quency bandwidth of the present antenna, allowing it to radiate closer to the end-fire direction.

The frequency-scanning properties of these antennas may also find application. The scan angle associated with a given frequency change may be increased by operating in a more dispersive region, or by increasing the dispersion with dielectric or periodic loading.

Appendix

Dimensions of Leaky Wave Antenna

Length of aperture (H-plane), $L = 24$ inches
Width of aperture (E-plane), $W = 18$ inches
Wire diameter, $d/2 = 0.005$ inch
Design frequency, $f = 11.42$ kmc.
Design wavelength, $\lambda = 2.624$ cm $= 1.033$ inches
Design guide wavelength, $\lambda_g = 3.205$ cm $= 1.262$ inches
At the design frequency,

$$\sin\theta_0 = \frac{\lambda}{\lambda_g} = 0.81883,$$

$$\theta_0 = 54.97°$$

TABLE I

z/L	$L\alpha(z)$ nepers	$\alpha(z)$ nepers/inch	C/a inches^{-1}	a inches	s inches	z inches
0	0	0	∞	0.900	0.005	0
0.05	0.04163	0.001735	73.8	0.886	0.0617	1.2
0.10	0.16323	0.006801	36.9	0.8732	0.0943	2.4
0.15	0.35682	0.01487	24.8	0.8607	0.122	3.6
0.20	0.61266	0.02553	18.7	0.8486	0.148	4.8
0.25	0.92113	0.03838	15.0	0.837	0.1723	6.0
0.30	1.2735	0.05306	12.55	0.8262	0.195	7.2
0.35	1.6622	0.06926	10.8	0.816	0.216	8.4
0.40	2.0790	0.08663	9.5	0.8062	0.237	9.6
0.45	2.5154	0.10481	8.5	0.7973	0.256	10.8
0.50	2.9565	0.12319	7.70	0.7892	0.274	12.0
0.55	3.3797	0.14081	7.08	0.7817	0.291	13.2
0.60	3.7460	0.15608	6.62	0.7759	0.304	14.4
0.65	3.9923	0.16635	6.36	0.772	0.314	15.6
0.70	4.0265	0.16778	6.30	0.7712	0.316	16.8
0.75	3.7409	0.15587	6.63	0.7762	0.305	18.0
0.80	3.0696	0.12790	7.55	0.7874	0.278	19.2
0.85	2.0894	0.08687	9.51	0.8063	0.237	20.4
0.875	1.5503	0.06460	11.23	0.8185	0.210	21.0
0.90	1.0441	0.04350	14.03	0.8332	0.180	21.6
0.925	0.60815	0.02534	18.8	0.849	0.147	22.2
0.95	0.27605	0.01150	28.15	0.8652	0.112	22.8
0.975	0.06972	0.002905	56.2	0.882	0.073	23.4
1.00	0	0	∞	0.900	0.005	24.0

The Unidirectional Equiangular Spiral Antenna*

JOHN D. DYSON†

***Summary*—Circularly polarized unidirectional radiation, over a bandwidth which is at the discretion of the designer, is obtainable with a single antenna. The antenna is constructed by wrapping balanced equiangular spiral arms on a conical surface. The non-planar structure retains the frequency-independent qualities of the planar models, and, in addition, provides a single lobe radiation pattern off the apex of the cone. Practical antennas have been constructed with radiation patterns and input impedance essentially constant over bandwidths greater than 12 to 1 and there is no reason to assume that these cannot be readily extended to more than 20 or 30 to 1.**

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Introduction

The principle of scaling in electromagnetic theory has been used extensively for many years in the testing of antennas. More recently, it has been applied to the design of antennas that have unique and useful characteristics as the frequency of operation is varied.[1] The characteristics of these antennas are essentially frequency independent over bandwidths which, although they are not unlimited, may be easily extended to 20 to 1, or more. The general approach is to design

[1] V. H. Rumsey, "Frequency Independent Antennas," 1957 IRE National Convention Record, pt. 1, pp. 114–118.

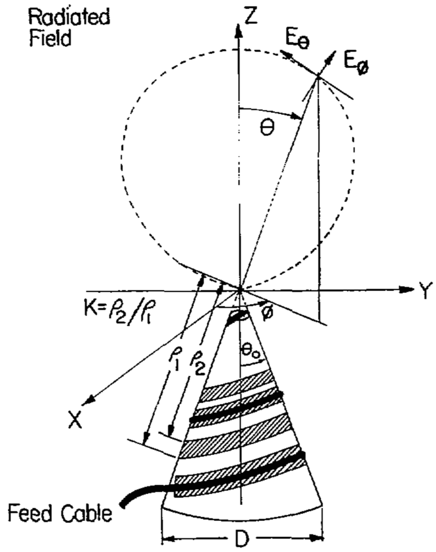


Fig. 1—Balanced conical antenna with radiation pattern coordinate system.

a radiating structure such that successive applications of a scaling factor result in structures which are identical with the original, or which, at most, differ by some rotation about an axis through the origin of the original structure. Although the structure must be infinite in size to fulfill the scaling condition of exact equivalence, it has been found that there is a class of structures that may be truncated and still retain, over an extremely wide range of frequencies, the characteristics of infinite size. This class of antennas is based upon a plane curve, the equiangular or logarithmic spiral, which possesses the property of having a change of scale merely equivalent to a rotation. Several such curves can be used to outline antennas on plane or conical surfaces. The basic characteristics of the planar antenna have been covered elsewhere;[2] the present paper is concerned with the nonplanar antenna.

It is possible to retain the desirable frequency-independent characteristics of the planar antenna and, in addition, confine the radiation to one hemisphere by forming the planar arms down onto a conical surface.[3] The nonplanar antenna so constructed, with a small cone angle, has a unidirectional rotationally-symmetric radiation pattern. The maximum radiation occurs on the antenna axis off the apex of the cone. These balanced conical spirals can be constructed to operate over bandwidths comparable to those of the basic planar structure.

[2] J. D. Dyson, "The Equiangular Spiral Antenna," IRE Trans. on Antennas and Propagation, vol. AP-7, pp. 181–187; April, 1959.

[3] J. D. Dyson, "The Unidirectional Equiangular Spiral Antenna," University of Illinois Antenna Lab. Tech. Rep. No. 33, Contract AF 33(616)3220, Wright Air Dev. Cen., Wright-Patterson Air Force Base, Ohio, July 10, 1958.

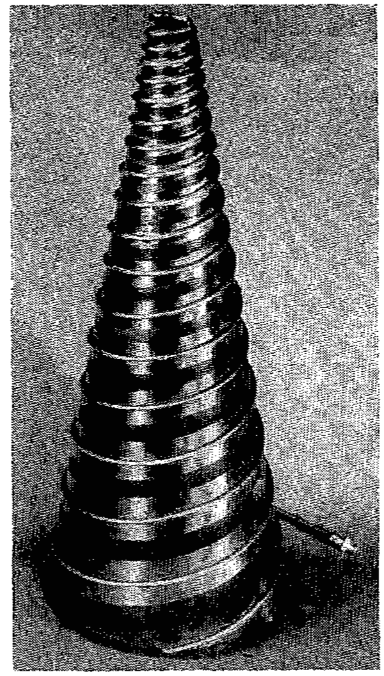


Fig. 2—Conical antenna, $D \simeq 20.7$ cm, $d \simeq 3$ cm, $b = 0.053$, $\alpha = 78°$, $K = 0.925$, $\theta° = 10°$.

The Nonplanar Antenna

The conical spiral antenna is defined as in Fig. 1. On the surface of revolution $\pi - \theta = \theta_0$; the edges of the arms are defined by

$$\rho_1 = e^{(a \sin \theta_0)\phi} = e^{b\phi}$$

and

$$\rho_2 = e^{b(\phi - \delta)} = K\rho_1.$$

A rotation of both curves by 180 degrees will provide the second arm of a balanced structure. The edges of the antenna, so defined, will maintain a constant angle α with the radius vector for any cone angle θ_0; however, the antenna will spiral more tightly; *i.e.*, b will decrease, as θ_0 decreases from $\pi/2$.

The antennas may be constructed by forming metal arms on a suitable conical support or may conveniently be etched on a flexible teflon-impregnated fiberglass material and then formed into a cone. Fig. 2 shows an etched antenna fed with RG 141/U cable bonded to the arms. As previously indicated for the planar antennas, these are balanced structures and a balanced feed is necessary for optimum performance. The feed may be brought in along the axis of the antenna by a balanced feed line or by an unbalanced line and balancing transformer or balun. The bandwidth of this latter method, of course, depends upon the bandwidth of the balun. The rapid decay of the current along the arms, observed in the planar case, is present in a modified form in these nonplanar antennas. This makes possible the same method of feeding the structure; *i.e.*, the cable is bonded to one antenna arm and is carried to the origin

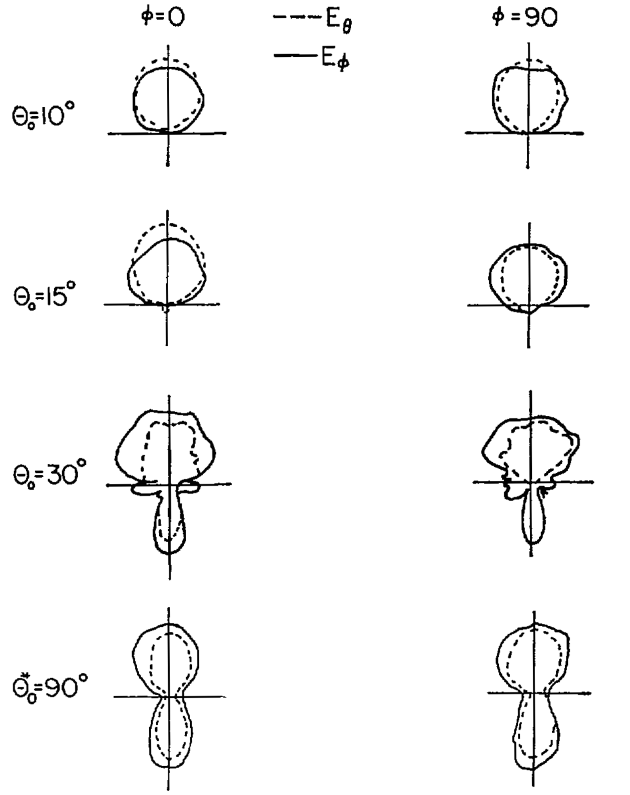


Fig. 3—Electric radiation patterns as a function of θ_0. $K=0.925$, $L=150$ cm, $\alpha=73°$, $b=(0.30 \sin \theta_0)$, $f=2000$ mc. * Dissymmetry between front, back lobes ($\theta_0=90°$) due to mount.

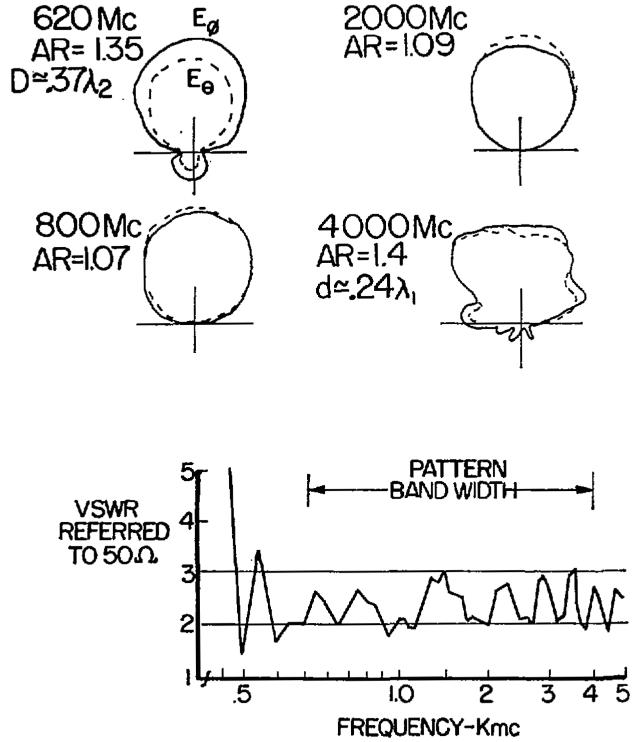


Fig. 4—Electric field patterns and VSWR of conical antenna. $\theta=10°$, $K=0.925$, $D=17.7$ cm, $\alpha=73°$, $b=0.053$, $\phi=0°$, AR=axial ratio on axis.

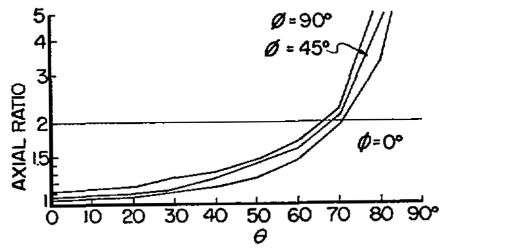


Fig. 5—Polarization of the e field of typical conical antenna as a function of angle off axis.

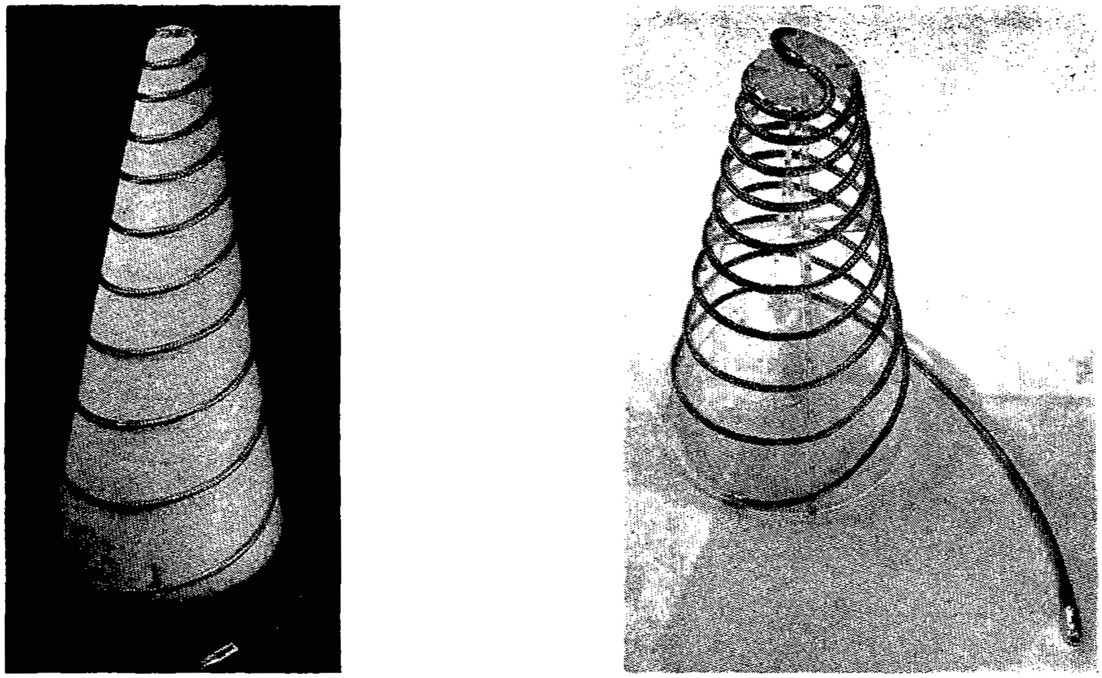
where the center conductor is tied to the opposite arm. A dummy cable on the opposite arm will maintain symmetry. Since the ends of the antenna arms do not carry appreciable antenna currents except at the very lowest frequency of operation, the arms themselves act as an "infinite balun," the feed terminals are isolated from ground in a balanced manner and the outside of the feed cable beyond the antenna arms does not carry a significant amount of antenna current. However, as the frequency of operation is decreased, a point will be reached where these conditions are violated; this should be considered below the range in which the antenna may be expected to operate satisfactorily.

Operation in Free Space

The change in the radiation pattern of a balanced antenna as it is depressed from a plane onto the surface of a cone as a function of the cone angle, θ_0, is indicated in Fig. 3 above. There is a marked increase in the front-to-back ratio for included cone angles of 30° (θ_0 of 15°) or less. The antenna radiates off the apex of the cone. There is no basic tilt to the pattern, and for small b (order of 0.05) the lobe is rotationally symmetric. The pattern rotates with frequency (as previously observed in the planar case), but this rotation is masked by the pattern symmetry. For $\theta_0=10°$ and $\alpha=73°$, the half-power beamwidths are typically 70° for E_θ polarization and 90° for E_ϕ polarization.

Typical radiation patterns are shown in Fig. 4. As is true with the planar antenna, the pattern bandwidth is at the control of the designer. The upper frequency, f_1, depends upon the diameter, d, of the truncated apex and the lowest useable frequncy, f_2, upon the base diameter, D. For the smaller sizes of feed cable (RG 141/U or smaller), $\theta_0=10°$ and parameters of the order of $K=0.85$ to 0.9, and $\alpha=73°$, an apex diameter of $\lambda_1/4$ and a base diameter of $3\lambda_2/8$ should provide a front-to-back ratio of 15 db, or greater, and an axial ratio well below 2 on axis. It is worth noting that the radiated field is circularly polarized well off axis, as indicated in Fig. 5.

The method of feeding the conical antenna, by embedding the cable on the antenna arms, leads to a very interesting, and what may well be the most useful



(a) (b)

Fig. 6—(a) Wire version of conical antenna, $D \simeq 30$ cm, $d \simeq 5.5$ cm, $\alpha = 73°$, $\theta = 10$, RG 8/U cable. (b) Wire version of a conical antenna, supported by polystyrene ribs. $D \simeq 30$ cm, $d \simeq 8.5$ cm, $\alpha = 73°$, $\theta_0 = 10°$, RG 8/U cable.

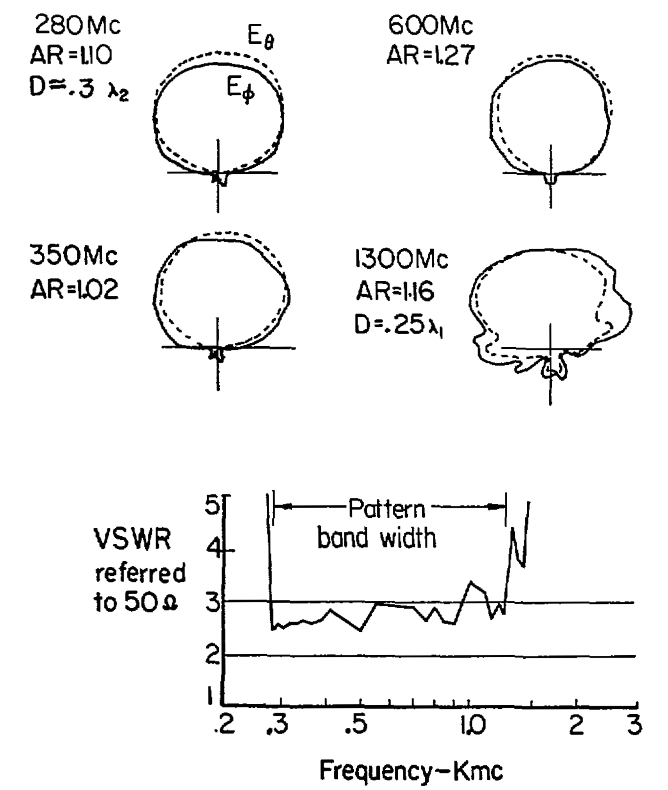


Fig. 7—Electric field radiation patterns and VSWR of wire antenna [Fig. 6 (a)]. $\theta = 10°$, $b = 0.053$, $D = 30$ cm, $\alpha = 73°$, $\phi = 0°$, K = not constant, cable = RG 8/U, AR = axial ratio on axis.

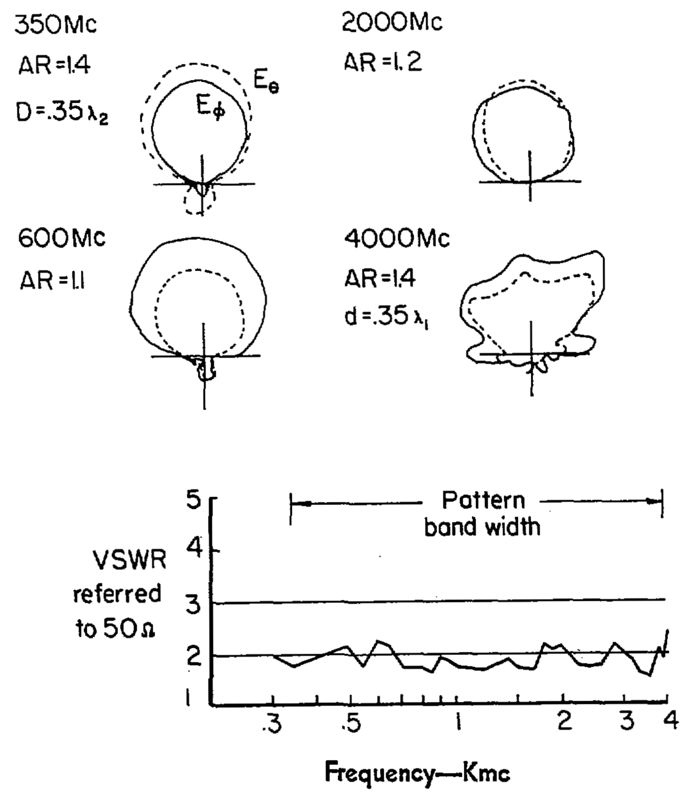


Fig. 8—Electric field radiation patterns and VSWR of conical antenna with RG 141/U arms. $\theta = 10°$, $b = 0.053$, $D = 30$ cm, $\alpha = 73°$, $\phi = 0°$, K = not constant, AR = axial ratio on axis.

version of the antenna. If the expanding arms are decreased to a narrow width and allowed to degenerate into constant width structures, the arms may be formed by the cable alone, as in Figs. 6(a) and 6(b). In this wire version, the feed cable, RG 8/U, becomes one arm. At the apex, the center conductor is carried over and bonded to the outer braid of a dummy cable which forms the opposite arm of a balanced antenna. Radiation pattern characteristics are indicated in Fig. 7. For this particular antenna, patterns were excellent down to a base diameter of approximately $0.3\lambda_2$, and, as for the true equiangular antennas referred to previously, up to an apex diameter of approximately $\lambda_1/4$. An apex diameter of approximately four centimeters appears to be a practical limit for cable of this size, which indicates an upper useful frequency of around 1600 or 1700 mc. However, the bandwidth can, as before, be extended downward to any desired frequency by a suitable extension of the cone.

Since the pattern range at this laboratory has a practical limit from 0.3 to 12 kmc, it is convenient to show the bandwidth potential by going up in frequency. Fig. 8 shows the patterns over an 11 to 1 range of frequencies for an antenna identical except for the fact that it was constructed of RG 141/U cable. The apex could then be carried to a diameter of approximately two centimeters. As indicated in Figs. 7 and 8, for a given antenna size, those structures with wider arms (large cable in the wire version, or smaller K in the true equiangular version) may be operated down to a slightly longer wavelength.

Antennas such as the one in Fig. 6 could most conveniently be constructed of solid wall $\frac{3}{8}$ inch coaxial cable and should be most useful structures in the VHF and UHF regions for telemetering and similar purposes.

The input impedance of these antennas appears to be well behaved over at least as wide a bandwidth as the radiation pattern. The mean impedance level appears to slowly decrease with decreasing cone angle. The variation in the measured impedance of the antennas referred to in Fig. 3, over the frequency range from 0.5 to 2.5 kmc, is indicated in Table I.

TABLE I

MEASURED INPUT IMPEDANCE OF BALANCED CONICAL ANTENNAS AS A FUNCTION OF θ_0($K=0.925$, $L=150$ cm, $b=(0.303 \sin \theta_0)$, $\alpha=73°$)

θ_0	Approximate Mean Impedance	Maximum SWR Referred to Mean
10°	129	1.9 :1
15°	147	1.9 :1
30°	153	1.95:1
90°	164	2.1 :1

The impedance is influenced by the construction at the terminal region. Since these antennas were fed with a coaxial cable of 0.140-inch diameter, this cable dominates the terminal region and undoubtedly contributes to the mean impedance level. However, this appears to be the only practical method of feeding these structures if the full bandwidth potentiality is to be realized. Hence, the input impedance with a practical size feed cable bonded to the arms is of interest.[4]

As indicated in Figs. 4, 7 and 8, these antennas would be a better match to a 100-ohm cable than to the 50-ohm cable used. However, the stability of the VSWR with frequency, over the pattern bandwidth, is apparent.

OPERATION OVER A GROUND PLANE

A previous investigation of the unbalanced conical antenna fed against a ground plane with the apex of the cone on the ground plane had demonstrated the pattern rotation and some of the characteristics of this version.[5] However, the radiation pattern of the unbalanced antenna was tilted off axis, and the pattern rotation caused the antenna to be of limited usefulness.

The balanced antenna referred to in Fig. 4 was placed with its base on a 12-foot square ground plane and the arms bonded to the ground plane. The presence of the ground plane narrows the beamwidth somewhat at the lower frequencies but does not deteriorate the radiation pattern, and, above 600 mc, the VSWR of the antenna was essentially unaffected by the ground plane.

OPERATION IN A FLUSH-MOUNTED CAVITY

A very limited investigation indicates that it is possible to place this antenna in a conical cavity and obtain essentially unaffected operation over the lowest portion of the bandwidth of the antenna. However, the cavities investigated have imposed on over-all frequency bandwidth of about 4 to 1.[6]

INCREASING THE BEAMWIDTH

The pattern beamwidth appears to be related to the rate of spiral and varies from around 70° for an angle α of 74°, to approximately 180 or 190° for an α of 45°. Preliminary patterns for an antenna with this latter angle are shown in Fig. 9. These are of some interest because they are within approximately 6 db of being those of a circularly polarized isotropic source in one hemisphere.

[4] Since preparation of this paper a new balun has been proposed that should find wide use with these structures. See R. H. DuHamel and F. R. Ore, "Log periodic feeds for lens and reflectors," 1959 IRE NATIONAL CONVENTION RECORD, vol. 7, pt. 1, pp. 128–138.

[5] R. L. Carrel, "Experimental Investigation of the Conical Spiral Antenna," University of Illinois Antenna Lab. Tech. Rep. No. 22, Contract AF 33(616)-3220, University of Illinois, Urbana, May, 1957.

[6] J. D. Dyson, "The Non-Planar Equiangular Spiral Antenna," *Proc. Eighth Annual Symp. on the USAF Antenna Res. and Dev. Program*, Robert Allerton Park, Monticello, Ill., October, 1958.

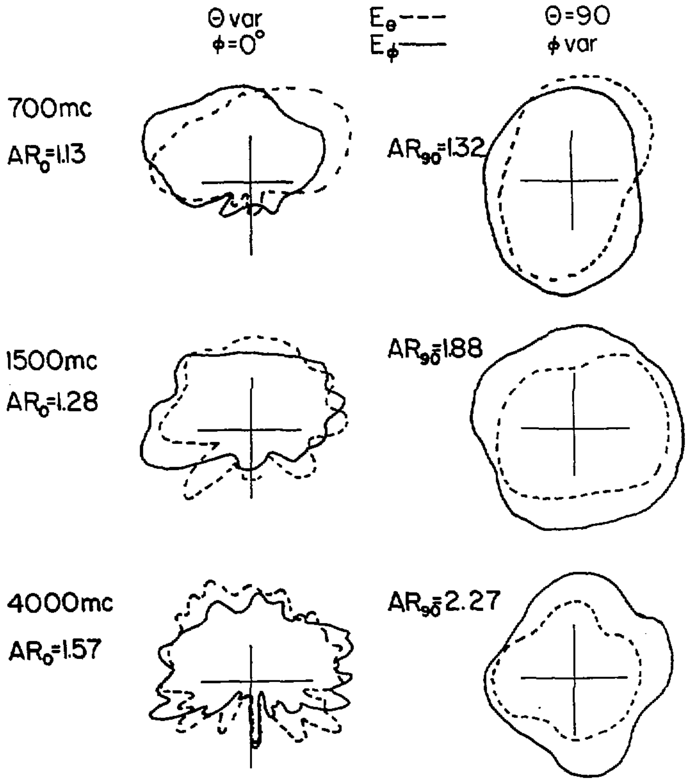


Fig. 9—Radiation patterns of balanced conical antenna with $L=78.3$ cm, $D=19.5$ cm, $H=54.5$ cm, $b=0.174$, $K=0.75$, $\theta_0=10°$, $\alpha=45°$, AR_0=axial ratio at $\theta=0°$, AR_{90}=axial ratio at $\theta=90°$.

Conclusion

An investigation of the balanced equiangular spiral antenna formed on a conical surface has shown that unidirectional, circularly-polarized single-lobe radiation patterns may be obtained over bandwidths that are at the discretion of the designer. The input impedance remains relatively constant over this pattern bandwidth.

Antennas of practical form may be constructed by shaping metal arms on a conical surface or by printed circuit techniques. A simple and most useful form is the wire version where the antenna arms consist of coaxial cable only. Such structures should find wide use at the VHF and UHF frequencies.

Operation of these nonplanar antennas is possible in free space, or with the antenna base over a ground plane. They need not be isolated from the ground plane. Flush mounted operation is possible with some sacrifice in bandwidth.

Acknowledgment

The author is pleased to acknowledge the helpful discussions he had with Dr. P. E. Mayes and the assistance he received from R. L. Jones, P. W. Arnold and other members of the University of Illinois Antenna Laboratory who contributed to the measurements