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The Use of Discrete Dyadic Wavelets in Image Processing

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Abstract—In this paper, using the discrete Walsh transform, we construct orthogonal and biorthogonal wavelets for complex periodic sequences similar to those studied earlier for the Cantor group. Results of numerical experiments demonstrate the effectiveness of the use of constructed discrete wavelets in image processing.

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1. DYADIC WAVELETS IN $\mathbb{C}_N$

For a positive integer number n we set $N = 2^n$ and denote $\mathbb{Z}_N = \{0, 1, \dots, N-1\}$. The set $\mathbb{Z}_N$ is an Abelian group, where the operation of bitwise addition modulo 2 is defined by the formula

$$k \oplus j := \sum_{\nu=0}^{n-1} |k_\nu - j_\nu|\, 2^\nu, \quad k_\nu, j_\nu \in \{0, 1\};$$

$k = \sum\limits_{\nu=0}^{n-1} k_\nu 2^\nu$ and $j = \sum\limits_{\nu=0}^{n-1} j_\nu 2^\nu$. The space $\mathbb{C}_N$ consists of complex sequences

$$x = (\dots, x(-1), x(0), x(1), x(2), \dots)$$

such that $x(j+N) = x(j)$ for any $j \in \mathbb{Z}$. An arbitrary sequence $x \in \mathbb{C}_N$ is determined if values $x(j)$ are given for $j \in \mathbb{Z}_N$. Therefore we sometimes identify x with the vector $(x(0), x(1), \dots, x(N-1))$. The scalar product and the norm in $\mathbb{C}_N$ are defined by formulas

$$\langle x, y\rangle := \sum_{j=0}^{N-1} x(j)\overline{y(j)}, \quad \|x\| := \langle x, x\rangle^{1/2}.$$

In this Section, using the discrete Walsh transform, we construct for the space $\mathbb{C}_N$ analogs of orthogonal and biorthogonal wavelets which were studied in [1–5] for the Cantor group and on the positive half-axis $\mathbb{R}_+$. The finite-dimensional case has a specific feature, namely, the proofs are simpler and there is more freedom in choosing parameters of the wavelet bases.

Denote by $\mathbb{Z}_+$ the set of integer nonnegative numbers. The system of Walsh functions $\{w_l \mid l \in \mathbb{Z}_+\}$ on the real axis $\mathbb{R}$ is defined by equalities

$$w_0(t) \equiv 1, \quad w_l(t) = \prod_{j=0}^{\mu} (w_1(2^j t))^{l_j}, \quad l \in \mathbb{N}, \quad t \in \mathbb{R},$$

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