the open-wire line, apply the sections consecutively until the last section is approached. It is seldom the line length is a multiple of the section length, so the remainder should be a special section by itself, if over half the length of a standard section, or should be added to the last section, making a special section longer than the standard. The distances between transposition poles will be shortened or lengthened sufficiently to apply the standard section.

When there are several cross-arms it will simplify the construction if the transposition types are so chosen, in laying out a standard section, that there will be as few different types of transposition poles as possible. The junction of two lines which separate, or become joint, is a point which should be made
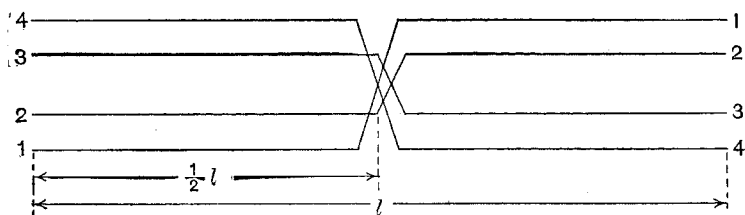
Fig. 14.

the junction of the transposition sections, as well. The 15 types shown in Fig. 11 will be sufficient for a 40-wire line, because there will be certain diagonal exposures requiring transposition only at long intervals, and these extra transpositions may be located at the junction poles between sections.

Single-wire grounded lines cannot be transposed, and for this reason are not extensively used. They may be operated for short lengths of a mile or so, without interference from cross-talk, if very low-power receivers are used. For greater lengths, there will be serious cross-talk; the low-power receiver is still necessary to remove extraneous inductive disturbances.

Phantom circuits, built up from metallic circuits, are not in extensive use. The ordinary transposition is of no effect, and unless specially transposed such circuits will cross-talk and will be subject to extraneous induction. The transposition of a phantom may be done in several ways. Fig. 14 shows a transposition in which the phantom is transposed, and its two com-

ponent circuits, while not transposed with respect to each other, are transposed with respect to any other parallel circuits but are also moved in position, thereby partly offsetting the latter transposition.

A second method is shown in Fig. 15, where the two component circuits are not transposed with respect to each other, or to any additional parallel circuits.

Another method, in which the component circuits are transposed with respect to each other, is shown in Fig. 16.

If the regular transposition sections are not too long, the
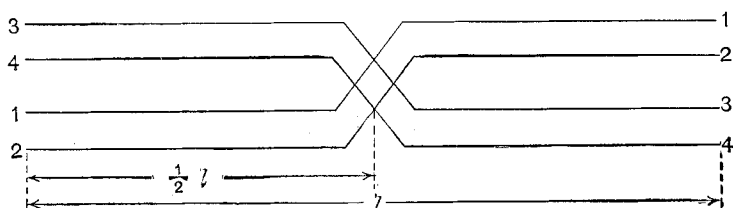
Fig. **15.**

phantom transposition may be placed at a pole forming the junction between two sections, where it will have no effect on the regular sectional system. If it is placed within a section it will upset the tabular exposures shown in Fig. 12. If many phantoms are employed, it will be best to lay out a section with only the phantom transpositions and then to superpose
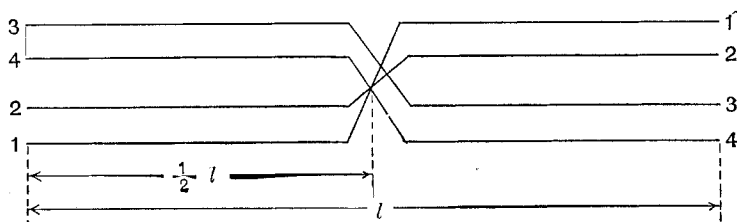
Fig. 16.

on it the additional simple transpositions to eliminate the cross-talk between the metallic circuits themselves.

When telephone lines are paralleled by power lines the transposition problem becomes more complicated. It is necessary to recognize the manner in which the disturbing circuits alter the magnitude of their influence, by reason of abrupt changes in line pressure or line current, and changes in the wire spacings. If circuit 1-2, in Fig. 10, is the power circuit and 3-4 is the telephone circuit, there will be no mutual interferences if the

distance l is not great.   That is, the percentage pressure drop in 1-2 must be small enough so that the total induced charges on the portion $a\,b$ are sensibly equal and opposite to those on $b\,c$; likewise, the percentage current loss in 1-2 must be so small that the magnetically induced pressure in the portion $a\,b$ is equal and opposite to that in $b\,c$.   The question as to whether electric or magnetic induction is the greatest is complicated by the radical differences between the systems.   In long telephone lines the electric induction may predominate; the impedance of an indefinitely long line is usually several hundred or a thousand ohms, and therefore the pressure is very much greater in magnitude than the current.   In power circuits this ratio is dependent largely on the load, in any given case, but there are many instances where the ratio of pressure to current is small. The magnetic induction is usually predominant to a large degree, in the latter cases.   There have been instances of exposure to circuits giving 24-hour service, from which the induction was heavy only during the period of load.

If the circuit 1-2, in Fig. 10, has connected to it a transformer or a branch circuit within $a\,c$, the effect of the transposition is no longer to neutralize the induction.   If the transformer is at b the transposition will have the least effect.   If 1-2 is a constant-current circuit and an arc lamp is cut in, the effect will be the same, as regards the efficiency of the transposition.   A change in the spacing of the wires 1-2 will also decrease the effect of the transposition.

Therefore that portion of the disturbing line within which the conditions, affecting the phase or the magnitude of the induced currents and pressures, are constant should be treated as a section and all the telephone circuits should be transposed opposite its centre.   The regular cross-talk transpositions should be removed from the section.   If the total external exposure to the power line involves several consecutive sections, the cross-talk transpositions should be opposite the junction of two sections, at the point where the disturbing current and pressure change in magnitude or phase.

An illustration of this is shown in Fig. 17 and 18, which shows the case in which the writer first tried this method, in 1902, on a telephone line paralleling two three-phase circuits.   Fig. 17 shows the initial situation, which caused service complaints. Fig. 18 shows the remedy, which eliminated the interferences entirely.

A more general case is shown in Fig. 19; this is hypothetical but illustrates various conditions. This method works well until the character of the exposure becomes complicated, and then the transpositions become so frequent as to be imprac-
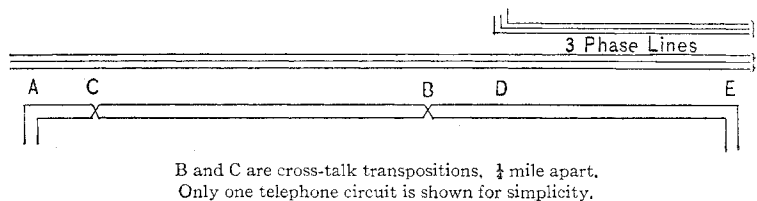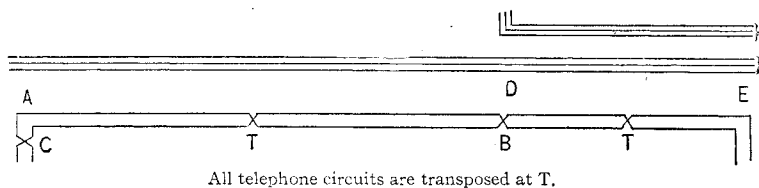

B and C are cross-talk transpositions, ¼ mile apart.
Only one telephone circuit is shown for simplicity.

FIG. 17.

All telephone circuits are transposed at T.

FIG. 18.

ticable and, further, they frequently cannot be properly situated because of physical limitations. If the electric light and power system is developing rapidly the exposures are changed frequently by new construction. Again the telephone line may

1–2 = Constant potential circuit.
3–4 = Constant current circuit.
5–6 = Telephone circuit.
T = Induction transpositions.
T' = Cross-talk transpositions.
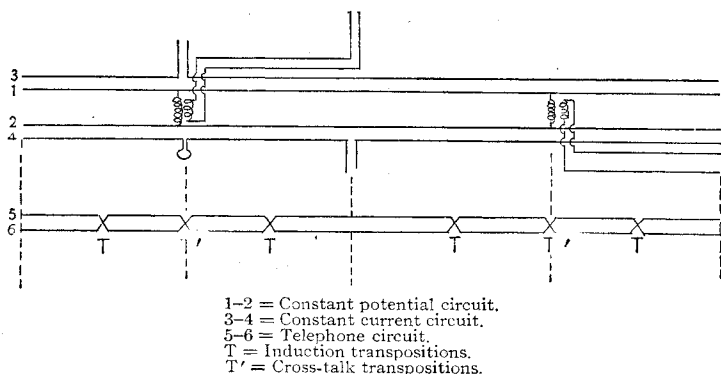
FIG. 19.

be separated from the power line by the width of a street; here the induction, while not excessive, is yet troublesome; and the theoretical transposition system to meet the conditions of ex-

posure is just as elaborate as though one line were beneath
the other.   Twisted pairs or cable will meet this condition,
although either remedy impairs the telephonic line efficiency.
A prior consideration is whether the power wires are not sep-
arated, with respect to each other, farther than is necessary
with the pressure employed and whether the separations are
uniform.   Fig. 20 shows a case in point, where the change in
the separation of the wires *a b* from several feet to less than
two feet so reduces the field of the power circuit that the cross-
talk transpositions in the telephone line practically take care
of the induction.
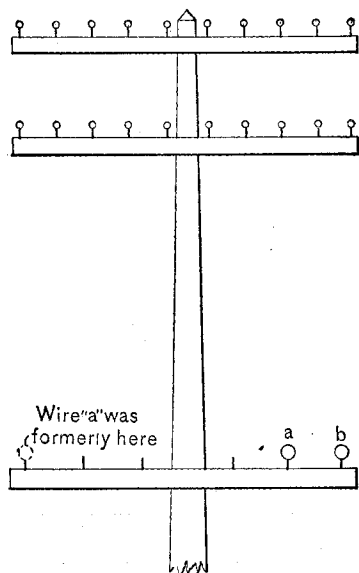
FIG. 20.

Where telephone lines have paralleled transmission lines along
highways, separated by 15 or 20 feet, or by the width of the high-
way, the following practice has been employed.   All the telephone
circuits are transposed at every tenth pole—40 poles per mile—the
cross-talk transpositions occurring midway, so that transpo-
sition poles are five spans apart, the cross-talk and the induction
transposition poles alternating.   This practice, while it is some-
what uncertain of producing the best results, is probably best
suited to certain requirements,—where the method of sections
would not produce any better results because of the physical
unevenness and crookedness of the right-of-way.

Not infrequently high-pressure systems are transposed for their internal protection. When telephone circuits are on such pole lines, the transpositions in the power circuit may be availed of to reduce induction in the telephone circuits, but in the case of separate telephone circuits, on a parallel pole line of separate ownership, it seems wiser to treat such transpositions as neutral points—to be opposite the junction of two transposition sections of the telephone line. The question of where to locate telephone wires on high-pressure lines and how to protect them contains material for a paper by itself. In general the induction from a three-phase line is slightly greater than from a single-phase line having the same wire spacing and current per wire and a pressure equal to the $\triangle$ pressure. The separation between the wires should be as small as consistent with the length of span and their plane suitably arranged with respect to the plane or planes of the power circuits.
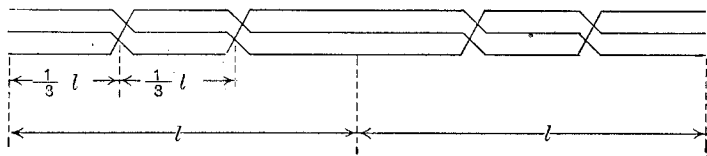

FIG. 21.

A three-wire, three-phase line will have two transpositions within a section, as in Fig. 21; two complete sections are shown.

It seems to be fairly general practice to transpose high-pressure systems, but it has been the writer's observation that the length of a section, l in Fig. 21, is usually three miles at least. On that account it is difficult to avail of the power line transpositions in transposing the telephone line, and it is usually necessary to treat them as the junction points between adjacent sections.

On account of the very low frequency and the high ratio of pressure to current customary in high-pressure practice, there are usually no inductive interferences if the power and the telephone lines are separated by the width of a highway or 30 to 40 feet, because the regular cross-talk transpositions are sufficient.

In the case of a single telephone circuit paralleling a power circuit, the frequency of transpositions is a problem requiring calculation in any given case, because of the lack of standard

practice in regard to wire spacing and circuit location. The writer made a series of listening tests with a bipolar receiver of 70 ohms resistance to determine the permissible current at low frequencies which would not interfere with telephonic transmission over the longest lines—1000 miles and upwards. The circuit is shown in Fig. 22.
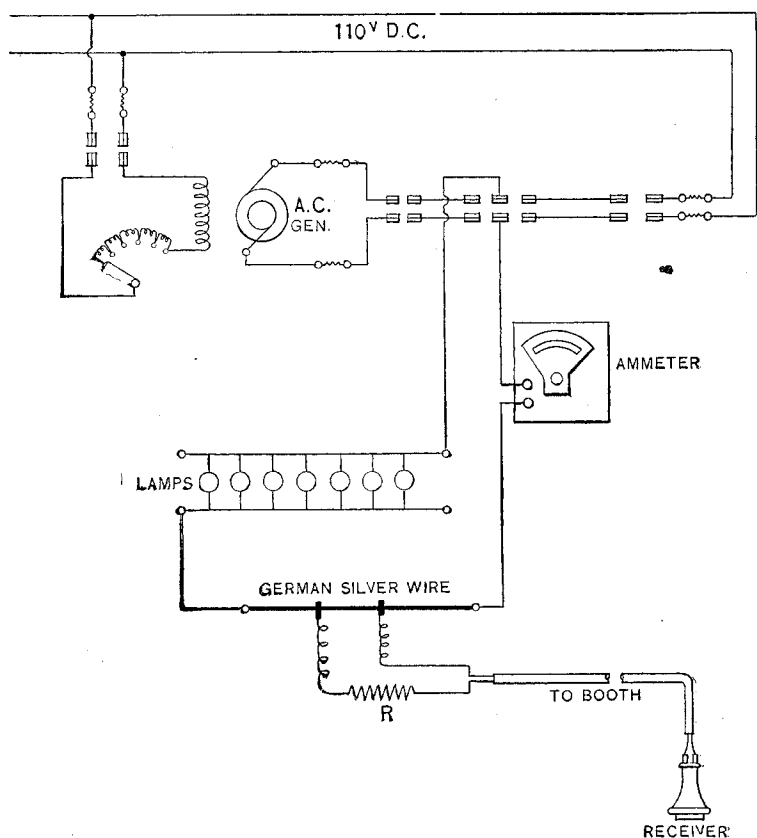

FIG. 22.

The method was that of deriving a very small known pressure from a non-inductive resistance and inserting a very large non-inductive resistance R (1000 to 20 000 ohms) in series with the shunt circuit containing the receiver. The listening tests were made by four observers, who adjusted R to the minimum permissible value, in their estimation. The average results are plotted in Fig. 23.

Naturally these results, based on ear measurements, are not highly precise, but they represent very small disturbances and are safe practice. In the case of short telephone circuits on power lines, used for private purposes, the receiver power may be much reduced and the permissible terminal current greatly increased. The method is shown in Fig. 24; the condenser is not wholly essential, but it increases the selective action.
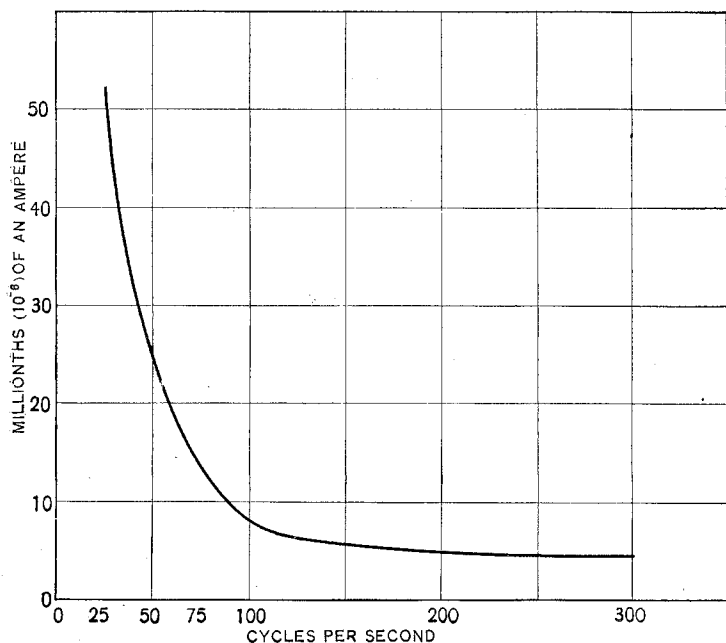
The general theory of mutual disturbances in parallel cir-

FIG. 23.

cuits, in complete form, is too elaborate to introduce here. It may be found in Vol. I. of Heaviside's Electrical Papers.*

The question of leakage is a consideration prior to that of transposition, for the theory of transposition rests on the hypothesis of electrically balanced circuits. The ordinary pony glass insulator seems to be sufficient where there are no high pressures on the same pole line. In cases of joint lines and of telephone circuits on high-pressure lines additional insulation, in the way of heavier insulators, often seems to be necessary. An elabo-

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*Electrical Papers, vol. I, on "Induction Between Parallel Wires."

rate discussion of what constitutes good insulation is not in place here, but the best construction in this respect is the first essential to successful transposition.

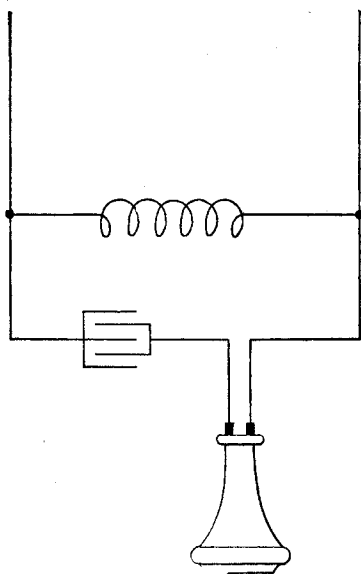There are two types of construction in use, in making trans-

FIG. 24.

positions.   One of these, the old-fashioned square transposition made by dead-ending both wires, each way, on two-piece insulators, and cross-connecting, is familiar to all.   The other method is shown in Fig. 25; it requires two spans in which to
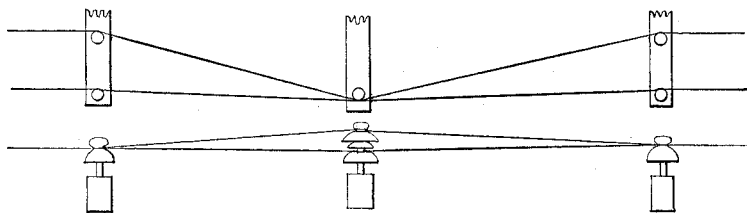

FIG. 25.

complete the entire transposition.   It is known as the " Murphy " and the " single-pin " transposition, and is coming into general use.

It has the comparative advantages of less first cost and

simpler construction. It can be cut in at any time, cut out, or moved several poles, at less cost and with much less work than in the case of a square transposition. If transpositions occur frequently, every $\frac{1}{2}$ or $\frac{1}{4}$ mile, the line capacity is increased a few per cent. and the line inductance diminished, with a consequent slight increase in attenuation. The square transposition has the advantage of concentrating the entire transposition within a very short length and of not altering the plane or the separation of the wires. While the single-pin transposition changes the plane of the circuit, the wire separation is greatly reduced and this is an advantage. Since it requires two spans in which to make this transposition, it is possible to transpose only at every other pole, as a maximum, in case of excessive induction, against every pole for the square transposition. For regular cross-talk transpositions the single-pin type is now preferred.

In conclusion, it seems evident that the recommendation of standards, in the matter of wire separations, standard cross-arms, and standard systems of transposition, would greatly facilitate the removal of inductive interferences and materially assist the builders of telephone circuits throughout the country. This matter is suggested for the consideration of the INSTITUTE.

DISCUSSION ON "THE TRANSPOSITION OF ELECTRICAL CON-
DUCTORS."

W. S. FRANKLIN: The speaker's objection to these estab-
lished methods is that in both cases the fundamental equations
are based upon ideal conditions; which even as ideal conditions
are impossible and meaningless. Precisely the place where the
mathematics is hitched on to the physics, there is no physics
at all, nothing in fact but an unconstrained fiction. Now, the
speaker knows of no case in which physically significant math-
ematics cannot be established on the basis of real conditions.
It is physically meaningless to consider a long straight wire as a
circuit, and the idea of inductance of such a wire is absurd.
It is also physically meaningless, in all strictness, to consider a
lone electrical charge; and the capacity of an isolated sphere, so
universally discussed in treatises on physics, is very nice and
simple, but the student who is drilled in that idea never can
quite understand a condenser. The real ideas are inductance
of *circuits*, capacities of *condensers*; that is, of two bodies in
the simplest cases, and mutual inductance of circuits and co-
efficients of mutual capacity of condensers. Of course, the
speaker does not mean to say that Maxwell's theory of a system
of charged bodies is essentially wrong, but he does mean to say
that it is not as intelligible, not as thinkable, if you please, as
it would be if it did not involve the highly fictitious idea of the
electrical pressure coefficient of a body with respect to itself.
and of the capacity coefficient of a body with respect to itself,
The speaker cannot think of such a thing, and he is sure that a
careful analysis of the ideas of anyone who thinks he thinks of
the pressure coefficient of a body with respect to itself or of the
capacity of a sphere, would reveal the fact that the ideas of
that person really refer to mathematical expressions or to words,
not to physical things at all.

Mr. Fowle says that cable ends on a long telephone line serve
to reduce cross-talk, to increase the effectiveness of the transpo-
sition on the middle portions of the line. Does he mean to say
that it is because of the attenuation which takes place in the
cable ends? or because of the very complete transposition which
is accomplished in a twisted cable?

F. F. FOWLE: The decrease in cross-talk, which occurs when
there is a terminal cable, is due, in the opinion of the speaker, to
the attenuation in the cable, which diminishes the power of
the cross-talk currents coming from the distant open-wire line.

W. J. LANSLEY: What does Mr. Fowle regard as the per-
missible distance, the ordinary minimum distance between tele-
phone circuits and single phase 2000 volt power circuits?

F. F. FOWLE: Does Mr. Lansley mean from the standpoint
of the hazard to the circuit or the standpoint of induction?

W. J. LANSLEY: Standpoint of induction.

F. F. FOWLE: It depends not only on the pressure of the
circuit, but on the load and on the distance between the wires
of the power circuit and the distance between the wires of the

telephone circuit. In the opinion of the speaker it is not possible to answer Mr. Lansley's question generally. Perhaps a specific case could be answered approximately.

W. J. LANSLEY: Take the pin distance between the wire. Suppose the 2000 volt wires are eight inches apart, and your telephone wires are the same distance apart, how far away should the telephone wire be from the power wire, presumably on the same pole line?

F. F. FOWLE: The speaker's opinion, in this case, is that the circuits should be separated at least two or three feet and that the telephone circuit should be transposed every ten poles, or every quarter mile. The size of the wires, the load on the alternating current circuit and the frequency are factors in determining the distance between transpositions. If the two circuits are on the same cross-arm, there is likely to be more leakage than there would be if they were on separate cross-arms. The use of extra heavy insulators on the telephone circuit is advisable.

W. S. FRANKLIN: The speaker understands the difficulty from leakage is partly balanced by transposition. Mr. Fowle do you depend on transposition at all for obviating bad insulation? Do you find it practicable?

F. F. FOWLE: The speaker is not aware that transpositions have been depended on in such cases. In telephone work it is essential to make frequent insulation tests; the insulation should be the same between each wire and earth and should be high. The speaker recalls an insulation measurement on a line nearly 1000 miles long which gave between 40 and 50 megohms per mile, on a dry, clear, cold day. An insulation resistance of 50 megohms per mile in good weather indicates first class construction, with the pony glass in ordinary use. In wet weather the insulation may fall to one megohm per mile, or less; but at one megohm per mile the attenuation is not increased to a noticeable extent and the distortion is diminished.

F. F. FOWLE (*by letter*): Certain of Professor Franklin's remarks do not seem to the writer to be fully justified.

The connection between the physical and the mathematical sides of any question is seldom obvious, in the step by step development, without careful study. This study is obviously the occupation of the student, but the engineer is after results by the shortest paths; and his education and experience enable him to conduct his researches without a too laborious attention to the physical interpretation of each step. The object of the mathematical investigation of inductance and capacity of aerial wires is obvious in Figs. 1 to 4, 8, and 9, and in the application of its results to the choice of types from Fig. 11 in treating a real situation, as exemplified in Fig. 13.

The writer has never seen elsewhere the deduction of equations (40), (41), and (42). The fact that the investigation is an extension of Heaviside's method and is based on Maxwell's fundamental equations, seems an advantage rather than otherwise. If the results are inaccurate or if there are more rational modern methods the writer will gladly learn of it.