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The universality of the stellar initial mass function

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ABSTRACT

We propose that the stellar initial mass function (IMF) is universal in the sense that its functional form arises as a consequence of the statistics of random supersonic flows.

A model is developed for the origin of the stellar IMF that contains a dependence on the average physical parameters (temperature, density and velocity dispersion) of the large-scale site of star formation. The model is based on recent numerical experiments of highly supersonic random flows that have a strong observational counterpart.

It is shown that a Miller–Scalo-like IMF is naturally produced by the model for the typical physical conditions in molecular clouds. A more 'massive' IMF in starbursts is also predicted.

Key words: stars: formation – stars: luminosity function, mass function – ISM: kinematics and dynamics.

1 INTRODUCTION

Star formation is a central problem of astrophysics and cosmology. It is very difficult to interpret observations of galaxies, or to predict their evolution, without any theoretical idea about the process of star formation.

While star formation rates and efficiencies can be constrained, because of the negative feedback of the process (young massive stars are able to disperse the star-forming gas), the mass distribution of the stars is not easily constrained theoretically.

Moreover, observing the stellar initial mass function (IMF) is difficult. In old systems, most massive stars have already evolved into cold white dwarfs and are hardly detectable. The masses of very small and long-lived stars are not easily inferred from the photometry. Bound stellar systems (globular clusters and open clusters) undergo a strong dynamical evolution (mass segregation, evaporation) that can significantly affect the IMF. In very young systems (e.g. young embedded clusters), the relation between the IMF and the luminosity function (LF) is strongly dependent on the assumed star formation history (e.g. initial burst or continuous). Finally, the determination of the IMF cut-off at the smallest masses requires very deep stellar counts, since it may be located at masses smaller than 0.1 $M_{\odot}$.

Most theoretical attempts to predict the IMF have been based on the idea of gravitational fragmentation. This idea is a direct consequence of linear gravitational instability: in a system with very small density and velocity fluctuations, gravitational instability causes the collapse of structures larger than a critical mass, that is the mass for which the thermal energy of the gas is comparable with its gravitational energy. During the collapse, if cooling is efficient, the critical mass becomes smaller, and substructures can collapse inside the collapsing object.

The picture of gravitational fragmentation depends on several idealized assumptions. The collapse of substructures and its final result are highly dependent on the presence of suitable perturbations in the density or velocity field. Moreover, fragmentation is stopped at some point, when opacity becomes important, but this occurs at a mass-scale that depends on unknown geometrical factors, which affect the rate of radiative loss. Finally, the whole idea of gravitational fragmentation relies on the linear gravitational instability, that is on the assumption that the density field is initially almost uniform, and the velocity field irrelevant. This is meaningful in the study of the formation of galaxies in the Universe, since we know that the Universe is initially very uniform. When we discuss smaller scales, instead, we are normally dealing with a contracting background where nonuniformities get amplified, rather than an expanding one. On scales smaller than galaxies, for example on the scale of giant molecular cloud complexes (10^5–10^6 $M_\odot$), the density and the velocity fields are highly non-linear and hierarchical, so that the idea of gravitational instability cannot be applied directly, using mean values of the physical parameters.

The next level of complexity is to still maintain the idea of a critical mass for gravitational instability, but using a distribution of the values of the physical parameters for its definition. The distribution of the physical parameters should be as close as possible to the actual distribution in the star-forming system, so that the complexity of the non-linear velocity and density field is bypassed using a statistical approach.

A further step is that of arguing that several ways of injecting and transferring kinetic energy in star-forming systems exist, such as gravity, magnetic fields, fluid turbulence, supernovae and H II regions, winds from young stars, tidal fields, thermal and magnetic instabilities, and galactic shear. All these sources of energy contribute to the generation of random motions, that, mediated by fluid turbulence, establish universal flow statistics.

When cooling is very efficient, the most important statistic is the density distribution. Once this is determined, the critical mass for gravitational instability can be defined along that distribution, resulting in a distribution of collapsing objects, or protostars.

In the present work, which is an improvement on Padoan (1995), we make use of recent numerical and observational results, which allow us to describe the density distribution in supersonic random flows (such as the ones in molecular clouds), and therefore to derive the mass distribution of protostars.

Previous physical models for the origin of the stellar IMF do not include a description of the effect of supersonic random flows in the star formation sites, based on the solution of the compressible fluid equations. Nevertheless, the idea that supersonic motions play an important part in the dynamics of molecular clouds, and in the formation of protostars, has already been expressed in the literature (e.g. McCrea 1960, Arny 1971, Larson 1981, Hunter & Fleck 1982, Léorat, Passot & Pouquet 1990, Elmegreen 1993).

The paper is organized as follows. In the next section we present the numerical and observational results on the density distribution in molecular clouds. Section 3 contains the derivation of the protostar mass function, the dependence on physical parameters of which is discussed in Sections 4 and 5. We then proceed to make comparisons with observations in Section 6. The paper ends with a general discussion, followed by a summary.

2 THE DENSITY FIELD IN RANDOM SUPERSONIC FLOWS

In this section we give a statistical description of the density field that emerges from randomly forced supersonic flows. Such motions are present in dark clouds, where stars are formed.

The statistical description is based on numerical experiments, but it is also confirmed by stellar extinction observations in dark clouds.

2.1 Random supersonic flows in numerical experiments

Norlund & Padoan (in preparation) have recently discussed the importance of supersonic flows in shaping the density distribution in the cold interstellar medium (ISM).

They have run numerical simulations of isothermal flows randomly forced to high Mach numbers. Their experiments are meant to represent a fraction of a giant molecular cloud, where in fact such random supersonic motions are observed. Most details about the numerical code, which solves the equations of magnetohydrodynamics in three dimensions and in a supersonic regime, and about the experiments are given in Nordlund & Padoan; here we only summarize the main results.

The physical parameters of the simulated system are: $\sigma_v = 2.5$ km s^{-1}, $T = 10$ K (therefore an rms Mach number of about 10), $M = 4000$ $M_\odot$, $L = 6$ pc, where L is the linear size of the periodic box.

It is found that the flow develops a complex system of interacting shocks, and these are able to generate very large density contrasts, up to 5 orders of magnitude, $\rho_{max}/\rho_{min} \approx 10^5$. In fact, most of the mass concentrates in a small fraction of the total volume of the simulation, with a very intermittent distribution. The probability density function of the density field is well approximated by a log-normal distribution:

$$p(\ln x)\,d\ln x = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \overline{\ln x}}{\sigma}\right)^2\right] d\ln x, \quad (1)$$

where x is the relative number density:

$$x = n/\bar{n} \quad (2)$$

and the standard deviation σ and the mean $\overline{\ln x}$ are functions of the rms Mach number of the flow, $\mathcal{M}$:

$$\overline{\ln x} = -\frac{\sigma^2}{2} \quad (3)$$

and

$$\sigma^2 = \ln(1 + \mathcal{M}^2\beta^2) \quad (4)$$

or, for the linear density:

$$\sigma_{linear} = \beta\mathcal{M} \quad (5)$$

where $\beta \approx 0.5$. Therefore, the standard deviation grows linearly with the rms Mach number of the flow.

It is also found that the power spectrum, $S(k)$, of the density distribution is consistent with a power law:

$$S(k) \propto k^{-2.6} \quad (6)$$

where k is the wavenumber.

The fact that the standard deviation of the linear density field, σ_{linear}, grows linearly with the rms Mach number of the flow can be easily understood. The density contrast behind an isothermal shock is proportional to $\mathcal{M}^2$, that is the square of the Mach number, but the dense shocked gas occupies only a fraction $\mathcal{M}^{-2}$ of the original volume. Since the standard deviation is a volume average, the two effects result in a linear growth of σ_{linear} with $\mathcal{M}$. This is in fact the result if one computes σ_{linear} for the simple case of a single strong, isothermal, plane shock that sweeps all the mass of the system.

2.2 Random supersonic flows in dark clouds

An observational counterpart of the numerical experiments on supersonic random flows has been recently indicated by Padoan, Jones & Nordlund (1997a), re-interpreting the observational results by Lada et al. (1994).

Lada et al. performed infrared stellar extinction measurements, through the dark cloud IC 5146 in Cygnus. They obtained values of extinction for more than 1000 stars, sampled the observed area with a regular grid, and measured the mean and the dispersion of the extinction determinations in each bin of the grid. They found that the dispersion grows with the mean extinction.

This result is an indication that the absorbing material in the dark cloud has structure well below the resolution of the extinction map. Padoan, Jones & Nordlund have shown that an intermittent 3D distribution in the cloud, in particular a log-normal distribution, explains in a natural way the growth of dispersion with mean extinction. They have also shown that the observational data can be used to constrain the value of the standard deviation and of the spectral index (power-law power spectrum) of the 3D density distribution. The observational constraints are in good agreement with the numerical predictions.

Therefore, both numerical results and observations show that the random supersonic flows, known to be present in dark clouds, result in a very intermittent density distribution, well described by a log-normal statistic. Both the values of the standard deviation and of the spectral index of such a distribution are predicted numerically and confirmed observationally.

3 THE DERIVATION OF THE STELLAR IMF

A simple way to define a mass distribution of protostars is that of identifying each protostar with one local Jeans mass. In this way the protostar MF is simply a Jeans mass distribution. Since the gas is cooling rapidly, the temperature is uniform, and the Jeans mass distribution is just determined by the density distribution.

The concept of the 'local' Jeans mass is meaningful in our scenario for molecular clouds (MCs), because random supersonic motions (cascading from a larger scale) are present, and are responsible for shaping the density field. Strong density enhancements, that is to say the local convergence of the flow, are a result of non-linear hydrodynamical interactions, rather than a result of the local gravitational potential. We therefore suggest a description of star formation where random motions are first creating a complex and highly non-linear density field (through isothermal shocks), and gravity then takes over, when each 'local' Jeans mass (defined with the local density) collapses into a protostar.

The statistic of the density field is not sufficient in general to predict the protostar MF. Some extra knowledge on the topology of the density field is necessary.

For example, the distribution of mass in a complex system of interacting shocks may be hierarchical. The mass distribution in MCs is also found to be hierarchical over a very large range of scales (Scalo 1985; Falgarone & Pérault 1987; Vázquez-Semadeni 1994). This brings a considerable difficulty when trying to define a mass distribution of cores inside MCs (Myers, Linke & Benson 1993; Blitz 1987; Carr 1987; Loren 1989; Stutzki & Güsten 1990; Lada, Bally & Stark 1991; Nozawa et al. 1991; Langer, Wilson & Anderson 1993; Williams & Blitz 1993) and indeed any mass distribution estimated from molecular emission line maps is ill-defined if the hierarchical structure is not taken into account.

The main uncertainty in the Jeans mass distribution, derived as a transformation of the density distribution, is related to the density fluctuations that are smaller than their Jeans mass. If many fluctuations are smaller than their Jeans mass, the transformation of density into Jeans mass overestimates the number of collapsing protostars with that mass.

Nevertheless, in our numerical experiments we find that isolated density fluctuations, smaller than their Jeans mass, are extremely rare, and do not account for more than 1 per cent in mass for any level of density considered. We conclude therefore that the transformation of the density field into the distribution of Jeans masses gives an estimate of the mass distribution of collapsing objects, which is not in error by more than a few per cent.

The density distribution per unit volume is given by equation (1). If we multiply that function with the relative density x, we get the density distribution per unit mass, that is the mass fraction at any given density:

$$f(x)\,\mathrm{d}x = xp(x)\,\mathrm{d}x. \tag{7}$$

The fraction of the total mass in collapsing structures of mass $<M$ is integral of the distribution $f(x)$ over relative densities $x > x_J$:

$$\int_{x_J}^{\infty} f(x)\,\mathrm{d}x$$

where x_J is the Jeans density for the mass M. The Jeans mass distribution is the derivative along mass of the previous integral:

$$F(M_J) = f(x_J)\,\frac{\mathrm{d}x_J}{\mathrm{d}M_J} \tag{8}$$

The Jeans mass can be written as:

$$M = M_J = 1\,\mathrm{M}_{\odot} B x^{-1/2} \tag{9}$$

where:

$$B = 1.2\left(\frac{T}{10\ \mathrm{K}}\right)^{3/2}\left(\frac{\bar{n}}{1000\ \mathrm{cm}^{-3}}\right)^{-1/2} \tag{10}$$

is the average Jeans mass, i.e. the Jeans mass for the average relative density $x = 1$.

Here we use the simplest definition of the Jeans mass: without turbulent pressure or rotation, because the gas has just been shocked and is dissipating its kinetic energy in a short time; without magnetic pressure; we will discuss the role of the magnetic field in such random flows in a subsequent work (Padoan & Nordlund 1997).

Using equations (1), (7), (8), (9) and (10) we get the protostar MF:

$$F(M)\,\mathrm{d}M=\frac{2B^2}{(2\pi\sigma^2)^{0.5}}M^{-3}\exp\left[-\frac{1}{2}\left(\frac{2\ln M-A}{\sigma}\right)^2\right]\mathrm{d}M \quad (11)$$

where M is in units of $\mathrm{M}_\odot$, and:

$$A=2\ln B-\overline{\ln x} \quad (12)$$

One can also express the MF in terms of the average Jeans mass, rather than of $\mathrm{M}_\odot$:

$$F(M/B)\,\mathrm{d}(M/B)=\frac{2}{(2\pi\sigma^2)^{0.5}}\left(\frac{M}{B}\right)^{-3}\times\exp\left[-\frac{1}{2}\left(\frac{2\ln(M/B)-|\overline{\ln x}|}{\sigma}\right)^2\right]\mathrm{d}(M/B)$$

A linear plot of the protostar MF is shown in Fig. 1, for $T=10$ K. One recognizes a long tail at large masses and an exponential cut-off at the smallest masses, inherited from the log-normal distribution of density. This shape is an important result, because most models for the origin of the stellar IMF are not able to reproduce the cut-off at the smallest masses that should be present in any reasonable IMF.

In the coming sections we will discuss the dependence of the MF on the average physical parameters of the star-forming gas, and we will then compare our results with the observations.

4 THE DEPENDENCE OF THE IMF ON THE PHYSICAL PARAMETERS

The protostar MF depends on the density distribution that arises from random supersonic motions, through a complex system of interacting shocks, and on the definition of the Jeans mass. The first dependence brings into the MF the dependence on the average temperature, T, and velocity dispersion of the flow, σ_v, through the rms Mach number of the flow, which is the only parameter of the density distribution in random supersonic flows. The dependence on the Jeans mass translates into a dependence of the MF on the average density, n, and on the temperature. Therefore our model for the MF may be applied to different sites of star formation, identified by their mean values of density, temperature and velocity dispersion.

In Figs 2, 3 and 4, we have plotted mass distributions for different values of the physical parameters. We have chosen to plot the exponent of the power-law approximation of the MF, rather than the actual MF. The exponent is defined as:

$$X=\frac{\partial\ln[F(\ln M)]}{\partial\ln M}=\left(\frac{2A}{\sigma^2}-3\right)-\frac{4}{\sigma^2}\ln M \quad (13)$$

The Salpeter MF has $X=-1.35$, and the Miller–Scalo MF (Miller & Scalo 1979) has $X=-1.0-0.43\ln M$, where M is in units of $\mathrm{M}_\odot$.

The most probable stellar mass per logarithmic mass interval, that is the stellar mass that contributes most to the MF, is defined by $X(M_{\max})\equiv0$, along the curves $X(M)$ plotted in the figures.

In Fig. 2 we see that a growing T produces a flattening of the MF at large masses, and a growth of $M_{\max}$, which is the typical stellar mass. The effect of the growth of density is illustrated in Fig. 3; its effect is the opposite of the effect of the temperature. Fig. 4 shows the dependence on velocity dispersion.

Note that, although the effect of temperature and density is qualitatively as expected from the definition of the Jeans mass, the effect of T on the MF is more complicated than through the Jeans mass, because T also affects the density distribution through the Mach number.

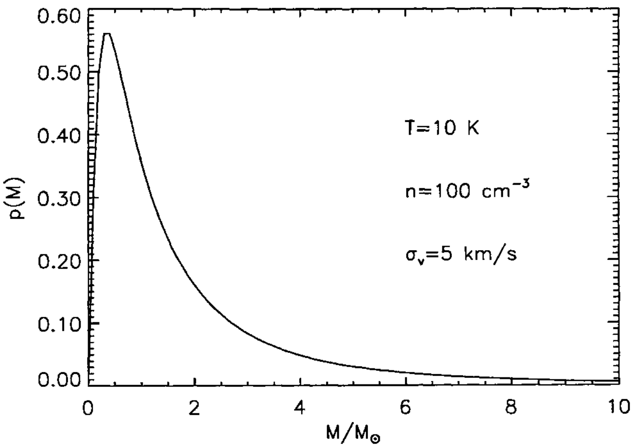

Figure 1. A linear plot of the theoretical MF. The linear shape is characterized by a maximum, with an exponential cut-off for smaller masses. The exponential cut-off is an important feature, because it could be identified in the observations without the ambiguities arising from uncertainties in the mass–luminosity relation.

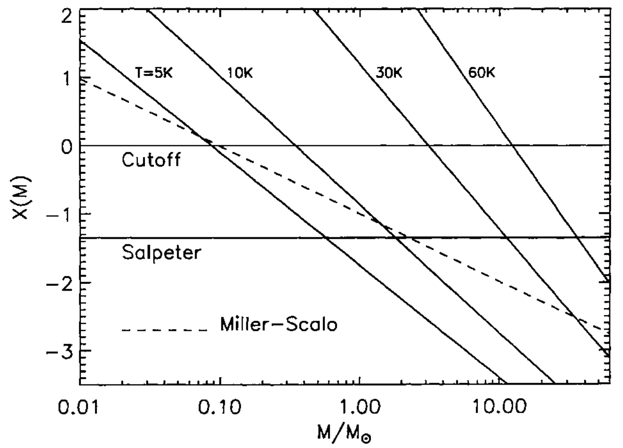

Figure 2. The power-law exponent of the theoretical MF is plotted versus the mass, for different temperatures. The Miller–Scalo MF (dashed line) is also plotted for comparison. The Salpeter value $X=-1.35$ and the cut-off value $X=0$ are also shown. The Miller–Scalo exponent is fitted by low temperatures at low masses, and by high temperatures at large masses. The mean density and velocity dispersion have been taken to be $n=1000\ \mathrm{cm}^{-3}$ and $\sigma_v=2.5\ \mathrm{km\ s}^{-1}$, typical of molecular cloud cores.

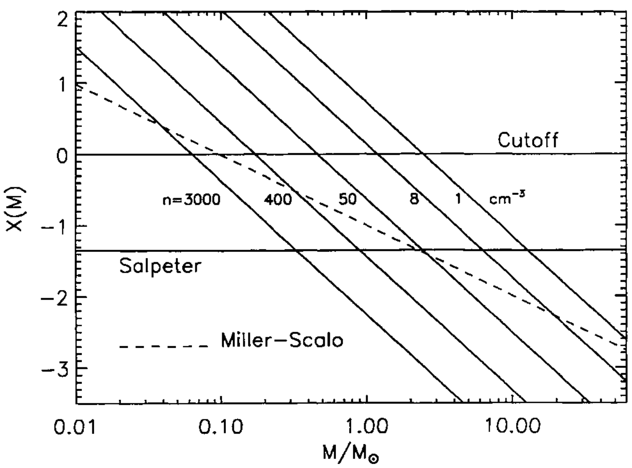

Figure 3. The same as in Fig. 2, but for different values of density. The temperature and velocity dispersion have been taken to be $T=10$ K and $\sigma_v=2.5$ km s^{-1}. The exponent $X(M)$ varies with mass always faster than in the Miller–Scalo MF, which is an indication that the Miller–Scalo MF emerges from a mixed population of stars formed in clouds with different temperatures.

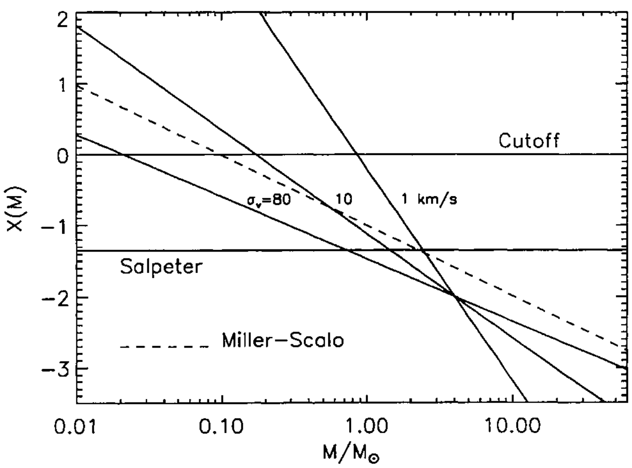

Figure 4. The same as in Fig. 2, but for different velocity dispersions. The temperature and density have been taken to be $T=10$ K and $n=1000$ cm^{-3}. Very large velocity dispersions, probably typical of large primordial clouds (protogalaxies and protoglobular clouds), can fit the Miller–Scalo MF very well, even for a single temperature.

We will now discuss the variation of the position of the cut-off (or of the typical stellar mass) with the physical parameters.

5 THE TYPICAL STELLAR MASS

The main result of a theory of star formation should be the prediction of the typical stellar mass. From this point of view, all models resulting in a power-law MF are unsuccessful, because power laws are featureless.

In the present work we have shown that the random supersonic motions present in molecular clouds produce a protostar MF with an exponential cut-off at the smallest masses, just below the most probable protostellar mass, $M_{\rm max}$.

The position of the maximum in the MF is given by imposing

$$X(M_{\rm max})\equiv 0$$

in equation (13). By using the definition of A from (12), of σ from (4), and of $M_{\rm J}$ from (9) and (10), we get:

$$M_{\rm max}=1\,{\rm M}_\odot B{\rm e}^{[-(1/2)\sigma^2]} \qquad (14)$$

where $1\,{\rm M}_\odot B$ is the average Jeans mass, i.e. the Jeans mass for the average density. Therefore:

$$M_{\rm max}=0.2\,{\rm M}_\odot\left(\frac{n}{1000\ {\rm cm}^{-3}}\right)^{-1/2}\left(\frac{T}{10\ {\rm K}}\right)^{2}\left(\frac{\sigma_v}{2.5\ {\rm km\ s}^{-1}}\right)^{-1}. \qquad (15)$$

We can read the result as a modified Jeans mass. The modification is quite important. In fact this modified Jeans mass is more sensitive to temperature than the traditional Jeans mass, and is also quite sensitive to the velocity dispersion σ_v.

This result is not surprising, because it looks like the Jeans mass at constant external pressure (Spitzer 1978, p. 241) if turbulent ram pressure is considered. Nevertheless it is an important result because it has been obtained from a realistic statistical description of random supersonic flows, which allows the prediction of the whole shape of the MF.

Another way to interpret the modified Jeans mass is to use equation (14), and substitute the standard deviation of the logarithmic density distribution, σ, with the linear standard deviation, $\sigma_{\rm linear}$, from equations (4) and (5). We obtain:

$$M_{\rm max}=\frac{1\,{\rm M}_\odot B}{\sigma_{\rm linear}} \qquad (16)$$

where $\sigma_{\rm linear}$ is about one half of the rms Mach number of the flow (cf. equation 5), and B is the Jeans mass for the mean density, in ${\rm M}_\odot$. Therefore we may conclude that *the most probable Jeans mass is equal to the Jeans mass for the mean density divided by half of the rms Mach number.* As an example, a typical molecular cloud with rms Mach number of about 10, and with a Jeans mass of the mean density of about $1\,{\rm M}_\odot$, has a most probable Jeans mass of $0.2\,{\rm M}_\odot$.

In Fig. 5, we show contours of constant $M_{\rm max}$, on the plane $n-T$, for $\sigma_v=3$ km s^{-1} typical of molecular clouds.

6 THE OBSERVED IMF

In Fig. 6 the theoretical MF for $T=10$ K (dashed line) is compared with the Miller–Scalo MF (MSMF) (dotted line). The shape of the theoretical MF is different from the shape of the MSMF. In fact, for the typical parameters of MCs, or of MC cores, the MF is always less broad than the MSMF. On the other hand, the models with low T (say 5 K) give the correct slope for low masses, while the models with high T (say 40 K) give the correct slope for the large masses.

Therefore the MSMF can be reproduced only if the solar neighbourhood stars are assumed to be born in clouds with temperatures in the range 5–40 K, which is a reasonable assumption, since these temperature values are measured in cloud cores. It is likely that the solar neighbourhood stars are a mixed population coming from different cloud cores, or even from different giant molecular cloud complexes,

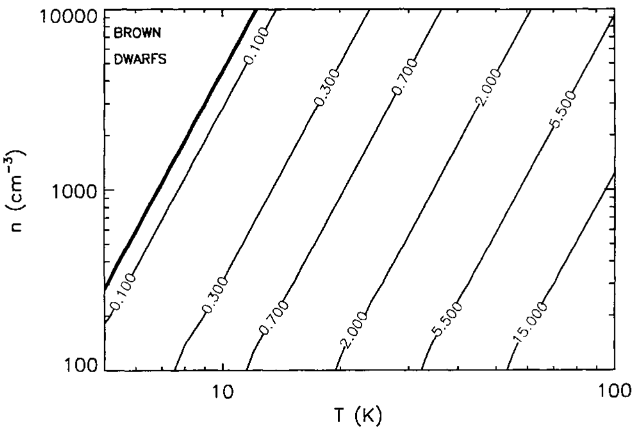

Figure 5. Lines of constant value of the cut-off, or typical stellar mass, in $M_{\odot}$, in the density–temperature plane. The velocity dispersion is $\sigma_v = 3.0$ km s^{-1}, typical of molecular clouds.

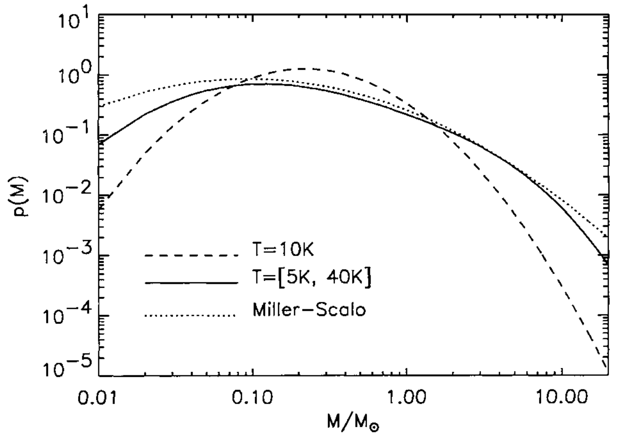

Figure 6. Log–log plot of the theoretical MF (dashed line), for a temperature $T = 10$ K. The Miller–Scalo MF is plotted for comparison (dotted line). The single temperature MF cannot be made to fit the Miller–Scalo MF. Once the theoretical MF is generated from a distribution of temperatures in the range 5–40 K (continuous line), it is practically coincident with the Miller–Scalo MF.

with temperatures in the range observed in present-day molecular clouds.

To illustrate the origin of a MF that contains a mixed population, coming from clouds with different temperatures, we integrate our theoretical MF along a temperature distribution, $g(T)\,dT$:

$$F_{\text{mixed}}(M)\,dM = \int_T F(M, T)\,dM g(T)\,dT.$$

Fig. 6 shows the result of the temperature integration (continuous line). The temperature distribution has been taken to be $g(T) \propto T^{-1}$, which means that there are more cold clouds than warm ones.

One can see that the temperature integration improves the shape of the single temperature MF, making the theoretical MF practically coincident with the MSMF.

We may therefore conclude that the model is consistent with the MSMF, as long as most of the solar neighbourhood stars are formed in molecular clouds similar to the ones that are the sites of present-day star formation, with temperatures between 5 and 40 K.

It has been claimed by many authors, on both theoretical and observational grounds, that the IMF in starburst regions is more 'massive' than in the solar neighbourhood. Models of the stellar populations in starbursts suggest an MF with the low-mass cut-off at a few $M_{\odot}$ (e.g. Augarde & Lequeux 1985; Doane & Mathews 1993; Riecke et al. 1993; Doyon, Joseph & Wright 1994).

These 'massive' MFs are in agreement with our theoretical prediction. In fact, a value of 4 $M_{\odot}$ is predicted for the cut-off in the MF, for $T \approx 60$ K, which is reasonable in environments with strong UV and X-ray radiation fields, and with enhanced (even by a factor 100) cosmic-ray flux. We also predict a slope of the MF considerably smaller than in the MSMF. For example, $X = -0.9$ [the value found by Malumuth & Heap (1994) in the core of 30 Doradus, R136a, which is a local example of a starburst event] is predicted by the model for $T = 60$ K, or slightly warmer (Fig. 2).

Padoan, Jimenez & Jones (1997b) have studied the hypothesis of a primordial origin of GCs, by applying the present model of star formation to protoglobular clouds of a few 10^8 $M_{\odot}$ in baryons. In their model the GCs originate from the star formation process in the core of the large cloud, at density $n \approx 10^4$ cm^{-3} and temperature $T \approx 100$ K (due to H_2 cooling), while most of the halo stars are the stars formed in the rest of the protoglobular cloud, which does not result in a bound system. For the halo stars the parameters are $n = 250$ cm^{-3} and $T = 100$ K. The assumed velocity dispersion is some fraction of the virial velocity, $\sigma_v \approx 50$ km s^{-1}.

It is found that the GC MF matches the Miller–Scalo MF very well. In particular, the exponent of the MF, in the interval [0.1, 0.6] $M_{\odot}$, is X = [0.5, −0.5], in agreement with the most recent results on the MF in NGC 6397 (Paresce, De Marchi & Romaniello 1995; d'Antona & Mazzitelli 1996) and in contrast with previous results (Fahlman et al. 1989; Richer et al. 1990, 1991).

7 DISCUSSION

A purely statistical description of the origin of the IMF has been given by several authors, as an attempt to model the process of gravitational fragmentation (e.g. Auluck & Kothari 1954, Kruszewski 1961, Kiang 1966, Reddish 1962, 1966, Fowler & Hoyle 1963, Belserene 1970, Larson 1972, Elmegreen & Mathieu 1983, Zinnecker 1984, Di Fazio 1986).

Other works tried to relate the MF of molecular cloud cores to the stellar MF [see Zinnecker (1993) for a discussion].

Most of the physical models for the origin of the IMF have been based on the concept of opacity-limited fragmentation (Silk 1977a,b; Yoshii & Saio 1985, 1986).

In all the cited models, the presence of random supersonic motions in the clouds is not taken into account. Models where such motions are considered in a semi-empirical way are Myers & Fuller (1993) and Silk (1995). The effect of turbulent motions are also considered by Arny (1971) as a source of internal pressure, and by Hunter & Fleck (1982) as a source of compression.

Takebe, Unno & Hatanaka (1962) assumed, as in the present work, that the mass function is set by the distribution of the Jeans mass in the cloud, but, missing a physical model for the cloud structure, they inferred this from the IMF.

In the present work, as in Padoan (1995), we suggest that all stars are formed as a consequence of turbulent fragmentation, that is the fragmentation due to a complex system of strong interacting shocks, formed in a field of random supersonic motions.

Such an approach has been made possible only very recently, thanks to new numerical simulations of 3D highly supersonic magnetohydrodynamic random flows (Nordlund & Padoan, in preparation), such as the ones observed in molecular clouds, and to the recognition of their observational counterpart (Padoan, Jones & Nordlund 1997; Padoan & Nordlund 1997).

We have seen that the theoretical MF depends on the values of temperature, density and velocity dispersion, averaged over the large-scale star-forming system. Since the velocity dispersion and the density can be very different, when measured at different scales, the application of the present model might seem ambiguous: how do we define the scale over which to perform the average of the physical parameters?

This, in fact, is not a problem, because one is likely to find the same IMF when the average is performed on different scales. The reason is that in general the star-forming gas has a hierarchical structure, such that the average density on a large scale is smaller than the density on small scales. The velocity, instead, grows with the scale.

As an example we consider the well-known scaling relations for the ISM (Larson 1979, 1981; Leung, Kutner & Mead 1982; Myers 1983; Quiroga 1983; Sanders, Scoville & Solomon 1985; Dame et al. 1986; Falgarone & Pérault 1987; Fuller & Myers 1992), which are approximately:

$$n(L) \propto L^{-1}$$

where $N(L)$ is the density averaged on the linear scale L, and:

$$\sigma_v(L) \propto L^{1/2}$$

where $\sigma_v(L)$ is the velocity dispersion averaged on the scale L.

We can now see how the typical stellar mass changes, when the average is performed on different scales. Equation (15) becomes:

$$M_{\rm max} \approx 0.1\,{\rm M}_\odot \left(\frac{T}{10\,{\rm K}}\right)^2 \qquad (17)$$

that is the dependence on velocity dispersion and density cancel each other.

Therefore, the scalings found in the ISM are such that performing the average on a molecular cloud core of 10 $M_\odot$, or on the whole giant molecular cloud complex of $10^6\,M_\odot$, gives a prediction, for the typical stellar mass, that is the same. Nevertheless, the shape of the distribution is affected by the choice of the velocity dispersion, in the sense that the MF becomes broader when the velocity dispersion is increased. Therefore some care must be taken when estimating the velocity dispersion of the star formation site.

Note that the scaling (17) does not apply to primordial clouds where globular clusters are formed, however. The turbulent ram pressure is there much larger than in molecular clouds, but since the temperature is also larger (H_2 cools the gas down to 100 K), a characteristic stellar mass close to the one in molecular clouds is obtained.

As we discussed above, the protostar MF can be obtained directly from the density distribution of the gas, because most high-density structures formed in the supersonic random flow are larger than their Jeans mass. This does not imply that the expected star formation efficiency of molecular clouds should be close to 100 per cent. In fact, it takes about two dynamical times before the log-normal density distribution is achieved, that is about 10^7 yr for a typical molecular cloud of $10^5\,M_\odot$. It is therefore possible that the first supernova explosions are able to disrupt the clouds before a large fraction of protostars are formed.

Moreover, it is well known that not all the gas that starts to collapse into a protostar will finally accrete on to the star; some fraction of it will be expelled by stellar winds. Although we have not gone into these details in the formulation of the MF, it is clear that this process alone can further reduce the star formation efficiency.

Finally, some parts of the cloud are strongly magnetized, and therefore their collapse may be hindered, or delayed, until the cloud is disrupted by supernova explosions. This influence of magnetic fields is not discussed here, but it is the subject of another work (Padoan & Nordlund, in preparation).

8 SUMMARY AND CONCLUSIONS

In the present work we have proposed a new physical model for the origin of the stellar IMF. The model is based on a new statistical description of star formation on a large scale, which focuses on the importance of random supersonic flows observed in the sites of star formation.

Recent numerical and observational results, concerning the density distribution that arises from random supersonic motions, are implemented in the theoretical model for the MF of protostars. The main results of the present work are as follows.

(i) The MF is quantified without free parameters, with its dependence on the mean temperature, density and velocity dispersion of the star-forming gas.

(ii) The shape of the protostar MF has a single maximum, a long tail of massive stars, and an exponential cut-off below the maximum. Such a shape is inherited directly from the density distribution in random supersonic flows.

(iii) The typical protostellar mass is

$$M_{\rm max} \approx 0.2\,{\rm M}_\odot \left(\frac{n}{1000\,{\rm cm}^{-3}}\right)^{-1/2} \left(\frac{T}{10\,{\rm K}}\right)^2 \left(\frac{\sigma_v}{2.5\,{\rm km\,s}^{-1}}\right)^{-1}$$

and

$$M_{\rm max} \approx 0.1\,{\rm M}_\odot \left(\frac{T}{10\,{\rm K}}\right)^2$$

using the ISM scaling laws.

(iv) A Miller–Scalo IMF is predicted for the solar neighbourhood stars, if they are formed in molecular clouds, similar to the ones observed in the sites of present day star formation, with temperatures in the range 5–40 K.

(v) Globular clusters are expected to have a MF similar to the Miller–Scalo MF, with a typical stellar mass of 0.1 $M_{\odot}$.

(vi) Starburst regions should have flatter IMFs, with a more massive cut-off, because of their higher mean temperature.

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