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THE UNIFORM METHOD TO THE SOLUTIONS OF TYPE OF THE FIFTH-ORDER SAWADA-KOTERA EQUATIONS

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Abstract. In this paper the exact explicit traveling wave solutions to the fKdV Sawada-Kotera equations are given by using a uniform method. We obtained some new forms of the solutions more than that appeared in Wazwaz [9]. The results in this paper are significant extension.

1. Introduction

The standard fifth-order KdV equation (fKdV) of the form

$$u_t + \alpha u^2 u_x + \beta u_x u_{xx} + \gamma u u_{xxx} + u_{xxxxx} = 0 \tag{1}$$

where α, β and γ are arbitrary nonzero parameters. Lots of forms of the fKdV equation can be constructed by changing these parameters, such as the Lax equation [1], the type of the Kaup-Kupershmidt equation [2], [3], [4], the

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Ito equation [5], and the type of the Sawada-Kotera equations [4], [6], [7], [8]. The fKdV equation (1) describes motions of long waves in shallow water under gravity and in a one-dimensional nonlinear lattice, and has wide applications in quantum mechanics and nonlinear optics [1-6]. Several different approaches, such as Bäcklund transformation, the Hirota's bilinear methods, the Lax pair methods, the tanh and extended tanh methods, etc., have been used to obtain soliton and multi-soliton solutions.

In this paper, we will investigate the exact solutions of the type of the Sawada-Kotera equations (SK equations for short) by using a uniform method. This type includes:

(i) The Sawada-Kotera equation [7] is given by

$$u_t + 5u^2u_x + 5u_xu_{xx} + 5uu_{xxx} + u_{xxxxx} = 0. \tag{2}$$

(ii) The Sawada-Kotera-Parker-Dye equation [4] is given by

$$u_t + 45u^2u_x - 15u_xu_{xx} - 15uu_{xxx} + u_{xxxxx} = 0. \tag{3}$$

(iii) The KdV-Sawada-Kotera-Ramani equation [6], [8] is given by

$$u_t + a(3u^2 + u_{xx})_x + b(15u^3 + 15u_xu_{xx} + u_{xxxx})_x = 0, \quad b \neq 0. \tag{4}$$

Recently, the type of the SK equations has been studied by several authors using various methods. In [9] Wazwaz obtained some forms of exact traveling wave solutions for equations (2) and (3) by tanh and extended tanh method. He obtained only two soliton solutions to (2) as follows:

$$u(x,t) = \frac{16}{3}\mu^2 - 8\mu^2\tanh^2(\mu x - \frac{128}{3}\mu^5 t) - 8\mu^2\coth^2(\mu x - \frac{128}{3}\mu^5 t),$$
$$u(x,t) = \mu^2 - 6\mu^2\tanh^2(\mu x + 19\mu^5 t) - 6\mu^2\coth^2(\mu x + 19\mu^5 t),$$

and solutions to (3) as follows:

$$u(x,t) = -\frac{16}{9}\mu^2 + \frac{8}{3}\mu^2\tanh^2(\mu x - \frac{128}{3}\mu^5 t) + \frac{8}{3}\mu^2\coth^2(\mu x - \frac{128}{3}\mu^5 t),$$
$$u(x,t) = \mu^2 + 2\mu^2\tanh^2(\mu x - 181\mu^5 t) + 2\mu^2\coth^2(\mu x - 181\mu^5 t),$$

by using the first and second criterion respectively. Li and Zhang in [10] obtained some exact traveling wave solutions for equation (4) for some special case. We notice that the traveling wave equations of the type of the SK equations has the uniform form by using suitable transformations, so we can use a uniform method to get the transformed exact solutions for this type of equations.

To study the traveling wave solutions of (2), (3) and (4), letting $u(x,t) = y(x - vt) = y(\xi)$, where v stands for wave velocity. Then integrating the result equations with respect to ξ once, respectively, equations (2), (3) and (4) become the following correspondingly traveling wave equations:

$$u'''' = -5uu'' - \frac{5}{3}u^3 + vu - 6\beta, \tag{5}$$

$$u'''' = 15uu'' - 15u^3 + vu + 2\beta, \tag{6}$$

$$u'''' = -15uu'' - 15u^3 - \frac{a}{b}(3u^2 + u'') + \frac{v}{b}u + 2g, \tag{7}$$

where β and g are two integral constants. We notice that by using suitable transformations these three types of the Sawada-Kotera-equations have the following uniform model of traveling wave equation:

$$y'''' = 30yy'' - 60y^3 + \alpha y + \beta \tag{8}$$

which just meet the F-IV form of Cosgrove's higher-order Painlevé equation [11], where $\alpha = v$ for (5), (6) by using the transformations $u = -6y$ to (5), $u = 2y$ to (6), and $\alpha = \frac{a^2}{5b^2} + \frac{v}{b}, \beta = \frac{a^3}{225b^3} + \frac{av}{30b^2} - g$ by using $u = -2y - \frac{a}{15b}$ to (7). Therefore we can study the exact solutions of the type of the SK equations uniformly by using Cosgrove's work. In this paper, we will investigate the exact explicit traveling wave solutions to (8) for the cases (1). $\beta = \frac{1}{9}\alpha\sqrt{\alpha}$ and (2). $\beta = -\frac{1}{9}\alpha\sqrt{\alpha}, \alpha > 0$ in the next two sections, respectively. We notice that in [10] the authors only studied equation (8) for $\beta = 0$. Hence the solutions obtained in the present paper are new exact traveling wave solutions for the original equations (2)-(4). We will show that for the equation (8), their traveling wave solutions correspond to some orbits in a 4-dimensional phase space of 4-dimensional dynamical system. These traveling wave solutions lie in a two-dimensional global homoclinic manifold to a hyperbolic equilibrium and in a two dimensional center manifold to a center-center equilibrium point.

2. The Exact Traveling Wave Solutions Of Equation (8) For $\beta = \frac{1}{9}\sqrt{\alpha^3}$ And Their Geometric Property

Now we begin to state and prove our theorem. For $\beta = \frac{1}{9}\sqrt{\alpha^3}$ we have the following conclusion.

Theorem 2.1. *The type of SK equations* (2)-(4) *have the uniformly transformed traveling wave equation* (8) *which has the nontrivial exact explicit solutions given by* (11)-(15)(*see below*). *These solutions are the correspondingly changed singular or regular solitary wave solutions of the original equations* (2)-(4). *Geometrically, the solution curves of the equation* (8) *defined by* $(x_1(\xi) = y(\xi), x_2(\xi) = y'(\xi), x_3(\xi) = y''(\xi), x_4(\xi) = y'''(\xi))$ *lie in the intersection of two level homoclinic manifolds*

$$\Phi_1(x_1, x_2, x_3, x_4) = 0, \qquad \Phi_2(x_1, x_2, x_3, x_4) = 0$$

of system (9)(*see below*) *for* $\beta = \frac{1}{9}\sqrt{\alpha^3}$, $\alpha > 0$.

Proof. For convenience, we take $\alpha = v = 16\mu^4, \mu > 0$ and then $\beta = \frac{64}{9}\mu^6$ to equation (8). Let

$$x_1 = y, x_2 = y', x_3 = y'', x_4 = y''',$$

then equation (8) for $\alpha = 16\mu^4,\ \beta = \frac{64}{9}\mu^6$ is equivalent to the four-dimensional system

$$\begin{aligned} &x_1' = x_2,\ x_2' = x_3,\ x_3' = x_4, \\ &x_4' = 30x_1x_3 - 60x_1^3 + \alpha x_1 + \beta \end{aligned} \tag{9}$$

which has two first integrals(see [11])

$$\begin{aligned} \Phi_1(x_1, x_2, x_3, x_4) &= (x_4 - 6x_1x_2)^2 \\ &\quad - 24(x_3 - 3x_1^2 + \frac{1}{12}\alpha)(x_1x_3 - \frac{1}{2}x_2^2 - 2x_1^3 + \frac{1}{12}\beta), \\ \Phi_2(x_1, x_2, x_3, x_4) &= x_1(x_4 - 6x_1x_2)^2 - x_2(x_4 - 6x_1x_2)(x_3 + 6x_1^2 - \frac{1}{6}\alpha) \\ &\quad + 12x_1(x_3 - 3x_1^2 + \frac{1}{12}\alpha)(x_1x_3 - \frac{1}{2}x_2^2 - 2x_1^3 + \frac{1}{12}\beta) \\ &\quad - 36(x_1x_3 - \frac{1}{2}x_2^2 - 2x_1^3 + \frac{1}{12}\beta)^2 \\ &\quad + 3(x_1x_3 - \frac{1}{2}x_2^2 - 2x_1^3 + \frac{1}{12}\beta)(12x_1^3 - \alpha x_1 + \beta) \\ &\quad + \frac{1}{3}(x_3 - 3x_1^2 + \frac{1}{12}\alpha)(x_3 + 6x_1^2 - \frac{1}{6}\alpha)^2. \end{aligned}$$

For given two constants K_1 and K_2, the two level sets defined by

$$\Phi_1(x_1, x_2, x_3, x_4) = K_1$$

and

$$\Phi_2(x_1, x_2, x_3, x_4) = K_2$$

determine two three-dimensional invariant manifolds of system (9). By using the method given in Li and Zhang [10], we first discuss the number and position of the equilibria of (9) in the phase space. For a known equilibrium, we compute the eigenvalues of the coefficient matrix of the linearized system of (9) at the equilibrium point in order to understand its local dynamical behavior. It is easy to see that (9) has a unique real equilibrium point $E_1(\frac{2}{3}\mu^2, 0, 0, 0)$ for which the eigenvalues of the coefficient matrix $M_1(\frac{2}{3}\mu^2, 0, 0, 0)$ of the linearized system of (9) are two real pairs $\pm 2\mu,\ \pm 4\mu$, the equilibrium point E_1 is a saddle-saddle. Let

$$K_1 = \Phi_1\left(\frac{2}{3}\mu^2, 0, 0, 0\right) = 0, \quad K_2 = \Phi_2\left(\frac{2}{3}\mu^2, 0, 0, 0\right) = 0.$$

Thus the two level sets defined by $\Phi_1(x_1,x_2,x_3,x_4)=0$ and $\Phi_2(x_1,x_2,x_3,x_4)=0$ pass through the equilibrium point $E_1(\frac{2}{3}\mu^2,0,0,0)$. Their intersection lies in the homoclinic manifold of E_1. By using the method given in Cosgrove [11], we know that the equation (8) admits solutions as the following form:

$$y(\xi)=U(\xi)+V(\xi),$$

where $U(\xi)$ and $V(\xi)$ are elliptic functions defined by the differential equations

$$\begin{aligned}(U')^2&=4U^3-\frac{1}{12}\alpha U+\frac{1}{24}\beta=4(U+\frac{2}{3}\mu^2)(U-\frac{1}{3}\mu^2)^2,\\(V')^2&=4V^3-\frac{1}{12}\alpha V+\frac{1}{24}\beta=4(V+\frac{2}{3}\mu^2)(V-\frac{1}{3}\mu^2)^2.\end{aligned}\tag{10}$$

In the $(U,\dot{U})-$phase plane and $(V,\dot{V})-$phase plane, the two equations defined by (10) determine the same cubic algebraic curve which is shown in Fig.1. Clearly, these equations of (10) give rise to a homoclinic orbit to the equilibrium point $(\frac{1}{3}\mu^2,0)$ and an open orbit.

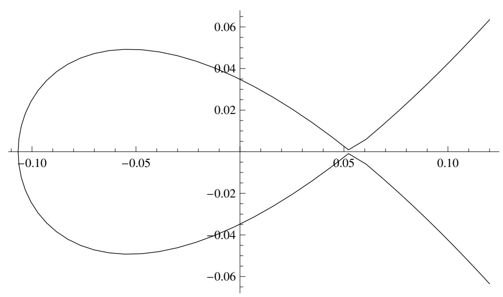

Fig.1. The phase curve defined by (10).

By using (10) to do integration, we obtain the following results.
(1) Corresponding to the homoclinic orbit in Fig.1, we have the parametric representation

$$U_1(\xi)=V_1(\xi)=-\frac{2}{3}\mu^2+\mu^2\tanh^2(\mu\xi).$$

(2) Corresponding to the open orbit in Fig.1, we have the parametric representation

$$U_2(\xi)=V_2(\xi)=-\frac{2}{3}\mu^2+\mu^2\coth^2(\mu\xi).$$

(3) Corresponding to the equilibrium point $(\frac{1}{3}\mu^2,0)$ in Fig.1, we have the parametric representation

$$U_3(\xi)=V_3(\xi)=\frac{1}{3}\mu^2.$$

Therefore, we obtain the exact explicit parametric representations of the nontrivial solutions to (8) for $\beta = \frac{64}{9}\mu^6$ as follows:

$$y_1(\xi) = x_1(\xi) = U_1 + V_1 = 2U_1 = -\frac{4}{3}\mu^2 + 2\mu^2 \tanh^2(\mu\xi), \tag{11}$$

$$y_2(\xi) = x_1(\xi) = U_1 + V_2 = -\frac{4}{3}\mu^2 + \mu^2[\tanh^2(\mu\xi) + \coth^2(\mu\xi)], \tag{12}$$

$$y_3(\xi) = x_1(\xi) = U_1 + V_3 = -\frac{1}{12}p^2 + \mu^2 \tanh^2(\mu\xi), \tag{13}$$

$$y_4(\xi) = x_1(\xi) = U_2 + V_2 = 2U_2 = -\frac{4}{3}\mu^2 + \mu^2 \coth^2(\mu\xi), \tag{14}$$

$$y_5(\xi) = x_1(\xi) = U_2 + V_3 = -\frac{1}{3}\mu^2 + \mu^2 \coth^2(\mu\xi), \tag{15}$$

$$y_6(\xi) = x_1(\xi) = U_3 + V_3 = 2U_3 = \frac{2}{3}\mu^2.$$

This completes the proof of Theorem 2.1. □

By utilizing Theorem 2.1, let $u(\xi) = -6y(\xi)$, and $\alpha = 16\mu^4, \beta = \frac{1}{9}\sqrt{\alpha^3} = \frac{64}{9}\mu^6$ we obtain the nontrivial exact explicit traveling wave solutions to (2) as follows:

$$\begin{aligned}
u_1(\xi) &= 8\mu^2 - 12\mu^2 \tanh^2(\mu\xi),\\
u_2(\xi) &= 8\mu^2 - 6\mu^2[\tanh^2(\mu\xi) + \coth^2(\mu\xi)],\\
u_3(\xi) &= 2\mu^2 - 6\mu^2 \tanh^2(\mu\xi),\\
u_4(\xi) &= 8\mu^2 - 12\mu^2 \coth^2(\mu\xi),\\
u_5(\xi) &= 2\mu^2 - 6\mu^2 \coth^2(\mu\xi).
\end{aligned}$$

We see that $u_1(\xi)$, $u_3(\xi)$ are regular solitary wave solutions, while $u_2(\xi), u_4(\xi)$, $u_5(\xi)$ are singular solitary wave solutions of equation (2). Notice that Wazwaz in [9] obtained only two soliton solutions akin to $u_2(\xi)$ for equation (2), hence the results in this work are significant extension and we obtain else some new ones.

By $u(\xi) = 2y(\xi)$, and $\alpha = 16\mu^4, \beta = \frac{1}{9}\sqrt{\alpha^3} = \frac{64}{9}\mu^6$ we obtain the nontrivial exact explicit traveling wave solutions to (6) as follows:

$$
\begin{aligned}
u_1(\xi) &= -\frac{8}{3}\mu^2 + 4\mu^2 \tanh^2(\mu\xi),\\
u_2(\xi) &= -\frac{8}{3}\mu^2 + 2\mu^2[\tanh^2(\mu\xi) + \coth^2(\mu\xi)],\\
u_3(\xi) &= -\frac{2}{3}\mu^2 + 2\mu^2 \tanh^2(\mu\xi),\\
u_4(\xi) &= -\frac{8}{3}\mu^2 + 4\mu^2 \coth^2(\mu\xi),\\
u_5(\xi) &= -\frac{2}{3}\mu^2 + 2\mu^2 \coth^2(\mu\xi).
\end{aligned}
$$

Similarly, we see that $u_1(\xi)$, $u_3(\xi)$ are regular solitary wave solutions, while $u_2(\xi)$, $u_4(\xi)$, $u_5(\xi)$ are singular solitary wave solutions of equation (3). Notice that Wazwaz in [9] obtained only two soliton solutions akin to $u_2(\xi)$ for equation (3), hence the results in this work are significant extension and we obtain else some new ones. For

$$\mu = \sqrt[4]{\frac{a^2 - 5bv}{80b^2}}, \quad a^2 - 5bv > 0, \quad g = \frac{a^3}{225b^3} + \frac{av}{30b^2} - \frac{1}{45|b|^3}\sqrt{(a^2 - 5bv)^3},$$

then $\alpha = 16\mu^4$ and $\beta = \frac{1}{9}\sqrt{\alpha^3} = \frac{64}{9}\mu^6$, by $u(\xi) = -2y(\xi) - \frac{a}{15b}$ we obtain the nontrivial exact explicit traveling wave solutions to (4) in parametric representations as follows:

$$
\begin{aligned}
u_1(\xi) &= \frac{8}{3}\mu^2 - \frac{a}{15b} - 4\mu^2 \tanh^2(\mu\xi),\\
u_2(\xi) &= \frac{8}{3}\mu^2 - \frac{a}{15b} - 2\mu^2[\tanh^2(\mu\xi) + \coth^2(\mu\xi)],\\
u_3(\xi) &= \frac{2}{3}\mu^2 - \frac{a}{15b} - 2\mu^2 \tanh^2(\mu\xi),\\
u_4(\xi) &= \frac{8}{3}\mu^2 - \frac{a}{15b} - 4\mu^2 \coth^2(\mu\xi),\\
u_5(\xi) &= \frac{2}{3}\mu^2 - \frac{a}{15b} - 2\mu^2 \coth^2(\mu\xi).
\end{aligned}
$$

Similarly, we see that $u_1(\xi), u_3(\xi)$ are regular solitary wave solutions, while $u_2(\xi), u_4(\xi), u_5(\xi)$ are singular solitary wave solutions of equation (7) which is the traveling wave solutions of equation (4).

3. The Exact Traveling Wave Solutions To (8) For $\beta = -\frac{1}{9}\sqrt{\alpha^3}$ And Their Geometric Property

For $\beta = -\frac{1}{9}\sqrt{\alpha^3}$, we have the following conclusion.

Theorem 3.1. *The type of SK equations* (2)-(4) *have the uniformly transformed traveling wave equation* (8) *which has the nontrivial exact explicit solutions given by* (17), (18)(*see below*). *These solutions are the correspondingly changed singular periodic wave solutions of the original equations* (2)-(4). *Geometrically, the solution curves of the equation* (8) *defined by* $(x_1(\xi) = y(\xi), x_2(\xi) = y'(\xi), x_3(\xi) = y''(\xi), x_4(\xi) = y'''(\xi))$ *lie in the intersection of two level center manifolds*

$$\Phi_1(x_1, x_2, x_3, x_4) = 0, \qquad \Phi_2(x_1, x_2, x_3, x_4) = 0$$

of system (9) (*see Theorem* 2.1) *for* $\beta = -\frac{1}{9}\sqrt{\alpha^3}, \alpha > 0$.

Proof. We take $\alpha = 16\mu^4,\ \beta = -\frac{1}{9}\sqrt{\alpha^3} = -\frac{64}{9}\mu^6$ to (8). Then system (9) has a unique real equilibrium point $E_2(-\frac{2}{3}\mu^2, 0, 0, 0)$ for which the eigenvalues of the coefficient matrix $M_2(-\frac{2}{3}\mu^2, 0, 0, 0)$ of the linearized system of (9) are two purely imaginary pairs $\pm\mu^2 i,\ \pm 2\mu^2 i$, the equilibrium point E_2 is a center-center.

At this point, we have $K_1 = \Phi_1(-\frac{2}{3}\mu^2, 0, 0, 0) = 0,\ K_2 = \Phi_2(-\frac{2}{3}\mu^2, 0, 0, 0) = 0$, hence the two level sets defined by

$$\Phi_1(x_1, x_2, x_3, x_4) = 0$$

and

$$\Phi_2(x_1, x_2, x_3, x_4) = 0$$

pass through the equilibrium point E_2. Their intersection lies on the center manifold of $E_2(-\frac{2}{3}\mu^2, 0, 0, 0)$. In this manifold, we know that the equation (8) admits solutions as the following form:

$$y(\xi) = U(\xi) + V(\xi),$$

where $U(\xi)$ and $V(\xi)$ are elliptic functions defined by the differential equations

$$\begin{aligned} (U')^2 &= 4U^3 - \frac{1}{12}\alpha U + \frac{1}{24}\beta = 4(U - \frac{2}{3}\mu^2)(U + \frac{1}{3}\mu^2)^2, \\ (V')^2 &= 4V^3 - \frac{1}{12}\alpha V + \frac{1}{24}\beta = 4(V - \frac{2}{3}\mu^2)(V + \frac{1}{3}\mu^2)^2. \end{aligned} \tag{16}$$

In the $(U, \dot{U})-$phase plane and $(V, \dot{V})-$phase plane, the two equations defined by (16) determine the same cubic algebraic curve which is shown in Fig.2. Clearly, these equations of (16) give rise to an open curve passing through the point $(\frac{2}{3}\mu^2, 0)$ and, the equilibrium point $(-\frac{1}{3}\mu^2, 0)$.

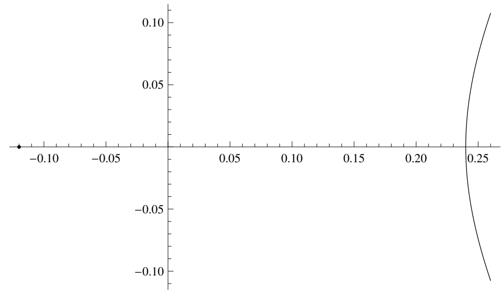

Fig.2. The phase curve defined by (16).

By using (16) to do integration, we obtain the following results.
(1) Corresponding to the open orbit in Fig.2, we have the parametric representation

$$U_1(\xi) = V_1(\xi) = \frac{2}{3}\mu^2 + \mu^2 \tan^2(\mu^2\xi).$$

(2) Corresponding to the equilibrium point $(-\frac{1}{3}\mu^2, 0)$ in Fig.2, we have the parametric representation

$$U_2(\xi) = V_2(\xi) = -\frac{1}{3}\mu^2.$$

Therefore, we obtain the exact explicit parametric representations of the nontrivial solutions to (11) for $\beta = -24p^2$ as follows:

$$y_1(\xi) = x_1(\xi) = U_1 + V_1 = 2U_1 = \frac{4}{3}\mu^2 + 2\mu^2 \tan^2(\mu\xi), \quad (17)$$

$$y_2(\xi) = x_1(\xi) = U_1 + V_2 = \frac{1}{3}\mu^2 + \mu^2 \tan^2(\mu\xi), \quad (18)$$

$$y_3(\xi) = x_1(\xi) = U_2 + V_2 = -\frac{2}{3}\mu^2.$$

This completes the proof of Theorem 3.1. □

Remark 3.2. Correspondingly, by $u(\xi) = -6y(\xi)$, and $\alpha = 16\mu^4, \beta = -\frac{1}{9}\sqrt{\alpha^3} = -\frac{64}{9}\mu^6$ we obtain the nontrivial exact explicit traveling wave solutions to (2) as follows:

$$u_1(\xi) = -8\mu^2 - 12\mu^2 \tan(\mu\xi),$$
$$u_2(\xi) = -2\mu^2 - 6\mu^2 \tan(\mu\xi).$$

Hence the equation (2) has noncompact singular periodic wave solutions $u_1(\xi)$ and $u_2(\xi)$ for $\beta = -\frac{1}{9}\sqrt{\alpha^3}$ to equation (8).

By $u(\xi) = 2y(\xi)$, and $\alpha = 16\mu^4, \beta = -\frac{1}{9}\sqrt{\alpha^3} = -\frac{64}{9}\mu^6$ we obtain the nontrivial exact explicit traveling wave solutions to (6) as follows:

$$u_1(\xi) = \frac{8}{3}\mu^2 + 4\mu^2\tan(\mu\xi),$$
$$u_2(\xi) = \frac{2}{3}\mu^2 + 2\mu^2\tan(\mu\xi).$$

Hence the equation (3) has noncompact singular periodic wave solutions $u_1(\xi)$ and $u_2(\xi)$ for $\beta = -\frac{1}{9}\sqrt{\alpha^3}$ to equation (8).

For $\mu = \sqrt[4]{\frac{a^2-5bv}{80b^2}}$, $a^2 - 5bv > 0$, $g = \frac{a^3}{225b^3} + \frac{av}{30b^2} + \frac{1}{45|b|^3}\sqrt{(a^2-5bv)^3}$, then $\alpha = 16\mu^4$ and $\beta = -\frac{1}{9}\sqrt{\alpha^3} = -\frac{64}{9}\mu^6$, by $u(\xi) = -2y(\xi) - \frac{a}{15b}$ we obtain the nontrivial exact explicit traveling wave solutions to (4) in parametric representations as follows:

$$u_1(\xi) = -\frac{8}{3}\mu^2 - \frac{a}{15b} - 4\mu^2\tan(\mu\xi),$$
$$u_2(\xi) = -\frac{2}{3}\mu^2 - \frac{a}{15b} - 2\mu^2\tan(\mu\xi).$$

Hence the equation (4) has noncompact singular periodic wave solutions $u_1(\xi)$ and $u_2(\xi)$ for $\beta = -\frac{1}{9}\sqrt{\alpha^3}$ to equation (8).

4. Conclusion

In this paper, we have obtained many exact explicit wave solutions for the type of the SK equations (2)-(4) by uniformly employing the Cosgrove's method. Some exact explicit traveling wave solutions are obtained. The local dynamical behavior of some known equilibria are discussed. The obtained solutions included the types of regular or singular solitary waves, noncompact periodic singular waves. The results in this work are significant extension to that in [9] and we obtain else some new forms of the solutions.

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