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THE UNIQUENESS PROBLEM OF MEROMORPHIC MAPPINGS WITH DEFICIENCIES

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Abstract. In this paper we mainly study the effect of the existence of deficient divisors in the sense of Nevanlinna to the uniqueness problem of meromorphic mappings into a projective algebraic manifold M. We give some uniqueness theorems for families of dominant meromorphic mappings from the complex m-space into M with the same preimages of divisors under the additional conditions on Nevanlinna's deficiencies.

Introduction. The main purpose of this paper is to study how the existence of deficient divisors affects the uniqueness problem of meromorphic mappings. The uniqueness problem of meromorphic mappings under conditions on the preimages of divisors was first studied by G. Pólya, R. Nevanlinna and H. Cartan, and they proved classical uniqueness theorems for meromorphic functions on the complex plane $\boldsymbol{C}$ (cf. [7]). There have been a number of detailed researches on the uniqueness problem of meromorphic functions on $\boldsymbol{C}$. In the multidimensional case, we also have many studies. On the other hand, the defect relation for meromorphic mappings implies that the deficient divisors in the sense of Nevanlinna are very few. In fact, the set of these divisors is at most countable. Furthermore, we have the following conjecture: Almost all meromorphic mappings have no Nevanlinna's deficient divisor (cf. [6]). It therefore seems that the existence of deficient divisors imposes a strong restriction on the behavior of meromorphic mappings. In this paper we prove unicity theorems for some families of meromorphic mappings from the complex m-space $\boldsymbol{C}^m$ into projective algebraic manifolds with the same inverse images of divisors under the additional conditions on Nevanlinna's deficiencies. We note here that the unicity theorems for meromorphic functions on $\boldsymbol{C}$ under the conditions on Nevanlinna's deficiencies were already studied and some interesting results were obtained (cf. [8], [13], [14] and [15]).

Let M be a projective algebraic manifold and K_M the canonical bundle of M. For a line bundle L over M, we denote by $\Gamma(M, L)$ the space of all holomorphic sections of $L \to M$.

DEFINITION 0.1. A line bundle L over M is said to be *big* provided that

$$\dim \Gamma(M, \nu L) \geq C\nu^{\dim M}$$

for all sufficiently large positive integers ν and for some positive constant C.

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We denote by $\mathrm{Pic}(M)$ the Picard group over M. Let $F \in \mathrm{Pic}(M) \otimes \boldsymbol{Q}$ and $\gamma \in \boldsymbol{Q}$. We simply write γF for $F^{\otimes \gamma}$. Then F is said to be *big* provided that a line bundle $\nu F \in \mathrm{Pic}(M)$ is big for some positive integer ν. We fix a big line bundle $L \to M$. Now we set

$$\left[\frac{F}{L}\right] = \inf\{\gamma \in \boldsymbol{Q}; \gamma L \otimes F^{-1} \text{ is big}\}.$$

It is easy to see that $[F/L] < 0$ if and only if F^{-1} is big.

DEFINITION 0.2. A meromorphic mapping $f : \boldsymbol{C}^m \to M$ is said to be *dominant* if $\mathrm{rank} f = \dim M$.

Throughout this paper, we assume that there exists at least one dominant meromorphic mapping $f_0 : \boldsymbol{C}^m \to M$. We note that K_M is not big in our case (cf. [5, p. 143]). Let $D_1, \dots, D_q$ be divisors in $|L|$ such that $D_1 + \cdots + D_q$ has only simple normal crossings, where $|L|$ denotes the complete linear system defined by L. Let $E_1, \dots, E_q$ be hypersurfaces in $\boldsymbol{C}^m$ such that $\dim E_i \cap E_j \leq m-2$ for $i \neq j$. Assume that there exists a positive integer k_0 such that the union of all irreducible components of $f_0^* D_j$ with the multiplicities at most k_0 is equal to E_j for each j. Let

$$\mathcal{E} = \mathcal{E}(f_0; k_0; (\boldsymbol{C}^m, \{E_j\}), (M, \{D_j\}))$$

be the set of all *dominant* meromorphic mappings $f : \boldsymbol{C}^m \to M$ such that the union of irreducible components of $f^* D_j$ with the multiplicities at most k_0 coincides with E_j and $f = f_0$ on E_j for all $1 \leq j \leq q$. We also define the subfamily $\mathcal{E}_0$ of $\mathcal{E}$ by

$$\mathcal{E}_0 = \{f \in \mathcal{E}; \delta_{f_0}(D_j) \leq \delta_f(D_j) \text{ for all } 1 \leq j \leq q\}.$$

Let $\boldsymbol{P}_n(\boldsymbol{C})$ be the n-dimensional complex projective space and $\Phi : M \to \boldsymbol{P}_n(\boldsymbol{C})$ a nonconstant meromorphic mapping. In this paper, we always assume that $\mathrm{rank}\, \Phi = \dim M$. Set

$$G_0 = M - (\{w \in M - I(\Phi); \mathrm{rank}\, d\Phi(w) < \dim M\} \cup I(\Phi)),$$

where $I(\Phi)$ is the locus of indeterminacy of Φ.

DEFINITION 0.3. A set $\{D_j\}_{j=1}^q$ of divisors is said to be *generic with respect to* f_0 *and* Φ provided that

$$f_0(\boldsymbol{C}^m - I(f_0)) \cap \mathrm{Supp} D_j \cap G_0 \neq \emptyset$$

for at least one $1 \leq j \leq q$, where $I(f_0)$ denotes the locus of indeterminacy of f_0.

We denote by H the hyperplane bundle over $\boldsymbol{P}_n(\boldsymbol{C})$. We define $F_0 \in \mathrm{Pic}(M) \otimes \boldsymbol{Q}$ by

$$F_0 = \frac{qk_0}{k_0+1} L \otimes \left(-\frac{2k_0}{k_0+1}\right) \Phi^* H.$$

If F_0 is sufficiently big, we can conclude $\mathcal{E} = \{f_0\}$ as follows:

THEOREM 0.4. *Suppose that* $\{D_j\}_{j=1}^q$ *is generic with respect to* f_0 *and* Φ. *If* $F_0 \otimes K_M$ *is big, then the family* $\mathcal{E}$ *contains just one mapping* f_0.

In the definition of the family $\mathcal{E}$ we impose the strong condition on the meromorphic mappings contained in $\mathcal{E}$, that is, every mapping in $\mathcal{E}$ must be equal to f_0 on all E_j. We note that this condition cannot be simply dropped (see Remarks 2.8 in Section 2). In the case where $F_0 \otimes K_M$ is not big, we cannot prove $\sharp\mathcal{E} = 1$ in general. However we can show the unicity theorem for $\mathcal{E}$ under an additional condition on the existence of Nevanlinna's deficient divisors. Indeed, we have the following unicity theorem, which is our main result in this paper:

THEOREM 0.5. *Suppose that* $\{D_j\}_{j=1}^q$ *is generic with respect to* f_0 *and* Φ, *and*

$$\left[\frac{F_0^{-1} \otimes K_M^{-1}}{L}\right] = 0 .$$

If $\delta_{f_0}(D_j) > 0$ *for at least one* $1 \le j \le q$, *then the family* $\mathcal{E}$ *contains just one mapping* f_0.

We note the following: In the case where $[F_0^{-1} \otimes K_M^{-1}/L]$ is positive, we cannot conclude $\mathcal{E} = \{f_0\}$ under the condition on the existence of deficient divisors in the sense of Nevanlinna (see Remarks 2.25 in Section 2). For the family $\mathcal{E}_0$, we have the following unicity theorem:

THEOREM 0.6. *Suppose that* $\{D_j\}_{j=1}^q$ *is generic with respect to* f_0 *and* Φ, *and*

$$\left[\frac{F_0^{-1} \otimes K_M^{-1}}{L}\right] < \frac{1}{k_0 + 1} \sum_{j=1}^{q} \delta_{f_0}(D_j) .$$

Then the family $\mathcal{E}_0$ *contains just one mapping* f_0.

We give the proofs of the above theorems in Section 2 by proving more general results.

1. Preliminaries. In this section we recall some known facts on Nevanlinna theory of dominant meromorphic mappings into projective algebraic manifolds. Let $z = (z_1, \dots, z_m)$ be the natural coordinate system in $\boldsymbol{C}^m$, and set

$$\|z\|^2 = \sum_{\nu=1}^{m} z_\nu \bar{z}_\nu , \quad B(r) = \{z \in \boldsymbol{C}^m ; \|z\| < r\} ,$$

$$d^c = \frac{\sqrt{-1}}{4\pi}(\bar{\partial} - \partial) , \quad \alpha = dd^c \|z\|^2 .$$

For a $(1, 1)$-current φ of order zero on $\boldsymbol{C}^m$ we set

$$N(r, \varphi) = \int_1^r \langle \varphi \wedge \alpha^{m-1}, \chi_{B(t)} \rangle \frac{dt}{t^{2m-1}} ,$$

where $\chi_{B(r)}$ denotes the characteristic function of $B(r)$.

Let M be a compact complex manifold and $L \to M$ a line bundle over M. We denote by $\mathrm{Bs}|L|$ the base locus of $|L|$. Let $\{\varphi_0, \dots, \varphi_n\}$ be a basis for $\Gamma(M, L)$. Then we define a meromorphic mapping $\Phi_L : M \to \boldsymbol{P}_n(\boldsymbol{C})$ by

$$\Phi_L(z) = (\varphi_0(z) : \cdots : \varphi_n(z)) , \quad z \in M - \mathrm{Bs}|L| .$$

Let $|\cdot|$ be a hermitian fiber metric in L, and let ω be its Chern form. Let $f : \boldsymbol{C}^m \to M$ be a meromorphic mapping. We set

$$T_f(r, L) = N(r, f^*\omega) ,$$

and call it the characteristic functin of f with respect to L. In the case where $M = \boldsymbol{P}_n(\boldsymbol{C})$ and $L = H$ is the hyperplane bundle, we simply write $T_f(r)$ for $T_f(r, H)$. Furthermore, we also define $T_f(r, F)$ for $F \in \mathrm{Pic}(M) \otimes \boldsymbol{Q}$ in the following way. If ν is a positive integer with $\nu F \in \mathrm{Pic}(M)$, then we set

$$T_f(r, F) = \frac{1}{\nu} T_f(r, \nu F).$$

It is easy to see that $T_f(r, F)$ is well-defined. Then we have Nevandinna's inequality for meromorphic mappings as follows (cf. [11, Theorem 2.3]):

THEOREM 1.1. *Let $L \to M$ be a line bundle over M and $f : \boldsymbol{C}^m \to M$ a meromorphic mapping. Then*

$$N(r, f^*D) \leq T_f(r, L) + O(1)$$

for $D \in |L|$ with $f(\boldsymbol{C}^m) \not\subseteq \mathrm{Supp}D$, where $O(1)$ stands for a bounded term as $r \to +\infty$.

Let E be an effective divisor on $\boldsymbol{C}^m$ such that $E = \sum_j \nu_j E_j$ for distinct irreducible hypersurfaces E_j in $\boldsymbol{C}^m$ and for nonnegative integers ν_j, and let k be a positive integer. We set

$$N_k(r, E) = \sum_j \min\{k, \nu_j\} N(r, E_j).$$

Then we have the following second main theorem for dominant meromorphic mappings (cf. [10, Theorem 2] and [11, Theorem 3.2]):

THEOREM 1.2. *Let M be a projective algebraic manifold with $m \geq \dim M$ and L a big line bundle over M. Let $D_1, \dots, D_q$ be divisors in $|L|$ such that $D_1 + \cdots + D_q$ has only simple normal crossings. Let $f : \boldsymbol{C}^m \to M$ be a dominant meromorphic mapping. Then*

$$qT_f(r, L) + T_f(r, K_M) \leq \sum_{j=1}^{q} N_1(r, f^*D_j) + S_f(r),$$

where $S_f(r) = O(\log T_f(r, L)) + o(\log r)$ except on a Borel subset $E \subseteq [1, +\infty)$ with finite measure.

Let $f : \boldsymbol{C}^m \to M$ be a meromorphic mapping, and let $D \in |L|$. We define Nevanlinna's deficiency $\delta_f(D)$ by

$$\delta_f(D) = 1 - \limsup_{r \to +\infty} \frac{N(r, f^*D)}{T_f(r, L)}.$$

It is clear that $0 \leq \delta_f(D) \leq 1$ and $\delta_f(D) = 1$ if $f(\boldsymbol{C}^m) \cap \mathrm{Supp}D = \emptyset$. If $\delta_f(D) > 0$, then D is called a *deficient divisor in the sense of Nevanlinna*. Finally we state the following fact as lemma (cf. [10, p. 566]):

LEMMA 1.3. *Let $L \to M$ be a big line bundle and $f : \boldsymbol{C}^m \to M$ a dominant meromorphic mapping. Then there exists a positive constant C such that*

$$C \log r \leq T_f(r, L) + O(1).$$

2. Unicity theorems for families of dominant meromorphic mappings. In this section we prove unicity theorems for some families of dominant meromorphic mappings of $\boldsymbol{C}^m$

into a projective algebraic manifold M. Let $L \to M$ be a big line bundle. Let $D_1, \dots, D_q$ be divisors in $|L|$ such that $D_1 + \cdots + D_q$ has only simple normal crossings. Let E be an effective divisor on $\boldsymbol{C}^m$, and let k be a positive integer. If $E = \sum_j \nu_j E'_j$ for distinct irreducible hypersurfaces E'_j in $\boldsymbol{C}^m$ and for nonnegative integers ν_j, then we define the support of E with order at most k by

$$\mathrm{Supp}_k E = \bigcup_{0<\nu_j \le k} E'_j .$$

Let $E_1, \dots, E_q$ be hypersurfaces in $\boldsymbol{C}^m$ such that $\dim E_i \cap E_j \le m-2$ for $i \ne j$. Let $k_1, \dots, k_q$ be positive integers with $k_1 \ge \cdots \ge k_q$. Assume that there exists a dominant meromorphic mapping $f_0 : \boldsymbol{C}^m \to M$ with $\mathrm{Supp}_{k_j} f_0^* D_j = E_j$ for all $1 \le j \le q$. Let

$$\mathcal{F} = \mathcal{F}(f_0; \{k_j\}; (\boldsymbol{C}^m, \{E_j\}), (M, \{D_j\}))$$

be the set of all *dominant* meromorphic mappings $f : \boldsymbol{C}^m \to M$ such that

$$\mathrm{Supp}_{k_j} f^* D_j = E_j \quad \text{and} \quad f = f_0 \text{ on } E_j$$

for all $1 \le j \le q$. We define $F_1 \in \mathrm{Pic}(M) \otimes \boldsymbol{Q}$ by

$$F_1 = \left(\sum_{j=1}^{q} \frac{k_j}{k_j+1} \right) L \otimes \left(-\frac{2k_1}{k_1+1} \right) \Phi^* H .$$

Let $\mathcal{F}_0$ be the subfamily of $\mathcal{F}$ defined by

$$\mathcal{F}_0 = \{ f \in \mathcal{F}; \delta_{f_0}(D_j) \le \delta_j(D_j) \text{ for all } 1 \le j \le q \} .$$

We first show the following unicity theorem:

THEOREM 2.1. *Suppose that* $\{D_j\}_{j=1}^q$ *is generic with respect to* f_0 *and* Φ. *If* $F_1 \otimes K_M$ *is big, then the family* $\mathcal{F}$ *contains just one mapping* f_0.

PROOF. Let f be an arbitrary mapping in $\mathcal{F}$. We first show that $\Phi \circ f \equiv \Phi \circ f_0$. We note that

$$N_1(r, f^*D) \le \frac{1}{k+1} \{ kN(r, \mathrm{Supp}_k f^*D) + N(r, f^*D) \} \tag{2.2}$$

for any positive integer k (cf. [3, p. 126]). We also note that

$$\frac{k_j}{k_j+1} \le \frac{k_1}{k_1+1}$$

for all $1 \le j \le q$. By Theorem 1.1, Theorem 1.2 and (2.2), we have

$$\begin{aligned} qT_f(r, L) + T_f(r, K_M) &\le \sum_{j=1}^{q} N_1(r, f^*D_j) + S_f(r) \\ &\le \sum_{j=1}^{q} \frac{1}{k_j+1} \{ k_j N(r, \mathrm{Supp}_{k_j} f^*D_j) + N(r, f^*D_j) \} + S_f(r) \\ &\le \frac{k_1}{k_1+1} \sum_{j=1}^{q} N(r, E_j) + \left(\sum_{j=1}^{q} \frac{1}{k_j+1} \right) T_f(r, L) + S_j(r) . \end{aligned}$$

We define an effective divisor E on $\boldsymbol{C}^m$ by $E = E_1 + \cdots + E_q$. Then it follows from $\dim E_i \cap E_j \leq m-2$ $(i \neq j)$ that

$$N(r, E) = \sum_{j=1}^{q} N(r, E_j).$$

Hence we obtain

$$\left(\sum_{j=1}^{q} \frac{k_j}{k_j+1}\right) T_f(r, L) + T_f(r, K_M) \leq \frac{k_1}{k_1+1} N(r, E) + S_f(r).$$

For brevity, we set $T(r, F) = T_f(r, F) + T_{f_0}(r, F)$ for $F \in \mathrm{Pic}(M) \otimes \boldsymbol{Q}$. We also set $S(r) = S_f(r) + S_{f_0}(r)$. Then we have

$$\left(\sum_{j=1}^{q} \frac{k_j}{k_j+1}\right) T(r, L) + T(r, K_M) \leq \frac{2k_1}{k_1+1} N(r, E) + S(r). \tag{2.3}$$

Now we assume that $\Phi \circ f \not\equiv \Phi \circ f_0$. Set $\boldsymbol{P}_n(\boldsymbol{C})^2 = \boldsymbol{P}_n(\boldsymbol{C}) \times \boldsymbol{P}_n(\boldsymbol{C})$. We denote by $\pi_j : \boldsymbol{P}_n(\boldsymbol{C})^2 \to \boldsymbol{P}_n(\boldsymbol{C})$ $(j = 1, 2)$ the natural projections on j-th factor. We define the line bundle $\tilde{H} \to \boldsymbol{P}_n(\boldsymbol{C})^2$ by $\tilde{H} = \pi_1^* H \otimes \pi_2^* H$. Let Δ be the diagonal of $\boldsymbol{P}_n(\boldsymbol{C})^2$. We define a meromorphic mapping $\varphi : \boldsymbol{C}^m \to \boldsymbol{P}_n(\boldsymbol{C})^2$ by $\varphi = (\Phi \circ f, \Phi \circ f_0)$. Since $\Phi \circ f \not\equiv \Phi \circ f_0$, there exists a holomorphic section $\tilde{\sigma}$ of $\tilde{H} \to \boldsymbol{P}_n(\boldsymbol{C})^2$ such that $\varphi^* \tilde{\sigma} \not\equiv 0$ and $\Delta \subseteq \mathrm{Supp}(\tilde{\sigma})$ (cf. [2, p. 354]). It follows from Theorem 1.1 that

$$N(r, \varphi^*(\tilde{\sigma})) \leq T_f(r, \Phi^* H) + T_{f_0}(r, \Phi^* H) + O(1). \tag{2.4}$$

Since $f = f_0$ on E and $\Delta \subseteq \mathrm{Supp}(\tilde{\sigma})$, it is clear that

$$N(r, E) \leq N(r, \varphi^*(\tilde{\sigma})). \tag{2.5}$$

By (2.3), (2.4) and (2.5), we have

$$T(r, F_1) + T(r, K_M) \leq S(r). \tag{2.6}$$

Since $F_1 \otimes K_M$ is big, there exists a positive constant C such that

$$CT(r, L) \leq T(r, F_1) + T(r, K_M) + O(1). \tag{2.7}$$

Indeed, by Kodaira's Lemma (cf. [5, Lemma 2]), there exists a positive integer μ such that the line bundle $\mu(F_1 \otimes K_M) \otimes L^{-1} \to M$ is big. Thus there exists a nonzero holomorphic section $\tau \in \Gamma(M, \nu(\mu(F_1 \otimes K_M) \otimes L^{-1}))$ for a sufficiently large positive integer ν. By Theorem 1.1, we have

$$\begin{aligned} N(r, f^*(\tau)) &\leq T_f(r, \nu(\mu(F_1 \otimes K_M) \otimes L^{-1})) + O(1) \\ &= \mu\nu\{T_f(r, F_1) + T_f(r, K_M)\} - \nu T_f(r, L) + O(1). \end{aligned}$$

Hence

$$\frac{1}{\mu} T_f(r, L) \leq T_f(r, F_1) + T_f(r, K_M) + O(1).$$

This shows (2.7). By (2.6) and (2.7), we see

$$T(r, L) \le S(r).$$

Thus, by Lemma 1.3, we have a contradiction. Therefore $\Phi \circ f \equiv \Phi \circ f_0$.

We now conclude $\mathcal{F} = \{f_0\}$ in the following way. Let G_0 be as in the Introduction, that is,

$$G_0 = M - (\{w \in M - I(\Phi); \operatorname{rank} d\Phi(w) < \dim M\} \cup I(\Phi)).$$

By the assumption, we have $R_j := f_0(\boldsymbol{C}^m - I(f_0)) \cap \operatorname{Supp} D_j \cap G_0 \neq \emptyset$ for some j. Take a point $p \in R_j$. Then there exists an open neighborhood U of p such that $\Phi\big|_U : U \to \Phi(U)$ is biholomorphic. Set $U' = f_0^{-1}(U)$ and take an arbitrary mapping f in $\mathcal{F}$. It follows from $\Phi \circ f = \Phi \circ f_0$ and $f = f_0$ on E_j that $f = f_0$ on U'. Thus we see $f \equiv f_0$ by uniqueness of analytic continuation. Q.E.D.

We give here some remarks on the above theorem.

REMARKS 2.8. (1) In the definition of the family $\mathcal{F}$, we assume that $f = f_0$ on all E_j for every $f \in \mathcal{F}$. The following simple example shows that this hypothesis cannot be simply removed (cf. [2, p. 357]): Let $M = \boldsymbol{P}_2(\boldsymbol{C})$ and $\Phi : \boldsymbol{P}_2(\boldsymbol{C}) \to \boldsymbol{P}_2(\boldsymbol{C})$ the identity mapping. Let D be a Fermat curve of degree d defined by

$$w_0^d + w_1^d + w_2^d = 0,$$

where $\{w_0, w_1, w_2\}$ is a homogeneous coordinate system in $\boldsymbol{P}_2(\boldsymbol{C})$. We define distinct dominant meromorphic mappings $f, g : \boldsymbol{C}^2 \to \boldsymbol{P}_2(\boldsymbol{C})$ by

$$f = (\varphi : \psi : 1) \quad \text{and} \quad g = (\psi : \varphi : 1),$$

where φ and ψ are distinct holomorphic functions on $\boldsymbol{C}^2$. Then it is clear that $f^*D = g^*D$ as divisors. Hence $\operatorname{Supp}_k f^*D = \operatorname{Supp}_k g^*D$ for all positive integers k. Note that $F_1 \otimes K_{\boldsymbol{P}_2(\boldsymbol{C})}$ is positive if $d > 8$ (see the proof of Theorem 2.9 below). Thus we cannot conclude $f = g$ under conditions depending only on d.

(2) Let $e_0 = \sharp\Phi^{-1}(\Phi(w))$ for $w \in G_0$. In the case where $\{D_j\}_{j=1}^q$ is not generic with respect to f_0 and Φ, we can conclude $\sharp\mathcal{F} \le e_0$ as follows. Assume that there exist mutually distinct mappings $f_0, \dots, f_p$ in $\mathcal{F}$. Let

$$G_0' = \{z \in \boldsymbol{C}^m; f_j(z) \in G_0 \text{ and } f_j(z) \neq f_{j'}(z) \text{ for } 0 \le j < j' \le p\}.$$

Then G_0' is an open dense subset of $\boldsymbol{C}^m$. For $z_0 \in G_0'$, we have

$$\{f_0(z_0), \dots, f_p(z_0)\} \subseteq \Phi^{-1}\Phi(f_0(z_0)).$$

Therefore $p + 1 \le e_0$. In the particular case where Φ is bimeromorphic mapping, we always have $\mathcal{F} = \{f_0\}$ without the generic condition on $\{D_j\}_{j=1}^q$.

(3) Since L is big, there exists a positive integer q_0 depending only on L and Φ such that the number of mappings in the family $\mathcal{F}$ is bounded by e_0 if $q \ge q_0$. Furthermore, there exists a positive integer q_1 depending only on L such that the family $\mathcal{F}$ contains just one mapping f_0 if $q \ge q_1$. Indeed, if we take the smallest positive integer q_0 such that $(q_0/2)L \otimes (-2)\Phi^*H \otimes K_M$ is big, then q_0 has the desired property. Let ν be the smallest

positive integer such that $\Phi_{\nu L} : M \to \Phi_{\nu L}(M)$ is bimeromorphic (cf. [4, Theorem 5]). If we define $q_1 = q_0$ for $\Phi = \Phi_{\nu L}$, then we have $\mathcal{F} = \{f_0\}$ provided that $q \geq q_1$.

Now we have Theorem 0.3 as an immediate consequence of Theorem 2.1. In the case of $M = \boldsymbol{P}_1(\boldsymbol{C})$, we have the unicity theorem due to Gopalakrishna and Bhoosnurmats (cf. [3, Theorem 1]). In the case where $M = \boldsymbol{P}_n(\boldsymbol{C})$ and $q = 1$, we have the following unicity theorem (cf. [2, Theorem 4.1]):

THEOREM 2.9. *Let D be a hypersurface in $\boldsymbol{P}_n(\boldsymbol{C})$ with simple normal crossings. Suppose that the degree d of D is greater than $n + 3 + (n+1)/k$. Then the family $\mathcal{F}(f_0; \{k\}; (\boldsymbol{C}^m, \{E\}), (\boldsymbol{P}_n(\boldsymbol{C}), \{D\}))$ contains just one mapping f_0.*

PROOF. Let $M = \boldsymbol{P}_n(\boldsymbol{C})$ and $L = dH$ in Theorem 2.1. Let $\Phi : \boldsymbol{P}_n(\boldsymbol{C}) \to \boldsymbol{P}_n(\boldsymbol{C})$ be the identity mapping. Since $K_{\boldsymbol{P}_n(\boldsymbol{C})} = -(n+1)H$, we have

$$F_1 \otimes K_{\boldsymbol{P}_n(\boldsymbol{C})} = \left(\frac{k(d-2)}{k+1} - n - 1\right) H . \tag{2.10}$$

Hence $F_1 \otimes K_{\boldsymbol{P}_n(\boldsymbol{C})}$ is positive provided that $d > n + 3 + (n+1)/k$. Thus we have the desired conclusion. Q.E.D.

For the family $\mathcal{F}_0$, we have the following unicity theorem:

THEOREM 2.11. *Suppose that $\{D_j\}_{j=1}^q$ is generic with respect to f_0 and Φ, and*

$$\left[\frac{F_1^{-1} \otimes K_M^{-1}}{L}\right] < \frac{1}{k_1 + 1} \sum_{j=1}^{q} \delta_{f_0}(D_j) .$$

Then the family $\mathcal{F}_0$ contains just one mapping f_0.

PROOF. If $F_1 \otimes K_M$ is big, we have our assertion by Theorem 2.1. Hence we may assume that $[F_1^{-1} \otimes K_M^{-1}/L] \geq 0$. Let f be an arbitrary mapping in $\mathcal{F}_0$. For the proof, it suffices to show that $\Phi \circ f \equiv \Phi \circ f_0$. As in the proof of Theorem 2.1, we have

$$\begin{aligned} & qT(r, L) + T(r, K_M) \\ & \quad \leq \frac{2k_1}{k_1 + 1} N(r, E) + \sum_{j=1}^{q} \frac{1}{k_j + 1} \{N(r, f^* D_j) + N(r, f_0^* D_j)\} + S(r) . \end{aligned} \tag{2.12}$$

By the definition of Nevanlinna's deficiency, for any $\varepsilon > 0$, there exists $r_0 > 0$ such that

$$N(r, f^* D_j) + N(r, f_0^* D_j) < (1 - \delta_f(D_j) + \varepsilon) T_f(r, L) + (1 - \delta_{f_0}(D_j) + \varepsilon) T_{f_0}(r, L)$$

for all $r \geq r_0$ ($r \notin I$), where $I \subseteq [1, +\infty)$ is a Borel subset with finite measure. We may assume that the exceptional set for $S(r)$ is included in I. Now assume that $\Phi \circ f \not\equiv \Phi \circ f_0$. Then we have

$$\begin{aligned} qT(r, L) + T(r, K_M) \leq & \frac{2k_1}{k_1 + 1} \{T_f(r, \Phi^* H) + T_{f_0}(r, \Phi^* H)\} \\ & + \sum_{j=1}^{q} \frac{1}{k_j + 1} \{(1 - \delta_f(D_j) + \varepsilon) T_f(r, L) + (1 - \delta_{f_0}(D_j) + \varepsilon) T_{f_0}(r, L)\} + S(r) . \end{aligned} \tag{2.13}$$

By the definition of the family $\mathcal{F}_0$, it is clear that

$$(\delta_{f_0}(D_j) - \varepsilon)T(r, L) \leq (\delta_f(D) - \varepsilon)T_f(r, L) + (\delta_{f_0}(D) - \varepsilon)T_{f_0}(r, L)\,.$$

Hence we have

$$\begin{aligned} T(r, F_1) + T(r, K_M) &\leq -\sum_{j=1}^{q} \frac{1}{k_j + 1}(\delta_{f_0}(D_j) - \varepsilon)T(r, L) + S(r) \\ &\leq \left(-\frac{1}{k_1 + 1}\sum_{j=1}^{q} \delta_{f_0}(D_j) + q\varepsilon\right) T(r, L) + S(r)\,. \end{aligned}$$

Thus we see

$$T(r, F_1) + T(r, K_M) + \left(\frac{1}{k_1 + 1}\sum_{j=1}^{q} \delta_{f_0}(D_j) - q\varepsilon\right) T(r, L) \leq S(r)\,.$$

Take a rational number γ so that $\gamma > [F_1^{-1} \otimes K_M^{-1}/L]$. Then we have

$$-\gamma T(r, L) < T(r, F_1) + T(r, K_M) + O(1)\,.$$

Hence

$$\left(\frac{1}{k_1 + 1}\sum_{j=1}^{q} \delta_{f_0}(D_j) - q\varepsilon - \gamma\right) T(r, L) \leq S(r)\,.$$

Thus we see

$$\frac{1}{k_1 + 1}\sum_{j=1}^{q} \delta_{f_0}(D_j) \leq \left[\frac{F_1^{-1} \otimes K_M^{-1}}{L}\right].$$

This contradicts the definition of $\mathcal{F}_0$. Therefore $\Phi \circ f \equiv \Phi \circ f_0$. Q.E.D.

REMARK 2.14. We define the subfamily $\mathcal{F}_1$ of $\mathcal{F}$ by

$$\mathcal{F}_1 = \left\{ f \in \mathcal{F};\, \left[\frac{F_1^{-1} \otimes K_M^{-1}}{L}\right] < \frac{1}{k_1 + 1}\sum_{j=1}^{q} \min\{\delta_f(D_j), \delta_{f_0}(D_j)\} \right\}.$$

Then by an argument similar to the above proof, we can show that the family $\mathcal{F}_1 = \{f_0\}$ if $\{D_j\}_{j=1}^{q}$ is generic with respect to f_0 and Φ. Note that $\mathcal{F}_0 \subseteq \mathcal{F}_1$ if the assumption of Theorem 2.11 is satisfied.

We have Theorem 0.4 as a special case of Theorem 2.11. Next we consider the case where $[F_1^{-1} \otimes K_M^{-1}/L] = 0$.

THEOREM 2.15. *Suppose that $\{D_j\}_{j=1}^{q}$ is generic with respect to f_0 and Φ, and*

$$\left[\frac{F_1^{-1} \otimes K_M^{-1}}{L}\right] = 0\,.$$

If $\delta_{f_0}(D_j) > 0$ for at least one $1 \leq j \leq q$, then the family $\mathcal{F}$ contains just one mapping f_0.

PROOF. Let f be an arbitrary mapping in $\mathcal{F}$. Assume that $\Phi \circ f \not\equiv \Phi \circ f_0$. Then there exist positive constants C_1 and C_2 such that

$$C_1 \leq \frac{T_f(r, L)}{T_{f_0}(r, L)} \leq C_2 \tag{2.16}$$

for all sufficiently large r with $r \notin I$. For the proof of (2.16), we first show the following: For any positive constant $\nu < 1$, there exists a positive number r_1 such that

$$\nu\{T_f(r, \Phi^*H) + T_{f_0}(r, \Phi^*H)\} < N(r, E) \tag{2.17}$$

for all sufficiently large $r \geq r_1$ with $r \notin I$. Assume the contrary. Then there exist a positive constant $\nu_0 < 1$ and a monotone increasing sequence $\{r_k\}$ with $r_k \notin I$ such that $r_k \to +\infty$ and

$$N(r_k, E) \leq \nu_0\{T_f(r_k, \Phi^*H) + T_{f_0}(r_k, \Phi^*H)\}.$$

We may assume that $\nu_0 \in \boldsymbol{Q}$. Since (2.12) also holds for $f \in \mathcal{F}$, we have

$$\begin{aligned} qT(r_k, L) + T(r_k, K_M) &\leq \frac{2\nu_0 k_1}{k_1 + 1}\{T_f(r_k, \Phi^*H) + T_{f_0}(r_k, \Phi^*H)\} \\ &+ \sum_{j=1}^{q} \frac{1}{k_j + 1}\{(1 - \delta_f(D_j) + \varepsilon)T_f(r_k, L) + (1 - \delta_{f_0}(D_j) + \varepsilon)T_{f_0}(r_k, L)\} + S(r_k). \end{aligned}$$

We define $F_2 \in \mathrm{Pic}(M) \otimes \boldsymbol{Q}$ by

$$F_2 = \left(\sum_{j=1}^{q} \frac{k_j}{k_j + 1}\right) L \otimes \left(-\frac{2\nu_0 k_1}{k_1 + 1}\right) \Phi^*H.$$

Then we have

$$\begin{aligned} &T(r_k, F_2) + T(r_k, K_M) \\ &+ \sum_{j=1}^{q} \frac{1}{k_j + 1}\{(\delta_f(D_j) - \varepsilon)T_f(r_k, L) + (\delta_{f_0}(D_j) - \varepsilon)T_{f_0}(r_k, L)\} \leq S(r_k). \end{aligned} \tag{2.18}$$

Since Φ^*H is big, there exists a positive integer μ such that $\mu\Phi^*H \otimes L^{-1}$ is big. Then it is easy to see that

$$\frac{1}{\mu} T(r, L) \leq T(r, \Phi^*H) + O(1). \tag{2.19}$$

Set

$$\mu_0 = \frac{2(1 - \nu_0)k_1}{k_1 + 1}.$$

Note that $\mu_0 > 0$. Since $F_2 = F_1 \otimes \mu_0\Phi^*H$, it is clear that

$$\begin{aligned} T(r, F_2 \otimes K_M) &= T(r, F_1 \otimes K_M) + \mu_0 T(r, \Phi^*H) + O(1) \\ &\geq T(r, F_1 \otimes K_M) + \frac{\mu_0}{\mu} T(r, L) + O(1). \end{aligned} \tag{2.20}$$

It follows from $[F_1^{-1} \otimes K_M^{-1}/L] = 0$ that $F_1 \otimes K_M \otimes (\mu_0/\mu)L$ is big. Hence there exists a positive constant C such that

$$CT(r, L) \leq T(r, F_1) + T(r, K_M) + \frac{\mu_0}{\mu}T(r, L) + O(1). \tag{2.21}$$

By (2.18), (2.20) and (2.21), we see

$$CT(r_k, L) + \sum_{j=1}^{q} \frac{1}{k_j + 1}\{(\delta_f(D_j) - \varepsilon)T_f(r_k, L) + (\delta_{f_0}(D_j) - \varepsilon)T_{f_0}(r_k, L)\} \leq S(r_k).$$

This is absurd. Thus we have (2.17).

We next note the following:

$$N(r, E) \leq qT_{f_0}(r, L) + O(1). \tag{2.22}$$

Indeed, (2.22) is an immediate consequence of Theorem 1.1. By (2.17), (2.19) and (2.22), we see

$$\frac{\nu}{\mu}T(r, L) \leq qT_{f_0}(r, L) + O(1).$$

This shows (2.16). Now we assume that $\delta_{f_0}(D_l) > 0$ for some l. By (2.13), we have

$$\begin{aligned} &T(r, F_1) + T(r, K_M) \\ &\quad \leq -\sum_{j=1}^{q} \frac{1}{k_j + 1}\{(\delta_f(D_j) - \varepsilon)T_f(r, L) + (\delta_{f_0}(D_j) - \varepsilon)T_{f_0}(r, L)\} + S(r) \\ &\quad \leq q\varepsilon T_f(r, L) - \left(\frac{\delta_{f_0}(D_l)}{k_l + 1} - q\varepsilon\right)T_{f_0}(r, L) + S(r). \end{aligned}$$

Hence

$$T(r, F_1) + T(r, K_M) + \left(\frac{\delta_{f_0}(D_l)}{k_l + 1} - q\varepsilon\right)T_{f_0}(r, L) - q\varepsilon T_f(r, L) \leq S(r).$$

Take a rational number $\gamma > 0$. It follows from $[F_1^{-1} \otimes K_M^{-1}/L] = 0$ that

$$-\gamma T(r, L) + \left(\frac{\delta_{f_0}(D_l)}{k_l + 1} - q\varepsilon\right)T_{f_0}(r, L) - q\varepsilon T_f(r, L) \leq S(r). \tag{2.23}$$

By (2.16), (2.23) and Lemma (1.3), we see

$$-C_3\gamma + \frac{\delta_{f_0}(D_l)}{k_l + 1} - C_4\varepsilon \leq 0,$$

where C_3 and C_4 are positive constants independent of ε and γ. This implies $\delta_{f_0}(D_l) = 0$ and hence we have a contradiction. Therefore $\Phi \circ f \equiv \Phi \circ f_0$. Q.E.D.

We now obtain Theorem 0.5 as a special case of Theorem 2.15. In the case where $M = \boldsymbol{P}_1(\boldsymbol{C})$ and $\Phi : \boldsymbol{P}_1(\boldsymbol{C}) \to \boldsymbol{P}_1(\boldsymbol{C})$ is the identity mapping, we have Ueda's unicity theorem ([14, Theorem 1]) by Theorem 2.11 and Remark 2.14. In the case where $M = \boldsymbol{P}_n(\boldsymbol{C})$ and $q = 1$, we have the following:

COROLLARY 2.24. *Let D be a hypersurface in $\boldsymbol{P}_n(\boldsymbol{C})$ of degree $n+4$ with simple normal crossings. If $\delta_{f_0}(D) > 0$, then the family $\mathcal{F}(f_0; \{n+1\}; (\boldsymbol{C}^m, \{E\}), (\boldsymbol{P}_n(\boldsymbol{C}), \{D\}))$ contains just one mapping f_0.*

PROOF. By (2.10), $F_1 \otimes K_{\boldsymbol{P}_n(\boldsymbol{C})}$ is trivial. Hence $[F_1^{-1} \otimes K_{\boldsymbol{P}_n(\boldsymbol{C})}^{-1}/L] = 0$. Thus we have our assertion. Q.E.D.

REMARKS 2.25. (1) In the case where $[F_1^{-1} \otimes K_M^{-1}/L]$ is positive, we cannot conclude $\mathcal{F} = \{f_0\}$ under the condition on the existence of deficient divisors. We now give the following counter example: Let $f_0 : \boldsymbol{C} \to \boldsymbol{P}_1(\boldsymbol{C})$ be a meromorphic function defined by $f_0(z) = \exp z$. Set $D_1 = 0$, $D_2 = \infty$, $D_3 = 1$ and $D_4 = -1$. Then it is clear that D_1 and D_2 are Picard's deficient divisors of f_0. Let $k_j = 1$ and put $E_j = \mathrm{Supp}_1 f_0^* D_j$ for $1 \leq j \leq 4$. Let $\Phi : \boldsymbol{P}_1(\boldsymbol{C}) \to \boldsymbol{P}_1(\boldsymbol{C})$ be the identity mapping. In this case $L = H$ and $[F_1^{-1} \otimes K_{\boldsymbol{P}_1(\boldsymbol{C})}^{-1}/L] = 1$. Now we see $\delta_{f_0}(D_1) = \delta_{f_0}(D_2) = 1$ but $\sharp\mathcal{F} \geq 2$. Indeed, $f(z) = \exp(-z) \in \mathcal{F}$ and $f_0 \not\equiv f$. Note that the proofs of the above theorems also work in the case where some of E_j are empty sets.

(2) We now consider the case where $k_j = +\infty$ for some j. We first note that $\mathrm{Supp} f^* D = \mathrm{Supp}_{k_j} f^* D$ if $k_j = +\infty$. Set $k_j/(k_j+1) = 1$ and $1/(k_j+1) = 0$ for $k_j = +\infty$. Then it is easy to see that the proofs of Theorems 2.1 and 2.11 also work in the case where $k_j = +\infty$ for some j. In the case where $[F_1^{-1} \otimes K_M^{-1}/L] = 0$, we have the conclusion of Theorem 2.15 if we assume that $\delta_{f_0}(D_l) > 0$ with $k_l \neq +\infty$ for some l. Indeed, the proof of Theorem 2.15 is still valid under this condition. We note here that the condition $k_l \neq +\infty$ cannot be simply dropped in this case. Indeed, let $D_1, \ldots, D_4$, f_0 and Φ be as in (1). Now let $k_j = +\infty$ for all $1 \leq j \leq 4$. Then we see $[F_1^{-1} \otimes K_{\boldsymbol{P}_1(\boldsymbol{C})}^{-1}/L] = 0$ but $\sharp\mathcal{F} \geq 2$.

We give here some examples of families of dominant meromorphic mappings with deficiencies that satisfy the assumptions of Theorem 2.15.

EXAMPLE 2.26. Let $M = \boldsymbol{P}_2(\boldsymbol{C})$ and $\Phi : \boldsymbol{P}_2(\boldsymbol{C}) \to \boldsymbol{P}_2(\boldsymbol{C})$ the identity mapping. Let $\{w_0, w_1, w_2\}$ be a homogeneous coordinate system in $\boldsymbol{P}_2(\boldsymbol{C})$. Let D_1 be a Fermat curve of degree two in $\boldsymbol{P}_2(\boldsymbol{C})$ defined by

$$w_0^2 + w_1^2 + w_2^2 = 0\,.$$

We define a dominant meromorphic mapping $f_0 : \boldsymbol{C}^2 \to \boldsymbol{P}_2(\boldsymbol{C})$ by

$$f_0(z_1, z_2) = (\cos z_1 : \sin z_1 : z_2)\,.$$

Then $f_0^* D_1$ is defined by $z_2^2 + 1 = 0$. Since $f_0^* D_1$ is an algebraic curve in $\boldsymbol{C}^2$, we have $N(r, f^* D_1) = O(\log r)$. On the other hand, we have $T_{f_0}(r) = (2/3\pi)r + o(r)$. Indeed, we first note that f_0 has a reduced representation

$$\left(\frac{1}{2}(\exp\sqrt{-1}z_1 + \exp(-\sqrt{-1}z_1)) : \frac{1}{2\sqrt{-1}}(\exp\sqrt{-1}z_1 - \exp(-\sqrt{-1}z_1)) : z_2\right)\,.$$

We define a subset $\mathcal{P}(z)$ of $\boldsymbol{C}$ by $\mathcal{P}(z) = \{\sqrt{-1}z_1, -\sqrt{-1}z_1, 0\}$, where $z = (z_1, z_2)$. We let $C(\mathcal{P}(z))$ denote the circumference of the convex hull of $\mathcal{P}(z)$ in $\boldsymbol{C}$. We define

$$K(\mathcal{P}) = \frac{1}{2\pi}\int_S C(\mathcal{P}(z))d\sigma(z),$$

where σ is the invariant measure on the unit sphere $S = S^3 \subseteq \boldsymbol{C}^2$ normalized so that $\sigma(S) = 1$. By [12, Lemma 3], we have $T_{f_0}(r) = K(\mathcal{P})r + o(r)$. In our case, it is easy to see that $C(\mathcal{P}(z)) = 2|z_1|$. Furthermore, by [9, p. 14], we see

$$\begin{aligned} K(\mathcal{P}) &= \frac{1}{2\pi}\int_S 2|z_1|d\sigma(z) \\ &= \frac{1}{2\pi}\int_B 2|w|\frac{dw}{\pi} \\ &= \frac{2}{3\pi}, \end{aligned}$$

where $B = \{w \in \boldsymbol{C}; |w| < 1\}$. Now it is clear that $\delta_{f_0}(D_1) = 1$. Let $q = 4$ and let $k_j = 1$ for $1 \le j \le 4$. In this case, we have $L = 2H$. It follows from $K_{\boldsymbol{P}_2(\boldsymbol{C})} = -3H$ that $F_1 \otimes K_{\boldsymbol{P}_2(\boldsymbol{C})}$ is trivial. Thus $[F_1^{-1} \otimes K_{\boldsymbol{P}_2(\boldsymbol{C})}^{-1}/L] = 0$. It we take $D_2, D_3, D_4 \in |2H|$ to be generic and set $E_j = \mathrm{Supp}_1 f_0^* D_j$ for $1 \le j \le 4$, then $D_1 + \cdots + D_4$ has only simple normal crossings and $\dim E_i \cap E_j = 0$ $(i \neq j)$. By Theorem 2.15, the family $\mathcal{F}(f_0; \{k_j\}; (\boldsymbol{C}^2, \{E_j\}), (\boldsymbol{P}_2(\boldsymbol{C}), \{D_j\}))$ contains just one mapping f_0.

The following example is due to B. Shiffman (cf. [11]):

EXAMPLE 2.27. Let $\{w_0, w_1, w_2\}$ and $\Phi : \boldsymbol{P}_2(\boldsymbol{C}) \to \boldsymbol{P}_2(\boldsymbol{C})$ be as in Example 2.26. Let d be a positive integer not less than three. We define a dominant meromorphic mapping $f_0 : \boldsymbol{C}^2 \to \boldsymbol{P}_2(\boldsymbol{C})$ by

$$f_0(z_1, z_2) = (\exp z_2 + \exp(1-d)z_1^2 : 1 : \exp z_1^2).$$

Let C be a curve in $\boldsymbol{P}_2(\boldsymbol{C})$ defined by $w_1^d - w_0 w_2^{d-1} = 0$. Then the singular locus of C consists of the single point $(1 : 0 : 0)$ and $f_0(\boldsymbol{C}^2) \cap C = \emptyset$. We define a divisor D on $\boldsymbol{P}_2(\boldsymbol{C})$ by $D = H_1 + \cdots + H_d$, where $H_1 = \{w_0 - w_1 = 0\}$ and $H_2, \ldots, H_d$ are projective lines such that $D \cup C - (1 : 0 : 0)$ has only simple normal crossings and $(1 : 0 : 0) \notin D$. Then we have $\delta_{f_0}(D) = d^{-2}$. For details, see [11, pp. 179–181]. Let $q = 1$ and $k_1 = 3$. Now we assume that $d = 6$. Then $L = 6H$ and hence $F_1 \otimes K_{\boldsymbol{P}_2(\boldsymbol{C})}$ is trivial. Set $E = \mathrm{Supp}_3 f_0^* D$. Note that $E \neq \emptyset$. Indeed, $f_0^* H_1$ is defined by $\exp z_2 + \exp(1-d)z_1^2 - 1 = 0$. Hence it is clear that $\mathrm{Supp}_3 f_0^* H_1 \neq \emptyset$. Thus we see $E \neq \emptyset$. By Corollary 2.24, the family $\mathcal{F}(f_0; \{3\}; (\boldsymbol{C}^2, \{E\}), (\boldsymbol{P}_2(\boldsymbol{C}), \{D\}))$ contains just one mapping f_0.

We now give the final remark. In the proofs of the above theorems, we use the second main theorem for dominant meromorphic mappings due to Sakai and Shiffman. We note here the following: If we use a second main theorem of another type, we can obtain results similar to the above theorems. For example, in the case of meromorphic mappings of $\boldsymbol{C}^m$ into $\boldsymbol{P}_n(\boldsymbol{C})$ with hyperplanes as divisors, we can prove some unicity theorems under certain conditions on Nevanlinna's deficiencies by making use of the second main theorem for meromorphic mappings into $\boldsymbol{P}_n(\boldsymbol{C})$ with hyperplanes as divisors. See [1] for these results.

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