The Use of Laser Altimetry in the Orbit and Attitude Determination of Mars Global Surveyor

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Abstract. Altimetry from the Mars Observer Laser Altimeter (MOLA) which is carried on board Mars Global Surveyor (MGS) has been analyzed for the period of the MGS mission known as Science Phasing Orbit 1 (SPO-1). We have used these altimeter ranges to improve orbit and attitude knowledge for MGS. This has been accomplished by writing crossover constraint equations that have been derived from short passes of MOLA data. These constraint equations differ from traditional crossover constraints and exploit the small foot print associated with laser altimetry.

Introduction

The full potential of many investigations in the space-geodetic and planetary sciences is limited by the accuracy with which spacecraft ephemerides can be computed. Such investigations include gravity estimation, limb occultation experiments and altimetric mapping of topography. Usually, ephemerides are produced by orbit solutions which rely on ground based tracking and possibly satellite-to-satellite tracking. For some Earth orbiting satellites, radar altimetry over oceans has proven to be of use when added to conventional tracking data in orbit solutions. This paper reports the first use of satellite altimeter data in an orbit solution (a) from a laser over solid topography and (b) for a spacecraft orbiting a planetary body other than the Earth. Laser altimetry has some advantages over radar altimetry which are very useful in orbit determination, especially for planetary orbiters. This paper also reports on a technique for exploiting the advantages of laser altimetry in orbit and attitude determination.

Radar vs. Laser Altimetry in Orbit and Attitude Solutions

Altimetry in the form of "crossovers" is what is usually used for orbit determination. Using altimetry in this form requires the least knowledge about the surface at which the altimeter is pointed. *Shum et al.* [1990] give a good description of the use of altimeter crossover constraint equations in orbit determination.

Typically, crossovers are thought of as a way to assess or improve only the radial component of a satellite's orbit. That is because in most cases, the horizontal location of the crossover and the pair of times at which each crossover occurs are predetermined from orbits that have been already well determined. Most crossover constraint equations that have been used with radar altimetry are formulated in terms of height discrepancies.

We have just finished analyzing the use of MOLA altimetry in MGS orbit solutions during he period known as SPO-1 (March 27 -April 28, 1998). The SPO-1 phase of the MGS mission and the characteristics of the MGS orbit during this period are described by *Albee* [1998]. The MOLA instrument [*Afzal*, 1994] has some attributes which are different from radar altimeters [*Zuber et al.*, 1992]. Because of this, we have formulated our crossover constraint equations to describe the minimum distance between two curves that

have been traced out on the planet surface instead of a height discrepancy at a predetermined point.

This requires that each crossover constraint equation takes into account a whole series of altimeter ranges from each of the two altimeter passes (ascending and descending) surrounding the location where a conventional (height discrepancy) crossover occurs. These ranges are "geolocated" (the planet fixed coordinates of the bounce points are determined) by using knowledge of the spacecraft orbit and instrument pointing. For each pair of nearly intersecting passes we determine the two planet fixed locations (and therefore the times) at which the passes come closest to intersecting. The distance between these two points is our crossover discrepancy.

In order to describe how these crossover constraints interact in an orbit solution which relies predominately on ground based tracking, it is important to point out that orbit determination is an iterative procedure. On each iteration, as the estimate of the orbit evolves, we redetermine the planet fixed locations (and equivalently the times) at which the pairs of passes of MOLA altimetry come closest to intersecting. This redetermination is necessary since MOLA is returning data with high horizontal resolution over sloping terrain. In fact, the sloping terrain is taken into account in our crossover constraint equation through the use of short passes of altimetry (as opposed to single points). The use of terrain allows the crossover constraints to contribute to the change in crossover locations (times) on subsequent iterations. The change in crossover locations facilitates the improvement in resolution of the horizontal components of the orbit as well as the radial component.

Another facet of our crossover modeling is that we do not assume that the altimeter is nadir pointing. Laser altimeters typically operate at off nadir angles. As a result we need to use spacecraft attitude and laser pointing information as part of the crossover computation (in the geolocation of bounce points). That makes it possible to have our crossover constraint equations contribute to the refinement of spacecraft attitude and laser pointing parameters. With radar altimetry, pointing information is obtained by analyzing the waveform of the return pulse e.g. [*Hayne et al.*, 1994].

The goals of this paper are to give a general description of the above formulation of crossover constraints and to demonstrate that this formulation can be used to exploit two unique possibilities of satellite laser altimetry: the refinement of attitude parameters directly from crossover analysis and the improvement in the resolution of the horizontal components of a satellite's orbit.

Formulation of Laser Crossover Constraint Equations

In a least squares batch estimator, a constraint equation can be treated just like an observation equation. In order to sum an observation into a set of normal equations, all that is needed is a residual (residual = observation - computed observation), a weighting factor and the partial derivatives of the computed observation with respect to all of the adjusting parameters. For crossovers, we let the observed value equal zero. The computed value is the crossover discrepancy (minimum distance between passes) and so the residual is the negative of the minimum distance. The weighting factor is the square of the reciprocal of a user assigned crossover standard deviation. This standard deviation corresponds roughly to the expected crossover discrepancy after adjustment. For MGS we have used a value of five meters for the standard deviation of each

crossover constraint equation. In order to explain how the partial derivatives are computed, it is necessary to give a few more details about the procedure for computing crossover discrepancies.

The geolocation process first finds the inertial coordinates of each bounce point. The laser pulse is rigorously traced from the satellite at transmit time along a ray the orientation of which is given by a combination of telemetered attitude information and adjustable attitude parameters. The bounce point is found along that ray at the point where the distance to the satellite at receive time combined with the distance back to the satellite at transmit time matches the travel time of the altimeter range. That gives the time tag and the inertial coordinates of the bounce point which are then rotated to planet fixed using IAU parameters [*Davies et al.*, 1996]. At this point time varying corrections such as tides can be considered so that the bounce point can be referenced to a mean surface. However, we have neglected this effect in our current study.

Once every range in a MOLA pass has been geolocated, the X, Y and Z planet fixed coordinates of the bounce points can be fit to polynomials (three polynomials) in time. These three polynomials describe a curve in space which can be compared to the curve (three more polynomials) on the other side of the crossover. The six polynomials are used to write a distance function. Given a pair of times (one from each pass) the distance between the passes at the associated points can be found. This function in two variables is easily minimized. The times giving the minimum distance are found and this gives the crossover discrepancy.

Using polynomials allows the computation of minimum distance to be an easily automated procedure. It also allows the computation of partial derivatives of minimum distance with respect to adjusted parameters to take surrounding topography into account. For a parameter, P, the partial derivative of minimum distance with respect to P is computed by summing up all products of the following form:

$$\frac{\partial(\min\_ dis \tan ce)}{\partial(polynomial\_coeff)} \times \frac{\partial(polynomial\_coeff)}{\partial(P)}$$

(for every coefficient of all six polynomials). The partial derivatives $\partial(min\_distance)/\partial(polynomial\_coeff)$ are computed numerically. In other words, each polynomial coefficient is altered slightly, a new minimum distance is recomputed (allowing times to readjust) and then the desired partial is represented by a difference quotient. The partial derivatives $\partial(polynomial\_coeff)/\partial P$ are easily computed once the partial derivatives $\partial X/\partial P$, $\partial Y/\partial P$ and $\partial Z/\partial P$ are known for each bounce point $[X,Y,Z]$ in each of the two passes that are being used for each crossover. For example, when a polynomial is fit to the $\partial X/\partial P$ for all the bounce points in the pass, the coefficients of this new polynomial are themselves partial derivatives of the form $\partial(polynomial\ coeff)/\partial P$. The partial derivatives $\partial X/\partial P$, $\partial Y/\partial P$, $\partial Z/\partial P$ are easily computed with information that is naturally available as part of the orbit determination process.

Orbit and Attitude Improvement for MGS Using Crossovers

All of the SPO-1 orbit solutions described in this section rely on ground based tracking including two-way ramped range [*Moyer*, 1995], two-way and three-way ramped Doppler [*Moyer*, 1987] and 1-way Doppler [*Moyer*, 1987]. We used

the GEODYN orbit determination and geodetic parameter estimation software [*Pavlis et al.*, 1998] for our orbit solutions. We demonstrate below that the addition of crossover constraint equations as described in the above sections improves these orbit solutions. We also demonstrate that when knowledge of instrument pointing has been refined using crossovers, the orbit solutions are further improved.

The SPO-1 period was subdivided into six orbit solutions (arcs) each covering a little over six days. The start and stop time of each arc was chosen so that adjacent arcs would overlap by twelve hours which is just larger than the orbital period for MGS during SPO-1 (11 hr 38 min). We will refer to three distinct solutions for these six arcs: SX0, SX5 and SX5A. These three solutions use the same data (ground tracking and altimetry) and solve for the same set of parameters. SX0 differs from SX5 only in the weight assigned to the 535 crossover constraint equations. In SX5, the standard deviation for the crossovers constraints was set at five meters. In SX0 the standard deviation for crossovers was set at one million meters (no practical contribution). SX5A differs from SX5 only in the values assigned to certain pointing parameters. In none of the three SX solutions was any type of pointing parameter adjusted. Three types of attitude information were used in all three solutions:

1. Telemetered quaternions that describe the spacecraft orientation.
2. A time tag bias for (1) above.
3. Constant roll, pitch and yaw offset parameters that describe the offset in orientation of the MOLA instrument to the spacecraft body. Preflight values of these are available.

Solution SX5A differs from SX5 only in the values for roll, pitch and yaw instrument pointing offset that were used. SX5 uses the preflight values. SX5A uses values for these that were adjusted from crossovers in a preliminary solution that is described in the next paragraph. Other preliminary solutions for orbits also solved for a (MOLA) observation timing bias. From these earlier orbit solutions we have adopted a value of 0.112 seconds. All of the SX solutions used this value.

We produced separate solutions for attitude and orbit parameters. This is because the crossovers that contribute most to attitude information (where the satellite is pointing well off nadir for either the ascending or descending pass) are the least desirable to use for orbit improvement. We made a solution for attitude parameters using only 279 of the most off nadir crossovers. In this solution, crossovers were allowed to contribute only to the solution of attitude parameters. The adjusted parameters were a telemetered spacecraft attitude timing bias and a constant offset in roll and pitch.

It is necessary to solve for a attitude timing bias. Our adjusted value is 1.160 seconds. By looking at tracks of geolocated altimetry from passes where the spacecraft had roll outs, it is easy to see that the telemetered attitude has a timing bias of at least one second. Even the discrepancies of crossovers from "quiet" passes improve with the application of the 1.160 second attitude timing bias. By comparing the results of SX5 and SX5A (below) it can be seen that our adjusted value of roll and pitch improve the orbit solution and therefore improves the geolocation process in two ways (better orbits and better pointing).

In this section we will gauge orbit quality in four ways:
1. Orbit overlap statistics
2. Formal standard deviations of adjusted parameters
3. Fit to ground tracking data
4. Crossover discrepancies

Table I shows the five 12 hour overlaps between the six arcs in SPO-1 for the three SX solutions. It shows that inclusion of MOLA altimetry improves total positioning of the satellite, mainly through horizontal improvement. The improvement is slightly better if MOLA is allowed to contribute pointing information. In evaluating the improvements it is important to note that MGS was in a highly eccentric orbit during SPO-1 and altimetry observations could only be made for approximately one half hour of each 11 hour 38 minute revolution. It is also worth noting that the altimeter only returned data during periapsis when the satellite was usually well tracked from the Earth. As a result, overlap discrepancies during these periapsis pass portions of the orbit tend to be smaller than elsewhere whether or not altimetry is included. The inclusion of MOLA data should be even more useful when it can be applied over the entire orbit (after the MGS orbit is circularized at the end of aerobraking).

Table II gives the formal standard deviation of the six initial kepler state parameters of the six arcs. As would be expected, the standard deviations are improved by the addition of constraint equations. However, it is interesting to note that the parameters that seem to be generally the most improved are inclination (I), right ascension of the node (Ω) and the argument of perigee (ω). These correspond to cross track (I and Ω) and along track (ω) components of the orbit and this correlates well with the results shown in table I. There is only one arc (5) for which the standard deviation of the semi-major axis (A) is dramatically improved. That corresponds to an improvement in the radial component and seems to correlate well with the only overlap for which there is a dramatic improvement in the radial component (overlap 4/12-4/418 between arcs 4 and 5). The agreement between tables (I) and (II) is encouraging.

The inclusion of crossover constraints into an orbit solution that contains observation equations from ground tracking can not improve the fit of the solution to the ground tracking observations. However, in these SPO-1 arcs the fit was not seriously degraded. The (approximately) 8200 1-way Doppler measurements were fit about 18% higher (from 0.182 Hz to 0.216 Hz) in the constrained solutions and the (approximately) 24000 2-way Doppler measurements were fit about 10% higher (from 0.0328 Hz to 0.0361 Hz). The (approximately) 3450 range measurements did not change in fit (about 30 meters). The range measurements are not very useful in this comparison since we solve for pass by pass range biases.

Table III gives the L1 and L2 discrepancies of the 1185 SPO-1 crossovers that occur under circumstances that are considered suitable for the computation of crossover discrepancies. The L2 statistic is just the Root Mean Square (RMS) of the 1185 crossovers and the L1 is the RMS of the crossovers within one standard deviation of the mean. These are presented for the three SX orbit solutions. In all cases the telemetered attitude timing bias of 1.160 seconds was applied. For each orbit solution the crossovers are presented with the pointing done two ways: with the preflight values of roll and pitch offsets and with the adjusted values. Either way there is improvement from SX0 to SX5 and from SX5 to SX5A. For each orbit solution there is improvement when the adjusted values of pointing are used. Table III further supports the claim that the crossovers have improved the orbits and that pointing adjustment has improved both the orbit and the geolocation.

Conclusions

1. Satellite laser altimetry in the form of crossover constraint equations described in this paper can be used to improve orbit and attitude solutions.
2. The horizontal resolution (small footprints) associated with laser altimetry, can be exploited to improve the horizontal aspects of a satellite orbit.
3. There is a 1.160 second timing bias associated with the telemetered attitude information on MGS.
4. There is a 0.112 second observation timing bias associated with the MOLA observations.
5. We have improved the geolocation of MOLA data by using MOLA data in orbit and attitude solutions.

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Table 1. Orbit overlap comparisons

Overlap	Radial (m)			Cross-Track (m)			Along-Track (m)			Total (m)		
	SX0	SX5	SX5A	SX0	SX5	SX5A	SX0	SX5	SX5A	SX0	SX5	SX5A
3/27 - 4/01	11.7	16.3	13.8	738	107	91	27	45	36	785	117	99
4/01 - 4/07	4.6	11.1	8.9	1385	413	419	623	179	184	1519	450	458
4/07 - 4/12	1.9	13.0	10.6	362	165	208	197	82	106	413	185	233
4/12 - 4/18	30.7	17.2	16.1	419	635	560	262	416	367	495	759	670
4/18 - 4/24	24.7	24.6	24.6	1158	496	251	883	326	146	1457	594	292
RMS	18.5	17.1	15.8	907	414	347	505	253	201	1045	486	402

Table 2. Kepler epoch state vector recovered sigmas

Arc	A (m)		e (10^{-8})		I (10^{-5} deg)		Ω (10^{-5} deg)		ω (10^{-5} deg)		M (10^{-6} deg)	
	SX0	SX5	SX0	SX5	SX0	SX5	SX0	SX5	SX0	SX5	SX0	SX5
3/27	14.3	14.2	34	33	117	47	47	24	30	10	365	362
4/01	0.6	0.6	23	20	92	37	38	15	31	12	19	16
4/07	2.4	2.1	11	8	60	31	22	11	25	13	80	65
4/12	2.5	2.2	13	9	103	36	38	13	52	18	44	35
4/18	5.6	1.1	17	9	55	27	19	10	32	16	31	13
4/24	3.5	3.1	17	9	38	31	12	10	27	22	97	91
RMS	16.1	14.9	51	42	202	87	78	36	83	38	390	381

Table 3. Crossover discrepancy

Pointing offset	L1 (m)			L2 (m)		
	SX0	SX5	SX5A	SX0	SX5	SX5A
Pre-flight	xx.x	xx.x	xx.x	xx.x	xx.x	xx.x
Adjusted	xx.x	xx.x	xx.x	xx.x	xx.x	xx.x