

## THE VIRTUOUS TORTOISE

There is no philosophically interesting distinction to be made between inference-rules and premises. That there is such a distinction is often held to follow from the possibility of infinite regress illustrated by Carroll's story of Achilles and the tortoise. I will argue that this is wrong on three separate grounds. Consequently, Carroll's fable provides no motivation to abandon the traditional logical separation of arguments into their premises and conclusions. There is no proposition that must be taken to be a rule and must not be taken as a premise.

### 1. Introduction

The common interpretation of Carroll's story of Achilles and the tortoise presents it as a problem concerning logical form and proposes to solve it with the help of inference-rules to which some special logical status or function is given.<sup>1</sup> By this means a new logical category – that of inference-rules – is introduced to supplement the traditional categories of premises and conclusions. Some propositions that historically were considered to be premises must, on this new scheme, be re-interpreted as inference-rules. If this were so, it certainly would be something of philosophical significance.

On the other hand, if construing a proposition as a premise or as an inference-rule were just different ways of looking at the same thing – that is to say, if the inference-rule, although perhaps not unreasonable in itself as a way of construing the proposition, were bereft of its special logical status – the distinction would not be of

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<sup>1</sup> Carroll, L. (1895). "What the Tortoise Said to Achilles." *Mind*, vol.4, pp.278–280. I am not saying that this is the only reason for which inference-rules have been introduced. Another popular source of motivation comes from Mill's theory of the syllogism, where inference-rules are sometimes invoked to avoid *petitio principii*. I have argued elsewhere that this also is mistaken and rests on a misunderstanding of Mill.

any particular significance, or any threat to the traditional view of logical arguments as composed of premises and a conclusion. This is the position that I want to defend by arguing that:

- 1) The infinite regress is not the logical regress it has been held to be, but an epistemological regress. Discussion of this regress, its ramifications and possible solutions, are outside the scope of this paper. It is enough for my purposes that the use of inference-rules is not, or even imputed to be, a solution to this epistemological regress.
- 2) Even if (1) is wrong and it is a logical regress being illustrated, this logical regress is entirely virtuous and does not require any solution, let alone by the introduction of inference-rules.
- 3) Even if (1) and (2) are wrong and there is a vicious logical regress, the introduction of inference-rules still would not solve it.

I will start by giving a fairly lengthy excerpt from Carroll's paper and mark significant points in square brackets for later discussion:

"That beautiful First Proposition of Euclid!" the Tortoise murmured dreamily. . . .  
"Well, now, let's take a little bit of the argument in that First Proposition – just two steps, and the conclusion drawn from them. Kindly enter them in your note-book. And, in order to refer to them conveniently, let's call them A, B, and Z:  
(A) Things that are equal to the same are equal to each other.  
(B) The two sides of the Triangle are things that are equal to the same.  
(Z) The two sides of this Triangle are equal to each other."  
"Readers of Euclid will grant, I suppose, that Z follows logically from A and B, so that anyone who accepts A and B as true, must accept Z as true?"  
"Undoubtedly! . . ." [A1]  
"And if some reader had not yet accepted A and B as true, he might still accept the sequence as a valid one, I suppose?"  
"No doubt such a reader might exist. He might say 'I accept as true the Hypothetical Proposition that, if A and B be true, Z must be true; but I don't accept A and B as true.' [A2]  
"And might there not also be some reader who would say 'I accept A and B as true, but I don't accept the Hypothetical'?"  
"Certainly there might. . . ." [A3]

“And neither of these readers,” the Tortoise continued, “is as yet under any logical necessity to accept Z as true?”

“Quite so,” Achilles assented. [A4]

“Well, now, I want you to consider me as a reader of the second kind, and to force me, logically, to accept Z as true.” . . .

“I’m to force you to accept Z, am I?” Achilles said musingly. “And your present position is that you accept A and B, but you don’t accept the Hypothetical – ”

“Let’s call it C,” said the Tortoise.

“ – but you don’t accept:

(C) If A and B are true, Z must be true.”

“That is my present position,” said the Tortoise.

“Then I must ask you to accept C.”

“I’ll do so,” said the Tortoise, “as soon as you’ve entered it in that note-book of yours. . . . Now write as I dictate:

(A) Things that are equal to the same are equal to each other.

(B) The two sides of this triangle are things that are equal to the same.

(C) If A and B are true, Z must be true.

(Z) The two sides of this Triangle are equal to each other.”

“You should call it D, not Z,” said Achilles. “It comes next to the other three. If you accept A and B and C, you must accept Z.”

“And why must I?”

“Because it follows logically from them. If A and B and C are true, Z must be true. You don’t dispute that, I imagine?”

“If A and B and C are true, Z must be true,” the Tortoise thoughtfully repeated.

“That’s another Hypothetical, isn’t it? And, if I failed to see its truth, I might accept A and B and C, and still not accept Z, mightn’t I?”

“You might,” the candid hero admitted; “though such obtuseness would certainly be phenomenal.” [A5]

“Still, the event is possible. So I must ask you to grant one more Hypothetical.”

“Very good. I’m quite willing to grant it, as soon as you’ve written it down. We will call it

(D) If A and B and C are true, Z must be true.”

“Have you entered that in your note-book?”

“I have!” Achilles joyfully exclaimed . . . . “And at last we’ve got to the end of this ideal race-course! Now that you accept A and B and C and D, of course you accept Z.”

“Do I?” said the Tortoise innocently. “Let’s make that quite clear. I accept A and B and C and D. Suppose I still refuse to accept Z?”

“Then Logic would take you by the throat, and force you to do it!” Achilles triumphantly replied. “Logic would tell you ‘You can’t help yourself. Now that you’ve accepted A and B and C and D, you must accept Z!’” So you’ve no choice, you see.”

“Whatever Logic is good enough to tell me is worth writing down,” said the Tortoise.

”So enter it in your book, please. We will call it

(E) If A and B and C and D are true, Z must be true.”

“Until I’ve granted that, of course, I needn’t grant Z. So it’s quite a necessary step, you see?”

“I see,” said Achilles; and there was a touch of sadness in his tone. [A6]<sup>2</sup>

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2 Carroll (ibid.)

Achilles seems to have conceded defeat at A6, having come to realise that his concessions A1-A5 has initiated an infinite regress.

2. What kind of regress is it?

In my view the regress is not a logical regress but an epistemological regress, and the problem it poses is not a logical problem but an epistemological problem.

The common understanding is that it is a logical regress, and I can see why people see it this way. After all,

(A);(B); therefore (Z)

(A);(B);(C); therefore (Z)

(A);(B);(C);(D); therefore (Z)

(A);(B);(C);(D);(E); therefore (Z)

are all logical arguments, and it cannot be denied that we can go on to generate an infinite number of analogous arguments by continually insisting that the conditional involved be “written down.” Thus, it is perhaps natural to think that by refusing the request to “write it down” one stops the regress and solves the problem; the conditional must be given a special normative status such that by definition it should never be written down – it must be either an inference-rule or an implicit premise that can never be made explicit. This analysis has been adopted from Ryle and taken by him to motivate the idea of an ‘inference-ticket’ which was later to influence Toulmin in his idea of a warrant.<sup>3</sup> If Ryle’s analysis is wrong this undercuts to some extent

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<sup>3</sup> Avoiding Carroll’s regress is not the only reason that Ryle believes that conditionals should be treated as rules rather than statements. A fuller defence of my thesis that there is no interesting distinction between premises and rules would take account of Ryle’s other reasons. For reasons of space and convenience this paper concentrates on the regress objection.

Toulmin's motivation for his conception of a warrant and his attack on traditional logic.

I think it is wrong, and I reject this understanding of what Carroll's story illustrates. The main reason that I reject it is that, although there undoubtedly is an infinite regress here, I fail to see anything vicious in this regress. Consider the following two arguments:

p; therefore, q

p; p; therefore, q

Quite obviously we can generate an infinite number of premises by reduplicating the one in the original argument, but although tedious I don't see how this creates any kind of logical problem. There would only be a problem were the added premise an ampliation of what was in the original argument, but that is not the case here or in Carroll's argument – if the argument (A);(B); therefore (Z) is valid, then (C) is a tautological consequence of (A) and (B),<sup>4</sup> and if it is not valid and hence we need to concede the necessity of adding (C), every conditional added thereafter must be non-ampliative since the argument (A);(B);(C); therefore (Z) *must* be valid and (D) *must* be a tautological consequence of (A) and (B) and (C). In the absence of such ampliation it is no problem at all that there could be an infinite number of premises. In fact, it is arguable that these are not different arguments, since they each have exactly

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Ryle, G. (1945-46). "Knowing How and Knowing That." *Proceedings of the Aristotelian Society*, pp.1-16.

Toulmin, S.E. (2003). *The Uses of Argument, Updated Edition*. Cambridge: Cambridge University Press.

4 Thomson also points out the tautology involved and the redundancy involved in adding it as a premise. According to him, however, this is a reason not to add it as a premise. According to me, it is a reason why there is no vice involved with adding it as a premise because the regress it potentially generates is not vicious. Anything that is in the informational content of the premises can be made into an explicit premise or "written down," even when doing so serves no purpose.

Thomson, J. F. (1960). "What Achilles should have said to the Tortoise." *Ratio*, vol.3, pp.95-105.

the same informational content. Making explicit a tautological consequence of a set of premises by adding it as a premise may be redundant but does not create any logical problem.

It should also be noted that adding (C) is not necessary (logically or otherwise) to make the conclusion (Z) follow necessarily from the premises (A) and (B), and Carroll does not say otherwise. The logical necessity Carroll speaks about is the logical necessity of accepting a proposition, not the logical necessity of its being true. In other words, he does not deny the logical necessity of “p; if p then q; therefore, q” but only points out that this does not make it true that “I accept p; I accept that if p then q; therefore, I accept q.” It is certainly logically possible to accept the premises without accepting the conclusion, and it seems to be physically possible as well.

This leads me neatly into my own understanding of Carroll’s argument: there is a genuine problem here, but not one of logical form but of justification. For perhaps someone might think that what is missing from “I believe p; I believe that if p then q; therefore, I believe q” is a grasp of the logical connection between the premises and the conclusion. This logical connection can be expressed as a conditional and added to the premises. But if the person failed to grasp the logical connection in the first argument there is absolutely no reason to suppose that he will grasp it in the second even if he accepts the conditional that expresses it for the preceding argument. The moral is that we cannot justify logical truths to someone who fails to grasp logical connections in the first place. We can draw an analogy to induction: while induction is circular because it requires a Principle of Induction that can only be justified inductively, deduction is similarly circular because its axioms can only be justified deductively, and although deductive justification guarantees the truth of what it

justifies it does not guarantee that anyone believes it to be true. Any attempt to justify deduction necessarily involves appealing to its own axioms, and this is circular.<sup>5</sup>

This position is similar to that of Haack.<sup>6</sup> However, the justification I am talking about is not, I think, the same as what Haack is talking about. I am not sure that deduction itself requires further justification, but even if it does, whatever justification we give it will not bridge the gap between the logical necessity of the truth and the contingency of believing the truth. What I am talking about is justification from the Tortoise's point of view, in this case the point of view of someone who is deductively blind. This is sometimes called *doxastic* justification. One way of putting the moral of Carroll's argument is that we can get a regress of *doxastic* justification similar in form to the more familiar epistemic regress. We might respond to the question why we are justified in believing  $r$  by saying it can be validly inferred from  $p$  and  $q$  and we are justified in believing  $p$  and  $q$ . The usual regress proceeds by questioning why we are justified in believing  $p$  and  $q$ . But you could also proceed by asking why we are justified in believing the inference to be valid. Such a question might indicate deductive blindness but is nonetheless possible, and if we are not justified from our own point of view in believing the inference to be valid then plausibly we are not justified in believing  $r$ . This question, however, can only be answered in a question-begging way by appealing to deduction. We can put it this way: deductive skepticism is a logically coherent position.

Toulmin describes the kind of situation involved nicely without, however,

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5 We can also draw an analogy for rationality itself; if someone does not see why they should be rational it is fruitless to give them reasons to be rational, because by definition they will not accept reasons as constraining their behaviour.

6 Haack, S. (1976). "The justification of deduction." *Mind*, vol.84 no.337, pp.112-119

connecting it to Carroll's problem.<sup>7</sup> Someone asked to provide a proof of some mathematical fact must set out a set of steps, each of which is individually a simple calculation, going from undisputed facts to what he wants to prove. He concedes that it is still possible to doubt the correctness of the calculation, or that it has been performed correctly. Nonetheless, it is not necessary, for example, to break the calculations down even further. This would only be "a formality, for how could one be confident that a man who questions '5 times 7 is 35' will accept '1 and 1 make 2'?" Ordinarily, when this stage is reached, there is no more room for 'proof' or 'grounds'" This, it seems to me, is the situation that Achilles finds himself in in his argument with the tortoise, and is, as Toulmin seems to say here, something he just has to live with.

### 3. Where do inference-rules fit in?

On my interpretation, nowhere; whether the conditional (a particular case of *modus ponens*) be treated as a premise or an inference-rule does not make the justification of deduction any less circular – the issue is completely irrelevant. Suppose, though, that my interpretation is wrong, that is to say, discarding the first of the three arguments, consider the other two that accept the more common interpretation. Does the inference-rule view succeed then? Is there a genuine problem to which it is the solution? I have already argued why I think that there is no genuine problem here: the logical regress involved is virtuous. However, for the time being let us discard this second argument along with the first and suppose that the logical regress is a genuine problem. Does treating the conditional as a rule rather than a premise really solve the problem?

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<sup>7</sup> Toulmin (ibid.): 225.



Suppose Achilles says “No, I won’t write it down because it is a rule.” If this succeeds then it is only by fiat, and amounts to retracting his concession that anything Logic is good enough to tell him is worth writing down. Suppose the Tortoise graciously accepts this but continues: “Very well! I can see how writer’s cramp affects your sword-arm. Can I not concede the premises, concede the rule, and yet still refuse to concede the conclusion?” Using inference-rules saves Achilles’ notebook but does nothing to solve his problem,<sup>8</sup> and it would be strange if it did because this would amount to saying that whether something is implicit or explicit, written down or not written down, is a fundamental logical distinction and not just a difference in mode of expression – a demand that something remain implicit and unexpressed *as a point of logic* is not intelligible. In short, we are asked to just accept that rules should not be written down and to further accept that only about what has been written down can doubt be expressed, that by making something a rule we make it immune from any questioning or skeptical attack. Unless an independent motivation is given for these things, this solution is *ad hoc* and not a real solution at all.

Consider also that we can always turn an inference-rule into a premise by conditionalizing the conclusion on the minor premise. For instance, in the argument

$\forall x. P(x) \supset Q(x)$

P(a)

Q(a)

the major premise (the universal conditional) could be interpreted as an inference-rule rather than a premise. But in

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8 Hollis seems to share this view.

Hollis, Martin. (1975). “A retort to the Tortoise.” *Mind*, vol.84 no.336, pp.610-616

$\forall x. P(x) \supset Q(x)$

$P(a) \supset Q(a)$

the universal conditional is clearly and could only be a premise. Equally clearly, the fact that it has become a premise does not mean that we have to support it by adding another conditional. It is not the case that there was no regress in the first argument (because it contained an inference-rule) but there is a regress in the second argument (because it contains only a premise); it would be bizarre if a simple weakening of the argument could create a regress where there was none before.

In his discussion of Carroll's argument Thomson maintains that there is an important distinction between premises and inference-rules (he calls them principles) but he denies that the regress argument (which regress he denies occurs) has anything to do with it. His reasoning seems to be that the conditional is a comment on the argument's validity and is hence not a part of the argument.<sup>9</sup> I agree with Thomson that the conditional, because it is true when the argument is valid and can be seen as a particular instance of the deduction theorem, does make such a comment.

Nevertheless, I think that this conditional is a part of the argument. Thomson's conclusion would only hold if the following dilemma also holds: it is a comment on the argument or a part of the argument but never both. This dilemma is false, however – what is part of the argument depends only on the informational content of the premises. Thomson's conditional is not an ampliation but is part of the argument's content and hence part of the argument. Being a comment on the argument does not disqualify a proposition from also being a part of the argument.

#### 4. Conclusion

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<sup>9</sup> Thomson (ibid.)

Consideration of Achilles and the tortoise does not support making a logical distinction between inference-rules and premises. Inference-rules do not solve the quasi-Carroll problem of arguments becoming indefinitely long [so (3) is true] and no solution is actually required because this is entirely harmless [so (2) is true]. The tortoise is virtuous: at worst he points out that deductive skepticism is a logically coherent position. Lastly, inference-rules do not have anything to do with solving the epistemological problem Carroll is really expressing concerning the justification of deduction [so (1) is true]. Carroll's argument gives us no reason to introduce inference-rules as a distinct logical category with a distinct logical status and function.

Furthermore, the introduction of inference-rules has the effect of casting every argument whatsoever into the form of a practical syllogism that, when the reasoning is theoretical, amounts to demanding that something is to be believed, implying that our beliefs are more under the control of our wills than they really are. It becomes a weird kind of *ad baculum* with logic or rationality becoming sticks meant to force you to have some particular belief or accept some particular proposition. This, indeed, seems to be Achilles' strategy when he says that logic will take you by the throat, and the Tortoise shows that it fails; if anything, the Tortoise shows that the rules of logic are *not* really rules and cannot be given the kind of force that rules can be given, but instead only describe relations of implication. Having instead a premise in the form of a declarative makes the point that what is at issue is the implicational relations between the propositions, which relations define what it is rational (or at least truth-conducive) to believe. Only the propositional contents of speech-acts belong in reasoning; neither illocutionary forces, nor what Toulmin calls the 'force' of the warrant, nor any other feature of the speech-acts by which we may express them belong to the *logos* of the argument.