The Use of Shunts and Transformers with Alternate Current Measuring Instruments

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XIV. *The Use of Shunts and Transformers with Alternate Current Measuring Instruments.* *By* Charles V. Drysdale, *D.Sc.**

[Plate IX.]

The growing demand for instruments for alternate-current measurement, of considerable range and accuracy, has called attention to some of the difficulties attending their production, especially to their limited range. As alternate-current instruments measure the root-mean-square values of the P.D. or current, it necessarily follows that the deflecting torque falls very rapidly as the P.D. or current is decreased, For most commercial instruments, it may be taken that a ten-fold range of torque is the utmost possible for accurate reading, so that when the deflexion is proportional to the torque the range of an A.C. instrument can only be of the order of 3 to 1 except by the employment of double coils or auxiliary devices. The auxiliary devices which may be employed are transformers for electromagnetic ammeters, voltmeters, and wattmeters ; shunts for ammeters and wattmeters, series resistances for electromagnetic voltmeters, and condensers and split resistances for electromagnetic voltmeters. Of these, the use of shunts and transformers may give rise to serious errors, and it is proposed here to investigate the amount of these errors and the conditions for their elimination.

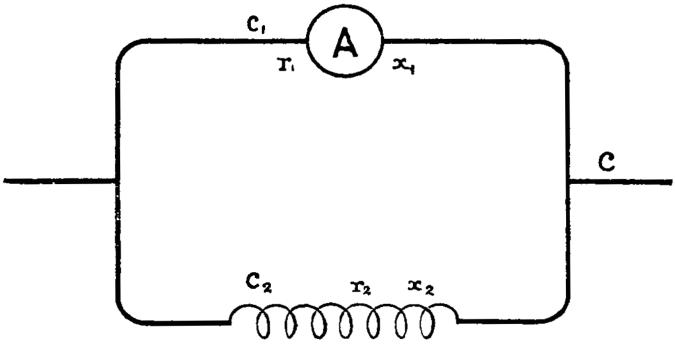
In dealing with any shunting or transforming device we must bear in mind that it may cause errors in two ways:—(*a*) by the multiplying power of the shunt or the ratio of the transformer being affected by frequency, &c. ; and (*b*) by a phase displacement being introduced between the current or P.D. on the instrument and the main current or voltage. Ammeters and voltmeters are of course only affected by the first error, but the phase-displacement may be of much greater importance in wattmeters.

* Read March 27, 1908.

Shunts.

Multiplying Power.—In fig. 1, we have an instrument of resistance r_1 and reactance x_1 shunted by a circuit of resistance r_2 and reactance x_2. Then, if V is the P.D. between

Fig. 1.



the terminals, and C_1 and C_2 are the currents in the instrument and shunt respectively, we have

$$C_1 = \frac{V}{I_1} = \frac{V}{r_1 - jx_1} \quad \text{and} \quad C_2 = \frac{V}{I_2} = \frac{V}{r_2 - jx_2}.$$

Hence the total current in the circuit $C = C_1 + C_2$

$$= V\left\{\frac{1}{r_1 - jx_1} + \frac{1}{r_2 - jx_2}\right\} = V\frac{r_1 + r_2 - j(x_1 + x_2)}{(r_1 - jx_1)(r_2 - jx_2)}, \quad (1)$$

and the virtual current $\overline{C} = \overline{V}\frac{I}{I_1 I_2}$, where I is the impedance of the whole circuit formed by the instrument and shunt *in series*, and I_1 and I_2 are the impedances of the instrument and shunt respectively.

Consequently, since $\overline{C_1} = \frac{V}{I_1}$ the multiplying power of the shunt

$$M' = \frac{\overline{C}}{\overline{C_1}} = \frac{I}{I_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence

$$M' = \sqrt{\frac{(r_1 + r_2)^2 + (x_1 + x_2)^2}{r_2^2 + x_2^2}} = \sqrt{\frac{\left(\frac{r_1 + r_2}{r_2}\right)^2 + \left(\frac{x_1 + x_2}{r_2}\right)^2}{1 + \left(\frac{x_2}{r_2}\right)^2}}$$

But the multiplying power of the shunt for direct currents $M = \frac{r_1 + r_2}{r_2}$ from which $r_1 = (M-1)r_2$. In addition $\frac{x_1}{r_1} = T_1 p$, where T_1 is the time constant of the instrument and p is 2π times the frequency as usual. Similarly $\frac{x_2}{r_2} = T_2 p$, where T_2 is the time constant of the shunt.

Putting these values in the expression above we have

$$M' = \sqrt{\frac{M^2 + \{(M-1)T_1 + T_2\}^2 p^2}{1 + T_2^2 p^2}},$$

which after a little further simplification reduces to

$$M' = M\sqrt{1 + \frac{M-1}{M^2} \frac{(M-1)T_1 + (M+1)T_2}{1 + T_2^2 p^2} (T_1 - T_2)p^2}. \quad (3)$$

This formula at once shows that if $T_1 = T_2$ the shunt has the same multiplying power for both direct and alternate currents, as is well known. It further shows that if

$$(M-1)T_1 + (M+1)T_2 = 0 \qquad \text{or} \qquad \frac{T_2}{T_1} = -\frac{M-1}{M+1}$$

the shunt is again correct. This would be the case if the instrument is shunted with a resistance r_2 and capacity K such that $Kr_2 = \frac{M-1}{M+1} T_1$ *.

If the instrument is inductive, and is shunted with a non-inductive resistance, $T_2 = 0$ and the formula reduces to

$$M' = M\sqrt{1 + \left(\frac{M-1}{M}\right)^2 T_1^2 p^2}. \quad . \quad . \quad . \quad (4)$$

As an example, a Kelvin centiampere balance was found on test to have a resistance of 62·4 ohms and an inductance (by secohmmeter) of 161 millihenrys. The time constant of this instrument was therefore ·00258 second. The instrument was then shunted with a non-inductive resistance, and it was found that a current of ·878 amp. gave a reading of ·17 amp. on the balance with direct current, thus giving a multiplying

* Mr. Alexander Russell has cast doubts on this formula, and I have found that it is only true in special cases. It was based on the assumption that a shunted condenser could be treated as an impedance with negative time constant, which is not strictly true.

power M of 5·16. Hence at 50 periods, which is the usual frequency of the supply, M′ should be

$$5{\cdot}16\sqrt{1+\frac{4{\cdot}16^2\times{\cdot}00258^2\times 314^2}{5{\cdot}16^2}} = 6{\cdot}17;$$

while on testing a current of ·927 amp. gave a reading of ·15 amp. or a multiplying power of 6·18. It should be noticed that the formula may be expressed in the form $T_1 = \frac{\sqrt{M'^2-M^2}}{(M-1)p}$, which gives us a convenient method of testing the inductances of ammeters, &c.

For commercial work, therefore, we see that the only legitimate method of employing shunts is to make the time constants of the instruments and shunts either negligible or approximately equal. If T_1 and T_2 are nearly equal since $\sqrt{1+h} = 1+\frac{h}{2}$ nearly when h is small,

$$M' = M\left\{1+\frac{M-1}{2M^2}\frac{(M-1)T_1+(M+1)T_2}{1+T_2{}^2p^2}(T_1-T_2)p^2\right\}. \quad (5)$$

It will be noticed that for a given difference in the time constants the correcting term is greater the greater the value of M and the less that of T_2. Consequently, if we take in our formula M so large that M, (M+1) and (M−1) may be regarded as equal and at the same time neglect T_2p in comparison with unity, we have:

$$M'=M\left\{1+\frac{T_1+T_2}{2}(T_1-T_2)p^2\right\}=M\{1+(T_1-T_2)Tp^2\}, \quad (6)$$

where T is the average value of the two time constants T_1 and T_2.

This may be written in the form $\frac{\Delta M}{M}=T\,.\,\Delta T\,.\,p^2$, and finally

$$\frac{\Delta M}{M} = T^2p^2\frac{\Delta T}{T}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Consequently, to ensure that the ratio M shall not be altered by more than x per cent. with alternating current of given frequency, the time constants must be adjusted to equality within $\frac{x}{T^2p^2}$ per cent.

In the case of the Kelvin centiampere balance before mentioned, $T = \cdot 00258$ at a frequency of 50, therefore $Tp = 314 \times \cdot 00258 = \cdot 810$ and $T^2p^2 = \cdot 655$. Consequently the time constant or the self-induction of the shunt must be adjusted to $\frac{1}{\cdot 655} = 1\cdot 53$ per cent. for an accuracy of 1 per cent. in the multiplying power. The simplest method of adjusting or checking the shunt is of course to test the ratio with both D.C. and A.C. or with A.C. of two different frequencies.

Phase Displacement.—Returning to formula (1) and rationalizing the denominator, we get

$$C = \frac{V}{I_1^2 I_2^2}\left\{(r_1+r_2)-j(x_1+x_2)\right\}(r_1+jx_1)(r_2+jx_2)$$

$$= \frac{V}{I_1^2 I_2^2}\left\{(r_1+r_2)(r_1r_2-x_1x_2)+(x_1+x_2)(r_1x_2+r_2x_1)\right.$$
$$\left. +j\left[(r_1+r_2)(r_1x_2+r_2x_1)-(x_1+x_2)(r_1r_2-x_1x_2)\right]\right\},$$

and consequently

$$\tan\phi = \frac{(r_1+r_2)(r_1x_2+r_2x_1)-(x_1+x_2)(r_1r_2-x_1x_2)}{(r_1+r_2)(r_1r_2-x_1x_2)+(x_1+x_2)(r_1x_2+r_2x_1)}$$

where ϕ is the angle of lag of the main current behind V.

Inserting the values of M, T_1, and T_2 as before, we get

$$\tan\phi = \frac{T_1+(M-1)T_2+\{(M-1)T_1+T_2\}T_1T_2p^2}{M+\{(M-1)T_1^2+T_2^2\}p^2}p.$$

Similarly $\tan\phi_1 = T_1p$ where ϕ_1 is the lag in the instrument. Hence

$$\tan(\phi_1-\phi) = \frac{\tan\phi_1-\tan\phi}{1+\tan\phi_1\tan\phi}$$

is the tangent of the angle of lag of the current in the instrument behind the main current. Putting in the values of $\tan\phi_1$ and $\tan\phi$ and simplifying, we have as the result

$$\tan\psi = \tan(\phi_1-\phi)$$
$$=(M-1)(T_1-T_2)p\frac{1+T_1^2p^2}{M+\{MT_1^2+(M-1)T_1T_2+T_2^2\}p^2+\{(M-1)T_1+T_2\}T_1^2T_2p^4}$$
$$=\frac{M-1}{M}(T_1-T_2)p\left\{1-\frac{(1+T_1^2p^2)\{(M-1)T_1+T_2\}T_2p^2}{M+\{MT_1^2+(M-1)T_1T_2+T_2^2\}p^2+\{(M-1)T_1+T_2\}T_1^2T_2p^4}\right\} \quad (8)$$

The phase-displacement is consequently zero for either $M=1$ or $T_1=T_2$ as is obvious, and also for $1+T_1^2p^2=0$, which is impossible. If the shunt is non-inductive $T_2=0$ and $\tan\psi=\frac{M-1}{M}T_1p$; while if the instrument is non-inductive and the shunt inductive,

$$\tan\psi=-\frac{M-1}{M+T_2^2p^2}T_2p.$$

Again, if both T_1p and T_2p are small compared with unity, the expression for $\tan\psi$ reduces to $\frac{M-1}{M}(T_1-T_2)p$. This of course means that the tangent of the phase-displacement between the instrument and mains is the fraction $\frac{M-1}{M}$ of that between the instrument and shunt, as is obvious geometrically. Finally, if T_1 and T_2 are nearly equal,

$$\tan\psi=\psi=\frac{M-1}{M(1+T^2p^2)}(T_1-T_2)p \quad . \quad . \quad . \quad (9)$$

where T is the mean time constant as before. When T is small this reduces to the last expression.

As an illustration of the error produced by shunting a wattmeter, we may assume a case where the time constant of the main circuit has a value of ·0025 as in the Kelvin balance above cited, and suppose that it is shunted with a non-inductive shunt of a nominal multiplying power of 10. We then have $M=10$, $T_2=0$, and T_1p at 50∼

$$=314\times{\cdot}0025={\cdot}785.$$

Hence

$$M'=M\sqrt{1+\left(\frac{M-1}{M}\right)^2T_1^2p^2}=10\sqrt{1+{\cdot}81\times{\cdot}785^2}=12{\cdot}25.$$

The phase-displacement is

$$\tan\psi_1=\frac{M-1}{M}T_1p=\frac{9}{10}\times{\cdot}785={\cdot}705\,;$$

so that the current in the wattmeter lags about 35° behind that in the main circuit. Hence the instrument would read 22·5 per cent. too low owing to the error in the ratio, while the phase-displacement would cause the power factor to be apparently about ·5 when actually zero.

This strikingly illustrates the enormous errors which may be produced by shunts in wattmeters. For any given small displacement ψ we have from (9)

$$\frac{T_1 - T_2}{T} = \frac{\Delta T}{T} = \frac{M}{M-1}\left(\frac{1 - T^2 p^2}{Tp}\right)\psi. \quad . \quad . \quad (10)$$

and ψ will then be the maximum error in the power factor for loads nearly in quadrature. Consequently, in the above case, if the error in the power factor is not to exceed ·01,

$$\frac{\Delta T}{T} = \frac{10}{9}\frac{(1 - \cdot785^2)}{\cdot785} \times \cdot01 = \cdot00542 \quad \text{or} \quad \cdot54 \text{ per cent.};$$

and we saw before that for the ratio to be correct to 1 per cent.

$$\frac{\Delta T}{T} = 1\cdot53 \text{ per cent.}$$

TRANSFORMERS.

In dealing with the application of transformers to instruments, we shall have to consider separately their employment as "current transformers" or as "voltage transformers"; the former being employed in conjunction with ammeters for the purpose of utilizing instruments of convenient range, and to insulate the instrument from high voltage circuits; the latter with electromagnetic or hot-wire voltmeters to avoid undue waste of energy, and again as with ammeters to disconnect them from the high-pressure circuits. In wattmeters and energy-meters of the induction type, transformers are frequently employed in both the main and shunt circuits simultaneously.

The relations between the primary and secondary currents and voltages have been obtained by Steinmetz and others from the well-known vector diagram, but it will be well to treat the problem *ab initio* as we require the results to be exhibited in the most convenient form.

Current Transformers.—It is most convenient to deal at first with the magnitudes and phase relations of the currents, and we can afterwards apply the results to the investigation of the voltage relations.

Let A_p = primary ampere turns $= C_p n_p$.

A_s = secondary ,, ,, $= C\,n_s$.

A_m = magnetizing ,, ,, $= C_m n_p$.

A_c = core loss ,, ,, $= C_c n_p$.

A_0 = no load ,, ,, $= C_0 n_p$.

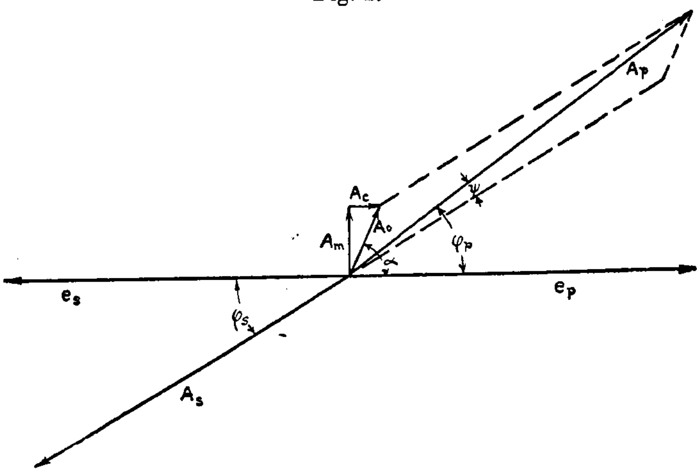
ϕ_p = lag of primary current behind core E.M.F.

ϕ_s = lag of secondary ,, ,, ,, ,,

α = lag of no load ,, ,, ,, ,,

ψ = phase displacement between primary and secondary currents.

Fig. 2.



Then we have

$$\hat{A}_p + \hat{A}_s = \hat{A}_0 \text{ vectorially;}$$

or writing the currents in the Steinmetz notation,

$$\hat{A}_s = -A_s \cos\phi_s - jA_s \sin\phi_s \quad . \quad . \quad . \quad . \quad (11)$$

$$\hat{A}_0 = A_c + jA_m.$$

Hence

$$\hat{A}_p = A_0 - A_s = A_s \cos\phi_s + A_c + \ (A_s \sin\phi_s + A_m) \quad . \quad (12)$$

and

$$\overline{A}_p = \sqrt{(A_s \cos\phi_s + A_c)^2 + (A_s \sin\phi_s + A_m)^2}$$

$$= \sqrt{A_s^2 + 2A_s(A_c \cos\phi_s + A_m \sin\phi_s) + A_0^2}.$$

If A_0 is small in comparison with A as it should always be in practice, we may write

$$\frac{A_p}{A_s} = 1 + \frac{A_c}{A_s}\cos\phi_s + \frac{A_m}{A_s}\sin\phi_s. \quad . \quad . \quad (13)$$

If R_c is the ratio of the primary to secondary currents, and R_t the ratio of secondary to primary turns, we have

$$R_c = R\left(1 + \frac{A_c}{A_s}\cos\phi_s + \frac{A_m}{A_s}\sin\phi_s\right). \quad . \quad . \quad (14)$$

Hence the ratio of the currents can only be equal to the transformation ratio if A_c and A_m are both zero or

$$\tan\phi_s = -\frac{A_c}{A_m} = -\cot\alpha.$$

This implies that the secondary current should be in quadrature with the no load current, as is geometrically obvious.

To test the constancy of transformation we have

$$\Delta R_c = R_t\left\{\Delta\left(\frac{A_c}{A_s}\right)\cos\phi_s + \Delta\left(\frac{A_m}{A_s}\right)\sin\phi_s\right\}, \quad . \quad (15)$$

which shows that for constant ratio of transformation both A_c and A_m must be proportional to A_s or that

$$\tan\phi_s = -\frac{\Delta\left(\frac{A_c}{A_s}\right)}{\Delta\left(\frac{A_m}{A_s}\right)}. \quad . \quad . \quad . \quad . \quad . \quad (16)$$

For constant secondary impedance A_s is proportional to the core E.M.F. and hence to the induction density in the core, B. Hence the current vector should be perpendicular to the curve of which the ordinate is $\frac{A_m}{A_s}$, representing the reluctance of the circuit, and the abscissa is $\frac{A_c}{A_s}$ or the ratio of the core-loss current to B. It is obvious that this is when the secondary load is a leading one, and this result is contrary to the frequently quoted experiments of Mr. Campbell *, who

* Phil. Mag. vol. xlii. p. 271; Journal I. E. E. vol. xxxiii. p. 654.

is claimed to have found experimentally that better constancy of ratio is obtained with an inductive instrument. In the case, however, where there is an air-gap in the transformer, the curve connecting $\frac{A_m}{A_s}$ with $\frac{A_c}{A_s}$ may be practically horizontal, and in this case the regulation will be worst on a non-inductive instrument, and better the greater is either the lag or lead of the secondary current. In estimating the value of Mr. Campbell's experiments it must not be forgotten that the two Kelvin balances employed by him must have been of very different impedance and consequently the core inductions would have been decidedly different. This of itself would probably have been sufficient to produce the effect found. Experimental evidence will be found below on this point.

Effect of variation of frequency.—Increase of frequency increases the impedance of the instrument if inductive, but at the same time increases the E.M.F. for the same core induction. With a fully inductive instrument, therefore, and if eddy currents are absent, variations in frequency should have no effect upon the ratio. This agrees with Mr. Campbell's theoretical and experimental conclusions.

Phase Difference.—Reverting to equations (11) and (12), we have

$$\tan \phi_p = \frac{A_s \sin \phi_s + A_m}{A_s \cos \phi_s + A_c},$$

Hence

$$\tan \beta = \tan(\phi_p - \phi_s) = \frac{\frac{A_m}{A_s} \cos \phi_s - \frac{A_c}{A_s} \sin \phi_s}{1 + \frac{A_c}{A_s} \cos \phi_s + \frac{A_m}{A_s} \sin \phi_s} \quad . \quad (17)$$

which is zero for A_m and A_c both zero or for $\tan \phi_s = \frac{A_m}{A_c}$ as is obvious.

Hence for zero phase-displacement the instrument should be so inductive that the ratio of its reactance to resistance equals the ratio of the magnetizing to the core-loss currents of the transformer.

As the relation between $\frac{A}{A_s}$ and $\frac{A_c}{A_s}$ is not in general linear it follows that the phase-displacement cannot be constant, but it is fairly evident that the best constancy would be approximately obtained when the actual displacement is smallest, *i.e.* with an inductive instrument.

With variation of frequency it is again obvious that with a fully inductive instrument, without eddy currents in the transformer, the phase-displacement should be unaffected by frequency. The less inductive the instrument the more rapidly should the phase-displacement decrease with frequency, owing to the drop in the induction density.

P.D. Relations.—Writing the secondary current in the form $A_s(\cos\phi_s + j\sin\phi_s)$ and the primary

$$A_s\cos\phi_s + A_c + j(A_s\sin\phi_s + A_m),$$

let r_p and r_s be the primary and secondary resistances and x_p and x_s the primary and secondary reactances for coils of a single turn each.

Then

$$V_p = E + \{A_s\cos\phi_s + A_c + j(A_s\sin\phi_s + A_m)\}(r_p - jx_p)$$

$$-V_s = E - A_s(\cos\phi_s + j\sin\phi_s)(r_s - jx_s),$$

from which

$$V_p = E + A_s\left\{r_p\left(\cos\phi_s + \frac{A_c}{A_s}\right) + x_p\left(\sin\phi_s + \frac{A_m}{A_s}\right)\right.$$
$$\left. -j\left[x_p\left(\cos\phi_s + \frac{A_c}{A_s}\right) - r_p\left(\sin\phi_s + \frac{A_m}{A_s}\right)\right]\right\} . \quad (18)$$

and

$$-V_s = E - A_s\{r_s\cos\phi_s + x_s\sin\phi_s - j(x_s\cos\phi_s - r_s\sin\phi_s)\} \quad (19)$$

Voltage Ratio.—Taking the ratio of $\frac{V_s}{V_p}$, simplifying, and neglecting squares of small quantities, we have

$$R_v = R_t\left\{1 - \frac{A_s}{E}\left[(r_p + r_s)\cos\phi_s + (x_p + x_s)\sin\phi_s\right] - \frac{A_c}{E}r_p - \frac{A_m}{E}x_p\right\},$$

which will be seen to be correct from the vector diagram.

Denoting $r_p + r_s$ by r the equivalent resistance,

$x_p + x_s$ by x ,, reactance,

and $\sqrt{r^2 + x^2}$ by I ,, impedance,

$$R_v = R_t\left\{1 - \frac{A_s}{E}(r\cos\phi_s + x\sin\phi_s) - \frac{A_c}{E}r_p - \frac{A_m}{E}x_p\right\}. \quad (20)$$

For an electrostatic or high-resistance voltmeter, $A_s = 0$ and we have

$$R_v = R_t\left\{1 - \frac{A_c r_p + A_m x_p}{E}\right\}. \quad . \quad . \quad . \quad (21)$$

For a hot-wire instrument $\phi = 0$ and

$$R_v = R_t\left\{1 - \frac{A_s r + A_c r_p + A_m x_p}{E}\right\}. \quad . \quad . \quad (22)$$

Returning to formula (20) we have

$$\frac{\Delta R_v}{R_v} = r_p \Delta\left(\frac{A_c}{E}\right) + x_p \Delta\left(\frac{A_m}{E}\right)$$

if the instrument and internal impedances are constant. This is independent of the phase of the secondary current and can only be zero if A_c and A_m are proportional to E, which is only the case for an air-core transformer. For an iron-core transformer the best constancy of ratio is thus obtained by making A_c and A_m and also r_p and x_p as low as possible, the core being of the best quality iron, and the coils subdivided and intercalated.

P.D. Phase Relations.—From equations (18) and (19) we obtain

$$\tan\psi_p = -\frac{x_p\left(\cos\phi_s + \frac{A_c}{A_s}\right) - r_p\left(\sin\phi_s + \frac{A_m}{A_s}\right)}{E + A_s\left\{r_p\left(\cos\phi_s + \frac{A_c}{A_s}\right) + x_p\left(\sin\phi_s + \frac{A_m}{A_s}\right)\right\}} A_s$$

$$\tan\psi_s = \frac{x_s\cos\phi_s - r_s\sin\phi_s}{E - A_s\{r_s\cos\phi_s + x_s\sin\phi_s\}} A_s.$$

If, as should be the case with voltage transformers, the resistance and inductive drop in the transformer are small compared with E, we have

$$\psi_p - \psi_s = \beta = -\frac{A_s(x\cos\phi_s - r\sin\phi_s) + A_c x_p - A_m r_p}{E} \quad . \quad (23)$$

as is geometrically obvious.

For the phase-difference to be zero it follows that either the resistance and inductance must be zero, or that

$$\tan\phi_s = \frac{A_s^2 rx \pm (A_c x_p - A_m r_p)\sqrt{A_s^2(x^2 + r^2) - (A_c x_p - A_m r_p)^2}}{A_s r^2 - (A_c x_p - A_m r_p)^2}. \quad (24)$$

In this expression if A_s is large compared with A_c or A_m $\tan\phi_s = \frac{x}{r}$, or the ratio of the reactance to the resistance of the instrument should be the same as for the transformer, and the instrument should therefore be fairly inductive.

With a non-inductive instrument

$$\beta = \frac{A_s x + A_c x_p - A_m r_p}{E}, \quad . \quad . \quad . \quad . \quad (25)$$

and if the instrument takes no appreciable current compared with A_c or A_m,

$$\beta = \frac{A_c x_p - A_m r_p}{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

To find the variation of phase-displacement with voltage we have evidently from (23)

$$\Delta\beta = x_p \Delta\left(\frac{A_c}{E}\right) - r_p \Delta\left(\frac{A_m}{E}\right),$$

and this is evidently zero for an air-core transformer.

Relation between the Secondary Current and the Magnetizing and Core-Loss Currents.—If l is the length of the magnetic path, a its area of cross section, and I_s the total secondary impedance (reduced to 1 turn), we have

Effective core E.M.F. $\bar{e} = \frac{a\bar{B}p}{10^8} = \frac{a\hat{B}p}{\sqrt{2}\times 10^8}$ volts per turn;

Secondary ampere turns $A_s = \frac{\bar{e}}{I_s} = \frac{a\hat{B}p}{\sqrt{2}\times 10^8 I_s}$,

from which

$$\hat{B} = \frac{\sqrt{2}\times 10^8 I_s A_s}{ap} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Assuming the core loss to be $\eta\hat{B}^c$ ergs per c.c. per cycle:—

Total core loss

$$w_m = \frac{\eta\hat{B}^c V_n}{10^7} \text{ watts,}$$

and the core-loss ampere turns

$$A_c = \frac{w_m}{e} = \frac{5\sqrt{2}}{\pi}\eta l \left\{\frac{\sqrt{2}\times 10^8 I_s A_s}{ap}\right\}^{(c-1)} \quad . \quad . \quad (28)$$

For the magnetizing current

$$A_m = \frac{10}{4\pi}\frac{\overline{B}l}{\mu}$$

or

$$A_m = \frac{10^9}{4\pi}\frac{l}{\mu a p} I_s A_s. \quad . \quad . \quad . \quad . \quad . \quad (29)$$

These relations are sufficient to enable the magnetizing and core-loss currents to be calculated when the total secondary impedance is known. The resistances are of course readily calculable, while the equivalent inductance of the whole transformer for a single primary and secondary winding may be calculated from the following formula, deduced from that given by Prof. Kapp :

$$L = C\frac{\sqrt{2}}{10^{10}}\frac{P}{LK^2}(3KS + T) \quad . \quad . \quad . \quad . \quad (30)$$

P being the mean perimeter of the coils in inches,
L the length of gap from iron to iron, parallel to insulation between coils,
S the thickness of the insulation in inches,
K the number of sections per coil,
T the length of the winding space,
and C a constant given as 44 for core and 55 for shell type transformers.

From personal experiment, however, the author has found that the value 44 is more nearly correct for shell type transformers.

Experimental.—As some of the conclusions arrived at are of importance, and are somewhat at variance with existing ideas, some experimental evidence will be of value. A brief reference only will be necessary here as some of these tests have already been published *, together with a description of the methods by which the results were obtained.

The principal transformer tested was made for the tests, and was of the shell type ; the core having a magnetic path of 35·3 cm. and a sectional area of 71 sq. cm. Four independent coils, each of 150 turns of No. 18 D.C.C. copper wire, were wound side by side, the resistance of each coil being 1·2 ohms. Fig. 3 shows the core, and fig. 4 the

* "Phase Displacements in Resistances and Transformers," Electrician, No. lviii. p. 160.

Fig. 3.—Isometric view of Transformer Core.

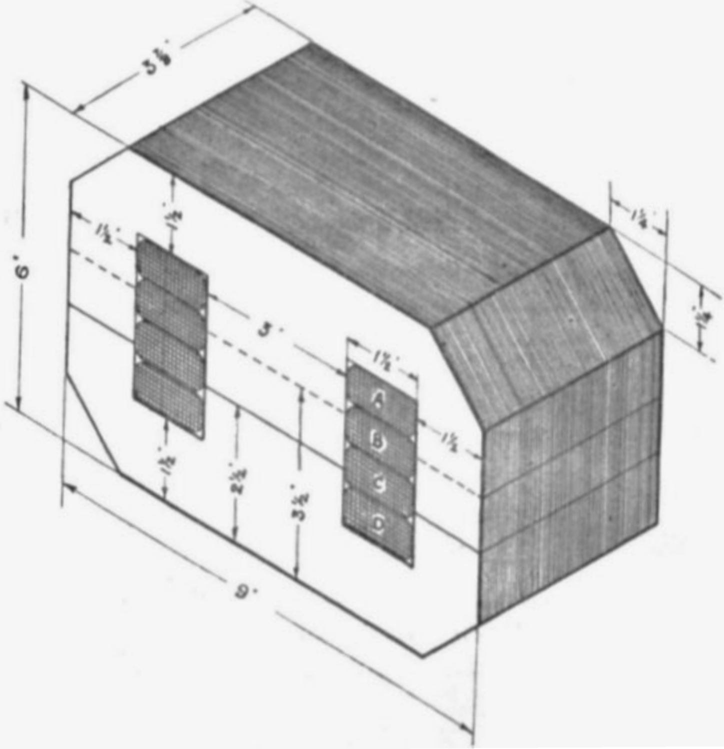
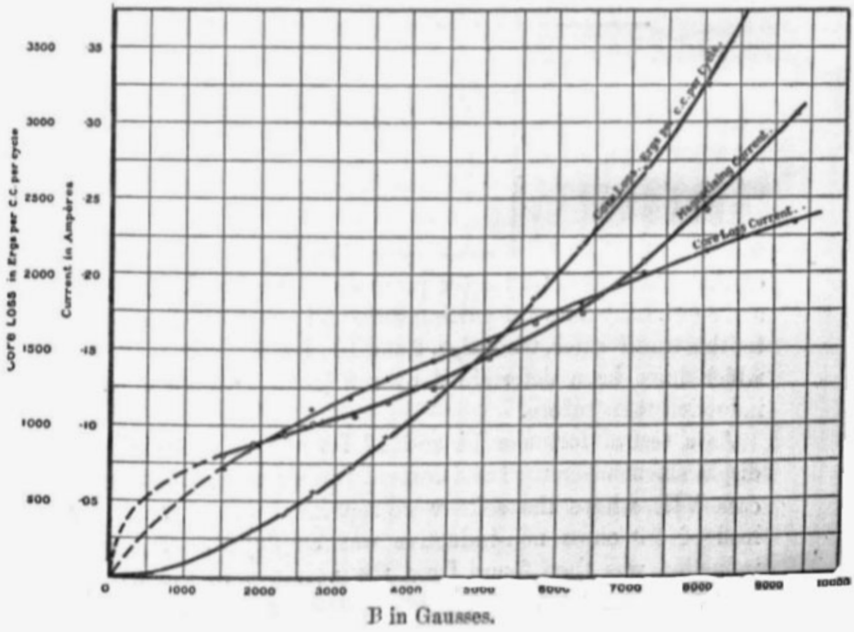


Fig. 4.—Relation of Magnetizing and Core-loss Currents to Induction.



relation of the magnetizing and core-loss currents to the core induction, determined experimentally.

Figs. 5 and 6 show the connexions for determining the current and P.D. ratios and phase displacements. The current ratio is determined by an ordinary wattmeter instead

Fig. 5.—Connexions for Current Test.

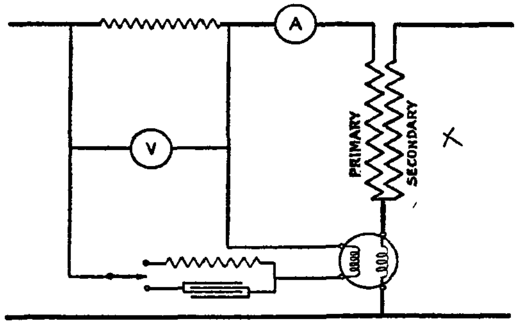
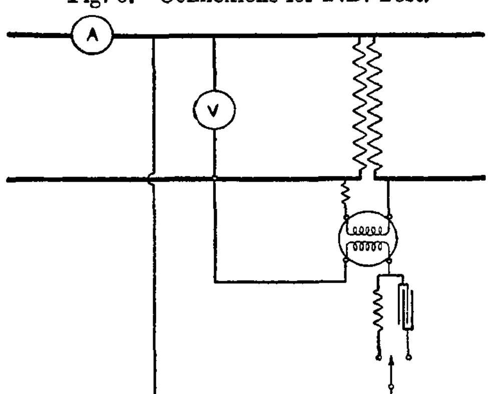


Fig. 6.—Connexions for P.D. Test.



of the differential wattmeter previously employed. Figs. 7 and 8 (Pl. IX.) show the current ratio and phase-displacements for this transformer, while figs. 9 and 10 give the P.D. relations which have been determined over a greater range of core induction than before.

As a test of formulæ 14 and 17 for the ratio and phase-displacement when used as a current transformer, we will take case VII., where the coils were separated and a secondary load of 5·9 ohms non-inductive was employed. The core induction was then found from the voltages in the unused coils and was taken at 1565 for a secondary load of 5 amps. On measuring the equivalent reactance of the transformer it came out at 5·12 ω or 2·56 ω for the secondary leakage reactance. From this we get a total secondary resistance of 7·1 ω, reactance of 2·56 ω, and impedance of 7·5 ω approx. $\cos\phi_s = \cdot 94$ and $\sin\phi_s = \cdot 34$.

Formula 14 gives us as the loss of ratio per cent.

$$100\frac{R_t - R_c}{R_t} = 100\left\{\frac{A_c}{A_s}\cos\phi_s + \frac{A_m}{A_s}\sin\phi_s\right\},$$

which works out at 1·92 per cent., while the experimental value is 1·8 per cent.

The phase-displacement in the same case may be taken as

$$\frac{A_m}{A_s}\cos\phi_s - \frac{A_c}{A_s}\sin\phi_s,$$

which works out to ·58°, the experimental value being ·75°.

In the case of a lagging load as in curve VI. but with the coils close together, the calculated loss of ratio is 1·87 per cent. as against 2·7 per cent. by experiment, and the phase-displacement works out at ·57° * which agrees with the experimental value. The agreement between calculation and experiment is not always therefore very close, but this is probably due to the difficulty of determining the core induction. In a current transformer the magnetic leakage may be quite comparable with the main working flux, owing to the small value of the latter, and hence the distribution of flux in the core is very irregular. The mean value of the induction, however, as calculated by formula 27, agrees very fairly with the experimental value, being 1575 as against 1565 in case VII., and 1345 as against 1385 in case VI. Calculations have not been made in other cases, as the induction densities are too low.

The statement on p. 243 that a non-inductive secondary circuit should give a better ratio than an inductive one of the same impedance, is amply borne out by comparing curves V. and VI. The advantage of a low induction density is also manifest.

Calculations of P.D. ratio and phase-displacement show an

* The curves for current ratio and phase-displacement appeared in the 'Electrician' (*loc. cit.*), but in case VI. the phase-displacement was unfortunately given with reversed sign.

agreement within ·1 per cent. in the ratio and ·01° in the displacement, which is sufficient for most purposes. The theory is in each case confirmed.

Tests made with instrument transformers of various makers have in no case shown anything like such good results, and the writer is of opinion that these results represent nearly the best that can be obtained without using some special alloy of low core-loss, and also of high permeability. In view of the importance which magnetic leakage assumes in current transformers, the secondary current nearly always lags 20° to 30°, and the magnetizing current is therefore nearly as important as the core loss-current.

Fig. 11 (Pl. IX.) gives the relation between A_e and A_m and is consequently the locus of the end of the no load vector A_0 for various inductions. This vector is therefore over a considerable part of the range, at about 45°, and consequently the best ratio should be obtained with a secondary circuit leading by about 45°. In fig. 12 a curve is given for the relation of $\frac{A_e}{B}$ to $\frac{A_m}{B}$ which is of use in working out the P.D. ratio and phase displacement.

Figs. 13 and 14 show curves of ratio and phase-displacement for three current transformers made by Messrs. Everett & Edgcumbe and Messrs. Nalder Bros. & Thompson. In the former, the primary and secondary windings were close together, giving good ratio but large phase displacement; and it is noticeable that throughout lower phase-displacement is got by greater loss of ratio, as indicated in the tests on the experimental transformer. In the Nalder transformer, the primary and secondary coils are wound on thick porcelain bobbins side by side. The leakage is therefore somewhat large, producing a variation of ratio of about 8 per cent. between 2 and 18 amperes; but this is accompanied by a phase-displacement which is less than ·2° over the range from 6 to 18 amperes or the upper two-thirds of the range. Such a transformer therefore, if used with a wattmeter, *and calibrated with it*, should do fairly well for commercial testing.

In conclusion it should be noted that the tests on all four transformers give results which are not very widely different and the results may therefore be taken as fairly representative of the behaviour of instrument transformers.

The writer wishes to express his thanks to his senior Demonstrator, Mr. A. C. Jolley, for great assistance in the experimental work, and to Mr. A. F. Burgess, B.Sc., for checking the calculations.

Discussion.

Mr A. Campbell expressed his interest in the thorough manner in which Dr Drysdale had gone into the question of the use of transformers with measuring instruments. With regard to the Author's criticism of his (Mr Campbell's) work on the subject (which was carried out twelve years ago) he should like to mention several points. The main object of the first quoted paper was to show that with air-core transformers the transformation ratio becomes more and more constant (for various frequencies) the higher we make the time-constant of the secondary circuit; thus high inductance and low resistance are wanted. Mr Campbell stated that in his paper he also gave several experiments to show that iron-ring transformers "may in many cases be used in a similar way," care being taken to have the resistance of the secondary circuit small enough. In a later paper he stated that to make the ratio sufficiently constant and independent of frequency we require relatively low resistance and high inductance in the secondary circuit. Dr Drysdale showed that for the special case of constant frequency, relatively high inductance does not give the most constant ratio. His (Mr Campbell's) experiments were not complete enough to settle this point and he was careful not to dogmatize on the matter. He was glad that Dr Drysdale had elucidated it.

Dr A. Russell expressed his interest in the paper. He suggested that the Author should take the mutual inductance between the shunt and the instrument into account. If L_1, R_1 be the constants of the instrument and L_2, R_2, those of the shunt, and if M be the mutual inductance between them, the multiplying factor for the reading will be the same whatever the frequency provided that $\frac{L_1 - M}{R_1} = \frac{L_2 - M}{R_2}$.

Mr Kenelm Edgcumbe referred to the fact that the Author recommended small core-losses, and pointed out that he had seen it stated that in special cases it was possible to improve the ratio and phase errors by increasing the core-losses.

Dr Drysdale, in reply to Mr Campbell, said his praise of the air-core transformer was justified, but in practice instrument-makers were forced to use iron for commercial reasons. Similarly, the ring form of transformer was generally impossible as in practice instrument transformers were used to insulate the observing instrument from high pressure mains, and considerable insulation between the windings was necessary. The reason why an iron-cored transformer behaved differently to one with an air core chiefly resided in the core-loss current which was not proportional to the magnetizing current. The method of testing the transformers would be shown in operation in the laboratories.

Fig. 7.—Ratio Tests on Current Transformer.

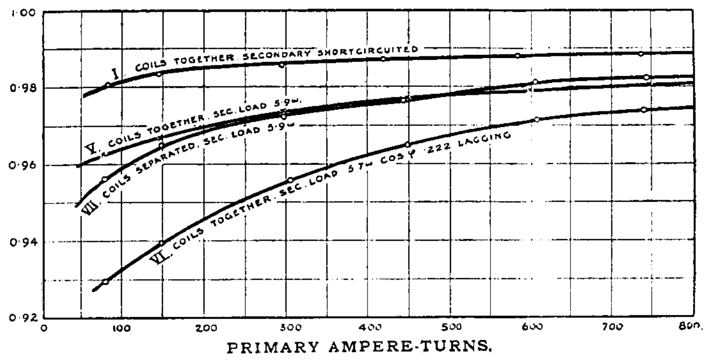


Fig. 8.—Phase Displacement on Current Transformer.

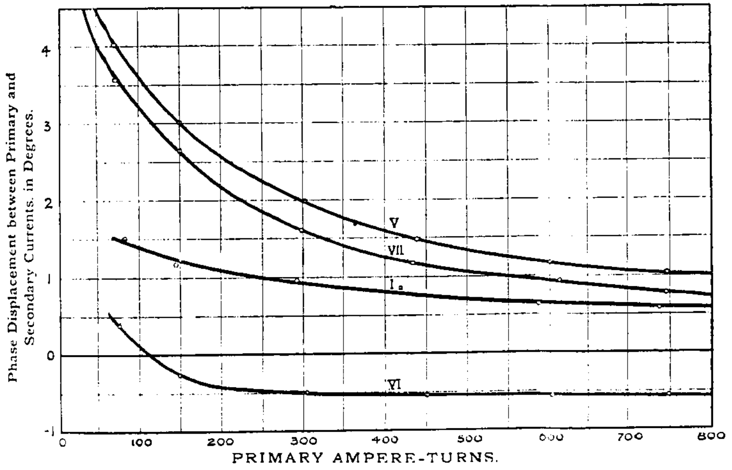


Fig. 9.—Ratio Tests on Voltage Transformer.

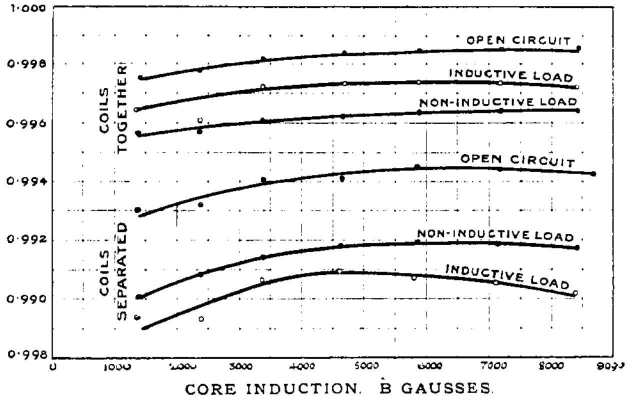


Fig. 10.—Displacement Tests on Voltage Transformer.

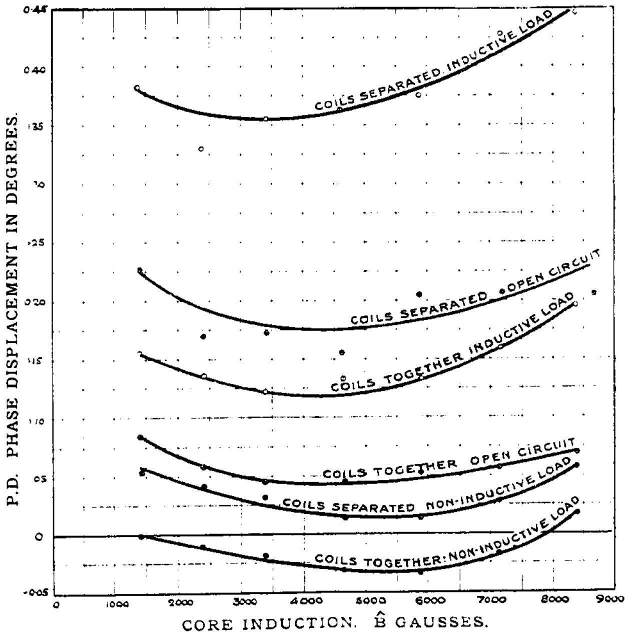


Fig. 11.—Relation between Core Loss and Magnetizing Current.

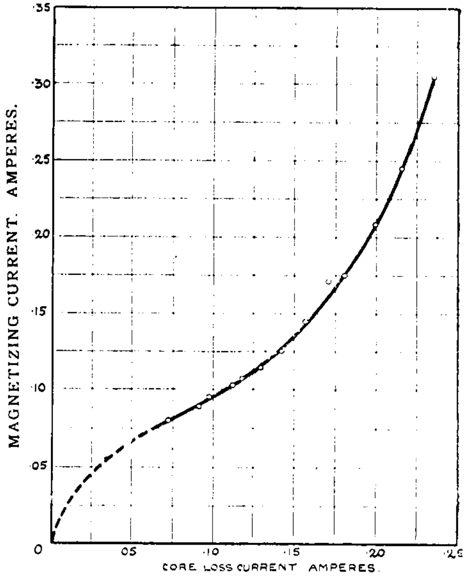


Fig. 12.—Relation between $\frac{A_c}{B}$ and $\frac{A_m}{B}$.

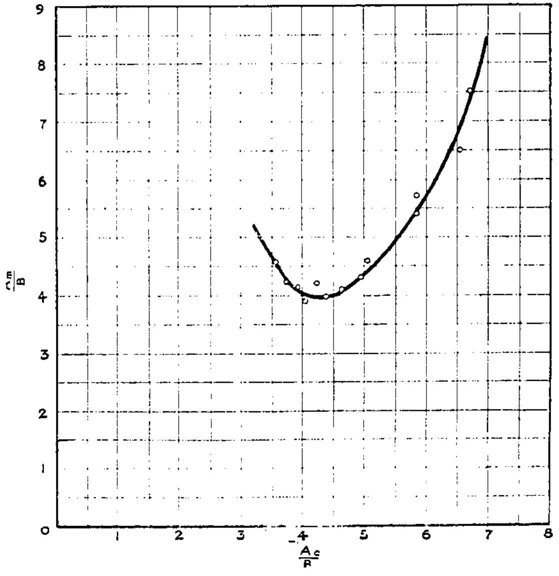


Fig. 13.—Ratio Tests on Current Transformers.

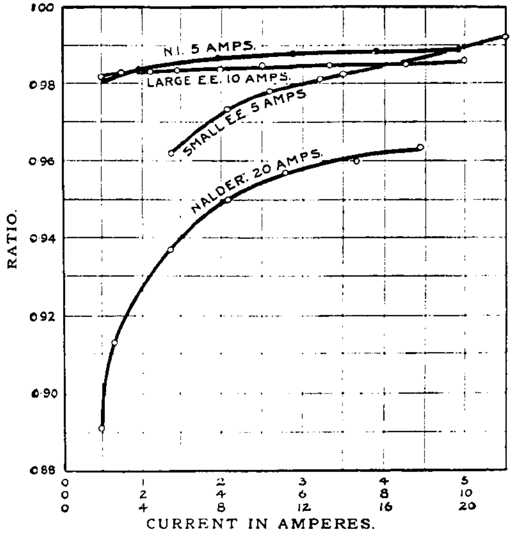


Fig. 14.—Displacement Tests on Current Transformers.