THE WELFARE EFFECTS OF THE INTRODUCTION OF STORAGE*

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This paper examines the welfare effects of introducing storage into a market with stochastic supply in which all agents are competitive profit-maximizers with rational expectations. These welfare effects are the net result of the initial increase in demand for stock-building and the partial and asymmetric reduction in the dispersion of consumption brought about by storage. The distributional impacts depend crucially on the information available to producers before storage is introduced, the elasticity of supply, the specification of the consumption demand curve, and the cost of storage.

In this paper we study the welfare effects of introducing storage as a competitive economic activity in a market with stochastic supply. Despite the obvious importance of the inventions of canning, refrigeration, and chemical preservation, not to mention the substantial research seeking to improve storage further or expand the set of goods that can be stored, the effects of such technical advances on the welfare of market participants are not well understood. We show how these welfare effects depend on the specification of demand, the elasticity of supply, the cost of storage, and the information available to producers before storage is introduced. In addition, we explore the magnitude of these effects with some simple examples.

In focusing on the initiation of storage, we are actually addressing the subject of many analytical and numerical studies that have claimed the objective of assessing the introduction of market stabilization, while implicitly assuming the absence of private storage (see Turnovsky [1978] for a survey of this literature). Nearly all of the analytical studies in the tradition of Waugh [1944], Oi [1964], and Massell [1969] evaluate complete stabilization of either price or consumption via some market intervention involving storage, but the storage activity itself is never explicitly considered. Unfortunately, competitive storage and complete stabilization are not analytically equivalent. In the first place, a commodity stockpile can never be expected to stabilize future consumption completely because there is always a chance that a series of bad harvests will exhaust the

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stockpile.[1] Second, because of the costs of storage, it is by no means optimal even to try to stabilize consumption and price completely. And those analytical studies (e.g., Newbery and Stiglitz [1979]) that consider partial stabilization ignore the essential asymmetry in the effects of storage on the dispersion of consumption. (See Gustafson [1958a and b] and Wright and Williams [1982a].)

Storage is explicitly modeled in another set of studies that concentrate on the stochastic simulation of various storage rules for a public storage authority. Unfortunately, these rules are most often arbitrary (see, for example, Duloy and Parish [1964], Reutlinger [1976]). Further, where supply is economically responsive, the results reflect the irrational way in which producers' expectations about price are said to be formed (see, for example, Sharples [1980]). In addition, the welfare effects of both the analytical and numerical studies depend at least in part on the unexplained absence of private storage. (For an illustration of the implications of this omission, see Wright and Williams [1982b].)

Instead of following the welfare implications of arbitrary behavioral specifications, we examine the effects of the introduction of storage as performed by competitive profit-maximizers, who form expectations that are rational in the sense that they are consistent with knowledge by all agents of the structure of the model. The same behavioral assumptions are attributed to producers and are rendered meaningful by allowing for response to economic incentives. In other models of optimal storage, including Gustafson [1958a] and Johnson and Sumner [1976], the assumption that supply is completely inelastic makes the price expectations of producers irrelevant.[2]

The introduction of storage is an inherently dynamic phenomenon, although previous analytical studies of market stabilization have not recognized this fact. The approach described in this paper could be used to calculate the welfare effects of changes in storage costs, spoilage, and so forth. The results would depend on the amount already in store in a relatively straightforward fashion. Economy of exposition dictates that we concentrate on the initial condition of nothing in store, which corresponds to the introduction of storage or storability.

1. See Townsend [1977] for a proof of the proposition that the stockpile will be exhausted with probability one in finite time.

2. Helmberger and Weaver [1977] also considered private competitive storage. Their description of market behavior, however, is founded on the erroneous assumption [p. 640] that if current storage is positive, future storage will always be positive. A simple model with integer storage and rationally responsive supply is found in Gardner [1979].

I. A MODEL OF A MARKET WITH STORAGE

In our model of a closed competitive economy, consumers, who are assumed to be identical, have an inverse consumption demand for the commodity in question of

(1) $$P_t = \alpha + \beta q_t^{1-C}, \quad P > 0, \quad P'(q) < 0,$$

where P_t is price in period t, and q_t is the amount consumed. In a study of ideal stabilization, Wright [1979] demonstrated the sensitivity of welfare changes to the parameter C, which is the Pratt-Arrow measure of curvature used to assess relative risk aversion:

(2) $$C \equiv -(qP''(q)/P'(q)).$$

Assuming that the coefficient of relative risk aversion of consumers with respect to income is approximately equal to the income elasticity of demand for the commodity, or that the budget share of the commodity is small relative to the absolute value of its demand elasticity, consumers are "commodity risk averse" if C is greater than 1.0. That is, consumers would pay for a mean-preserving decrease in the dispersion of consumption of the commodity. If C is below 1.0, the demand curve displays commodity risk preference.[3]

3. To see the significance of the parameter C, consider the indirect utility function $V(p,\pi,Y)$, where Y is income and π is the vector of prices of all other commodities, all assumed to have infinitely elastic, instantaneous, deterministic supplies. Then the sign of the effect on expected utility of a marginal destabilization of consumption q of the commodity in question is the sign of the second derivative of V with respect to q:

$$\frac{\partial^2 V}{\partial q^2} = V_{pp}\left(\frac{\partial P}{\partial q}\right)^2 + V_P \frac{\partial^2 P}{\partial q^2}.$$

Using Roy's identity, and (2), we see that

$$\begin{aligned} \frac{\partial^2 V}{\partial q^2} &= V_{pp}\left(\frac{\partial P}{\partial q}\right)^2 + CV_y \frac{\partial P}{\partial q} \\ &= -V_Y \frac{\partial P}{\partial q} - qV_{PY}\left(\frac{dp}{dq}\right)^2 + CV_y \frac{\partial P}{\partial q} \\ &= -V_y \frac{\partial P}{\partial q} + \frac{q\gamma}{P} V_y(\eta^Y - R)\left(\frac{\partial P}{\partial q}\right)^2 + CV_y \frac{\partial P}{\partial q} \\ &= \frac{PV_Y}{q\eta^D}\left[C - 1 + \gamma \frac{(\eta^Y - R)}{\eta^D}\right], \end{aligned}$$

where R is the coefficient of relative risk aversion with respect to income, η^Y is the income elasticity of demand, and γ the expenditure share. Thus, the measure of relative commodity risk aversion with respect to quantity consumed ρ_c is

$$\rho_c \equiv -\frac{q\,\partial^2 v}{\partial q^2}\frac{\partial v}{\partial q} = C - 1 + \frac{\gamma(\eta^Y - R)}{\eta^D}.$$

If the last term is negligible ($R \to \eta^Y$ or $\gamma/\eta^D \to 0$), then consumers are "commodity risk averse" (they would pay for a mean-preserving decrease in the dispersion of consumption) if $C > 1$. If $C < 1$ they are "commodity risk-preferring." The significance of the parameter C was identified in Wright [1979]; we are grateful to David Newbery for suggesting the above derivation. ρ_c is obviously distinct from the coefficient of relative price risk aversion discussed in Turnovsky *et al.* [1980], p 143.

The quantity consumed is

$$q_t = I_t - S_t, \tag{3}$$

where S_t is the amount stored from period t to period $t + 1$, and I_t is the amount on hand in period t:

$$I_t = x_t + S_{t-1}, \tag{4}$$

where x_t is production in period t.

Some disturbance such as weather disrupts production in each period. Realized production has the following relationship to planned production:

$$x_t = \hat{x}(P_t^r)[1 + v_t], \tag{5}$$

where v_t is a random, serially uncorrelated disturbance with probability density function $f(v)$ of finite variance. Most sources of production instability, including weather, have multiplicative effects on output, rather than the additive effects assumed in much of the literature on production stabilization. P_t^r is the producer incentive at time $t - 1$ when commitment to planned production $\hat{x}(P_t^r)$ must be made; production is perfectly inelastic within period t. Producer i maximizes expected profits,

$$E[\Pi_{it}] \equiv E[x_{it}P(q_t)] - g_i(\hat{x}_{it}), \tag{6}$$

where x_{it} is the output of producer i, g_i is his total cost of production, and E denotes the conditional expectation given the information available to all agents in period $t - 1$. Each producer has the same multiplicative disturbance v_t. An atomistic producer perceives P_t, which is a function of aggregate production, to be independent of x_{it}, but recognizes the correlation between the disturbance in his own production and the disturbance in aggregate production.[4] The first-order condition for competitive profit maximization is

$$\frac{\partial E[\Pi_{it}]}{\partial \hat{x}_{it}} = \frac{\partial E[x_{it}P(q_t)]}{\partial \hat{x}_{it}} - \frac{\partial g_i(\hat{x}_{it})}{\partial \hat{x}_{it}}. \tag{7}$$

The rational producer incentive, sometimes called the "action certainty equivalent price," is expected revenue of producer i per unit of his planned production:

$$P_t^r = \frac{\partial E[x_{it}P(q_t)]}{\partial \hat{x}_{it}} \tag{8}$$

$$= \frac{\partial E[[1 + v_t]\hat{x}_{it}P(\hat{x}_t[1 + v_t] + S_{t-1} - S_t)]}{\partial \hat{x}_{it}} = \frac{\partial E[P(q_t)x_{it}]}{\partial \hat{x}_{it}}.$$

4. Given risk neutrality, any additional individual disturbances independent of aggregate disturbances would not alter the results presented below.

If the disturbances were additive instead of multiplicative and shared in a way that was independent of each producer's planned production, P_t^r would simply equal the expected price $E[P_t]$ (see Wright [1979]).

Like production, private storage is assumed to be competitive and expected profit maximizing. Although entrepreneurs could specialize in the storage business, there is no reason in this model to suppose they would have any inherent advantage over producers or consumers who store on their own. Whoever does the storing must consider the costs of storing an amount S_t from period t to period $t + 1$, which are, as of period $t + 1$,

$$(9) \qquad K(S_t) = \xi(S_t) + P_tS_t + r(P_tS_t + \xi(S_t)),$$

where $\xi(S_t)$ is the cost of storage services, paid for in period t, and r is the interest rate. The term P_tS_t represents the value of what is put into store and $r(P_tS_t + \xi(S_t))$ the opportunity cost of capital tied up.

As long as the marginal cost of storage services is constant, there are no rents, in expectation, associated with storage activity. This simplifies calculations of the changes in expected welfare due to storage and makes it unnecessary to specify who actually does the storing.

Paul [1970] indicates unit physical storage costs for commodities such as grain are remarkably constant over the observed range of storage. However, the spread between spot and futures prices, known as the "price of storage," is highly nonlinear and is quite negative at small amounts stored (see Telser [1958] or Brennan [1958]). This phenomenon arises because intermediate users pay to keep inventories on hand to deal with the stochastic nature of transportation, processing, and final demand (see Working [1949] and Williams [1980]). Since this paper is focusing on production uncertainty, these other stochastic elements, as well as their considerable complications for the specification of final consumption demand and the calculation of changes in welfare, are ignored.[5] Accordingly, we specify the cost of storage services as

$$(10) \qquad \xi(S_t) = kS_t, \quad k \geq 0.$$

Profit-maximizing competitive storage, if positive, will set the expected marginal revenue from storage equal to the marginal storage

5. We have solved for the market equilibrium in cases where storage costs have the general nonlinear form observed in grain markets, and the allocative results are quite similar to those derived from the model used in this paper.

costs. That is to say,

$$(11)\qquad \begin{array}{ll} 0 \geq (1+r)^{-1}EP_{t+1} - (P_t + k), & S_t = 0 \\ 0 = (1+r)^{-1}EP_{t+1} - (P_t + k), & S_t > 0. \end{array}$$

As Samuelson [1971] shows, these private arbitrage conditions are precisely the Kuhn-Tucker necessary conditions for socially optimal storage in an undistorted economy with an individualistic social welfare function.

EP_{t+1} is a function of current storage and planned production, which in turn depends on current storage. Of course, P_t is a function of S_t and I_t because what is not stored out of the amount available is consumed. Thus, the arbitrage conditions (11) implicitly contain current storage as a function of the amount available. If the horizon is infinite, given appropriate regularity and transversality conditions,[6] storage is a stationary function of availability:

$$(12)\qquad S_t = f(I_t), \quad 0 \leq f' \leq 1, \quad S_t \geq 0.$$

Given this relation, P_t can be expressed as a function of the amount in store alone. If one uses equation (1), the inverse consumption demand function, and equations (3) and (12),

$$(13)\qquad P_t = P(f^{-1}(S_t) - S_t) \equiv \phi(S_t),$$

where $\phi(S_t)$ is the inverse demand function for storage. Because competitive private storage is also welfare-maximizing storage in an undistorted economy with an individualistic social welfare function, the area under the inverse demand function $\phi(S_t)$ can be interpreted as the present value of the expected future welfare from current storage. Whatever is stored in the current period will be used optimally at some point in the future, and hence its current marginal valuation accurately reflects expectations of its future use.

II. THE EFFECTS OF THE INTRODUCTION OF STORAGE: AN ILLUSTRATION

The introduction of competitive storage in the market model presented above induces an interrelated set of responses in the path of prices, output, and consumption. A heuristic example, by illustrating the nature of these responses and their welfare implications,

6. Two conditions are (see Samuelson [1971])

$$\lim_{t\to\infty}(1+r)^{-t}EP_t = 0 \quad \text{and} \quad \lim_{t\to\infty}(1+r)^{-t}S_t = 0.$$

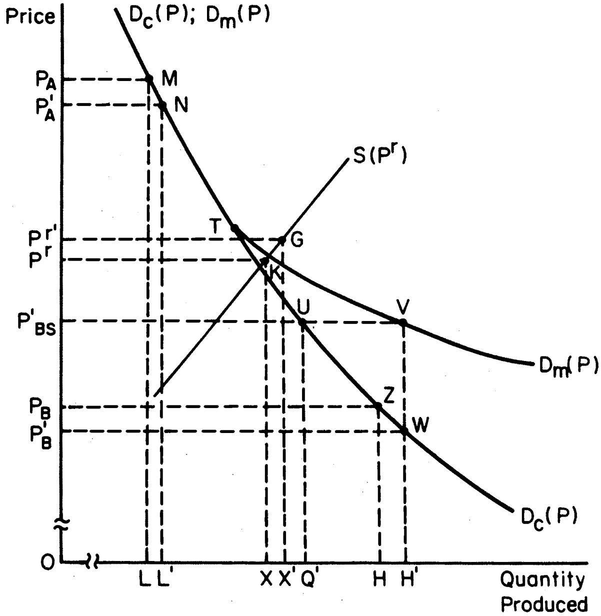

FIGURE I
Market Equilibrium with and without Storage
(primes denote values under storage)

can help explain the economics behind the more general results presented in the next section.

In the example depicted in Figure I, the disturbance v_t takes on two values, $\pm\zeta$, each with probability of 0.5.[7] The consumption demand curve is $D_c(P)$, and the (one-year-lagged) supply curve is $S(P^r)$. If storage is impossible, planned production is X, and realized production and consumption are either $L \equiv (1 - \zeta)X$ or $H \equiv (1 + \zeta)X$, with realized prices of P_A or P_B, respectively. P^r, the equilibrium production incentive, equals $\frac{1}{2}[P_A L + P_B H]/X$, and is constant from year to year. Clearly P^r is distinct from both the mean price and the certainty price.

If profit-maximizing storage is made economically possible from period 1, the consumption demand curve $D_c(P)$ is horizontally augmented below T by the storage demand curve $\phi^{-1}(P)$ to yield the market demand curve $D_m(P)$. Expected revenue increases at any level

7. To show the qualitative changes due to storage more clearly, Figure I is not drawn strictly to scale.

of planned production. Assuming the introduction of storage and its effects are accurately anticipated in period 0, one can see that the equilibrium producer incentive is $P^{r'}$ and the equilibrium production planned for period 1 is X'. Realized production will be either $L' \equiv (1 - \zeta)X'$ or $H' \equiv (1 + \zeta)X'$.

When production is H', Q' is consumed, and $(H' - Q')$ is carried over to period 2. The market demand curve for production in the next period is shifted to the left by the carryover. In period 1 the anticipation of this demand shift will reduce the price expected in period 2, as well as the incentive for production commitments in period 1 for output in period 2. The market price in subsequent periods at any given realization of the production disturbance will reflect the two offsetting effects of the introduction of storage: augmentation of the consumer demand curve by storage demand, and any carryover from the previous period. But the net result will be cushioned by the endogenous response in planned production.

This description of market equilibrium with storage sets the stage for illustration of the effects of introducing storage on the welfare of producers and consumers. We maintain two assumptions that are adopted in most studies of market stabilization, often implicitly. The first is that there are no market distortions or sources of instability in the economy outside of the market under study. The second is that welfare effects can be measured by changes in consumer and producer surplus; thus all distributional questions, other than comparisons of aggregate producer and consumer gains, are ruled out. We make the further assumption that the relevant surplus measures are adequately approximated by calculations based on market supply and demand curves. The conditions under which this procedure is appropriate seem acceptable for most commodities in a developed economy, but may well be violated in less developed countries for agricultural staples that have large shares of consumers' budgets (see Willig [1976]).

Because the effect of introducing storage is dynamic, it is useful to begin the welfare analysis by considering the implications of advancing the introduction of storage from period 2 to period 1, assuming storage remains possible for all later periods. The welfare change from advancing the introduction of storage can be divided into period 1 effects and subsequent effects. In period 1, if $v = -\zeta$ and supply is L', the increase in consumer surplus due to the increase in planned supply from X to X' in response to the increase in the producer incentive from P^r to $P^{r'}$ is area $P_AMNP'_A$ in Figure I. When $v = +\zeta$, production is higher under storage by $(H' - H)$, but consumption is reduced by the carryover $(Q' - H')$. Consequently, the decrease in consumer surplus relative to the no-storage situation is $P'_{BS}UZP_B$. The expected change in producer surplus in period 1 is given by $P^{r\prime}GKP^{r}$.[8]

The carryover from period 1 will raise consumption and consumer surplus, and depress producer surplus, in some period(s) in the future. The induced changes in expected future consumer and producer surplus cannot be separately indicated, even for the simple heuristic case illustrated here, although they are separated in the numerical calculations in the next section. But the net effect of carryover on the present value of the expected sum of consumer and producer surplus in all future periods, net of the cost of what is put into store, is given by the surplus under the derived demand curve for storage,

$$\int_0^{H'-Q'} \phi(S_t)\, dS - P'_{BS}(H' - Q'),$$

which is represented in Figure I by area TVU.

The total welfare effects of a one-period advance in the introduction of storage for either producers or consumers are the present values of the effects expected in the future plus the effects in period 1. The welfare effects of introducing storage now, rather than never having storage at all, are equal to the present value of advancing the introduction of storage by one period in each of the periods from the infinite future to the present. The present value of the many one-period effects, which are all equal, is easily calculated, given a constant interest rate r, by dividing the value in any one period by r.

III. The Effects of Storage: Numerical Results

Because the amount stored cannot be negative, derivation of competitive storage behavior is analytically intractable in general. However, using the numerical approach described in Appendix 1, it is possible to calculate the storage rule quite accurately. Numerical methods are also useful in deriving the distributive implications of storage. In Appendix 2 we describe an algorithm that can find a function $R(S)$, which relates the change in the present value of expected producer surplus from period 2 to infinity to the amount of storage carried over from period 1. The corresponding change in consumer surplus from storage in period 1 is the difference between

8. By construction there is a one-period gap between expenditure on production and receipts. Producer surplus is calculated as a net present value as of harvest time.

$R(S)$ and the surplus under the storage demand curve. These measures of surplus represent the expected distributional impacts of competitive storage.

We calculated these distributional impacts for various values of the supply elasticity and various specifications of demand. We chose an eighty-point discrete approximation to the normal density function for the distribution of the disturbance in production, with mean zero and standard deviation 0.05 (the right order of magnitude for cereal production).[9] The long-run (one period lagged) supply curve was constructed to be linear over the relevant range.[10] The real interest rate was 5 percent, and physical storage costs were set at zero.

3.1. Results for Constant Demand Elasticity

Consider initially the case where the introduction of storage and its implications are perfectly anticipated in the previous period when production commitments are made. The numerically calculated welfare effects of introducing storage, for selected demand and supply elasticities, are shown in Table I, where demand is assumed to have constant elasticity. (We denote by the shorthand term "welfare effects" the expected welfare changes conditional on the information available before the introduction of storage.) The present value of the effect of introducing storage is given as a percent of the expected annual revenue in each case.[11]

As shown in Table I, storage plays a significant allocative role even when production is of rather modest variability. To provide a familiar standard for evaluating the significance of the welfare changes caused by storage, the last column in Table I shows the excise tax rate that, if imposed on the commodity when storage is impossible, would incur a welfare cost equivalent in magnitude to the benefits of intro-

9. Because the error is multiplicative, the standard deviation of production changes as mean production changes. The standard deviation around a trend in world production of grain is just over 3 percent [Eaton, 1980]. The standard deviation for a particular crop or region would be higher. On the other hand, the observed fluctuation in production is partly a response to endogenous changes in economic incentives (see Wright and Williams [1981]). Results obtained using a symmetric triangular distribution with the same mean and variance were very similar.

10. All supply and demand elasticities are measured at the point that would denote equilibrium if production were not stochastic. The assumption that supply is linear is not crucial in this model because movements in planned production are small relative to movements along the consumption demand curve, as illustrated in Figure I.

11. Nonlinearities in supply outside the observed range of production affect land values but have no effect on the derivation of the optimal storage rule or on changes in welfare. Expected annual revenue without storage is preferred to land value because revenue is independent of otherwise irrelevant portions of the supply function.

TABLE I

PRESENT VALUE OF WELFARE EFFECTS WITH CONSTANT ELASTICITY OF DEMAND
(percent of expected annual revenue without storage)

		Net benefit of introducing storage now rather than never	Change in producers' surplus	Change in consumers' surplus	Annual tax with equivalent net welfare effect (percent rate)
$\eta^D = -0.2$	$\eta^S = 0.0$	4.1	−16.2	20.3	∞
$\eta^D = -0.2$	$\eta^S = 0.1$	4.5	−13.0	17.5	25.8
$\eta^D = -0.2$	$\eta^S = 0.2$	4.7	−10.3	15.0	21.9
$\eta^D = -0.2$	$\eta^S = 1.0$	5.5	−2.7	8.2	18.2
$\eta^D = -0.5$	$\eta^S = 0.0$	0.7	−0.8	1.5	∞
$\eta^D = -0.5$	$\eta^S = 0.1$	0.7	−0.7	1.4	9.5
$\eta^D = -0.5$	$\eta^S = 0.2$	0.8	−0.6	1.4	7.4
$\eta^D = -0.5$	$\eta^S = 1.0$	0.9	−0.1	1.0	5.2

ducing storage.[12] For example, if $\eta^D = -0.2$ and $\eta^S = 1.0$, prohibiting storage forever has the same social cost as the imposition of a hefty 18.2 percent excise tax forever. Note also that the net social value of storage increases with the supply elasticity, although not as much as does the cost of an excise tax. The net social benefit of storage, however, is much lower at higher elasticities of demand. For example at $\eta^D = -0.5$ and $\eta^S = 0$, the net social value of introducing storage is only one-sixth of the value for $\eta^D = -0.2$. As the demand elasticity increases, adjustment of consumption becomes an increasingly effective substitute for storage in accommodating fluctuations in supply.

But the net social value of storage tends to be dominated by the gross welfare changes, just as the distributional effects of an excise tax dominate its excess burden. For all the examples in Table I, consumers gain and producers lose from storage, a result that depends crucially on having constant elasticity of demand.

3.2. Results for More General Specifications

The distributive effects of the introduction of storage are illustrated in Figure II for a wide range of the demand curvature parameter C. The three curves in Figure II all show the approximately linear relation between the relative gains to producers and the demand curvature measured along the horizontal axis. The producers' share, less one, is the relative loss to consumers. Curve AB shows that when supply is completely inelastic and demand has constant elasticity of -0.2 ($C = 6$), producers can expect to lose about four and consumers to gain about five times the net social benefit of storage. But when demand is linear ($C = 0$), the gross distributive effects are reversed; producers gain about six and consumers lose about five times the net social benefit. Only for values of C between 3.0 and 3.6, do both groups benefit from the introduction of storage. Thus, when supply response is ruled out, Figure II lends support to the qualitative conclusions of

12. Excess burden of a tax T as a fraction of the total amount spent on a commodity is approximately

$$-\frac{\tfrac{1}{2}\Delta QT}{PQ} = \frac{1}{2}\left[\frac{T}{P}\right]^2\left[\frac{1}{|\eta^D|} + \frac{1}{\eta S}\right]^{-1},$$

where Q is the amount produced and consumed and P is the price. If b is set equal to social cost of going without storage forever divided by total expenditure, the annual tax rate with equivalent welfare cost is

$$\frac{T}{P} = t = \left[2br\left(\frac{1}{|\eta^D|} + \frac{1}{\eta^S}\right)\right]^{1/2}.$$

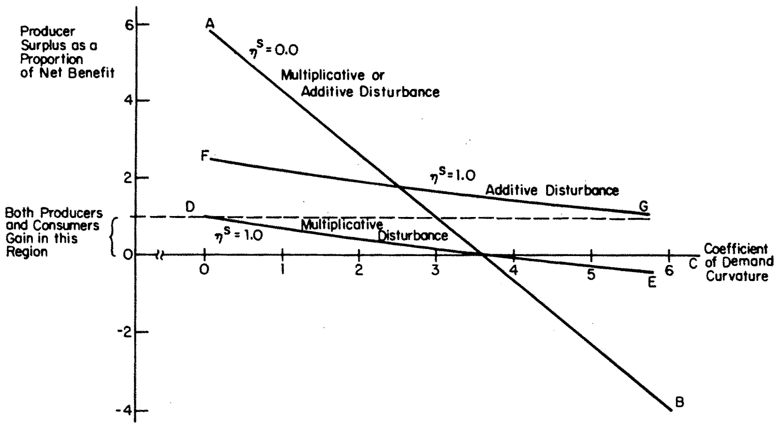

FIGURE II
Expected Distributive Effects of the Unanticipated Introduction of Storage
($\eta^D = -0.2$)

previous analytical studies that showed substantial redistributive effects from market stabilization.

When the assumption of zero supply elasticity is relaxed, two questions concerning specification immediately arise. The first concerns the form of the production disturbance, and the second involves the anticipations of producers regarding the date of the introduction of storage and its economic consequences. If these anticipations are accurate at the time of commitment to production and the disturbance in production is multiplicative, the distributive results for $\eta^S = 1$ are presented by curve *DE* in Figure II. A comparison of curve *DE* with curve *AB* shows that the gross distributive effects are weaker at the higher supply elasticity. Consumers share at least some of the benefit from introducing storage whether they are commodity-risk-preferring ($C = 0$) or commodity-risk-averse ($C = 6$). Producers never gain or lose an amount greater than the net benefit, over the range of C considered. Since the net benefit is less than 20 percent higher at $\eta^S = 1$ than at $\eta^S = 0$, the maximum redistribution at stake is much lower at the higher supply elasticity.

To illustrate the importance of the specification of the production disturbance, the results for $\eta^S = 1$ under the commonly assumed additive disturbance are shown by curve *FG* in Figure II. As in curve *DE*, the gross distributive effects are less sensitive to C at the higher supply elasticity. If the disturbance is really multiplicative, however, assumption of an additive specification uniformly overstates the benefit to producers and the loss to consumers by an amount greater than the net benefit of introducing storage, so that it would appear that consumers lose from the introduction of storage for all values of C shown, whereas in fact the opposite would be the case!

The differential gain to producers from storage under the additive disturbance is shown by Figure II to be approximately independent of the demand curvature parameter C, given the demand and supply elasticities. An analogous result can be derived analytically in the case of ideal stabilization. Wright [1979, pp. 1024–25] shows that the gains to producers from ideal stabilization of a small multiplicative disturbance are approximately

$$(14) \qquad \overline{Q}_M(P^s - P^r) = -\frac{\sigma^2\overline{Q}_M^3}{1 - P'\hat{x}'(P^r)}\left(\frac{P''}{2} + \frac{P'}{\overline{Q}_M}\right),$$

where $\overline{Q}_M$ is mean output in the unstabilized equilibrium with multiplicative disturbance and P' and P'' are the derivatives calculated at the equilibrium when production is not stochastic. When the disturbance is additive, the output of producers can be viewed as divided into two parts, one deterministic and responsive to incentives, with mean $\overline{Q}_A$, the other stochastic with mean zero, and zero supply response [Wright, 1979, p. 1029]. Ignoring terms higher than third order, one sees that the effect of stabilization corresponding to the latter part is just $-P'\sigma_A^2$, where σ_A^2 is the additive output variance. The other part of the effect of stabilization is the change in rents to stable production when the incentive moves from expected price $E(P)^A$ to the stable price P^s. The change is approximately $\overline{Q}_A(P^s - E(P)^A)$:

$$(15) \quad \overline{Q}_A[P^s - E(P)^A] = \overline{Q}_A\left[P'\hat{x}'(P^r)(P^s - E(P)^A) - \frac{P''}{2}\sigma_A^2\right] = -\frac{P''}{2}\frac{\sigma_A^2\overline{Q}_A}{(1 - P'\hat{x}'(P^r))}.$$

Since σ_A^2 is approximately equal to $\sigma^2\overline{Q}_M^2$, and $\overline{Q}_M$ is approximately equal to $\overline{Q}_A$, the difference between the sum of the two components of expected gains under additive disturbances and the expected gains under multiplicative disturbances is

$$(16) \quad \left\{-P'\sigma_A^2 - \frac{P''}{2}\frac{\sigma_A^2\overline{Q}_A}{1 - P'\hat{x}'(P^r)} + \frac{\sigma^2\overline{Q}_M^3}{1 - P'\hat{x}'(P^r)}\left[\frac{P''}{2} + \frac{P'}{\overline{Q}_M}\right]\right\} \approx P'\sigma^2\overline{Q}_M^3\left[-1 + \frac{1}{1 - P'x'(P^r)}\right].$$

The difference arises because the component of the gain containing

P' under additive disturbances does not elicit a supply response that moderates the gain, in contrast to the multiplicative case. (For example, when demand is linear, the atomistic, price-taking, and economically responsive producer perceives his profits to be a linear function of the disturbance when it is additive, but a concave function when the disturbance is multiplicative.) Dividing (16) by $-\frac{1}{2}P'\sigma^2\overline{Q}_M^2$, which is the expected net social benefit from ideal stabilization ignoring terms in σ^3 [Wright, 1979, p. 1027], we have

$$(17) \qquad -\frac{2P'\hat{x}'(P^r)}{1 - P'\hat{x}'(P^r)} = -\frac{2[\eta^S/\eta^D]}{1 - \eta^S/\eta^D}.$$

When $\eta^S = 1$ and $\eta^D = -0.2$, the above expression equals 1.67, about the size of the vertical gap between lines *FG* and *DE* in Figure II, which of course, refers to storage rather than ideal stabilization. Note also that dividing (14) by $-\frac{1}{2}P'\sigma^2\overline{Q}_M^2$, produces $(2 - C)(1 - P'\hat{X}'(P^r))^{-1}$. The apparent linearity of curves in Figure II is mirrored in analytical results for ideal stabilization.

The specification of producers' anticipations also has crucial implications. The results for $\eta^S = 1$ presented above were produced under the assumptions that the date of the innovation of storage, and the implications of storage, are fully anticipated, and that long-run supply responds fully in one year. It may well be that the possibility of storage becomes known after production has been committed for that year, or that producers take time to discern its economic consequences. The learning process is complicated because the augmented demand curve derived here is not valid until it is fully recognized, since the market demand curve includes the storage demand, which is itself dependent upon the producer response. Although the nature of the learning process and the conditions for its convergence on the rational expectations equilibrium with full knowledge are interesting questions, we shall not pursue them here.[13] Instead, we use just one example to show how sensitive the results are to the rather stringent expectational assumptions adopted above.

If the introduction of storage for period t is not announced until after production plans have been made for that period in period $t - 1$, and if the possibility of storage was previously unrecognized, planned production remains at the level with storage impossible. If we assume that storage and all its effects are fully recognized after its

13. For an interesting discussion of these issues in a simple production model (without storage), see DeCanio [1979]. He assumes that initially producers respond to an incentive formed by an adaptive expectations process, and studies the conditions under which this incentive converges on the rational expectation.

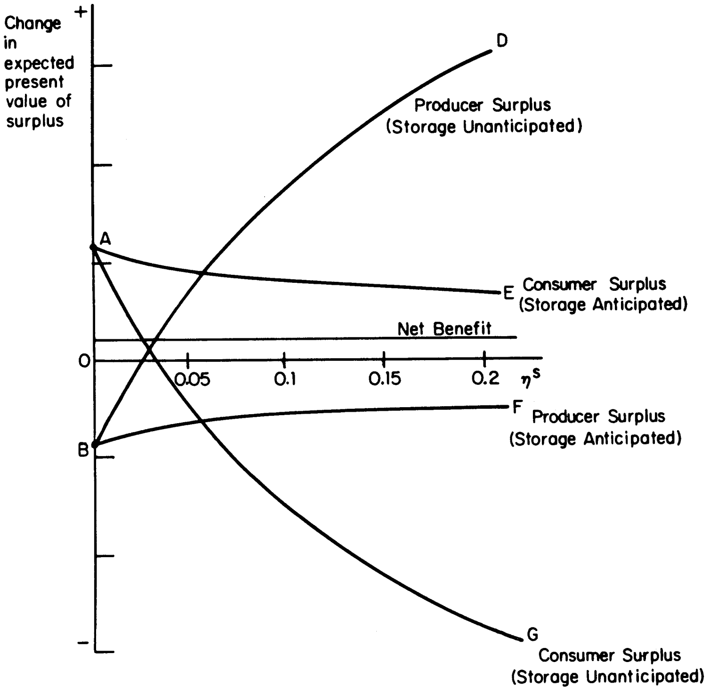

FIGURE III
Effect of Anticipations on the Distributional Effects of Introducing Storage (constant demand elasticity, $\eta^D = -0.2$)

announcement, the distributional impact is considerably different than when the possibility of storage was known in period $t - 1$. In Figure III lines AG and BD show the change in consumer and producer surplus, respectively, for various values of the supply elasticity η^S and constant elasticity of demand, $\eta^D = -0.2$. The signs of the distributional effects reverse as η^S increases, and unless supply is very inelastic, producers have a large gain from the unanticipated introduction of storage, because production cannot respond in the first year, even though storage increases market demand considerably.

Analytical studies of stabilization by simplified "stocking rules" that do not explicitly consider the implications of storage can lead to very different, and misleading, inferences. For example, Newbery and Stiglitz [1979] model the "immediate" impact of unanticipated storage as the analytical solution for the case where supply elasticity equals zero and the "long-run" impact as the case where supply elasticity has some positive value. Their principal conclusion [p. 816] is that "the

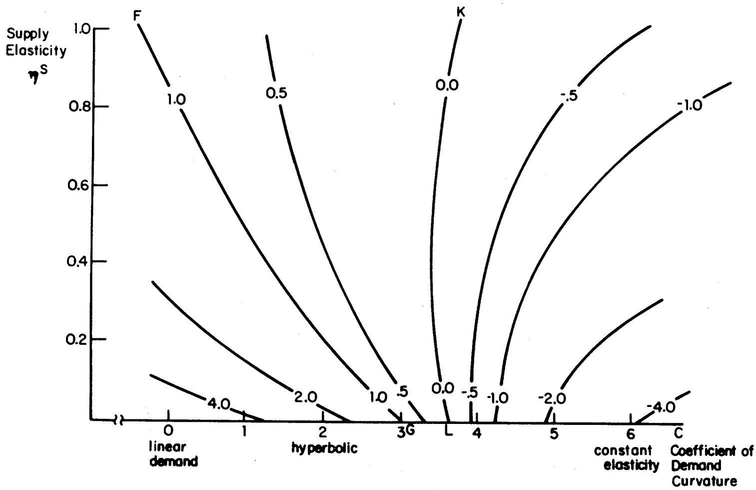

FIGURE IV
Expected Producer Gain as a Proportion of the Net Benefit of the Anticipated Introduction of Storage
($\eta^D = -0.2$)

full impact of price stabilization on identical producers is a simple positive fraction of the immediate impact (before supply adjusts), the fraction being $\epsilon/(\epsilon + \eta)$, where ϵ is the constant elasticity of demand, and η is the underlying supply elasticity." Although this conclusion implicitly refers to the dynamic question about the effects of introducing storage, it should properly relate to the comparative statics analysis of zero and positive long-run supply elasticities. In fact, as seen in Figure III, if supply does not adjust in the period when storage is introduced, both the sign and magnitude of the properly discounted long-run effect of introducing storage on producer welfare depend crucially on the long-run supply elasticity. On the other hand, if the introduction of storage is fully anticipated, the expected effect on producer surplus is always negative regardless of the long-run supply elasticity, as seen in curve *BF*. As was indicated in Figure II, the sign of the distributional effects is insensitive to the supply elasticity when *C* is large.

A more general view of the distributive effects of introducing storage is shown in Figure IV. On each of the contours, the ratio of the present value of the expected change in producer surplus to the expected present value of the net social benefit of introducing storage is constant at the fraction indicated. For parameter combinations to the left of curve *FG*, producers gain more than the expected net social benefit, at the expense of consumers. Between curves FG and KL, both producers and consumers gain from storage, while to the right of KL consumers gain more than the net social benefit, at the expense of producers.

Numerical results were also obtained for higher storage costs. Higher values of r or k result in a squeezing of the right-hand side of the map in Figure IV toward the left along with a displacement of the whole map to the left, reducing the range of C for which producers gain from storage, as well as the net benefits from introducing storage. Higher demand elasticities have similar effects.[14]

IV. Interpretation of the Results

The distributive consequences of the introduction of storage can be viewed as the net result of two effects. The first is the effect of the equilibrium change in the dispersion of consumption, and the second is the initial boost in demand arising from the need to build up a stock where none previously existed.

The first effect of introducing storage, an incomplete and asymmetric change in the dispersion of consumption, can be examined with reference to the effects of ideal stabilization, that is, complete and costless elimination of the production disturbance, which renders storage unnecessary. The effects of such complete stabilization are what is actually captured by studies in the tradition of Massell [1969]. Results derived in Wright [1979] indicate that producers and consumers both gain from ideal stabilization under combinations of parameters lying between the contours $F''G''$ and $K''L''$ in Figure V.[15] To obtain the effects of the change in the dispersion of consumption brought about by storage, we performed stochastic simulations of the same market with and without the capacity for storage. The contours $F'G'$ and $K'L'$ in Figure V, created by such a comparative statics exercise, abstract from the welfare effects of the initial building of stocks.[16] Comparison of $K'L'$ with $K''L''$ reveals that the maximum value of C for which the incomplete stabilization of consumption by storage favors producers is somewhat lower than under ideal stabili-

14. This was confirmed by generating a map similar to Figure IV for $\eta^D = -0.5$.

15. FG'' and KL'' in Figure V delineate the implications of $\eta^S/|\eta^D|$ for the welfare effects of ideal production stabilization for any demand elasticity. The effects of the less perfect stabilization achieved by competitive storage are sensitive not only to this ratio but also to the actual value of the elasticity of demand, as noted above.

16. For each set of parameter values we simulated 10,004 periods starting with $S = 0$ and discarded for first four. The same string of random numbers was used in each simulation.

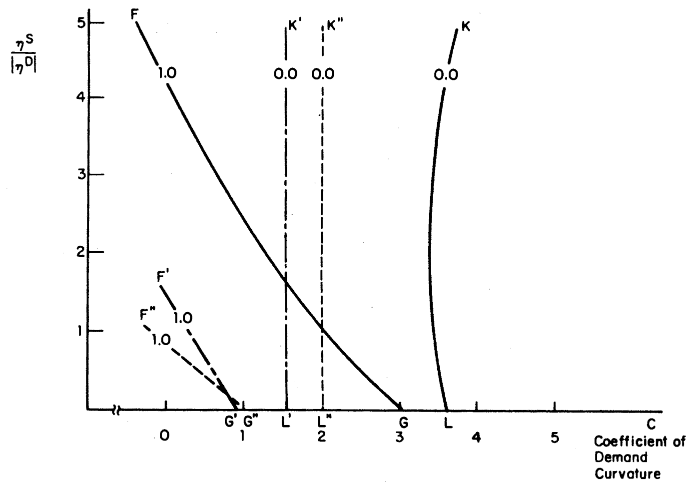

FIGURE V
A Comparison of Dynamic Results with Comparative Statics and Ideal Stabilization
($\eta^D = -0.2$)

zation. By a similar comparison of $F'G'$ and $F''G''$, it can be seen that at $C = 0$ (linear demand), under storage, producers have a higher share of the net benefit. (The net social benefit to be shared, however, is higher under ideal stabilization.) Results not shown here confirm that as the costs of storage fall, the contours for ideal stabilization represent the comparative statics effects of storage increasingly accurately.

The full dynamic welfare effects of introducing storage are shown by contours KL and FG, taken from Figure IV. The difference between these contours and the set $K'L'$ and $F'G'$ shows the importance of the boost in demand for initial stock-building. This second effect of the introduction of storage is highly favorable to producers at all combinations of parameters. For the cases shown here, it is much more important than the distinction between complete and partial stabilization of consumption. Yet in all analytical models of price stabilization, it has been ignored completely. Moreover, if supply is considerably more elastic than demand, both consumers and producers gain from the introduction of storage over much more of the relevant range of demand curvature than what is implied by a study of ideal stabilization or a comparative statics approach to storage.

V. CONCLUSION

In this paper we have shown the welfare effects of the introduction of rational competitive profit-maximizing storage in a market with stochastic supply. The major differences in welfare effects from those indicated in previous analyses of complete stabilization of consumption or ideal production stabilization are due to the initial buildup of stocks, which favors producers, and the fact that storage-induced stabilization is partial rather than complete, which tends to favor consumers. Our results also show the great importance of anticipations regarding the introduction of storage, and confirm previous analytical conclusions [Wright, 1979] regarding the importance of the demand specification and the elasticity of supply.

If long-run supply is completely inelastic, the numerical results presented here confirm the qualitative conclusion of previous studies that the net social benefit of introducing storage is dominated by its distributional effects. Even so, the net social benefit of introducing storage can be very significant relative to excess burdens of typical excise taxes. The distributional effects are strongly influenced by the parameter C, which reflects the curvature of the demand curve. Furthermore, under fixed long-run supply the perfectly anticipated introduction of storage can favor producers over a greater range of C than what is implied by studies of complete stabilization and related concepts that have been used as analytical analogues for storage. And when supply is elastic, the range over which producers and consumers both gain is greatly increased, and outside that range adverse effects are greatly moderated—an encouraging result for those interested in the political feasibility of measures to facilitate rational storage.

Because the topic of this paper is inherently dynamic, it is not surprising that the assumptions about expectations are crucial. If, for example, producers take time to recognize or react to the implications of storage, the distributional effects of its introduction can be reversed. In addition, factors that reduce the amount of storage, such as wastage or higher storage costs, as well as higher demand elasticities, tend to favor consumers at the expense of producers. Investigation of other issues, such as the implications of nonlinear storage costs, or extension to several production sectors, is beyond the scope of this study. Fortunately, the approach taken here, being rather flexible, is capable of handling these and other questions in the future.

APPENDIX 1: DERIVATION OF THE RELATIONSHIPS AMONG EQUILIBRIUM STORAGE, PLANNED PRODUCTION, AND CURRENT AVAILABILITY

The objective of our numerical algorithm is to choose, for any given availability I_t, levels of storage S_t and planned production $\hat{X}_{t+1}$

that would result from the actions of profit-maximizing storers and producers who have rational expectations about price in period $t + 1$.

The equilibrium relationship between storage S_t and availability I_t, for positive S_t, exists implicitly in the arbitrage condition:

$$\text{(a)} \qquad P(I_t - S_t) + k = (1 + r)^{-1}EP_{t+1}(S_t), \quad S_t > 0.$$

We do not directly approximate the storage rule; instead we numerically derive the function $EP_{t+1}(S_t)$, relating current storage to the expected price next year, and solve the arbitrage condition to obtain the S_t associated with any I_t. The derivation of $EP_{t+1}(S_t)$ in the computer program can be described as follows.

1. Choose a first guess $\psi(S_t)$ for $EP_{t+1}(S_t)$, where $\psi(S_t)$ is a fourth-order polynomial in S_t.

2. Choose an $N \times 1$ vector $\tilde{S}_t$ of discrete values S_t^i of S_t, $i = 1, \ldots, N$.

3. For each component S_t^i of $\tilde{S}_t$ choose a guess χ for the equilibrium value of planned production $\hat{X}_{t+1}^i$ associated with S_t^i.

4. Multiply χ by $(1 + v_j)$, for the M discrete values v_j, $j = 1, \ldots, M$, in the approximation to the probability density function $f(v)$.

5. Add the chosen storage S_t^i to each of the realized values of production generated in (4) to produce a vector $\tilde{I}_{t+1}^i$ of amounts available in the next period.

6. For each element I_{t+1}^{ij} of $\tilde{I}_{t+1}^i$ numerically solve the equation,

$$\text{(b)} \qquad P(I_{t+1}^{ij} - S_{t+1}^{ij}) + k = (1 + r)^{-1}\psi(S_{t+1}^{ij}),$$

for S_{t+1}^{ij}. If the solution is negative, set S_{t+1}^{ij} equal to zero.

7. For each pair $I_{t+1}^{ij}, S_{t+1}^{ij}$ calculate the associated market price $P(I_{t+1}^{ij} - S_{t+1}^{ij})$.

8. Calculate $P_{t+1}^r(S_t^i)$, the rational producer incentive, where

$$P_{t+1}^r(S_t^i) = \sum_{j=1}^{M} [1 + v_j]\chi f(v_j)P(I_{t+1}^{ij} - S_{t+1}^{ij})/\chi,$$

and the expected price,

$$E[P_{t+1}(S_t^i)] = \sum_{j=1}^{M} f(v_j)P(I_{t+1}^{ij} - S_{t+1}^{ij}).$$

9. Substitute $P_{t+1}^r(S_t^i)$ in the function for planned production:

$$\hat{X}_{t+1} = \hat{X}(P_{t+1}^r).$$

If $|\hat{X}_{t+1} - \chi| > \epsilon$, where ϵ is set at a small number, choose a new guess for χ and repeat steps (4)–(9).

10. If $|\hat{X}_{t+1} - \chi| < \epsilon$, we now have an approximately consistent set $S_t^i, \hat{X}_{t+1}^i, EP_{t+1}(S_t^i)$.

11. When this procedure has been repeated for each of the elements of $\tilde{S}_t$, the associated values of $EP_{t+1}(S_t^i)$, $i = 1, \ldots, N$, are fitted to a fourth-order polynomial $\psi^*(S)$, where the storage observations are the elements of $\tilde{S}_t$. In practice, the fourth-order polynomial used is, in the last iteration, accurate to the sixth significant figure; that

is, $R^2 \approx 1.000000$. If the fitted values of this polynomial differ by less than a certain chosen small amount from the values using the guess $\psi(S_t^i)$ from step (1), $\psi^*(S_t^i)$ is adopted as the equilibrium function $EP_{t+1}(S_t)$. If the convergence criteria are not satisfied, adopt $\psi^*(S)$ as the new guess for $\psi(S)$, and repeat steps (1)–(11).

The resulting stationary function $EP_{t+1}(S_t)$ is consistent with the profit-maximizing arbitrage condition (a). That is to say, the expected price resulting from storage is the one used in deciding how much to store; storers', as well as producers', expectations are internally consistent. Once $EP_{t+1}(S_t)$ has been derived, equilibrium storage S_t for any given value of the continuous variable I_t is found by numerical solution of the arbitrage condition (a). If the solution is negative, $S_t = 0$.

Appendix 2: Solving for the Expected Change in Future Producer Surplus

The effects of any one harvest on producer welfare can be thought of as having three components. The most obvious is the revenue from the harvest. But any carryover from the harvest will depress future revenue. Accordingly, the second component is the expected effect of this carryover on future producer surplus, represented by the function $R(S)$. The final component is the cost of production $G(\hat{x})$, which depends on planned production rather than realized production. Thus, producer surplus from one harvest is the sum of these three components discounted to the time of harvest, period $t + 1$. Its expected value, given the information available at planting time t is

$$E[\text{prod. surp.}] = E[x_{t+1}P(x_{t+1} + S_t - S_{t+1})] + \frac{E[R(S_{t+1})]}{1+r} - [1+r]G(\hat{x}_{t+1}).$$

Expected revenue and production costs are easily calculated. It is impossible, however, to derive an analytical expression for $R(S)$. Instead, $R(S)$ is obtained by the following method:

1. Choose a fifth-order polynomial $\rho(S)$ as a first guess for $R(S)$.
2. Choose an $N \times 1$ vector $\tilde{S}_t$ of discrete values S_t^i of $S_t, i = 1, \ldots, N$.
3. For each component S_t^i of $\tilde{S}_t$, find the associated equilibrium planned production $\hat{x}_{t+1}^i$ consistent with $EP_{t+1}(S_t^i)$, where $EP_{t+1}(S_t^i)$ is as derived in Appendix 1 above.
4. Generate realized values of output, $x_{t+1}^{ij} = (1 + v_j)\hat{x}_{t+1}^i$, $j = 1, \ldots, M$, where M is the number of discrete values in the approximation to $f(v)$.
5. Compute $I_{t+1}^{ij} = x_{t+1}^{ij} + S_t^i$.
6. Given I_{t+1}^{ij}, compute numerically S_{t+1}^{ij} from the arbitrage condition,

$$P(I_\tau^{ij} - S_\tau^{ij}) + k = (1+r)^{-1}EP_{\tau+1}(S_\tau^{ij}), \quad S > 0,$$

where $\tau = t + 1$.

7. Calculate $\gamma(S_t^i)$, where

$$\gamma(S_t^i) = \sum_{j=1}^{M} \{x_{t+1}^{ij} P(x_{t+1}^{ij} + S_t^{ij} - S_{t+1}^{ij}) f(v_j) + \rho(S_{t+1}^{ij}) f(v_j)[1 + r]^{-1}\} \\ -[1 + r]G(\hat{x}_{t+1}).$$

8. Fit the vector of values $[\gamma(S_t^i) - \gamma(0)]$ to a fifth-order polynomial $\rho^*(S)$. If the fitted values differ by less than a chosen small number from the values fitted by the previous guess $\rho(S)$, then $\rho^*(S)$ is adopted as the function $R(S)$. At equilibrium, in practice $R^2 \approx$ 1.000000. If convergence is not achieved, choose $\rho^*(S)$ as the new guess $\rho(S)$, and repeat steps (7)–(8).

This calculation of expected producer surplus using $R(S)$ can be duplicated by making a large set of long simulations of the storage model, each starting with the same initial store, and computing the mean present value of producer surplus. Although we have used this approach as a check on the routine, the same task is much more easily and inexpensively achieved by the method described here.

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