Microchim. Acta 148, 133–141 (2004)
DOI 10.1007/s00604-004-0278-3

Microchimica Acta
Printed in Austria

Original Paper

The Uncertainty of Weighing Data Obtained with Electronic Analytical Balances

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Received May 8, 2004; accepted July 10, 2004; published online November 26, 2004

Abstract. Weighing is a common task in any chemical laboratory and weighing data are associated with some uncertainty, as this is common with all other working procedures and their data. This paper presents the influence factors which are part of the combined measurement uncertainty of a mass determination and their interplay, namely the technical specifications of the balance (repeatability, nonlinearity, sensitivity tolerance, and temperature coefficient of the sensitivity) and the effect of air buoyancy. Depending on the net and gross weight, the densities involved (density of air and of the weighing object in relation to the density of the reference weights) and the uncertainties of these densities the relative uncertainty of weighing data is often in the 10^{-5} to 10^{-4} range (10 to 100 ppm). It must be kept in mind that such low values can only be obtained with simple weighing goods, i.e. in the absence of disturbances such as electrostatic charges, air drafts, evaporation, or water adsorption phenomena.

Key words: Mass determination; weighing; uncertainty; balance; air buoyancy.

Weighing, i.e. the determination of mass, is a common operation in the analytical laboratory and perhaps the one which can be performed with the lowest uncertainty. As it is the case with all measurement tasks, a weighing result has a certain uncertainty even though it is small. The uncertainty comes from a number of physical and technical features of the balance and from the air buoyancy which is the main bias effect. So far they have never been presented thoroughly in a journal devoted to analytical chemistry. A discussion can be found in the proceedings of a conference held in Broadbeach in 2001 [1]. The EURACHEM/CITAC Guide "Quantifying Uncertainty in Analytical Measurement" [2] presents the uncertainty of weighing in few words only and air buoyancy is not discussed at all. The book by Jones and Schoonover includes a short chapter on measurement uncertainty without discussing the details [3].

We present the various influence parameters and their interplay to the combined standard uncertainty as well as some practical examples.

Weighing Value and Mass

A contemporary electronic analytical balance does not directly measure the mass m_s of a sample s; instead, it measures its weight force. Therefore the display shows the so-called weighing value w_s. In most cases these data are not identical if the sample is weighed in air because the resulting air buoyancy (in the following termed buoyancy) gives rise to a deviation, i.e. a

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systematic influence. The effect can simply be modeled as follows:

$$m_s = Bu \cdot w_s \tag{1}$$

where Bu is the buoyancy correction factor for the sample. In practice, things are more complicated. Electronic precision balances are calibrated with reference weights whose mass is known with low uncertainty; however, they are also subject to buoyancy. They are made from a steel alloy with a density of 8000 kg m^{-3}. This results in the fact that the mass of a sample that has the same density can be determined without the necessity of a correction because the influence of buoyancy onto the sample and the calibration weight cancel each other. The weighing values of objects with a density other than 8000 kg m^{-3} deviate from their mass. In analytical laboratories (as well as in everyday life) the majority of the weighed objects have a lower density than steel and their weighing value is lower than their mass, i.e. Bu is >1. The opposite is true for the alloys and pure metals with densities higher than 8000 kg m^{-3}. Figure 1 shows the deviation of the weighing value from the mass as a function of weighing sample density.

The buoyancy correction factor Bu is defined by the following equation [4] using the densities of sample ρ_s, reference weight ρ_r and air ρ_a:

$$Bu = \frac{1-(\rho_a/\rho_r)}{1-(\rho_a/\rho_s)} = \frac{\rho_s(\rho_r-\rho_a)}{\rho_r(\rho_s-\rho_a)} \tag{2}$$

Equation (1) shows the multiplicative relationship between mass and weighing value. Therefore, the combined relative standard uncertainty of the mass $u_c(m_s)/m_s$ can be calculated in accordance to the laws of uncertainty propagation:

$$\frac{u_c(m_s)}{m_s} = \sqrt{\left(\frac{u(Bu)}{Bu}\right)^2 + \left(\frac{u(w_s)}{w_s}\right)^2} \tag{3}$$

where u(Bu) is the standard uncertainty of the buoyancy factor and $u(w_s)$ is the standard uncertainty of the weighing value. Both uncertainties depend themselves on a number of influence parameters.

For the following presentation we assume that the numerous possible bias effects such as water adsorption, drifts etc. are less prominent than the influence parameters discussed below.

The Influence Parameters on the Uncertainty of the Weighing Value

The weighing operation and the electro-mechanical design of the balance give rise to various effects which influence the data obtained. Among these influences are the repeatability, nonlinearity, sensitivity tolerance and the temperature coefficient of the sensitivity. Eccentric load can be another effect. The technical specifications presented in the following discussion are valid for the semi-micro electronic balance

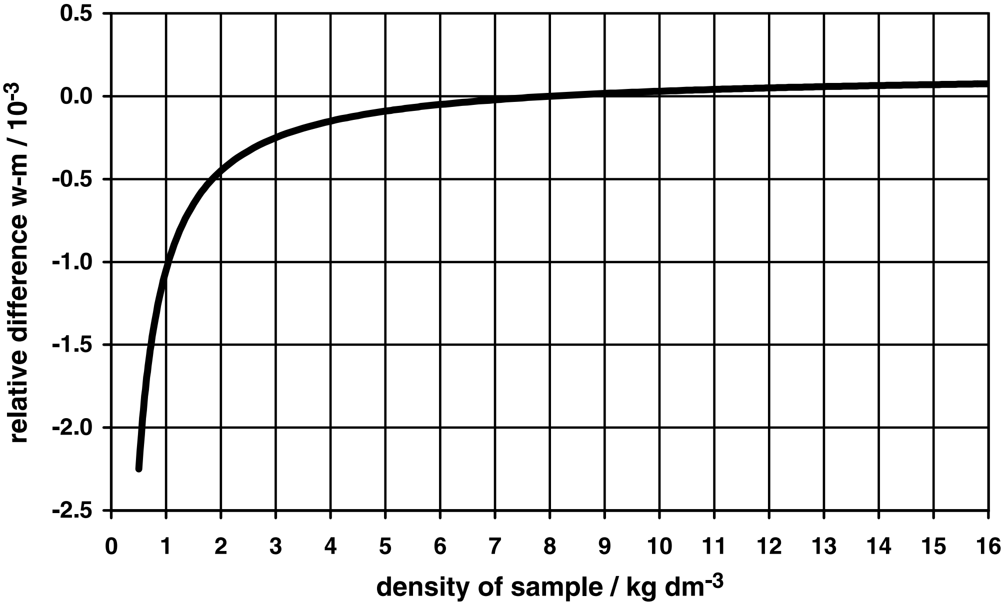

Fig. 1. The relative air buoyancy, i.e. the relative difference between weighing value and mass as a function of the sample density for weighing operations in air, performed on a contemporary electronic laboratory balance (air density = 1.01 kg m^{-3}). The reference weights have a density of 8000 kg m^{-3}, therefore no buoyancy correction is necessary if the sample has the same density. The weighing value of samples with lower density is smaller than their mass and vice versa

Table 1. Typical specifications of electronic analytical balances

Balance type	Micro	Semi-micro	Precision
Weighing capacity	5 g	200 g	1 kg
Repeatability (up to)	0.8 μg (<2 g) 0.9 μg (>2 g)	15 μg (<50 g) 40 μg (>50 g)	1 mg
Nonlinearity (within)	2 μg (<0.5 g) 4 μg (>0.5 g)	30 μg (<10 g) 120 μg (>10 g)	2 mg
Sensitivity tolerance	$10 \cdot 10^{-6}$	$2 \cdot 10^{-6}$	$3 \cdot 10^{-6}$
Temperature coefficient	$1.5 \cdot 10^{-6}\,\mathrm{K}^{-1}$	$1.5 \cdot 10^{-6}\,\mathrm{K}^{-1}$	$2 \cdot 10^{-6}\,\mathrm{K}^{-1}$
Calibration weight density	$8006\,\mathrm{kg\,m}^{-3}$	$8006\,\mathrm{kg\,m}^{-3}$	$7900\,\mathrm{kg\,m}^{-3}$
Standard uncertainty of reference weight density	$10\,\mathrm{kg\,m}^{-3}$	$10\,\mathrm{kg\,m}^{-3}$	$25\,\mathrm{kg\,m}^{-3}$

AT 201 (Mettler Toledo, Greifensee, Switzerland, www.mt.com) with a maximum load of 205 g but apply to similar balances of other manufacturers as well. The influence parameters as discussed below, however, are the same for other models of balances that work with electrodynamic compensation (or electromagnetic force restoration), irrespective of their construction for the milligram or the ton range. Table 1 presents typical data of analytical balances.

It is a matter of course that a balance needs to be calibrated in regular intervals or after a major change of the environmental parameters. Some types of modern balances have a built-in and self-activated calibration routine. The others must be calibrated by the laboratory personnel or by a technician of the manufacturing company.

Repeatability REP

Deviating results are not uncommon if the same object is weighed under repeatability conditions. The reasons for this behaviour are of physical nature: small air drafts present even within the draft shield, temperature non-equilibria, and electronic noise to name but a few. In addition there is the resolution of the digital display but from a practical point of view this effect is included in the experimental repeatability. Expressed as standard deviations s (which are identical with standard uncertainties u in this case), the following data are typical for an AT 201 balance if operated carefully:

- up to 50 g: u(REP) = s(REP) = 0.015 mg
- from 50 to 200 g: u(REP) = s(REP) = 0.04 mg

The load levels refer to the gross weight on the balance platform (tare plus net weight) and are valid for a complete weighing operation including the determination of the tare. (A tare operation is also performed when the empty platform is zeroed.) If the operator works less carefully or if the climatic conditions in the laboratory are unsatisfactory the repeatability can be worse, i.e. the value of s(REP) will be higher and should be determined experimentally. This is also true for critical weighing objects such as volatile or hygroscopic goods.

Nonlinearity NL

An ideal balance exhibits a perfectly linear relationship between displayed value and the load on the platform. In reality this characteristic curve is not straight but curved in a certain shape which is not determined for each individual instrument (although this could be done in principle). Figure 2 illustrates the ideal and real characteristic curves. Instead of individual data

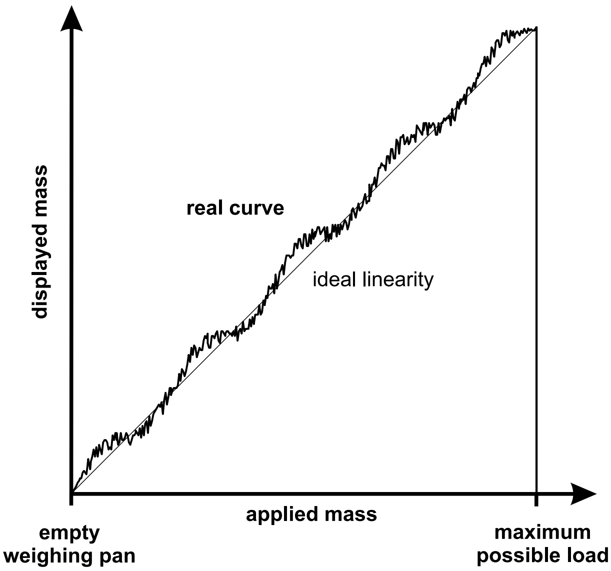

Fig. 2. A possible characteristic curve of a balance. For better clarity the deviations from linearity are disproportionately large

the manufacturers guarantee maximum deviations from linearity which will not be exceeded:

- within 10 g: $NL_{max} = 0.03$ mg (this is also the value to be used if a sample ≤ 10 g is weighed into a tare vessel of e.g. 150 g)
- within 200 g: $NL_{max} = 0.12$ mg

These numbers refer to the net mass and need to be considered twice for each weighing operation because the deviation from ideality may occur with the determination of the tare and of the gross weight as well. The data must be treated as rectangular distributions [5]. In order to convert them into standard uncertainties it is necessary to divide them by $\sqrt{3}$. Therefore the nonlinearity contribution of a single reading is:

$$u(NL) = \frac{NL_{max}}{\sqrt{3}} \tag{4a}$$

For the calculation of a combined measurement uncertainty it is necessary to use the squared value. The nonlinearity contribution of a complete weighing operation adds up to:

$$u^2(NL) = 2\frac{NL^2_{max}}{3} = 0.67 \cdot NL^2_{max} \tag{4b}$$

Note: The technical data given above are worst-case scenarios but they are recommended for everyday use. Non-linearity is a more complex feature of a balance as discussed elsewhere [6].

Sensitivity Tolerance ST

The slope of the characteristic curve, i.e. the sensitivity of the balance, has some tolerance or uncertainty. For an AT 201 the maximum deviation is $2 \cdot 10^{-6}$ of the net mass, e.g. 2 µg if a sample of 1 g is weighed in. The sensitivity tolerance is also treated as a rectangular distribution but it is proportional to the net weighing value:

$$u(ST) = w_{net}\frac{ST_{max}}{\sqrt{3}} = w_{net}\frac{2 \cdot 10^{-6}}{\sqrt{3}} \tag{5a}$$

$$u^2(ST) = 1.3 \cdot 10^{-12} \cdot w^2_{net} \tag{5b}$$

Note: This uncertainty parameter includes the uncertainty of the built-in reference weight as well as the uncertainty of the process by which the balance adjusts its sensitivity (the slope) with the help of this reference weight. This uncertainty term is also known as u(CAL) [7]; the mathematical treatment of u(ST) and u(CAL) is identical but the difference is more than a linguistic one. We tend to use the term "calibration" no longer because first, it does not cover all influences as just explained, and second, it means the comparison with a reference without any setting. However, what is done in the laboratory from time to time is the adjustment of the balance sensitivity, i.e. the determination of the sensitivity, including its proper setting if necessary.

Temperature Coefficient TC

The slope of the characteristic curve is temperature dependent with a maximum static deviation of $1.5 \cdot 10^{-6}\,°C^{-1}$ (static means that the balance is in the temperature equilibrium with its surroundings). What is described by TC is in fact the temperature drift of the sensitivity, and "temperature coefficient" is a somewhat sloppy description of this phenomenon. This value is proportional to the net weighing value and is treated as a rectangular distribution. It is advisable to describe the temperature deviation between calibration and weighing by a rectangular distribution as well:

$$u(TC) = w_{net}\frac{TC_{max}}{\sqrt{3}} \cdot \frac{\Delta T}{\sqrt{3}} = w_{net}\frac{1.5 \cdot 10^{-6}}{3}\Delta T \tag{6a}$$

$$u^2(TC) = 0.25 \cdot 10^{-12} w^2_{net} \cdot \Delta T^2 \tag{6b}$$

For operator-calibrated balances ΔT should be the maximum temperature fluctuation which occurs in the laboratory over the day or year. For self-calibrating balances ΔT will be lower; the operating instruction booklet or the manufacturer will give some information about the temperature drift which triggers a new calibration (this value can be preset with some balance models). ΔT is known in the form $\pm x$ °C (e.g. ± 3 °C) and it is the number x which is used in Eqs. (6).

Eccentric Load

Another effect which can contribute to the uncertainty budget of weighing data is eccentric load, a phenomenon which occurs when the center of gravity of a weighing object is not placed vertically above the center of the weighing pan. The deviation from the weighing value to the one found in the center can be as high as 0.2 mg for the type of balance discussed in this paper. However, it can be easily avoided by the careful placing of the object centrally onto the pan. Therefore eccentric load is not considered in our discussion.

The variances of repeatability, nonlinearity, sensitivity tolerance, and sensitivity temperature coefficient are additive since their causes are independent of each other. They are added up to the combined uncertainty of the weighing value:

$$u_c(w) = \sqrt{u^2(REP) + u^2(NL) + u^2(ST) + u^2(TC)} \tag{7a}$$

With u(REP) = s(REP), as discussed above, and the expressions presented as Eqs. (4b), (5b) and (6b) we obtain:

$$u_c(w) = \sqrt{\begin{aligned} & s^2(REP) + 0.67 \cdot NL_{max}^2 \\ & + 10^{-12} w_{net}^2 (1.3 + 0.25 \cdot \Delta T^2) \end{aligned}} \tag{7b}$$

The Influence Parameters on the Uncertainty of the Buoyancy Factor

The buoyancy correction factor has been introduced with Eq. (1) and defined with Eq. (2). Here follows a simplified derivation of its uncertainty (the accurate equations are presented in the Appendix): We assume that the air density and its uncertainty are identical for the calibration and weighing operations as implied by Eq. (2). For the calculation of the standard uncertainty none of the simple rules of uncertainty propagation can be used because this equation is a combination of additive and multiplicative relationships. It is necessary to determine its three partial derivatives:

$$\frac{\partial Bu}{\partial \rho_s} = \frac{\rho_a(\rho_a - \rho_r)}{\rho_r(\rho_s - \rho_a)^2} \tag{8a}$$

$$\frac{\partial Bu}{\partial \rho_r} = \frac{\rho_s \cdot \rho_a}{\rho_r^2(\rho_s - \rho_a)} \tag{8b}$$

$$\frac{\partial Bu}{\partial \rho_a} = \frac{\rho_s(\rho_r - \rho_s)}{\rho_r(\rho_s - \rho_a)^2} \tag{8c}$$

The standard uncertainty is then calculated as follows:

$$u(Bu) = \sqrt{\begin{aligned} & \left(\frac{\partial Bu}{\partial \rho_s}\right)^2 u^2(\rho_s) + \left(\frac{\partial Bu}{\partial \rho_r}\right)^2 u^2(\rho_r) \\ & + \left(\frac{\partial Bu}{\partial \rho_a}\right)^2 u^2(\rho_a) \end{aligned}} \tag{9}$$

We obtain:

$$u(Bu) = \frac{\rho_s}{\rho_r(\rho_s - \rho_a)} \sqrt{\begin{aligned} & \left(\frac{\rho_a(\rho_a - \rho_r)}{\rho_s(\rho_s - \rho_a)}\right)^2 u^2(\rho_s) + \left(\frac{\rho_a}{\rho_r}\right)^2 u^2(\rho_r) \\ & + \left(\frac{\rho_r - \rho_s}{\rho_s - \rho_a}\right)^2 u^2(\rho_a) \end{aligned}} \tag{10}$$

Now another simplification is possible: We can set $\rho_r - \rho_a = \rho_r$, or vice versa, because $(8000 - 1.2)\ \mathrm{kg\,m^{-3}} \approx 8000\ \mathrm{kg\,m^{-3}}$. This leads to the following representation of Eq. (10):

$$u(Bu) = \frac{\rho_s}{\rho_r(\rho_s - \rho_a)} \sqrt{\begin{aligned} & \left(\frac{-\rho_r \cdot \rho_a}{\rho_s(\rho_s - \rho_a)}\right)^2 u^2(\rho_s) + \left(\frac{\rho_a}{\rho_r}\right)^2 u^2(\rho_r) \\ & + \left(\frac{\rho_r - \rho_s}{\rho_s - \rho_a}\right)^2 u^2(\rho_a) \end{aligned}} \tag{11}$$

The negative sign in the term of $u(\rho_s)$ is irrelevant because it is squared for the uncertainty calculation.

It is obvious that it is necessary to know the densities of air, sample and reference weights as well as their respective uncertainties in order to calculate the uncertainty of the buoyancy factor.

Density of Air

The density of air increases with increasing pressure, decreasing temperature and decreasing relative humidity. The following empirical equation can be used [8]:

$$\rho_a/\mathrm{kg \cdot m^{-3}} = 10^{-3} \frac{A \cdot p - B \cdot h_r \cdot \exp(C \cdot T)}{273.15 + T} \tag{12}$$

p: air pressure in Pa
h_r: relative air humidity in %
T: air temperature in °C
A: 3.4848
B: 9.024
C: 0.0612

The equation is valid for the conditions $90{,}000\ \mathrm{Pa} \leq p \leq 110{,}000\ \mathrm{Pa}$, $10\,^\circ\mathrm{C} \leq T \leq 30\,^\circ\mathrm{C}$, $h_r \leq 80\%$. The possible deviations are less than 10^{-4}. (A finer approximation would also include the influence of the carbon dioxide content.)

For the uncertainty of the air density it is convenient to calculate the extreme values which may occur in the laboratory and to treat these data as the boundaries of a rectangular distribution. The extremes are the combination of high atmospheric pressure, low temperature and low humidity on the one hand, and low pressure, high temperature and high humidity on the other. For the calculation of the density according to Eq. (12) the mean values of pressure, temperature and humidity are used.

Density of the Reference Weights

As already mentioned, most reference weights are made today from a steel alloy with density

$\rho_{\mathrm{cal}} = 8000\,\mathrm{kg\,m^{-3}}$. Based on measurements made in the Mettler Toledo laboratories a reasonable standard uncertainty of this density is $10\,\mathrm{kg\,m^{-3}}$.

Density of the Sample to be Weighed

This topic is less trivial than it may seem. In many cases the density is not well known but nobody has the time and interest to determine it. Good data can be obtained for liquids quite easily but not for solids. The densities of formulated drugs, washing powder or textiles cannot be found in the literature. Even for many pure chemicals the density is unknown or the reliability of the published data is difficult to estimate. We assume that the object density is only rarely known to an uncertainty of ±1% and that ±10% or even worse is what can be expected in reality. It is then this uncertainty which dominates the combined standard uncertainty of a mass value.

The Equation for the Combined Standard Uncertainty of Mass Determination

A simple equation was already presented above:

$$\frac{u_c(m_s)}{m_s} = \sqrt{\left(\frac{u(Bu)}{Bu}\right)^2 + \left(\frac{u(w_s)}{w_s}\right)^2} \qquad (3)$$

The second term can be noted in more detail by using Eq. (7a):

$$\frac{u_c(m_s)}{m_s} = \sqrt{\left(\frac{u(Bu)}{Bu}\right)^2 + \frac{u^2(REP) + u^2(NL) + u^2(ST) + u^2(TC)}{w_s^2}} \qquad (13)$$

For the uncertainty calculation we can postulate the pseudo-identity of mass and weight value: $m \approx w$. This leads to:

$$u_c(m_s) = \sqrt{m_s^2\left(\frac{u(Bu)}{Bu}\right)^2 + u^2(REP) + u^2(NL) + u^2(ST) + u^2(TC)} \qquad (14)$$

or, with Eq. (7b) and considering that buoyancy is only relevant for the net weight:

$$u_c(m_s) = \sqrt{s^2(REP) + 0.67 \cdot NL_{\max}^2 + m_{s,\mathrm{net}}^2 \left[\left(\frac{u(Bu)}{Bu}\right)^2 + 10^{-12}\left(1.3 + 0.25 \cdot \Delta T^2\right)\right]} \qquad (15)$$

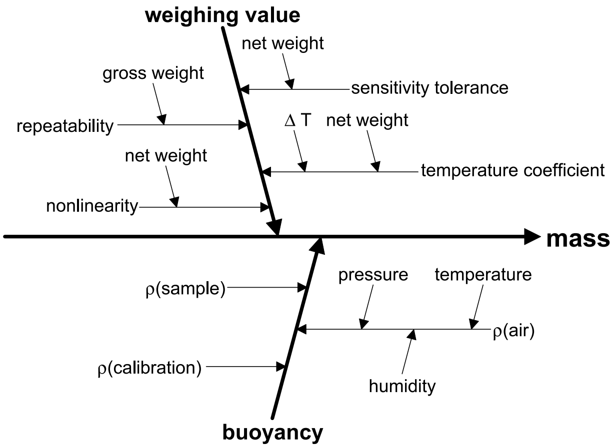

Fig. 3. The cause-and-effect diagram of the main uncertainty parameters of weighing on an electronic analytical balance

We refrain from including the equation for u(Bu), i.e. Eq. (11), into this description of the combined standard uncertainty of a weighing operation because the resulting equation looks rather complicated. To perform the calculation it is advisable to set up a spreadsheet table.

Equation (15) is valid for electronic semi-micro balances with a maximum load of 200 g. As already explained, we used a standard deviation of the repeatability of 0.015 mg or 0.04 mg, depending on the gross weight. The maximum nonlinearity was assumed to be 0.03 mg or 0.12 mg, depending on the net weight. If these data are used as mg values it is also necessary to put in the mg value of the net mass $m_{s,\mathrm{net}}$ in Eq. (15).

Unfortunately, it is impossible to set up simple rules of thumb for the combined standard uncertainty of weighing data. The final number is the result of an interplay of gross and net weight, the uncertainty of the air density (i.e. the possible extreme values of the air density in a certain laboratory), the density of the weighing object and the uncertainty of this density, to name but the most important influence parameters. They are presented as a cause-and-effect diagram in Fig. 3. The interplay of the technical balance parameters and buoyancy is shown in Fig. 4 for objects of different densities and with the technical data given in the legend. In this graph, the combined uncertainty is dominated by the nonlinearity of the balance up to a mass of 1 g and for all densities; for objects of 10 g and more the uncertainty of the buoyancy is pre-

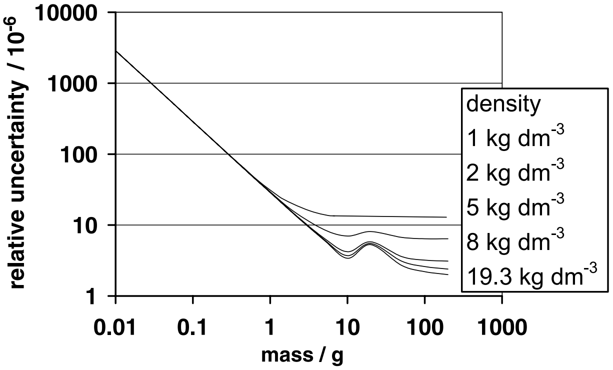

Fig. 4. The relative mass uncertainty of samples between 10 mg and 200 g if weighed on a semi-micro balance. Climatic conditions: Pressure 101300 Pa ± 1000 Pa, relative humidity 60% ± 10%, temperature 25 °C ± 1 °C, resulting in a mean air density of 1.176 kg m^{-3} with a standard uncertainty of 0.010 kg m^{-3}. The tare is 0 g and the weighing sample density has an uncertainty of 1%. The densities of the five curves are noted in the box in identical order (the lowest one is for gold). With other weighing and climate conditions the graph looks different. All curves converge below 1 g because the nonlinearity of the balance is the dominating parameter in this region (NL = 0.03 mg). The hump between 10 g and 100 g comes from the nonlinearity specification which changes at sample mass 10 g (0.03 mg or 0.12 mg, see text)

ponderant, mainly due to the uncertainty of the air density. However, if the relative uncertainty of the object density is higher than 1%, if the climatic properties differ from those of Fig. 4 (higher or lower variability of pressure, humidity and/or temperature), or if the tare mass is not 0 g but, e.g., 50 g, the curves have another position within the diagram.

Examples

Aqueous Solution

3.504 g of an aqueous solution of unknown composition is weighed into a vessel of 110 g. The mean atmospheric pressure at the location of the laboratory is 101,000 Pa (1010 mbar) with extremes over the year of ±1500 Pa. The temperature is 22 ± 3 °C, the relative humidity is 50 ± 25%.

With Eq. (13) the mean air density is calculated to be 1.19 kg m^{-3}. The extremes are 1.22 kg m^{-3} (102,500 Pa, 19 °C, 25%) and 1.15 kg m^{-3} (99,500 Pa, 25 °C, 75%). This gives a standard uncertainty of the air density of $(0.07/2 \cdot \sqrt{3})\,\mathrm{kg\,m^{-3}} = 0.02\,\mathrm{kg\,m^{-3}}$ (note that with a rectangular distribution, half of the span is divided by $\sqrt{3}$). The density of water at 22 °C is 998 kg m^{-3} [9]; we assume that this density is not known better than ±5% because the composition of the solution is unknown (rectangular distribution). This gives a standard uncertainty $u(\rho_s) = 30\,\mathrm{kg\,m^{-3}}$.

Equation (8) gives a buoyancy correction factor Bu = 1.00104. Therefore the mass of the solution is $3.504\,\mathrm{g} \cdot 1.00104 = 3.508\,\mathrm{g}$. The combined standard uncertainty of the buoyancy factor according to Eq. (11) is $u(\mathrm{Bu}) = 4.0 \cdot 10^{-5}$. 25% stem from the uncertainty term of the air density and 75% from the uncertainty term of the sample density whereas the uncertainty contribution of the reference weight density is negligible.

These data are needed for the calculation of the combined standard uncertainty of the mass of aqueous solution using Eq. (15):

$$u_c(m) = \sqrt{\begin{aligned}&0.04^2 + 0.67 \cdot 0.03^2 + 3504^2\big[(4.0 \cdot 10^{-5})^2\\ &+10^{-12}(1.3 + 0.25 \cdot 3^2)\big]\,\mathrm{mg}^2\end{aligned}}$$

This gives a combined standard uncertainty of 0.15 mg (relative uncertainty = $4.2 \cdot 10^{-5}$ or 42 ppm). It results mainly (almost 90%) from the uncertainty of the buoyancy, whereas u(REP) yields only 7.5% and u(NL) a mere 2.8%. The uncertainties of sensitivity tolerance and temperature coefficient are negligible in this case.

Aluminium Profile

848 mg of a profile made from an aluminium alloy are weighed into a vial of 3 g. The mean atmospheric pressure is 95,000 Pa with extremes of ±1200 Pa. The temperature is 20 ± 1 °C, the relative humidity is 60 ± 10%.

The mean air density is 1.12 kg m^{-3} with extremes of 1.14 and 1.10 kg m^{-3} (96,200 Pa, 19 °C, 50% and 93,800 Pa, 21 °C, 70%, respectively). The standard uncertainty of the air density is 0.011 kg m^{-3}. The density of the alloy is 2950 kg m^{-3} and we assume that this value is not more accurate than to ±1%, giving a standard uncertainty of 15.6 kg m^{-3}.

The buoyancy correction factor is 1.000240; therefore the weight value and mass of the aluminium are identical if a resolution of 1 mg is sufficient (the buoyancy has an effect of 0.20 mg). The combined standard uncertainty of the buoyancy factor is $4.6 \cdot 10^{-6}$ stemming to 77% from the uncertainty term of the air density and to 23% from the uncertainty term of the sample density; again, the uncertainty contribution of the reference weights is negligible (0.14%).

The combined standard uncertainty of the mass of the aluminium profile is 0.029 mg (relative uncertainty $= 3.4 \cdot 10^{-5}$ or 34 ppm). In this case, the repeatability and nonlinearity terms yield 27% and 71%, respectively, of the total uncertainty whereas the buoyancy term is small with 1.8%.

Due to the small influence of the uncertainty of buoyancy in this case it is not important if 848 mg of sodium chloride, aluminium, or gold are weighed under the same circumstances as described above; the mass uncertainty is 0.029 mg in all three cases. For gold with a density of 19300 kg m^{-3} the contributions of repeatability and nonlinearity are almost identical with 27% and 72.5% but the buoyancy term is even less, namely 0.2%. For sodium chloride with density 2170 kg m^{-3} the numbers are 26%, 70% and 3.2%, respectively.

The Relevance of Mass Uncertainties

The combined relative standard uncertainty of many analytical procedures is 1–5% if the matrix is simple or even absent (chemicals, drug formulations, alloys, drinking water). It can reach 30% for sophisticated analyses (clinical chemistry, forensic science, trace analysis of environmental samples). In such cases it is not necessary to consider the uncertainty of weighing data with typical relative values in the 10^{-5} to 10^{-4} range (10 to 100 ppm). If there is a doubt about the relevance of this uncertainty we recommend to use a value of 10^{-4} (100 ppm) relative standard uncertainty for each mass determination in the calculation of the combined standard uncertainty of the analytical procedure. If it is then found that the contribution of the weighings is not negligible, their real uncertainties should be calculated with Eqs. (11) and (15). If another type of balance is used the numerical values of Eq. (15) need to be adapted.

In addition it is necessary to perform the buoyancy correction with Eqs. (1) and (2) in all cases where the combined relative standard uncertainty of the analysis is in the 1‰ range. Instead of doing it by calculation it is also possible to use a mass artefact, i.e. a well-defined weighing object of known mass and density which shows preferably a high air buoyancy effect [7].

The uncertainties calculated above are probably the lowest possible values. In reality the uncertainty will often be larger, especially with volatile or hygroscopic objects or in cases of static electric charges on the surface of non-conducting samples. Objects which show such unfavourable behaviour must be handled with special precaution and techniques. Good weighing practice is a prerequisite for good and consistent results [10, 11]. This includes also the proper choice of the balance such as a micro-balance if the sample mass is small [12].

References

[1] Reichmuth A (2001) Weighing accuracy with laboratory balances. Proc 4th Biennial Conf Metrol Soc Australia, Broadbeach (QLD, AU), p 38

[2] EURACHEM/CITAC Guide, Quantifying Uncertainty in Analytical Measurement, 2nd edition (2000) Ellison S L R, Rösslein M, Williams A (eds) ISBN 0-948926-15-5. Free download from: http://www.measurementuncertainty.org/mu/guide

[3] Jones F E, Schoonover R M (2002) Mass measurement. CRC Press, Boca Raton, p 55

[4] Schwartz R (2000) Determination of mass in practice. In: Kochsieck M, Gläser M (eds) Comprehensive mass metrology. Wiley-VCH, Weinheim, p 400

[5] EURACHEM/CITAC Guide, Appendix E 1

[6] Reichmuth A (2000) NCSL Workshop and Conference, Toronto, CAN, July 17–20

[7] Wunderli S, Fortunato G, Reichmuth A, Richard P (2003) Anal Bioanal Chem 376: 384–391

[8] Schwartz R (2000) Mass determination with balances. In: Kochsieck M, Gläser M (eds) Comprehensive mass metrology. Wiley-VCH, Weinheim, p 232

[9] Jones F E, Harris G L (1992) J Res NIST 97: 335–340

[10] Scott M, co-ordinator, Guide to the Measurement of Mass and Weight (1998) Institute of Measurement and Control, ISBN 0-904457-27-3

[11] Weighing the Right Way (2002) Company publication, Mettler Toledo, Greifensee, Switzerland

[12] United States Pharmacopoeia, USP 27 – NF 22 (2004) General chapter ⟨41⟩ Weights and Balances, United States Pharmacopoeial Convention Inc., Rockville, MD, USA, p 2148

Appendix: The Detailed Description of the Uncertainty of the Buoyancy Correction Factor

Strictly speaking, Eq. (2) is not exactly true in many cases because the air density at the moment of calibration, $\rho_{\mathrm{a,cal}}$, is not necessarily identical with the density when the weighing operation is performed $\rho_{\mathrm{a,weigh}}$:

$$\mathrm{Bu} = \frac{\rho_{\mathrm{s}}(\rho_{\mathrm{r}} - \rho_{\mathrm{a,cal}})}{\rho_{\mathrm{r}}(\rho_{\mathrm{s}} - \rho_{\mathrm{a,weigh}})} \tag{16}$$

The four partial derivatives of this equation are:

$$\frac{\partial \mathrm{Bu}}{\partial \rho_{\mathrm{s}}} = \frac{\rho_{\mathrm{s}}(\rho_{\mathrm{a,cal}} - \rho_{\mathrm{r}})}{\rho_{\mathrm{r}}(\rho_{\mathrm{s}} - \rho_{\mathrm{a,cal}})^2} + \frac{\rho_{\mathrm{r}} - \rho_{\mathrm{a,cal}}}{\rho_{\mathrm{r}}(\rho_{\mathrm{s}} - \rho_{\mathrm{a,weigh}})} \tag{17a}$$

$$\frac{\partial \mathrm{Bu}}{\partial \rho_{\mathrm{r}}} = \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{r}}(\rho_{\mathrm{s}} - \rho_{\mathrm{a,weigh}})}\left(1 - \frac{\rho_{\mathrm{r}} - \rho_{\mathrm{a,cal}}}{\rho_{\mathrm{r}}}\right) \qquad (17\mathrm{b})$$

$$\frac{\partial \mathrm{Bu}}{\partial \rho_{\mathrm{a,cal}}} = -\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{r}}(\rho_{\mathrm{s}} - \rho_{\mathrm{a,weigh}})} \qquad (17\mathrm{c})$$

$$\frac{\partial \mathrm{Bu}}{\partial \rho_{\mathrm{a,weigh}}} = \frac{\rho_{\mathrm{s}}(\rho_{\mathrm{r}} - \rho_{\mathrm{a,cal}})}{\rho_{\mathrm{r}}(\rho_{\mathrm{s}} - \rho_{\mathrm{a,weigh}})^2} \qquad (17\mathrm{d})$$

This gives the following standard uncertainty expression:

$$\mathrm{u}(\mathrm{Bu}) = \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{r}}(\rho_{\mathrm{s}} - \rho_{\mathrm{a,weigh}})}\sqrt{\begin{array}{l}\left[(\rho_{\mathrm{r}} - \rho_{\mathrm{a,cal}})\left(\frac{1}{\rho_{\mathrm{s}}} - \frac{\rho_{\mathrm{s}} - \rho_{\mathrm{a,weigh}}}{(\rho_{\mathrm{s}} - \rho_{\mathrm{a,cal}})^2}\right)\right]^2 \mathrm{u}^2(\rho_{\mathrm{s}}) \\ +\left(\frac{\rho_{\mathrm{a,cal}}}{\rho_{\mathrm{r}}}\right)^2 \mathrm{u}^2(\rho_{\mathrm{r}}) + 1 \cdot \mathrm{u}^2(\rho_{\mathrm{a,cal}}) + \left(\frac{\rho_{\mathrm{r}} - \rho_{\mathrm{a,cal}}}{\rho_{\mathrm{s}} - \rho_{\mathrm{a,weigh}}}\right)^2 \mathrm{u}^2(\rho_{\mathrm{a,weigh}})\end{array}} \qquad (18)$$

This equation is identical with Eq. (11) if $\rho_{\mathrm{a,cal}} = \rho_{\mathrm{a,weigh}} = \rho_{\mathrm{a}}$, $\mathrm{u}(\rho_{\mathrm{a,cal}}) = \mathrm{u}(\rho_{\mathrm{a,weigh}}) = \mathrm{u}(\rho_{\mathrm{a}})$ and $\rho_{\mathrm{r}} - \rho_{\mathrm{a}} = \rho_{\mathrm{r}}$.