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# The wavelength dependence of optical polarization in the galaxy M82

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**Summary**. New linear polarization data are presented for the galaxy M82 in red light using the Durham polarimeter and a McMullan electronographic camera. The degree of polarization found was substantially smaller than for previously obtained blue data. In both colours the NW side is less highly polarized than the SE side. These results indicate a NW–SE asymmetry in the polarized intensity together with a dilution by redder unpolarized light.

If M82 is tilted at $\sim 10^\circ$ to the edge on position with the nearer half to the NW, scattered light off dust grains to the NW of the disc must pass through the disc dust before reaching the observer. Polarized light from both sides is diluted by unpolarized disc light. The required brightness distribution and optical thickness of the disc appear reasonable, and a fairly constant halo dust density is predicted.

If M82 is virtually edge on the NW–SE asymmetry in degree of polarization could still be obtained if there is more dust to the SE. Dilution could result from halo starlight or intrinsic gas emission. However, the latter is unlikely unless gas emission is extremely strong at intermediate distances along the minor axis of the galaxy.

## 1 Introduction

Although the nearby Ir II galaxy M82 has been the subject of much research during recent years, its morphology and evolutionary state are still poorly understood. Its extensive halo was discovered by Elvius (1962) to be highly polarized at optical wavelengths. H$\alpha$ measurements in the halo (Lynds & Sandage 1963) showed a redshift to the NE and a blueshift to the SW of the disc, which was taken to be tilted at $8^\circ$ to the edge on position. This suggested the presence of an explosive outflow of gas along the rotation axis. Early attempts to explain the polarization in terms of synchrotron radiation (Lynds & Sandage 1963) or electron scattering (Solinger 1969) proved unsatisfactory, and Elvius (1972) proposed that the polarization arose by dust scattering of light from the body of the galaxy in the halo. More recent polarimetry of line and continuum radiation of selected areas by Visvanathan &

★ Guest Observers, Wise Observatory, Israel, 1978 March.

Sandage (1972), abbreviated VS, and Visvanathan (1974) suggested that both types were polarized to a similar extent. It was therefore concluded that the emission line radiation observed in the halo was also produced by the scattering of light from the body of M82. Subsequently Solinger & Markert (1975) argued that the disc, rather than the nucleus of M82 was responsible for illuminating the dust grains. Complete polarization maps of the entire M82 region were presented by Schmidt, Angel & Cromwell (1976) photographically in the *V* waveband and by Bingham *et al.* (1976) (abbreviated BMPWAS) using a McMullan camera in the *B* waveband. Solinger, Morrison & Markert (1977), using a more sophisticated version of their 1975 model, attributed the total intensity data to light scattered off a large constant density cloud of dust particles illuminated by a composite nucleus and disc. In addition, an exponentially falling off component of intensity is required close in. They accounted for the variation of the H$\alpha$ Doppler shift in the halo as being a result of the drift of M82 through the dust cloud. Recent spectroscopic data presented by Axon & Taylor (1978) show considerable splitting of the H$\alpha$ line in parts of the halo. This would suggest the presence of some intrinsic H$\alpha$ emission in these regions from the surface of a radially expanding cone of gas. The reader is referred to Solinger *et al.* (1977) for a more detailed review of work on the halo of M82.

In this paper new polarization data are presented, and two possible models of the galaxy proposed to explain the observations.

## 2 Experimental details

In 1975 March, M82 was observed on the Wise 1-m telescope using the Durham electronographic polarimeter, and polarization measurements were made in the *B* waveband of the *UBV* system. These results have already been reported (BMPWAS).

In 1978 March, further measurements of the galaxy were made at the Wise Observatory using the electronographic polarimeter with an *R* bandpass filter. After allowing for the fall off in camera sensitivity, the effective response with this filter was from 0.6 to 0.7 $\mu$m. The reduction technique is described elsewhere (Warren-Smith 1979).

Plate 1 shows the *R* polarization map of M82 superimposed on a Palomar plate of the same object. The polarization has been mapped at a matrix of locations corresponding to a spatial resolution of 7.5 arcsec. The polarization of each area is represented by a line of length proportional to the degree of polarization and orientation parallel to the *E* vector of the linearly polarized component of the light.

The analysis of observations made in this paper draws on the authors' data in the *R* waveband and that of BMPWAS in the *B* waveband. Although surface variation in photocathode sensitivity was only corrected for with the *R* data, an estimate of the systematic error this could cause is included in the error bars for the *B* data.

## 3 Discussion of the observations

The *R* polarization map shows the characteristic M82 pattern of centrosymmetric vectors pointing back to the nuclear region of the galaxy, and the orientation of the vectors (typical error ~ 15°) does not differ significantly from the earlier *B* measurements (BMPWAS). The degree of polarization is noticeably smaller than the corresponding measurements by BMPWAS at *B* wavelengths however.

To display this feature more clearly, the degree of polarization in the two colours is plotted as a function of distance from the polarization centre in Fig. 1. A narrow strip parallel to the minor axis of the galaxy has been chosen which crosses the disc near the

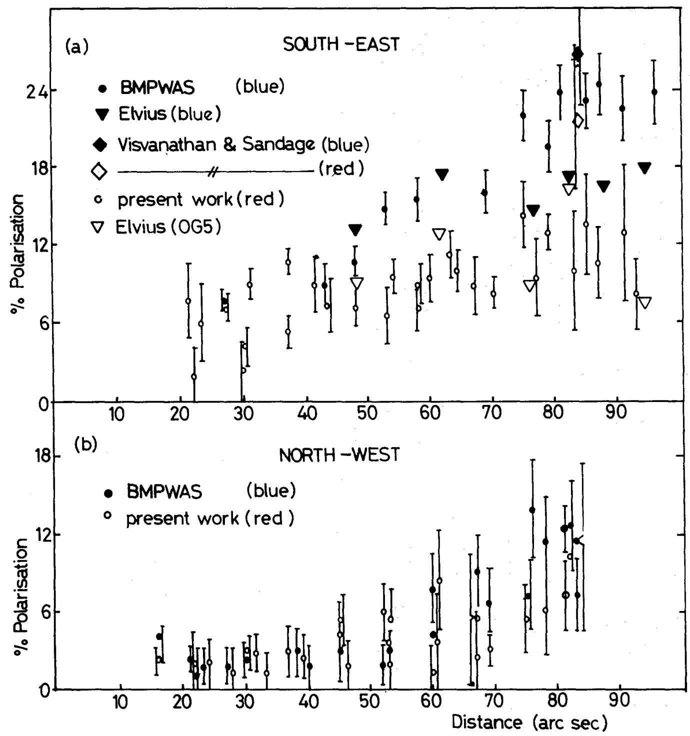


**Figure 1.** Measurements of the degree of polarization at points along the minor axis in the *B* waveband (BMPWAS) and *R* waveband (present work). Distances are taken from the centre of the polarization pattern. The $1\sigma$ error bars on the data points take account of both statistical and systematic effects. Data of other workers is given for comparison.

region of the optical bright spot. Also plotted in the same figure, are measurements by Elvius (1969) using BG12 and OG5 filters. Her measurements are in the central region of the M82 halo but do not correspond exactly to the minor axis. The single area measurement of VS for a waveband centred on 0.59 $\mu$m is also shown. This is the nearest value corresponding to our *R* waveband.

Inspection of the figure reveals two important features:

(a) The SE side of the galaxy is more highly polarized than the NW side in both colours, but the effect is more marked in the blue. The same effect is apparent, but was not commented on in the *V* waveband measurements described by Schmidt *et al.* (1976). From the measurements of VS the polarization on the two sides appears to tend to an asymptotic value of ~ 30 per cent at large distances from the nucleus (~ 150 arcsec).

(b) The SE side of the galaxy exhibits wavelength dependence of the polarization with the blue measurements 50–100 per cent greater than those in red. Although the numerical values of the different authors' measurements appear to be somewhat discrepant, especially the single point measurement of VS, the polarization decrease with increasing wavelength seems to be a common feature. Moreover, the continuum measurements of VS are definitely not inconsistent with those of BMPWAS and the authors when account is taken of their respective error bars. (Also the large error bars on the polarization continuum measurements of VS will fairly easily accommodate the wavelength dependence required by the present data.) Comparing the polarizations in H$\alpha$ + N II (27 ± 3 per cent) and the continuum (22 ± 6 per cent) at 86 arcsec, VS concluded that the line and continuum polarizations were the same, whereas the present *R* data (14 ± 4 per cent) suggest there is a real difference between them. The presence of wavelength dependent polarization in the NW side of the

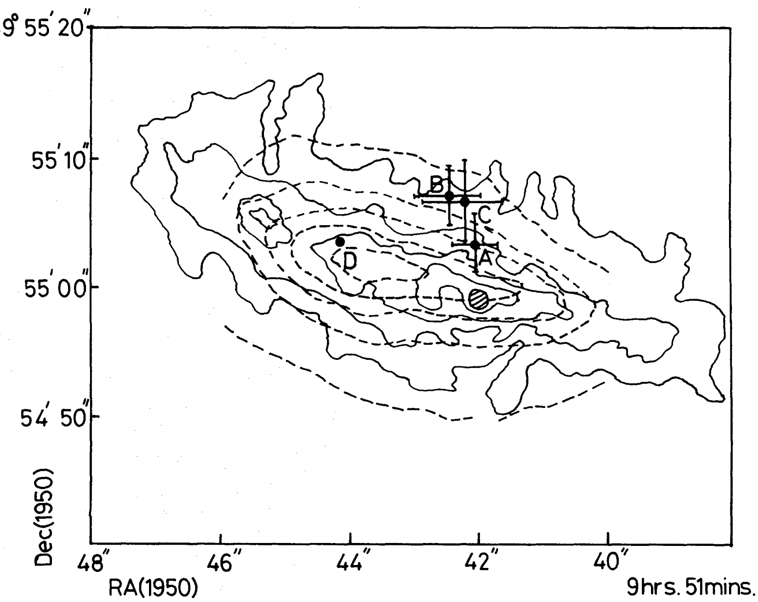


**Figure 2.** The central region of M82 showing 5 GHz contours of Hargrave (1974) (solid) and 2.2 μm contours of Abolins *et al.* (1979) (dotted). The positions marked are: (a) the centre of the optical polarization in *B* (BMPWAS); (b) the centre of symmetry (Solinger & Markert 1975); (c) the centre of symmetry of the optical polarization in *R* (this paper); (d) the optical bright spot.

galaxy is not detectable from the present observations because of the low polarizations and relatively high noise level.

Fig. 4 shows the total intensity of light in the two colours as a function of distance from the polarization centre. Both sets of intensity measurements were made in arbitrary units, but the *B* data were subsequently placed on an absolute scale by fitting the total intensity trace on to that measured absolutely by VS. However, no absolute measurement of *B–R* was possible.

The centre of symmetry of the polarization pattern in the *R* waveband (Fig. 2) was found to lie close to that in *B* waveband (BMPWAS) and to that estimated by Solinger & Markert (1975) at optical wavelengths from the data of Elvius (1962) and VS. The area of strongest radio emission lies to the south, and probably represents an active region of young stars (O'Connell & Mangano 1978). Near infrared emission is from a ridge extending in a north-easterly direction from this radio centre (Abolins *et al.* 1979).

## 4 Theoretical interpretation of the observations

Until recently, models of M82 (Solinger *et al.* 1977; Schmidt *et al.* 1976; Perkins, Pallister & Scarrott 1977) have attempted to explain the polarization and surface brightness in the halo solely in terms of the scattering of disc light from particles with a particular density distribution in the halo. This type of model predicts that the polarization is independent of wavelength, $\lambda$, for Rayleigh particles (radius $a \lesssim \lambda/2\pi$) of electrons, while for most types of dust grains in the Mie domain ($a > \lambda/2\pi$) the polarization would increase at longer wavelengths. Neither of these cases can therefore mimic the observed wavelength dependence.

We would like to describe two possible mechanisms which separately or in conjunction help to describe the observed form of the polarization data.

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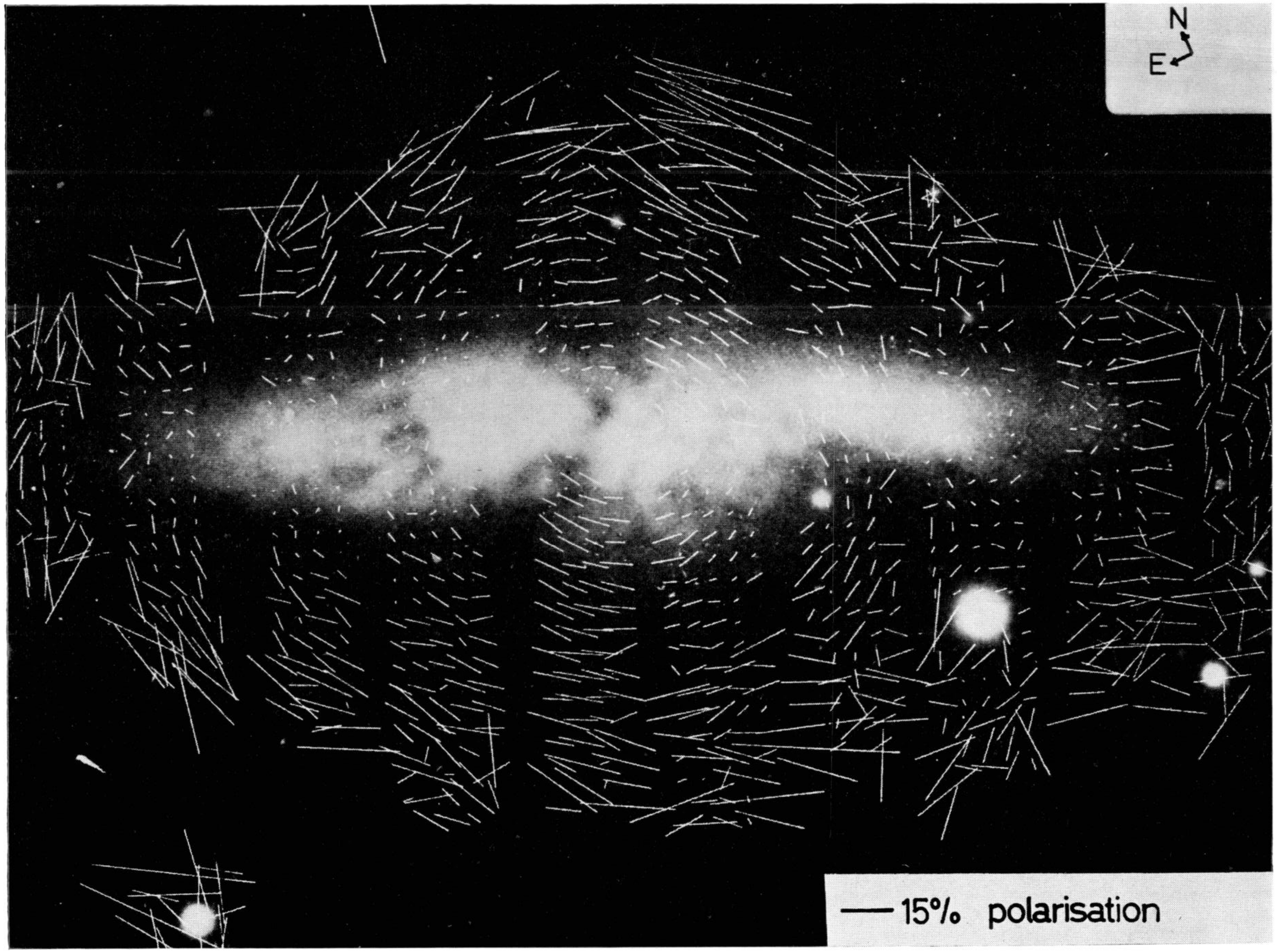


**Plate 1.** The linear optical polarization of M82 in the *R* waveband. This has been mapped at a matrix of locations corresponding to a spatial resolution of 7.5 arcsec. The polarization of each area is represented by a line of length proportional to the degree of polarization and orientation parallel to the '*E*' vector of the linearly polarized component of the light. ☆ Fiducial stars.

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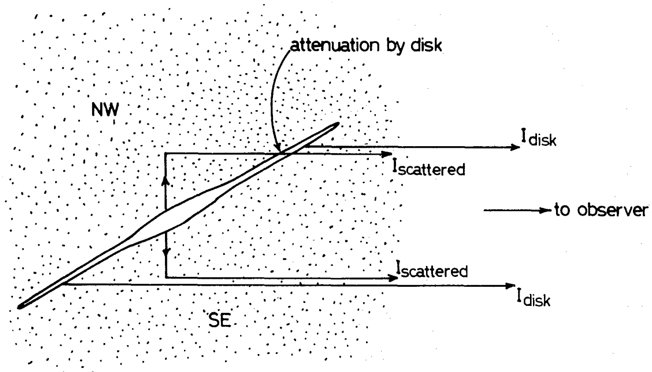


Figure 3. The tilted disc model. The dust surrounding M82 is distributed symmetrically with respect to the plane of the disc. Light from the bright central regions of the galaxy scattered off dust particles to the SE of the disc may reach the observer directly, while that off particles to the NW of the disc must first pass through the disc, where it is attenuated, before reaching the observer. The polarized scattered light ($I_{\text{scattered}}$) becomes mixed with unpolarized light intrinsic to the disc ($I_{\text{disc}}$).

### 4.1 TILTED DISC MODEL

In this case the M82 disc is tilted to the line of sight so that the effect of the extension of the disc luminosity in the direction of the minor axis must be considered. Using an axial ratio of 0.17 derived from a deep yellow exposure of M82, Lynds & Sandage (1963) concluded that M82 was tilted at $\sim 8^\circ$ to the line of sight and, from the asymmetry of the dust lanes, the NW side of the galaxy was nearer to the observer. The geometry is shown in Fig. 3, where we have assumed a tilt of $\sim 10^\circ$. It can be seen that practically all the partially polarized scattered light on the SE side travels directly to the observer, while the majority of that to the NW must first pass through the M82 disc where it will be attenuated. This introduces the large observed NW–SE asymmetry in the polarized intensity. The scattered intensity before attenuation is assumed to be symmetrical with respect to the major axis. The addition of redder light, intrinsic to the disc, which is assumed to be distributed symmetrically about the major axis leads to the observed fall off in polarization towards longer wavelengths.

Deductions can now be made concerning the optical thickness of the disc and the relative contributions to the intensity from disc and scattered light. Let the suffices N and S refer to the part of M82 appearing to the NW and SE of the major axis respectively. Suffices B and R refer to the blue and red filter wavebands used. From our data, quantities $t_{\text{B}}$, $t_{\text{R}}$, $b_{\text{B}}$, $b_{\text{R}}$, $C_{\text{s}}$ can be determined for each point along the minor axis and are defined as follows:

$$t = I_{\text{PN}}/I_{\text{PS}},$$

$$b = I_{\text{TN}}/I_{\text{TS}},$$

$$C_{\text{s}} = P_{\text{RS}}/P_{\text{BS}}.$$

Here $I_{\text{P}}$ is the polarized intensity, $I_{\text{T}}$ the total intensity and $P$ the percentage polarization of the light. Any contribution to the total intensity from halo stars or intrinsic gas emission will be included in the disc intensity. Considering the polarized light, it can be seen that $t$ is simply the fraction of light transmitted by the disc. Total intensities are related by the equation

$$b = \frac{I_{\text{TN}}}{I_{\text{TS}}} = \frac{tI_{\text{C}} + I_{\text{D}}}{I_{\text{C}} + I_{\text{D}}},$$

where $I_C$, $I_D$ are the components of intensity due to scattering (before attenuation) and disc light respectively. Defining

$$f = I_D/I_C, \tag{1}$$

as the ratio of disc intensity to scattered intensity,

$$b = \frac{t+f}{1+f}.$$

Solving for $f$,

$$f = \frac{b-t}{1-b}. \tag{2}$$

The optical thickness, $\tau'$, of the disc to light (incident obliquely at 10° to its plane) is given by

$$t = \exp(-\tau').$$

Finally the optical thickness of the disc normal to its plane

$$\tau = \tau'/\alpha,$$

where $\alpha$ is the factor by which the optical thickness is increased when light is transmitted obliquely. $\alpha$ is not exactly cosec 10° for a typical spiral galaxy, due to deviations from perfect disc geometry, but can be estimated as follows, using mean values for the overall extinction taken over a large number of galaxies.

The extra absorption in $B$-magnitudes at a particular angle of tilt over that expected when Sa–Sd galaxies are viewed face-on is given by (Heidmann, Heidmann & de Vaucouleurs 1972; Fesenko 1972)

$$\delta A_B = (0.70 \pm 0.16\,\text{mag}) \log \operatorname{cosec} \theta = 0.53 \pm 0.12\,\text{mag},$$

taking a tilt, $\theta$, of 10°.

Using this relationship and a similar one for the change in colour with tilt, the amount of absorption, $A_B(0)$, when the galaxy is viewed face-on can be estimated using various models of the dust density distribution in the galactic disc. This has been done separately by Homberg (1958) and Zasov (1978), and their results are given in Table 1. Taking a mean for these two workers and adding the extra absorption due to tilt, the total absorption in $B$-magnitudes,

$$A_B = A_B(0) + \delta A_B = 0.92 \pm 0.15\,\text{mag} \quad (\text{Sa–Sb})$$
$$= 0.97 \pm 0.29\,\text{mag} \quad (\text{Sc–Sd}).$$

Hence the ratio, $\alpha$, of total absorption to face-on absorption

$$= \frac{A_B}{A_B(0)} \quad \begin{matrix} \approx 2.4 & (\text{Sa–Sb}) \\ \approx 2.2 & (\text{Sc–Sd}). \end{matrix}$$

**Table 1.** Average internal absorption of blue light, $A_B(0)$, in the discs of spiral galaxies when viewed face-on.

| Spiral type | Sa–Sb | Sc | Sc–Sd |
|---|---|---|---|
| Homberg (1958) | 0.42 mag | 0.27 mag | |
| Zasov (1978) | 0.37 mag | – | 0.60 mag |
| Mean | 0.39±0.03 mag | 0.44±0.17 mag | |

The mean of $\alpha \approx 2.3$ is taken as being representative of Sa–Sd galaxies. It is assumed that $\alpha$ also takes on the same value in the $R$ waveband. Hence

$$\tau \approx \tau'/2.3.$$

The degree of polarization of the scattered light before dilution by halo light,

$$P_C = P_S \times \frac{I_{TS}}{I_{CS}} = P_S \times \frac{I_{DS} + I_{CS}}{I_{CS}} = P_S \times (f+1).$$

Hence

$$\frac{P_{CR}}{P_{CB}} = C_s \cdot \frac{f_R + 1}{f_B + 1}.$$

Table 2 shows the values of some of these quantities at three representative points up the minor axis of M82 distant 30, 60 and 90 arcsec from the optical centre. The values of quantities determined at 90 arcsec in the $R$ waveband have been omitted because of uncertainty in measuring the small degree of polarization. When conclusions are drawn from these results it should be remembered that the range of error bars on measurements of the degree of polarization, $P_N$, up to 60 arcsec on the NW side are such that $P_N$ could be somewhat smaller. In particular, this could make our estimates of optical thickness rather higher.

It can be seen that the optical thickness does not vary much with position along the minor axis in either colour. This is consistent with estimates of the distribution of dust in spiral galaxies as deduced from measurements of neutral hydrogen abundance from the 21-cm radio data. These studies (e.g. Rots 1975 (M81); Wright, Warner & Baldwin 1972 (M33); Burton 1976 (the Galaxy)) show in general a hydrogen surface density which varies only slowly over the disc (though with frequently a hole near the nucleus). If the distribution of hydrogen over a galactic disc in two dimensions is examined it can be seen that, superimposed on this fairly constant background, there are somewhat more dense regions corresponding to the spiral arms.

Now it is generally accepted that the gas-to-dust ratio in the disc of our Galaxy is nearly constant (Kerr & Knapp 1974; Morton 1974; Quiroga 1978). Assuming this to be a general feature of spiral galaxies, it follows that the inter-arm dust surface density is roughly constant over the disc. The more dense regions of H I which correspond to the spiral arms will have both more dust and more intrinsic starlight associated with them with the dust and stars distributed differently. This could account for the somewhat humpy nature of the polarization and total intensity traces in M82. The mean value for $\tau_B$ of 0.63 mag, when halved to give $A_B(0) = 0.32$ mag, may be compared with those for $A_B(0)$ in Table 1 and the agreement is fairly good. It can be seen from Table 2 that at 30 and 60 arcsec the optical thickness is rather grey to wavelength change, as would be expected for the size of Mie particles typically found in a galactic disc.

From our knowledge of $f_B, f_R$ and the total intensities in our $B$ and $R$ wavebands, the components of intensity from the disc and from scattering follow from equation (2) and are plotted in Fig. 4 at three representative points.

The disc intensity results can now be compared with what would be predicted assuming the disc of M82 is like that of a normal spiral galaxy. Various workers (e.g. de Vaucouleurs 1959; Freeman 1970) have found that outside the central region almost all spiral galaxies follow the radial surface brightness distribution

$$I(r) = I(0) \exp(-r/r_1)$$

**Table 2.** Parameters relating to the tilted disc model at various distances along the minor axis from the optical centre, using date of BMPWAS (*B* waveband) and the authors (*R* waveband).

| Distance along minor axis, $r$ (arcsec) | Fraction of light transmitted by disc, $t$ | | Optical thickness of disc: Viewed obliquely, $\tau'$ | | Optical thickness of disc: Viewed normally, $\tau$ | | Viewed normally in $B$-mags, $\tau_B$ | Ratio of disc intensity to scattered intensity on SE side, $f$ | | Percentage polarization of the scattered component, $P_C$ | | Ratio of optical thicknesses of disc in red and blue light, $\tau_R/\tau_B$ | Ratio of the percentage of the scattered components in red and blue light, $\frac{P_{CR}}{P_{CB}} = C_s \frac{(f_R+1)}{(f_B+1)}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *B* | *R* | *B* | *R* | *B* | *R* | | *B* | *R* | *B* | *R* | | |
| 30 | 0.33 | 0.30 | 1.1 | 1.2 | 0.49 | 0.52 | 0.53 | 2.6 | 6.9 | 27.0 | 47.0 | 1.1 | 1.8 |
| 60 | 0.22 | 0.35 | 1.5 | 1.1 | 0.65 | 0.46 | 0.71 | 1.1 | 2.9 | 33.0 | 37.0 | 0.7 | 1.1 |
| 90 | 0.25 | – | 1.4 | – | 0.60 | – | 0.65 | 0.44 | – | 35.0 | – | – | – |
| >130 | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | – | – | – | – |

in the disc, where $I$ is the surface brightness normal to the disc and $r$ is the distance from the nucleus along the major axis. '$r_1$' is a scale factor. As the optical thickness of disc dust varies only slowly with radius, it follows that the surface brightness distribution in the absence of disc dust would also vary exponentially. Consequently, along the minor axis a hypothetical disc without dust would show an intensity

$$I'_{\mathrm{D}} = I'_{\mathrm{D0}} \cdot \mathrm{cosec}\,\theta \exp(-r\,\mathrm{cosec}\,\theta/r_1) \tag{3}$$

where $I'_{\mathrm{D0}}$ is the intensity extrapolated to the centre in the absence of dust when viewed face-on.

In the presence of dust in the disc, the radiation must on average penetrate only half the disc dust, so that the intensity of the emerging disc light

$$I_{\mathrm{D}} = I'_{\mathrm{D0}}\,\mathrm{cosec}\,\theta \exp(-r\,\mathrm{cosec}\,\theta/r_1) \exp(-\tau'/2). \tag{4}$$

Taking logarithms in equation (3),

$$\log_{10} I'_{\mathrm{D}} = \log_{10}(I'_{\mathrm{D0}}\,\mathrm{cosec}\,\theta) - (r/r_1)\,\mathrm{cosec}\,\theta \cdot \log_{10} \mathrm{e}.$$

$I'_{\mathrm{D}}$ can be determined from the relationship

$$I'_{\mathrm{D}} = I_{\mathrm{D}} \exp(\tau'/2).$$

Log $I'_{\mathrm{D}}$ was now plotted against $r$ at our three points for the $B$ data. These lay very close to a straight line which supports the assumption that M82 has an exponential disc. The scale-length, assuming a tilt, $\theta$, of $10^\circ$, was deduced from the gradient and turned out to be

$$r_{1\mathrm{B}} = 1.8\,\mathrm{kpc}.$$

Here a distance to M82 of 3 Mpc was assumed (Tammann & Sandage 1968), so that 1 arcmin corresponds to 0.88 kpc. This value of $r_{1\mathrm{B}}$ applies to our range of 30–90 arcsec; i.e. a radial distance of 2.5–7.5 kpc. It may be compared with the value of 1.0 kpc obtained by Solinger & Markert (1975) from the flattening of the observed pattern of polarization vectors, for light originating at a radial distance of typically 2 kpc. The difference in the estimates may lie outside their errors and could be due to their referring to a different part of the disc.

The intercept of our graph on the log $I'_{\mathrm{d}}$ axis gives, assuming a tilt of $10^\circ$, a value for the extrapolated central disc intensity when viewed normally of

$$I'_{\mathrm{D0}} = 4.3 \times 10^{-28}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,arcsec^{-2}},$$

in the absence of disc dust. Taking a representative optical thickness from Table 2 of $\tau_{\mathrm{B}} = 0.65$, then

$$I_{\mathrm{D0}} = I'_{\mathrm{D0}} \exp(-0.65/2) = 3.1 \times 10^{-28}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,arcsec^{-2}}.$$

This may be converted to more convenient units as follows. Now

$$\log I_{\lambda\mathrm{D}}(B) = -0.40\,B_{\mathrm{D}} - 8.17 \tag{5}$$

(Allen 1973), where $I_{\lambda\mathrm{d}}(\mathrm{B})$ is the disc flux in erg cm$^{-2}$ Å$^{-1}$ s$^{-1}$ arcsec$^{-2}$ near 4400 Å and $B_{\mathrm{d}}$ is the disc brightness in mag arcsec$^{-2}$ in the $B$ waveband. Converting the flux measurement to 1 Hz intervals,

$$I_{\mathrm{D}}(B) = I_{\lambda\mathrm{D}}(B) \times (\bar{\lambda}^2/C) \times 10^8$$
$$= I_{\lambda\mathrm{D}}(B) \times 6.5 \times 10^{-12},$$

where $\bar{\lambda}$ is the typical $B$ wavelength in cm and $c$ is the speed of light. Substituting this into equation (5) and rearranging,

$$B_{D0} = -48.4 - 2.5 \log_{10} I_D(B). \tag{6}$$

Substituting our value for $I_{D0}$, the extrapolated central disc brightness

$B_{D0} = 20.37$ $B$-mag arcsec$^{-2}$.

Freeman (1970) determined $B_{D0}$ for 36 spiral galaxies and found that 28 of these were distributed fairly closely about a mean

$\bar{B}_{D0} = 21.65 \pm 0.30$ $B$-mag arcsec$^{-2}$.

Thus

$B_{D0} - \bar{B}_{D0} = -1.28$ $B$-mag arcsec$^{-2}$.

Our value for $B_{D0}$ is brighter than for Freeman's 28 galaxies, and corresponds to the least bright of Freeman's remaining eight galaxies of which seven have a type II luminosity profile characterized by the disc intensity dipping below the exponential value as the centre is approached. This, together with the fairly small scalelength of 1.8 kpc, suggests that M82 may have a type II luminosity profile.

Taking the absolute visual magnitude, $V$, of M82 to be $-19.6$ and the colour to be typically $B-V = 0.9$ mag (de Vaucouleurs 1961), the absolute blue magnitude

$B = -18.7$ mag.

Comparing this with Freeman's plot of $B$ against $\log_{10} r_{1B}$ for spiral galaxies, it was found that M82 lies in the mainstream of galaxies, whose $-B$ tend to increase with disc scale length, while the seven type II galaxies with large $-B_{D0}$ mentioned above are rather brighter than average for any particular scale length. However, the total luminosity has been underestimated because M82 is observed almost edge-on which results in a substantial extinction of light. This correction would once again place M82 in the category of brighter galaxies.

The disc intensity in the blue is now known as a continuous function of radius from equation (4), if it is assumed that the optical thickness of the disc when viewed obliquely at 10° is roughly constant at 1.5 with radius; this will only introduce a small error – see Table 2. The function is displayed in Fig. 4 and can be seen to give a good fit to the data.

The $R$ data give disc intensities at the two points which are displaced from the $B$ intensities by roughly the same distance on a logarithmic scale. The scale length is therefore of the same order in the $R$, and $B-R$ is roughly constant at different radii. This is consistent with the result (Freeman 1970) that $B-V$ is approximately constant in the discs of M33, M31, NGC 1332 and the LMC.

Let us now consider the scattered component of the intensity. First, the percentage polarization of the scattered component in the $B$ increases from 27 per cent at 30 arcsec to 34.5 per cent at 90 arcsec. This may be compared with light from an extended source scattered from a large optically thin cloud of constant dust density with Rayleigh grains. It is easily shown that this would give a percentage polarization approaching 33.3 per cent at large distances which falls off at smaller distances where the dust no longer 'sees' the source as non-extended.

The polarization of red scattered light is seen to fall off from 47 per cent at 30 arcsec to 37 per cent at 60 arcsec. However, because of the uncertainties in measurements, $P_R$ may not differ appreciably from $P_B$.

The degree of polarization in Hα + N II measured by VS at 86 arcsec (27 ± 3 per cent) also lies fairly close to 33.3 per cent, suggesting that there is little dilution at this wavelength. This is consistent with the scarcity of H II regions in the disc of M82 (O'Connell & Mangano 1978) and implies that any intrinsic Hα light in this region is far less intense than the scattered Hα light. The difference between the Hα + N II and continuum polarizations (27 ± 3, 14 ± 4 per cent respectively) may therefore be accounted for by the presence of dilution in the continuum but not in these spectral lines, due to the dearth of H II regions in the outer parts of the M82 disc. However, simple scattering models (Solinger *et al.* 1977; Schmidt *et al.* 1976; Perkins, Pallister & Scarrott 1977) which do not require dilution would be unable to accommodate this difference.

The presence of this more highly polarized Hα + N II line in the light transmitted by the *R* filter makes little difference to the measured continuum polarization at this point, as its equivalent width here is only 220 Å (VS). Moreover, in the inner halo at 35 arcsec SE it is still less important, with an equivalent width of only 145 Å (O'Connell & Mangano 1978).

The radial dependence of the intensity of the scattered light can be compared with that of an infinite, constant dust density model proposed by Solinger *et al.* (1977). This assumes the scattered light is from a source consisting of a disc with an exponential intensity fall off

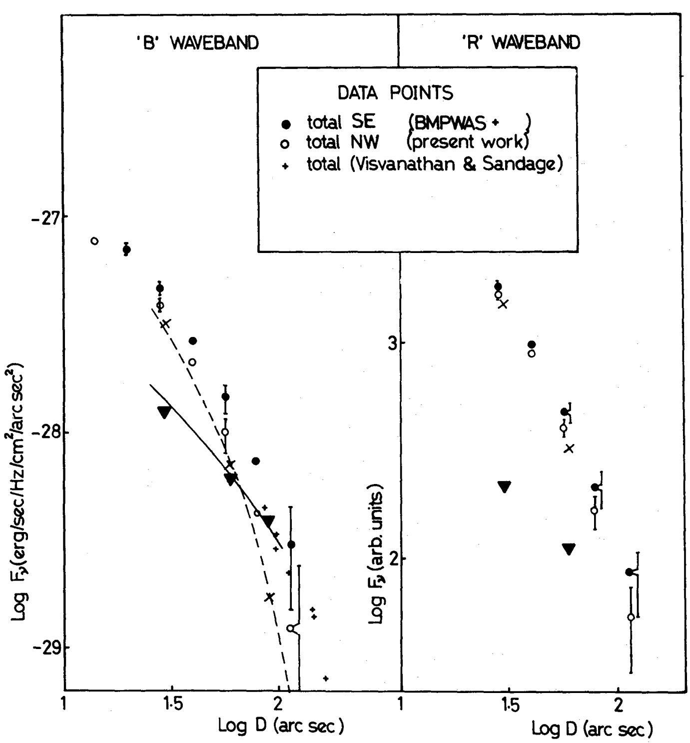


**Figure 4.** The total flux data as interpreted by the tilted disc model. The variation of the total flux of radiation, $F_\nu$, with distance from the polarization centre along the minor axis in the *B* waveband (BMPWAS 1976) and *R* waveband (present work) is shown. The 1σ error bars on our data points take account of both statistical and systematic effects. (For clarity of presentation only a proportion of representative error bars are displayed.) Also shown are the absolute *B* measurements by Visvanathan & Sandage (1972) which enable the *B* data to be fitted to an absolute scale. Using a tilted disc model, the total intensity has been separated at distances of 30, 60 and 90 arcsec into disc and SE scattered contributions, shown by X and V respectively. The dashed line shows the intensity fall off expected from a tilted exponential disc whose central brightness and scale length have been adjusted to give the best fit to the disc contribution. The solid line shows the scattered intensity predicted from a model of M82 proposed by Solinger *et al.* (1977) in which an edge-on exponential disc and nucleus illuminate a large, constant density cloud of scattering particles. The intensity has been scaled to give a fit to our data at 60 arcsec.

of scale length 1 kpc together with a concentric smaller disc with an exponential fall off of scale length 200 pc and a total luminosity of 0.1 of the big disc. The resulting intensity dependence is plotted in Fig. 4 for comparison with our scattered intensity points, the intensity scale being adjusted to give agreement at 60 arcsec. This model gives a reasonable fit to our model. Because the colour of the halo is fairly constant, the comparison with Solinger's results, which apply to the $V$ waveband, is justified.

### 4.2 HALO STAR MODEL

In this case, the tilt of the disc is assumed to be negligible, the high axial ratio being attributed to the large amount of scattered light above and below the disc. The geometry is illustrated in Fig. 5. The large NW–SE asymmetry in polarized intensity is now obtained by decreasing the dust density on the NW side by a constant factor so that the degree of polarization of the scattered light is still the same. This time, to get the required wavelength dependence of polarization, it is assumed that the polarized scattered light is diluted by an unpolarized halo contribution due to either Population II stars (Schmidt *et al.* 1976; Perkins 1979) or gas emission.

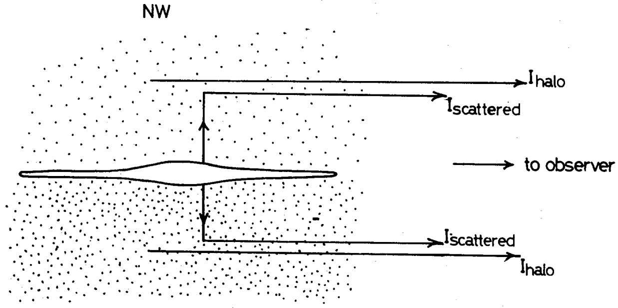


**Figure 5.** The halo star model. Light from the disc and nucleus scattered off dust particles and hence polarized ($I_{scattered}$) becomes mixed with unpolarized light ($I_{halo}$) direct from halo stars or gas emission before reaching the observer. There is roughly only half the density of dust on the NW side as on the SE side, so that the scattered light on the NW side is weaker.

In many ways this model performs an equivalent function to the 'tilted disc' model. As a result the intrinsic intensity and degree of polarization of the scattered light are the same in both cases, as is the scattered intensity on the SE side.

It is first necessary to determine the factor by which the dust density in the halo is smaller on the NW side. This is simply the quantity $t$ defined as for the disc model. Once again the definition

$$b = \frac{I_{TN}}{I_{TS}} = \frac{tI_{CS} + I_H}{I_{CS} + I_H}$$

is used, where the subscript H refers to the halo. This time we let

$$f = I_H/I_{CS}$$

so that

$$b = \frac{t+f}{1+f}.$$

leading as with the disc model to equation (2).

The values of the quantity, $t$, may be compared with the neutral hydrogen surface density distribution determined from 21-cm radio measurements (Gottesman & Weliachew 1977). The resolution is unfortunately far too low (3.5 arcmin) for a detailed comparison to be made. Nevertheless, it appears that the 21-cm radio maximum along the minor axis is displaced ~ 1 arcmin SE of the optical centre, and that the hydrogen density contours fall off roughly symmetrically on either side of this maximum. At 60 arcsec either side of the optical centre the hydrogen surface density ratio is ~ 0.86. This may be compared with the values in Table 2 of

$$t_B = 0.22; \quad t_R = 0.35.$$

It can be seen that there is some correlation between neutral hydrogen density and dust density along lines of sight 60 arcsec NW and SE of the centre along the minor axis.

There is a natural explanation for this asymmetry in dust and hydrogen density if it is assumed that M82 is drifting through a dust cloud (Solinger *et al.* 1977). The SE face of the disc must then have a velocity component along its axis of rotation which causes dust and gas to be swept up ahead of it. This leads to asymmetry in dust and hydrogen density.

The required contribution to the total intensity from unpolarized light of halo stars is shown in Fig. 6 and will be the same as the disc contribution in the tilted disc model (Fig. 4). As a comparison, the expected intensity of light from halo stars was estimated for a galaxy of the mass and dimensions of M82. This was done as follows.

Spinrad *et al.* (1978) present photometric measurements across the halo of the edge-on Sc spiral NGC 253 in the $R$ and $I$ wavebands. The $R$ data were fitted quite closely by the relationship

$$R = 18.21 + 7.59 \log r'_{253}$$

where $r'_{253}$ is the distance in arcmin from the Galactic Centre. Combining this with equation (6) it follows that

$$\log I(B) = -0.4(B-R + 7.59 \log r'_{253} + 66.6),$$

which relates the brightness in $B$ in NGC 253 to the distance from its centre. The colour index $R-I$ is tabulated as a function of distance, so that $B-R$ can readily be determined assuming the standard colours for giant stars. This variation of surface brightness for NGC 253 may now be scaled down to the smaller mass and radius of M82 to give an estimate of its corresponding surface brightness.

The masses of M82 and NGC 253 are taken to be

$$M_g = 3 \times 10^{10} M_\odot$$

and

$$M_{253} = 10^{11} M_\odot$$

respectively (Allen 1973) so that

$$M_g/M_{253} = 0.3.$$

Freeman (1970) concludes that the mass surface density, $\mu(r)$, of galactic discs falls off exponentially with radius, $r$, with an extrapolated central value $\mu_0$ which is probably approximately constant for different galaxies, i.e.

$$\mu(r) = \mu_0 \exp(-r/r_0)$$

where $r_0$ is the scale length.

This leads to a total disc mass

$$M_D = \int_0^\infty \mu_0 \exp\left(\frac{-r}{r_0}\right) 2\pi r\, dr = -\ 2\pi\mu_0 \int_0^\infty -r_0 \exp\left(\frac{-r}{r_0}\right) dr = 2\pi\mu_0 r_0^2.$$

Taking $M_D$ to be not far short of the total mass of the disc and bulge, $M_g$, then

$$M_g \mathrel{\tilde{\propto}} r_0^2.$$

From considerations of the evolution of a galaxy from an initial, approximately spherical distribution of Population II stars, the halo stars should have a distribution which falls off with a scale length

$$r_s \mathrel{\tilde{\propto}} r_0 \mathrel{\tilde{\propto}} M_g^{1/2}. \qquad (8)$$

Comparing the scale length for halo stars in M82, $r_s$, with that for NGC 253, $r_{s253}$,

$$\frac{r_s}{r_{s253}} \approx \frac{r_0}{r_{0253}} \approx \left(\frac{M_g}{M_{253}}\right)^{1/2} \approx 0.55,$$

where $r_{0253}$ is the radius of the disc of NGC 253. This equation is in agreement with the ratio of the radii given by Allen (1973) which turns out to be

$$\frac{r_0}{r_{0253}} = \frac{3.5}{6.5} = 0.54.$$

It is assumed for the moment that the ratio of mass of halo stars to total galactic mass is roughly the same for different galaxies. Then the halo star intensity at a point up the minor axis when integrated along a line of sight

$$\propto g/r_s r_0 = \text{ constant},$$

from equation (8), when comparing equivalent points for two different galaxies, these points having a distance

$$\propto r_s \propto M_g^{1/2}$$

up the minor axis. The surface brightness at any point along the minor axis of M82 when viewed edge-on was therefore estimated to be that in NGC 253 at

$$1/0.54 = 1.8$$

times the distance up the minor axis.

This now enables the halo intensity in the *B* waveband to be estimated. The resulting distribution is plotted in Fig. 6. The dilution required by the halo model is within half a magnitude of this estimate at our three points so gives satisfactory agreement. Now the relative importance of the spheroidal component in type I galaxies is not in fact a constant, as was assumed earlier; the ratio of the apparent radius of the spheroidal component to the radius of the disc component at 26.5 *B*-mag arcsec$^{-2}$, $R_{sph}/R_{26.5}$, ranges from 0.65 to 0.15 for Sa–Sd galaxies (Freeman 1970). As NGC 253 is of type Sc, it is therefore unlikely that M82 is of a much earlier type if the halo star model holds, as the dilution required would then be too small.

It is possible that intrinsic gas emission (e.g. H$\alpha$ and H$\beta$) supplements the halo star contribution. However, this is likely to be minimal in the region of the VS measurements of H$\alpha$ polarization at 86 arcsec on the SE side for the same reasons as those discussed for the tilted disc model. Moreover, at 35 arcsec SE the H$\alpha$ + N II equivalent width is only 145 Å

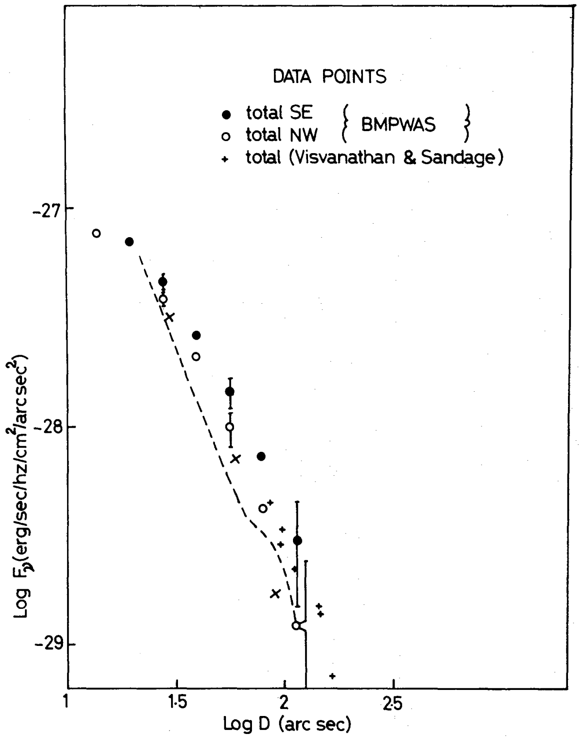


**Figure 6.** Total flux data as interpreted by the halo star model. Total flux data in the $B$ waveband (BMPWAS) is displayed as in Fig. 4. Using a halo star model, the total intensity has been separated at distances of 30, 60 and 90 arcsec into halo star and SE scattered contributions. These contributions must be identical with those for the tilted disc model. The intrinsic component is shown by X and may be compared with the dashed line showing the intensity fall off measured across the edge-on spiral galaxy NGC 253 (Spinrad *et al.* 1978), when scaled down to the mass, radius and distance of M82. The scattered contribution is once again compared with the model of Solinger *et al.* (1977) as shown in Fig. 4.

(O'Connell & Mangano 1978) so makes a negligible contribution to the brightness in $R$. No measurements are available at intermediate distances to show whether the H$\alpha$ polarization is sufficiently low to dilute the continuum polarization there or add significantly to the total $R$ intensity.

## 5 Colour measurements over M82

Artamonov (1978) presents measurements of $U$, $B$ and $V$ over the surface of M82 and interprets them in terms of a tilted disc model of similar inclination (9°) to that used in this paper (10°) and with the same half of the disc towards the observer. His traces up the minor axis use distances measured from the optical bright spot and this leads to a total intensity in the $B$ waveband having an opposite asymmetry to the results of BMPWAS which use the polarization centre. To check that the two sets of data were consistent, further intensity traces were obtained from the observations of BMPWAS using the optical centre and these then agreed with those of Artamonov. He has estimated the variation in optical depth in the $V$ waveband, $\tau_V/2$, of dust between stars and observer with distance along the major axis in a north-easterly direction from spectroscopic date (giving the underlying stellar brightness) and colour measurements (from which the reddening can be determined). This shows a linear fall off in $\tau'_V/2$ from 2.0 at 75 arcsec to 0.75 at 225 arcsec. A distance along the minor axis of 30 arcsec would correspond to a distance along the major axis of

$$30/\sin 10° = 173\,\text{arcsec}$$

at which Artamonov estimates $\tau'_V/2$ to be on average 1.1. This may be compared with our estimates of

$\tau'_B \approx \tau'_R \approx 1.2$

which are only about half that of Artamonov. This discrepancy is probably a result of the uncertainty in $P_N$, which could lead to $\tau'$ being substantially underestimated by us.

Artamonov is able to give a satisfactory interpretation of the variation in colour along the minor axis of M82 as measured in the $(U-B)-(B-V)$ plane using a tilted disc model similar to our own. In order to do this he has to take account of the asymmetry in the disc colour at small distances from the nucleus caused by the tilt. In an earlier paper (Artamonov & Notni 1976) the colour of the far halo near the minor axis is interpreted as due to highly reddened light from O and B stars in the nucleus which is scattered into the line of sight. Most of the energy produced by these stars would presumably be absorbed by the optically thick dust clouds in which they are embedded, and subsequently be reradiated in the infra-red. Harper & Low (1973) estimate the 3–3000 $\mu$m infrared luminosity of the nuclear region to be $2 \times 10^{44}$ erg s$^{-1}$ and more recently Kleinmann, Wright & Fazio (1976) obtain a total infrared luminosity in this region of $8 \times 10^{43}$ erg s$^{-1}$. Let us take $\sim 10^{44}$ erg s$^{-1}$ as being representative. After subtracting one-fifth of this as the contribution from late K and early M giant stars (Willner *et al.* 1977), the remaining infrared radiation is of the order of that which would be produced by the hot stars if nuclear light is to dominate over disc light in the far halo (O'Connell & Mangano 1978). The model is therefore self-consistent. Moreover, the decreasing value of $(U-B)$ on moving south along the minor axis (and to a lesser extent on moving north along this axis) supports the idea of a scattered component becoming more important further out in the halo particularly on the SE side and to a lesser extent to the NW side.

Our halo star model, which cannot produce asymmetry in the intrinsic intensity, offers no ready explanation for the NW–SE asymmetry in the $(U-B)-(B-V)$ plane, even after the asymmetry in polarized light has been allowed for. The intrinsic intensity could have asymmetry if gas emission were important in the halo. The H$\alpha$ filaments are known to be stronger on the SE side (Sandage & Miller 1964). The corresponding asymmetry in H$\beta$ could make the $B$ more intense to the SE. However, even supposing the H$\beta$ equivalent width to be appreciable, which is unlikely (Lynds & Sandage 1963), the resulting asymmetry would be in the opposite sense to that observed. This provides support for the tilted disc model.

## 6 Conclusions and discussion

The authors' polarization map of M82 in the $R$ waveband shows the degree of polarization to be substantially smaller than that measured by BMPWAS in $B$. This $B$ and $R$ data has been interpreted in terms of two alternative models.

Let us first consider the tilted disc model. A tilt of $\sim 10°$ is assumed and little variation from this can be tolerated. A much smaller angle would mean that the disc of radius 7 kpc (Homberg 1958) would not extend out sufficiently far along the minor axis to provide the required asymmetry in $P$ and $I_T$, while a substantially larger angle would require that the spiral structure could be observed in the halo which is not the case.

The model requires an exponential disc of scale length $\sim 1.8$ kpc and an extrapolated central brightness of 20.4 $B$-mag arcsec$^{-2}$, 1.3 $B$-mag arcsec$^{-2}$ brighter than the average for Freeman's 28 galaxies of normal brightness. This suggests that M82 could lie in Freeman's group of bright galaxies and may have a type II luminosity profile. The high total luminosity of M82 considering its small scale length supports this. The disc brightness in the $B$ and $R$ wavebands falls off at about the same rate between 30 and 60 arcsec which is what would be expected for a normal spiral (Freeman 1970).

The model predicts an optical thickness for the disc of $\tau_B \sim 0.6$ and $\tau_R \sim 0.5$ when viewed normally between 2.5 and 7.5 kpc from the disc centre. The value of $\tau_B$ compares quite favourably with the average values given by Homberg (1958) and Zasov (1978) for Sa–Sd spirals which range from 0.5 to 1.1 in the *B* waveband. However, Artamonov's estimate of $\tau_V$ from the reddening is roughly twice our value, which may be somewhat low due to having possibly overestimated the degree of polarization on the NW side through systematic errors in our observations. Nevertheless, our estimates are of the right order. The rather grey wavelength dependence of $\tau$ is consistent with a grain composition similar to that in our own Galaxy. Also, in a realistic tilted disc model the disc intensity should normally be asymmetrical NW to SE (Elvius 1956). However, this effect is unlikely to extend out into our range of interest (30–90 arcsec) unless the central bulge is unusually large. As there is no evidence for this, the asymmetry in the disc intensity, and its possible effects on estimates of the optical thickness of the disc have been ignored.

The tilted disc model predicts that the polarization of the scattered light is in the region of 35 per cent in both colours in the range 30–90 arcsec. This is close to the value expected from a point source in an infinite cloud with a constant density of Rayleigh grains which gives a degree of polarization of 33.3 per cent. Furthermore, results from the more precise *B* data of BMPWAS suggest that this has fallen off slightly nearer the disc to 27 per cent at 30 arcsec. This is to be expected, as dust grains nearer in are no longer illuminated by a quasi-point source; the extended disc source results in some dilution. Moreover, the scattered intensity distribution gives a fairly good fit in both wavebands to Solinger's model with an extensive constant density dust cloud.

The broad features of the colour distribution over M82 in the *UBV* system can be accounted for in terms of a tilted disc model (Artamonov 1978). The halo light near the minor axis can be interpreted as light from O and B stars in the nucleus which was reddened by passage through dust before being scattered into the line of sight. The infrared emission from this dust which would result from the absorption of ultraviolet radiation from the hot stars turns out to be of the same order as that observed.

The halo star model requires a distribution of halo star intensity along the minor axis similar to that found in a typical Sc spiral galaxy, when scaled down to the size and mass of M82.

It is possible that line-emission provides some of the unpolarized intensity in the halo in the region 40–80 arcsec, with presumably H$\alpha$ predominating in the *R* waveband and H$\beta$ in the *B* waveband. Measurement of the equivalent widths of the lines in this region along the minor axis would be useful in testing this hypothesis. Both models can account for the difference between the H$\alpha$ + N II and continuum polarizations as being due to the absence of significant dilution in these spectral lines by either the outer regions of the disc or normal halo stars. The colour asymmetry is less easy to account for in the halo star model as tilt effects no longer play a part, and the asymmetrically distributed scattered light is negligible at small distances. This gives added support to the tilted disc model, which the authors favour as being the more plausible of the two in the light of presently available observational evidence. The disc intensity in the tilted disc model will include any contribution from halo stars and gas emission in the halo, and the former at least is likely to be important.

The present data were obtained from a series of eight 10-min exposures. The future acquisition of deeper plates would improve precision in model fitting and allow the far halo to be examined, where dilution effects might be expected to be negligible. The wavelength dependence of polarization and the dust density distribution could then be determined more directly and thus extend the present novel conclusions concerning the optical properties of the M82 halo.

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