The Two-Stage Current Transformer

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This paper presents a brief discussion of the current transformer as used with measuring and controlling apparatus with special reference to the degree of accuracy which can be attained in the ratio and phase angle. A new type of current transformer is then described, in which it is possible to secure much higher accuracy with a given amount of iron and copper in the transformer. In this new device the transformation is effected in two stages, the first yielding in the usual way a secondary current which is approximately correct in magnitude and phase, and the second yielding an auxiliary corrective current which, when combined with the first secondary current, gives a resultant current which very closely approximates to the secondary current which would be furnished by an ideal current transformer having no errors. The two currents may easily be combined by having two like windings in the devices operated, one for the main and one for the auxiliary secondary current.

The mathematical theory of the two-stage current transformer is developed and applied. Experimental curves are given to compare the performance of the new transformer with that of an ordinary simple current transformer of good average performance. The effect of mutual inductance between the external secondary circuits is discussed, and some of the special advantages of the new transformer are given.

THE SIMPLE CURRENT TRANSFORMER

THE term "current transformer" as ordinarily used refers to a transformer used to deliver to electrical measuring and controlling devices a definite fraction of the line current. It consists essentially of a core of magnetic material on which are wound two coils, one of which, usually of a few turns of large wire, is connected in series with the high-voltage circuit, while the other coil (usually of a greater number of turns of smaller wire) supplies a secondary induced current which operates the measuring and controlling devices in the secondary circuit. The impedance of the external secondary circuit is properly referred to as the secondary burden.

In order that a secondary current may be induced, a certain component of the primary current must be used to produce the necessary magnetization, and to supply the core loss. The core being of iron it is readily appreciated that this component of current varies with (1) secondary burden, (2) frequency, (3) magnitude of the secondary current. Because this component does not vary directly with the secondary current, the ratio of the two currents varies with any changes in the above three factors occurring either separately or jointly. Also, the electrical phase difference between the primary current and the secondary current, which would be exactly 180 deg. in an ideal transformer, departs from 180 deg. by a small angle, the "phase angle," which varies with each of the three causes mentioned as affecting the ratio of currents. For the accurate operation of electrical measuring apparatus, especially wattmeters and watt-hour meters, it is necessary that the ratio of primary current to secondary current should always be constant in a fixed ratio and that the departure from the 180 deg. phase relation should be negligible. This should be true for all ordinary conditions of secondary burden, primary current and frequency. Changes in ratio affect the readings of instruments at all power factors while phase angles cause errors which greatly increase as the power factor is lowered. For example, in using a polyphase watt-hour meter for measuring the energy delivered over a three-phase system, a variation of 1 per cent in the ratio causes an error of 1 per cent in the registration, irrespective of the power factor. When the system is at 86.6 per cent or 50 per cent power factor, a phase angle of 20 minutes will cause errors of 0.3 per cent and 1.0 per cent respectively in the registration of the meter. While such errors in ratio and phase are known to exist, their effect upon the accuracy of the instruments to which they are connected is not always appreciated. In the past and even at the present day, many central-station men consider a current transformer as of absolute ratio and zero phase angle.

Besides the conditions already spoken of as affecting the ratio and phase angle of the ordinary current transformer, there is the question as to the magnetization of the core brought about during moments of opening and closing of the primary circuit or accidental opening of the secondary under load.

Voltage ("potential") transformers are inherently capable of a very much better performance than current transformers, especially as regards phase angle. The induction watt-hour meter has also been brought to a high state of development, and its performance on inductive and non-inductive loads is readily controlled by the user through the three standard adjustments (light-load, full-load, phase). The current transformer has lagged in development behind the other two essential elements of metering equipment. The only way to improve it radically, with the methods of construction commonly employed, is to use iron of magnetic qualities much superior to anything now commercially available. It is the purpose of this paper to show how the transformation of current for metering purposes can be brought up to an accuracy at least as high as that of the other component functions, by means of a device which we have called a "two-stage" current transformer.

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The two-stage current transformer (shown diagrammatically as two distinct transformers in Fig. 1) inherently and automatically effects a correction of current ratio and phase angle between primary and secondary currents to a high degree of accuracy and within wide limits of secondary burden. This is effected in a manner which may be called a "multi-

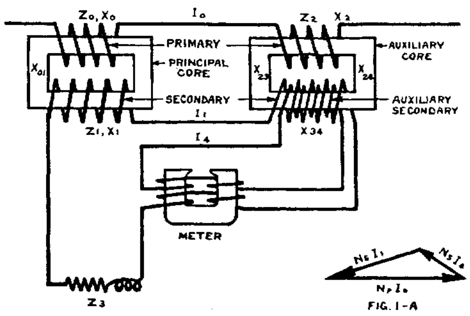

Fig. 1—Elementary Diagram of Electric and Magnetic Circuits of Two-Stage Current Transformer shown as Two Separate Transformers

stage" transformation in which one transformer (the one at the left in Fig. 1) is used to effect the transformation in the ordinary way, yielding a current which is approximately correct as to magnitude and phase. The primary current and this secondary current are then passed through two windings of the second current transformer in which the ratio of secondary turns to primary turns is equal to the desired ratio of primary current to secondary current, the two currents being sent through their respective windings in such a way that their magnetizing effects upon the core (in ampere-turns) tend to oppose each other. (This exact ratio of turns is in contrast to the fact that in ordinary current transformers as now constructed, in order to secure approximately the desired ratio, one or more turns of the secondary winding must be omitted from the number which would be required by an ideal transformer). This second current transformer is provided with another winding called the auxiliary secondary, having very approximately the same number of turns as the principal secondary winding.

It will be evident that if the first transformer is operating under conditions such that the secondary current happens to be exactly correct in magnitude and phase, the ampere-turns of the two windings of the second transformer will annul each other at every moment, and will produce no resultant magnetization in the core of the second transformer and as a result no current will flow in the auxiliary secondary winding.

If, however, as is usually the case in practise, the secondary current produced by the first transformer deviates from the desired ideal value in magnitude or phase angle, or in both, this current and the primary current flowing in opposite directions around the core of the second transformer produce a resultant magnetizing force which acts upon this core. If the auxiliary secondary be now connected to an external circuit, a current will flow which will tend to reduce the flux in the auxiliary core to zero. Under suitable conditions this auxiliary secondary current closely approximates in magnitude and phase to the current which must be vectorially added to the principal secondary current to produce a current such as would be given by an ideal transformer of exact ratio and zero phase angle.

The relations involved may be seen from the vector diagrams of Fig. 2, in which (a) is a simplified diagram of the action of an ordinary current transformer. $O F$ represents the direction of the flux in the core, $O E_1$ that of the induced secondary e. m. f.; their magnitudes are immaterial for the present discussion. With the usual case of a secondary circuit having resistance and inductance, the secondary current $O I_1$ will lag behind $O E_1$, and if the secondary coil has one turn, $O I_1$ may also represent the secondary ampere-turns. $O A$ is drawn of length equal to $O I_1$ and 180 deg. away from it, and represents the component of the primary ampere-turns which balances the secondary ampere-turns. To produce the flux and supply the core losses a magnetizing current I_m must flow through the one-turn primary, and I_m shows the magnitude and direction of this current and its magnetizing ampere-turns. Combining $O A$ and $O I_m$, we get the vector $O I_0$, which represents the primary current and its ampere-turns. It may be seen that since $O A$ is shorter than $O I_0$, I_1 is smaller than the desired value. (In practise, this is usually corrected, for any given set of conditions, by

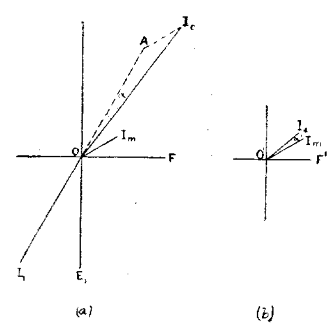

Fig. 2—Vector Relations of the Two-Stage Current Transformer

"dropping secondary turns"; that is, by making their number slightly less than the number required by an ideal transformer. However, for any other set of conditions, the current I_m will in general not change in such proportion to the other currents as to keep the ratio at the desired value.) Also, since $O A$ leads $O I_0$ by the angle α, the secondary current has a phase error ("phase angle") of this amount.

If we pass $O I_0$ and $O I_1$ through one-turn windings surrounding another core in such a way that their magnetizing effects are substantially in opposition, their resultant magnetizing force will be equal to $O I_m$. These two opposing windings may thus be regarded as equivalent to a one-turn primary winding traversed by the current I_m. Then in a third one-turn closed-circuit winding around this core (see Fig. 2 (b)) the current I_4 will be induced. It is evident that if this

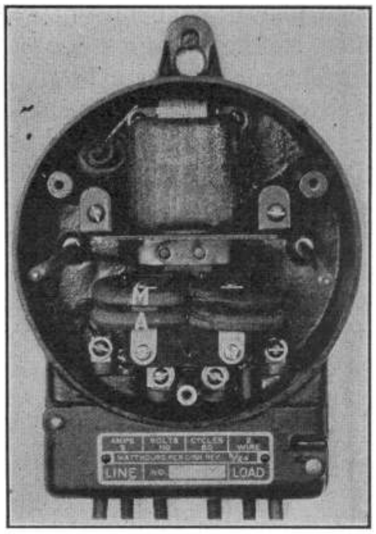

FIG. 3—MOTOR ELEMENT OF WATT-HOUR METER, SHOWING THE TWO CURRENT WINDINGS REQUIRED FOR USE WITH THE TWO-STAGE CURRENT TRANSFORMER

current be vectorially added to $O A$ of Fig. 2 (a) the resulting current will be very much closer to $O I_0$ in magnitude and phase than is $O A$.

The current from the auxiliary secondary may be readily utilized by providing the meter (or other de-

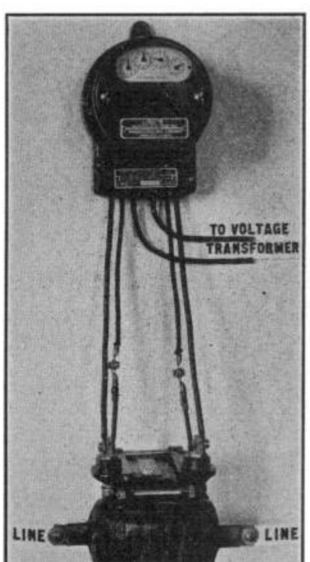

FIG. 4—TWO-STAGE CURRENT TRANSFORMER CONNECTED TO WATT-HOUR METER

vices) with two identical current windings connected respectively to the main and auxiliary secondary circuits, as shown in Figs. 1 and 3. Under such conditions the total ampere-turns in the windings of each instrument so connected to the transformer system will be for all practical purposes exactly equal to the ampere-turns derived from an ideal transformer. The mathematical treatment of the electric and magnetic network involved is given in the appendix to this paper.

Instead of the two physically distinct transformers shown diagrammatically in Fig. 1, it is more convenient to use a single primary winding and a single secondary winding encircling both cores, with the auxiliary secondary winding and a few turns of the main secondary winding surrounding the auxiliary core only.[1] This method of construction produces a two-stage transformer which is physically a single compact unit (see Fig. 4), which shows such a transformer connected to a watt-hour meter. The method of linking the electric and magnetic circuits is shown diagrammatically in Fig. 5, in which the numbers 1, 2, 3 represent the primary winding, the main secondary winding, and the auxiliary secondary winding respectively.

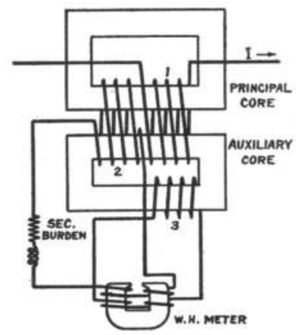

FIG. 5—ELECTRIC AND MAGNETIC CIRCUITS OF TWO-STAGE CURRENT TRANSFORMER MADE AS A STRUCTURAL UNIT

COMPARATIVE PERFORMANCE

Without going into details concerning the causes limiting the accuracy of current transformers having simple secondary windings, it is sufficient to recognize that a higher degree of accuracy in current transformers is desirable in order to bring up the accuracy of the readings of the meters and indication of the instruments which they operate. Let us consider the effect of such errors when the secondary is connected to a watt-hour meter which, for the purpose of this discussion, is assumed to be correct for all loads and power factors within the limits considered. If the meter were connected directly to the line the speed would be proportional to

$$S = E\, I \cos \theta$$

and if we assume next that a current transformer of nominal 1:1 ratio is interposed we would have the speed proportional to

$$S' = \frac{E\, I}{R} \cos (\theta - \alpha)$$

where R is the value, as taken from the calibration curve of the transformer, of the quotient, true ratio divided

1. This construction was suggested by Dr. F. B. Silsbee.

by marked ratio, and α is the small angle (the "phase angle") by which the reversed secondary current leads the primary current.

When operating at unity power factor the term cos $(\theta - \alpha)$ is almost exactly equal to unity, so that the per cent registration of the meter will be almost inversely proportional to R.

As the power factor decreases the effect of α is felt more and more. Since for inductive loads the value of θ is positive there will be a tendency for the meter to run faster as the power factor is lowered. As the load is lowered the values of both R and α increase and in

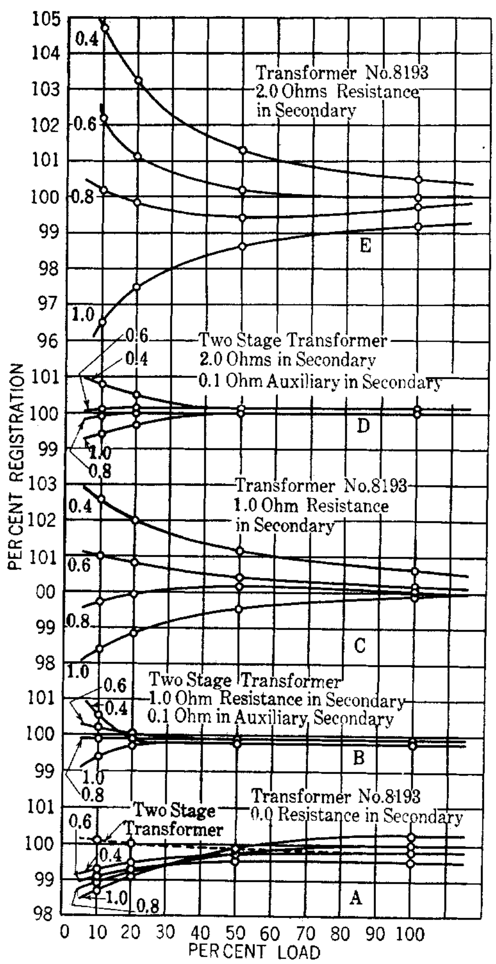

FIG. 6—COMPARATIVE PERFORMANCE OF SIMPLE CURRENT TRANSFORMER AND TWO-STAGE CURRENT TRANSFORMER

general there is a slight tendency for one to compensate the other, yet in most cases the meter will actually exhibit an increased per cent registration on inductive loads. For leading power factors the opposite is true and as the power factor is lowered the meter becomes slower and slower.

Fig. 6C shows this characteristic on inductive loads very clearly. The data as plotted show the degree to which the accuracy of a meter is affected when connected to a line through a modern simple current transformer. This transformer exhibits a good average performance. Transformers of considerably lower accuracy are in service and some of heavier modern designs show a higher accuracy. The curves show the per cent registration of the meter for various loads and at power factors as indicated. Without the transformer the per cent registration would in each case have been 100. The data were taken by direct measurement rather than by computation from ratio and phase-angle curves. In the case cited the secondary burden was 1 ohm resistance plus the resistance of the meter. This particular transformer had about 1200 ampere-turns at full load in the primary. A two-stage transformer was built having about the same amount of iron in its structure but using only one-half the number of ampere-turns. Data were taken on this transformer when connected to a meter and with various values of secondary burden. For zero secondary burden there was practically no deviation from 100 per cent registra-

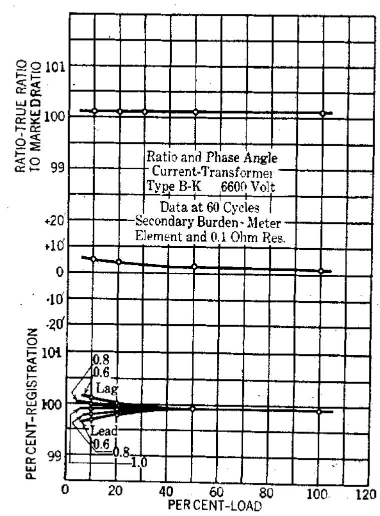

FIG. 7—RATIO, PHASE-ANGLE AND PERFORMANCE CURVES OF TWO-STAGE CURRENT TRANSFORMER

tion for all loads and power factors. Figs. 6B and 6D show the results using secondary burdens of one ohm and two ohms respectively and in both cases there is practically no deviation from 100 per cent registration for all loads above 20 per cent. Figs. 6C and 6E show results which do not even compare favorably with A, B and D, although the secondary burden in case C was very favorable to high accuracy. Fig. 6A shows in full lines the performance of the ordinary current transformer when the secondary burden is only a meter element and 0.1 ohm lead resistance. The dotted line shows the performance of the two-stage transformer under the same conditions. For the latter the curves for the various power factors were so nearly coincident that they are shown as one line.

Fig. 7 shows the conventional ratio and phase-angle curves for the two-stage current transformer with a secondary burden consisting of a watt-hour meter and 0.1 ohm resistance. The lower set of curves, like those of Fig. 6, shows the performance as a function of both ratio and phase-angle.

The above data show the great utility of the two-stage transformer in obtaining the highest degree of accuracy in electric metering when the use of a current transformer becomes necessary. When used on switchboards the auxiliary secondary need be connected only to the wattmeters and watt-hour meters, since in general an error of 1 to 2 per cent in the indications of ammeters is of no serious consequence.

Watt-hour meters operated from two-stage current transformers need not be calibrated to compensate for inaccuracies in the transformers themselves, since the main secondary and the auxiliary secondary together provide an effective current which is at all times in the proper phase with and ratio to the primary current. This condition is practically independent of any change of secondary burden, frequency, or aging effects in the main core, and should the main secondary become open-circuited the auxiliary secondary will still provide current in approximately the proper ratio.

Effect of External Mutual Inductance

In general the introduction of the auxiliary corrective current into any device means that the main secondary and auxiliary secondary circuits are magnetically coupled outside the transformer. This condition results in the introduction of e. m. fs. into the auxiliary secondary circuit in addition to those generated within the auxiliary secondary coil itself. Such e. m. fs. may become harmful to the successful operation of the transformer if they become sufficiently large. In order to show this effect an experiment was made on a two-stage transformer, as follows: The burden in both main secondary and auxiliary secondary circuits was approximately 0.25 ohm, and 0.79 mh. inductance. The constants of the transformer were first determined with the above secondary burdens and the test repeated using a mutual inductance of 0.21 mh. to couple magnetically the secondary and auxiliary secondary outside the transformer. The tests were made at 60 cycles.

The following table shows the change in constants for the condition with and without mutual inductance:

Without Mutual Inductance

Per Cent Load	10	20	40	60	100
Ratio	1.0017	1.0012	1.0010	1.0010	1.0010
Phase Angle	5.5′	3.5′	1.5′	0.7′	−0.7′

With Mutual Inductance

Ratio	1.016	1.014	1.011	1.010	1.007
Phase Angle	5.2′	2.0′	0.0′	−3.5′	−5.6′

The above figures show that the introduction of mutual inductance between the main secondary and auxiliary secondary circuits outside the transformer is at least harmful to the ratio of the transformer. It should be noted, however, that the mutual inductance used in the above experiment was about four times as great as that between the two current windings of a watt-hour meter. Furthermore, it is a simple matter to provide an external corrective mutual inductance of equal numerical value but of opposite sign, thus canceling the mutual induction taking place in the meter. This device would be a small laminated core with two windings.

The usual practise of keeping the secondary burden as low as possible should be adhered to in the case of the auxiliary secondary, and the corrective current should be applied only to apparatus where it is required from the standpoint of accuracy.

Some Features of Design

From a practical standpoint it is desirable to make the mutual reactances between primary and auxiliary secondary and between main secondary and auxiliary secondary the same and to arrange the coils so that both of these reactances will vary in the same ratio. This will not always be an easy matter when designing current transformers of high-voltage type, but in a laboratory standard transformer where the insulation between primary and secondary can be reduced to a minimum the problem is less difficult. By breaking up the primary and secondary into a number of sections and interleaving the sections on the core some very remarkable characteristics can be obtained. For example, a two-stage transformer of this type was built which had the primary and secondary built in two sections each and placed on the core in the following order: *P- S- P- S*; ampere-turns at full load, about 900.

The following table shows the characteristics of this transformer for 0.1 ohm resistance in the main secondary and auxiliary secondary circuits.

Per Cent Load	10	20	30	60	100
Ratio	1.0005	1.0005	1.0002	1.0002	1.0001
Phase Angle	2.0′	1.0′	0.5′	0.0′	0.0′

For the commercial testing of instrument transformers such a transformer could be considered as having a fixed ratio and negligible phase angle.

Testing Two-Stage Current Transformers

Almost any of the methods now in use for determining the constants of current transformers can, with slight modification, be applied to the two-stage current transformer.

The Agnew watt-hour meter method[2] will be of particular interest to the laboratory of limited facilities, since it gives results which are sufficiently accurate for all commercial purposes and requires no instruments or apparatus of precision except a current transformer whose constants are known. When testing a transformer of one-to-one ratio even this special transformer is not necessary. The ratio and phase angle as determined by any one of these methods will be termed the "effective" ratio and phase angle since they are determined from the vector sum of two currents.

Fig. 8 shows the arrangement for testing a one-to-one ratio two-stage current transformer. Two watt-hour

2. Agnew, Watt-hour Meter Method of Testing Instrument Transformers, Scientific Paper of the Bureau of Standards No 233, 1914; Craighead and Weller, *General Electric Review* Vol. 24, p. 642, 1921.

meters a and b are each equipped with two sets of series coils having the same number of turns as a standard five-ampere coil. Each disk is marked in hundredths of a revolution. The potential coils are connected in parallel to a source of e. m. f. of the same frequency as that which supplies current to the primary of the current transformer and arrangements should be made whereby the phase relations between the main

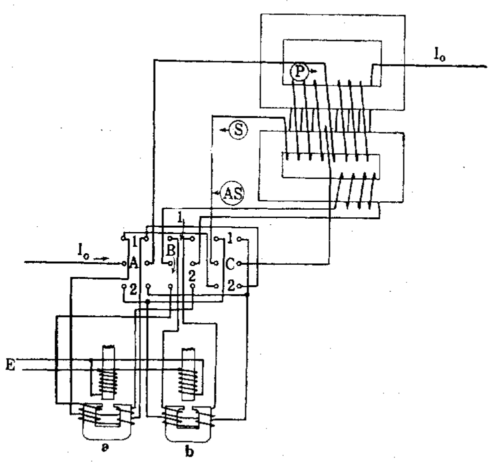

FIG. 8—DIAGRAM OF CONNECTIONS FOR AGNEW TWO-WATT-HOUR-METER METHOD OF TESTING A 1:1 RATIO TWO-STAGE CURRENT TRANSFORMER

current I_0 and applied e. m. f. E can be altered. It is not necessary that either of the watt-hour meters be in correct adjustment on unity power factor since it is shown that the constants of the two watt-hour meters do not enter into the computations. In many cases, however, the computations are somewhat lessened if the two meters are kept in rather close agreement as regards their constants.

When taking readings at low power factors in order to determine the phase angle it is desirable that the meters be in agreement as regards the angle by which the shunt-field flux lags behind the voltage. It is not necessary that the flux from each potential pole be exactly 90 deg. behind the voltage, but it is desirable that the angle be the same in both. For this reason it has been found desirable to adjust the meters to agreement at unity and at some low power factor; say 20 per cent.

Three double-pole double-throw switches are provided as shown. By throwing the switches first in the position $A_1 B_1 C_1$ meter a is in the primary of the transformer with one of the series coils disconnected while meter b has its windings connected to the secondary and auxiliary secondary respectively of the two-stage transformer. By throwing the switches into the position $A_2 B_2 C_2$ the relative position of the meters is interchanged.

If when the switches are in the position $A_1 B_1 C_1$ we designate by a_1 and b_2 the revolutions recorded on the meters a and b in a given time, and by a_2 and b_1 the revolutions recorded for the two meters a and b when the switches are in the position $A_2 B_2 C_2$, we have the ratio of the transformer

$$R = \sqrt{\frac{a_1 b_1}{a_2 b_2}} \quad (1)$$

if the applied e. m. f. E is in phase with the current I_0.

If the applied e. m. f. is not in phase with the current the quantity under the radical may be termed R', the *apparent ratio* of the transformer, since it is dependent not only upon the effective ratio of the currents but upon the phase angle as well.

From the standpoint of computations it is desirable to make b_1 and b_2 the same, in which case, neglecting terms of second and higher orders, we have

$$R = \sqrt{a_1/a_2} = 1 + \frac{a_1 - a_2}{2\, a_2} \quad (2)$$

When taken at power factors other than unity $1/R' \times 100$ would be the per cent registration which a meter would exhibit when placed in the secondary circuit of the current transformer if it were correct when placed in the primary circuit. Such data would be of particular interest to the practical meterman who desires to know the effect of both ratio and phase angle upon the accuracy of the meter. The curves shown in Fig. 6 are plotted on this basis.

Phase angles are determined by taking readings at both unity and low power factor for the same volt-

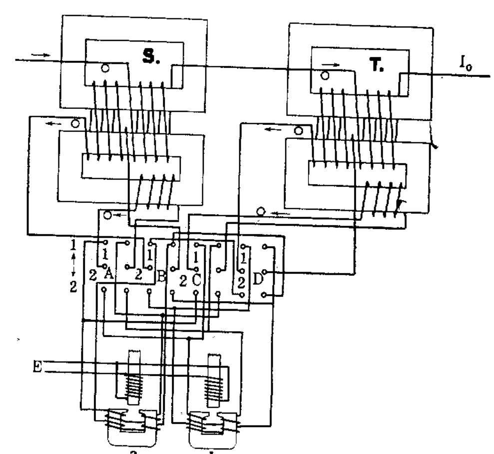

FIG. 9—DIAGRAM OF CONNECTIONS FOR AGNEW TWO-WATT-HOUR-METER METHOD OF TESTING TWO-STAGE CURRENT TRANSFORMERS OF RATIO OTHER THAN UNITY

ampere load, using a wattmeter, voltmeter and ammeter to determine the power factor as regards the primary current and applied voltage E. For this case we denote by a_1', a_2', b_1', b_2', the readings corresponding to a_1, a_2, b_1, b_2 in the test at unity power factor. Upon making $b_2' = b_1'$ we have the phase angle

$$\alpha = \tan^{-1}\left[\frac{1}{\tan\theta}\,(1 - R/R')\right] \quad (3)$$

where $R' = \sqrt{a_1'/a_2'}$ is the apparent ratio at the low power factor and R is the ratio at unity power factor while θ is the angle between E and I_0.

This formula gives the phase angle in radians. For practical purposes, since the angle is small, we may take the angle as equal to the tangent. Doing this and multiplying by 3438 to reduce to minutes, we have

$$\alpha = \frac{3438}{\tan\theta}\,(1 - R/R')\ \text{(minutes)} \qquad (4)$$

Fig. 9 shows the arrangement for testing a two-stage current transformer whose nominal ratio is different from unity. The transformer S is used as the standard and may be of either the two-stage or simple type. If of the simple type there will be no auxiliary secondary connections from this transformer to the switch as shown. If we let

R_1 = ratio of the standard transformer S
α_1 = phase angle of the standard transformer S
R_2 = ratio of the two-stage transformer B
α_2 = phase angle of the two-stage transformer B

then with the same meaning attached to the a and b symbols as before

$$R_2 = R_1\sqrt{a_1/a_2} = R_1\left(1 + \frac{a_1 - a_2}{2\,a_2}\right) \qquad (5)$$

$$\alpha_2 = \tan^{-1}\left[\frac{1}{\tan\theta}\left(1 - \frac{R_2}{R_1 R_2'}\right)\right] + \alpha_1 \qquad (6)$$

where $R_1 R_2' = R_1\sqrt{a_1'/a_2'}$ is the apparent ratio at the low power factor and $b_1' = b_2'$ as before.

From this we may derive the practical formula

$$\alpha_2 = \frac{3438}{\tan\theta}\left(1 - \frac{R_2}{R_1 R_2'}\right) + \alpha_1\ \text{(minutes)} \qquad (7)$$

Advantages of the Two-Stage Transformer

In conclusion it may be well to point out some additional advantages of the two-stage transformer.

From an engineering standpoint it is possible greatly to reduce the amount of copper and iron required to give results which are at least as good or better than now attained in the highest grade transformers produced. It is not necessary to have an accurate knowledge of the magnetic properties of the iron used. With reasonable amounts of materials used the inaccuracy of metering due to the presence of current transformers can be reduced to a negligible quantity. This is in contrast to the average simple current transformer whose accuracy under service conditions often leaves much to be desired. Even with a given secondary burden it cannot be compensated for all loads and power factors, as the curves clearly show.

The results of a questionnaire sent out recently to the large electric power companies by the Meter Committee of the National Electric Light Association[3] showed that, "The installation of separate current transformers was reported in practically all cases for watt-hour meters, while indicating instruments and other devices, including relays, are combined on the same transformer." The use of one two-stage transformer would give as good or better metering accuracy, and would save the cost of the extra transformer and the space occupied by it.

Current transformers of very large range, beyond the maker's facilities for precision testing, can be made with the assurance that by having approximately the correct number of turns in primary and principal secondary and the exact ratio of turns in primary and auxiliary secondary the ratio will be accurately correct and the phase angle negligible.

Transformers of ordinary construction of small number of secondary turns cannot be brought to exact ratio because it is not feasible to drop a fraction of a turn. This difficulty is absent in the two-stage transformer.

Since watt-hour meters operated from two-stage transformers do not need to be adjusted specially to offset transformer ratio and phase-angle errors, the work of testing such meters will be simplified and the cost of testing reduced.

An important possible application of the two-stage transformer is in the measurement of the output of a-c. generators on acceptance tests. In the case of large machines it is customary to stipulate a large penalty for each per cent by which the efficiency falls below the contract figure, and conversely a large bonus for an efficiency superior to that specified. It is therefore highly desirable to use current transformers which have a constant ratio and do not require troublesome corrections for phase-angle errors. The auxiliary winding can be applied to indicating wattmeters as well as to watt-hour meters.

Particularly on high-voltage circuits where accurate metering is so desirable the simple current transformers of best design show very poor characteristics. This is due to the separation of primary and secondary windings necessitated by the insulation requirements. This is particularly true of the Nicholson air-insulated transformer and the bushing type having low ampere-turns and long mean path of flux in the iron. In these cases the two-stage transformer will make possible a step well in advance of present-day methods of metering high-voltage systems.

Appendix

In the following discussion the mathematical relations underlying the action of the two-stage current transformer are established, and a comparison is made with the corresponding relations for the simple current transformer. In this connection the authors wish to acknowledge valuable assistance rendered by Dr. F. B. Silsbee.

The following symbols will be used:

I_0, I_1 and I_4 = primary, main secondary, and auxiliary secondary currents respectively

3. Report of N. E. L. A. Meter Committee, June 1921, p. 9, 44th Convention.

$X_0, X_1 \ldots X_4$ reactances of the several coils as indicated by the subscript and on Fig. 1

Z_3 = impedance of main burden plus resistance of coils 1 and 3

Z_4 = impedance of auxiliary burden plus resistance of coil 4

X_{01}, X_{23}, etc. = mutual reactances between coils indicated by the subscripts

$N_0, N_1 \ldots N_4$ = number of turns in coils 0, 1 4

μ_1, μ_4 = permeability of iron in main and auxiliary cores respectively

a_1, a_4 = cross-section of iron in respective cores

l_1, l_4 = length of magnetic circuit in respective cores

ω = 2π times frequency

j = $\sqrt{-1}$

D_1, D_4 = leakage factors (approx. equal to 1)

$\delta_{01}, \delta_{23}, \ldots$ etc. = leakage differences

c = $\dfrac{a_4 \mu_4 D_4 l_1}{a_1 \mu_1 D_1 l_4}$

z_3, z_4 = Z_3/X_1 and Z_4/X_1 respectively

$\eta_{34} \ldots$ etc. = $\delta_{34} \mu_4 \ldots$

ζ = Z_4/X_4

Throughout the development, the currents, impedances, and permeabilities will be regarded as plane vectors and the final ratio will be obtained as a complex number whose modulus is the true ratio and whose argument is the phase angle.

Applying Kirchoff's laws to the two secondary circuits shown in Fig. 1 we obtain

$$I_1 [Z_3 + j (X_1 + X_3)] + I_4 j X_{34} = - I_0 j (X_{01} + X_{23}) \tag{8}$$

$$I_1 j X_{34} + I_4 (Z_4 + j X_4) = - I_0 j X_{24} \tag{9}$$

Solving these two simultaneous equations gives

$$I_1 = - I_0 \frac{j (X_{01} + X_{23}) (Z_4 + j X_4) + X_{34} X_{24}}{(Z_4 + j X_4) [Z_3 + j (X_1 + X_3)] + X_{34}^2} \tag{10}$$

$$I_4 = - I_0 \frac{j X_{24} [Z_3 + j (X_1 + X_3)] + X_{34} (X_{01} + X_{23})}{(Z_4 + j X_4) [Z_3 + j (X_1 + X_3)] + X_{34}^2} \tag{11}$$

Now each of the reactances (X_{mn}) may be split up into a number of factors so that

$$\left.\begin{aligned}
X_1 &= \omega \frac{4\pi}{10} \frac{a_1}{l_1} \mu_1 N_1^2 D_1 \\
X_3 &= \omega \frac{4\pi}{10} \frac{a_4}{l_4} \mu_4 N_3^2 D_4 (1 - \delta_3) \\
X_4 &= \omega \frac{4\pi}{10} \frac{a_4}{l_4} \mu_4 N_4^2 D_4 \\
X_{01} &= \omega \frac{4\pi}{10} \frac{a_1}{l_1} \mu_1 N_0 N_1 D_1 (1 - \delta_{01}) \\
X_{23} &= \omega \frac{4\pi}{10} \frac{a_4}{l_4} \mu_4 N_2 N_3 D_4 (1 - \delta_{23}) \\
X_{24} &= \omega \frac{4\pi}{10} \frac{a_4}{l_4} \mu_4 N_2 N_4 D_4 (1 - \delta_{24}) \\
X_{34} &= \omega \frac{4\pi}{10} \frac{a_4}{l_4} \mu_4 N_3 N_4 D_4 (1 - \delta_{31})
\end{aligned}\right\} \tag{12}$$

In these expressions the leakage factors D_1 and D_4 are those by which the inductances of coils 1 and 4 respectively differ from those computed from the well-known simple formula for uniformly and closely wound ring coils. In the case of the other reactances the corresponding differences are allowed for by introducing the quantities δ which are in practical cases small compared with 1.

If we now limit ourselves to the practical case where $N_3 = N_4$ and $N_0 = N_2$, and for abbreviation let

$$c = \frac{a_4 \mu_4 D_4 l_1}{a_1 \mu_1 D_1 l_4} \tag{13}$$

and also let $Z_3/X_1 = z_3, \quad Z_4/X_1 = z_4$ (14)

we get, on making the various substitutions, the two equations:

$$I_1 = - I_0 \times$$

$$\frac{\dfrac{N_2}{N_1} \left\{ 1 - \delta_{01} + \dfrac{N_3}{N_1} c (- \delta_{23} + \delta_{31} + \delta_{24} - \delta_{34} \delta_{24}) - j \left[z_4 \left(\dfrac{N_1^2}{N_3^2 c} + \dfrac{N_1}{N_3} \right) - z_4 \left(\dfrac{\delta_{01} N_1^2}{N_3^2 c} - \dfrac{\delta_{23} N_1}{N_3} \right) \right] \right\}}{1 - \dfrac{N_3^2}{N_1^2} c (- \delta_3 + 2 \delta_{34} - \delta_{34}^2) - j \left[z_3 + z_4 \left(1 - \delta_3 + \dfrac{N_1^2}{N_3^2 c} \right) \right]} \tag{15}$$

and

$$I_4 = - I_0 \times$$

$$\frac{\dfrac{N_2}{N_1} \left[\dfrac{N_1 - N_3}{N_3} - \dfrac{N_1}{N_3} \delta_{24} + \delta_{01} + \delta_{34} + \dfrac{N_3}{N_1} c (-\delta_3 - \delta_{24} + \delta_{23} + \delta_{34} - \delta_{23}\delta_{24} + \delta_3\delta_{24}) - j z_3 \dfrac{N_1}{N_3} (1 - \delta_{24}) \right]}{1 - \dfrac{N_3^2}{N_1^2} c (- \delta_3 + 2\delta_{34} - \delta_{34}^2) - j [z_3 + z_4 (1 - \delta_3 + \dfrac{N_1^2}{N_3^2 c})]} \tag{16}$$

While these separately are very complicated functions of the leakage factors and burdens it will be noted that the δ's and z's are in practise small compared to 1 and hence we may neglect the higher powers and products of these quantities entirely. If this is done the division of numerator by denominator may be carried out explicitly and we obtain

$$I_1 = -I_0 \frac{N_2}{N_1}\Big[1 - \delta_{01} + \frac{N_3}{N_1} c\,(-\delta_{23} + \delta_{34} + \delta_{24}) - \frac{N_3^2}{N_1^2} c\,(2\,\delta_{34} - \delta_3) + \ldots + j\left(z_3 + z_4 \frac{N_3 - N_1}{N_3} + \ldots\right)\Big] \quad (17)$$

$$I_4 = -I_0 \frac{N_2}{N_1}\left\{ \frac{N_1 - N_3}{N_3} + \delta_{01} - \frac{N_1}{N_3}\delta_{24} + \delta_{34} + \frac{N_3}{N_1} c\,(\delta_{23} - \delta_{34} - \delta_{24}) + \frac{N_3^2}{N_1^2} c\,(2\,\delta_{34} - \delta_3) + \ldots - j\left[z_3 - z_4\left(1 + \frac{N_1^2}{N_3^2\, c}\right)\right]\right\} \quad (18)$$

It will be seen that most of the terms (e. g., δ_{01}) of (**18**) are equal in magnitude and opposite in sign to the corresponding terms in (**17**), which is of course the mathematical expression for the physical fact that I_4 has very nearly the correct value to compensate for the departure of I_1 from its ideal value of $-I_0 N_2/N_1$.

If therefore we compute the effective ratio in the usual form we get, after a rearrangement of terms

$$\text{Ratio} = \frac{I_0}{I_1 + I_4} = -\frac{N_3}{N_2}\left\{1 - \frac{N_3}{N_1}\,\delta_{34} + \delta_{24} + \ldots + j\left[z_4 \frac{(N_1 - N_3)\,N_1}{c\,N_3^2} + \ldots\right]\right\} \quad (19)$$

The corresponding expression for a single-stage transformer of the usual type is

$$\text{Ratio} = \frac{I_0}{I_1} = -\frac{N_1}{N_0}\,[1 + \delta_{01} + \ldots - j\,(z_3 + \ldots)] \quad (20)$$

A comparison of equations (**19**) and (**20**) shows at once the advantages of two-stage transformation. If N_1 is made equal to N_3 then the ratio in (**19**) becomes independent of the secondary burdens, and if in addition $\delta_{34} = \delta_{24}$ (*i. e.* if coils 3 and 2 have equal mutual inductances on coil 4) then the ratio becomes constant and equal to N_3/N_2.

To see in more detail the effect of various conditions on the operation of the apparatus we may make some further algebraic transformations and must introduce some physical assumptions. It is evident from (**20**) that if δ_{01} and z_3 were constants, ordinary current transformers would have a constant ratio and phase angle at any given frequency and burden. Consequently the entire variation of transformer ratio with current and the main part of its variation with frequency and burden are due to the fact that the permeability is not constant and the core loss does not vary in proportion to the square of the flux density. In the group of equations (**12**) no explicit mention was made of core loss but this may readily be taken care of by considering μ to be a complex quantity having a real component proportional to that component of the induced voltage in quadrature with the magnetizing current and an imaginary component proportional to the core-loss component of the induced voltage. The leakage differences δ are roughly a measure of the ratio of the air leakage flux peculiar to one coil to the total flux which is mainly in the iron. Consequently these quantities, roughly at least, will be inversely proportional to the permeability of the iron. Also the quantities z_3 and z_4 involving, as they do, X_1 or X_4 in the denominator will be inversely proportional to μ. We may therefore at least approximately set

$$\left.\begin{aligned} \delta_{34} &= \eta_{34}/\mu_4 \\ \delta_{24} &= \eta_{24}/\mu_4 \\ z_3 &= \zeta_3/\mu_4 \\ \frac{N_1^2\, z_4}{N_3^2\, c} &= \frac{\zeta_4}{\mu_4}, \quad \text{or} \quad \zeta_4 = \frac{Z_4}{X_4}\,\mu_4 \end{aligned}\right\} \quad (21)$$

Inserting these relations in (**13**) and (**14**) gives Ratio (two-stage) =

$$-\frac{N_3}{N_2}\left[1 + \left(\eta_{24} - \frac{N_3}{N_1}\,\eta_{34} + j\,\zeta_4 \frac{N_1 - N_3}{N_1}\right)\frac{1}{\mu_4}\right] \quad (22)$$

and

$$\text{Ratio (single-stage)} = -\frac{N_1}{N_0}\,[1 + (\eta_{01} - j\,\zeta_3)\frac{1}{\mu_1}] \quad (23)$$

The equations cannot be profitably pushed farther than this unless the permeability can be expressed as a definite function of the flux density. It may be noted, however, that the flux density at which a core works is proportional to the net induced voltage per turn and varies inversely as the frequency. Consequently for high frequencies or small currents and burdens μ_1 will be small but fairly constant while with higher flux densities corresponding to lower frequency or larger current and burden μ will be larger but will vary more rapidly with current. Since the auxiliary core has to circulate only the very small auxiliary current through the very moderate auxiliary burden, the flux density in it is low and μ_4 in equation (**22**) is fairly constant, though small. In the single-stage transformer, however, the flux must be sufficient to circulate the entire secondary current and μ_1 in equation (**23**) will vary rather rapidly with current.

The principal gain from two-stage transformation, however, is seen by the coefficient of $1/\mu_4$ in equation

(**22**), which involves only the *difference* of two nearly equal leakage factors instead of a single factor. Moreover, the main secondary burden does not enter at all into the first order terms in equation (**22**) for the two-stage transformer, and the auxiliary burden is multiplied by the factor

$$\frac{N_1 - N_3}{N_1}$$

which is always small and may be made zero if desired. To make $N_1 = N_3$ however in general will make I_4 larger than if N_1 is slightly less than N_3. The consequent increase in auxiliary flux density and in the variability of μ_4 and in second order terms may more than neutralize the improvement in the ζ_4 term.

Equation (**22**) shows very clearly the effect of external mutual inductance between the main and auxiliary secondary circuits, since such an effect is equivalent to increasing η_{34} and will directly destroy the balance between δ_{34} and δ_{24}. This effect is referred to at some length in the present paper.

On the whole it is seen that the errors of the single-stage transformer have been reduced to one order of magnitude smaller by the use of the two-stage transformation. This is confirmed by the experimental results which show that a transformer which operating single-stage has errors of several per cent will on two-stage operation show errors of only a tenth of this amount, or less.

Discussion

James R. Craighead: There are in addition to those outlined in this paper, several other methods of making phase angle correction in current transformers. The simplest is the use of a non-inductive shunt placed across the primary winding or across the secondary winding, which subtracts a certain amount of current either from the primary or the secondary side of the transformer. This current is in a phase position which is such that it tends to restore the remainder of the secondary current to the phase position of the primary current, and by so doing can diminish to some extent the usual leading (or positive) phase angle of the current transformer. Inductive reactance may be substituted for the non-inductive shunt where the phase angle is negative.

An extension of this method is the use of a separate winding on the core of the current transformer, which gives an opportunity of using other voltages than the voltage generated at the secondary terminals. This allows the use of condensance, reactance and resistance; and by using these three the sub-division of the current can be made so that the net current going through the meter for any given point can be brought more exactly into phase apposition. Both these methods produce a correct phase angle under only one condition, and change in current, frequency or secondary burden is usually accompanied by change in phase angle.

Consequently, this form also does not give a continuous or complete correction.

A third method requires the use of two current transformers. A main current transformer has the standard connection, with its primary in the primary line, and its secondary connected to the meter or other burden. An auxiliary current transformer is placed with its primary either in the primary line or the secondary line, and it has a ratio very different from the ratio of the first transformer.

This second transformer produces a very small current, roughly sufficient to produce a corrective result if applied either across the primary terminals of the main current transformer, or across the terminals of the burden. A combination can be made

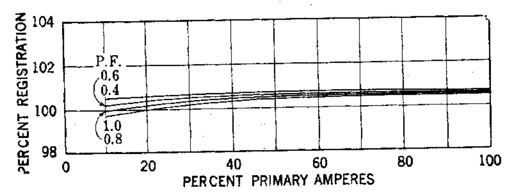

Fig. 1—Current Transformer for Metering Service—Accuracy at 60 Cycles

Burden of 1.8 ohms resistance (45 voltamperes)

to produce a result which is somewhat better than a simple point correction, by selecting main and auxiliary transformer so that their characteristics tend to offset errors through same range. As the auxiliary transformer has a comparatively large number of turns, more accurate correction of ratio may be made than with a transformer of standard type. Full correction can be obtained under only one condition, as in the preceding method.

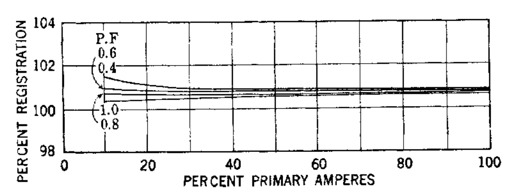

Fig. 2—Current Transformer for Metering Service—Accuracy at 60 Cycles

Burden of one watthour meter and leads (0.15 ohm resistance, 3.75 voltamperes)

The method proposed by Mr. Brooks, while resembling the previous method in the use of two transformers, is distinctly different in principle. If the auxiliary secondary, which I prefer to call the tertiary is disconnected, the transformer consists of a core subdivided into two parts, with a common primary and secondary on both parts, the secondary being connected to an external burden. If then we connect the tertiary to a burden of

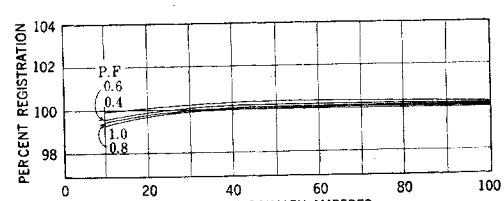

Fig. 3—Current Transformer for Metering Service—Accuracy at 60 Cycles

Burden of 0.97 ohm resistance, 4.03 milli henries inductance (45 voltamperes at 0.54 power factor at 5 amperes 60 cycles)

low impedance, the corrective current drawn reduces the flux in the auxiliary core. This increases the impedance of the secondary circuit and increases the voltage developed in the secondary by the main core, and consequently the flux in the main core and its exciting current. The error in the total secondary circuit is therefore increased, and we get a subdivision into two circuits, one whose accuracy has been largely increased by the redistribution of the flux, and the other whose accuracy has been decreased by the same cause. The auxiliary core is really excited by the difference between the primary and secondary currents as a true primary current, and the tertiary winding tends to deliver a proportionate current. Since this difference is the error in transformation of the main transformer, the corrective current changes in proportion to the need for correction.

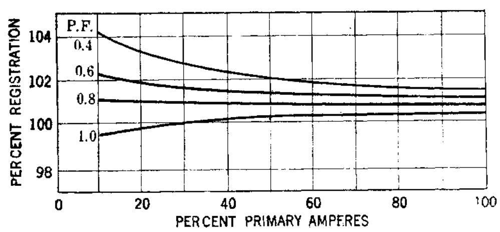

FIG. 4—CURRENT TRANSFORMER FOR METERING SERVICE—ACCURACY AT 25 CYCLES

Burden of 1.8 ohms resistance (45 voltamperes)

Consequently moderate changes in frequency, secondary burden or primary current make only very small changes in accuracy. From this the Brooks plan is evidently applicable where it s desirable to obtain high accuracy over a very small portion ofi the secondary burden, as a watthour meter. Where the purpose is to obtain better general accuracy on a large burden, such as curve drawing instruments, balanced relays, etc., the method is not applicable.

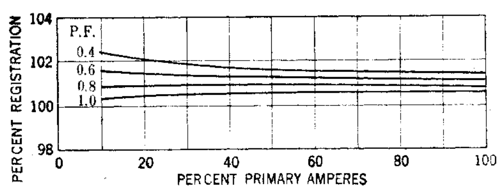

FIG. 5—CURRENT TRANSFORMER FOR METERING SERVICE—ACCURACY AT 25 CYCLES

Burden of one watthour meter and leads (0.15 ohm resistance 3.75 voltamperes)

Mr. Brooks has furnished curves showing the performance of an ordinary commercial type of transformer for comparison with his device. These results are not as good as may be obtained with the better grade of commercial transformers.

Figs. 1 to 6 show the accuracy of a meter (neglecting its internal error) operated from a current transformer of good standard type at power factors from unity to 0.4 lagging, with various secondary burdens and frequency.

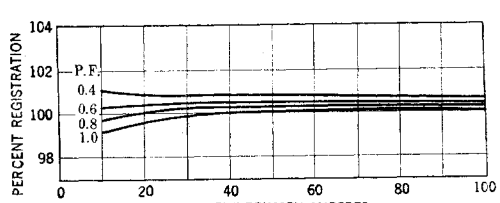

FIG. 6—CURRENT TRANSFORMER FOR METERING SERVICE—ACCURACY AT 25 CYCLES

Burden of 0.97 ohms resistance, 4.03 milli henries inductance, (45 voltamperes at 0.54 power factor at 5 amperes 60 cycles)

The first three show operation at 60 cycles, with burdens of (Fig. 1) 1.8 ohms non-inductive resistance, or 45 volt-amperes, at 5 amperes and 60 cycles; (Fig. 2) 0.15 ohms resistance or 3.75 volt-amperes (practically a watthour meter with its leads); and (Fig. 3) 0.97 ohms resistance with 4.03 milli-henries inductance or about 45 volt-amperes at 0.45 power factor. This last is the highest burden recommended for use with the transformer when a watthour-meter is included. The results show errors due to the transformer of less than 1 per cent, as compared with much larger errors shown by the transformer used as a comparison by the authors of the paper.

The next three figures show similar data for the same transformer and burdens at 25 cycles. In Fig. 4 is shown the effect of a non-inductive burden of 1.8 ohms—which is rare in metering practise—on the accuracy for comparison with Fig. 1. Fig. 5 shows the accuracy with burden of watthour meter and leads corresponding to Fig. 2, and Fig. 6 the accuracy with the full-rated burden for this service, corresponding to Fig. 3.

With the last condition, the maximum error is again reduced to close to one per cent, while the more severe condition shown in Fig. 4 causes a maximum error of over 4 per cent, approaching the amount shown in Mr. Brooks' example at 60 cycles. This, however, is a burden greater than is recommended for watthour meters to work with the transformer.

A study of the 25-cycle results shows that the average error through the ranges selected is in each case in the same direction. It may therefore be partly offset by adjustment of the potential transformer burden or of the watthour meter itself.

Perry A. Borden: In developing the two-stage current transformer, Mr. Brooks has eliminated what has been probably the worst feature in a-c. metering. Heretofore, about the only comfort we had lay in the possibility of the current transformer errors being to some degree compensated for by those of the voltage transformers. But this, even if true for one condition of the load could not be universally so.

While the two-stage principle, as applied to transformers of an inherently compact design and of high ampere-turns means a close approach to perfection, it would seem that its greatest application would lie in the field of air-insulated transformers for high-voltage work, and for bushing type transformers where the primary is, of necessity, limited to a single turn.

A limiting factor in the use of the two-stage transformer would appear to be introduced by the necessity of duplicating the secondary wiring both internally and externally to the meter. On large systems, where each installation is subject to the supervision of a trained meter-man, this would mean little difficulty; but on small utilities where the metering installations are made by a wireman with no court of appeal but the manufacturer's blueprint, the probability of error in the meter wiring would be double what it is now; and I think those who have had experience with meters installed by non-technical help, will agree that this is no small factor.

I should like to ask Mr. Brooks if he has made any studies or carried on experiments in the use of this transformer without the refinement of the extra winding in the meter. It would seem that the commendable features of the principle would be sacrificed but little by paralleling the secondary and the auxiliary windings in the meter, or even at the terminals of the transformer. If this could be done, the only objection to the device,—that of duplicate current circuits—would be at once removed.

F. C. Holtz: In reply to Mr. Craighead's remarks we might add that it was not our intention to show in this paper the characteristics of the best transformers to be had. We did, however, choose a transformer of good average characteristics—a type which represents a fair average of those in service today.

I should also like to inquire of Mr. Craighead the weight of the transformer whose characteristics were shown on the screen.

J. R. Craighead: It varies from 20 pounds up to about 1500. It is an average curve of the total amount of transformer.

F. C. Holtz: The two-stage current transformer can be made extremely light in weight. For example, we have made transformers of 200 amperes capacity with one turn, weighing approximately 13 pounds. This being the total weight of a transformer for 13,200-volt circuits.

The two-stage principle becomes particularly applicable for high voltages where it is necessary to obtain great separation between primary and secondary, and where the simple transformer cannot maintain the high standards of metering accuracy.

In reply to Mr. Borden's discussion, I should like to bring in an endorsement of his suggestion that an important application is that of the one-turn-primary current transformer. Where metering is done close to the generator bus, it is often found that the multiple turn transformer cannot withstand the effect of short circuits and a compromise is reached between the operating and meter departments through the installation of single-turn-primary current transformers. While such transformers do readily withstand the effects of short circuits, they very often fail to give satisfactory results in metering. This is particularly true of transformers of 200 amperes capacity and lower. Such transformers often show over 5 per cent variation in ratio from 10 per cent load to 100 per cent and have phase angles as high as 6. degrees on light loads. It is possible through the application of the two stage principle to build current transformers of the one turn primary type, which in addition to possessing the feature of indestructibility, have good electrical characteristics at low secondary burdens, such as for example, a meter element and .1 ohm resistance. Transformers of this type have been constructed which maintain correct ratio to within .3 per cent from 10 per cent to 100 per cent load, and whose phase angle is less than 10 minutes over the same range of load.

H. B. Brooks: Referring to Mr. Craighead's statement that the two-stage transformer is applicable only where it is desirable to obtain high accuracy over a very small portion of the secondary burden, we consider that it is only a question of design to obtain high accuracy over a large burden. In general, it is not necessary to do this, except in the case of watthour meters, for errors of even several per cent are not serious where the object is to obtain data as a guide in operating the plant. Of course, if the large burden introduces a relatively large mutual inductance between the main and auxiliary secondary circuits, corrective measures may have to be employed. The simplest is the use of an external mutual inductance of equal magnitude and opposite sign, by which the error in question may be reduced to zero.

Tests by Mr. Holtz show that in two-stage transformers having at least 800 ampere-turns the effective ratio and phase angle are not impaired by the introduction into the auxiliary secondary of burdens approximately equal to that of a graphic wattmeter and only to a slight extent by introducing a graphic ammeter or relay. He has made tests of an 800-ampere-turn two-stage current transformer and has found that a considerable burden can be introduced into the auxiliary secondary circuit without greatly affecting either ratio or phase angle, and that even at 25 cycles the results are quite satisfactory and superior to those obtainable in ordinary current transformers of the highest performance commercially obtainable.

While the use of duplicate current circuits is a drawback as Mr. Borden suggests, it is not nearly so serious as might at first appear. If the principal secondary circuit of a two-stage current transformer be opened under load, the auxiliary secondary winding will supply a current nearly equal to the desired total secondary current. This fact gives a means by which even an unskilled man may check the correctness of the connections of the auxiliary secondary coil, as follows. Assume that a three-phase watthour meter is to be operated by two two-stage current transformers. First the auxiliary secondary coils are disregarded, and it is immaterial whether their terminals are open or short-circuited. The principal secondary terminals are then connected to the current coils of the meter, and the correctness of the connections checked by diagram and polarity marks, or any other suitable method, just as if the current transformers were of the ordinary type. This done, the voltage is removed from one element, and the direction and approximate speed of rotation of the disk are noted. The auxiliary secondary terminals of the current transformer which is driving the meter are then connected to the meter terminals, and as a check, the principal secondary circuit is opened. If the auxiliary coil has been properly connected, the meter disk will continue to rotate in the same direction and at practically the same speed as before. If the auxiliary coil was connected backwards, the disk will run about the same speed as before, but in the opposite direction.

The fact above cited simply means that the auxiliary winding is a corrective device capable of functioning not merely over a limited range of error in the principal secondary current, but even in the extreme case of the absolute failure of the principal secondary current the meter will be kept running at nearly the correct speed.

It is obvious that other means may be used for facilitating correct connections, such as a cable of secondary wires of different colors, with corresponding markings on the meter terminals. In any case, the method of checking just outlined is simple and easily applied.

Answering Mr. Borden's question about the elimination of the extra winding in the meter, we would say that a considerable amount of work was spent on this point. By proper precautions, very accurate results may be obtained, but it is necessary for the best results to use a rather large burden in the principal (non-precision) secondary circuit, and furthermore, to have means for checking the accuracy. With the use of duplicate windings on the meter, low burdens may be used in the principal secondary circuit if desired, and if the transformer has been properly made and checked at the factory, no means of checking the accuracy are required, save only to see that the connections are properly made as above outlined.