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THE USES, VALUE AND LIMITATIONS OF
GAME THEORETIC METHODS IN DEFENSE ANALYSIS

M. Shubik

October 9, 1985

THE USES, VALUE AND LIMITATIONS OF
GAME THEORETIC METHODS IN DEFENSE ANALYSIS*

by

M. Shubik

1. INTRODUCTION

It has been approximately forty years since the publication of the seminal work on the theory of games (von Neumann and Morgenstern, 1944) and it has been approximately twenty-five years since the stimulating series of essays by Schelling (1960) raised a series of questions concerning the application of an intermixture of game theory and gamesmanship to strategic analysis.

The recognition of the need to study and formulate the principles of war taking into account the intermixture of behavioral and technological factors, dates back around twenty-five hundred years to the writings of Sun Tzu (circa 500 b.c.).

Wisdom often begins with the recognition of a problem and an evaluation of the importance of its solution. Unfortunately the recognition of the importance of a problem and our ability to find a solution may be separated by centuries if not millenia. Thus there has been a steady stream of

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invention and increase in the killing power of weaponry over the centuries together with an explosion in the means of communication in the last century. Yet it would be difficult to attribute the same level of progress to society's ability to control war.

Although the seeds for the mathematical formalization of tactical, strategical and diplomatic procedures are already present in Sun Tzu the specific development of a mathematical language for the study of conflict, cooperation and negotiation did not occur until the advent of the theory of games. Paradoxically the precision of the mathematical methods of game theory have helped to illustrate the imprecision and the elusiveness of many of the concepts at the basis of the science, art and social process of war.

The prodigious growth in the destructiveness of weapons has made it imperative that in order to survive we develop a capability to control the use of war. But this capability must be manifested in many ways such as the political and bureaucratic control of crisis instability; the ability to interpret ambiguous information and to prevent the propagation of error in command and control systems as well as the overall psychological and political ability to prevent passion or panic from dominating decisions. The dangers of going to red alert have been appreciated at least since the time of Xenophon (see Warner, 1949). Yet our ability to cope with tension and mutual suspicion is probably no greater today than it was over two thousand years ago.

The perception of and concern with the current dangers is manifested in the activities of physicists, senior defense officials, both active and retired, diplomats and political scientists, economists, game theorists, military operations researchers, lawyers, social-psychologists and others. Conferences involving interdisciplinary approaches serve to indicate that

there is an understanding that the control of conflict will not come from a single technological, legal, economic or socio-psychological fix. Reality is too complex to have our problems fully solved by a new piece of hardware, a change in costs, a new treaty or a modification to international law.

In spite of the recognition of basic problems requiring study and collaboration among individuals with highly diverse backgrounds and views, our success in developing an adequate understanding of threats or crisis control has been minimal.

Even some of the most elemental questions are not clear. Virtually all professionals concerned with defense foresee the horrors of nuclear war and subscribe to the desire to avoid it. Yet the goal of making the world free from nuclear war is not inconsistent with making the world safe for conventional limited war. Empires grow and decay. Nation States emerge and evolve. National and tribal boundaries change. No war should not be a synonym for no international change. The traditional way the map of the world has been remade has been by armies, navies, cavalry sweeps, sieges, bombardments and other acts of force. Possibly there are better ways but they have yet to be institutionalized.

The growth of civilization requires self-understanding and self-control. The necessities of this current stage in world history require a deep examination of our ability to built and control national and international systems for the command, control and communication over weaponry of a destructive power sufficient to endanger the survival of our societies.

Much of the analysis done by physical or social scientists is based upon an implicit or explicit assumption of rational behavior. The central actor in much of economic theory is homo oeconomicus (economic man), a direct descendant of Benthamite man. The actor in game theory is a direct descendant

of economic man. He knows what he wants, knows what actions are available to him, is able to calculate without passion or other distractions. He understands the logic of cross-purposes optimization in situations involving two or more strategic actors. In particular he knows that in general the concept of individual rationality does not generalize to situations with more than one actor.

The central contribution of game theory to defense analysis has been a language for the understanding of how to formulate and study strategic or cross-purposes optimization in situations involving two or more actors. It is suggested here in the remainder of this discussion that two fundamentally different classes of application of game theory to problems in defense have emerged. The first is the application of two-person zero sum game theory to military, primarily tactical situations which for the purposes at hand can be reasonably well modeled in this manner. The second is the application of two or more person nonconstant sum game theory to strategic problems involving threat analysis, crises control and the interface between international diplomatic relations and war.

When our interests are confined to applications of the two-person zero sum theory, a reasonably strong case can be made for homo ludens (game playing man) as an intelligent, calculating entity with no personality or psychological foibles playing against an equally bloodless opponent. It can even be argued that there is a natural way to extend the concept of individual rational behavior to the two-person zero-sum game.

When our concerns are strategic and the game is naturally a non zero sum cross purposes or mixed motive optimization, where there is neither total coincidence or total opposition of interests, the model of the individual actor raises many basic problems. The Soviet Union and the United States

may not be adequately described as two abstract players. Yet for some purposes such as the study of mass market behavior the model of many agents as intelligent economic men devoid of personality and concerned in a single-minded manner with the pursuit of profit may be adequate (see Shubik, 1982, 1984).

Implicit in setting up a model of some aspect of reality for analysis is a host of basic assumptions concerning the players and their environment. Once the implicit and explicit assumptions have been accepted analysis, calculation, logic and mathematics take over. The two critical sets of assumptions which must be made concern the players and their environment. Social psychologists, biologists and field commanders are concerned with individual differences. Yet underlying most analytical models more or less as a simplifying necessity for analytical purposes is the condition of external symmetry. Any difference in abilities of individuals must be specified within the model otherwise all nonspecified attributes are regarded as the same.

Unlike formal games such as chess or Bridge or Poker which have well defined beginnings and ends most models of strategic situations cannot be easily cut away from history or assigned a clear termination. In essence it is the responsibility of the modeler to establish the context in which the game is played. This is most clearly seen when a scenario is prepared for a war game or a politico-military exercise (P.M.E.).

The importance of context is such that rather than use the phrase "rational behavior" I suggest that the phrase "context rational behavior" provides the analyst with a reminder that behavior must be assessed in the context of the situation at hand and it warns against spurious generality.

The value of the assumption of external symmetry is enormous as a device for simplification. But it must be justified in an ad hoc manner. Thus for

example if we wish to apply two person zero sum game theory to a weapons evaluation problem involving a tank on tank duel the assumption that the opposing crews are equal in all respects appears to be reasonable.

In much of economic theory the assumption of ceteris paribus (all other things being equal) plays the same role as external symmetry. In both instances underlying the assumption is that unless otherwise specified individuals are fully rational perceptive optimisers. But the assumption of the existence of homo oeconomicus or his strategic cousin homo ludens should not be regarded as a step towards some ultimate abstraction of a Platonic rational man, but as a simplification for an individual who, no matter how intelligent or perceptive, is limited in his capacity to see, comprehend, process and act upon all of the information available.

Especially in the study of threats, bargaining and negotiation there has been a realization that it may be worth considering richer models of man. Yet when behavioral models are proposed phrases such as "limited rationality" are used to describe the actors. Unfortunately, the word "limited" frequently carries with it a pejorative connotation. I propose that the phrase "capacity constrained rationality" is more neutral. If we are willing to assume that information gathering and processing takes time and that no individual can know all that there is to be known then we need a definition of what we mean by rational behavior where the individual is aware that his decision must be made with limited knowledge concerning the prospects he faces.

A way of reconciling capacity constrained rationality with our model of rational behavior under complete information is provided by an interpretation of context rational behavior. If S_i is the largest set of prospects which can be handled by an individual i and S (where $S \supset S_i$) is the

total set, then culture, society, indoctrination and professional training may serve as some of the devices which map S onto S_i . Rational behavior is defined in reference to actions on S_i , but context has been set by the mapping of S to S_i . Thus for individuals with limited capacity rational behavior can only mean context rational behavior.

The work of Harsanyi (1967, 1968) on games with incomplete information takes the initial prior subjective probabilities as given with no explanation as to where they come from. Selten (1978) considers how to cope with structural uncertainty by considering a way to deal with unknown prospects. The valuation of the unknown is a parameter in his system whose setting could be explained by context. The work of Tversky and Kahneman (1981) on "framing" can also be interpreted as concern for context. Ellsberg's (1961) paper on the distinction between uncertainty and ambiguity must be considered in terms of context. An individual can choose between two urns. In one he knows that there are fifty black and fifty white balls. In the other all that he knows is that there are a hundred balls of color black or white. He obtains a prize if he selects a black ball. How does he frame his view of the experiment and the experimenter. If he thinks that the experimenter is saving on experimental funds the second urn may have a hundred white balls. If he is trying to reward daring needy graduate students it may have a hundred black balls.

2. GAME THEORY AND TACTICAL ANALYSIS

In a word it is easier to make a theory for tactics than for strategy.

Clausewitz

A rough rule of thumb is that if a conflict situation can be reasonably well modeled as a two-person zero sum game with complete information about the rules then it is amenable to a considerable body of game theoretic mathematical analysis which covers both statics and dynamics. If the situation is not well described as a two-person zero sum game then an ad hoc justification of the realism and relevance of the model, the motivations of the players and the solution concept must be given.

There have been stimulating and insightful analyses of arms races based on differential equation models (Richardson, 1960; Rapoport, 1961; Intriligator and Brito, 1984) and on control theory (see references in Ho and Olsder, 1983). But it must be stressed without qualification that for dynamic nonconstant sum games in general there is no generally agreed upon solution concept.

Although questions concerning arms races, crisis instability or the spread of nuclear weapons cannot be treated generally with great confidence by mathematical analysis, many problems at the level of tactics and weapons evaluation can be modeled with a high degree of confidence as two person zero sum games.

There is a substantial literature on duels, a survey of which has been given by Kimeldorf (1983). This work is of direct application in weapons evaluation where the relative vulnerabilities and advantages of weapons in one-on-one combat can be investigated. There is also a considerable body of literature on search and pursuit games (Dresher, 1961; Ho and Olsder, 1983). Game theoretic methods can be applied directly to the study of

optimal pursuit and evasion tactics such as the search for an enemy ship or submarine in a security zone. Tactical air support for land forces has also been considered (see Berkovitz and Dresher, 1960; Berkovitz, 1975).

Although not strictly game theory the mathematical work on models of combat attrition may be considered as closely related to two person zero sum games. Many variants of the Lanchester equations have been studied and applied in weapons evaluation and force and doctrine studies of combat simulation. The work of Taylor (1981) and the survey of Karr (1983) provide an extensive overview of the Lanchester attrition processes including their applications to theater level combat simulations.

Even for two person zero sum situations modeling problems arise. Weiss (1983) in his perceptive writing on the requirements for the theory of combat notes the problems with considering morale and the problems in characterizing communication and its failure.

There is a small but substantial community of those concerned with military operations research working at problems where two person zero sum game theory provides both appropriate models and conceptually satisfactory mathematical methods for direct application and computation of relevance to military problems primarily at a tactical level. This work is characterized by a maximin solution concept which pertains naturally to the problems under consideration. Unfortunately in the shift of concern from tactical to strategic problems neither the maximin solution nor the assumptions concerning external symmetry hold in general. An attempt to concentrate on worst case analysis can introduce a conservative bias verging on the paranoid.

In the shift from tactical to strategic problems the uses, limitations modeling problems, solution concepts and applications of game theory change radically. The stress tends to be more upon the clarification of concepts.

and the construction of adequate models than upon computation and the exploitation of specific mathematical structure.

3. STRATEGIC ANALYSIS AND THE THEORY OF GAMES

3.1. On Numbers

There are possibly four numbers which count in the study of nonconstant sum games. They are two, three, few and many. The economic studies of competition have concentrated heavily on duopoly (two), competitive markets (many), oligopoly (few) and to a lesser extent on triopoly (three). In strategic analysis where the players are nation states the economist's concern for anonymous mass competition does not appear. Two, three and few are the numbers of concern.

The division between two and more than two players is often of value. In particular the analyst must always be on guard against the dangers of "maximin, worst case or polarized opposition" thinking about two country strategic models.

Two person nonconstant sum games split naturally into three classes (1) no conflict games; (2) games with pure opposition and (3) mixed motive games. In a utopian world with the coincidence of joint optimality the problem of choice by individually rational players is solved. Games with pure opposition need not be zero sum. For example we can consider two players with strongly ordered preferences $a_1 > b_1 > c_1 > d_1$ and $a_2 > b_2 > c_2 > d_2$. The game shown in Table 1 is one of pure opposition

	1	2
1	a ₁ ,d ₂	b ₁ ,c ₂
2	c ₁ ,b ₂	d ₁ ,a ₂

TABLE 1

although $a_1 + d_2$ is not defined. Mixed motive games are the predominant type of interest. The two parties have neither a complete community nor opposition of interest, but an intermix of both.

When we raise the question of what we mean by a solution to a game the division between two player games and those with three or more becomes even more striking. In particular dynamic games with few (but larger than two) players are virtually intractable to formal mathematical game theoretic analysis for two reasons. The first is that there is no consensus as to what constitutes a good solution concept. The second is that if one selects a leading contender such as the "perfect equilibrium" (Selten, 1975) it is hard to compute and, in general, is not unique thus one cannot uniquely describe the development of play. The equilibrium concept of Kreps and Wilson (1982) is highly related to the perfect equilibrium but the equilibria are easier to compute, but the lack of uniqueness still remains.

3.2. The Modeling Checklist

In a previous note on the study of disarmament and escalation (Shubik, 1968) seven basic considerations in modeling were raised. They were: (1) Who are the players, (2) How is utility evaluated, (3) What do the payoff matrices mean, (4) What are the moves, (5) What knowledge do players have of each other's values, (6) What is the coding problem and, (7) How are coalitions, countries and institutions modeled? Together with the first chapters of Game Theory in the Social Sciences (Shubik, 1982) these two references provide a discussion of many of the problems in constructing formal mathematical models in "soft subjects" such as threats and negotiations.

Although many of the items on the 1968 checklist remain, further work on the theory of games, in experimental gaming, in the analysis of war and

the potential for war and in experimental psychology has suggested some modifications. In particular more emphasis is placed here upon the modeling of utility or preferences; the description of and calculations with uncertainty; the treatment of information and communication conditions and the meaning of misperception and irrationality in the context of a dynamic game.

Before discussing the many suggestive insights to be gleaned from examining two-by-two matrix representations of various strategic problems a brief exposition of the more important modeling problems is given.

(1) Who are the players

This poses problems in aggregation, historical context and preference representation. Although lip service may be paid to conceptualizing the United States and the Soviet Union as players these entities are hardly individuals with well ordered preferences. Any theory of war or crises control calls for a finer characterization requiring the specification of "games within the game" (Shubik, 1984). It is well known that there are considerable difficulties in constructing a completely ordered set of social or societal preferences from the ordered sets of preferences of individuals even without postulating the political parties and bureaucracies of a society which aggregate these preferences. The review of Yuen Foong Khong (1984) of Bueno de Mesquita's (1981) book The War Trap notes this particular problem.

A glance at the command and control systems of the U.S. or the Soviets should be enough to warn against too heavy an aggregation of players in the study of crises control (see Bracken, 198 , Blair, 1985).

Lesson: Aggregation can create players whose preferences are difficult to describe and in general will not form an ordered set. Thus what is "rational behavior" for a country is hardly a general operational concept.

(2) How is utility evaluated

In a weapons evaluation exercise it is not too difficult to construct a loss function based upon the number of tanks or aircraft knocked out on each side. The theory of duels or pursuit and evasion have natural payoff structures. Even in economics the structure of preferences considered in consumer choice has an underlying justification in frequently made choices by many individuals over a set of reasonably well defined goods or services. Preferences over objects of political or diplomatic choice are far more difficult to justify. Given that international relations in general and defense policy in particular are run by fiduciaries it is unlikely that the public even perceives of the choices seen by its fiduciaries, thus while the latter may have preferences defined over moves, the former may have preferences defined only over the few outcomes they perceive.

When we move to the evaluation of megadeaths the meaning of "acceptable casualties" becomes a topic for jesuitical debate. Even thinking about the concept depends upon how the question is framed. For example can we think about a war in which the percentage of world population killed equals that inflicted upon the world of Ghengiz Khan? It is estimated that his invasions may have wiped out between 8-12 percent of the world population. In modern terms this is around the combined population of the United States and the Soviet Union.

(3) How is uncertainty treated

Since the seminal work of von Neumann and Morgenstern (1944) the conceptualization of risk and preference for risky outcomes has been radically changed. Their key observation was logical, but with empirical overtones. If you are willing to believe the economists' assumptions concerning complete ordering of preferences over certain prospects then all you have to do is to swallow an extra couple of innocent looking axioms and you arrive at a utility scale or a utility function defined up to a linear transformation.

This observation provided a fundamentally new view of how to consider decisionmaking under uncertainty. The theory raised experimental, empirical and mathematical problems. Some forty years later it appears that the original assumptions concerning preference and the added assumptions concerning uncertainty were not as innocent as they might have seemed to be. In particular it is precisely in the domain of the problems faced in control of crises and the limiting of international political tension where many of the difficulties appear. What utilities are to be attached to one's own death, or the death of others. How are these values influenced by context. In the psychological literature it is well known that the way in which a question is framed may have considerable influence on the response. Tversky and Kahneman (1981) present an example of a question concerning the treatment of a disease. It was framed once in terms of individuals probably saved and once in terms of individuals who might die. The two questionnaires evoked considerably different responses. Another phenomenon observed by the psychologists is that the treatment of extremely low and high probability events does not appear to fit the expected utility hypothesis. Tversky and Kahneman (1981) offer a prospect theory to explain the divergence from expected utility theory. Einhorn and Hogarth (1984) offer a different

explanation based upon Ellsberg's distinction between ambiguity and uncertainty. In the early literature on insurance and lotteries Friedman and Savage (1948) distinguished between attitudes towards risk of loss and risk of gain. Tied in with the possibility of loss is the individual's attitude towards survival. Perhaps as the risk of failure to survive is increased there comes a point of discontinuity in behavior. Jervis (1976) has applied the observations on small probabilities to international politics.

Virtually everyone of these problems concerning risk behavior are central to the control of nuclear forces in the time of crises. If defense budgets are to be in tens to hundreds of billions it would appear that a major research program on risk behavior would be of considerable value.

The comments above are all concerned with individual risk behavior yet in major crises involving large bureaucracies group behavior may be of even greater importance. The comments of Janis (1972) on "groupthink" raise a new set of problems in crisis control.

(4) Information and communication; (5) Coding

Information and communication and the somewhat cryptic heading of coding are placed together here. Even though they are separate topics the link between them is at the nexus of many of the major difficulties in understanding threats and behavior in crises. In game theory information has an extremely precise context free meaning. There are a number of states which can be distinguished by an individual. An information set portrays the set of the states among which an individual is unable to distinguish. If A's information is a refinement of B's A has more information, he is more perceptive.

But crises between nations invariably take place in a context. The words and deeds must be interpreted in the context of the history and

institutions involved. Game theory or for that matter any other formal decision theory has little to contribute to how one is satisfied that the appropriate context has been reflected in the model and its analysis. Information is not the mere receipt of a signal but its interpretation.

One type of problem involving information in a more or less context free mode is the inspection problem which can be formulated in several variations dealing with attempts to verify limits on testing, numbers or locations of weapons (see Aumann and Maschler, 1966).

Another problem involving information and communication is retargeting. We may represent a defense command and control network as a graph with arcs being communication lines and nodes missile sites or command centers. An attack removes some nodes and arcs. After the attack an individual commander located at some node may only have limited knowledge of how much of the system has survived. How should his optimum response policy depend upon the state of the system? Bracken, Haviv, Shubik and Tulowitsky (1984) have examined some simple models and a far more extensive treatment has been given by Weiss (1983).

For the mathematical game theorist, military operations research analyst or technical systems designer once values have been assigned to arrays of surviving targets several interesting and difficult mathematical problems can be formulated. But the understanding of damage exchange rates in a first and second strike analysis counting targets destroyed on both sides provides only a partial insight into the analysis of grand tactics in a nuclear war. I use the phrase "grand tactics" to stress that in a strike and counterstrike analysis the time span is short and resources are fixed as in tactical analysis yet the consequences are clearly at the strategic level. Information that a launch has taken place is referred to as tactical warning

(Bracken, 1984). This indicates that if we believe that a first strike must be punished then damage exchange or zero sum thinking takes over. But it is precisely the difficulties encountered in attaching values to wiping out populations and in deciding why a threat to retaliate should be carried out, which raise substantive issues in reconciling the strategic with the tactical damage exchange view of nuclear exchange.

Of prime concern to the strategic analyst is the possibility for the compounding of human errors in a command and control system. Although there are many specific calculations called for to estimate how much redundancy is required to keep certain types of error below some specified level, for the most part the problems are preformal. The uses of game theory and other methods are to pose questions and provide examples in order to isolate phenomena.

3.3. The Search for Solution Concepts

For tactical problems involving two players the saddlepoint or maximin solution provides a reasonable conservative extension of individual rational behavior to a two person game of opposition. The solution extends to multi-stage games, and with appropriate qualifications to stochastic games of indefinite length.

Strategic problems have been considered primarily as games in extensive, strategic or coalitional form. The coalitional form is used as a parsimonious way to study the potentials for cooperation in a cartel or alliance. The format purposely suppresses details concerning information, process and strategy. There have been several attempts to devise a formal dynamic theory of coalition formation (for example Shenoy, 1979) but little application to defense. The various cooperative game solutions, the core, value, nucleolus and others (see Shubik, 1982) do not yield particularly

deep insights for two player games, and are essentially static solutions (although some formal connections between the core and strong noncooperative equilibrium points have been made: Aumann, 1959).

Except for what might be a highly useful core analysis of alliances; cooperative game theory with its emphasis on avoiding dynamics and any explicit consideration of strategy does not appear to be particularly promising for the analysis of defense problems.

The obvious candidates for models of defense problems are games in strategic or extensive form. Figures 1a and 1b show the extensive and strategic forms of a two move game where Player 2 is genuinely committed to a no first strike policy. Player 1 either instigates a first strike or not. Player 2 faces the option of a counterstrike if attacked.

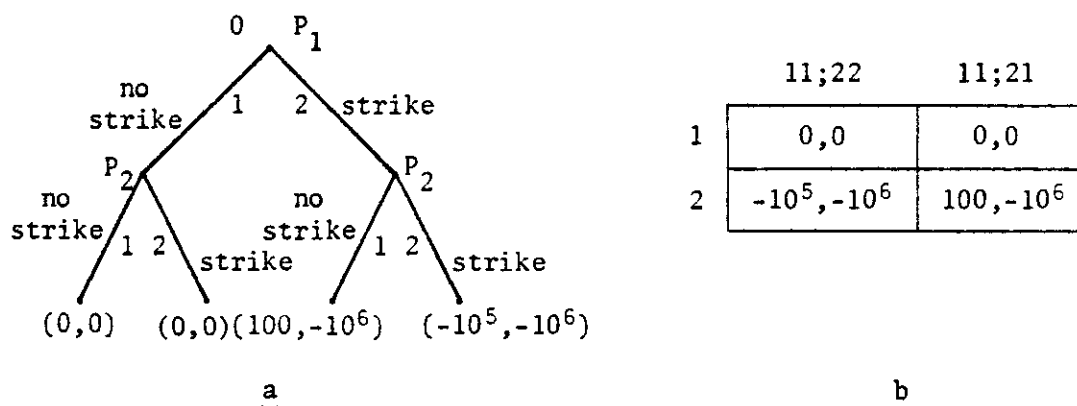


FIGURE 1

How realistic or valuable these representations are depends heavily on the problem. But even assuming that they are adequate the analyst requires a solution concept with which he can analyze the game. As is well known the noncooperative equilibrium (and several variants of the noncooperative equilibrium) is the dominant solution used to analyze games in strategic or extensive form. The main property of the noncooperative equilibrium is

optimal response. If A knows B's strategy then at an equilibrium (an N.E.) A will have no desire to change his strategy as he cannot improve. The same holds for B. But as attractive as the condition of optimal response may be there are others such as maximin, efficiency, symmetry which have been mathematized and still others such as honor, revenge, insecurity, fear and hate which have not yet been adequately characterized in formal models.

In Figure 1b there is a single N.E., at the strategy pair (1,11;22). This is interpreted as Player 1 uses his strategy to not strike. Player 2 uses his strategy which contains the contingent plan "If not struck do not initiate hostilities, if struck retaliate." Unfortunately, uniqueness of equilibria is not the rule especially for multimove games. Consider the matrix game shown in Table 2 played twice. In extensive form the game tree would have $3 + 3^2 + 3^3 + 3^4 = 120$ branches and each player has 3^4 different strategies. The matrix is the Prisoner's Dilemma bordered by a threat alternative. The one play game has two noncooperative equilibria at the

	1	2	3
1	5,5	-3,10	-10,-10
2	10,-3	0,0	-10,-10
3	-10,-10	-10,-10	-10,-10

TABLE 2

strategy pairs (2,2) and (3,3) with payoffs of (0,0) and (-10,-10) respectively. But if the game is repeated twice a strategy of "Play 1 to start, if he plays 1 then play 2, otherwise play 3" is such that if both use it they obtain (5,5) in the first period (0,0) in the second and it is in equilibrium. A repetition of (2,2) or (3,3) on each play gives two more equilibria.

We now may make the distinction between an equilibrium point and a

"perfect" equilibrium point in a sequential game. Figure 2 shows part of the game tree (in abbreviated form) for Table 2. The notation P_{12} indicates simultaneous moves as does the labeling of the branches by 1,1 to 3,3. A perfect equilibrium point (Selten, 1975) is not merely an equilibrium in the game as a whole but is also in equilibrium in every subgame, where a subgame can be regarded as an independent game. A subgame begins with a point of perfect information; each individual knows exactly where he is in the game tree. No information set in a subgame includes any node outside of the subgame. The dashes in Figure 2 enclose a subgame.

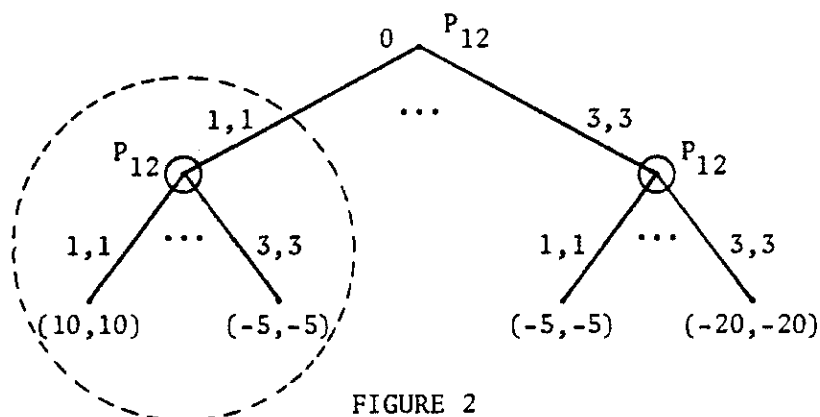


FIGURE 2

The threat strategy pair form a perfect equilibrium because after a doublecross, even though both players would lose 10 each by the use of (3,3) in the subgame this is an equilibrium. If we changes the payoffs slightly as is shown in Table 3 then the threat strategies no longer give rise to

	1	2	3
1	5,5	-3,10	-10,-11
2	10,-3	0,0	-10,-11
3	-11,-10	-11,-10	-11,-11

TABLE 3

a perfect equilibrium although they remain as an equilibrium. The key distinction involves the treatment of the plausibility of threats. The argument by those who support perfect equilibria is that the use of move 3 in the second play of the game after being doublecrossed is plausible for the game in Table 2 but not for the game in Table 3. But it is likely that the difference in plausibility between accepting payoffs of -10 and -11 is not great and the perfect equilibrium does not show this. The plausibility of a threat may depend significantly upon context.

The importance of context is nicely illustrated by an example suggested by Rosenthal. Consider the game shown in Figure 3 and Player 2's strategy.

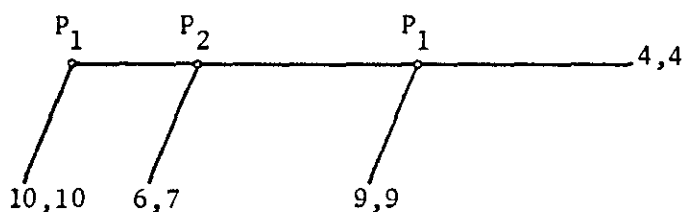


FIGURE 3

Clearly if Player 1 plays down both get 10 and the game is over. If he plays right then Player 2 can end the game with payoffs (6,7) or can give the move back to Player 1. The equilibrium should have Player 1 go down and Player 2 go right if he gets the move, followed by Player 1 going down to get (9,9). But if Player 2 ever gets the move Player 1, by the theory is either irrational or has made a blunder of some sort. Player 2's strategy may well depend on his interpretation of the context in which he obtained the move.

The lack of uniqueness* of N.E.'s makes the backward induction a somewhat

*Harsanyi and Selten (Harsanyi, 1982) have recognized the difficulties posed by multiple equilibria and have proposed a solution concept which selects a unique equilibrium. Justice cannot be done to its intricate reasoning here.

metaphysical exercise. Figures 4b and 4c show two different backward inductions on the simple two stage game shown in 4a. But if Player 2 were willing

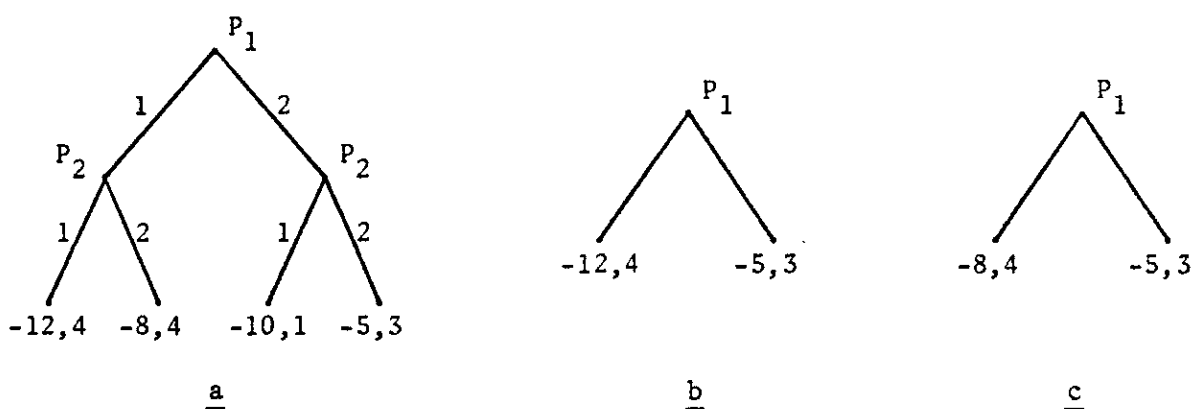


FIGURE 4

to commit to the "irrational" choice of move 1 if Player 1 chose 2 and 2 if he chose 1 then Figure 5 shows that this could be to his advantage.

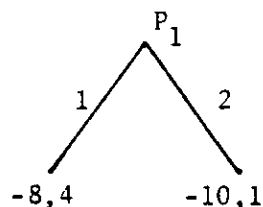


FIGURE 5

Of the 78 different 2×2 matrices with strong orderings on payoffs 3 are with pure opposition, 21 are no conflict games and the remaining lead to mixed motives for cooperation. Within these games the paradox of the failure of individual rational behavior to coincide with joint rationality appears in many different ways as is indicated. When ties or inability to distinguish between outcomes are also considered the number of cases becomes 726 and the need for rules concerning coordination and tie breaking procedures emerges. Many of the rogues gallery of examples have been discussed elsewhere (Schelling, 1960; Shubik, 1982, Ch. 10). Tables 4a, b, c, d, and e

	1	2
1	2,2	0,3
2	3,0	(1,1)

a Prisoner's Dilemma

	1	2
1	0,0	(1,2)
2	(2,1)	0,0

b Battle of Sexes

	1	2
1	0,0	3,2
2	2,3	1,1

c Chicken

	1	2
1	(1,1)	(2,1)
2	(1,2)	(2,2)

d Fate Control

	1	2
1	(1,1)	0,0
2	0,0	(1,1)

e Pure Coordination

TABLE 4

illustrate a few with the N.E. circled. In these games in (a) the equilibrium resulting from strictly dominating choices is not jointly optimal; in (b) a rule to break the impasse is required; in (c) there is no pure strategy equilibrium. In d and e coordination is a must. In d each player controls the other's payoffs. Each is hostage to the other.

To Schelling, myself and others the resolution of several of these problems is in context or beyond the formal model or in a richer model containing traditions, conventions, cultural norms, cues, prominence or other factors. If these are not added then a way of looking at the lack of uniqueness is that the mathematical description is not rich enough to narrow down the outcomes to a single one. Harsanyi adopts the view that there should be enough there to make the selection without invoking outside factors.

A promising new development which represents a considerable departure from the standard noncooperative equilibrium analysis has been put forward by Rubinstein (1985). He considers that a matrix game is to be played by

two finite automata known as Moore Machines. The players select the machines and must pay a fee for every state that is used for some percentage of the time in an indefinitely repeated game. This cost enters into the payoffs and should be minimized (actually the payoff he defines has it enter lexicographically into the payoff hence the optimization is somewhat complicated).

An interesting point in defense studies is raised. Should one keep paying for a force that is never used (see also Downs, 1964)? In Rubinstein's model the answer is no. As the stockpiles of nuclear weapons grow what constitutes use and value becomes both an academic and operational question.

For those who wish to apply formal game theory to negotiation and nuclear brinkmanship, the formal problems encountered in developing solution theories serve as a warning that calculation without context or specific ad hoc justification of the solution concept chosen may not be justified.

4. BARGAINING AND THREATS, ARMS RACES AND CRISIS STABILITY

Three topics of broad application and interest to those concerned with defense at the diplomatic and strategic level are noted.

4.1. Bargaining and Threats

The literature is large and diverse. A partial bibliography is provided by Shubik (1982, Ch. 12). For the defense analyst it is possibly worth noting three different broad sets of work (1) the formal theory, (2) experimental work and behavioral approaches and (3) an intermixture of essay analysis combined with predominantly 2×2 game examples.

The formal theory essentially started with Nash's (1953) analysis of a bargaining game with threats. Although Zeuthen's (1930) early work pre-dating formal game theory contained similar ideas. Since Nash's work many alternative models of bargaining and fair division have been developed.

Roth (1972) gives a valuable discussion of many of the alternative axiom sets. Ståhl (1972) attempts to lay out a formal dynamic theory. But in essence the formal work on bargaining and fair division appears to have had little direct application to strategic defense problems.

There is a growing literature in experimental gaming primarily in economics and social psychology. In some instances the relationship between theory and experimental results has begun to grow (see Smith, 1970 and Sauermann, 1967) but the perceptive book on the art and science of negotiation by Howard Raiffa (1982) shows how wide is the gap between formal theory and application. Context, socio-psychological and organizational factors all of which are usually carefully excluded from the economic and game theory models appear to play important roles.

Policymakers do not have the luxury of being able to wait for theory to take a decade to justify or explain what they may have to do tomorrow morning. They tend to be more literate than numerate. Thus although massive calculations may have little impact, game theoretic reasoning and simple but striking examples such as the prisoner's dilemma matrix have much to teach. The intellectual basis for Mutually Assured Destruction (MAD) is clearly game theoretic but the problem of how well it might work is heavily an empirical one concerning how well the simple game theoretic model fits the facts. This is even true of 2×2 matrix analogies as is shown in Table 5 below (a more detailed discussion is given in Shubik, 1983, see also Strauch, 1983).

Table 5a shows a simple situation in which Blue can retaliate to Red's strike and guarantee to inflict a damage level of -100 (Problem: What do the -200 and -100 really mean and who evaluates them?). If Blue fails to retaliate the payoffs are -200 and +100. We evaluate the status quo at 0,0.

	R	1st Strike	No Strike
B			
Launch on Warning		-200 -100	0 0
No Retaliation		-200 +100	0 0

a

	R	1	2
B			
1		-200 to -100 -100 to +50	0 0
2		-200 to -100 50 to 100	0 0

b

	R	1	2
B			
1		-200 0	0 0
2		-200 +100	0 0

c

TABLE 5

The only equilibrium is at (1,2) or launch on warning and no strike. A similar analysis with a Blue first strike completes a picture of a simple MAD world.

The numbers in the matrices have to be calculated or estimated from somewhere. This calls for evaluating one's view of the worth of targets destroyed, megadeaths, victory, defeats, Armageddon or however else one wishes to portray the consequences of nuclear war.

Even if we assume that we have a measure for the consequences of war, another important problem concerning the representation of payoffs in the matrices is do we wish to display expected values or should we indicate the degree of uncertainty separately. In matrices 5a and 5c we interpret the outcomes as certain. In matrix 5b rather than indicate expected payoffs the ranges have been indicated. We can see that in cell (1,1) there is a range of 100 in the estimates of the damage caused by a first strike. Furthermore there is a swing in the effectiveness of the counterstrike from -100 to 50. But with these shifts in values the risk assessment could be such that for some evaluating 5b a first strike is worth the gamble and for others it is not. In 5c, with outcomes as certain, a first strike pays off as the counterstrike fails.

The concentration on the 2×2 analogy leaves out the important possibility

that nuclear threat is used to achieve moderate political goals thus although the dichotomous choice presented by a simple MAD analysis may serve to illustrate what is required to deter a first strike it cannot be used to study the important interfacing between nuclear threat and limited aggression.

The intermix of essay form with examples requiring little or no mathematics can be attributed first to Schelling (1960) this type of analysis has been continued with somewhat more formality by Brams and Wittman (1981), Brams and Hessel (1984), Zagare (1981), Moulin (1981) and others. Brams and Hessel apply their analysis to the Polish internal crisis of 1980-81.

In a perceptive article Strauch (1981) has suggested that "It is this transfer of understanding which should be considered as the primary product of planning." His observation applies to this form of analysis. The models may be perceptive and the conjectured chains of action and reaction raise interesting questions. For transmitting ideas, advice and ad hoc knowledge to aid in policymaking this approach appears to be valuable. But as a research tool it is limited. Schelling and others have raised many interesting questions but after twenty-five years few if any answers have been supplied by this mode of analysis.

4.2. Arms Races and Escalation

The work of Richardson (1960) and Rapoport (1961) on arms races has already been noted. Intriligator (1982) provides further references and the Intriligator and Brito (1984) model of arms races show the close analogy between certain economic models of duopolistic competition and war outbreak in armament build up or build down. Without going into mathematical detail the driving observation behind the Intriligator, Brito analyses and the Shubik, Thompson (1959) model of duopoly as a game of economic survival is that there can be too little armament or investment in the firm as well as

too much. An arms race takes away from the productive use of assets elsewhere. However in a world with complete or little armament the relative advantage given to the nation with a few extra arms becomes considerable. Thus in the zone near zero, instability increases as, given a slight advantage it may be worth the risk to start a fight. At the other end of the spectrum there is an upper bound at which the overkill possessed by each side is so large that even a large accidental fluctuation does not disturb stability.

Differential or stochastic game models which can be explicitly analyzed like business cycle models in economics or Lanchester attrition models tend to be narrow in terms of the number of variables which can be handled and the lack of qualification concerning qualitative events which may change the dynamics of a model (an example is provided by the difficulty in modeling the "breaking point" phenomenon in a battle when one side becomes demoralized). However there are still valuable lessons to be learned. If one is willing to accept the assumptions then the conclusions that there can be too little as well as too much armament are of interest.

A recurrent theme in arms race modeling has already been noted in a discussion of Rubinstein's work. If all nations stockpile weapons will they be used? If they are not to be used will they be kept? The theory of taking precaution postulates that an optimal outcome is one in which disaster equipment is never used. But before the event the optimal policy requires that the equipment be procured. A completely different view is that supply creates its own demand. The debate ranges from nuclear war down to individual crime prevention. Does the banning of the possession of privately owned lethal weapons improve or lower the level of safety? The answer is not clear and appears to depend on context. Does the banning of the possession

of nuclear weapons by nation states improve or lower the level of safety? Who is going to do the banning and how is it to be enforced?

A simple game which can be played at parties with amusing and paradoxical consequences is the dollar auction (Shubik, 1971). A dollar is auctioned with bids in units of five cents. The high bidder wins the dollar and pays his bid, but the second highest bidder also pays. It is easy to see that a dollar may be sold for considerably more than a dollar when individuals keep bidding in order to cut losses.

This game has been considered both in terms of addiction and escalation. Teger (1980) has used the game experimentally to study the phenomenon of "too much invested to quit." O'Neill (1985a) provides an analysis based upon both players having limited resources and each knowing the resource limitations of the other. He then contrasts his "rational perfect equilibrium solution" with twelve basic items as they appear to be handled in the solution he proposes, in experimentation and in international escalation. Table 6 is a reproduction of O'Neill's Table 1.

The contrast between the pure game theoretic model and the list of extra considerations noted by O'Neill serves to indicate the need for considerable investigation into both the psychological and bureaucratic aspects of escalation.

	Ideal Dollar Auction	Real Dollar Auction	Inter- National Escalation
Entrapment:	Non- Entrapping	Often Entrapping	Often Entrapping
Limited ability to look ahead	-	X*	X
Desire to win for the sake of winning	-	X	X
Investment made passively	-	-	X
Verbal self-commitment to future moves	-	-	X
Uncertainty above objective consequences	-	-	X
Crisis instability	-	X	X
Expectation of future interaction	-	-	X
Informational component in moves	-	X	X
Misperception of current state of escalation	-	-	X
No third-party intervention	X	X	-
No withdrawing resources once invested	X	X	-
No dropping out on equal terms	X	X	-

Features promoting escalation in the dollar auction played by rational, real opponents and in international escalation.

*The X indicates that the property noted is present.

TABLE 6

4.3. Crisis Stability

Crisis stability or instability is possibly one of the most important problems of societal concern at this time. An early formal somewhat game theoretic analysis was provided by Ellsberg (1961). Grotte (1980) and O'Neill (1985b) provides references to many of the game theoretic models developed in the last twenty years. O'Neill (1985b, c) using a set of five axioms derives a crisis instability index which he applies to the 2×2 model

in which both sides can either attack or refrain from attack. See Table 7.

	Refrian	Decision to Attack
Refrain	0,0	$-r_1, -a_2$
Decision to Attack	$-a_1, -r_2$	$\frac{-a_1-r_1}{2}, \frac{-a_2-r_2}{2}$

where $-r_i < a_i < 0$

TABLE 7

The lower right cell has payoffs which are a 50:50 mix of the two adjacent cells to take into account of the possibility not of simultaneous attack, but of simultaneous decision to attack and chance determining who gets the first blow. The index developed is:

$$P(\text{war}) = 1/[1 + \{4a_1a_2/(r_1-a_1)(r_2-a_2)\}^g]$$

where g is a responsiveness factor to the payoffs. If $g = 0$ then the index is independent of payoffs and has a value of $1/2$. Low values of g indicate concern about factors outside of the formal payoff description. When r_i approaches a_i the index goes to zero as there is no advantage in first strike.

One way of viewing the index is in terms of damage exchange and regret. Tables 8a, b and c illustrate this. The first matrix has an index of

0,0	-1001,-1
-1,-1001	-501,-501

a

0	1001
0	500

regret

b

0	-1000
1000	0

difference

c

TABLE 8

$1/(1 + 4/10^6)$ with $g = 1$ or .999. War is virtually certain. Matrix 8b provides a minimize regret argument and 8c a minimum the difference in pay-offs.

O'Neill (1985c) adopting a nuclear exchange model developed by Grotte (1982) carries out a computation of his index calculated by evaluating the entries in Table 7. He then performs a sensitivity analysis on various forms of arms control, and draws conclusions that space defenses could be destabilizing unless they were very effective.

The O'Neill papers are a nice supplement and well reasoned operations research game theoretic addition to the writings of Bracken (1983) and Blair (1985) who are more concerned with the error proneness and the difficulties of control of each side's politico-military-bureaucratic delivery systems than with simplified computational models where the error factors have been implicitly assumed or avoided.

Crisis control is not one problem, but many. Formal game theory possibly does more to illustrate where our knowledge of individual and institutional crisis behavior is lacking than it does to provide a dynamics of crisis control. Under behavioral assumptions concerning risk some of the implications of changes in force structure can be evaluated. But the essence of many of our current problems is in the dynamics and no adequate dynamic theory exists. Informal game theoretic reasoning and examples help to point out the difficulties and paradoxes in threat behavior and "brinkmanship." But the questions raised far outnumber the answers supplied.

5. FUTURE DIRECTIONS

The growth of the applications of game theory in many of the behavioral sciences in the past forty years (see Shubik, 1982, Ch. 12) has more than fulfilled its early promise. Yet it is by no means a panacea or cure-all. By providing better tools for analyses more questions as well as answers are found.

The meaning of relevance and value in application depends upon purpose and priority. The applications of game theory and closely related topics to defense problems have been and will remain diverse. There are at least seven distinctions which merit note in application. They involve various blends of policy advice giving, mathematical analysis and operations research human factors analysis, bureaucracy and social systems understanding and history and political science. They are (1) General operational advice giving; at a high policy level this involves an intermix of conversational game theory, gamesmanship and uncommon sense. The works of Ellsberg, Schelling and Herman Kahn provide examples. (2) Specific operational advice giving is harder as it involves not only setting up a context of discourse but operationalizing the examples used to illustrate paradoxes, threats and ploys. (3) The bread and butter direct uses of operations research and calculation come in the setting up of problems which can be described with reasonable accuracy as two person zero sum games. (4) There is room for calculation and formal models if they are used in an ad hoc manner with extreme care. Examples are provided by Dalkey's (1965) nuclear war model; Grotte (1980) on nuclear stability; Shubik and Weber (1981) on systems defense and Grotte and Brooks (1983) on measuring naval presence (see also Brewer and Shubik, 1979 for a coverage of the links among models, games and simulations).

The four items noted above may all be viewed primarily from an operational viewpoint at some level the remaining three call for an emphasis on basic research and theory development. They are (5) The understanding of individual principal party and fiduciary agent risk behavior under normal conditions and under stress. The psychologists have raised many questions which challenge our views of a "rational decision" theory. But at this time we appear to be far from having a satisfactory substitute. (6) The functioning of man-machine systems both under normal pressures and under stress was recognized in the 1950's as a key area of study. The Systems Development Corporation and the Rand logistics laboratory provide examples of a large commitment to the investigation of function and failure to function in warning systems. In the last twenty years the disconnect between technical systems analysis and the human, bureaucratic, sociological and political control systems appears to have increased. Yet the need to understand the control of error in massive command and control systems has become more important in this time span. It is my firm belief that expenditures in the range of hundreds of millions, if not billions of dollars are called for in the study of individual and systems risk behavior.

The last item is the most challenging and difficult. In spite of the attractiveness of the slogan "satisficing man" where many of us feel that we know intuitively what is meant, it has been extremely difficult to operationalize in a satisfactory manner the meaning of capacity constrained rational behavior. Some advances have been made in computer science in artificial intelligence. But it may be that the future development of relevance to the problems of defense calls for emphasis on models of "context rationality." Gaming experiments are needed to study the differences and causes of differences in situations where the game theoretic model is the

same, but the briefing or setting of context, the players (their training and background) and organizational structure and time pressures are varied.

The models of man of the general, politician and bureaucrat, the game theorist, political scientist, social psychologist, psychologist, historian and artificial intelligence expert are far apart. Both the operational needs of the time and the scientific challenge call for increased understanding and resolution of these differences in view.

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