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The use of quantum-correlated D^0 decays for ϕ_3 measurement

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Abstract. We report the results of the Monte Carlo study of the method to determine the CKM angle ϕ_3 using Dalitz plot analysis of D^0 produced in $B^\pm \to DK^\pm$ decay. Our main goal is to find the optimal strategy for a model-independent ϕ_3 extraction. We find that the analysis using decays of CP-tagged D mesons only cannot provide a completely model-independent measurement in the case of a limited data sample. The procedure involving binned analysis of $B^\pm \to DK^\pm$ and $\psi(3770) \to (K^0_S\pi^+\pi^-)_D(K^0_S\pi^+\pi^-)_D$ decays is proposed which, in contrast, allows not only to reach the ϕ_3 precision comparable to an unbinned model-dependent fit, but also provides an unbiased measurement with currently available data.

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1 Introduction

A measurement of the angle ϕ_3 (γ) of the unitarity triangle using Dalitz plot analysis of $\overline{D}^0 \to K^0_S\pi^+\pi^-$ decay from $B^\pm \to DK^\pm$ process, introduced by Giri et al. [1] and the Belle Collaboration [2] and successfully implemented by BaBar [3] and Belle [4], presently offers the best constraints on this quantity. However, this technique is sensitive to the choice of the model used to describe the three-body D^0 decay. Currently, this uncertainty is estimated to be $\sim 10°$ and due to a large statistical error does not affect the precision of ϕ_3 measurement. As the amount of B factory data increases, though, this uncertainty will become a major limitation. Fortunately, a model-independent approach exists (see [1]), which uses the data of the τ-charm factory to obtain missing information about the D^0 decay amplitude.

In our previous study of the model-independent Dalitz analysis technique [5] we have implemented a procedure proposed by Giri et al. that uses decays of D meson in CP eigenstate (we denote them as D_{CP}) to $K^0_S\pi^+\pi^-$. Such decay can be obtained at the e^+e^- machine operated at the $\psi(3770)$ resonance, which decays to a pair of D mesons. The antisymmetry of the wave function of the $D\overline{D}$ state induces quantum correlations between the decay amplitudes of two D mesons. In particular, if one D meson is reconstructed in a CP eigenstate (such as $\pi^+\pi^-$ or $K^0_S\pi^0$), the other D meson is required to have the opposite CP parity. The procedure we have studied involves the division of the $\overline{D}^0 \to K^0_S\pi^+\pi^-$ Dalitz plots from flavor D^0, D_{CP} and $B^\pm \to DK^\pm$ decay into bins. The value of ϕ_3 is then obtained by solving the system of equations that includes the

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numbers of events in these bins. We have shown that this procedure allows to measure the phase ϕ_3 with the statistical precision only 30%–40% worse than in the unbinned model-dependent case. We did not attempt to optimize the precision and mainly considered a high-statistics limit with an aim to estimate the sensitivity of the future super B factory.

Decays $\psi(3770) \to D^0\overline{D}^0$ with both neutral D mesons decaying to $K^0_S\pi^+\pi^-$ (we will refer to these decays as $(K^0_S\pi^+\pi^-)^2$) have also been shown to include the information useful for a model-independent ϕ_3 measurement [6]. These decays, together with the CP-tagged $\overline{D}^0 \to K^0_S\pi^+\pi^-$ decays, are presently available at the CLEO-c experiment [7, 8]. The first analyses using data collected at $\psi(3770)$ resonance involve $\sim 400\,\mathrm{pb}^{-1}$ data set, while by the end of CLEO-c operation the integrated luminosity at the $\psi(3770)$ will reach $750\,\mathrm{pb}^{-1}$ [9, 10]. This corresponds to ~ 1000 CP-tagged $\overline{D}^0 \to K^0_S\pi^+\pi^-$ events and $(K^0_S\pi^+\pi^-)^2$ events. The actual numbers may vary by a factor of two depending on the details of the particular analysis. In this paper, we report on studies of the model-independent approach with a limited statistics of both $\psi(3770)$ and B data, using the $D_{CP} \to K^0_S\pi^+\pi^-$ and $(K^0_S\pi^+\pi^-)^2$ final states. The technique described can be applied to other three-body D^0 final states, such as $\pi^+\pi^-\pi^0$ state recently used by the BaBar Collaboration for a model-dependent ϕ_3 measurement [11], or $K^0_S K^+K^-$ state.

In Sect. 2 we remind the basic idea of the model-independent technique of ϕ_3 determination and introduce the notation. Section 3 is devoted to the binned analysis using D_{CP} data sample; we propose a way to reach the statistical sensitivity comparable to the model-dependent technique and discuss the limitations of this approach related to a limited charm data set. In Sect. 4, we discuss how

the $(K_S^0\pi^+\pi^-)^2$ sample can be utilized in a most efficient way, and obtain quantitative estimate of the statistical sensitivity of this approach.

2 Model-independent approach

The density of $\overline{D}^0 \to K_S^0\pi^+\pi^-$ Dalitz plot is given by the absolute value of the amplitude f_D squared:

$$p_D = p_D(m_+^2, m_-^2) = \left| f_D(m_+^2, m_-^2) \right|^2. \qquad (1)$$

The effects of charm mixing are not included in our formulas. For the currently expected ϕ_3 accuracy and present limits on parameters of D^0 mixing ($x_D, y_D \sim 0.01$ [12]), these effects can be safely neglected [13], although it is possible to take them into account if they appear to be significant for future precision measurements.

In the case of no CP-violation in D decay the density of the D^0 decay $\bar{p}_D$ equals

$$\bar{p}_D = |\bar{f}_D|^2 = p_D(m_-^2, m_+^2). \qquad (2)$$

Then the density of the D decay Dalitz plot from $B^\pm \to DK^\pm$ decay is expressed as

$$\begin{aligned} p_{B\pm} &= \left| f_D + r_B e^{i(\delta_B \pm \phi_3)} \bar{f}_D \right|^2 \\ &= p_D + r_B^2 \bar{p}_D + 2\sqrt{p_D \bar{p}_D}(x_\pm c + y_\pm s), \end{aligned} \qquad (3)$$

where $x_\pm, y_\pm$ include the value of ϕ_3 and other related quantities, the ratio r_B of the absolute values of interfering $B^+ \to \overline{D}^0 K^+$ and $B^+ \to D^0 K^+$ amplitudes (or their charge-conjugate partners), and the strong phase difference δ_B between these amplitudes:

$$x_\pm = r_B \cos(\delta_B \pm \phi_3); \quad y_\pm = r_B \sin(\delta_B \pm \phi_3). \qquad (4)$$

The functions c and s are the cosine and sine of the strong phase difference $\Delta\delta_D$ between the symmetric Dalitz plot points:

$$\begin{aligned} c &= \cos(\delta_D(m_+^2, m_-^2) - \delta_D(m_-^2, m_+^2)) = \cos\Delta\delta_D; \\ s &= \sin(\delta_D(m_+^2, m_-^2) - \delta_D(m_-^2, m_+^2)) = \sin\Delta\delta_D. \end{aligned} \qquad (5)$$

The phase difference $\Delta\delta_D$ can be obtained from the sample of D mesons in a CP-eigenstate, either CP-even or CP-odd, decaying to $K_S^0\pi^+\pi^-$. The Dalitz plot density of such decays is

$$p_{\rm CP} = |f_D \pm \bar{f}_D|^2 = p_D + \bar{p}_D \pm 2\sqrt{p_D \bar{p}_D}c \qquad (6)$$

(the normalization is arbitrary).

Another possibility is to use a sample where both D mesons (we denote them as D and D') from the $\psi(3770)$ meson decay into the $K_S^0\pi^+\pi^-$ state [6]. Since the $\psi(3770)$ is a vector, two D mesons are produced in a P-wave, and the wave function of the two mesons is antisymmetric. Then the four-dimensional density of two correlated Dalitz plots is

$$\begin{aligned} p_{\rm corr}&(m_+^2, m_-^2, m_+'^2, m_-'^2) \\ &= |f_D \bar{f}'_D - f'_D \bar{f}_D|^2 \\ &= p_D \bar{p}'_D + \bar{p}_D p'_D - 2\sqrt{p_D \bar{p}_D p'_D \bar{p}'_D}(cc' + ss'), \end{aligned} \qquad (7)$$

This decay is sensitive to both c and s for the price of having to deal with the four-dimensional phase space.

In a real experiment, one measures scattered data rather than a probability density. To deal with real data, the Dalitz plot can be divided into bins. In what follows, we show that using appropriate binning, it is possible to reach the statistical sensitivity of ϕ_3 measurement equivalent to the model-dependent approach.

3 Binned analysis with $D_{\rm CP}$ data

Assume that the Dalitz plot is divided into $2\mathcal{N}$ bins symmetrically to the exchange $m_-^2 \leftrightarrow m_+^2$. The bins are denoted by the index i ranging from $-\mathcal{N}$ to $\mathcal{N}$ (excluding 0); the exchange $m_+^2 \leftrightarrow m_-^2$ corresponds to the exchange $i \leftrightarrow -i$. Then the expected number of events in the bins of the Dalitz plot of D decay from $B^\pm \to DK^\pm$ is

$$\langle N_i \rangle = h_B \left[K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}(xc_i + ys_i) \right], \qquad (8)$$

where K_i is the number of events in the bins in the Dalitz plot of the D^0 in a flavor eigenstate, h_B is the normalization constant. Coefficients c_i and s_i, which include the information about the cosine and sine of the phase difference, are given by

$$c_i = \frac{\int_{\mathcal{D}_i} \sqrt{p_D \bar{p}_D} \cos(\Delta\delta_D(m_+^2, m_-^2))\, d\mathcal{D}}{\sqrt{\int_{\mathcal{D}_i} p_D\, d\mathcal{D} \int_{\mathcal{D}_i} \bar{p}_D\, d\mathcal{D}}}, \qquad (9)$$

s_i is defined similarly with cosine substituted by sine. Here $\mathcal{D}_i$ is the bin region over which the integration is performed. Note that $c_i = c_{-i}$, $s_i = -s_{-i}$ and $c_i^2 + s_i^2 \le 1$ (the equality $c_i^2 + s_i^2 = 1$ being satisfied if the amplitude is constant across the bin).

The coefficients K_i are obtained precisely from a very large sample of D^0 decays in the flavor eigenstate, which is accessible at B-factories. The expected number of events in the Dalitz plot of $D_{\rm CP}$ decay equals to

$$\langle M_i \rangle = h_{\rm CP} \left[K_i + K_{-i} + 2\sqrt{K_i K_{-i}}c_i \right], \qquad (10)$$

and thus can be used to obtain the coefficient c_i. As soon as the c_i and s_i coefficients are known, one can obtain x and y values (hence, ϕ_3 and other related quantities) by a maximum likelihood fit using (8).

Note that now the quantities of interest x and y (and consequently ϕ_3) have two statistical errors: one due to a finite sample of $B^\pm \to DK^\pm$ data, and the other due to $D_{\rm CP} \to K_S^0\pi^+\pi^-$ statistics. We will refer to these errors as B-statistical and $D_{\rm CP}$-statistical, respectively.

Obtaining s_i is a major problem in this analysis. If binning is fine enough, so that both the phase difference $\Delta\delta_D$ and the amplitude $|f_D|$ remain constant across the area of each bin, expressions (9) reduce to $c_i = \cos(\Delta\delta_D)$ and $s_i = \sin(\Delta\delta_D)$, so s_i can be obtained as $s_i = \pm\sqrt{1 - c_i^2}$. Using this equality if the amplitude varies will lead to the bias in the x, y fit result. Since c_i is obtained directly, and s_i is overestimated by the absolute value, the bias will mainly affect y determination, resulting in lower absolute values of y.

Our studies [5] show that the use of the equality $c_i^2 + s_i^2 = 1$ is satisfactory for the number of bins around 200 or more, which cannot be used with presently available D_{CP} data. It is therefore essential to find a relatively coarse binning (the number of bins being 10–20) which a) allows to extract s_i from c_i with low bias, and b) has the sensitivity to the ϕ_3 phase comparable to the unbinned model-dependent case.

Fortunately, both the a) and b) requirements appear to be equivalent. To determine the B-statistical sensitivity of a certain binning, let's define a quantity Q – a ratio of a statistical sensitivity to that in the unbinned case. Specifically, Q relates the number of standard deviations by which the number of events in bins is changed by varying parameters x and y, to the number of standard deviations if the Dalitz plot is divided into infinitely small regions (the unbinned case):

$$Q^2 = \frac{\sum_i \left(\frac{1}{\sqrt{F_i}} \frac{\mathrm{d}F_i}{\mathrm{d}x}\right)^2 + \left(\frac{1}{\sqrt{F_i}} \frac{\mathrm{d}F_i}{\mathrm{d}y}\right)^2}{\int_{\mathcal{D}} \left[\left(\frac{1}{\sqrt{|f_B|^2}} \frac{\mathrm{d}|f_B|^2}{\mathrm{d}x}\right)^2 + \left(\frac{1}{\sqrt{|f_B|^2}} \frac{\mathrm{d}|f_B|^2}{\mathrm{d}y}\right)^2\right] \mathrm{d}\mathcal{D}}, \quad (11)$$

where $f_B = f_D + (x + \mathrm{i}y)\bar{f}_D$, $F_i = \int_{\mathcal{D}_i} |f_B|^2 \mathrm{d}\mathcal{D}$.

Since the precision of x and y weakly depends on the values of x and y [5], we can take for simplicity $x = y = 0$. In this case one can show that

$$Q^2|_{x=y=0} = \sum_i (c_i^2 + s_i^2) N_i / \sum_i N_i. \quad (12)$$

Therefore, the binning which satisfies $c_i^2 + s_i^2 = 1$ (i.e. the absence of bias if s_i is calculated as $\sqrt{1 - c_i^2}$) also has the same sensitivity as the unbinned approach ($Q = 1$). The factor Q defined this way is not necessarily the best measure of the binning quality (the binning with higher Q can be insensitive to either x or y, which is impractical from the point of measuring ϕ_3), but it allows an easy calculation and correctly reproduces the relative quality for a number of binnings we tried in our simulation.

The optimal binning that gives the best ϕ_3 precision is naturally model-dependent, but our goal is to find the analysis procedure that should give an unbiased result for any reasonable variation of the D^0 amplitude (i.e. the fit procedure should be model-independent). In our studies we use the two-body amplitude obtained in the latest Belle ϕ_3 Dalitz analysis [4].

From the consideration above it is clear that a good approximation to the optimal binning is the one obtained from the uniform division of the strong phase difference $\Delta\delta_D$. In the half of the Dalitz plot $m_+^2 < m_-^2$ (i.e. the bin index $i > 0$) the bin $\mathcal{D}_i$ is defined by the condition

$$2\pi(i - 1/2)/\mathcal{N} < \Delta\delta_D(m_+^2, m_-^2) < 2\pi(i + 1/2)/\mathcal{N}, \quad (13)$$

and in the remaining part ($i < 0$) the bins are defined symmetrically. We will refer to this binning as $\Delta\delta_D$-binning. As an example, such a binning with $\mathcal{N} = 8$ is shown in Fig. 1a. Although the phase difference variation across the bin is small by definition, the absolute value of the amplitude can vary significantly, so the condition $c_i^2 + s_i^2 = 1$ is not satisfied exactly. The values of c_i and s_i in this binning are shown in Fig. 3c with crosses.

Figure 1b shows the division with $\mathcal{N} = 8$ obtained by continuous variation of the $\Delta\delta_D$-binning to maximize the factor Q. The sensitivity factor Q increases to 0.89 compared to 0.79 for $\Delta\delta_D$-binning.

We perform a toy MC simulation to study the statistical sensitivity of the different binning options. We use the amplitude from the Belle analysis [4] to generate decays of flavor D^0, D_{CP}, and D from $B^\pm \to DK^\pm$ decay to the $K_S^0\pi^+\pi^-$ final state according to the probability density given by (1), (6) and (3), respectively. In the present study we use the errors of parameters x and y rather than ϕ_3 as a measure of the statistical power since they are nearly independent of the actual values of ϕ_3, strong phase δ and amplitude ratio r_B. The error of ϕ_3 can be obtained from these numbers given the value of r_B. To obtain the B-statistical error we use a large number of D^0 and D_{CP} decays, while the generated number of D decays from the $B^\pm \to DK^\pm$ process ranges from 10^2 to 10^5. For each number of B decay events, 100 samples are generated, and the statistical errors of x and y are obtained from the spread of the fitted values. A study of the error due to D_{CP} statistics is performed similarly, with a large number of B decays, and the statistics of D_{CP} decays varied. Both errors are checked to satisfy the square root scaling.

The binning options used are $\Delta\delta_D$-binning with $\mathcal{N} = 8$ and $\mathcal{N} = 20$, as well as "optimal" binnings with maximized Q obtained from these two with a smooth variation of the bin shape. For comparison, we use the binnings with the uniform division into rectangular bins (with $\mathcal{N} = 8$ and
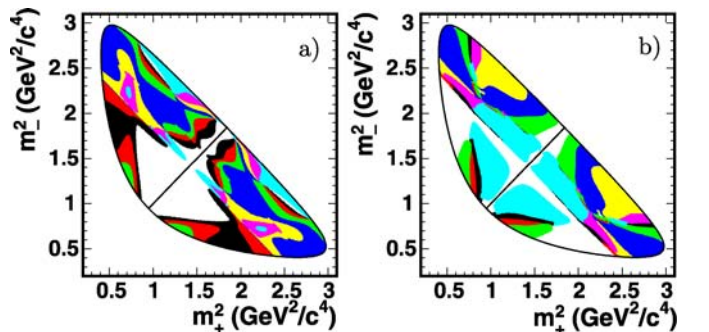

Fig. 1. Divisions of the $\overline{D}^0 \to K_S^0\pi^+\pi^-$ Dalitz plot. Uniform binning of $\Delta\delta_D$ strong phase difference with $\mathcal{N} = 8$ (**a**), and the binning obtained by variation of the latter to maximize the sensitivity factor Q (**b**)

$\mathcal{N} = 19$ in the allowed phase space, the ones which are denoted as 3×3 and 5×5 in [5]).

The B- and $D_{\rm CP}$-statistical precision of different binning options, recalculated to 1000 events of both B and $D_{\rm CP}$ samples, as well as their calculated values of the factor Q, are shown in Table 1. The factor Q reproduces the ratio of the values $\sqrt{1/\sigma_x^2 + 1/\sigma_y^2}$ for the binned and unbinned approaches with the precision of 1%–2%. Note that the "optimal" binning with $\mathcal{N} = 20$ offers the B-statistical sensitivity only 4% worse than an unbinned technique. While the binning with maximized Q offers better B-statistical sensitivity, the best $D_{\rm CP}$-statistical precision of the options we have studied is reached for the $\Delta\delta_D$-binning. However, for the expected amount of experimental data of B and $D_{\rm CP}$ decays the B-statistical error dominates, therefore, slightly worse precision due to $D_{\rm CP}$ statistics does not affect significantly the total precision.

We have considered the choice of the optimal binning only from the point of statistical power. However, the conditions to satisfy low model dependence are quite different. Since the bins in the binning options we have considered are sufficiently large, the requirement that the phase does not change over the bin area is a strong model assumption. We have performed toy MC simulation to study the model dependence. While the binning was kept the same as in the statistical power study (based on the phase difference from the default D^0 amplitude), the amplitude used to generate D^0, $D_{\rm CP}$ and $B^\pm \to DK^\pm$ decays was altered in the same way as in the Belle study of the model dependence in the unbinned analysis [4]. As a result, the same bias of $\Delta\phi_3 \sim 10°$ is observed as in unbinned analysis. The magnitude of the bias in x and y (for initial $x = 0$, $y = 0.1$) is demonstrated in Fig. 2. This bias is apparently caused by a fixed relation between the c_i and s_i, and it affects mainly the y variable.

In a real analysis, one can control the model error by testing if the amplitude used to define binning is compatible with the observed $D_{\rm CP}$ data. This can be done, e.g., by dividing each bin and comparing calculated values of c_i in its parts, or by comparing the expected and observed numbers of events in each bin. The first results by the CLEO-c Collaboration are available [14] that show good agreement of experimental data with c_i calculated from two-body amplitude for $\Delta\delta_D$-binning.
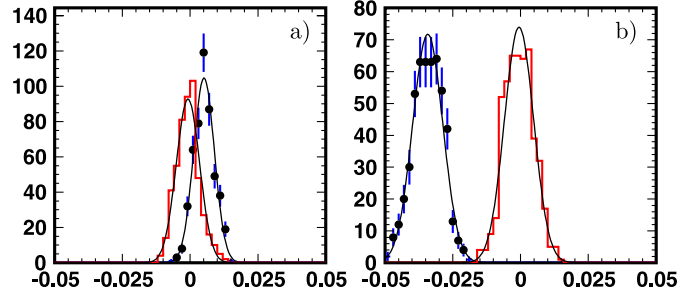
Fig. 2. Toy MC study of the analysis using $D_{\rm CP}$ data. Difference between the fitted and generated **a** x and **b** y values. Result of the toy MC study with $\Delta\delta_D$ binning, 10^5 B decays and 10^4 $D_{\rm CP}$ decays. *Histogram* shows the fit result with the same D^0 decay amplitude used for event generation and binning, the *points with the error bars* show the case with different amplitudes

We conclude that the method of ϕ_3 determination using only $D_{\rm CP}$ data is only asymptotically model-independent, since for any finite bin size the calculation of s_i is done using model assumptions of the $\Delta\delta_D$ variations across the bin. Increasing the $D_{\rm CP}$ data set, however, allows to apply a finer binning and therefore reduce the model error due to the variation of the phase difference.

4 Binned analysis with correlated $D^0 \to K_S^0 \pi^+ \pi^-$ data

The use of $\psi(3770)$ decays where both neutral D mesons decay to the $K_S^0 \pi^+ \pi^-$ state allows to significantly increase the amount of data useful to extract phase information in D^0 decay. It is also possible to detect events of $\psi(3770) \to (K_S^0\pi^+\pi^-)_D(K_L^0\pi^+\pi^-)_D$, where K_L^0 is not reconstructed, and its momentum is obtained from kinematic constraints. The number of these events is approximately twice that of $(K_S^0\pi^+\pi^-)^2$. However, it is impossible to simply combine these samples since the phases of the doubly Cabibbo-suppressed components in $\overline{D}^0 \to K_S^0\pi^+\pi^-$ and $\overline{D}^0 \to K_L^0\pi^+\pi^-$ amplitudes are opposite. In the analysis of B data only $K_S^0\pi^+\pi^-$ state can be used, but it is possible to utilize $K_L^0\pi^+\pi^-$ data to better constrain

Table 1. Statistical precision of (x, y) determination using different binnings and with an unbinned approach. The errors correspond to 1000 events in both the B and $D_{\rm CP}$ $((K_S^0\pi^+\pi^-)^2)$ samples. The D^0 amplitude used is the result of the Belle analysis [4]

Binning	Q	B-stat. err.		$D_{\rm CP}$-stat. err.		$(K_S^0\pi^+\pi^-)^2$-stat. err.	
		σ_x	σ_y	σ_x	σ_y	σ_x	σ_y
$\mathcal{N} = 8$ (uniform)	0.57	0.033	0.060	0.005	0.010	0.015	0.032
$\mathcal{N} = 8$ ($\Delta\delta_D$)	0.79	0.027	0.037	0.004	0.007	0.005	0.010
$\mathcal{N} = 8$ (optimal)	0.89	0.023	0.032	0.006	0.011	0.008	0.011
$\mathcal{N} = 19$ (uniform)	0.69	0.027	0.055	0.004	0.011	0.013	0.019
$\mathcal{N} = 20$ ($\Delta\delta_D$)	0.82	0.027	0.035	0.005	0.007	0.004	0.008
$\mathcal{N} = 20$ (optimal)	0.96	0.022	0.029	0.008	0.011	0.004	0.010
Unbinned	–	0.021	0.028	–	–	–	–

the $\overline{D}^0 \to K_S^0 \pi^+ \pi^-$ amplitude using model assumptions based on SU(3) symmetry [14]. In what follows, we will consider the use of $K_S^0 \pi^+ \pi^-$ data only.

In the case of a binned analysis, the number of events in the region of the $(K_S^0 \pi^+ \pi^-)^2$ phase space is

$$\langle M \rangle_{ij} = h_{\mathrm{corr}} \big[K_i K_{-j} + K_{-i} K_j \\ - 2 \sqrt{K_i K_{-i} K_j K_{-j}} (c_i c_j + s_i s_j) \big] . \quad (14)$$

Here two indices correspond to two D mesons from $\psi(3770)$ decay. It is logical to use the same binning as in the case of D_{CP} statistics to improve the precision of the determination of c_i coefficients, and to obtain s_i from data without model assumptions, contrary to D_{CP} case. Note that in the case of using $(K_S^0 \pi^+ \pi^-)^2$ decays, the parameters c_i and s_i are treated as independent variables. The obvious advantage of this approach is its being unbiased for any finite $(K_S^0 \pi^+ \pi^-)^2$ statistics (not only asymptotically as in the case of D_{CP} data).

Note that in contrast to D_{CP} analysis, where the sign of s_i in each bin is undetermined and has to be fixed using model assumptions, $(K_S^0 \pi^+ \pi^-)^2$ analysis has only a four-fold ambiguity: change of the sign of all c_i or all s_i. In combination with D_{CP} analysis, where the sign of c_i is fixed, this ambiguity reduces to only two-fold. One of the two solutions can be chosen based on a weak model assumption (incorrect s_i sign corresponds to complex-conjugate D decay amplitude, which violates a causality requirement when parameterized with the Breit–Wigner amplitudes).

The coefficients c_i, s_i can be obtained by minimizing the negative logarithmic likelihood function

$$-2 \log \mathcal{L} = -2 \sum_{i,j} \log P(M_{ij}, \langle M \rangle_{ij}) , \quad (15)$$

where $P(M, \langle M \rangle)$ is the Poisson probability to get M events with the expected number of $\langle M \rangle$ events.

The number of bins in the 4-dimensional phase space is $4\mathcal{N}^2$ rather than $2\mathcal{N}$ in the D_{CP} case. Since the expected number of events in correlated $K_S^0 \pi^+ \pi^-$ data is of the same order as for D_{CP}, the bins will be much less populated. This, however, does not affect the precision of the c_i, s_i determination since the number of free parameters is the same and each of the parameters is constrained by many bins.

The coefficients c_i, s_i obtained this way can then be used to constrain x, y with the maximum likelihood fit of the B decay data using (8). To correctly account for the errors of the c_i, s_i determination, this likelihood should include distributions of these quantities, in addition to Poisson fluctuations in the B data bins. A more convenient way is to use the common likelihood function, covering both B and $K_S^0 \pi^+ \pi^-$ data:

$$-2 \log \mathcal{L} = -2 \sum_{i,j} \log P(M_{ij}, \langle M \rangle_{ij}) \\ - 2 \sum_i \log P(N_i, \langle N \rangle_i) , \quad (16)$$

with x, y, h_B, h_{corr}, c_i and s_i as free parameters. This approach is also more optimal in the case of large B data

sample, since it imposes additional constraints on c_i, s_i values.

The toy MC simulation was performed to study the procedure described above. Using the amplitude from the Belle analysis [4], we generate a large number of $D^0 \to K_S^0 \pi^+ \pi^-$ decays and several sets of $(K_S^0 \pi^+ \pi^-)^2$ decays (according to the probability density given by (7)) and B decays (3). We use the same binning options as in the D_{CP} study. The combined negative likelihood (16) is minimized in the fit to each toy MC sample. We constrain $c_i^2 + s_i^2 < 1$ in the fit to avoid entering an unphysical region with a negative number of events in the bin. For low number of $(K_S^0 \pi^+ \pi^-)^2$ decays this constraint introduces asymmetric tails in the x, y distributions. For 10^3 events and more this asymmetry becomes negligible. Since the number of $(K_S^0 \pi^+ \pi^-)^2$ decays we expect in the experiment is of the order of 10^3, we do not expect this effect to cause a significant problem.

The number of $(K_S^0 \pi^+ \pi^-)^2$ and B decays in our study of statistical sensitivity ranges from 10^3 to 10^5. The errors of x and y parameters are calculated from the spread of the fitted values. If the number of $(K_S^0 \pi^+ \pi^-)^2$ decays is comparable or larger than the number of B decays, the x and y errors can be represented as quadratic sums of two errors, each scaled as a square root of $(K_S^0 \pi^+ \pi^-)^2$
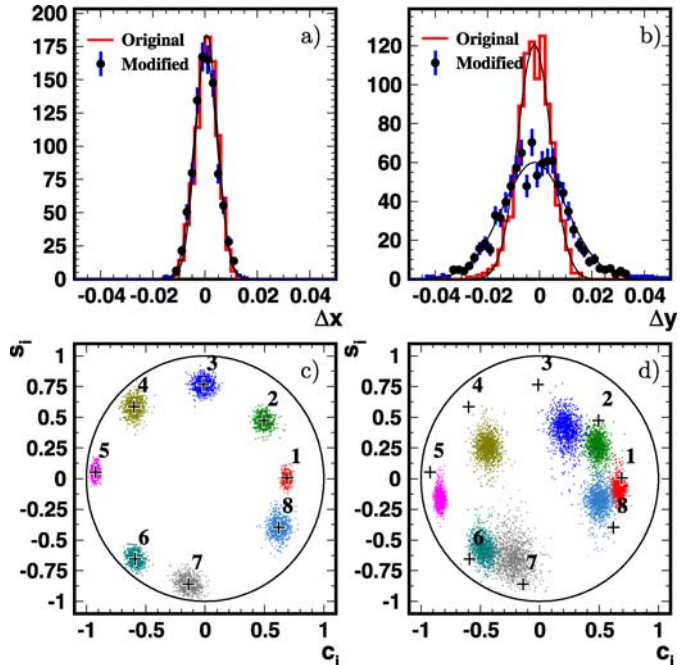
Fig. 3. Toy MC study of the analysis using $(K_S^0 \pi^+ \pi^-)^2$ data. *Top line*: difference between the fitted and generated **a** x and **b** y values. Result of the toy MC study with $\Delta\delta_D$ binning, 10^5 B decays and 10^4 $(K_S^0 \pi^+ \pi^-)^2$ decays. The *histogram* shows the fit result with the same D^0 decay amplitude used for event generation and binning, the *points with the error bars* show the case with different amplitudes. *Bottom line*: coefficients c_i, s_i obtained in the fit to toy MC sample. Different colors correspond to different bins. Cases with the same amplitude (**c**) and different amplitudes (**d**) used for event generation and binning

and B statistics, respectively. However if the number of B decays is large, the errors of c_i and s_i depend also on B decay statistics, so separating the total error into B- and $(K_S^0\pi^+\pi^-)^2$-statistical errors becomes impossible.

The best $(K_S^0\pi^+\pi^-)^2$-statistical error is obtained for $\Delta\delta_D$-binning and recalculated to 1000 events yields $\sigma_x = 0.005$, $\sigma_y = 0.010$, which is only slightly worse than the error obtained with the same amount of D_{CP} data (see Table 1 for comparison). We also check that significant change of the model used to define the binning does not lead to the systematic bias (although it does decrease the statistical precision). Figure 3 demonstrates the precision of the determination of c_i, s_i coefficients in our toy MC study and the absence of the systematic bias for both x and y when the model is varied.

The numbers of $(K_S^0\pi^+\pi^-)^2$ and D_{CP} decays in the $\psi(3770)$ data are comparable, and so are the statistical errors due to the $\psi(3770)$ data sample for the two approaches. However, the approach based on $(K_S^0\pi^+\pi^-)^2$ data allows to extract both c_i and s_i without additional model uncertainties, so it can be used to check the validity of the constraint $c_i^2 + s_i^2 = 1$ and therefore to test the sensitivity of the particular binning. The same binning can be used in both $(K_S^0\pi^+\pi^-)^2$ and D_{CP} approaches, therefore improving the accuracy of the c_i determination. Technically it can be done in a straightforward way by adding the third term related to the number of D_{CP} decays into the likelihood (16).

5 Conclusion

We have studied the model-independent approach to ϕ_3 measurement using $B^\pm \to DK^\pm$ decays with the neutral D decaying to $K_S^0\pi^+\pi^-$. The analysis of $\psi(3770) \to D\bar{D}$ data allows to extract the information about the strong phase in the $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay, whereas this phase is fixed by model assumptions in a model-dependent technique. We consider the case with a limited $\psi(3770) \to D\bar{D}$ data sample which will be available from CLEO-c in the near future.

In the binned analysis, we propose a way to obtain the binning that offers an optimal statistical precision (close to the precision of an unbinned approach). Two different strategies of the binned analysis are considered: using the $D_{CP} \to K_S^0\pi^+\pi^-$ data sample, and using decays of $\psi(3770)$ to $(K_S^0\pi^+\pi^-)_D(K_S^0\pi^+\pi^-)_D$. The strategy using D_{CP} decays alone cannot offer a completely model-independent measurement: it provides only the information about c_i coefficients, while s_i for low D_{CP} statistics has to be fixed using model assumptions. However, as the D_{CP} data sample increases, model-independence can be reached by reducing the bin size. The strategy using the $\psi(3770) \to (K_S^0\pi^+\pi^-)_D(K_S^0\pi^+\pi^-)_D$ sample, in contrast, allows to obtain not only c_i coefficients with an accuracy comparable to D_{CP} approach, but also s_i in a model-independent way. Both strategies can use the same binning of the $\overline{D}^0 \to$

$K_S^0\pi^+\pi^-$ Dalitz plot and therefore can be used in combination to improve the accuracy due to $\psi(3770)$ statistics.

The expected sensitivity to ϕ_3 is obtained based on the two-body $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay amplitude measured by Belle [4]. For the CLEO-c statistics of $750\,\text{pb}^{-1}$ ($\sim 1000 D_{CP}$ and $(K_S^0\pi^+\pi^-)^2$ events) the expected errors of the parameters x and y due to $\psi(3770)$ statistics are of the order of 0.01. For $r_B = 0.1$ it gives the ϕ_3 precision $\sigma_{\phi_3} = \sigma_{x,y}/(\sqrt{2}r_B) \simeq 5°$, which is well below the expected error due to current B data sample (the total integrated luminosity of the two B-factories, BaBar and Belle, slightly exceeds $1\,\text{ab}^{-1}$, which corresponds to $\sim 1000\,B^\pm \to DK^\pm$ decays and ϕ_3 precision of about 20° for $r_B = 0.1$). Further improvement of ϕ_3 precision at the super B factory [15] and LHCb [16] will require a larger charm dataset, which can be provided by the BES-III experiment [17, 18].

In our study, we did not consider the experimental systematic uncertainties, e.g. due to imperfect knowledge of the detection efficiency or background composition. We believe these issues can be addressed in a similar manner as in already completed model-dependent analyses.

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