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Report No. 1970 22 January 1971

THE UNSTEADY HYDRODYNAMICS AND CONTROL OF HYDROFOILS NEAR A FREE SURFACE

Joseph I. Smullin
Erich K. Bender

Final Report — 1 August 1969 - 30 April 1970
Contract No. N00014-70-C-0032

Department of Defense Distribution Statement No. 1

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Naval Ship Research and Development Center
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Attention: Subprojects S-46-06
Task 1700

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ABSTRACT

In this Report we discuss (1) the development of a digital computer program to compute hydrofoil loads and (2) some aspects of hydrofoil control. The program computes the lift, pitching moment, and flap hinge moment on a two-dimensional hydrofoil with a trailing edge flap operating near a free surface with waves. The computational approach involves the numerical solution of an integral equation relating an upwash distribution to a kernel function and pressure distribution. The pressure distribution is expanded in a truncated Glauert series, the integration is carried out numerically using a Gaussian quadrature, and the coefficients of the Glauert series are evaluated by a minimum error collocation method.

The control problem investigated involves the positioning of a pivoted hydrofoil by means of a servo-controlled trim tab. When the foil is pivoted at its quarter chord and control is implemented solely by means of a servo tab, the system is virtually uncontrollable. However, by pivoting the foil off the quarter chord point or by augmenting the servo tab with a servo attached directly to the foil, the system can be controlled.

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1. INTRODUCTION

One of the principal features of a hydrofoil craft with fully submerged foils is its ability to maneuver in a seaway with greater isolation from the sea surface than virtually every other type of (surface) vessel. A hydrofoil craft is able to "platform" or traverse the short waves of a rough sea without undergoing significant vertical motion and to "contour" over high-amplitude long wavelength swells. During turning maneuvers the craft leans or banks into the turn much like an aircraft or bicycle, thereby maintaining a shipboard environment in which the lateral loads are minimal. This is particularly desirable for personnel on board.

To control a hydrofoil craft, the foils must be continuously adjusted. When the craft is travelling a straight course at a constant mean height in waves, the orbital motion of the water results in a time-varying upwash and lift fluctuations on the foil. Typically, the foil angle of attack or the deflection of flaps is varied to counter the lift fluctuations due to wave motion. Foils or foil control surfaces must be similarly moved to contour over waves or to make coordinated (i.e., properly banked) turns.

Varying the foil angle of attack to control lift forces requires especially large, heavy, special-purpose servomechanisms. These servos require a great deal of power (as much as 800 hp) and high-pressure, high-volume hydraulic lines which are often hazardous. In order primarily to reduce the servomechanism requirements, we have undertaken a study of the unsteady hydrodynamics and control of hydrofoils.

The four hydrofoil configurations we studied are illustrated in Fig. 1. Lift on the basic foil, illustrated as configuration 1, is

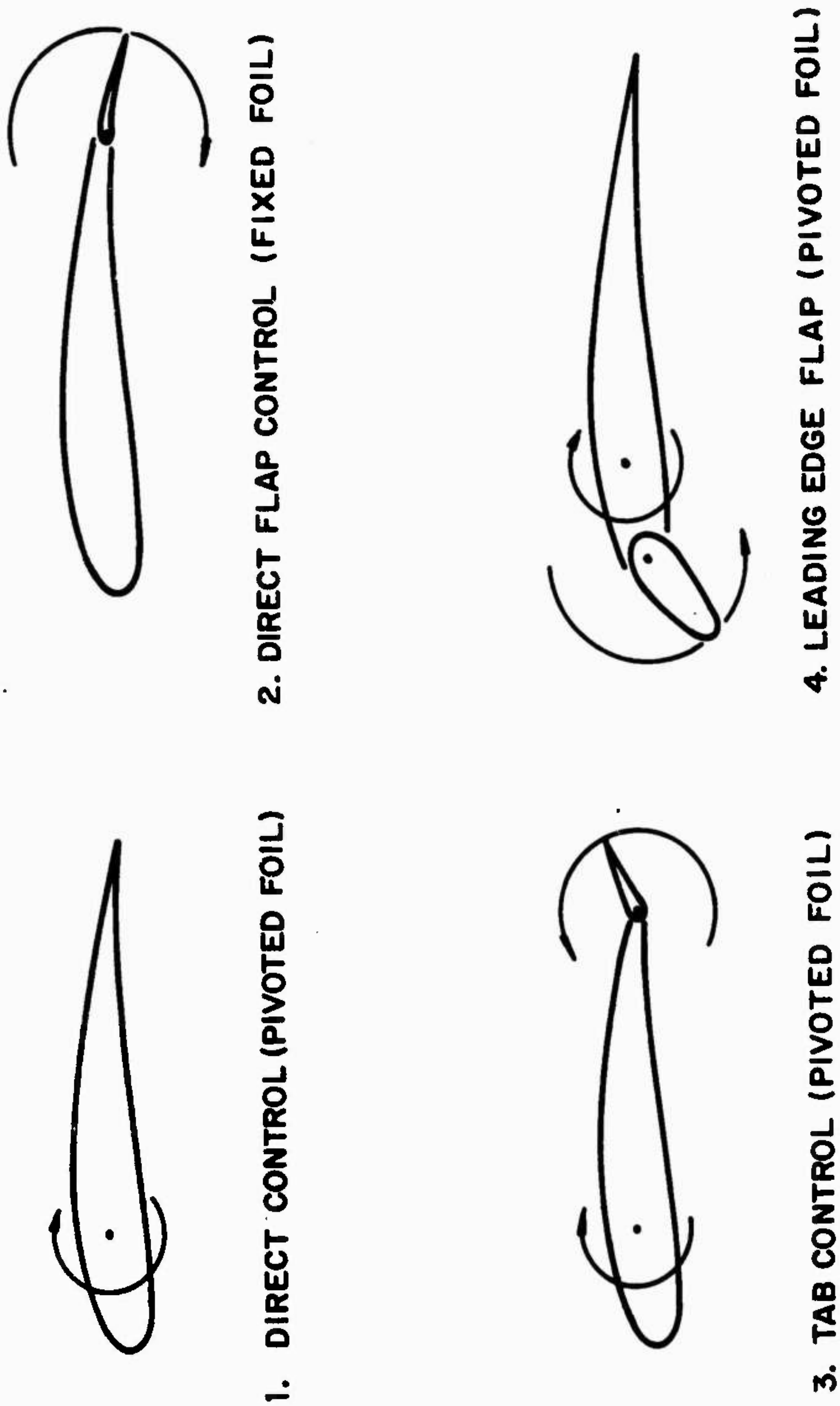

FIG.1 FOIL CONFIGURATIONS

controlled by varying the angle of attack of the entire foil. The foil in configuration 2 is rigidly attached to the craft and lift is controlled by deflecting a trailing-edge flap. Both of these configurations are presently used on hydrofoil craft. Configuration 3 uses a trailing-edge tab to rotate the foil about a pivot point. This configuration has the potential for very low power consumption but possesses certain inherent controllability problems. The foil in configuration 4 uses a leading-edge flap to rotate the foil. It has potentially low-power requirements and ease of control but is probably more susceptible to damage by impact with submerged debris.

The greatest part of this Report deals with the development of numerical methods and a digital computer program to determine the unsteady loads on a hydrofoil with a flap operating in waves in the vicinity of a free surface. This represents an extension of an earlier work [1] which did not include the effects of a flap. We do not explicitly develop results for a leading-edge flap; however, this configuration is simply the superposition of a rigid foil rotation plus a flap deflection. We also discuss a study of the control of hydrofoils, with emphasis on the tab-control system described by configuration 3.

2. UNSTEADY HYDRODYNAMICS

2.1 Hydrodynamic Coefficients

Assuming that the flow/foil interaction is linear (which is valid for surface wave heights that are small compared with the foil depth), the net forces and moments are equivalent to the sum of the loads generated by the upwash on a foil moving steadily at constant speed U through rough water plus the loads caused by the heave and rotary motion of a foil moving through calm water. The lift and moments due to upwash velocity V are given by

$$L_V = C_{LV}(k)V(k) \tag{1a}$$

$$M_{\alpha V} = C_{M_\alpha V}(k)V(k) \tag{1b}$$

$$M_{\beta V} = C_{M_\beta V}(k)V(k) \tag{1c}$$

where most of the variables are defined in Fig. 2 and k is the reduced frequency ($k = \omega b/U$), nondimensionalized by the semichord b and by U.

The forces and moments owing to the motion of the foil are

$$L = C_{L\alpha}(k)\alpha + C_{L\beta}(k)\beta + C_{L\dot{h}}(k)\dot{h} \tag{2a}$$

$$M_\alpha = C_{M_{\alpha\alpha}}(k)\alpha + C_{M_{\alpha\beta}}(k)\beta + C_{M_{\alpha\dot{h}}}(k)\dot{h} \tag{2b}$$

$$M_\beta = C_{M_{\beta\alpha}}(k)\alpha + C_{M_{\beta\beta}}(k)\beta + C_{M_{\beta\dot{h}}}(k)\dot{h} \; . \tag{2c}$$

The purpose of our hydrodynamic analysis is to evaluate the twelve functions $C_{ij}(k)$ in Eqs. 1 and 2. This has been done by

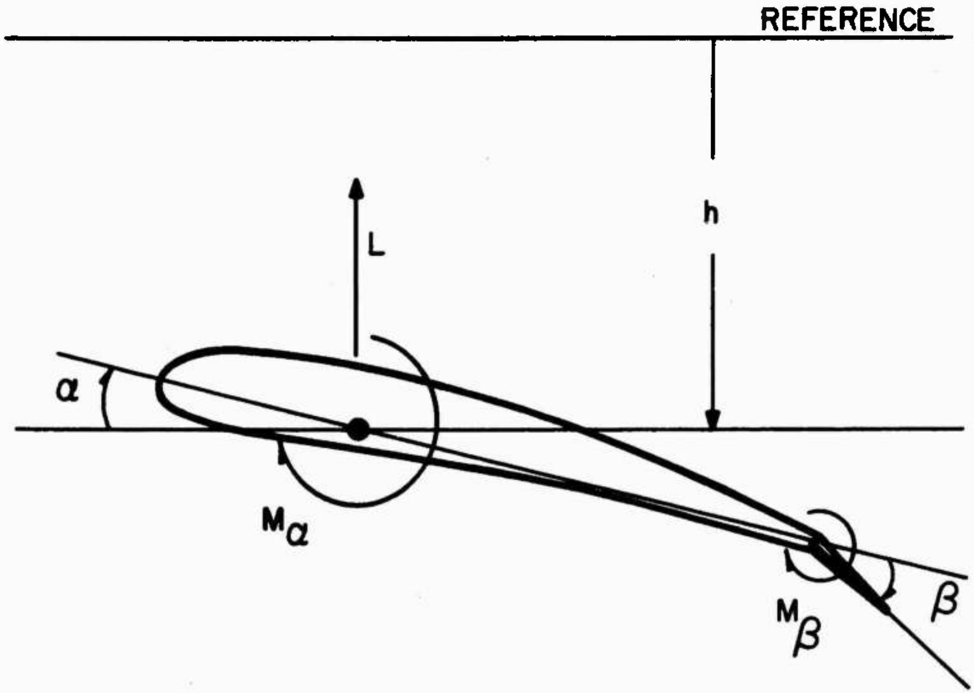

FIG.2 HYDROFOIL NOMENCLATURE

modifying an existing digital program [1], which computes the unsteady lift and moment on a two-dimensional foil near the surface, to include the effects of a flap and the upwash caused by surface waves.

2.2 Hydrodynamic Analysis

The unsteady pressure distribution over the surface of a hydrofoil is related to the upwash by an influence function, the kernel function [1], which is a complex function of reduced frequency, depth, and Froude number. For a foil which has its leading edge at x = -1 and its trailing edge at x = +1 as shown in Fig. 3, this relation can be written as:

$$V(x) = \frac{1}{2\pi} \int_{-1}^{1} \Delta P(\xi) K(x-\xi) d\xi \ . \qquad (3)$$

Here, ΔP is the pressure difference across the foil, V is the upwash nondimensionalized by the velocity U, and K is the kernel.

For a foil operating at finite depth, the kernel function must satisfy the boundary conditions of the free surface, as well as those of the foil itself. The free surface is modeled by including a virtual image of the foil pressure distribution on the opposite side of the surface. During operation at low Froude numbers, the moving pressure disturbance from the foil causes a large enough gravity wave to affect the kernel significantly. The complex kernel as a function of depth and Froude number for two-dimensional flow is developed in detail by Widnall [1]; the result is given here in Appendix A. For a rigid foil oscillating in heave or pitch, the upwash V(x) is known, as it is simply the local vertical velocity of the foil. That is:

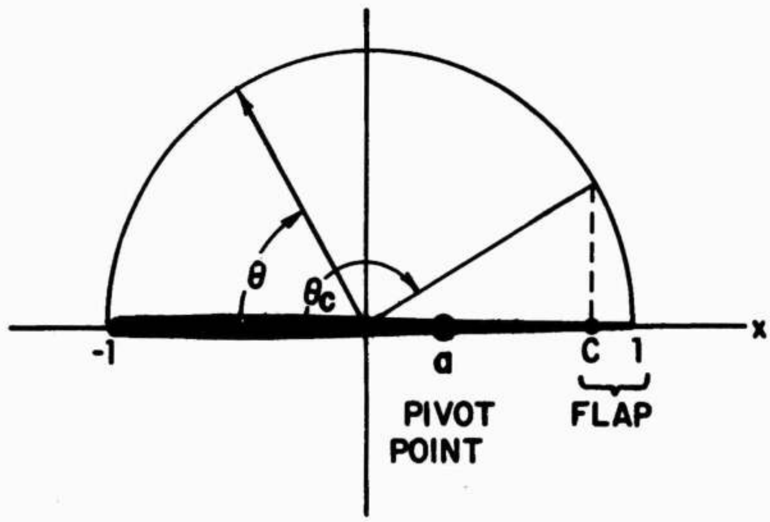

FIG.3 HYDROFOIL COORDINATES USED IN UNSTEADY HYDRODYNAMIC COMPUTATION

heave: $V(x) = \dot{h} = \left(\bar{h}\, e^{i\omega t}\right)$

pitch: $V(x) = \alpha + \dot{\alpha}(x-a)\,\frac{b}{U} = \left(\bar{\alpha}\, e^{i\omega t}\right)[1 + ik(x-a)]$.

Here, $\dot{h}$ is nondimensionalized by U and a is the location of the foil pivot point (see Fig. 3). With the kernel and upwash known, Eq. 3 can be solved for $\Delta P(x)$.

Numerical technique for solving ΔP

For attached flow on the hydrofoil, the Kutta condition at the trailing edge must be satisfied. This boundary condition is satisfied by writing the pressure distribution as a Glauert series:

$$\Delta P = \sum_{n=1}^{\infty} p_n a_n(x) \tag{4}$$

where

$$a_n = \begin{matrix} \cot(\theta/2) \\ \sin(n-1)\theta \end{matrix} \quad , \quad \begin{matrix} n = 1 \\ n \geq 2 \end{matrix}$$

and

$$x = -\cos\theta \ .$$

By combining Eqs. 3 and 4, the velocity may be written:

$$V(x) = \sum_{n=1}^{\infty} p_n \int_{-1}^{+1} a_n(\xi) K(x-\xi) d\xi \ . \tag{5}$$

If this series is truncated at NOLT terms* and the integration carried out, Eq. 5 can be written as

---

*We use somewhat cumbersome terms such as NOLT in this text to be consistent with expressions used in the computer program.

$$V(x) = \sum_{n=1}^{NOLT} p_n I_n(x) \tag{6}$$

where

$$I_n = \int_{-1}^{+1} a_n(\xi) K(x-\xi) d\xi \ .$$

If, in the numerical solution for ΔP, the velocity is specified at NP collocation points, there will be NP equations of the form of Eq. 6. Using a notation $A\{^B\}$, where { } is a matrix with A rows and B columns, this set of equations can be written

$$_{NP}\{V\}^{1} = _{NP}\{I_n\}^{NOLT} \ _{NOLT}\{p_n\}^{1} \ . \tag{7}$$

By making the number of collocation points, NP, larger than the number of terms in the Glauert series, NOLT, the set of equations is overdetermined and the pressure modes can be solved for by a method which minimizes the mean square error of the resultant downwash at the collocation points. The coefficients p_n which minimize the mean square error satisfy the following equation:

$$\frac{\partial}{\partial p_n}\left\{\sum_{r=1}^{NP}\left[V(x_r) - \sum_{n=1}^{NOLT} p_n I_n(x_r)\right]^2\right\} = 0 \qquad \text{for } n = 1,2,\cdots,NOLT \tag{8}$$

The result [2] is

$$\{p_n\} = [\{\bar{I}_n\}^T \{I_n\}]^{-1} \{\bar{I}_n\}^T \{V\} \tag{9}$$

where $\{\bar{I}\}^T$ is the conjugate transpose of $\{I_n\}$.

The lift and moment acting on the foil can be found by integrating the pressure and its first moment over the chord of the foil. The lift and the moment about the foil quarter-chord in terms of the Glauert series are given by

$$C_L = \pi(p_1 + 1/2\ p_2) \tag{10a}$$

$$C_M = \frac{\pi}{8}(-p_2 + p_3)\ . \tag{10b}$$

<u>Foil with trailing-edge flap</u>

Addition of a hinged trailing-edge flap to the hydrofoil expands the set of unknown force and moment parameters for unsteady motions to include the hinge moment due to pitch and heave and the effect of flap oscillation on lift, and moments at the quarter chord and flap hinge. (The geometry of the flapped hydrofoil is shown in Fig. 2.)

<u>Hinge moment</u>

The moment at the flap hinge can be computed by integrating the moment of each of the pressure modes over the flapped portion of the foil chord, $\Theta_c \to \pi$. The result is:

$$C_{M_\beta} = p_1 \int_{\Theta_c}^{\pi} \cot\frac{\Theta}{2}\,(\cos\Theta - \cos\Theta_c)\sin\Theta d\Theta$$

$$+ \sum_{n=2}^{NOLT} p_n \int_{\Theta_c}^{\pi} \sin(n-1)\Theta(\cos\Theta - \cos\Theta_c)\sin\Theta d\Theta$$

$$= p_1 \int_{\Theta_c}^{\pi} (1+\cos\Theta)\cos\Theta d\Theta$$

$$- p_3\left[\frac{\pi-\Theta_c}{4} - \frac{\sin(4\Theta_c)}{16}\right] + p_2\,\frac{\sin^3(\Theta_c)}{3}$$

$$+ \sum_{n=4}^{NOLT} p_n\ \frac{1}{4}\left[\frac{\sin(n-3)\Theta_c}{(n-3)} - \frac{\sin(n+1)\Theta_c}{n+1}\right]$$

$$+ \frac{1}{2}\,p_2\,\cos\Theta_c[\pi-\Theta_c-\sin\Theta_c\cos\Theta_c]$$

$$+ \sum_{n=3}^{NOLT} \frac{1}{2}\cos\Theta_c p_n\left[\frac{\sin n\Theta_c}{n} - \frac{\sin(n-2)\Theta_c}{n-2}\right]. \tag{11}$$

Oscillating flap

A foil with an oscillating trailing-edge flap has a pressure distribution with a logarithmic singularity at the flap hinge, which is fit by the pressure series [3].

$$\frac{p}{\rho U^2} = \sum_{n=1}^{\infty} p_n a_n + p_c a_c\ , \tag{12}$$

where

$$p_c = \frac{\bar{\beta}}{\pi \sin\theta_c} \qquad \bar{\beta} = \text{amplitude of angular deflection of flap}$$

$$a_c = \sin\theta \ \ln(x-c)^2$$

and $\sum_{n=1}^{\infty} p_n a_n$ is the Glauert series.

A relation for the oscillating flap analogous to Eq. 5 is,

$$V = \sum_{n=1}^{\infty} p_n \int_{-1}^{1} a_n K d\xi + p_c \int_{-1}^{1} a_c K d\xi \ . \tag{13}$$

This can be written:

$$V^* = \sum_{n=1}^{\infty} p_n \int_{-1}^{1} a_n K d\xi \tag{14}$$

with the "equivalent" upwash V*,

$$V^* = V - p_c \int_{-1}^{1} a_c K d\xi \ , \tag{15}$$

which can be calculated directly since p_c is known. Using this equivalent upwash, the Glauert pressure modes can be computed in the same manner as was used for the plain foil, Eq. 9.

In order to find the equivalent upwash, the integration over the chord for the product of the flap hinge pressure mode times the kernel must be performed. This integral,

$$\int_{-1}^{1} a_c K d\xi = \int_{-1}^{1} \sin\theta \ \ln(\xi-c)^2 \, K d\xi \ , \tag{16}$$

has a logarithmic singularity at the hinge point which can be isolated by writing

$$\int_{-1}^{1} a_c K d\xi = \int_{-1}^{1} (\sin\Theta K - \sin\Theta_c K_c) \ln(\xi - c)^2 \, d\xi$$

$$+ \sin\Theta_c K_c \int_{-1}^{1} \ln(\xi - c)^2 \, d\xi \; . \tag{17}$$

Writing the kernel as $K(x-\xi) = \frac{-1}{2\pi}\frac{1}{x-\xi} + \overline{K}(x-\xi)$, where $\overline{K}$ is a function of frequency, depth, and Froude number, and integrating in the second term, Eq. 17 becomes

$$\int_{-1}^{1} a_c K d\xi = \int_{-1}^{1} H(\xi) d\xi + \sin\Theta_c \left[\overline{K}_c - \frac{1}{2\pi(x-c)}\right]$$

$$\times \left[-4 + 2c \ln\left(\frac{1+c}{1-c}\right) + 2 \ln(1-c)^2\right] \tag{18}$$

where

$$H(\xi) = \left\{\sin\Theta\left[\overline{K} - \frac{1}{2\pi(x-\xi)}\right] - \sin\Theta_c\left[\overline{K}_c - \frac{1}{2\pi(x-c)}\right] \ln(\xi - c^2)\right\}. \tag{19}$$

When the last singularity at $x = \xi$ is evaluated analytically, the equation can be written

$$\int_{-1}^{1} a_c K d\xi = \int_{-1}^{x-\varepsilon} N(\xi) d\xi + \int_{x+\varepsilon}^{c-\varepsilon} N(\xi) d\xi + \int_{c+\varepsilon}^{1} N(\xi) d\xi - Q(\xi) \qquad \text{for } x < c$$

$$\tag{20}$$

$$\int_{-1}^{1} a_c K d\xi = \int_{-1}^{c-\varepsilon} N(\xi)d\xi + \int_{c+\varepsilon}^{x-\varepsilon} N(\xi)d\xi + \int_{x+\varepsilon}^{1} N(\xi)d\xi - Q(\xi) \qquad \text{for } x > c \tag{21}$$

where

$$N(\xi) = H(\xi) + \frac{\sin\Theta}{2\pi(x-\xi)} \ln(x-c)^2 \tag{22}$$

$$Q(\xi) = -\frac{x}{4\pi} \ln(x-c)^2 + \sin\Theta_c \left[\overline{K}_c - \frac{1}{2\pi(x-c)}\right]$$
$$\times \left[-4 + 2c \ln\left(\frac{1+c}{1-c}\right) + 2 \ln(1-c^2)\right]. \tag{23}$$

The integrals in Egs. 20 and 21, having no singularities, may be evaluated by a simple numerical scheme.

The equivalent upwash, V* as given in Eq. 15, may now be solved, using the values for the upwash due to flap oscillation:

$$V = \begin{cases} 0 & -1 \leq x \leq c \\ -\beta - \dot{\beta}(x-c)b/U & c \leq x \leq 1 \end{cases}.$$

The remainder of the solution for the p_n pressure modes is the same as developed in Eqs. 4 – 9.

The lift and moment coefficients are the same as those expressed in Eqs. 10 and 11, plus the effect of integrating the flap pressure mode, p_c (Eq. 12), over the foil. An equivalent set of equations is

$$C_L = \pi(p_1 + \tfrac{1}{2}\, p_2) + p_c \left[\int_{-1}^{1} a_c d\xi \right] \tag{24a}$$

$$C_{M_\alpha} = -\frac{\pi}{8}\,(p_2 - p_3) - \frac{1}{2}\, p_c \left[\int_{-1}^{1} a_c \left(\xi + \frac{1}{2}\right) d\xi \right] \tag{24b}$$

$$C_{M_\beta} = -\frac{1}{2} \sum_{1}^{\infty} p_n \left[\int_{c}^{1} a_n \xi \, d\xi - c \int_{c}^{1} a_n d\xi \right]$$

$$- \frac{1}{2}\, p_c \left[\int_{c}^{1} a_c (\xi - c) d\xi \right] . \tag{24c}$$

The singularity in the lift coefficient integration can be isolated leaving a nonsingular integral (Eq. 24a) which can be evaluated numerically.

$$\int_{-1}^{1} a_c d\xi = \int_{-1}^{1} \sin\theta \, \ln(\xi - c)^2 \, d\xi$$

$$= \int_{-1}^{1} (\sin\theta - \sin\theta_c) \ln(\xi - c)^2 \, d\xi + \sin\theta_c \int_{-1}^{1} \ln(\xi - c)^2 \, d\xi$$

$$= \left[\int_{-1}^{c} + \int_{c+\varepsilon}^{1} \right] (\sin\theta - \sin\theta_c) \ln(\xi - c)^2 \, d\xi$$

$$+ \sin\theta_c \left[-4 + 2c \ln\left(\frac{1+c}{1-c}\right) + 2 \ln(1 - c^2) \right] . \tag{25}$$

The integral in Eq. 24c for the flap contribution to the quarter-chord moment is:

$$\int_{-1}^{1} a_c\left(\xi+\frac{1}{2}\right)d\xi = \int_{-1}^{1} a_c(\xi-c)d\xi + \left(c+\frac{1}{2}\right)\int_{-1}^{1} a_c d\xi$$

$$= \int_{-1}^{c-\varepsilon} \sqrt{1-\xi^2}\,\ln(\xi-c)^2\,(\xi-c)d\xi$$

$$+ \int_{c+\varepsilon}^{+1} \sqrt{1-\xi^2}\,\ln(\xi-c)^2\,(\xi-c)d\xi + \left(c+\frac{1}{2}\right)\int_{-1}^{1} a_c d\xi \ , \tag{26}$$

and the contribution to the flap hinge moment is simply

$$\int_{c}^{1} a_c(\xi-c)d\xi = \int_{c}^{1} \sqrt{1-\xi^2}\,\ln(\xi-c)^2\,(\xi-c)d\xi \ . \tag{27}$$

Unsteady lift in waves

Superposed on the hydrofoil steady velocity U are the vertical and horizontal components of the orbital motion of water associated with surface-wave propagation. Although the lift owing to the horizontal component of this orbital motion has been shown by O'Neill to be present for foils with a mean value of lift or camber [4], it is usually small and shall be neglected here. Let us evaluate the vertical component or upwash with respect to a foil moving in a seaway.

The upwash (nondimensionalized by U) at depth D associated with plane waves of amplitude A, traveling at an angle ϕ with respect to a stationary x',y' coordinate frame (see Fig. 4) is given by

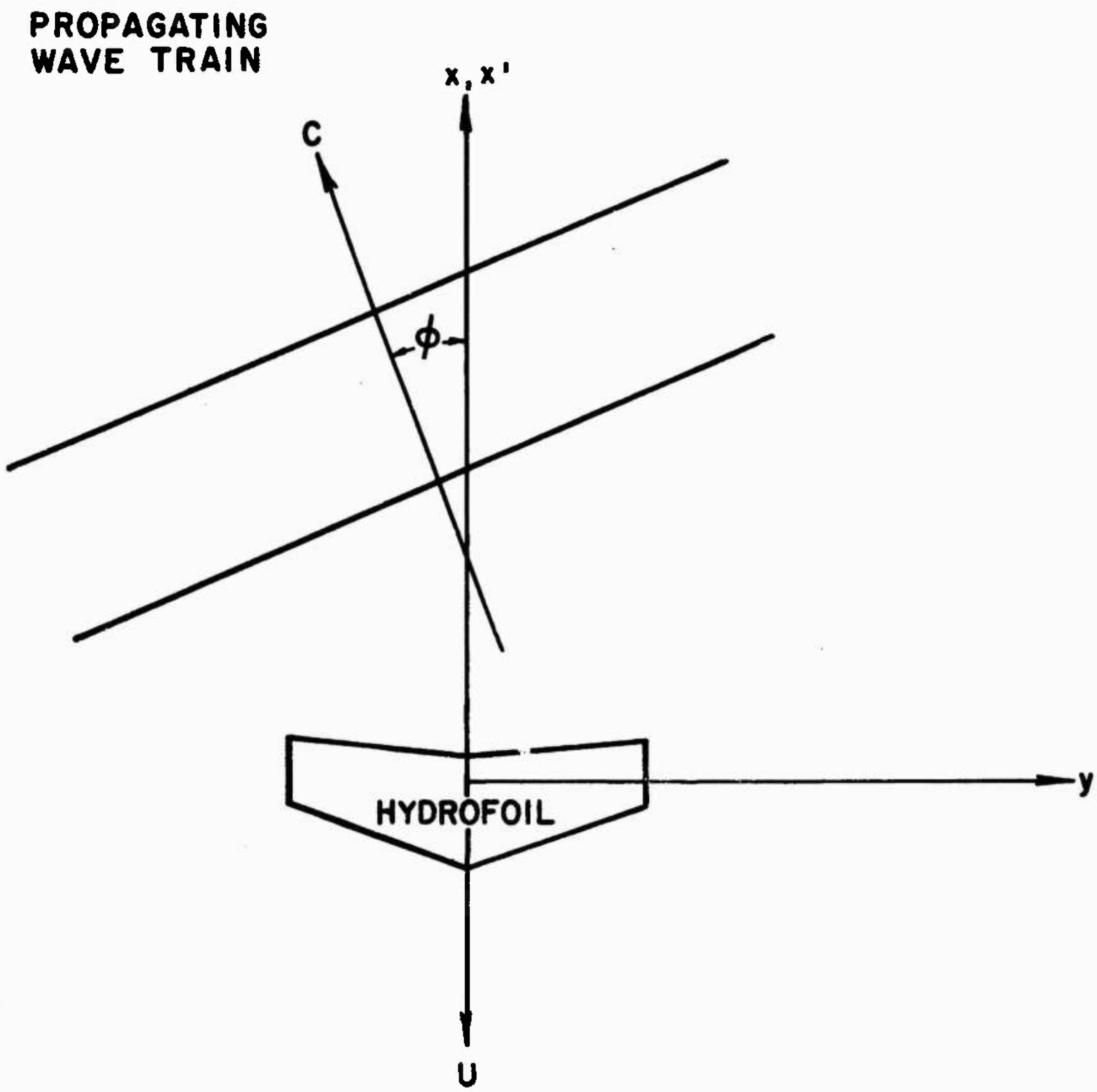

FIG.4 RELATION OF HYDROFOIL TO SURFACE WAVES

$$V(x'y't) = \frac{A_0\sqrt{k_w g}}{U}\, e^{-k_w D + i\,(\sqrt{k_w g}\; t\; -\; k_{x'}x' + k_{y'}y')} \tag{28}$$

where $k_{x'} = k_w \cos\phi$

$k_{y'} = k_w \sin\phi$

k_w is the wavenumber, 2π/wavelength, and g is the gravitational constant.

To account for ship velocity through the water, we may define a codirectional coordinate system moving at the speed of the ship along the *negative* x axis with velocity U and write

$$x = +\,x' - Ut\ . \tag{29}$$

Substituting this equation into Eq. 28 for V(x',y',t), and neglecting y' variations gives

$$V(x',t) = \frac{A_0\sqrt{k_w g}}{U}\, e^{-k_w D + i[(\sqrt{k_w g} + Uk_w\; \cos\phi)t\; -\; k_w x'\; \cos\phi]}\ . \tag{30}$$

The kx'cosϕ term describes the spatial variation of upwash which will depend on the angle of the foil with respect to the waves.

The lift associated with this upwash is solved in the same manner used for the heave and pitch upwash.

3. COMPUTER PROGRAMS

Two Fortran II programs for the calculation of the unsteady hydrodynamics of two-dimensional hydrofoils operating near a free surface are presented. The first program computes the unsteady lift, quarter chord moment, and flap hinge moment owing to foil pitch and heave, and to flap oscillations. The second program computes the unsteady lift and the moments at the flap hinge and the foil quarter chord for a hydrofoil platforming in waves. Both of the programs are based on an NSRDC program (see Ref. 1 for a description) which computes the unsteady hydrodynamics for a foil without a flap, oscillating in pitch and heave. Results are computed and plotted for specific cases and found to be in good agreement with theory.

The programs are written in Fortran II for use on the SDS 940 research computer in conjunction with the Bolt Beranek and Newman time sharing system.* The subroutines used are the same as those in the NSRDC version of the program by Widnall except for changes necessary in going from Fortran IV to II. These changes include the addition of an arctan function, THETA (X,Y) and, to reduce programming time and storage space, the substitution of a less sophisticated version of the exponential integral subroutine, EXPINT. In the subroutine for complex matrix inversion, CMPINV, the variable field, BLANK, has been broken down into its real number and interger components, PIVOT and INDEX. This was done because the SDS-940, unlike many other computers, allocates more space for the storage of real numbers than for intergers. The subroutine storage as changed, is still compatible with systems

*Number 1.85, using the 21 January 1969 Fortran operating system.

using the same amount of space for the storage of real and interger numbers.

3.1 Flap Program

Extending the NSRDC program to compute the flap hinge moment, foil pitch moment, and lift, owing to flap deflection, consists of two steps. The first is the integration of each term of the Glauert series over the flap to determine the moment about the hinge; the second step is the computation of the complex pressure distribution over the entire foil due to flap oscillation.

The integration for the hinge moment is done using the expression derived analytically in Eq. 11.

The computation of the pressure due to flap deflection, a_c in Eq. 12, is done in two separate steps. The first step is the calculation of the amplitude of the logarithmic pressure term for the flap (Eq. 12). The next step is the computation of the Glauert pressure series due to the equivalent upwash (Eq. 15). In order to calculate the lift and moment generated by the flap, a numerical integration of the flap mode must be done, as was seen in Eq. 20. This integration is done by a Gaussian quadrature [5] in the same manner as all the numerical integration in the original NSRDC program.

The program divides the specified number of collocation points evenly between the flapped and unflapped portions of the chord. In each portion of the foil the points are distributed sinusoidally in order to concentrate them at edges where the computed functions change most rapidly. The equations governing the distribution of collocation points are:

$$x_i = \frac{c-1}{2} - \left(\frac{1+c}{2}\right) \cos \frac{\pi i}{N} \quad \text{for } -1 < x < c$$

$$x_i = \frac{c+1}{2} - \left(\frac{1-c}{2}\right) \cos \frac{\pi i}{N} \quad \text{for } c < x < 1 . \qquad (31)$$

3.2 Wave Program

To compute the hydrodynamic coefficients of a hydrofoil operating in waves, we have expanded the program to determine the upwash on the foil as a function of the input parameters: foil depth, Froude number, heading angle (see Fig. 4), and wavelength. This is done by application of Eq. 30, modified to give the phase of the coefficients relative to the maximum vertical wave velocity at a specified point on the foil chord (e.g., quarter chord point). The computations for this upwash are handled in exactly the same manner as those for heave and pitch in the original program. The integral for flap hinge moment as a function of hinge location (Eq. 11) has been added. The method of collocation point location is the same as used for the flapped foil.

3.3 Program Flow Charts and Print-Out Listings

Appendix B contains functional flow charts of the programs for hydrodynamic coefficients of a foil with an oscillating flap and for a foil operating in waves. There has been an attempt here to identify variables by their symbolic names in the program and their name' 'om the theoretical development. Appendices C, D, and E contains listings of the two main programs plus a listing of the subroutines.

3.4 Input-Output

The inputs to the program are adapted to the BBN console-operated time sharing computer system. Some of the input that was introduced many times was stored on disk files for ease of operation. In a punch card system, this input would probably be more easily handled by reading the data from cards for each run. The inputs for the flap program, which follow the standard Fortran convention for interger and real number identification, are:

file NAME
(which reads from a file
NP, NOLT, IM, IFF, RF, FR, D)

The operator types in the values of the variables,
CFLAP, NFLP, NLE, NTE

using FORMAT (F 1Ø.4, 3I 1Ø).

The inputs to the wave program from file are the same as those for the flapped foil. The typed-in variables are,

WL, THET, FR, PPA, CFLAP, NLE, NTE

using

FORMAT (5 F1Ø.4, 2I4).

The Froude number, FR, which had been read in from file was typed in again to facilitate easier variation of this parameter.

Table 1 lists and describes the inputs to the both programs. Figures 5 and 6 are sample runs of the FLAP and WAVE programs.

The underlined portions were typed by the operator.

TABLE 1. Input for Flap and Wave Programs

NP	total number of collocation points (even number, greater than 2 NOLT)
NOLT	number of terms in the Glauert Series
IM	determine whether or not virtual image on opposite side of free surface is accounted for in kernel. Set IM = Ø to exclude free surface effects of image. Set IM = any integer other than zero to include free surface effects of virtual image.
IFF	=Ø: ($Fr = \infty$) Froude effect is not included in kernel ≠Ø: Froude effect is included in kernel
RF	reduced frequency ($k = [\omega b]/[U_0]$) of oscillation (heave, pitch or flap)
FR	Froude number, $F = (U_0)/(\sqrt{gb})$
D	depth in semichords
CFLAP	fraction of the foil occupied by flap
NFLP	number of terms in the integration for flap pressure term
NLE	number of terms in the integration for I_n in the region $\xi < x$ (see Eq. 6)
NTE	number of terms in the integration for I_n in the region $\xi > x$ (see Eq. 6)
WL	wavelength in semichords
THET	angle (radians) of foil heading
PPA	coordinate on foil to which wave phase is related (e.g., the PPA for the quarter chord is -0.5)

```
1 IMAGE IFF      K    FROUDE NUM  DEPTH IN SEMCHDS  NP   NOLT
    0    0     .2000    1.0000     1.0000              20  6

   FLAP CHORD      .25      FRACTION OF AIRFOIL CHORD
 -.14550E  1  .37504E  0 -.15301E  1  .28404E  0 -.35105E  0  .20663E  0

  .13395E- 3 -.39738E  0  .39955E- 1 -.79736E  0 -.74317E  0 -.15215E  0

  .14821E- 3  .52333E- 4 -.19511E- 1  .74768E- 4  .24946E  0  .11459E  0

 -.10429E- 3  .20587E- 4 -.10917E- 3 -.30692E- 5 -.27619E- 1 -.58644E- 1

 -.54742E- 4  .13830E- 4 -.59580E- 4  .10334E- 4 -.13913E  0  .15656E- 1

 -.23280E- 4  .57129E- 5 -.24219E- 4  .53858E- 5  .14798E  0  .18182E- 1
0     HEAVING    OSCILLATION

CLR= -4.5709    CLI=   .5540    CMR=   .0000    CMI=   .1561
CHR=   .0257    CHI=   .0059
0 COMPARISON WITH CORNELL HR= -.72749362 HI=,-.01182340
0   PITCHING OSCILLATION

CLR= -4.7441    CLI=  -.3602    CMR=  -.0234    CMI=   .3132
CHR=   .0248    CHI=   .0201
      FLAP OSCILLATION

CLR= -2.8148    CLI=   .4102    CMR=   .6462    CMI=   .1047
CHR=   .0528    CHI=   .0100
   -4.57087051      .55401701     .00000560     .15607327 .201.001.0020 6
   -4.74411224     -.36015309    -.02335243     .31315113 .201.001.0020 6
```

FIG. 5 OUTPUT FOR SAMPLE PROBLEM OF HYDROFOIL WITH OSCILLATING FLAP

```
1 IMAGE  IFF      K    FROUDE NUM   DEPTH IN SEMCHDS   NP    NOLT
    0     0     .400110000.0000     1.0000              20   6


   FLAP CHORD       .25       FRACTION OF AIRFOIL CHORD
WAVE LENGTH=  15.708 SEMI-CHORDS.  HEADING=     .000

 -.12493E  1  .32670E  0 -.13803E  1  .22688E  0 -.72378E  0  .26071E  0

  .11264E- 2 -.79565E  0  .16081E  0 -.15951E  1  .88772E- 3  .24788E- 2

  .12775E- 2  .34434E- 4 -.77353E- 1  .30591E- 3 -.70198E- 4  .26125E- 4

 -.17977E- 3 -.80877E- 5 -.18853E- 3 -.20852E- 3 -.10482E- 3  .31185E- 4

 -.98315E- 4  .24245E- 4 -.11936E- 3  .15447E- 4 -.51934E- 4  .19133E- 4

 -.37201E- 4  .13129E- 4 -.41007E- 4  .16225E- 4 -.22330E- 4  .70627E- 5
O     HEAVING     OSCILLATION

CLR= -3.9230     CLI=   -.2235     CMR=    .0001     CMI=    .3125
CHR=    .0221     CHI=    .0193
O COMPARISON WITH CORNELL HR= -.62438072 HI=,-.23559697
0   PITCHING OSCILLATION

CLR= -4.0837     CLI= -1.7929     CMR=   -.0935     CMI=    .6265
CHR=    .0155     CHI=    .0462

IN WAVES.

CLR= -2.2724     CLI=    .8229     CMR=   -.0004     CMI=   -.0010
CHR=    .0128     CHI=   -.0047
    -3.92300816      -.22345817       .00005934       .31246508 .400.001.0020 6
    -4.08365353     -1.79285942      -.09352663       .62652297 .400.001.0020 6
```

FIG. 6 OUTPUT FOR SAMPLE PROBLEMS OF HYDROFOIL IN WAVES

3.5 Flap Sample Problem

A sample problem using the FLAP program for a hydrofoil with a 25% flap, reduced frequency of 0.2, and no effect from the free surface (IM=IFF=Ø) is shown in Fig. 5. Twenty collocation points and six Glauert terms are used, while eight terms are used in the numerical integration (NLE=NTE=NFLP=8). The depth and Froude number are arbitrarily read in as 1.Ø, and will be set infinite because we have specified IM=IFF=Ø. Tne input read fr ... .ne file is printed and identified. The fraction of the chord occupied by the flap is printed out after the data from the file. The six columns of numbers in exponential form which follow are the complex values for NOLT terms of the Glauert series. Here row number represents the order of the term and the columns are alternately the real and imaginary parts of the terms for heave, pitch and flap oscillations, in that order from left to right. Thus, for flap oscillation the amplitude of the third term in the Glauert series is P_3 = .24946 + i.11459. The lift and moment coefficients for the three cases follow. The R and I suffixes stand for the real and imaginary parts respectively, while L stands for the lift, M for quarter chord moment and H for the hinge moment. The "Comparison with Cornell" is the heave lift coefficient divided by 2π (i.e., the coefficient at zero frequency without free surface affects). The last two rows of output are a summary of the heave and then pitch output in the following order:

CLR, CLI, CMR, CMI, RD, D, NP, NOLT.

3.6 Wave Sample Problem

For the sample problem utilizing the WAVE program, we have taken a case with the foil operating at a very high Froude corresponding to a foil speed that is much larger than the wave speed. This condition approximates a foil moving through a stationary sinusoidal gust. Again the free surface effects have been ignored (IM and IFF are zero). The program computes values in waves based upon unity orbital velocity at the surface. Setting D = Ø makes the amplitude of the sinusoidal upwash at the foil unity. The wavelength is 15.71 semi-chords while the heading, 0.0, is directly into the seas. The Froude number is 10^4, phase is computed relative to the upwash at -0.5 (quarter chord), and the flap is 25% of the chord. The output format is the same as for the flap oscillation, except that the third set of coefficients is for operation in waves, and the wavelength and the heading are printed out.

3.7 Operation

For a given computation the operator must select the values of NOLT, NP, NFLP, NLE, and NTE. Running time is affected by all of these parameters, being most sensitive to the first two. The minimum number of Glauert coefficients, NOLT, is determined by the flap size and is critical for accuracy of the flap hinge moment. For flaps which are shorter than half of the chord, this minimum value will be

$$\text{NOLT} = 2\pi/(\pi-\theta_c).$$

(rounded <u>up</u> to an interger).

For flaps which are hinged forward of the mid chord NOLT must be at least three.

A general rule for NP is that it should be greater than 2 × NOLT. The numerically integrated functions are fairly smooth and values of eight for NFLP, NLE, and NTE gave good results. Values up to thirty-two may be used.

3.8 Results

We have computed the twelve hydrodynamic functions for representative values of depth and Froude number; the results are illustrated in Figs. 7 through 18. The functions are plotted in the complex plane with several values of reduced frequency indicated on each curve. Phase and amplitude may be found directly from the plots as the angle and magnitude of the vector from the origin to the point of interest on the curve.

Figures 8 and 9 show pitch and heave functions for foils in waves for $D = \infty$. Strictly speaking, there is no influence of waves at infinite depth; this notation merely means that the free surface effects have not been included but that the foil penetrates a sinusoidal gust. We have compared this solution with the Sears function [6] and have also compared the functions for foil motion at infinite depth with Theodorsens function [7]. The agreement is within one percent.

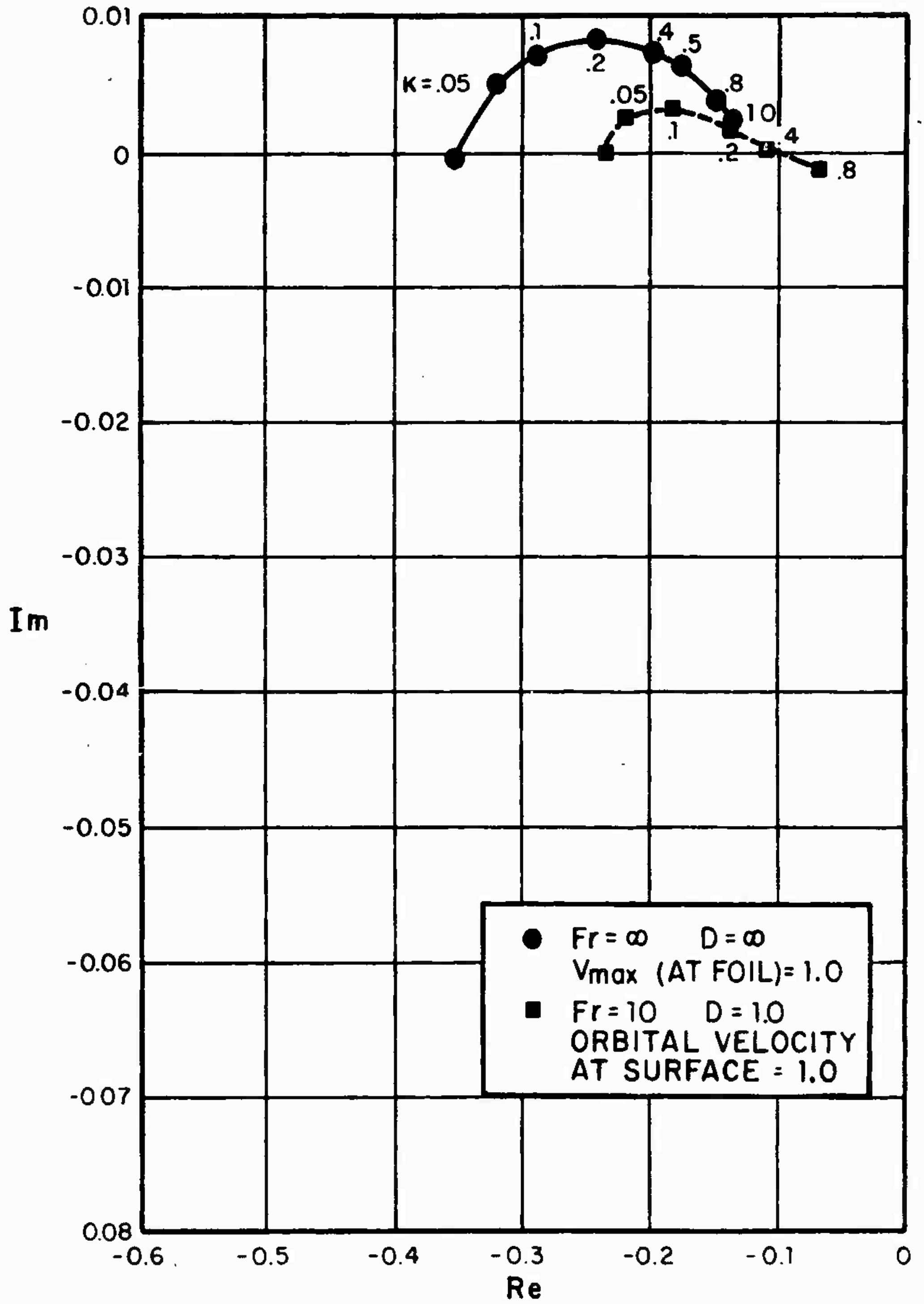

FIG. 7 $CM_{\beta}v$ OPERATION IN WAVES 25% FLAP, PHASED TO 1/4 CHORD DOWN WASH

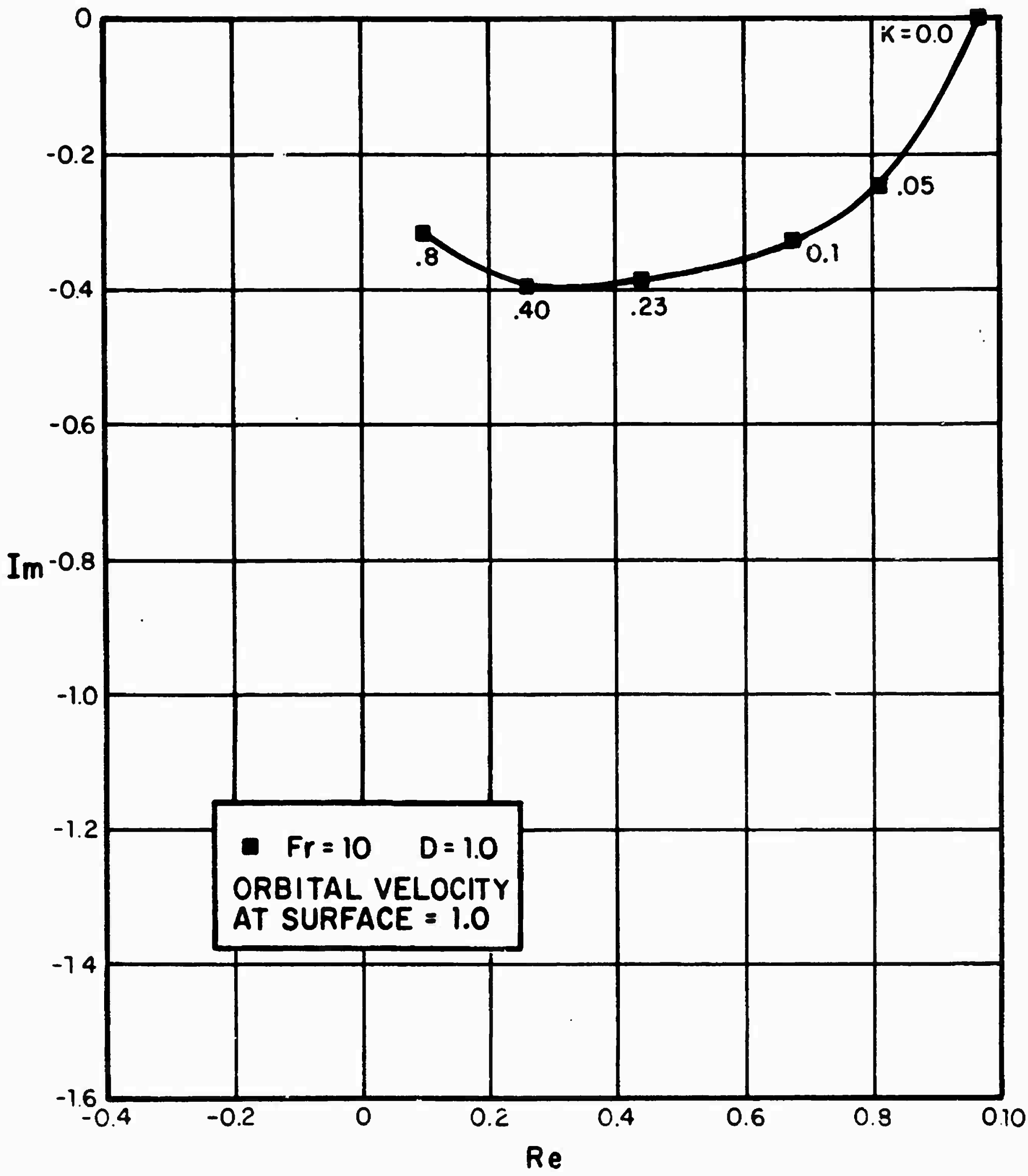

FIG. 8 $C_{M_\alpha v}$ OPERATION IN WAVES (PHASED TO 1/4 CHORD DOWNWASH)

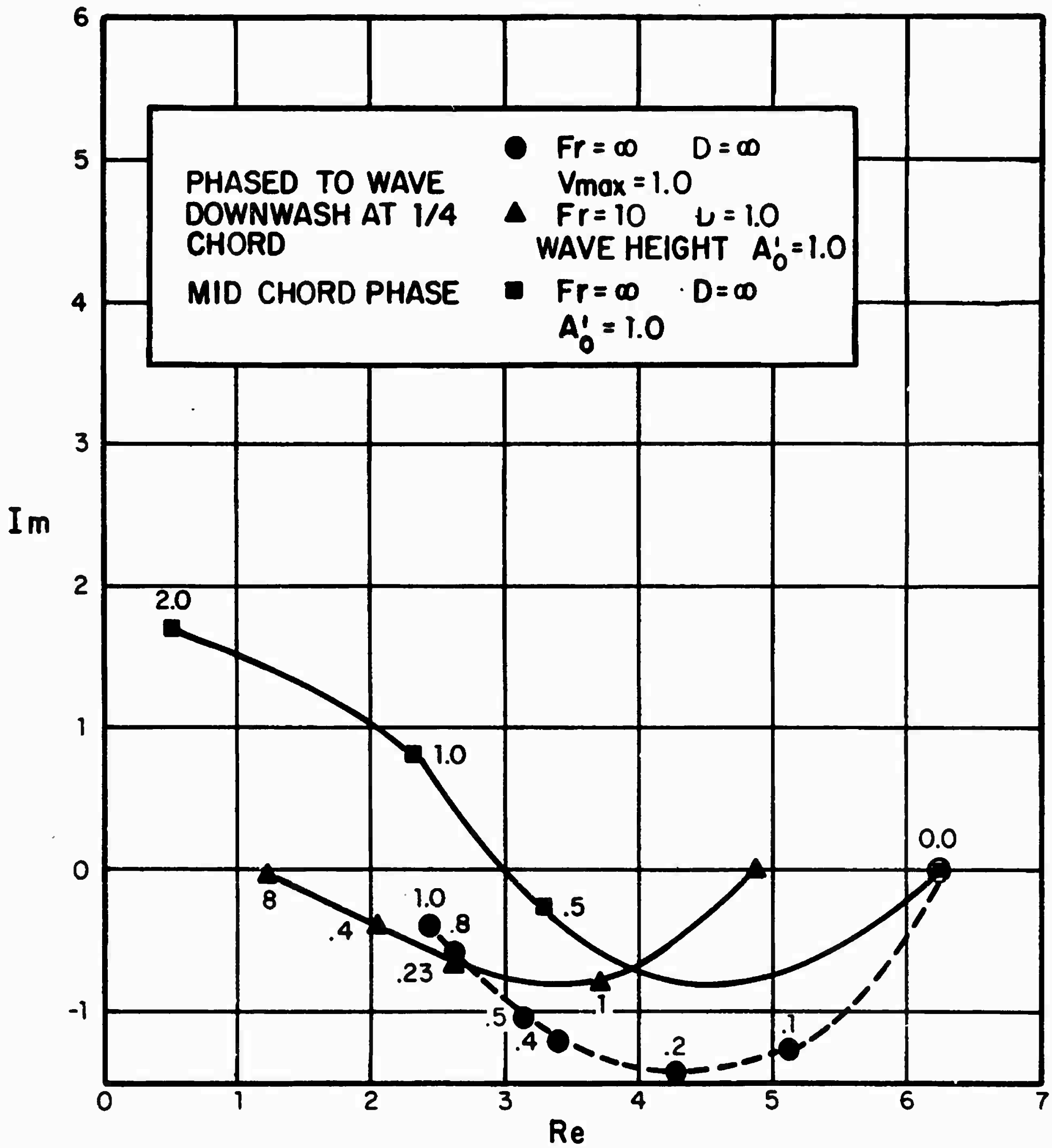

FIG. 9 C_{Lv} OPERATION IN WAVES

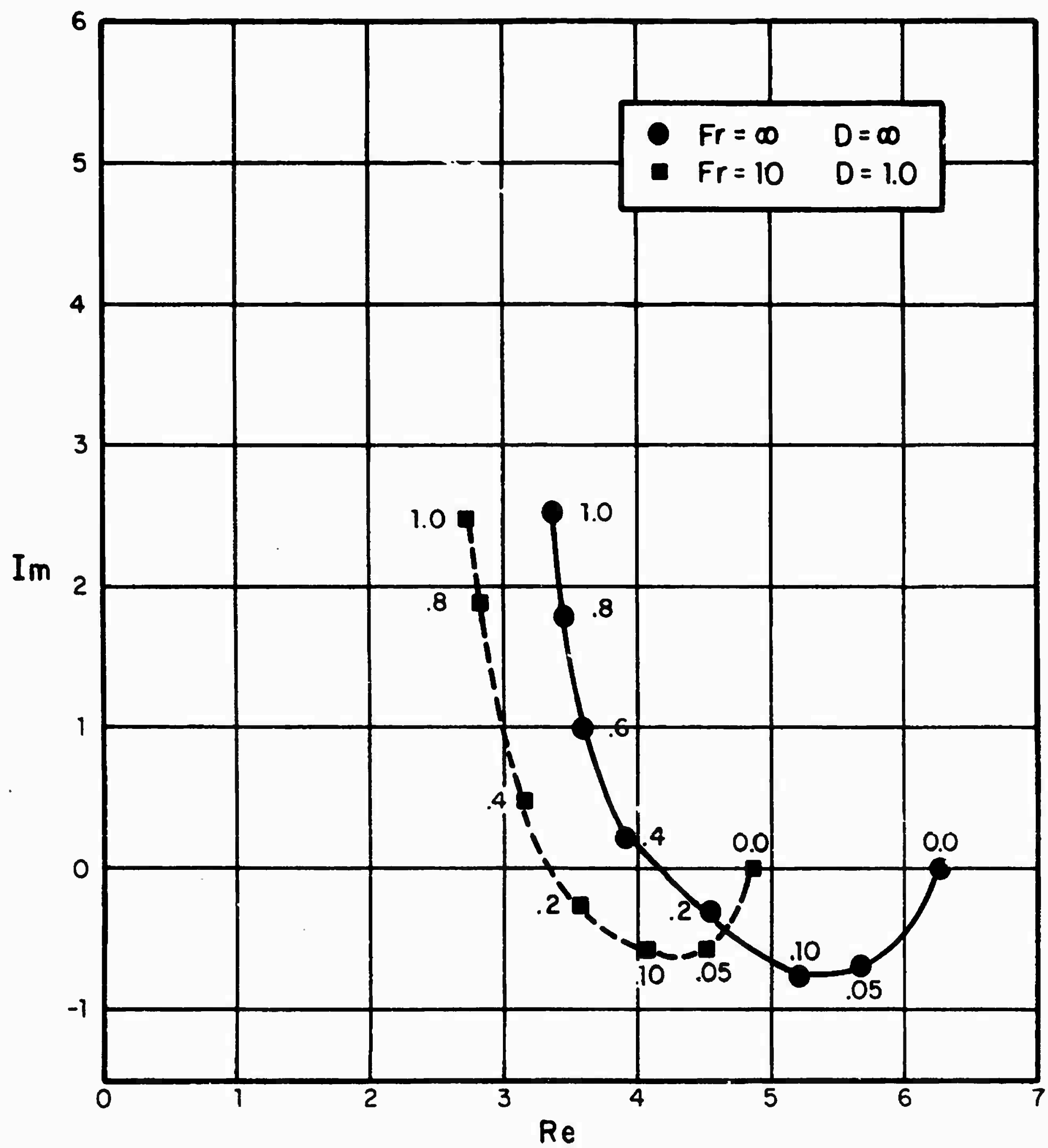

FIG. 10 $C_{L\dot{h}}$ HEAVE OSCILLATION

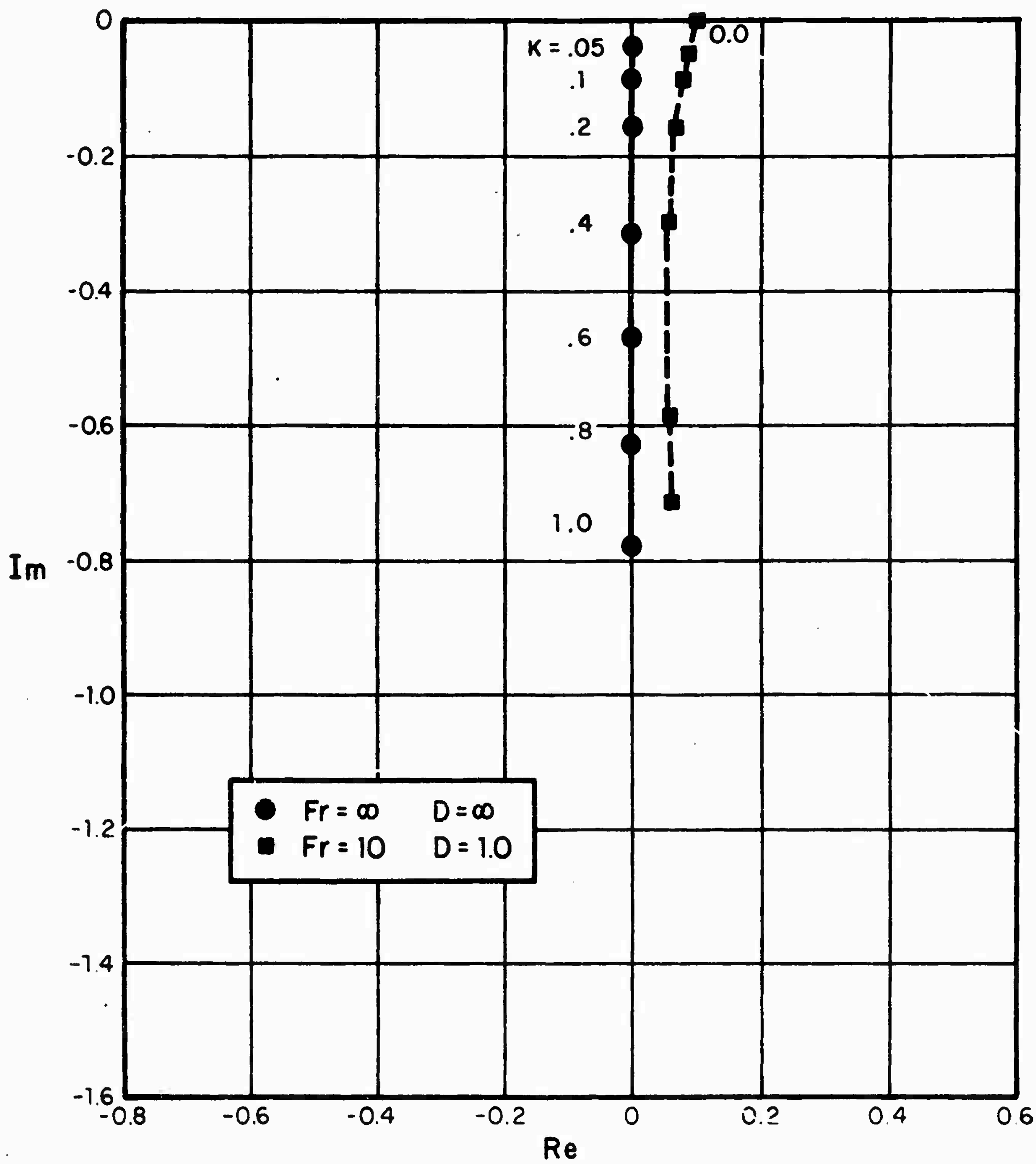

FIG. 11 $C_{M_\alpha \dot{h}}$ HEAVE OSCILLATION

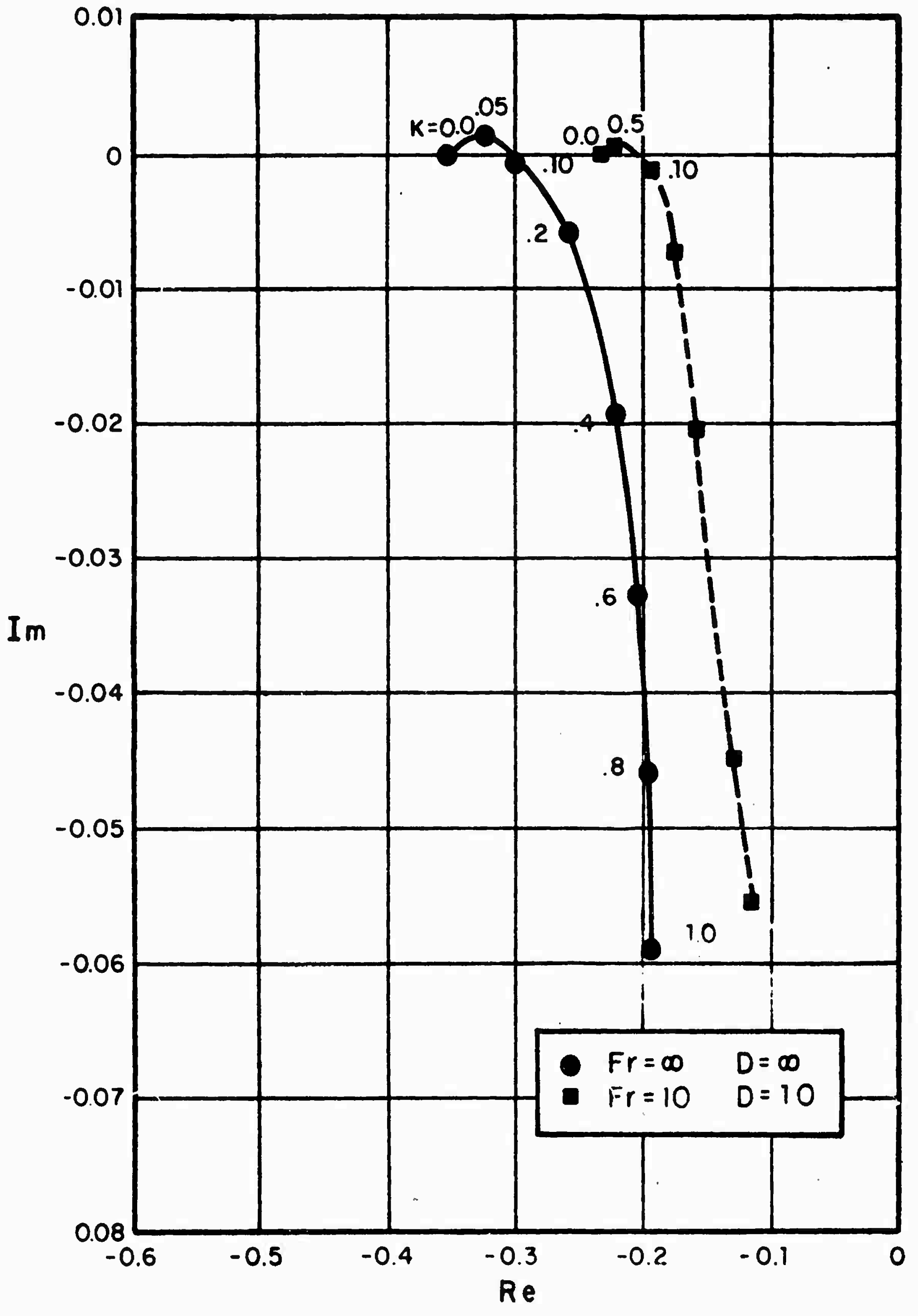

FIG. 12 $C_{M_\beta \dot{h}}$ HEAVE OSCILLATION 25% FLAP

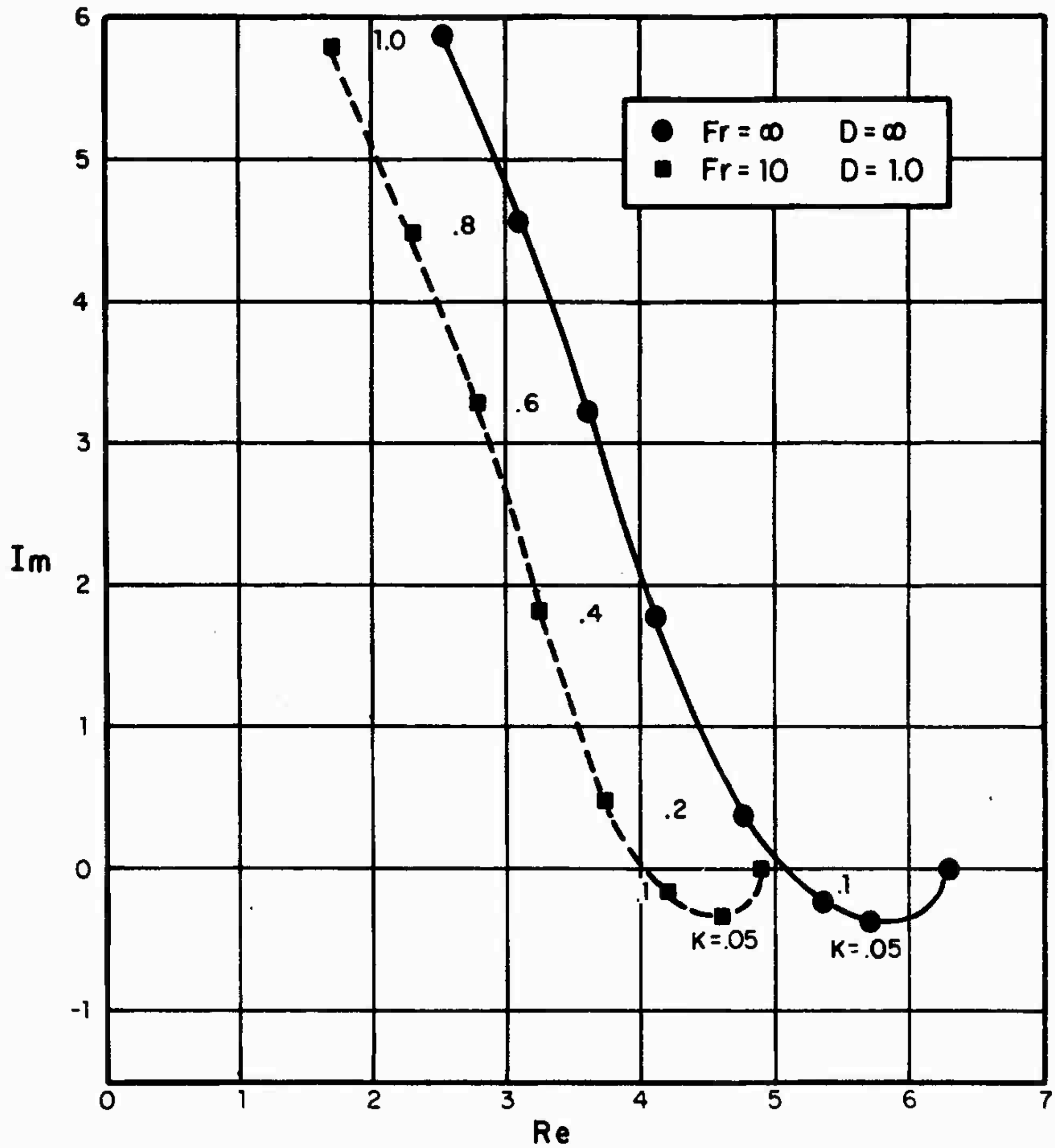

FIG. 13 $C_{L\alpha}$ PITCH OSCILLATION

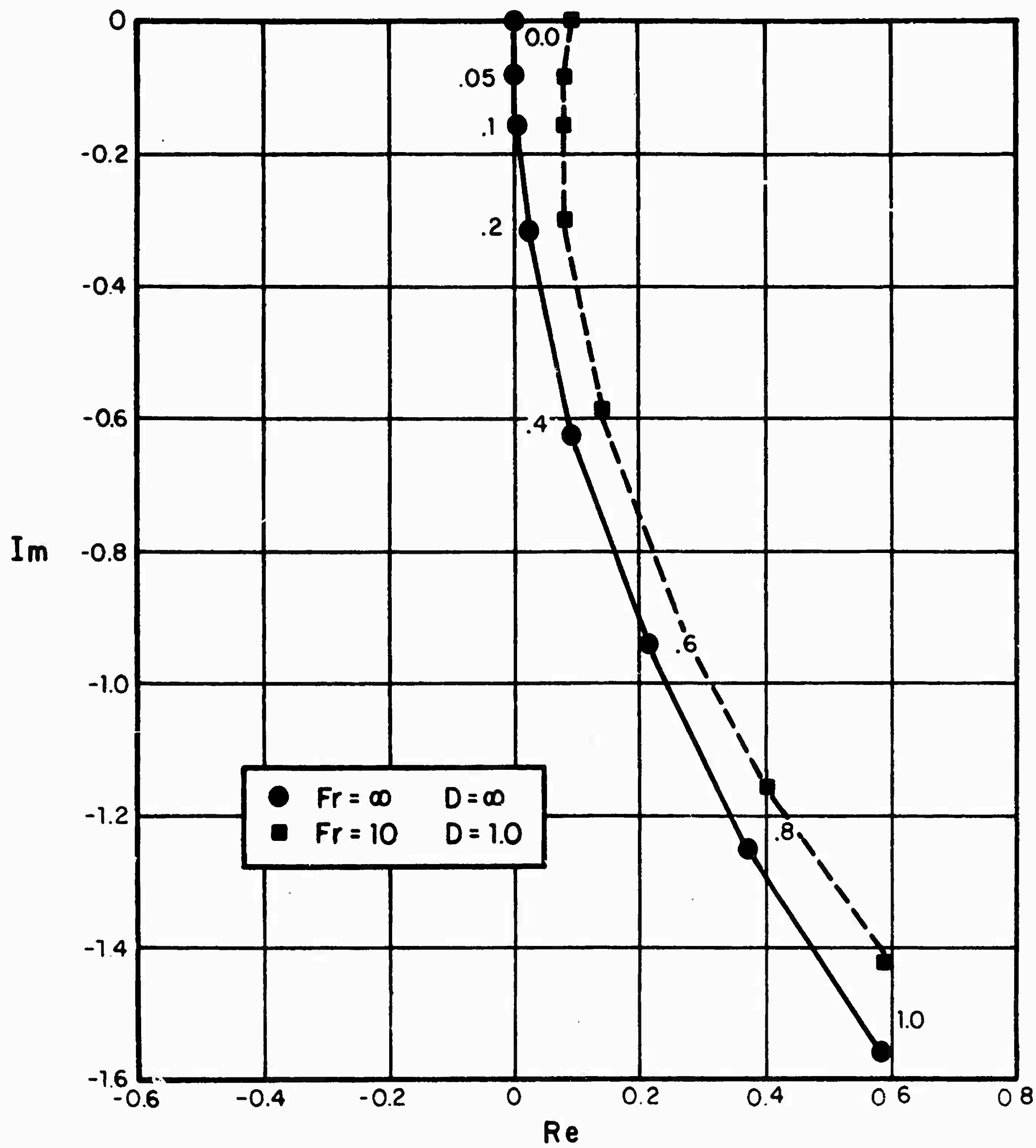

FIG. 14 $C_{M_\alpha \alpha}$ PITCH OSCILLATION

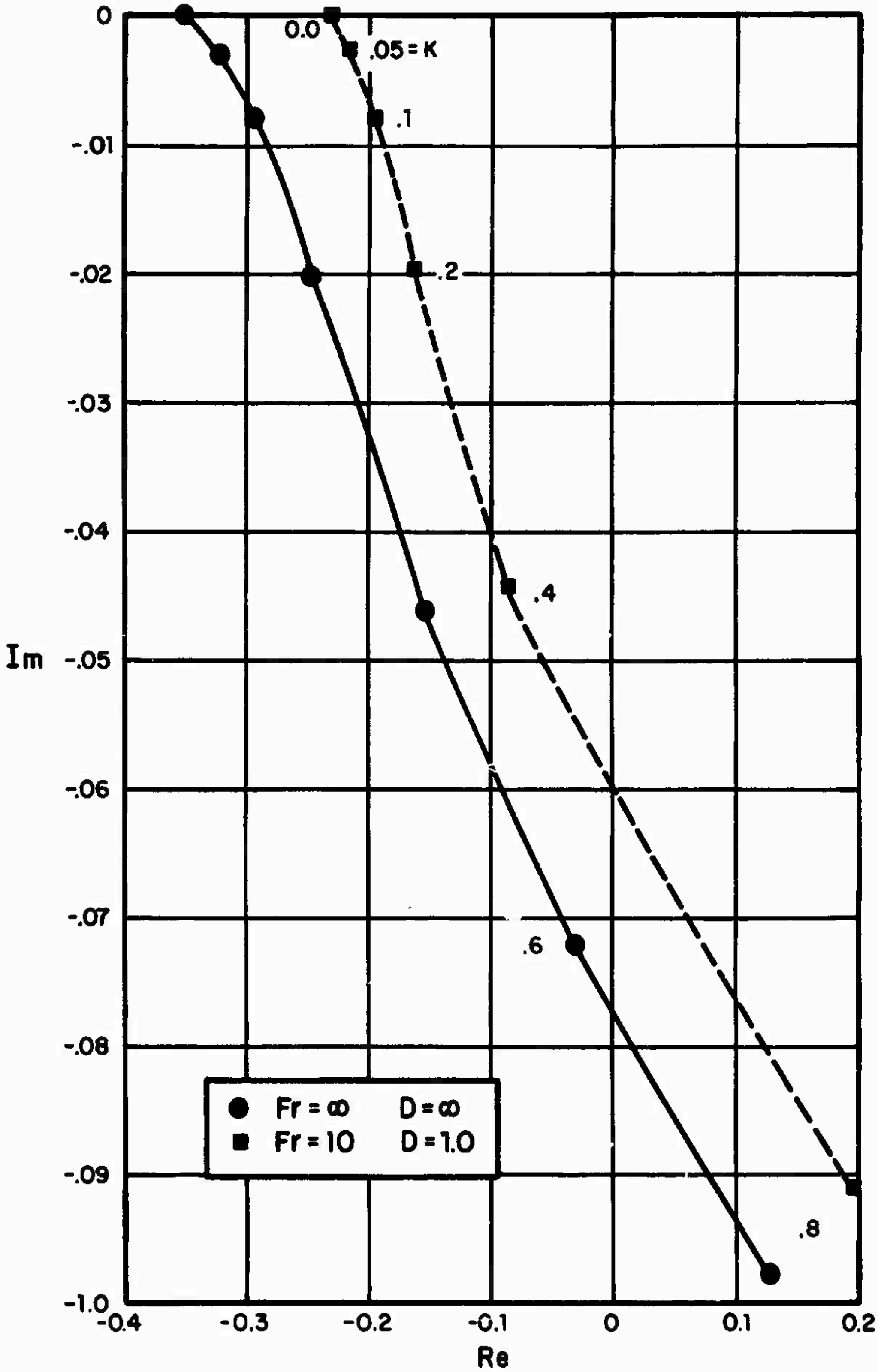

FIG. 15 $C_{M_{\beta\alpha}}$ PITCH OSCILLATION

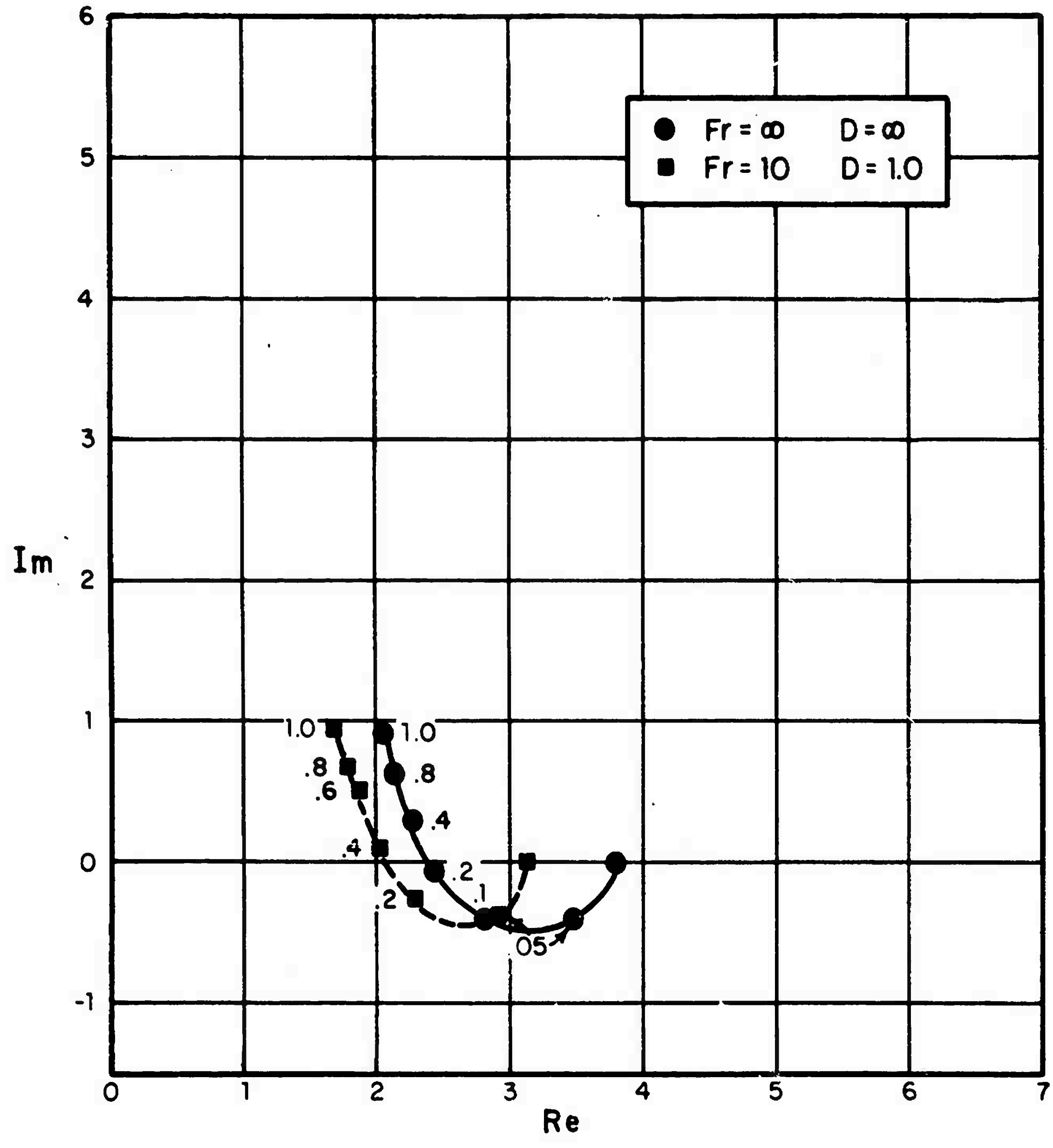

FIG. 16 $C_{L\beta}$ FLAP OSCILLATION. 25% FLAP

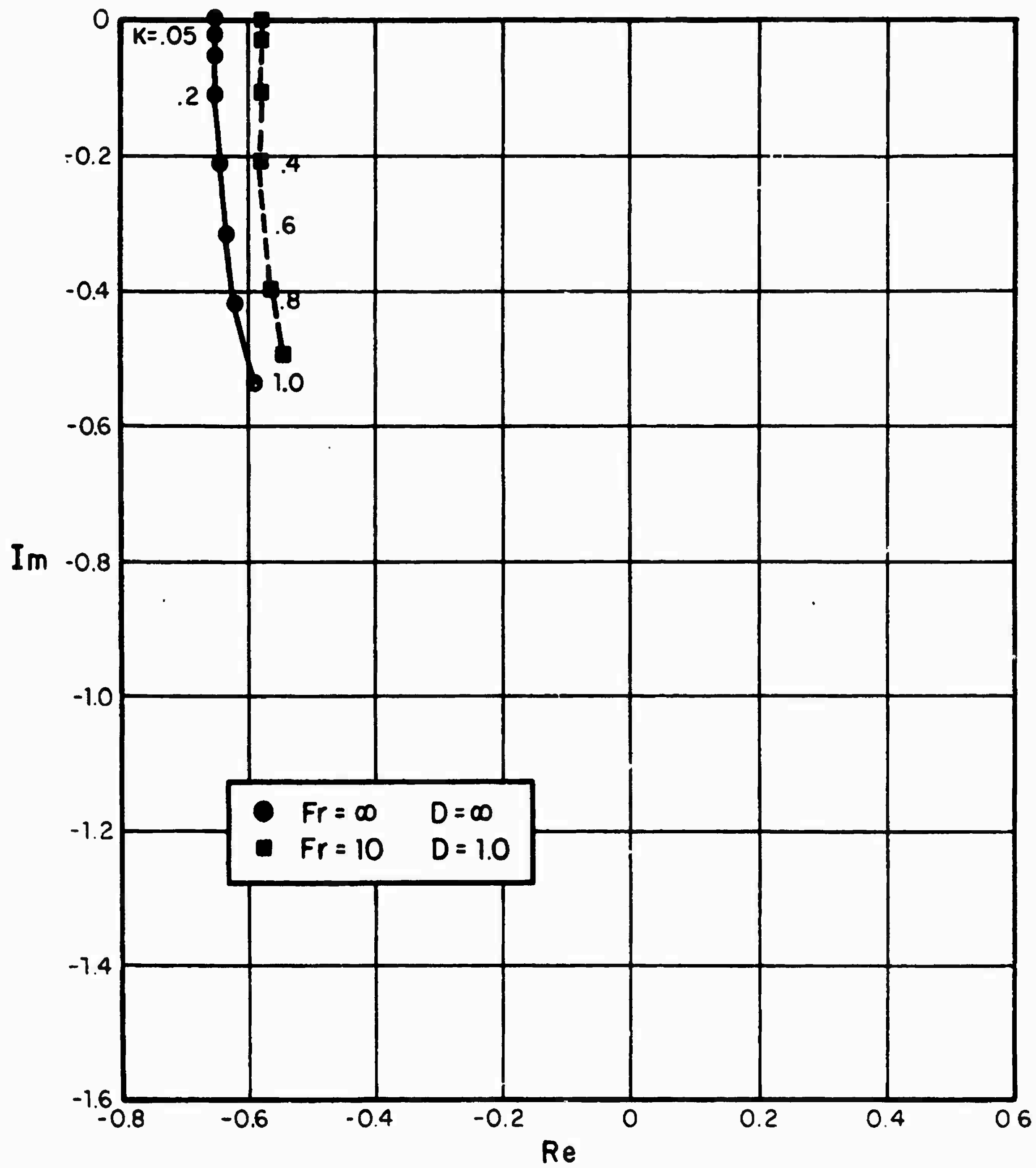

FIG. 17 $C_{M_{\alpha}\beta}$ FLAP OSCILLATION

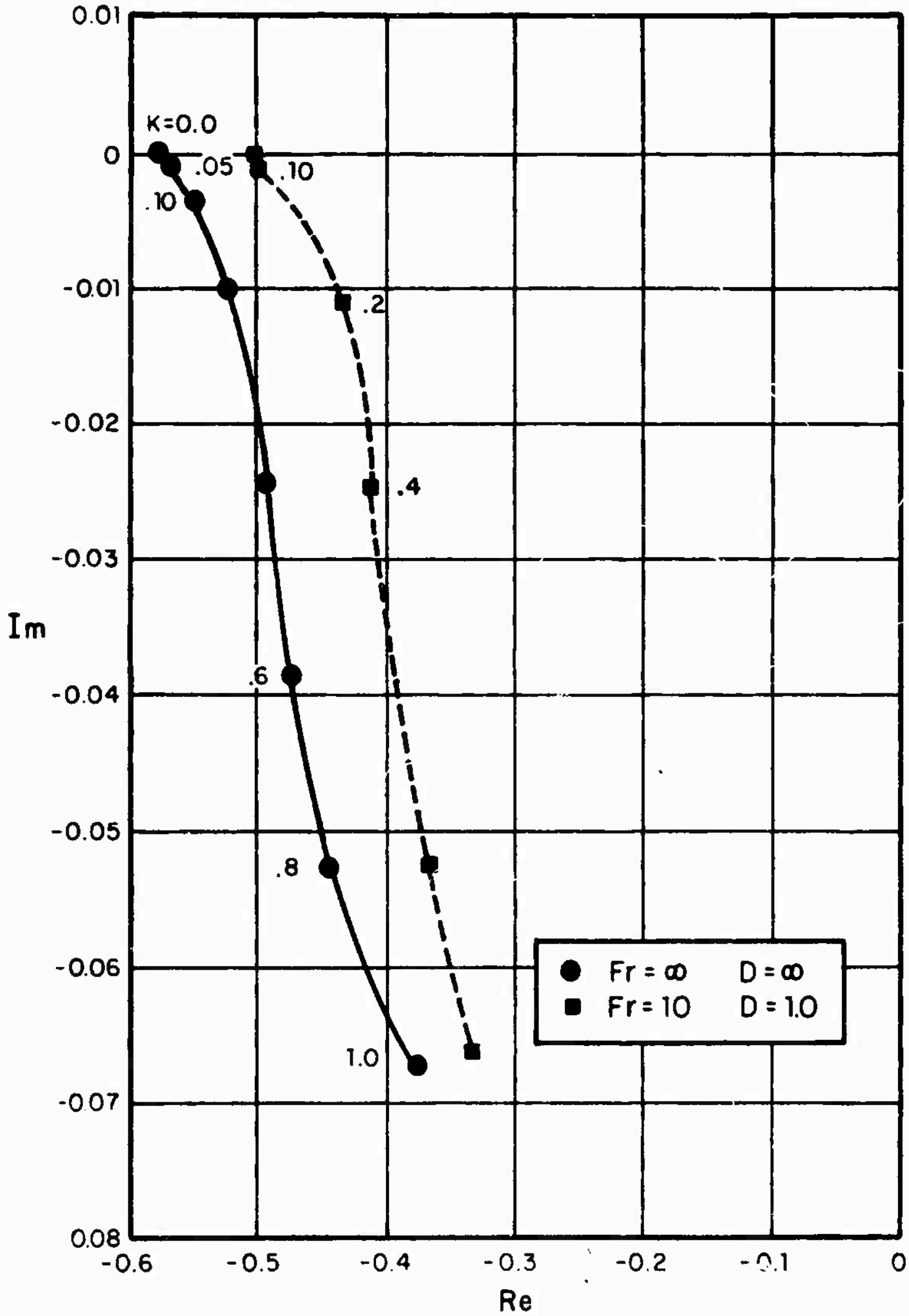

FIG. 18 $C_{M_\beta}\beta$ FLAP OSCILLATION 25% FLAP

4. CONTROL PROBLEMS

Freely pivoted foils, controlled by trailing edge servo tabs, exhibit the undesirable feature of initially developing lift in the direction opposite to that commanded. For example, when the tab is deflected downward, the lift is upward until the foil rotates sufficiently far to generate negative lift. This behavior leads one to suspect that the control of such foils may not be particularly straightforward and that there may, in fact, be some fundamental control system limitations affecting the suitability of pivoted tab-controlled foils for hydrofoil craft. Consequently, we have very briefly examined this problem to determine its nature and to outline a possible course of investigation.

We have conducted this study concurrently with the development of the hydrodynamics program discussed above. Consequently, the results of the program were not available for the control system study and we use Theodorsens equations for unsteady two-dimensional flow without free-surface effects [7]. Nondimensionalizing the equations for lift and foil moment, utilizing Newton's laws for the rigid body response of the foil and ship, and putting the equations into Laplace transform notation gives

$$-[(m+1)\phi+2]\phi h = [-a\phi^2+2(1-a)\phi+2]\alpha$$
$$-\pi^{-1}[T_1\phi^2+(T_4-T_{11})\phi-2T_{10}]\beta \tag{32}$$

and

$$-M_s+\left[\left(I+\frac{1}{8}+a^2\right)\phi^2-a(1-2a)\phi-2\left(a+\frac{1}{2}\right)\right]\alpha$$
$$=[a\phi+(2a+1)]\phi h+\frac{1}{\pi}\{[T_7+(c-a)T_1]\phi^2$$
$$+[aT_{11}-T_1+T_8+(c-a)T_4]\phi+[2aT_{10}-T_4]\}\beta\ . \tag{33}$$

Here m is the ship mass per unit span of the foil divided by $\pi\rho b^2$, ϕ is the Laplace operator nondimensionalized by b/U, a is the foil pivot position varying linearly from -1 at the leading edge to +1 at the trailing edge, M_s is a nondimensional moment applied directly to the foil by a servomechanism, I is the foil moment of inertia per unit span divided by $\pi\rho b^4$, and T's are functions of the tab pivot point defined by c.

For a flap comprising 10% of the foil chord, the coefficients of β in Eqs. 32 and 33 may be approximated by

$$+\frac{1}{2\pi}(\phi+5) \qquad \text{for Eq. (32)}$$

$$+\frac{a}{2\pi}(\phi+5) \qquad \text{for Eq. (33)}$$

We also may assume that I is negligible.

4.1 Control

The above equations, in block diagram form, are shown in Fig. 19. Lifts and moments owing to foil and tab angles and to vertical motion are indicated. In addition, a feedback gain K is shown, representing a loop in which a moment proportional to vertical position is applied directly to the foil. The transfer function for this system is

$$\frac{h}{b\beta} = -\frac{\frac{1}{16\pi}(\phi+5)\left[\phi^2+8a\phi-8\right]}{\left\{\left[\frac{1}{8}(m+1)+ma^2\right]\phi^4+\left[\frac{1}{4}-ma(1-2a)\right]\phi^3 + [1-m(2a+1)-aK]\phi^2+2K(1-a)\phi+2K\right\}} . \qquad (34)$$

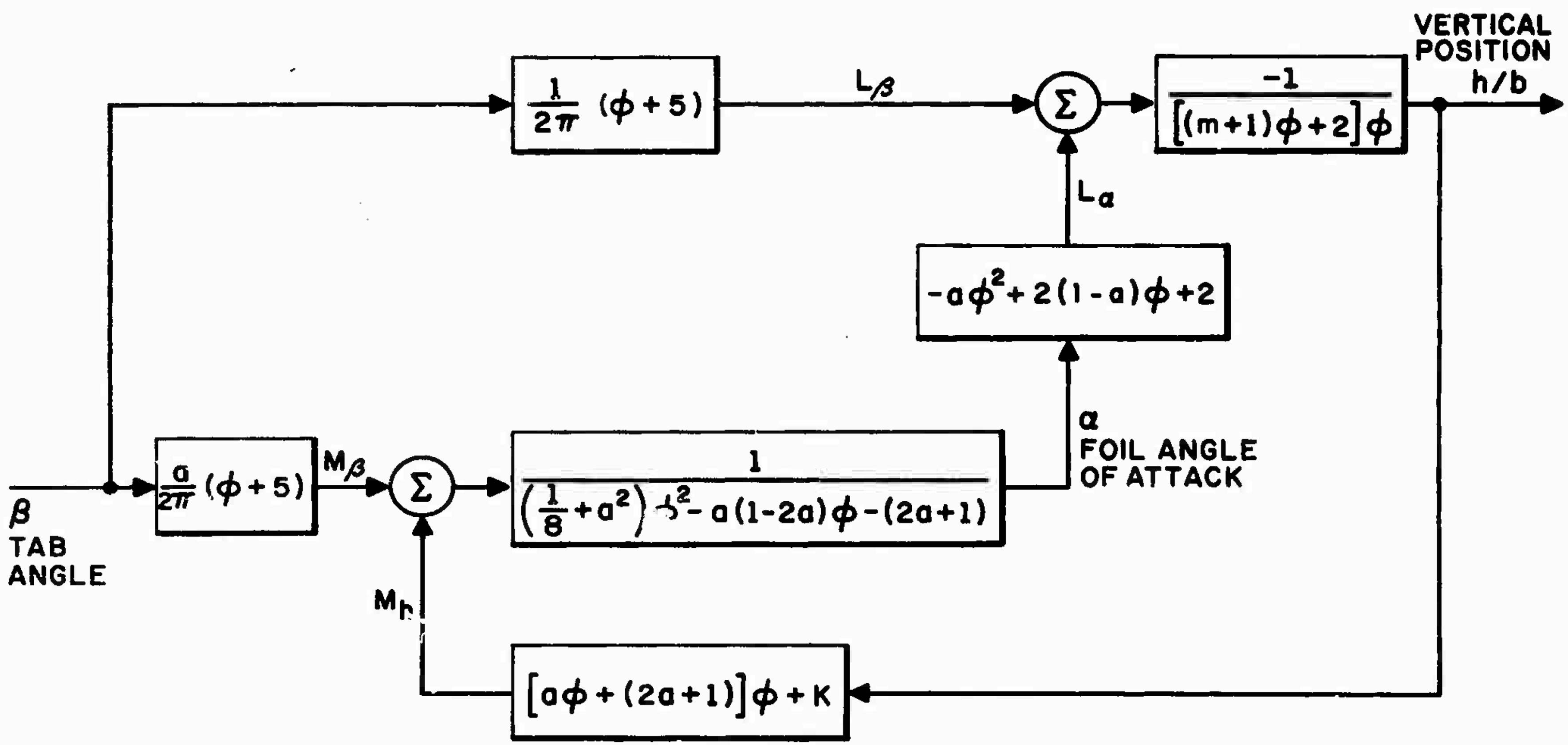

FIG.19 BLOCK DIAGRAM OF HYDROFOIL DYNAMICS WITH A SERVO TAB COMPRISING 10% OF THE FOIL CHORD

There are several interesting aspects of Eq. 34. First, the quadratic polynomial in the numerator always has one positive and one negative real root (i.e., open loop zeroes). Thus a root locus always has a branch ending in the right half plane, despite the form of compensation network employed. Secondly, if K is zero, Eq. 34 will have two poles at the origin. This means that an impulse in β will result in a steady velocity in h. Thus, a momentary deflection of the tab causes a transient that does *not* end with the foil realigning itself with the inflow vector $\vec{U}$. Instead, the foil becomes aligned with the vector sum of U and $\vec{h}$, defining an "effective" angle of attack of zero. If, in addition to K being zero, m is very large (corresponding in the limit as $m \to \infty$ to a fixed pivot point) the denominator of Eq. 34 becomes

$$m\phi^2\left[\left(\frac{1}{8} + a^2\right)\phi^2 - a(1-2a)\phi - (2a+1)\right]$$

which is always stable for $a < -1/2$ and unstable for $a > -1/2$ (corresponding to pivoting the foil upstream and downstream respectively of the quarter chord point).

If we assume a = -/12 and that m = 1 (a good approximation for typical hydrofoil craft) and apply Routh's stability criterion to the denominator of Eq. 34, all roots will lie in the left half plane (off the real axis) for $0 < K < 1/4$. Thus, we see that by choosing an appropriate value of feedback gain the open loop transfer function can be made controllable.

5. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

In this Report we have developed a program to compute the unsteady hydrodynamic lift and moment functions for a two-dimensional hydrofoil operating near a free surface, and have briefly investigated the control of hydrofoils. In several example computations, utilizing the developed program, we found that the effect of the free surface is to modify the magnitude of phase of the load on the foil. Lift due to heave and pitch oscillations, for example, decrease with decreasing depth as one might expect. The control studies indicate certain limitations in controlling foil lift by means of servo tabs.

These studies are largely preliminary to the primary objective of designing hydrofoil systems that have good response characteristics and low power consumption. Based on these results there are several control-system studies that ought to be undertaken as well as refinements in the hydrodynamic analysis. They are to:

1. Design a hydrofoil control system configuration that maximizes bandwidth and minimizes control power. This involves the choice of appropriate feedback loops, compensation functions, and foil parameter values.
2. Extend the unsteady hydrodynamic analysis to three dimensional hydrofoils. This would be particularly suitable to foils of low aspect ratio.

ACKNOWLEDGEMENTS

This Report reflects not only the efforts of the authors but also the contributions of several other people. We are grateful to Professor Sheila Widnall and Mr. Wayne Johnson of the Massachusetts Institute of Technology for their contributions to the hydrodynamic analysis of a hydrofoil with a trailing-edge flap. We also appreciate the helpful comments of Mr. William C. O'Neill, the technical monitor of this project, and Mr. Peter Besch; both are of the Naval Ship Research and Development Center.

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4. W.C. O'Neill, "Unsteady Lift and Hinge Moment Characteristics of the AG(EH) Main Foil and Strut Assembly", NSRDC Report 2805, July 1968.

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7. T. Theodorsen, "General Theory of Aerodynamic Instability and the Mechanism of Flutter", N.A.C.A Report 496, 1934.

APPENDIX A: KERNEL FUNCTION FOR TWO DIMENSIONAL FLOW WITH A FREE SURFACE [2]

$$K = -\frac{x}{2\pi(x^2+4d^2)} - \frac{ik}{4\pi} e^{-q_0}\left[Ei(q_0) + \pi i\left(1 + \frac{|x|}{x}\right)\right]$$

$$- \frac{ik}{4\pi} e^{q_0} Ei(-q_0^*) + \frac{ia_1}{4\pi} e^{-q_1}\left[Ei(q_1) + \pi i\left(1 + \frac{|x|}{x}\right)\right]$$

$$+ \frac{ia_2}{4\pi} e^{-q_2}\left[Ei(q_2) + \pi i\left(1 + \frac{|x|}{x}\right)\right] + \frac{ia_3}{4\pi} e^{q_3} Ei(-q_3)$$

$$+ \frac{ia_4}{4\pi} e^{q_4} Ei(-q_4) ,$$

where a star denotes complex conjugate and

$$a_1 = k[1 + f^{-1} + (3 + f^{-1})(4f + 1)^{-\frac{1}{2}}]$$

$$a_2 = k[1 + f^{-1} - (3 + f^{-1})(4f + 1)^{-\frac{1}{2}}]$$

$$a_3 = k[1 - f^{-1} + i(3 - f^{-1})(4f - 1)^{-\frac{1}{2}}]$$

$$a_4 = k[1 - f^{-1} - i(3 - f^{-1})(4f - 1)^{-\frac{1}{2}}]$$

$$q_0 = k(2d + ix)$$

$$q_1 = -s_1(2d + ix)$$

$$q_2 = -s_2(2d + ix)$$

$$q_3 = -s_3(2d - ix)$$

$$q_4 = -s_4(2d - ix)$$

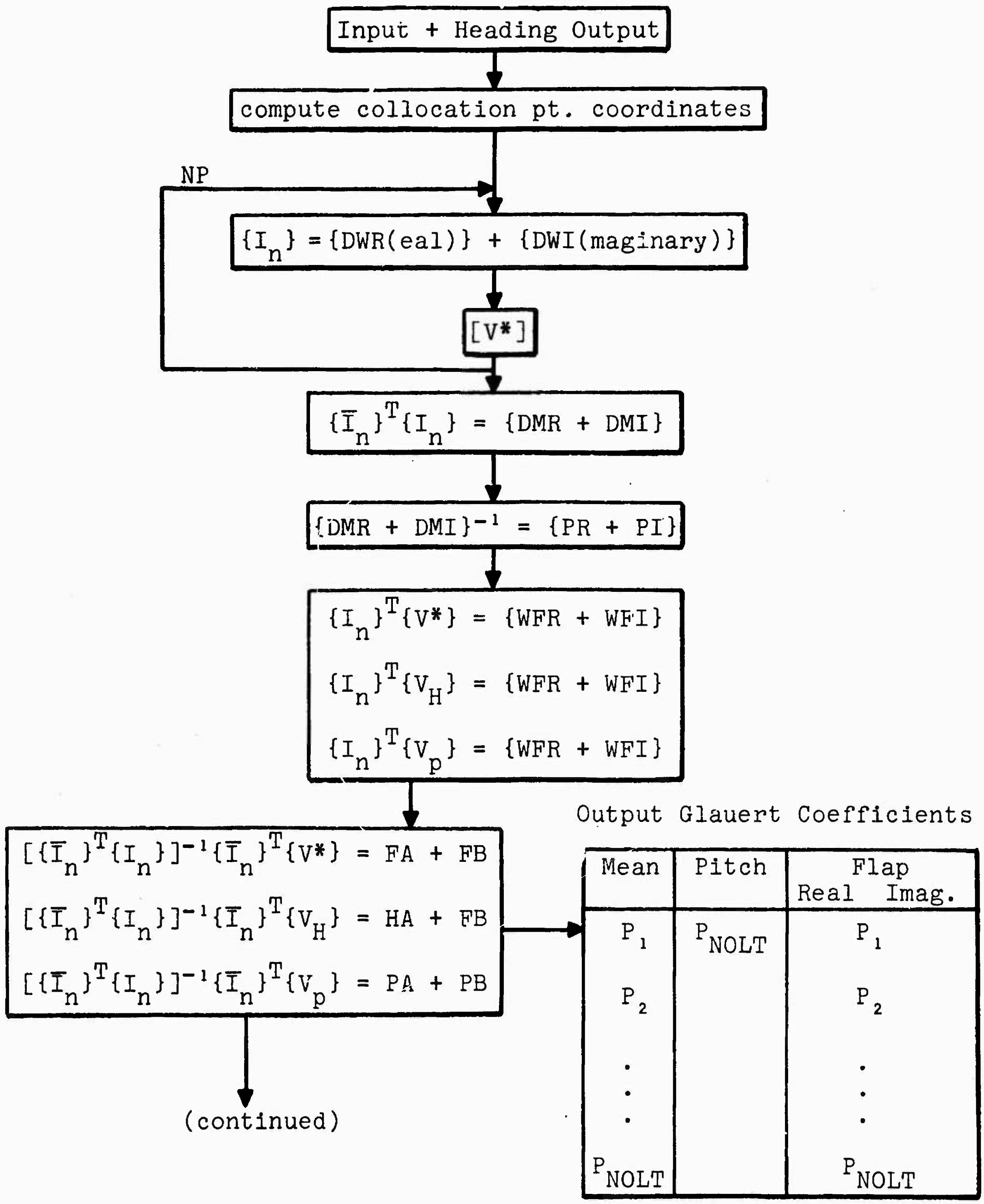
$$f = kF_r^2 = \frac{\omega U}{g}$$

$$Ei(q) = \int_{-\infty}^{q} \frac{e^t}{t}\, dt \ .$$

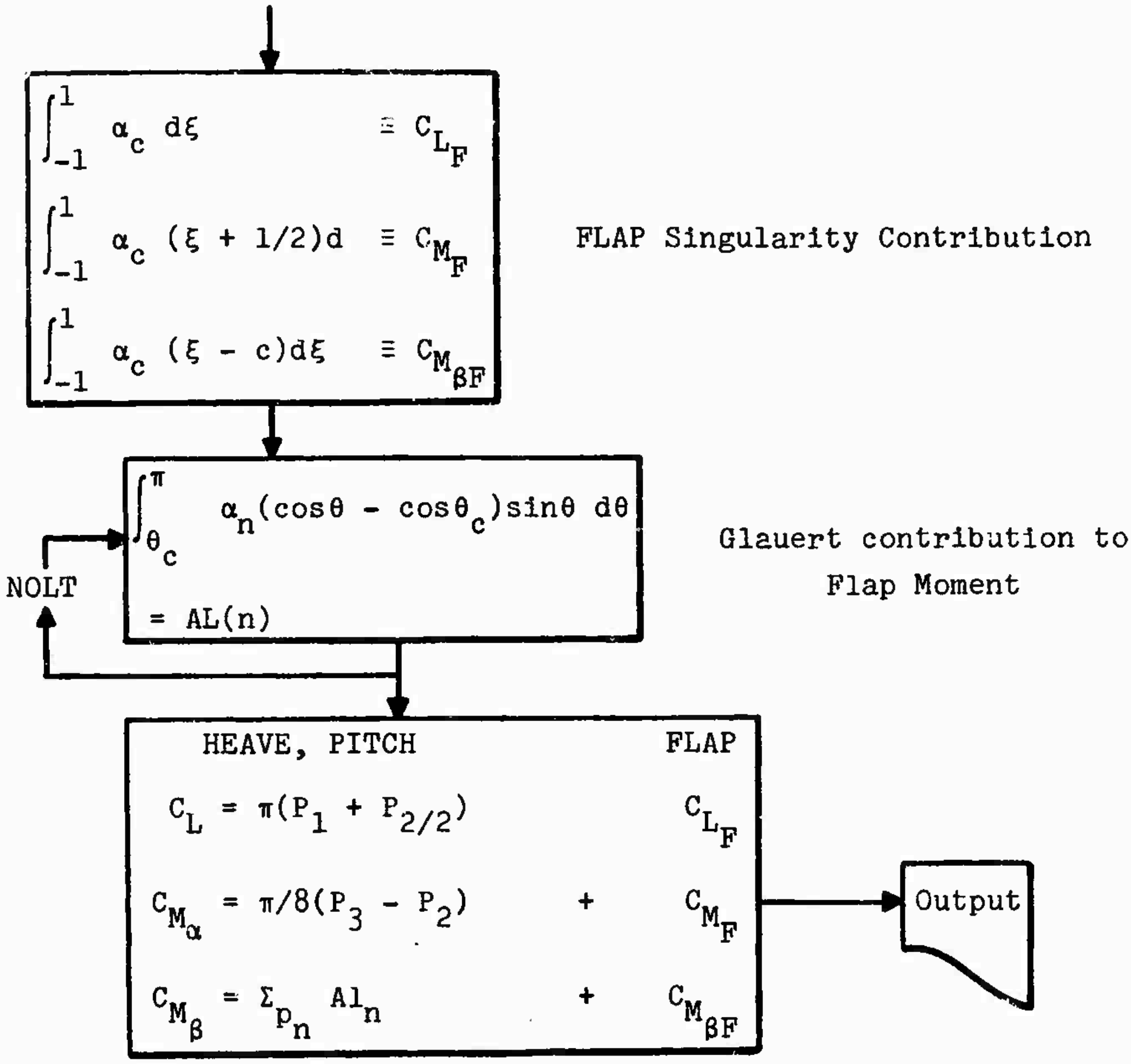
APPENDIX B

Flow Charts for Hydrodynamic Programs

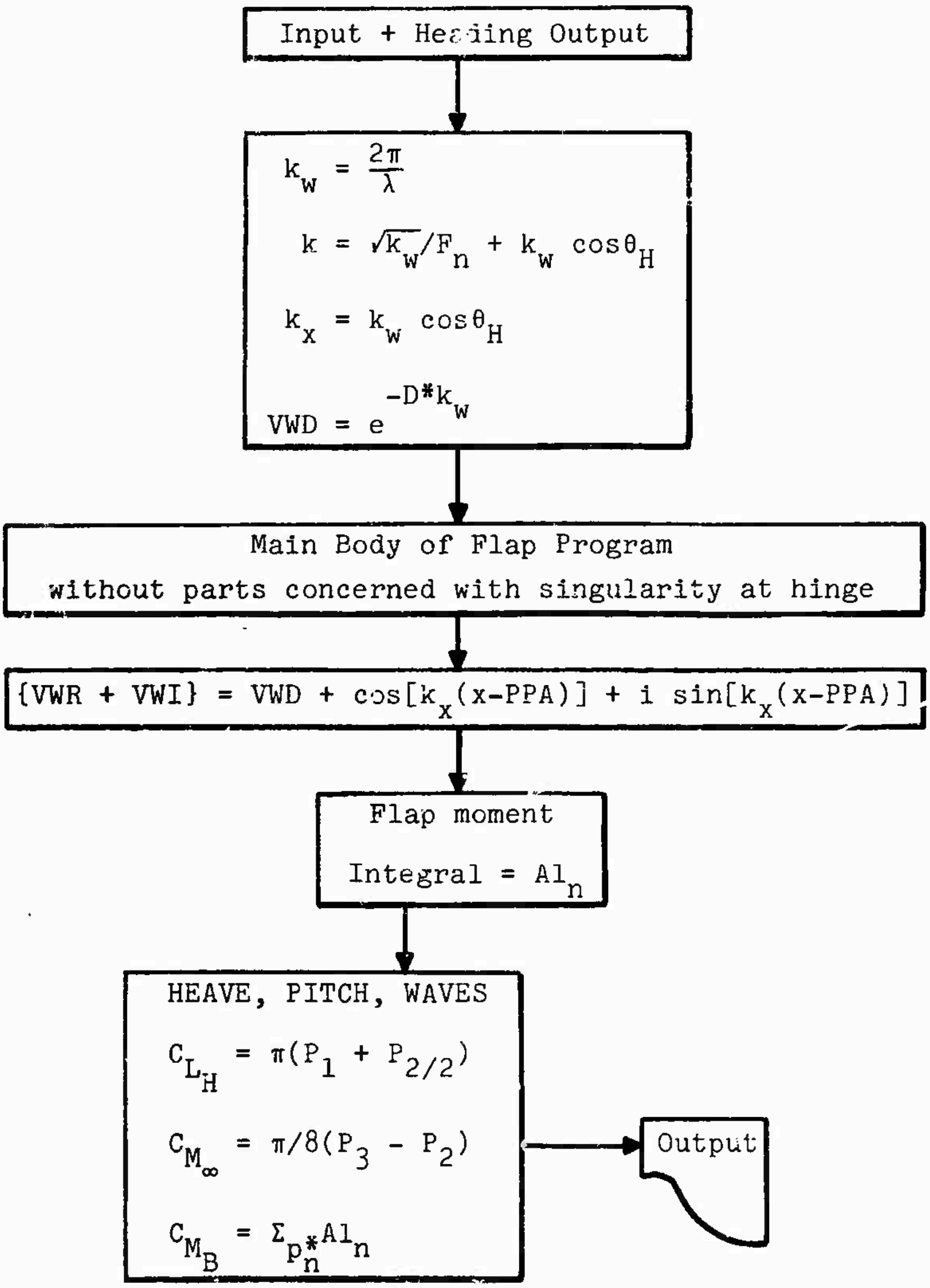
FUNCTIONAL FLOW CHART OF FLAP PROGRAM
(using notation from text and program)



FUNCTIONAL FLOW CHART OF FLAP PROGRAM *(continued)*



FLOW CHART OF MODIFICATIONS FOR HYDRODYNAMICS IN WAVES



APPENDIX C: PROGRAM TO COMPUTE THE HYDRODYNAMIC COEFFICIENTS FOR A FLAPPED FOIL OPERATING IN WAVES

$$C_{L_{\dot{h}}},\ C_{L_P},\ C_{L_V}$$

$$C_{M_{\dot{h}}},\ C_{M_{\alpha P}},\ C_{M_{\alpha V}}$$

$$C_{M_{\beta\dot{h}}},\ C_{M_{\beta P}},\ C_{M_{\beta V}}$$

```
C     TWO DIMENSIONAL PROGRAMME FOR FOIL IN WAVES
      DIMENSION AL(10),CR(10),CI(10),GR(32),GI(32),THETA(32),GX(32),
     1XO(32),DWR(20,6),DWI(20,6),DMR(6,6),DMI(6,6),X(100),AM(10),
     2PA(10),PB(10),HA(10),HB(10),PR(6,6),PI(6,6)
     3,WPR(10),WPI(10),WHR(10),CPVC(10),BANGL(20),NLE1(20),NTE1(
     420),WHI(10),WWR(10),WWI(10),WA(10),WB(10)
      COMMON XO,GR,GI,PO2,T,JEST,IM,IFF,D,FR,NT,S1,S2,S3R,S3I,S4R,S4I,
     1A1,A2,A3R,A3I,A4R,A4I
      PO2=1.57079633
      TI=-.5
      TYPE 1001
1001  FORMAT(/$  INPUT DATA FROM FILE:$)
      CALL OPENR(5)
      CALL READB(5,NP)
999   FORMAT(I5)
998   FORMAT(F10.4)
1     CALL READB(5,NOLT)
      CALL READB(5,NT)
      CALL READB(5,JEST)
      CALL READB(5,IM)
      CALL READB(5,IFF)
      CALL READB(5,RF)
      CALL READB(5,FR)
      CALL READB(5,D)
      CLOSE
      ACCEPT 996,WL,THET,FR,PPA,CFLAP,NLE,NTE
996   FORMAT(5F10.4,2I4)
      XKW=4.*PO2/WL
      RF=SQRT(XKW)/FR+XKW*COS(THET)
      WK=XKW*COS(THET)
      VWD=EXP(-XKW*D)
3     FORMAT (5I5,3F10.5)
      IF(NOLT)501,501,500
500   TYPE 2
2     FORMAT(/62H1 IMAGE IFF     K    FROUDE NUM   DEPTH IN SEMCHDS   NP
     1   NOLT    )
      TYPE 5,IM,IFF,RF,FR,D,NP,NOLT
5     FORMAT(/2I5,3F10.4,12X,2I3)
      XFLAP=1.-2.*CFLAP
      NP2=NP/2
      PI2=4.*PO2
      DO 31 J=1,NP2
      DX=COS(J*PI2/(NP+1.))
      X(J)=(-(XFLAP+1.)*DX+XFLAP-1.)/2.
      I=NP+1-J
31    X(I)=((-XFLAP+1.)*DX+XFLAP+1.)/2.
      DO 32 L=1,NP
      BANGL(L)=ANGL (-X(L),SQRT(1.-X(L)*X(L)))/(2.*PO2)
      NLE1(L)=NLE
```

```
32      NTE1(L)=NTE
        TFLAP=ANGL (-XFLAP.SQRT(1.-XFLAP*XFLAP))
        TYPE 9001,CFLAP
9001    FORMAT(///$   FLAP CHORD$,F8.2,5X,$FRACTION OF AIRFOIL CHORD$)
        SNTF=SIN(TFLAP)
        TYPE 51,WL,THET
51      FORMAT(/,$WAVE LENGTH=$,F8.3,$ SEMI-CHORDS.  HEADING=$,F8.3,/)
        DO 100 J=1,NP
        ANGLE=BANGL(J)
        NLE=NLE1(J)
        NTE=NTE1(J)
4       FORMAT(F10.4,2I10)
        THMAX=PO2*2.*ANGLE
        X(J)=-COS(THMAX)
        DO 6 I=1,NLE
        THETA(I)=THMAX/2.*(1.-GN(I,NLE))
6       XO(I)=X(J)+COS(THETA(I))
        DO 7 K=1,NOLT
        CR(K)=0.0
7       CI(K)=0.0
        CALL KER2(RF,NLE)
        DO 10 I=1,NLE
        AL(1)=COS(THETA(I)/2.)/SIN(THETA(I)/2.)
        DO 71 N=2, NOLT
71      AL(N)=SIN((N-1)*THETA(I))
        CW=WN(I,NLE)*THMAX/2.*AL(2)
        DO 8 K=1,NOLT
        CR(K)=AL(K)*CW*GR(I)+CR(K)
8       CI(K)=AL(K)*CW*GI(I)+CI(K)
        IF (J-5) 10,9,10
9       IF(JEST) 80,10,80
80      TYPE 22,XO(I),GR(I),GI(I),CW,(AL(K),K=1,3),(CR(K),K=1,3)
10      CONTINUE
        DO 15 I=1,NTE
        THETA(I)=THMAX+(PO2-THMAX/2.)*(1.-GN(I,NTE))
15      XO(I)=X(J)+COS(THETA(I))
        CALL KER2(RF,NTE)
        DO 25 I=1,NTE
        AL(1)=COS(THETA(I)/2.)/SIN(THETA(I)/2.)
        DO 151 N=2, NOLT
151     AL(N)=SIN((N-1)*THETA(I))
        CW=WN(I,NTE)*(PO2-THMAX/2.)*SIN(THETA(I))
        DO 20 K=1,NOLT
        CR(K)=AL(K)*CW*GR(I)+CR(K)
20      CI(K)=AL(K)*CW*GI(I)+CI(K)
        IF (JEST) 301,25,301
301     IF (J-5) 25,21,25
21      TYPE 22,XO(I),GR(I),GI(I),CW,(AL(K),K=1,3),(CR(K),K=1,3)
22      FORMAT(//5E12.4,/,5E12.4,/)
```

```
25      CONTINUE
        CPVC(1)=TI
        DO 251 N=2, NOLT
251     CPVC(N)=TI*COS((N-1)*THMAX)
        DO 26 K=1,NOLT
        DWR(J,K)=CR(K)+CPVC(K)
26      DWI(J,K)=CI(K)
        IF(J-5)100,27,100
27      IF(JEST)28,100,28
28      TYPE 29,(CPVC(I),I=1,3)
29      FORMAT(/3E20.8)
100     CONTINUE
        DO 101 I=1,NOLT
        DO 101 K=1,NOLT
        DMR(I,K)=0.0
101     DMI(I,K)=0.0
        DO 110 I=1,NOLT
        DO 110 K=1,NOLT
        DO 110 J=1,NP
        DMR(I,K)=DWR(J,I)*DWR(J,K)+DWI(J,I)*DWI(J,K) +DMR(I,K)
109     DMI(I,K)=-DWI(J,I)*DWR(J,K)+DWR(J,I)*DWI(J,K)+DMI(I,K)
110     CONTINUE
        CALL CMPINV(DMR,DMI,PR,PI,NOLT,INDEX1)
        IF(JEST) 111,115,111
111     TYPE 112,((DMR(I,J),DMI(I,J),J=1,NOLT),I=1,NOLT)
        TYPE 112,((PR(I,J),PI(I,J),J=1,NOLT),I=1,NOLT)
112     FORMAT(//3E20.8,/,3E20.8,/)
        TYPE 112,((DWR(J,K),DWI(J,K),K=1,NOLT),J=1,NP)
115     VPR=1.0
        VHR=1.0
        DO 150 I=1,NOLT
        WPR(I)=0.0
        WPI(I)=0.0
        WHR(I)=0.0
        WHI(I)=0.0
        WWR(I)=0.0
        WWI(I)=0.0
        DO 140 J=1,NP
        VPI=(X(J)+.5)*RF
        VWR=VWD*COS(WK*(X(J)-PPA))
        VWI=-VWD*SIN(WK*(X(J)-PPA))
        WHR(I)=WHR(I)+VHR*DWR(J,I)
        WHI(I)=WHI(I)-VHR*DWI(J,I)
        WPR(I)=WPR(I)+VPR*DWR(J,I)+VPI*DWI(J,I)
        WPI(I)=WPI(I)-VPR*DWI(J,I)+VPI*DWR(J,I)
        WWR(I)=WWR(I)+VWR*DWR(J,I)+VWI*DWI(J,I)
140     WWI(I)=WWI(I)-VWR*DWI(J,I)+VWI*DWR(J,I)
        IF (JEST) 141,150,141
141     TYPE 142,WHR(I),WHI(I),WPR(I),WPI(I)
142     FORMAT(/4E18.8)
```

```
150     CONTINUE
        DO 200 I=1,NOLT
        PA(I)=0.0
        PB(I)=0.0
        HA(I)=0.0
        HB(I)=0.0
        WA(I)=0.0
        WB(I)=0.0
        DO 200 J=1,NOLT
        PA(I)=PA(I)+PR(I,J)*WPR(J)-PI(I,J)*WPI(J)
        PB(I)=PB(I)+PR(I,J)*WPI(J)+PI(I,J)*WPR(J)
        HA(I)=HA(I)+PR(I,J)*WHR(J)-PI(I,J)*WHI(J)
        HB(I)=HB(I)+PR(I,J)*WHI(J)+PI(I,J)*WHR(J)
        WA(I)=WA(I)+PR(I,J)*WWR(J)-PI(I,J)*WWI(J)
200     WB(I)=WB(I)+PR(I,J)*WWI(J)+PI(I,J)*WWR(J)
        TYPE 201,(HA(I),HB(I),PA(I),PB(I),WA(I),WB(I),I=1,NOLT)
201     FORMAT(/6E12.5)
        AL(1)=PO2*2.-TFLAP-SNTF
        AL(2)=PO2-.5*TFLAP-.5*XFLAP*SNTF
        DO 35 N=3, NOLT
35      AL(N)=.5*(SIN(N*TFLAP)/N-SIN(N-2)*TFLAP)/(N-2))
        AM(1)=-PO2+.5*TFLAP+SNTF-.5*SNTF*XFLAP
        AM(2)=(SNTF**3)/3.
        AM(3)=-.5*PO2+.25*TFLAP-.0625*SIN(4.*TFLAP)
        DO 36 N=4, NOLT
36      AM(N)=.25*(SIN(N-3)*TFLAP)/(N-3)-SIN((N+1)*TFLAP)/(N+1))
        DO 1102 I=1,NOLT
1102    AL(I)=AM(I)-XFLAP*AL(I)
        CMHR=PO2/4.*(HA(3)-HA(2))
        CMHI=PO2/4.*(HB(3)-HB(2))
        CLHR=2.*PO2*(HA(1)+.5*HA(2))
        CLHI=2.*PO2*(HB(1)+.5*HB(2))
        CHHR=0.
        CHHI=0.
        DO 1201 I=1,NOLT
        CHHR=CHHR-.5*HA(I)*AL(I)
1201    CHHI=CHHI-.5*HB(I)*AL(I)
        TYPE 300
300     FORMAT(/30H0    HEAVING    OSCILLATION      )
        TYPE320,CLHR,CLHI,CMHR,CMHI,CHHR,CHHI
320     FORMAT(//$CLR=$,F8.4,$     CLI=$,F8.4,4X,$CMR=$,F8.4,4X,
       1$CMI=$,F8.4,/$CHR=$,F8.4,4X,$CHI=$,F8.4)
        CALL COMPR(CLHR,CLHI,RF)
        TYPE 350
350     FORMAT(/30H0   PITCHING OSCILLATION        )
        CMPR=PO2/4.*(PA(3)-PA(2))
        CMPI=PO2/4.*(PB(3)-PB(2))
        CLPR=2.*PO2*(PA(1)+.5*PA(2))
        CLPI=2.*PO2*(PB(1)+.5*PB(2))
        CHPR=0.
        CHPI=0.
        DO 1202 I=1,NOLT
        CHPR=CHPR-.5*PA(I)*AL(I)
```

```
1202   CHPI=CHPI-.5*PB(I)*AL(I)
       TYPE 320,CLPR,CLPI,CMPR,CMPI,CHPR,CHPI
       CMWR=PO2/4.*(WA(3)-WA(2))
       CMWI=PO2/4.*(WB(3)-WB(2))
       CLWR=2.*PO2*(WA(1)+.5*WA(2))
       CLWI=2.*PO2*(WB(1)+.5*WB(2))
       CWHR=0.
       CWHI=0.
       DO 1204 I=1,NOLT
       CWHR=CWHR-.5*WA(I)*AL(I)
1204   CWHI=CWHI-.5*WB(I)*AL(I)
       TYPE 351
351    FORMAT(//$IN WAVES.$)
       TYPE 320,CLWR,CLWI,CMWR,CMWI,CWHR,CWHI
360    FORMAT(//10H  CL REAL=,5XF12.8,10H  CL IMAG=,5XF12.8,/,
      110H  CM REAL=,5XF12.8,10H  CM IMAG=5XF12.8,/)
       TYPE 997,   CLHR,CLHI,CMHR,CMHI,RF,FR,D,NP,NOLT
       TYPE 997,   CLPR,CLPI,CMPR,CMPI,RF,FR,D,NP,NOLT
997    FORMAT(/4F14.8,3F4.2,2I2)
41     CONTINUE
       GO TO 1
501    CONTINUE
       STOP
       END
```

APPENDIX D: PROGRAM TO COMPUTE HYDRODYNAMIC COEFFICIENT FOR A FOIL WITH AN OSCILLATING FLAP.

$$C_{L_{\dot{h}}} , C_{L_P} , C_{L_B}$$

$$C_{M_{\alpha\dot{h}}} , C_{M_{\alpha P}} , C_{M_{\alpha B}}$$

$$C_{M_{\beta\dot{h}}} , C_{M_{AP}} , C_{M_{\beta B}}$$

```
C       TWO DIMENSIONAL PROGRAMME
        DIMENSION VFR(2Ø),VFI(2Ø),FKR(2Ø),FKI(2Ø),TT(4),WFR(1Ø)
       1,WFI(1Ø),FA(1Ø),FB(1Ø),AM(1Ø)
        DIMENSION AL(1Ø),CR(1Ø),CI(1Ø),GR(32),GI(32),THETA(32),GX(32),
       1XO(32),DWR(2Ø,6),DWI(2Ø,6),DMR(6,6),DMI(6,6),X(2Ø)
       2PA(1Ø),PB(1Ø),HA(1Ø),HB(1Ø),PR(6,6),PI(6,6)
       3,WPR(1Ø),WPI(1Ø),WHR(1Ø),CPVC(1Ø),BANGL(2Ø),NLE1(2Ø),NTE1(
       42Ø),WHI(1Ø)
        COMMON XO,GR,GI,PO2, T,JEST,IM,IFF,D,FR,NT,S1,S2,S3R,S3I,S4R,S4I,
       1A1,A2,A3R,A3I,A4R,A4I
        PO2=1.57Ø79633
        TI=-.5
        TYPE 1ØØ1
1ØØ1    FORMAT(/$  INPUT DATA FROM FILE:$)
        CALL OPENR(5)
        CALL READB(5,NP)
999     FORMAT(I5)
998     FORMAT(F1Ø.4)
1       CALL READB(5,NOLT)
        CALL READB(5,NT)
        CALL READB(5,JEST)
        CALL READB(5,IM)
        CALL READB(5,IFF)
        CALL READB(5,RF)
        CALL READB(5,FR)
        CALL READB(5,D)
        CLOSE
3       FORMAT (5I5,3F1Ø.5)
        ACCEPT 4 ,CFLAP,NFLP,NLE,NTE
        IF(NOLT) 5Ø1,5Ø1,5ØØ
5ØØ     TYPE 2
2       FORMAT(/62H1 IMAGE IFF      K    FROUDE NUM   DEPTH IN SEMCHDS   NP
       1   NOLT    )
        TYPE 5,IM,IFF,RF,FR,D,NP,NOLT
5       FORMAT(/2I5,3F1Ø.4,12X,2I3)
        XFLAP=1.-2.*CFLAP
        NP2=NP/2
        PI2=4.*PO2
        DO 31 J=1,NP2
        DX=COS(J*PI2/(NP+1.))
        X(J)=(-(XFLAP+1.)*DX+XFLAP-1.)/2.
        I=NP+1-J
31      X(I)=((-XFLAP+1.)*DX+XFLAP+1.)/2.
        DO 32 L=1,NP
        BANGL(L)=ANGL (-X(L),SQRT(1.-X(L)*X(L)))/(2.*PO2)
        NLE1(L)=NLE
32      NTE1(L)=NTE
        TFLAP=ANGL (-XFLAP,SQRT(1.-XFLAP*XFLAP))
        TYPE 9ØØ1,CFLAP
```

```
9ØØ1   FORMAT(///$   FLAP CHORD$,F8.2,5X,$FRACTION OF AIRFOIL CHORD$)
       SNTF=SIN(TFLAP)
       DO 1ØØ2 I=1,NP
1ØØ2   XO(I)=-COS(PO2*2.*BANGL(I))-XFLAP
       CALL KER2(RF,NP)
       DO 1ØØ3 I=1,NP
       FKI(I)=GI(I)*SNTF
1ØØ3   FKR(I)=(GR(I)-1./(4.*PO2*XO(I)))*SNTF
       CNST=-4.+2.*ALOG(1.-XFLAP**2)+2.*XFLAP*ALOG((1.+XFLAP)/(1.-XFLAP))
       PC=+1./(4.*PO2*SNTF)
       DO 1ØØ J=1,NP
       ANGLE=BANGL(J)
       NLE=NLE1(J)
       NTE=NTE1(J)
4      FORMAT(F1Ø.4,3I1Ø)
       THMAX=PO2*2.*ANGLE
       X(J)=-COS(THMAX)
       DO 6 I=1,NLE
       THETA(I)=THMAX/2.*(1.-GN(I,NLE))
6      XO(I)=X(J)+COS(THETA(I))
       DO 7 K=1,NOLT
       CR(K)=Ø.Ø
7      CI(K)=Ø.Ø
       CALL KER2(RF,NLE)
       DO 1Ø I=1,NLE
       AL(1)=COS(THETA(I)/2.)/SIN(THETA(I)/2.)
       DO 71 N=2, NOLT
71     AL(N)=SIN((N-1)*THETA(I))
       CW=WN(I,NLE)*THMAX/2.*AL(2)
       DO 8 K=1,NOLT
       CR(K)=AL(K)*CW*GR(I)+CR(K)
8      CI(K)=AL(K)*CW*GI(I)+CI(K)
       IF (J-5) 1Ø,9,1Ø
9      IF(JEST) 8Ø,1Ø,8Ø
8Ø     TYPE 22,XO(I),GR(I),GI(I),CW,(AL(K),K=1,3),(CR(K),K=1,3)
1Ø     CONTINUE
       DO 15 I=1,NTE
       THETA(I)=THMAX+(PO2-THMAX/2.)*(1.-GN(I,NTE))
15     XO(I)=X(J)+COS(THETA(I))
       CALL KER2(RF,NTE)
       DO 25 I=1,NTE
       AL(1)=COS(THETA(I)/2.)/SIN(THETA(I)/2.)
       DO 151 N=2, NOLT
151    AL(N)=SIN((N-1)*THETA(I))
       CW=WN(I,NTE)*(PO2-THMAX/2.)*SIN(THETA(I))
       DO 2Ø K=1,NOLT
       CR(K)=AL(K)*CW*GR(I)+CR(K)
2Ø     CI(K)=AL(K)*CW*GI(I)+CI(K)
       IF (JEST) 3Ø1,25,3Ø1
3Ø1    IF (J-5) 25,21,25
```

```
21      TYPE 22,XO(I),GR(I),GI(I),CW,(AL(K),K=1,3),(CR(K),K=1,3)
22      FORMAT(//5E12.4,/,5E12.4,/)
25      CONTINUE
        CPVC(1)=TI
        DO 251 N=2, NOLT
251     CPVC(N)=TI*COS((N-1)*THMAX)
        DO 26 K=1,NOLT
        DWR(J,K)=CR(K)+CPVC(K)
26      DWI(J,K)=CI(K)
        IF(J-5)291,27,291
27      IF(JEST)28,291,28
28      TYPE 29,(CPVC(I),I=1,3)
29      FORMAT(/3E20.8)
291     ALNXC=ALOG((X(J)-XFLAP)**2)
        VFR(J)=0.
        VFI(J)=0.
        TT(1)=0.
        TT(2)=THMAX
        TT(3)=TFLAP
        TT(4)=PO2*2.
        IF (TT(3)-TT(2)) 1011,1012,1012
1011    VFR(J)=1.
        VFI(J)=(X(J)-XFLAP)*RF
        TT(2)=TFLAP
        TT(3)=THMAX
1012    CONTINUE
         DO 1020 K=1,3
        DO 1021 I=1,NFLP
        THETA(I)=TT(K)+.5*(TT(K+1)-TT(K))*(1.-GN(I,NFLP))
1021    XO(I)=X(J)+COS(THETA(I))
        CALL KER2(RF,NFLP)
        DO 1022 I=1,NFLP
        SN=SIN(THETA(I))
        CW=WN(I,NFLP)*.5*(TT(K+1)-TT(K))*SN
        ALNZC=ALOG((XFLAP+COS(THETA(I)))**2)
        VFR(J)=VFR(J)-PC*CW*((SN*(GR(I)-1./(PO2*4.*XO(I)))
       1-FKR(J))*ALNZC+SN/(PO2*4.*XO(I))*ALNXC)
1022    VFI(J)=VFI(J)-PC*CW*((SN*GI(I)-FKI(J))*ALNZC)
1020    CONTINUE
        VFR(J)=VFR(J)-PC*(-.5*X(J)*ALNXC +FKR(J)*CNST)
        VFI(J)=VFI(J)-PC*(FKI(J)*CNST)
100     CONTINUE
        DO 101 I=1,NOLT
        DO 101 K=1,NOLT
        DMR(I,K)=0.0
101     DMI(I,K)=0.0
        DO 110 I=1,NOLT
        DO 110 K=1,NOLT
        DO 110 J=1,NP
        DMR(I,K)=DWR(J,I)*DWR(J,K)+DWI(J,I)*DWI(J,K) +DMR(I,K)
```

```
109   DMI(I,K)=-DWI(J,I)*DWR(J,K)+DWR(J,I)*DWI(J,K)+DMI(I,K)
110   CONTINUE
      CALL CMPINV(DMR,DMI,PR,PI,NOLT,INDEX1)
      IF(JEST) 111,115,111
111   TYPE 112,((DMR(I,J),DMI(I,J),J=1,NOLT),I=1,NOLT)
      TYPE 112,((PR(I,J),PI(I,J),J=1,NOLT),I=1,NOLT)
112   FORMAT(//3E20.8,/,3E20.8,/)
      TYPE 112,((DWR(J,K),DWI(J,K),K=1,NOLT),J=1,NP)
115   VPR=1.0
      VHR=1.0
      DO 150 I=1,NOLT
      WFR(I)=0.
      WFI(I)=0.
      WPR(I)=0.0
      WPI(I)=0.0
      WHR(I)=0.0
      WHI(I)=0.0
      DO 140 J=1,NP
      VPI=(X(J)+.5)*RF
      WFR(I)=WFR(I)+VFR(J)*DWR(J,I)+VFI(J)*DWI(J,I)
      WFI(I)=WFI(I)-VFR(J)*DWI(J,I)+VFI(J)*DWR(J,I)
      WHR(I)=WHR(I)+VHR*DWR(J,I)
      WHI(I)=WHI(I)-VHR*DWI(J,I)
      WPR(I)=WPR(I)+VPR*DWR(J,I)+VPI*DWI(J,I)
140   WPI(I)=WPI(I)-VPR*DWI(J,I)+VPI*DWR(J,I)
      IF (JEST) 141,150,141
141   TYPE 142,WHR(I),WHI(I),WPR(I),WPI(I)
142   FORMAT(/4E18.8)
150   CONTINUE
      DO 200 I=1,NOLT
      FA(I)=0.
      FB(I)=0.
      PA(I)=0.0
      PB(I)=0.0
      HA(I)=0.0
      HB(I)=0.0
      DO 200 J=1,NOLT
      FB(I)=FB(I)+PR(I,J)*WFI(J)+PI(I,J)*WFR(J)
      FA(I)=FA(I)+PR(I,J)*WFR(J)-PI(I,J)*WFI(J)
      PA(I)=PA(I)+PR(I,J)*WPR(J)-PI(I,J)*WPI(J)
      PB(I)=PB(I)+PR(I,J)*WPI(J)+PI(I,J)*WPR(J)
      HA(I)=HA(I)+PR(I,J)*WHR(J)-PI(I,J)*WHI(J)
200   HB(I)=HB(I)+PR(I,J)*WHI(J)+PI(I,J)*WHR(J)
      TYPE 201 ,(HA(I),HB(I),PA(I),PB(I),FA(I),FB(I),I=1,NOLT)
201   FORMAT ((/,6E12.5))
      AFL=0.
      AFM=0.
      AFF=0.
      DO 1101 I=1,NFLP
      THTA=.5*TFLAP*(1.-GN(I,NFLP))
```

```
      SN=SIN(THTA)
      CS=COS(THTA)
      ALNXC=ALOG((XFLAP+CS)**2)
      CW=WN(I,NFLP)*.5*TFLAP*SN
      AFL=AFL+CW*(SN-SNTF)*ALNXC
      AFM=AFM+CW*SN*ALNXC*(-CS-XFLAP)
      THTA=TFLAP+(PO2-TFLAP*.5)*(1.-GN(I,NFLP))
      SN=SIN(THTA)
      CS=COS(THTA)
      ALNXC=ALOG((XFLAP+CS)**2)
      CW=WN(I,NFLP)*(PO2-TFLAP*.5)*SN
      AFL=AFL+CW*(SN-SNTF)*ALNXC
      AFF=AFF+CW*SN*ALNXC*(-CS-XFLAP)
11Ø1  CONTINUE
      AFL=AFL+SNTF*CNST
      AFM=(AFM+AFF)+(XFLAP+.5)*AFL
      AL(1)=PO2*2.-TFLAP-SNTF
      AL(2)=PO2-.5*TFLAP-.5*XFLAP*SNTF
      DO 35 N=3, NOLT
35    AL(N)=.5*(SIN(N*TFLAP)/N-SIN(N-2)*TFLAP)/(N-2))
      AM(1)=-PO2+.5*TFLAP+SNTF-.5*SNTF*XFLAP
      AM(2)=(SNTF**3)/3.
      AM(3)=-.5*PO2+.25*TFLAP-.0625*SIN(4.*TFLAP)
      DO 36 N=4, NOLT
36    AM(N)=.25*(SIN(N-3)*TFLAP)/(N-3)-SIN((N+1)*TFLAP)/(N+1))
      DO 11Ø2 I=1,NOLT
11Ø2  AL(I)=AM(I)-XFLAP*AL(I)
      CMHR=PO2/4.*(HA(3)-HA(2))
      CMHI=PO2/4.*(HB(3)-HB(2))
      CLHR=2.*PO2*(HA(1)+.5*HA(2))
      CLHI=2.*PO2*(HB(1)+.5*HB(2))
      CHHR=Ø.
      CHHI=Ø.
      DO 12Ø1 I=1,NOLT
      CHHR=CHHR-.5*HA(I)*AL(I)
12Ø1  CHHI=CHHI-.5*HB(I)*AL(I)
      TYPE 3ØØ
3ØØ   FORMAT(/30HO     HEAVING     OSCILLATION      )
      TYPE32Ø,CLHR,CLHI,CMHR,CMHI,CHHR,CHHI
32Ø   FORMAT(//$CLR=$,F8.4,$     CLI=$,F8.4,4X,$CMR=$,F8.4,4X,
     1$CMI=$,F8.4,/$CHR=$,F8.4,4X,$CHI=$,F8.4)
      CALL COMPR(CLHR,CLHI,RF)
      TYPE 35Ø
35Ø   FORMAT(/30H2    PITCHING OSCILLATION        )
      CMPR=PO2/4.*(PA(3)-PA(2))
      CMPI=PO2/4.*(PB(3)-PB(2))
      CLPR=2.*PO2*(PA(1)+.5*PA(2))
      CLPI=2.*PO2*(PB(1)+.5*PB(2))
      CHPR=Ø.
      CHPI=Ø.
      DO 12Ø2 I=1,NOLT
      CHPR=CHPR-.5*PA(I)*AL(I)
```

```
1202  CHPI=CHPI-.5*PB(I)*AL(I)
      TYPE 320,CLPR,CLPI,CMPR,CMPI,CHPR,CHPI
      TYPE 1301
1301  FORMAT(/,$       FLAP OSCILLATION$)
      CMFR=PO2/4.*(FA(3)-FA(2))-.5*PC*AFM
      CMFI=PO2/4.*(FB(3)-FB(2))
      CLFR=2.*PO2*(FA(1)+.5*FA(2))+PC*AFL
      CLFI=2.*PO2*(FB(1)+.5*FB(2))
      CHFR=-.5*PC*AFF
      CHFI=0.
      DO 1203 I=1,NOLT
      CHFR=CHFR-.5*FA(I)*AL(I)
1203  CHFI=CHFI-.5*FB(I)*AL(I)
      TYPE 320, CLFR,CLFI,CMFR,CMFI,CHFR,CHFI
      TYPE 997,   CLHR,CLHI,CMHR,CMHI,RF,FR,D,NP,NOLT
      TYPE 997,   CLPR,CLPI,CMPR,CMPI,RF,FR,D,NP,NOLT
997   FORMAT(/4F14.8,3F4.2,2I2)
      GO TO 1
501   CONTINUE
      STOP
      END
```

APPENDIX E: SUBROUTINES

```
      SUBROUTINE COMPR(CR,CI,RF)
      PI=3.1415193
      COR=CR/(2.*PI)
      COI=CI/(2.*PI)-RF/2.
      TYPE  10,   COR,COI
10    FORMAT(/30HO COMPARISON WITH CORNELL HR= ,F10.8,5H HI=,F10.8)
      RETURN
      END
      SUBROUTINE KER2(RF,NCP)
      DIMENSION GR(32),GI(32),XO(32)
      COMMON XO,GR,GI,PO2,X,JEST,IM,IFF,D,FR,NT,S1,S2,S3R,S3I,S4R,S4I,
     1A1,A2,A3R,A3I,A4R,A4I
      CM3R(A,B,C,D,P,Q) =A*C*P-A*D*Q-C*B*Q-P*B*D
      CM3I(A,B,C,D,P,Q) =A*C*Q+P*C*B+A*P*D-B*D*Q
      PI2=PO2*4.
3     DO 10 I=1,NCP
      XK=XO(I)*RF
      AB=(1.+ABS (XK)/XK)*PO2
      DK=2.*D*RF
      SINK=SIN (XK)
      COSK=COS (XK)
      EXPK=EXP (DK)
      CALL EXPINT(NT,0.0,XK,ER,EI)
      GR(I)=RF/PI2*CM3R (0.0,1.0,COSK     ,-SINK     ,ER,EI+AB )
      GI(I)=RF/PI2*CM3I (0.0,1.0,COSK     ,-SINK     ,ER,EI+AB )
      IF(IM)    4,10,4
4     IF (IFF) 6,5,6
5      CALL EXPINT(NT, DK, XK,ER,EI)
      GR(I)=GR(I)-XO(I)/(PI2*(XO(I)**2+4.*D**2))+RF/(2.*PI2)/EXPK
     1*CM3R (0.0,1.0,COSK     ,-SINK     ,ER,EI+2.*AB   )
      GI(I)=GI(I)+RF/(2.*PI2)/EXPK        *CM3I (0.0,1.0,COSK,
     1-SINK     ,ER,EI+2.*AB   )
      CALL EXPINT(NT,   -DK     ,XK,ER,EI)
      GR(I)=GR(I)+EXPK     /(PI2*2.)*CM3R (0.0,RF,COSK,
     1-SINK     ,ER,EI)
      GI(I)=GI(I)+EXPK     /(PI2*2.)*CM3I (0.0,RF,COSK,
     1-SINK     ,ER,EI)
      GO TO 10
6      CALL EXPINT(NT, DK     ,XK,ER,EI)
      GR(I)=GR(I)-XO(I)/(PI2*(XO(I)**2+4.*D**2))-RF/(2.*PI2)/EXPK
     1*CM3R (0.0,1.0,COSK     ,-SINK     ,ER,EI+2.*AB)
      GI(I)=GI(I)-RF/(2.*PI2)/EXPK        *CM3I (0.0,1.0,COSK,
     1-SINK     ,ER,EI+2.*AB   )
      CALL EXPINT(NT,    -DK     ,XK,ER,EI)
      GR(I)=GR(I)-EXPK      /(PI2*2.)*CM3R (0.0,RF,COSK,
     1-SINK     ,ER,EI)
      GI(I)=GI(I)-EXPK      /(PI2*2.)*CM3I (0.0,RF,COSK,
     1-SINK     ,ER,EI)
      CALL WAVES(RF,I)
```

```
10    CONTINUE
      RETURN
      END
      SUBROUTINE WAVES(RF,I)
      DIMENSION XO(32),GR(32),GI(32)
      COMMON XO,GR,GI,PO2,X,JEST,IM,IFF,D,FR,NT,S1,S2,S3R,S3I,S4R,S4I,
     1A1,A2,A3R,A3I,A4R,A4I
      CM3R(A,B,C,D,P,Q)=A*C*P-A*D*Q-C*B*Q-P*B*D
      CM3I(A,B,C,D,P,Q)=A*C*Q+P*C*B+A*P*D-B*D*Q
      PI4=PO2*8.
      DT=2.*D
      EX=XO(I)
      ABX=PO2*2.*(1.+ABS (EX)/EX)
2      F=RF*FR**2
      S1=-RF*(1.+1./(2.*F)*(1.+SQRT (4.*F+1.)))
      S2=-RF*(1.+1./(2.*F)*(1.-SQRT (4.*F+1.)))
      A1=RF*(1.+1./F+(3.+1./F)*1./SQRT (4.*F+1.))
      A2=RF*(1.+1./F-(3.+1./F)*1./SQRT (4.*F+1.))
      CALL EXPINT(NT,-S1*DT,-S1*EX,ER,EI)
      GR(I)=GR(I)+EXP (S1*DT)/PI4*CM3R (0.0,A1,  COS (S1*XO(I)),
     1SIN (S1*XO(I)),ER,EI+ABX)
      GI(I)=GI(I)+EXP (S1*DT)/PI4*CM3I (0.0,A1,    COS (S1*XO(I)),
     1SIN (S1*XO(I)),ER,EI+ABX)
      CALL EXPINT(NT,-S2*DT,-S2*EX,ER,EI)
      GR(I)=GR(I)+EXP (S2*DT)/PI4*CM3R (0.0,A2,    COS (S2*XO(I)),
     1SIN (S2*XO(I)),ER,EI+ABX)
      GI(I)=GI(I)+EXP (S2*DT)/PI4*CM3I (0.0,A2,    COS (S2*XO(I)),
     1SIN (S2*XO(I)),ER,EI+ABX)
      IF (F-.25) 10,100,20
100   STOP
10    R=1.-4.*F
      RAD =SQRT (R)
      S3= -RF*(1.-.5/F*(1.-RAD))
      S4= -RF*(1.-.5/F*(1.+RAD))
      A3= RF*(1.-1./F-(3.-1./F)/RAD)
      A4= RF*(1.-1./F+(3.-1./F)/RAD)
      Q3R=DT*S3
      Q3I=-EX*S3
      Q4R=DT*S4
      Q41=-EX*S4
      ABXM=(1.-ABS (EX)/EX)*PO2*2.
      CALL EXPINT(NT,Q3R,Q3I,ER,EI)
      GR(I)=GR(I)+EXP (-Q3R)/PI4*CM3R (0.0,A3,COS (-Q3I),SIN (-Q3I),
     1ER,EI+ABXM)
      GI(I)=GI(I)+EXP (-Q3R)/PI4*CM3I (0.0,A3,COS (-Q3I),SIN (-Q3I),
     1ER,EI+ABXM)
      CALL EXPINT(NT,Q4R,Q4I,ER,EI)
      GR(I)=GR(I)+EXP(-Q4R)/PI4*CM3R (0.0,A4,COS (-Q4I),SIN (-Q4I),
     1ER,EI-ABX)
      GI(I)=GI(I)+EXP (-Q4R)/PI4*CM3I (0.0,A4,COS (-Q4I),SIN (-Q4I),
     1ER,EI-ABX)
      GO TO 99
```

```
20    S3R=-RF*(1.-1./(2.*F))
      S3I=-RF*(1./(2.*F)*SQRT (4.*F-1.))
      S4R=S3R
      S4I=-S3I
      A3R=RF*(1.-1./F)
      A3I=RF*(3.-1/F)*1./SQRT (4.*F-1.)
       A4R=A3R
       A4I=-A3I
       Q3R=-S3R*DT-S3I*XO(I)
       Q3I=S3R*XO(I)-S3I*DT
       CALL EXPINT(NT,-Q3R,-Q3I,ER,EI)
       IF (F-.5) 21,22,22
21     AB=PO2*2.*(1.-ABS (Q3I)/Q3I)
       GO TO 23
22     AB=0.
23     GR(I)=GR(I)+EXP (Q3R)/PI4*CM3R (-A3I,A3R,COS (Q3I),SIN (Q3I),ER,
     1 EI+AB)
       GI(I)=GI(I)+EXP (Q3R)/PI4*CM3I (-A3I,A3R,COS (Q3I),SIN (Q3I),ER,
     1 EI*AB)
       Q4R=-S4R*DT-S4I*XO(I)
       Q4I=S4R*XO(I)-S4I*DT
       CALL EXPINT(NT,-Q4R,-Q4I,ER,EI)
       IF (F-.5) 24,25,25
24     AB=PO2*2.*(1.+ABS (Q4I))/Q4I
       GO TO 26
25     AB=0.
26     GI(I)=GI(I)+EXP (Q4R)/PI4*CM3I (-A4I,A4R,COS (Q4I),SIN (Q4I),ER,
     1 EI-AB)
       GR(I)=GR(I)+EXP (Q4R)/PI4*CM3R (-A4I,A4R,COS (Q4I),SIN (Q4I),ER,
     1 EI+AB)
99     RETURN
      END
C      COMPLEX MATRIX INVERSION

       SUBROUTINE CMPINV(A,B,C,D,N,INDEX1)

       DIMENSION A(6,6),B(6,6),C(6,6),D(6,6),PIVOT(25),INDEX(50),
     1IPIVOT(25),SPACE(200)
       COMMON SPACE,PIVOT,INDEX ,IPIVOT

C      SAVE A AND INVERT -A.

5      M=N
7      L=1
10     DO 30 I=1,M
20     DO 30 J=1,M
30     D(I,J)=-A(I,J)
40     CALL MATINV (D,M,DUMMY,0,DUMMY)
```

```
C       CHECK IF A WAS NON-SINGULAR.

50      DO 70 I=1,M
60      IF (IPIVOT(I)-1) 250, 70, 250
70      CONTINUE

C       COMPUTE C=INVERSE OF (A+BA(INVERSE)B).

80      CALL SQMULT (B,D,D,M)
90      CALL SQMULT (D,B,C,M)
100     DO 120 I=1,M
110     DO 120 J=1,M
120     C(I,J)=A(I,J)-C(I,J)
130     CALL MATINV (C,M,DUMMY,0,DUMMY)

C       CHECK THAT C EXISTS.

140     DO 170 I=1,M
150     IF (IPIVOT(I)-1) 155, 170, 155

C       INVERSE DOES NOT EXIST, SET SIGNAL AND RETURN.
155     INDEX1=2
160     RETURN
170     CONTINUE

C       COMPUTE D=-CBA(INVERSE).

180     CALL SQMULT (C,D,D,M)
220     GO TO (230,330), L

C       SUCCESSFUL INVERSION

230     INDEX1=1
240     RETURN


C       A IS SINGULAR, SO INTERCHANGE A AND B AND TRY AGAIN.
250     DO 300 I=1,M
280     DO 300 J=1,M
290     DUMMY=A(I,J)
        A(I,J)=B(I,J)
300     B(I,J)=DUMMY
305     IF (L-2) 310, 370, 370
310     L=2
        GO TO 10

C       INTERCHANGE A AND B, C AND D WITH CHANGED SIGNS.
```

```
330     DO 350 I=1,M
340     DO 350 J=1,M
        DUMMY=A(I,J)
        A(I,J)=B(I,J)
        B(I,J)=DUMMY
        DUMMY=-C(I,J)
        C(I,J)=-D(I,J)
350     D(I,J)=DUMMY
360     GO TO 230

C       A AND B BOTH SINGULAR. CANNOT FIND INVERSE. SET SIGNAL AND RETURN.
370     INDEX1=3
380     RETURN
        END
C       MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS

        SUBROUTINE MATINV(A,N,B,M,DETERM)

        DIMENSION IPIVOT(25),A(6,6),B(25,1),INDEX(25,2),PIVOT(25),
      1 SPACE(200)
        COMMON SPACE,PIVOT,INDEX,IPIVOT
        EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)

C       INITIALIZATION

10      DETERM=1.0
15      DO 20 J=1,N
20      IPIVOT(J)=0
30      DO 550 I=1,N

C       SEARCH FOR PIVOT ELEMENT

40      AMAX=0.0
45      DO 105 J=1,N
50      IF (IPIVOT(J)-1) 60, 105, 60
60      DO 100 K=1,N
70      IF (IPIVOT(K)-1) 80, 100, 740
80      IF (ABS (AMAX)-ABS (A (J,K))) 85, 100, 100
85      IROW=J
90      ICOLUM=K
95      AMAX=A(J,K)
100     CONTINUE
105     CONTINUE
110     IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1

C       INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
```

```
130     IF (IROW-ICOLUM) 140, 260, 140
140     DETERM=-DETERM
150     DO 200 L=1,N
160     SWAP=A(IROW,L)
170     A(IROW,L)=A(ICOLUM,L)
200     A(ICOLUM,L)=SWAP
205     IF(M) 260, 260, 210
210     DO 250 L=1, M
220     SWAP=B(IROW,L)
230     B(IROW,L)=B(ICOLUM,L)
250     B(ICOLUM,L)=SWAP
260     INDEX(I,1)=IROW
270     INDEX(I,2)=ICOLUM
310     PIVOT(I)=A(ICOLUM,ICOLUM)
320     DETERM=DETERM*PIVOT(I)

C       DIVIDE PIVOT ROW BY PIVOT ELEMENT

330     A(ICOLUM,ICOLUM)=1.0
340     DO 350 L=1,N
350     A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
355     IF(M) 380, 380, 360
360     DO 370 L=1,M
370     B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)

C       REDUCE NON-PIVOT ROWS

380     DO 550 L1=1,N
390     IF(L1-ICOLUM) 400, 550, 400
400     T=A(L1,ICOLUM)
420     A(L1,ICOLUM)=0.0
430     DO 450 L=1,N
450     A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
455     IF(M) 550, 550, 460
460     DO 500 L=1,M
500     B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
550     CONTINUE

C       INTERCHANGE COLUMNS
```

```
600     DO 710 I=1,N
610     L=N+1-I
620     IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
630     JROW=INDEX(L,1)
640     JCOLUM=INDEX(L,2)
650     DO 705 K=1,N
660     SWAP=A(K,JROW)
670     A(K,JROW)=A(K,JCOLUM)
700     A(K,JCOLUM)=SWAP
705     CONTINUE
710     CONTINUE
740     RETURN
        END
C       SQUARE MATRIX MULTIPLICATION

        SUBROUTINE SQMULT(A,B,C,N)

        DIMENSION A(6,6),B(6,6),C(6,6),D(6,6),COLUMN(25),
      1 SPACE(200)
        COMMON SPACE,COLUMN

5       M=N
10      DO 50 J=1,M
20      DO 25 K=1,M
25      COLUMN(K)=B(K,J)
30      DO 50 I=1,M
35      C(I,J)=0.0
40      DO 50 K=1,M
50      C(I,J)=C(I,J)+A(I,K)*COLUMN(K)
60      RETURN
        END
        FUNCTION GN(I,N)
        DIMENSION G(32)
1      IF(N-6) 2,60,2
2      IF(N-8) 3,8,3
3      IF(N-10) 4,10,4
4      IF (N-16) 6,16,6
6      IF (N-32) 7,32,7
60      G(1)=.9324695142
        G(2)=.6612093865
        G(3)=.238619161
        IF (I-3) 61,61,62
61      GN=G(I)
        GO TO 7
62      M=7-I
        GN=-G(M)
        GO TO 7
```

```
8       G(1)=.9602808565
        G(2)=.7966664774
        G(3)=.5255324099
        G(4)=.1834346425
        IF (I-4) 9,9,89
9       GN=G(I)
        GO TO 7
89      M=9-I
        GN=-G(M)
        GO TO 7
10      G(1)=.973906529
        G(2)=.8650633667
        G(3)=.6794095683
        G(4)=.43339539
        G(5)=.1488743389
        IF (I-5) 11,11,12
11      GN=G(I)
        GO TO 7
12      M=11-I
        GN=-G(M)
        GO TO 7
16      G(1)=.989400935
        G(2)=.944575
        G(3)=.8656312
        G(4)=.7554044
        G(5)=.617862
        G(6)=.4580167B
        G(7)=.2816035
        G(8)=.0950125
        IF (I-8) 17,17,18
17      GN=G(I)
        GO TO 7
18      M=17-I
        GN=-G(M)
        GO TO 7
32      G(1)=.9972639
        G(2)=.985611512
        G(3)=.9647623
        G(4)=.9305
        G(5)=.89632116
        G(6)=.849367614
        G(7)=.7944838
        G(8)=.732182119
        G(9)=.663044267
        G(10)=.5877157577
        G(11)=.506899909
        G(12)=.42135128
        G(13)=.331868602
        G(14)=.239287362
        G(15)=.144471962
        G(16)=.048307666
        IF (I-16) 33,33,34
```

```
33      GN=G(I)
        GO TO 7
34      M=33-I
        GN=-G(M)
        GO TO 7
7       RETURN
        END
        FUNCTION WN(I,N)
        DIMENSION W(32)
1       IF(N-6) 2,60,2
2       IF (N-8) 3,8,3
3       IF (N-10) 4,10,4
4      IF(N-16) 6,16,6
6       IF(N-32) 7,32,7
60      W(1)=.1713244924
        W(2)=.3607615731
        W(3)=.4679139346
        IF (I-3) 61,61,62
61      WN=W(I)
        GO TO 7
62      M=7-I
        WN=W(M)
        GO TO 7
8       W(1)=.1012285363
        W(2)=.2223810345
        W(3)=.3137066459
        W(4)=.3626837834
       IF (I-4) 9,9,89
9       WN=W(I)
        GO TO 7
89      M=9-I
        WN=W(M)
        GO TO 7
10      W(1)=.0666713443
        W(2)=.1494513492
        W(3)=.2190863625
        W(4)=.2692667193
        W(5)=.2955242247
        IF (I-5) 11,11,12
11      WN=W(I)
        GO TO 7
```

```
12      M=11-I
        GO TO 7
16      W(1)=.027152459
        W(2)=0.062253524



        W(3)=.0951585117
        W(4)=.1246289713
        W(5)=.1495959888
        W(6)=.1691565194
        W(7)=.182603415
        W(8)=.1894506105
        IF (I-8) 17,17,18
17      WN=W(I)
        GO TO 7
18      M=17-I
        WN=W(M)
        GO TO 7
32      W(1)=.00701861
        W(2)=.016274395
        W(3)=0.02539207
        W(4)=.034273863
        W(5)=.042835898
        W(6)=.050998059
        W(7)=.058684093
        W(8)=.065822223
        W(9)=.072345794
        W(10)=.078193896
        W(11)=.083311924
        W(12)=.087652093
        W(13)=.091173879
        W(14)=.0938444
        W(15)=.093872
        W(16)=.099654009
        IF (I-16) 33,33,34
33      WN=W(I)
        GO TO 7
34      M=33-I
        WN=W(M)
        GO TO 7
7       RETURN
        END
C    MODIFIED PROGRAM FOR EXPONENTIAL INTEGRAL
       SUBROUTINE EXPINT(NT,X,Y,ER,EI)
       R=X**2+Y**2
1      PI=3.14159265
```

```
50    SUMR=X
      SUMI=Y
      TEMR=X
      TEMI=Y
      EN=1.0
53    EN=EN+1.0
      FACT=(EN-1.0)/(EN**2)
      TERMR=(TEMR*X-TEMI*Y)*FACT
      TERMI=(TEMI*X+TEMR*Y)*FACT
      TEMR=TERMR
      TEMI=TERMI
      SUMR=SUMR+TERMR
      SUMI=SUMI+TERMI
      IF(R*FACT**2-.001) 60,60,53
60    ER=.5772157+.5*ALOG(R)+SUMR
      EI=-PI+ ANGL(X,Y)+SUMI
      NT=EN
      RETURN
      END
      FUNCTION ANGL(X,Y)
C      ANGL BETWEEN 0. AND 2.
      IF (Y) 10,20,30
10    ANGL=ATAN(-X/Y)+4.721388975
      RETURN
20    IF(X) 21,22,22
21    ANGL=3.14159265
      RETURN
22    ANGL=0.
      RETURN
30    ANGL=ATAN(-X/Y)+1.57079633
      RETURN
      END
      STOP
```

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13. ABSTRACT
In this Report we discuss (1) the development of a digital computer program to compute hydrofoil loads and (2) some aspect of hydrofoil control. The program computes the lift, pitching moment, and flap hinge moment on a two-dimensional hydrofoil with a trailing edge flap operating near a free surface with waves. The computational approach involves the numerical solution of an integral equation relating an upwash distribution to a kernel function and pressure distribution. The pressure distribution is expanded in a truncated Glauert series; the integration is carried out numerically using a Gaussian quadrature; and the coefficients of the Glauert series are evaluated by a minimum error collocation method.

The control problem investigated involves the positioning of a pivoted hydrofoil by means of a servo-controlled trim tab. When the foil is pivoted at its quarter chord point and control is implemented solely by means of a servo tab, the system is virtually uncontrollable. However, by pivoting the foil off the quarter chord point or by augmenting the servo tab with a servo attached directly to the foil, the system can be controlled. ( )

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