The Welfare Performance of Sequential Pricing Mechanisms

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August 6, 2004

Abstract

Consumers are commonly required to subscribe to particular tariff options before uncertainty regarding their future purchases gets resolved. Since the general comparison of welfare performance of different pricing mechanisms is ambiguous, this paper empirically evaluates the expected welfare associated to standard nonlinear pricing and optional tariffs by using information directly linked to the type of individual consumers. Results shows that tariffs composed of nonlinear options does not necessarily outperforms simpler pricing strategies in terms of expected profits. Furthermore, the evidence suggests that a menu of optional two–part tariffs dominates any other pricing strategy from an expected welfare perspective. JEL: D42, D82, L96.

Keywords: Optional Nonlinear Pricing; Quantity Discounts; Expected Profits and Welfare; Nonparametric Simulation.

[1] Thanks are due to Petra Todd (editor), two anonymous referees, and seminar participants at Universidad Autónoma de Barcelona, Carlos III, New York, Pennsylvania, Pompeu Fabra, Princeton, Rutgers, Stern School, Wisconsin–Madison, and Wissenschaftszentrum Berlin für Sozialforschung, as well as those attending the 1999 Workshop on Empirical Industrial Organization in Vigo, the 2000 Workshop on Applied Microeconomics and Econometrics at Johns Hopkins University, and the 2000 Winter Meeting of the Econometric Society in Boston. I am also grateful to José Pernías for speeding up my simulation routines. Partial funding from the Ameritech Foundation through the Consortium for Research on Telecommunications Strategy and Policy is gratefully acknowledged. A previous version of this paper circulated as CEPR Discussion Paper No. 2699: "Quantity Discounts for Taste–Varying Consumers." I am responsible for any errors that may still remain. Please address correspondence to: Eugenio J. Miravete, Department of Economics, University of Pennsylvania, McNeil Building / 3718 Locust Walk, Philadelphia, PA 19104-6297. E–mail: *miravete@ssc.upenn.edu*.

1 Introduction

Consumers have to choose frequently among sets of class of services in situations where they are uncertain about their future consumption needs. This is the case of subscription markets such as utilities, cable, or telecommunications. Consumers first sign up for one among a set of alternative tariff options and later decide how much to consume. For instance, telephone customers have to choose among different long distance plans offered by competing firms, or among different subscription contracts to the local telephone monopolist. In both cases, the subscription and consumption decisions are separated in time. Similarly, internet access providers also allow choosing among different connection plans depending on the expected usage of the network. Cable companies offer a variety of channel options for monthly subscription at different rates and bundling discounts. Car rental rates depend on the duration of the lease, mileage, and/or fuel option chosen. Public transportation systems offer the possibility of advance purchase of passes of varied duration at different discount rates depending on the expected usage of the system. Banks ask their customers to select one among few checking and savings accounts depending on their average expected balance and number of monthly checks drawn. Also, health clubs charge different monthly rates depending on registration fees related to the duration of the contract.

All these situations, typical of subscription markets, are characterized by a two–stage decision process. First, consumers decide which class of service they sign up for based on their expectation of future usage. Later, once their needs are known with certainty, they decide how much to buy from the firm, conditional on the rates of the tariff plan previously chosen. Although consumers might be initially motivated to signing up a contract to commit to a particular level of consumption, they may end up purchasing more or less than their predicted consumption.[2] Similarly, firms, either because of reputation, repeated interaction with consumers, or legal restrictions, cannot renege the contract and switch customers from one class of service to a different one, neither to take advantage of customers consumption decisions, or to favor them. Thus, firms can only profit from the stochastic dimension of consumers' demands through the design of the offered options.

Why cannot the above examples be properly addressed with the existing nonlinear pricing theory? The concept of self–selecting tariff has been incorrectly used as synonym of optional tariffs. This is particularly true in many works dealing with pricing of telecommunications services. A common mistake present in most of the related empirical literature is to neglect the existence of the two decision stages and assume that consumers make purchases and choose among class of services simultaneously.[3] If this were the case, the only relevant information for consumers to make that decision would be known at the time of consumption, and therefore the "choice" of the corresponding self–selecting tariff plan would be exactly dual to the usage decision.

This paper addresses two issues. First, it characterizes the design of fully nonlinear tariff options in a sequential screening environment. Traditional nonlinear pricing models need to be adapted to address the distinction between an *ex ante* and an *ex post* individual consumer types. Second,

[2] Within the present framework, tariff options do not serve any purpose as commitment device to target some level of future consumption. It is only the distinction between the subscription and the consumption decision what motivates the present model of sequential screening.

[3] See for instance Hobson and Spady (1988), Kling and van der Ploeg (1990), MacKie–Mason and Lawson (1993), and Mitchell and Vogelsang (1991, §8).

even in environments where consumers are uncertain about their future consumption, firms still offer tariff options that in most cases, ignore screening for the type components linked to the randomness of the realization of individual demand, *i.e.*, two–part tariffs. In order to address the rationale of this common practice, this paper evaluates empirically the expected welfare associated to standard nonlinear pricing and optional tariffs. In particular, I compare the relative performance of these alternative pricing mechanisms using direct observations of consumer types from the 1986 Kentucky telephone tariff experiment. This comparison shows that a tariff made of complex nonlinear options does not necessarily outperform two–part tariffs as measured by expected profits and welfare. The evidence suggests that evaluated *ex ante*, a menu of optional two–part tariffs dominates any other pricing strategy from a welfare perspective.

Rochet and Stole (2003, §8) document that, unlike many multidimensional screening models, sequential screening provides with a framework where some characterizations of equilibrium tariffs are still possible. Optimal tariffs were first analyzed by Clay, Sibley, and Srinagesh (1992). Miravete (1996) extended the framework to include a continuum of types when a single seller offered only two–part tariff options. This setup was later used by Miravete (2002) to empirically identify the magnitude of the different sources of asymmetric information in the demand for local telephone service using pooled data. Courty and Li (2000) analyzed a more general framework of sequential screening where the type of consumers, instead of the standard single–dimensional parameter is represented by a family of distributions. The present paper specifically addresses the design of *Optional Nonlinear Tariffs* (*ONLTs*) under the assumption that different type components lie on the real line, thus avoiding the problem of bunching, so common to multidimensional screening models, as well as facilitating its empirical evaluation.

In order to deal with the stochastic nature of consumer demand, the model of this paper assumes that consumers' types have two components: the *ex ante* type θ_1 and the type shock θ_2. Together they define the *ex post* type θ_0 that drives purchase decisions. The *ex ante* type is always known to consumers, and it determines the choice of the class of service. This type dimension is private information and could be linked to something similar to the average consumption level for each consumer (or expected valuation of the product). The type shock θ_2 represents departures from the expected consumption due to unpredictable events (or unexpected changes in valuation due to any general or individual circumstances). This type shock is different for each individual and remains private information. The monopolist designs each tariff option to maximize his expected profits given the information set of consumers at each stage. The realization of θ_1 critically conditions the choice among tariffs, while the value of θ_2 together with the tariff plan chosen determines the actual level of usage in the second stage of the game.

Given the absence of general results regarding the comparison of the welfare implications of the different tariffs, the empirical analysis of this paper intends to shed some light on the relative welfare performance of alternative pricing mechanisms dealing with sequential screening problems. The seller may find that the profit maximizing strategy is to implement a *pay–as–you–go* system and ignore the sequential structure of the problem. This is the case of the *Standard Nonlinear Tariff* (*SNLT*), where consumers face a single fully nonlinear tariff where payments depend on the actual purchasing decisions. In this case there is no distinction between subscription and consumption decisions. Alternatively, the seller may offer either a menu of *Optional Two–Part Tariffs* (*OTPTs*) or a more general menu of *Optional Nonlinear Tariffs* (*ONLTs*). While *OTPTs* are common, they fail to address part of the asymmetry of information associated to consumption. Consumers are not given any additional incentive to reveal their type shock as all the screening process takes place exclusively through the design of the linear tariff option. This is remedied with the use of the more complex *ONLTs*. Although *ONLTs* are more powerful mechanisms potentially leading to efficiency gains relative to *OTPTs*, they are rarely used in practice. In this paper I provide with a first characterization of the optimal *ONLT* and present empirical evidence that supports the idea that the spread use of simpler tariff options responds mostly to profitability considerations.

In this paper I am interested in evaluating the relative performance of the described pricing mechanisms *ex ante*, *i.e.*, when θ_1 is known to consumers but before θ_2 is realized. The comparison of mechanisms needs to be made *ex ante* because the monopolist has to decide today –before individual demands are realized– which options to offer to his customers in order to price consumption tomorrow. Two issues make the evaluation of optional tariffs a difficult task. First, contrary to the literature on the optimality of linear contracts, the stochastic elements of demand enter nonlinearly into the agents' objective functions instead of as additive shocks to their participation and individual rationality constraints.[4] Second, it is well known that the hazard rate of the distribution of types plays a key role in the characterization of the optimal nonlinear tariffs. But most importantly in the present case, the hazard rate of the distribution of *ex ante* and *ex post* types may differ significantly for θ_0 and its components θ_1 and θ_2. Thus, severe nonlinearities impede general results regarding *ex ante* evaluations of welfare. To overcome this lack of general results, I conduct an empirical evaluation that uses a unique data set where *ex ante* and *ex post* types of each consumer in the sample can be reasonably linked to the available individual information.

An area where optional tariffs are prevalent is telecommunications. I use data from the 1986 Kentucky local telephone tariff experiment to illustrate the empirical implications of the model and make policy evaluations using the suggested type–varying model. The interesting feature of this data set is that it includes information such as actual and consumers' reported expectations of weekly telephone use at the end of a historic period where individual local calls where not priced beyond the fixed monthly subscriber charge. Thus, price considerations are absent and the actual and expected number of calls can credibly be linked to θ_0 and θ_1 in the model.

This paper adopts a quite unique empirical approach. The empirical literature of asymmetric information models has attempted to recover the underlying distribution of the asymmetric information parameters from observed actions. In the empirical auction literature, the distribution of valuations is recovered from the observed bids making use of a structural model that characterizes the optimal bidding function. Still, results are many times contingent on the particular specification of the model and great effort has been made to identify nonparametrically as many elements of the model as possible. In the present paper, I do not need to rely on a particular family of distributions of consumer types because individual indicators linked to the different type components are directly available.[5] Thus, for instance, I can compute Anderson's (1996) nonparametric test of stochastic

[4] See for instance the works of Caillaud, Guesnerie, and Rey (1992) or Laffont and Tirole (1986).

[5] Empirical models as those of Ivaldi and Martimort (1994), Miravete (2002), Wolak (1996), as well as most of the empirical auction literature, identify the effects of asymmetric information through some structural restrictions and/or distribution assumptions. It is then difficult to acknowledge whether the estimates actually isolate the effect of asymmetry of information or those of the misspecification of the structural model.

dominance to provide with evidence in favor of the suggested type–varying model. Furthermore, in evaluating the ambiguous welfare results I simulate consumer surplus, profits, and welfare using kernel density estimates of consumer types, thus reducing the possibility of misspecification of the distribution of types. To my knowledge, this is the first attempt to evaluate models of asymmetric information using indicators directly linked to individual types.

My empirical strategy assumes a particular demand function that encompasses the common features of the demand for telecommunication services while using a general empirical distribution of types. In particular, the demand will allow for satiation, *i.e.*, a bounded consumption level at zero marginal charge. This demand formulation fulfills all standard regularity conditions necessary for a well behaved nonlinear pricing solution. Thus, theoretical results are still robust to functional form assumptions and valid for any demand function that fulfills the single–crossing property. Contrary to this specification of demand, I do not assume any particular distribution to deal with the asymmetric information parameter, and instead I use the empirical kernel distributions of the number of expected and actual calls as the general distributions of *ex ante* and *ex post* consumer types, respectively. For this flexible formulation I then compute the expected consumer surplus, profits, and welfare of the three suggested tariffs: *SNLT*, *OTPT*, and *ONLT* for two cities of Kentucky (Bowling Green and Louisville). A separate evaluation of these cities is interesting because the features of the estimated distributions of calls are quite different for each local exchange. These are genuine policy evaluations using structural elements such as the empirical distribution of consumer types. Results indicate that overall, a menu of two–part tariffs outperforms any other in terms of expected welfare.

The paper is organized as follows. Section 2 characterize *SNLT*, *OTPT*, and *ONLT* for a particular demand function with bounded maximum consumption (as in the case of telecommunications) but general distribution of types. I briefly describe the underlying assumptions of the model and study sufficient conditions for these tariffs to be characterized by quantity discounts. Section 3 discusses the ambiguous effect of general distributional orderings on the expected welfare ranking of the suggested pricing mechanism. Section 4 first presents nonparametric tests of stochastic dominance that support the fundamental assumptions of the type–varying model, and later uses the kernel estimates of the densities of θ_0, θ_1, and θ_2 to evaluate the different tariff solutions for a particular demand specification in two separate local exchanges in Kentucky. Section 5 concludes.

2 Standard and Optional Nonlinear Pricing

There are some potential profits from having consumers locked–in in a particular tariff option whenever the subscription and consumption decisions are separated in time. The distinction between subscription and consumption also allows us to differentiate the information that consumers have at each decision stage. While tariff choices are made conditional on their *ex ante* type θ_1, their purchase decision will be made conditional on their *ex post* type θ_0, once the demand randomness –the type shock θ_2– is realized.

This sequential framework opens several possibilities for sellers to engage in price discrimination beyond the standard pay–as–you–go system. I consider the case where a monopolist can either screen consumers by offering a menu of two–part tariff options, or alternatively, fully nonlinear options. In the first case, the monopolist ignores screening consumers with respect to θ_2, and he only attempts to extract rents associated to θ_1 through the design of two–part tariffs. Nonlinear tariff options lead to potential welfare gains because they also include incentives to reveal θ_2 at the consumption stage. However, these more complicated options are rarely used and the empirical application will show that in expectations, such complicated tariff options add little to expected profits or welfare.

In order to provide with the necessary analytical solutions, this section characterizes three alternative equilibrium tariffs: the continuum of self–selecting two–part tariffs that solves the standard nonlinear pricing problem, the optional two–part tariffs case, and the problem of designing fully nonlinear options. The goal of this section is to isolate sufficient constraints on demand and distribution of consumer's single–dimensional taste index so that screening of different types of consumers is achieved by means of a concave tariff. Concavity of tariffs ensure that by lowering the marginal charge that larger consumers face (quantity discounts), they are given enough incentives to avoid bunching, *i.e.*, that different consumers end up being treated in a similar manner.

In a sequential environment, the hazard rate properties of the distributions of type components condition the overall profit and welfare evaluation of the different nonlinear pricing strategies. The following subsections make explicit assumptions about utility functions and the stochastic structure of demand to solve two different, but analytically similar, pricing problems. I then point out that the increasing hazard rate property (*IHR*) of the corresponding distribution is key to ensure the existence of a separating equilibrium through a concave tariff. Still, I also show that as we move from *OTPT* to *ONLT*, further regularity conditions are needed for the lower envelope of *ONLT* to be concave. The last part of this section discusses the ambiguous outcome of comparing the welfare induced by the suggested tariff solutions.

2.1 Asymmetric Information Parameters

Assume that consumers' preference heterogeneity is captured by a single–dimensional index, θ_0. This taste indicator is private information for consumers while the monopolist only knows the distribution of such index, $F_0(\theta_0)$. The monopolist then designs a fully nonlinear tariff to maximize his expected profits given $F_0(\theta_0)$, extracting consumer surplus in varying proportions depending on consumers' purchase levels. Thus, consumers are given incentives to self–select into their purchase levels according to their preference intensity, θ_0.

This setup is appropriate for the standard nonlinear pricing problem because the choice of consumption is simultaneous to the dual choice of marginal tariff. However, in the case of sequential screening the stochastic structure of demand is richer because the information set of consumers differ at the time of subscribing the tariff option and when they decide on consumption. Thus, the *ex post* type θ_0 includes two components: the single–dimensional *ex ante* type θ_1, and the type shock θ_2, so that:

$$\theta_0 = \theta_1 + \theta_2 \tag{1}$$

Assumption 1: Types θ_i, $i=0,1,2$, have a differentiable probability density function $f_i(\theta_i)\geq 0$, $i=0,1,2$, on $\Theta_i=[\underline{\theta}_i,\overline{\theta}_i]\subseteq\mathbb{R}$, such that the cumulative distribution function given by:

$$F_i(\theta_i)=\int_{\underline{\theta}_i}^{\theta_i} f_i(z)dz \qquad ; \qquad i=0,1,2, \tag{2}$$

is absolutely continuous. Types remain private information for each consumer while their distribution is common knowledge.

In order to solve the three pricing problems, I also need to assume that $F_i(\theta_i)$ is *IHR* to ensure a separating equilibrium and avoid bunching of types at any given consumption or marginal tariff levels. This property characterizes most common distributions used in economics, and such assumption should not be considered restrictive.

Definition 1: If a univariate random variable θ_i has density $f_i(\theta_i)$ and distribution function $F_i(\theta_i)$, then the hazard rate of $F_i(\theta_i)$ is the ratio: $r_i(\theta_i)=f_i(\theta_i)/[1-F_i(\theta_i)]$ on $\{\theta_i\in\Theta_i:F_i(\theta_i)<1\}$. A univariate random variable θ_i or its cumulative distribution function $F_i(\theta_i)$ are said to be increasing hazard rate if $r_i'(\theta_i)>0$ on $\{\theta_i\in\Theta_i:F_i(\theta_i)<1\}$.

The distribution of θ_0 should be the result of combining the distributions of its components θ_1 and θ_2. This coherence condition will allow us to compare the solutions of the *OTPT* with the *SNLT*. In order to write $F_0(\theta_0)$ explicitly as the convolution of $F_1(\theta_1)$ and $F_2(\theta_2)$, we need to assume that type components are independently distributed.

Assumption 2: Type components θ_1 and θ_2 are independent random variables.

Definition 2: Let θ_1 and θ_2 be independent, univariate, random variables with cumulative distribution functions $F_i(\theta_i):\Theta_i\to[0,1]$, $i=1,2$. The cumulative distribution function of $\theta_0=\theta_1+\theta_2$ is then given by the Fourier convolution: [6]

$$F_0(\theta_0)=\int_{\Theta_2} F_1(\theta_0-\theta_2)f_2(\theta_2)d\theta_2. \tag{3}$$

Therefore, given any arbitrary, but well behaved, distribution function for the *ex ante* type θ_1 and the type shock θ_2, it is always possible to identify the distribution of *ex post* types θ_0 up to a linear transformation.[7]

2.2 Demand

The monopolist sells a single product x at a marginal tariff p. Consumers' income is taken as *numeraire*. In addition, and for simplicity, I assume that there are no income effects for consumers or

[6] Notice that the distribution of the aggregate type θ_0 is also defined on the real line instead of on a unit square. Keeping the definition of the aggregate type single–dimensional avoids problems of bunching and optimal exclusion in the lower bound of the support found by Armstrong (1996) in the case of multidimensional type spaces.

[7] It is not difficult to envision situations where large consumers also make more or less mistakes than small consumers. However, convolution (3) is only defined for independently distributed variables, *e.g.*, Karlin (1968, §1). Explicit expressions for the aggregation of correlated random variables rarely exists, and it is not possible in general to link the survival properties of the distribution of the aggregate to those of the components. The results of this paper should therefore be qualified for cases where type components could be correlated.

capacity constraints for the monopolist. For analytical convenience, let consumers choose a particular two–part tariff $\{\hat{A}(\theta_0), \hat{p}(\theta_0)\}$ uniquely characterized by a fixed fee $\hat{A}(\theta_0)$ and a marginal charge $\hat{p}(\theta_0)$. For the *SNLT* problem, this is equivalent to choosing $\hat{x}(\theta_0)$, but in the case of *OTPTs* the *ex ante* choice $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$ is not necessarily the cost minimizing choice for the *ex post* consumption level $\tilde{x}(\theta_1+\theta_2)$. Thus, the assumed indirect utility function net of fixed fee payment A is:

$$V(p, A, \theta) = v(p, \theta) - A = \frac{\theta}{\alpha} \exp[-\alpha p] - A \qquad ; \qquad \alpha > 0, \tag{4}$$

so that Roy's identity ensures that:

$$-V_p(p, \theta, A) = -v_p(p, \theta) = x(p, \theta) = \theta \exp[-\alpha p]. \tag{5}$$

This specification of demand has been used before in several telecommunications studies because it is bounded under the flat rate option, something that occurs in the present data. If $p = 0$, consumers make their satiation number of calls, $x(0, \theta) = \theta$. Similarly, when $p = 0$, the expected usage (number of calls) equals $E_2[\theta_0] = \theta_1$ after integrating out (1) with respect to $dF_2(\theta_2)$, since $E_2[\theta_2] = 0$. This assumption will allow us to identify the realized expectation bias of each consumer later in the empirical application.

In order to ensure the existence of a separating equilibrium, it is necessary that consumers' demands of different types do not cross each other so that consumers can be ranked by their preference intensity, θ_0. This is the well known *single–crossing property* (*SCP*). Observe that this is the case for the exponential demand function (5) because $-V_{pp}(\cdot) = -\theta \exp[-\alpha p] < 0$ and $-V_{p\theta}(\cdot) = -v_{p\theta}(\cdot) = x_\theta(\cdot) = \exp[-\alpha p] > 0$. Although for convenience the indirect utility function (4) leads to a bounded demand even at $p = 0$, the theoretical results of this section are robust to this functional form specification of demand as it fulfills the SCP requirement.

2.3 Standard Nonlinear Tariff

A monopolist with zero marginal cost maximizes his expected profits using the distribution of consumers' *ex post* types $F_0(\theta_0)$. In this standard problem, consumers implicitly reveal their type as they decide over consumption. Therefore, there is a one–to–one correspondence between the chosen amount $x(\theta_0)$ and the self–selecting two–part tariff $\tilde{\hat{T}}(\theta_0)\{\hat{A}(\theta_0), \hat{p}(\theta_0)\}$. But there is no real tariff choice as a subscription in advance to a particular optional contract. The time line of this game is:

- t_0: Nature reveals *ex post* valuations θ_0 to consumers.
- t_1: A monopolist offers a nonlinear tariff schedule defined as the lower envelope of a continuum of two–part tariff options $\hat{T}(\theta_0) = \{\hat{A}(\theta_0), \hat{p}(\theta_0)\}$ taking into account the distribution of *ex post* valuations, $F_0(\theta_0)$.
- t_2: Each consumer truthfully reveals her *ex post* valuation θ_0 and the monopolist assigns her a particular contract $\{\hat{A}(\theta_0), \hat{p}(\theta_0)\}$.
- t_3: Individual consumption and payments are realized.

$\Longrightarrow$ INSERT Figure 1: Standard Nonlinear Tariff $\Longleftarrow$

Figure 1 represents an example of *SNLT*. Under the assumed regularity conditions the optimal tariff is an increasing and concave function that can be implemented through a continuum of self–selecting two–part tariffs. The tariff offers quantity discounts, *i.e.*, it prices high valuation customers closer to marginal cost, thus enhancing welfare. Observe that as the *ex post* type θ_0 increases, payments of consumers move along the concave lower envelope of the self–selecting two–part tariffs. The optimal *SNLT* is the solution of the following optimal control problem:[8]

$$(6a) \qquad \max_{p(\theta_0)} \int_{\Theta_0} \Big[A(\theta_0) + p(\theta_0)\theta_0 \exp[-\alpha p(\theta_0)]\Big] dF_0(\theta_0),$$

$$(6b) \qquad \text{s.t.} \quad V(\theta_0) = \frac{\theta_0}{\alpha}\exp[-\alpha p(\theta_0)] - A(\theta_0),$$

$$(6c) \qquad V'(\theta_0) = \frac{1}{\alpha}\exp[-\alpha p(\theta_0)],$$

$$(6d) \qquad V(\underline{\theta}_0) = \frac{\underline{\theta}_0}{\alpha}\exp[-\alpha p(\underline{\theta}_0)] - A(\underline{\theta}_0) \geq 0,$$

where (6*c*) and (6*d*) represent the incentive compatibility (*IC*) and individual rationality (*IR*) constraints, respectively.[9] The *IR* constraint (6*d*) suffices to ensures that all consumer types participate in the market because in equilibrium, the optimal marginal tariff $\hat{p}(\theta_0)$ is a monotonic decreasing function of θ_0 while $v_{p\theta}(\cdot) < 0$ because of the *SCP*.[10] The solution of the *ex post SNLT* problem is a pair of functions $\{\hat{A}(\theta_0), \hat{p}(\theta_0)\}$ relating each optimal two–part tariff offered by the monopolist to consumers of different *ex post* types θ_0:

$$(7a) \quad \hat{p}(\theta_0) = -\frac{1}{r_0(\theta_0)}\left[\frac{v_{p\theta}(\hat{p}(\theta_0),\theta_0)}{v_{pp}(\hat{p}(\theta_0),\theta_0)}\right] = \frac{1}{\alpha\theta_0 r_0(\theta_0)},$$

$$(7b) \quad \hat{A}(\theta_0) = v(\hat{p}(\theta_0),\theta_0) - \int_{\underline{\theta}_0}^{\theta_0} v_\theta(\hat{p}(z),z)dz = \frac{\theta_0}{\alpha}\exp[-\{\theta_0 r_0(\theta_0)\}^{-1}] - \int_{\underline{\theta}_0}^{\theta_0} \frac{1}{\alpha}\exp[-\{zr(z)\}^{-1}]dz.$$

Equation (7*a*) presents the classical result that only the highest consumer type is efficiently priced since $r(\theta_0) \to \infty$ as $\theta_0 \to \overline{\theta}_0$. The magnitude of the price distortion –markup for each type θ_0– critically depends on the monopolist's knowledge of the population distribution of types. The hazard rate of this distribution captures the economic effect of informational asymmetries and plays an key role in defining the magnitude of the optimal markup for each *ex post* consumer type. As the following proposition shows, if $F_0(\theta_0)$ is *IHR* the *SNLT* is characterized by quantity discounts, a common feature among actual pricing strategies.[11] All proofs are presented in the Appendix.

Proposition 1: *The SNLT is concave if* $F_0(\theta_0)$ *is IHR.*

[8] See Appendix for a brief highlight of the solution of this problem.

[9] Incentive compatibility requires that each consumer maximizes her utility at the chosen consumption level. Since in equilibrium, $A(\theta_0)$, $p(\theta_0)$, and $x(\theta_0)$ are strictly monotonic and almost everywhere differentiable, *IC* can also be enforced locally –Tirole (1989, §3.5)–, that is:

$$\theta_0 \in \arg\max_{\theta_0'} \left\{\frac{\theta_0}{\alpha}\exp[-\alpha p(\theta_0')] - A(\theta_0')\right\}.$$

The *IC* constraint in (6*c*) is written after applying the envelope theorem to this consumer's first order maximization condition in the *ex post* choice of tariffs, and substituting it into $V'(\theta_0)$.

[10] Observe that, by assumption, all the market is served. This assumption simplifies the analysis and is also justified in the application of this paper because the Federal Communications Commission has pursued an active policy to achieve Universal Service in local telephony and thus over 90% of the residents in Louisville and Bowling Green had access to local telephony in 1986.

[11] Results regarding the concavity of the tariffs are robust to demand functions other than (5) as long as $v_{pp\theta}(\cdot) = v_{p\theta\theta}(\cdot) = 0$. Concavity of the tariff when these conditions do not hold require further constraints on the sign and magnitude of these third derivatives relative to how increasing is the hazard rate function $r_0(\theta_0)$.

2.4 Optional Two–Part Tariffs

For the *SNLT* problem, the purchase decision $\hat{x}(\theta_0)$ and the choice among self–selecting two–part tariffs $\{\hat{A}(\theta_0), \hat{p}(\theta_0)\}$ are simultaneous, and solving the *SNLT* could have been done as in Tirole (1989, §3.5), where the variational problem is stated in terms of $\hat{x}(\theta_0)$. This duality is lost when consumers first choose an optional tariff characterized by a fixed payment A and a particular marginal tariff p, and later, once θ_2 is realized, they decide on the purchase level x. The structure of the game can be summarized in the following time line:

- t_0: Nature reveals *ex ante* valuations θ_1 to consumers.
- t_1: A monopolist offers a continuum of optional two–part tariffs $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$ taking into account the distribution of *ex ante* valuations, $F_1(\theta_1)$ and integrating out the effect of the type shock, θ_2.
- t_2: Each consumer truthfully reveals her *ex ante* valuation θ_1 and the monopolist assigns her a particular contract $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$.
- t_3: Nature reveals type shocks θ_2 to consumers, thus defining *ex post* valuations $\theta_0 = \theta_1 + \theta_2$ for each consumer.
- t_4: Individual consumption and payments are realized according to the subscribed tariff option.

$\Longrightarrow$ INSERT Figure 2: Optional Two–Part Tariffs $\Longleftarrow$

Observe that at the time of choosing among tariffs, consumers are not fully aware of their preferences. They only know θ_1 and the distribution of θ_2. Although, consumers do not commit to a particular future purchase level, the choice of tariff plan is final, and neither the monopolist can take advantage by switching consumers to a different plan, nor the consumer can request such a change in the interim between the tariff subscription and the consumption decision.[12] In Figure 2, the mathematical lower envelope of the different tariff options (dotted line) does not represent the payments for different realizations of θ_0. Consumers first choose the optimal two–part tariff conditional on their *ex ante* valuation θ_1, and thus, different realizations of θ_2 will move consumers payments away from the mathematical lower envelope, and along the chosen two part tariff option.

In the case of *OTPTs*, given consumers' expectations on type shocks, they choose the tariff plan $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$ that maximizes their expected net rents. Integrating out the effect of θ_2, the *OTPT* is the solution of the following optimal control problem:[13]

$$(8a) \qquad \max_{p(\theta_1)} \int_{\Theta_1} \Big[A(\theta_1) + p(\theta_1)\theta_1 \exp[-\alpha p(\theta_1)] \Big] dF_1(\theta_1),$$

[12] Since renegotiation is not allowed and θ_1 and θ_2 are independent, revealing θ_1 to the monopolist when subscribing to a particular tariff option does not facilitate the *ex post* screening of consumers as in, for instance, Lewis and Sappington (1994).

[13] In this case the *IC* and *IR* conditions only hold in expectations, since consumers do not yet know their final valuation of the product. Thus, the objective function of the monopolist, as well as the individual constraints integrate out the effect of the unknown θ_2. As in the *SNLT* case, the *IC* condition can be enforced locally. Therefore:

$$\theta_1 \in \arg\max_{\theta_1'} \int_{\Theta_2} \Big\{ \frac{\theta_1 + \theta_2}{\alpha} \exp[-\alpha p(\theta_1')] - A(\theta_1') \Big\} dF_2(\theta_2).$$

Again, the *IC* constraint in (8*c*) is written after applying the envelope theorem to this consumer's first order condition in the *ex ante* choice of tariffs, and substituting it into $V'(\theta_1)$. Notice that expected *IC* and *IR* constraints might be violated *ex post*.

$$(8b)\qquad \text{s.t.}\quad V(\theta_1) = \frac{\theta_1}{\alpha}\exp[-\alpha p(\theta_1)] - A(\theta_1),$$

$$(8c)\qquad V'(\theta_1) = \frac{1}{\alpha}\exp[-\alpha p(\theta_1)],$$

$$(8d)\qquad V(\underline{\theta}_1) = \frac{\underline{\theta}_1}{\alpha}\exp[-\alpha p(\underline{\theta}_1)] - A(\underline{\theta}_1) \geq 0,$$

Observe that according to (6d) all consumers whose *ex post* valuation exceeded the minimum price asked by the monopolist participated in the market. For the optional tariff case, (8d) only requires that those whose expected valuation exceeds the minimum asked price participate in the market. Besides this difference affecting the participation decision, problems (6) and (8) share a formal similarity with the caveat that in the latter case, the uncertainty associated to the type shock is integrated out as consumers are required to subscribe to an optional tariff before they fully learn their *ex post* type.

Once the tariff option has been chosen, consumers learn their *ex post* type through the realization of an individual type shock. The value of θ_2 conditions whether consumers with *ex ante* type θ_1 purchase at all. Their consumption level is decided, conditional on the previously chosen tariff plan $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$. The optimal consumption decision maximizes their actual rent subject to the previous tariff choice, which leads to the following *ex post IC* and *IR* constraints for each *ex ante* type θ_1:

$$(9a)\qquad V'(\theta_1+\theta_2) = v_\theta(\tilde{p}(\theta_1), \theta_1+\theta_2) = \frac{1}{\alpha}\exp[-\alpha\tilde{p}(\theta_1)],$$

$$(9b)\qquad V(\theta_1+\underline{\theta}_2(\theta_1)) = \frac{\theta_1+\underline{\theta}_2(\theta_1)}{\alpha}\exp[-\alpha\tilde{p}(\underline{\theta}_1)] \geq 0,$$

where $\underline{\theta}_2(\theta_1)$ is the minimum type shock necessary for a consumer with *ex ante* type θ_1 who subscribed to the *OTPT* $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$ to purchase a positive amount of the product x. Each *ex ante* consumer type who chose a particular option $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$ faces a different *ex post* participation constraint. If the type shock is small enough, $\theta_2 \leq \underline{\theta}_2(\theta_1)$, consumers with *ex ante* type θ_1 do not buy anything and thus the monopolist only makes the already paid fixed fee $\tilde{A}(\theta_1)$ from such customers.[14] The monopolist maximizes his expected profits based exclusively on the distribution of *ex ante* types. After accounting for the above *ex ante IC* and *IR* constraints and integrating out the effect of θ_2, the *OTPT* solution becomes:

$$(10a)\qquad \tilde{p}(\theta_1) = -\frac{1}{r_1(\theta_1)}\left[\frac{E_2[v_{p\theta}(\tilde{p}(\theta_1), \theta_1+\theta_2)]}{E_2[v_{pp}(\tilde{p}(\theta_1), \theta_1+\theta_2)]}\right] = \frac{1}{\alpha\theta_1 r_1(\theta_1)},$$

$$\tilde{A}(\theta_1) = E_2\left[v(\tilde{p}(\theta_1), \theta_1+\theta_2) - \int_{\underline{\theta}_1}^{\theta_1} v_\theta(\tilde{p}(z), z+\theta_2)dz \;\middle|\; \theta_2 \geq \underline{\theta}_2(\theta_1)\right]$$

$$(10b)\qquad = \frac{\theta_1}{\alpha}\exp[-\{\theta_1 r_1(\theta_1)\}^{-1}] - \int_{\underline{\theta}_1}^{\theta_1} \frac{1}{\alpha}\exp[-\{z r_1(z)\}^{-1}]dz.$$

[14] The *ex ante*, type dependent, cut–off shock $\underline{\theta}_2(\theta_1)$ is uniquely defined in (9b) for each θ_1 due to continuity of all functions involved and monotonicity of the indirect utility function as $v_\theta(\cdot) > 0$. It is important to notice the distinction between the *ex ante IC* and *IR* that determine which plan consumers subscribe to and the *ex post IC* and *IR* that conditions the choice of consumption level and actual payments.

This solution resembles that of the *ex post* pricing very closely. With the exception of the *ex post* participation constraint, the menu of optional two–part tariffs does not screen consumers with respect to their *ex post* type, since θ_2 is also integrated out in the monopolist's objective function. Thus, the monopolist just screens consumers with respect to θ_1 by offering them a menu of optional two–part tariffs that accounts for consumer differences before θ_2 is realized. Regardless of their different individual type shock all *ex post* consumer types are treated similarly (they all face the same marginal charge) as long as they share the same *ex ante* type. The type shock only determines the amount that consumers purchase depending on the tariff option previously chosen. However, this approach still allows us to identify the existence of discounts for individuals with higher expected consumption levels.[15]

Proposition 2: *The lower envelope of the optimal OTPT is concave if $F_1(\theta_1)$ is IHR.*

Proposition 2 shows that there is a concave, lower envelope function underlying the optional tariffs. This is illustrated in Figure 2. This concave function $\tilde{T}(\theta_1) = \{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$ is the mathematical lower envelope of the menu of two–part tariffs that consumers have to choose from before they totally know their consumption needs. But this function is not the tariff lower envelope in the traditional sense and does not coincide with the lower envelope of Figure 1 unless the distribution of θ_2 is degenerate. For each *ex ante* type θ_1 and tariff choice $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$ there is a unique type shock $\theta_2 = \theta_2^\star(\theta_1)$ so that total payments equal those of the lower envelope $\tilde{T}(\theta_1)$. We know that $\theta_2^\star(\theta_1)$ is unique because the *SCP* requires that demand is increasing in the type, $x_\theta(\cdot) = \exp[-\alpha p] > 0$, and the marginal tariff $\tilde{p}(\theta_1)$ is given. Therefore, if consumers receive any other shock different from $\theta_2^\star(\theta_1)$ they will move along the tariff option chosen and will always pay more under the chosen tariff regime than if the had correctly anticipated their future consumption. If we now repeat the analysis for any other *ex ante* type who chooses a different tariff option, we could easily check that the shape of the actual *ex post* tariff depends on the choice of a particular tariff and the realization of demand. The actual payment function, which depends on combinations of θ_1 and θ_2, is not ensured to be concave unless we unrealistically restrict the behavior of θ_2.[16] However, since the distribution of θ_1 is *IHR*, *OTPTs* are such that they lead to quantity discounts in the sense of offering a lower marginal rate associated to higher fixed fees. Thus, consumers with larger expected consumption subscribe tariff options with higher fixed fee but lower marginal rate.[17]

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[15] In the regulation literature, Caillaud, Guesnerie, and Rey (1992) and Laffont and Tirole (1986) among others, prove that linear contracts in reported costs are robust to the existence of additive shocks in the cost functions of firms. In these models, firms' objective functions are linear in any cost noise that might exist. Thus, substituting its expected value, firms' *IC* and participation constraints are unchanged. The realization of the cost shock still affects total payments (as in the present model), but the lower envelope of the linear contracts remains unchanged. This is not the case for the model presented here. Uncertainty enters nonlinearly in consumers' objective function, thus affecting the *ex ante IC* and *IR* constraints $(8a)$–$(8b)$. Neither the tariff's lower envelope or the two–part tariff options are immune to the existence of uncertainty, but even if this is the case, the tariff can still be implemented by a menu of linear options represented by $(10a) - (10b)$.

[16] That was the case of Clay, Sibley, and Srinagesh (1992), where θ_1 is defined on a grid, and θ_2 is assumed to be small enough so that the *ex ante* and *ex post* ordering of consumers types are identical.

[17] Baron and Besanko (1999) address the equivalence of solutions when the type of an alliance θ_0 comprises the types of the alliance members θ_1 and θ_2 and where the type of the alliance is defined as in equation (1). This is not the case in the present model because of the sequential nature of the screening process, as well as for the fact that the unresolved uncertainty about θ_2 affects the *IC* and participation constraint of each consumer. As mentioned before, a direct mechanism $\{\hat{A}(\theta_1+\theta_2), \hat{p}(\theta_1+\theta_2\}$ is not equivalent to $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1\}$ unless the distribution of θ_2 becomes degenerate.

2.5 Fully Nonlinear Options

When tariff options consist only of two–part tariffs, the monopolist ignores screening consumers with respect to θ_2. In this subsection I solve the more complex problem of fully nonlinear tariff options designed to minimize the *ex post* informational rents of consumers who revealed their *ex ante* type θ_1 through the choice of a particular optional nonlinear tariff. I characterize *ONLT* starting from the *OTPT* solution of the previous section. Because of the independence assumption of the distribution of θ_1 and θ_2, screening consumers can be divided into two stages, the first of which is characterized by the optimal screening of consumers with respect to the *ex ante* type given by the *OTPT*. This section improves over Spulber (1992) in the sense that the monopolist is not restricted to offer only baseload contracts that are contingent upon the realization of the shock θ_2. Here for each θ_1, a new menu of contracts is offered and the seller effectively screens consumers sequentially with respect to θ_1 –through the choice of tariff plan– as well as θ_2, through the consumption decision.

Optimal *OTPTs* screen consumers with respect to their *ex ante* valuation θ_1. Consumer of *ex ante* type θ_1 subscribes to $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$ and for each θ_1 there is a single realization of the shock $\theta_2^\star(\theta_1)$ for which the subscribed tariff option $\{\tilde{A}(\theta_1), \tilde{p}(\theta_1)\}$ proves to be the least expensive one *ex post*. If a consumer of *ex ante* type θ_1 receives a type shock of exactly $\theta_2^\star(\theta_1)$ two things happens. First, her *ex ante* tariff choice is renegotiation proof (regardless of whether renegotiation is feasible or not). And second, no other incentives are necessary to induce such consumer to reveal her type shock $\theta_2^\star(\theta_1)$. Thus, this must be a point in common between the optimal *OTPT* and *ONLT* mechanisms.

Solving the *ONLT* consists of characterizing the additional optimal incentive that the seller has to provide, relative to *OTPT*, in order to extract as much informational rent from the type shock component of individuals as the realized shock θ_2 departs from $\theta_2^\star(\theta_1)$. If each nonlinear tariff option is concave they are implementable by a continuum of self–selecting two–part tariffs. The task of the monopolist is now to design the *optimal menu of menus of optional two–part tariffs* that best screens consumers sequentially. At stage 1, when consumers only know θ_1 they choose a nonlinear tariff option $\tilde{\tilde{T}}(. \mid \theta_1)$, *i.e.*, a particular continuum of *ex post*, self–selecting, two–part tariffs $\{\tilde{\tilde{A}}(. \mid \theta_1), \tilde{\tilde{p}}(. \mid \theta_1)\}$. Given consumers' private information θ_1, their expectation on type shocks, and their knowledge of the "shapes" of tariff options for each θ_1, they choose the tariff plan that maximizes their expected net rent. Later, once θ_2 is realized, the mechanism determines consumption and payments conditional on the previous choice of tariff. The new time line is:

- t_0: Nature reveals the *ex ante* valuation θ_1 to each consumer.
- t_1: A monopolist offers a continuum of optional nonlinear tariffs $\{\tilde{\tilde{A}}(.\mid\theta_1), \tilde{\tilde{p}}(.\mid\theta_1)\}$ taking into account the distribution of *ex ante* valuations, $F_1(\theta_1)$ and integrating out the effect of the type shock, θ_2.
- t_2: Each consumer truthfully reveals her *ex ante* valuation θ_1 and the monopolist assigns her a particular contract $\{\tilde{\tilde{A}}(.\mid\theta_1), \tilde{\tilde{p}}(.\mid\theta_1)\}$.
- t_3: Nature reveals the type shock θ_2 to each consumer, thus defining the *ex post* valuation $\theta_0 = \theta_1 + \theta_2$ for each consumer.

t_4: Each consumer truthfully reveals her *ex post* valuation θ_0 and the monopolist assigns her a particular contract $\{\tilde{\tilde{A}}(\theta_2 \,|\, \theta_1), \tilde{\tilde{p}}(\theta_2 \,|\, \theta_1)\}$ among the options of the previously subscribed tariff $\{\tilde{\tilde{A}}(.\,|\,\theta_1), \tilde{\tilde{p}}(.\,|\,\theta_1)\}$.

t_5: Individual consumption and payments are realized.

General characterizations of the menu of nonlinear tariffs are difficult and cumbersome. However, the fact that type components are statistically independent from each other proves to be very useful in obtaining the solution to the *ONLT* problem. Since the shock is independent of the *ex ante* type $\tilde{T}(\theta_1)$, the mathematical lower envelope of the *OTPT* still captures the optimal incentive mechanism to screen consumers with respect to their *ex ante* type dimension regardless of whether tariff options are two–part tariffs or more general nonlinear functions. Thus, $\tilde{\tilde{T}}(.\,|\,\theta_1)$ can be thought of being composed of two elements: one that screens consumers with respect to θ_1, whose optimal solution is represented by $(10a)-(10b)$; and another that induces self–selection of *ex post* types given the optimal tariff choices of each *ex ante* type θ_1, *i.e.*, truthful revelation of the type shock θ_2 through the consumption decision. This second component of the *ONLT* is aimed to reduce consumers' informational rents exclusively related to θ_2.[18]

$\Longrightarrow$ INSERT Figure 3: Optional Nonlinear Tariffs $\Longleftarrow$

Figure 3 illustrates this argument. The dotted line is the mathematical lower envelope $\tilde{T}(\theta_1)$, the result of screening consumers with respect to *ex ante* types θ_1 only. The straight line represent a particular two–part tariff option that implements such solution as characterized by $(10a) - (10b)$. This option is tangent to $\tilde{T}(\theta_1)$ when the realized type shock of a consumer with *ex ante* type θ_1 equals $\theta_2^\star(\theta_1)$. There is a continuous concave function representing a particular nonlinear tariff option that is also tangent to $\tilde{T}(\theta_1)$ at the same point. If the realized shock equals $\theta_2^\star(\theta_1)$, the chosen *OTPT* and *ONLT* are equally powerful in screening consumers. If the realized type shock diverges from this critical value, the *ONLT* provides further incentives over those of the *OTPT* for consumers to reveal their realized demand. As can easily be seen in Figure 3, the *OTPT* is one of the self–selecting two–part tariffs that implements the *ONLT*. Therefore, the characterization of the *ONLT* can be made by finding the optimal change in tariff –fixed fee and marginal charge– from each one *OTPT* for each *ex ante* type θ_1 and for different consumption levels induced by θ_2. In brief, let denote the rent of a consumer of type $\theta_0 = \theta_1 + \theta_2$ that subscribe to the *ONLT* $\{\tilde{\tilde{A}}(.\,|\,\theta_1), \tilde{\tilde{p}}(.\,|\,\theta_1)\}$ as:

$$(11) \qquad \tilde{\tilde{V}}(\theta_2 \,|\, \theta_1) = \frac{\theta_1 + \theta_2}{\alpha} \exp[-\alpha \tilde{\tilde{p}}(\theta_2 \,|\, \theta_1)] - \tilde{\tilde{A}}(\theta_2 \,|\, \theta_1),$$

And similarly, let define the difference between the fixed fee and marginal rate that a consumer of type $\theta_0 = \theta_1 + \theta_2$ faces when confronted to the *ONLT* and *OTPT*, respectively:

$$(12a) \qquad \Delta\tilde{\tilde{p}}(\theta_2 \,|\, \theta_1) = \tilde{\tilde{p}}(\theta_2 \,|\, \theta_1) - \tilde{p}(\theta_1),$$

$$(12b) \qquad \Delta\tilde{\tilde{A}}(\theta_2 \,|\, \theta_1) = \tilde{\tilde{A}}(\theta_2 \,|\, \theta_1) - \tilde{A}(\theta_1).$$

[18] Obviously, if θ_1 and θ_2 were not independent it would be impossible to separate the origin of the rent extraction as screening for θ_1 should also account for the related distribution of θ_2. In this latter case, a truly multidimensional screening approach as that studied by Rochet and Choné (1998) is needed. However, and contrary to the model of the present paper, these multidimensional screening models lead easily to non–monotonic solutions as well as exclusion at the bottom of the type support.

The fact that *ONLT* and *OTPT* are equally powerful in screening consumers with respect to θ_1 defines a new boundary condition for the screening problem with respect to θ_2: consumers will be indifferent between these two mechanisms if they choose the tariff plan that is *ex post* the least expensive one, *i.e.*, at $\theta_2^\star(\theta_1)$:

$$\Delta\tilde{\tilde{V}}(\theta_2^\star(\theta_1)\,|\,\theta_1) = \tilde{\tilde{V}}(\theta_2^\star(\theta_1)\,|\,\theta_1) - \tilde{V}(\theta_1) = 0, \tag{13}$$

where $\tilde{V}(\theta_1)$ is defined in (8*b*). This constraint is the major difference between *ONLT* and the *OTPT* or *SNLT* mechanism design problems. For any other value of θ_2, the *ONLT* problem solves how much *ex post* informational rent increase needs to be left to consumers. Evidently, this will depend on the properties of the distribution of the type shock alone. As before, integrating out the effect of θ_2, each optimal *ONLT* is the solution of the following optimal control problem:[19]

$$\max_{p(\theta_2|\theta_1)} \int_{\Theta_1} E_2\Big[A(\theta_2\,|\,\theta_1) + p(\theta_2\,|\,\theta_1)(\theta_1 + \theta_2^\star(\theta_1))\exp[-\alpha p(\theta_2\,|\,\theta_1)]\Big]dF_1(\theta_1), \tag{14a}$$

$$\text{s.t.}\quad V(\theta_1) = E_2\left[\frac{\theta_1+\theta_2}{\alpha}\exp[-\alpha p(\theta_2\,|\,\theta_1)] - A(\theta_2\,|\,\theta_1)\right], \tag{14.b}$$

$$V'(\theta_1) = E_2\left[\frac{1}{\alpha}\exp[-\alpha p(\theta_2\,|\,\theta_1)]\right], \tag{14c}$$

$$V(\theta_2^\star(\theta_1)\,|\,\theta_1) = \frac{\underline{\theta}_1 + \theta_2^\star(\theta_1)}{\alpha}\exp[-\alpha p(\theta_2^\star(\theta_1)\,|\,\theta_1)] - A(\theta_2^\star(\theta_1)\,|\,\theta_1) \geq 0, \tag{14d}$$

$$\Delta\tilde{\tilde{V}}(\theta_2^\star(\theta_1)\,|\,\theta_1) = 0, \tag{14e}$$

$$V'(\theta_2\,|\,\theta_1) = \frac{1}{\alpha}\exp[-\alpha\tilde{\tilde{p}}(\theta_2\,|\,\theta_1)], \tag{14f}$$

$$\tilde{\tilde{V}}(\underline{\theta}_2(\theta_1)\,|\,\theta_1) = \frac{\theta_1 + \underline{\theta}_2(\theta_1)}{\alpha}\exp[-\alpha\tilde{\tilde{p}}(\underline{\theta}_1)] \geq 0. \tag{14g}$$

Similarly to the *OTPT* problem, equations (14*c*)–(14*d*) represent the *ex ante IC* and *IR* constraints affecting consumers's choice of nonlinear tariff options, while (14*f*)–(14*g*) are *ex post* counterparts determining consumption and payments. Contrary to the *OTPT* problem, these constraints appear in the statement of the problem because the monopolist is now designing a tariff that provides additional incentives to reveal the type shock of consumers, once their individual uncertainty is realized.

Equation (13) defines a new boundary condition for the screening of consumers with respect to θ_2 so that for each θ_1, only a single two–part tariff from the menu that defines the particular nonlinear tariff option $\tilde{\tilde{T}} = \{\tilde{\tilde{A}}(.\mid\theta_1), \tilde{\tilde{p}}(.\mid\theta_1)\}$ coincides with the optional two–part tariff of the *OTPT* solved in Section 2.4. Given all these constraints the monopolist's problem solves, for each

[19] The *IC* and *IR* conditions affecting the subscription decisions hold in expectations. As in the *OTPT* case, the *IC* condition can be enforced locally. Therefore:

$$\theta_1 \in \arg\max_{\theta_1'} \int_{\Theta_2} \left\{\frac{\theta_1+\theta_2}{\alpha}\exp[-\alpha\tilde{\tilde{p}}(\theta_2\,|\,\theta_1')] - \tilde{\tilde{A}}(\theta_2\,|\,\theta_1')\right\}dF_2(\theta_2),$$

which leads to (14*c*), after applying the envelope theorem to each consumer's first order maximization condition in the *ex ante* choice of tariffs, and substituting it into $\tilde{\tilde{V}}'(\theta_2\,|\,\theta_1)$.

possible nonlinear option, the change in marginal rate and fixed fee that maximize the increase in revenues from the corresponding "boundary two–part tariff" option. By pointwise maximization of the constrained optimal control problem (14), the first order necessary conditions are:[20]

$$(15a)\qquad \Delta\tilde{\tilde{p}}(\theta_1+\theta_2)\alpha(\theta_1+\theta_2)\exp[-\alpha\tilde{\tilde{p}}(\theta_1+\theta_2)]f_2(\theta_2) - \lambda_2(\theta_2)\exp[-\alpha\tilde{\tilde{p}}(\theta_1+\theta_2)] = 0,$$

$$(15b)\qquad f_2(\theta_2) = \lambda_2'(\theta_2),$$

$$(15c)\qquad \lambda_2[\theta_2^*(\theta_1)] = 0,$$

where λ_2 is the Lagrange multiplier of the boundary constrain (13). Observe that this transversality condition does not bind at $\theta_2^\star(\theta_1)$ since $V(\theta_2^\star(\theta_1)) > 0$ whenever $\theta_2^\star(\theta_1) > \underline{\theta}_2$, which ensures a unique tangency of each *ONLT* to the corresponding *OTPT*. Therefore:

$$(16)\qquad \lambda_2(\theta_2) = \int_{\theta_2^\star(\theta_1)}^{\theta_2} f_2(z)dz = F_2(\theta_2) - F_2(\theta_2^\star(\theta_1)),$$

and thus, the optimal changes of the marginal tariff and fixed fee relative to the optimal two–part tariff option chosen by an *ex ante* type θ_1 are:

$$(17a)\qquad \Delta\tilde{\tilde{p}}(\theta_2\,|\,\theta_1) = \frac{F_2(\theta_2^\star(\theta_1)) - F_2(\theta_2)}{\alpha(\theta_1+\theta_2)f_2(\theta_2)} = \frac{1}{\alpha(\theta_1+\theta_2)}\left[\frac{1}{r_2(\theta_2)} - \frac{1-F_2(\theta_2^\star(\theta_1))}{f_2(\theta_2)}\right],$$

$$\Delta\tilde{\tilde{A}}(\theta_2\,|\,\theta_1) = \frac{\exp\left[-\{(\theta_1+\theta_2^\star(\theta_1))r_1(\theta_1)\}^{-1}\right]}{\alpha}\Bigg\{(\theta_1+\theta_2)\left(\exp\left[\frac{F_2(\theta_2^\star(\theta_1)) - F_2(\theta_2)}{(\theta_1+\theta_2)f_2(\theta_2)}\right] - 1\right)$$

$$(17b)\qquad - \int_{\theta_2^\star(\theta_1)}^{\theta_2}\left(\exp\left[\frac{F_2(\mu_2) - F_2(z)}{(\theta_1+z)f_2(z)}\right] - 1\right)dz\Bigg\},$$

These two equations in conjunction with $(10a)-(10b)$ characterize a menu of optional nonlinear tariffs $\tilde{\tilde{T}}(\theta_2\,|\,\theta_1) = \{\tilde{\tilde{A}}(\theta_2\,|\,\theta_1), \tilde{\tilde{p}}(\theta_2\,|\,\theta_1)\}$ for each value of θ_1. Observe that equation (17a) implies that consumers with *ex ante* type θ_1 face higher marginal charges than $\tilde{p}(\theta_1)$ if they receive a small shock $\theta_2 < \theta_2^\star(\theta_1)$, but on the contrary, marginal tariffs will be smaller than $\tilde{p}(\theta_1)$ if $\theta_2 > \theta_2^\star(\theta_1)$. This is also the case in Figure 3 where a single *OTPT* is a supporting hyperplane of both, one particular *ONLT* and the lower envelope of *OTPT*. The following proposition isolates *sufficient* conditions for *each* nonlinear option of the *ONLT* solution to be concave.

PROPOSITION 3: *If the lower envelope of OTPT is concave, for each nonlinear option of the ONLT solution to be concave it suffices that the following two conditions hold simultaneously:*

a) $r_2'(\theta_2) \geq f_2'(\theta_2)[1 - F_2(\theta_2^\star(\theta_1))]/[1 - F_2(\theta_2)]^2$,

b) $\theta_2 \leq \theta_2^\star(\theta_1)$.

These are sufficient, not necessary, conditions for *each* nonlinear tariff option to be concave. However, these conditions do not determine whether any of these *ONLTs* is more or less concave than the lower envelope of *OTPT*. Thus, it is possible that the two concave lines represented in Figure 3 cross each other because for the *ONLT* case, the optimal markup for each consumption

[20] See the Appendix and Kamien and Schwartz (1991, §II.7) for the derivation of these optimality conditions

level depends on how increasing are the hazard rate of the distribution of type components relative to each other.

Proposition 3 presents more complex conditions than those of Proposition 2 because we now take into account the effect of the type shock θ_2 in addition to the properties of the distribution of θ_1. Thus, for instance, the first condition requires not only that $F_2(\theta_2)$ is *IHR*, but also that such distribution be *sufficiently increasing*. This is only a restrictive condition for realizations of the type shock very close to the lower bound of the support of θ_2. If θ_2 is uniformly distributed, or if $f_2'(\theta_2) \leq 0$ for the whole support of the distribution, the first condition coincides with the *IHR* requirement.[21] As for the second condition, it requires that demand shocks are negative. If they are positive nonlinear options could still be concave, but the larger θ_2 is relative to $\theta_2^\star(\theta_1)$, the more increasing $r_2(\theta_2)$ should be to compensate such effect.

Therefore, the *IHR* property remains critical for the model to be well behaved, but it is no longer sufficient to ensure that each nonlinear tariff option leads to quantity discounts. If we just require that $r_2'(\theta_2) > 0$, we may find an asymmetric treatment of consumers with different θ_1: nonlinear tariff options chosen by high θ_1 are most likely concave, while on the contrary, low θ_1 choosing tariff options targeting low consumption levels would suffer important premia if they consume more than they expected.

3 Ambiguous Welfare Comparisons

The previous section has shown how to solve in isolation either *SNLP*, *OTPT*, and the more complex *ONLT*. In order to compare the relative profitability and welfare associated to different tariffs, I proceed by first comparing the markups of each one of them. Markups are inversely related to the hazard rate of the distribution of types, as this statistic enters all optimal nonlinear pricing solutions presented above. A comparison between *ex ante* and *ex post* pricing is possible because there is a well defined relationship between the magnitudes of the hazard rates of the random variables of (1) that define the convolution (3). Hazard rate dominance suffices to ensure first order stochastic dominance (*FOSD*). In general, if the tariff function is increasing, *FOSD* of θ_0 over θ_1 suffices to ensure that the monopolist will obtain higher profits using *ex ante* pricing. Second order stochastic dominance (*SOSD*) of θ_0 over θ_1 also lead to higher expected profits if the tariff is increasing and concave, which is certainly the case for *SNLT* and *OTPT*. Unfortunately, none of these general results hold for welfare comparisons, which motivates the empirical analysis of the remaining sections of this paper.

3.1 Preservation of *IHR* under Convolution

In order to study the relationship between the features of the *ex ante* optional and the *ex post* standard nonlinear tariffs, I will first show that the *IHR* property of the distributions of the

[21] This is the case of the exponential or Weibull distribution with shape parameter less than one. An interesting case is the beta distribution of the second kind defined on $[0, 1]$ with parameters $p = 1$ and $q > 0$. This distribution is *IHR* as long as $q > 0$, and the density function is always decreasing when $q > 1$. The hazard rate of this distribution varies from q to ∞. Thus, it is always possible to find a large enough value of q to ensure that the nonlinear tariff option is concave, even when $\theta_2 > \theta_2^\star(\theta_1)$.

components of the type $\{\theta_1, \theta_2\}$ is passed through to the distribution of the *ex post* type, θ_0. The following Proposition summarizes a key result for the proper characterization of the *ex ante* and *ex post* tariffs.

PROPOSITION 4: *If the distribution of the components of the type, $F_1(\theta_1)$ and $F_2(\theta_2)$ are both IHR, then the convolution distribution $F_0(\theta_0)$ is also IHR.*

A sufficient condition to compare the optimal solutions of the *ex ante* and *ex post* nonlinear pricing mechanisms is to require a particular hazard rate ordering of the involved distributions, such as in Laffont and Tirole (1993, §1.4). Since optimal nonlinear solutions critically depend on the value of the hazard rate of the corresponding distribution I have next to establish how large is the hazard rate of the convolution distribution $F_0(\theta_0)$ relative to those of the components of the *ex post* type, and thus determine whether a type–varying model may lead to a unambiguous ordering of hazard rates and markups of each pricing mechanism. Proposition 5 shows that for the present type–varying model, θ_0 dominates in hazard rate to θ_i if these distributions have a common support.

PROPOSITION 5: *Let $F_i(\theta_i)$ be IHR, i.e., $r_i'(\theta_i) > 0$ on $\{\theta_i \in \Theta_i \subset \mathbb{R} : F_i(\theta_i) < 1\}$, for $i = 1, 2$. Let $F_0(\theta_0)$ denote the convolution distribution of $\theta_0 = \theta_1 + \theta_2$, with hazard rate $r_0(\theta_0)$. Then $r_0(\theta) \leq \min\{r_1(\theta), r_2(\theta)\}$ on $\{\theta \in \Theta \subset \mathbb{R} : F_i(\theta) < 1; i = 0, 1, 2\}$.*

Proposition 5 implies that the distribution $F_0(\cdot)$ always puts more weight on higher type values than the distribution $F_1(\cdot)$. Therefore, given some value xi, the probability that $\theta_0 > \xi$ always exceeds the probability that $\theta_1 > \xi$. This intuitive result is formalized in the following proposition and corollary.

PROPOSITION 6: *If $r_0(\theta) \leq r_i(\theta)$ on $\{\theta \in \Theta \subset \mathbb{R} : F_i(\theta) < 1; i = 0, 1, 2\}$, then $F_0(\theta)$ first order stochastically dominates $F_i(\theta)$.* [22]

COROLLARY 1: *If $r_0(\theta) \leq r_i(\theta)$ on $\{\theta \in \mathbb{R}_+ : F_i(\theta) < 1; i = 0, 1, 2\}$, then $F_0(\theta)$ first order stochastically dominates $F_i(\theta)$.*

According to Laffont and Tirole's interpretation (1993, §1.4–1.5), Proposition 5 means that the distribution $F_0(\theta)$ is more favorable than the distribution $F_1(\theta)$. Maskin and Riley (1984, §4) also considered the effect of exogenous changes in the distribution of consumer types on the shape of nonlinear tariffs. Proposition 6 shows that this result could be endogenously obtained within the type–varying framework whenever the distribution of the aggregate type θ_0 and those of its type components share the same support. Within the more restricted framework of Corollary 1, this requires the existence of an independent, but systematically positive type shock to ensures that the actual purchase is always higher than the expected purchase.

3.2 Ex post Welfare Comparison

The monopolist has to make an strategic choice between the *ex post* and *ex ante* tariffs. There are several nonlinearities that turn the outcome of such comparison quite ambiguous unless we

[22] The converse is not true. Maskin and Riley (1984, §4) show that the hazard rate ordering is necessary to rank the profitability of screening mechanisms. They show that stochastic dominance alone does not lead to higher expected profits just because FOSD does not necessarily imply hazard rate dominance.

make specific demand and distribution assumptions. If we approach the problem from an *ex post* perspective, the monopolist should always prefer *SNLP* to *OTPT* options because in this latter case he would not be screening consumers with respect to θ_2. The same comparison between *SNLP* and *ONLT* is however not so straightforward because although in both cases all type components are used in the design of the tariff, in the latter case, the monopolist screens them sequentially rather than simultaneously. An unambiguous result can be stated when different *ex post* signals are distributed differently, the monopolist will always prefer the distribution with uniformly lower hazard rate. Proposition 7 states this well known result.

PROPOSITION 7: *Let the distributions $F_0(\theta)$ and $F_1(\theta)$ be IHR, i.e., such that $r_0'(\theta) > 0$ in θ on $\{\theta > 0 : F_0(\theta) < 1\}$, and $r_1'(\theta) > 0$ in θ on $\{\theta > 0 : F_1(\theta) < 1\}$. Assume also that $r_0(\theta) \leq r_1(\theta)$, $\forall\theta$. Then, the price mark–up and the marginal tariff will be uniformly higher under the $F_0(\theta)$ distribution than under the $F_1(\theta)$ distribution.*

Distribution $F_0(\theta)$ puts more weight on types close to the highest than distribution $F_1(\theta)$. Therefore, since $r_0(\theta) < r_1(\theta)$ the pricing mechanism based on the distribution $F_0(\theta)$ is more powerful than if $F_1(\theta)$ is used. It reduces the informational rent of inframarginal agents to avoid that the numerous agents close to the highest type $\overline{\theta}$ imitate the less concentrated inframarginal types, thus overall increasing the expected payoff of the monopolist.

3.3 Ex ante Welfare Comparison

The monopolist, as well as the regulator (if any approval is needed), have to evaluate the choice among alternative ways of screening consumers *ex ante* instead of *ex post*. This complicates such evaluation considerably because consumers' expectations affect the IC and IR constraints (they are not additively separable), and thus integrating out the effect of θ_2 still affects the shape of the *ex ante* tariff.

Information structures that lead to the hazard rate ordering provide with a unique case where different nonlinear tariffs can be sorted. Under the conditions of Proposition 6 the optimal *ex post* markup will always exceed the *ex ante* one. However, in the case of sequential screening, the existence of type shocks makes consumers to move along the tariff option chosen away from the *OTPT*'s lower envelope $\tilde{T}(\theta_1)$. Thus, *ex post* billing according to *OTPT* may well exceed those of *SNLP* for that same purchase level. But more frequently, comparison among informational structures will not lead to situations in which one distribution is more favorable than the other over the whole support of the distribution of types. Without strict hazard rate dominance, markups can be higher under one tariff only for a given range of consumption, making the comparison even more difficult.

Results regarding profits are conclusive if such hazard rate ordering of the distributions can be established . Optimal transfer functions $T(\theta)$ are necessarily increasing, $T'(\theta) = p(\theta) > 0$ as $x'(\theta) > 0$. Furthermore, if the problem is well behaved, tariff functions will be concave as shown in Propositions 1 and 2, $T''(\theta) = p'(\theta) < 0$. The monopolist generally expects an increase in profits by introducing optional pricing as the following proposition shows.

Proposition 8: *Expected profits are higher under ex ante pricing if any of the following conditions hold:*

(i) $T'(\cdot) > 0$ *and* $F_0(\cdot)$ *FOSD* $F_1(\cdot)$,

(ii) $T'(\cdot) > 0$, $T''(\cdot) < 0$, *and* $F_0(\cdot)$ *SOSD* $F_1(\cdot)$.

Therefore, more favorable distributions, *i.e.*, *FOSD* induced by hazard rate dominance as shown in Proposition 6, increase expected profits even for cases where the pricing problem does not fulfill all required conditions to discriminate among consumers by means of quantity discounts. But if these quantity discounts are optimal, then less restrictive stochastic orderings –*SOSD*– also lead to the same conclusion. The commonly observed practice of using optional nonlinear tariffs is therefore profit maximizing under very general conditions, which should suffice to explain its widespread use.

Unfortunately, such conclusive results cannot be made extensive to consumers' rents. The indirect utility function (4) is increasing in θ. But the effect on the net rent $v(p(\theta), \theta) - T(\theta)$ remains ambiguous as $T'(\theta) > 0$. If $v(\cdot)$ is more increasing than $T(\cdot)$, then part *(i)* of Proposition 8 could still hold, and consumers will prefer optional pricing to mandatory *ex post* pricing. However, it is also possible to encounter that type shocks are so biased that $F_0(\cdot) \leq F_1(\cdot)$ and consumers still may prefer *SNLT* to any optional tariffs. In this case, preferences just fail to be increasing enough in θ. A similar analysis could be made for the case of SOSD in order to apply part *(ii)* of Proposition 8. In addition to $v(\cdot)$ being more increasing than $T(\cdot)$, it would also be needed that $v(\cdot)$ is more concave than $T(\cdot)$. Thus, even more restrictive preferences are necessary to obtain a definite ordering of pricing strategies under increasingly less restrictive stochastic environments. Results will therefore depend on the particular demand and distribution functions assumed for each particular case study. This motivates the empirical analysis of the following sections of the paper where I will not assume any particular distribution of consumer types to evaluate welfare, but rather use the available empirical distributions of types from a telecommunications tariff experiment.

4 Empirical Evidence

This section has two objectives. First, I document that the theoretical prerequisites of the model are fulfilled in a particular case where actual data provides with a good explicit indicators for θ_0, θ_1, and θ_2. I study the reliability properties of the corresponding distributions that condition the optimal markup at different usage levels. A most remarkable feature of this analysis it that unlike many applied studies on empirical auctions, the source of asymmetric information in the application studied here is not identified through the specification of some distribution of unobserved characteristics, but rather using direct observations of consumers' taste parameters. Thus, the second objective is to use these indicators to obtain kernel estimates of their distributions and use them to evaluate welfare effects associated to each of pricing solution. The idea is to evaluate *ex ante* what is the relative performance of each screening mechanism. In pursuing this task, I will focus on the case of a continuum of tariff options. The goal of this paper is to evaluate the performance of sequential screening that makes use of either linear or fully nonlinear options with respect to the standard nonlinear pricing alternative.[23]

[23] I ignore here that in real life firms only offer a few tariff options to implement approximately the nonlinear solution. Miravete (2004) evaluates the foregone welfare and profits due to the use of only few tariff options using

4.1 Data

Data used in this paper come from the 1986 local telephone tariff experiment conducted by South Central Bell (*SCB*) on behalf of the Kentucky Public Service Commission. Data are described in detail in Miravete (2002, §2). What makes this data unique is that in addition to demographics and individual usage information, *SCB* also collected information related to customers' telephone usage expectations. *SCB* explicitly requested customers' own estimates of their weekly average number of calls. The use of survey data is often not considered a wise empirical strategy. However, these individual estimates are particularly useful because local calls were never priced before and consumers were not aware of the tariff experiment that was going to be held in the second half of the year. Thus, neither marginal tariffs or strategic considerations influence these estimates of customers' own satiation levels. Even if the formation of individual expectations may be subject to the effect of unobserved individual heterogeneity, this statistic is the best summary available of expected individual usage upon which households conditioned their tariff choice decisions.[24]. Furthermore, this information, available for most households of the sample, can be compared with the actual number of weekly phone calls for every month in the study, and to estimate the same empirical distribution of types that *SCB* could have constructed with this information in order to evaluate the profitability of introducing optional calling plans.

$\Longrightarrow$ INSERT TABLE 1: Descriptive Statistics $\Longleftarrow$

Table 1 presents the descriptive statistics of the sample observations. There is significant difference between usage and expected usage of local telephone service across the two local exchanges. While the number of calls is higher in Louisville than in Bowling Green, the expected consumption is much more accurate in the latter exchange. On average, Bowling Green residents underestimate telephone usage by 2% and Louisville residents underestimate their usage by 29%.

4.2 Are Data Consistent with the Type–Varying Model?

I focus on the spring months of 1986, where the present data provide us with an uncommonly available direct indicator for θ_1, the expected number of weekly calls, and also for θ_0, the actual number of weekly calls. I ignore the data from the fall months of 1986 because as many consumers face positive marginal charges, the choice of consumption and the marginal tariff are simultaneous, *e.g.*, MacKie–Mason and Lawson (1993, §3.2). This is not the case during the spring months because all local telephone customers were placed under a mandatory flat rate regime. Price was a relevant economic variable for the decision to subscribe the telephone service, but any additional call involves a zero marginal charge, and consequently local telephone customers should consume at their satiation levels.

$\Longrightarrow$ INSERT TABLE 2: Consumption Expectation Bias $\Longleftarrow$

a different data set for the cellular telephone industry. Evaluating the fully nonlinear case addresses the maximum welfare and profit gains from price discrimination. Few tariff options are needed to converge towards this upper bound. See Wilson (1993, §8.3).

[24] The econometric analysis of subscription decisions performed by Miravete (2003) using this same data confirms that choosing among tariff options critically depends on individual estimates of future usage.

There is important heterogeneity in the magnitude of the usage expectation bias across cities. The first column of Table 2 shows the average usage expectation bias, $\theta_2 = \theta_0 - \theta_1$, which while being positive for customers of these two local exchanges, it is about seventeen times larger in Louisville than in Bowling Green. A more detailed analysis by demographic strata shows further differences between residents of these two exchanges. While in Louisville the bias is always positive and large, independently of the demographic characteristic considered, in Bowling Green it is more balanced and in several occasions it takes negative values. In both cities consumers tend to underestimate their future usage, but in Louisville they do it by more than an order of magnitude. The smaller average bias in Louisville (single and male household) is still more than seven times larger than the average bias in Bowling Green. Figures 4.5–4.6 show the empirical density functions of type shocks. Although these expectation bias are quite disperse, small mistakes around the mean are the most frequent event.

All this appears to support the idea that consumers can be ordered differently before and after consumption is realized. While expected and actual number of calls are related, the average correlation is only 0.34 and thus, considering a second source of asymmetric information –the type shock– appears to be justified. In Table 2, the Pearson's analog goodness of fit test provides further evidence that distributions $F_0(\cdot)$ and $F_1(\cdot)$ cannot be considered identical. That hypothesis is always rejected and therefore we can conclude that the distribution of θ_2 is not degenerate, thus supporting the idea that the type varying model is an appropriate representation of consumers preferences.

The evidence supports the suggested type–varying model as *SOSD* cannot be rejected –see Table 3–, which according to Proposition 8 leads to higher expected profits under *OTPT* or *ONLT* than *SNLT*, a result that is confirmed for most cases in Table 4. The hypothesis of *FOSD* is much more restrictive than *SOSD* because it implies that consumers systematically underestimate their future consumption. Thus, if the distribution of expected calls first order stochastically dominates the distribution of actual calls or *vice versa*, we can easily conclude which of the two mechanisms is more profitable.[25] In both cities there is evidence (stronger in Louisville) in favor of a mean increasing spread of the distribution of θ_0 relative to that of θ_1. However, a systematic ordering of the means of θ_0 and θ_1 (through a positive average θ_2) is not sufficient to ensure the stochastic dominance of θ_0 over θ_1, since the whole distribution matters.

$\Longrightarrow$ INSERT FIGURE 4: Empirical Distributions $\Longleftarrow$

Figures 4.1–4.2 present the empirical frequency distributions of actual and expected weekly number of local calls for the spring months of the experiment in the local exchanges of Bowling Green and Louisville respectively. More informative is the empirical cumulative distribution functions shown in Figures 4.3–4.4, which clearly indicate that in both cities telephone customers tend to underestimate their future local telephone usage, which leads to the relative ordering of the averages of θ_0 and θ_1 discussed in Table 2. Figure 4.4 appears to indicate that θ_0 first order stochastically dominates θ_1 in Louisville, although Figure 4.3 fails to prove the same for Bowling Green. In order to test the hypotheses of *FOSD* and *SOSD*, I compute Anderson's (1996) nonparametric test of

[25] Notice that the markup for each usage level decreases with the hazard rate of the corresponding distribution as shown in equation ($7a$) for the *SNLT* and ($10a$) for the *OTPT*, respectively. If *FOSD* is not present, then the lower envelopes of these tariffs will cross each other, and not all consumers will be charged a larger markup under one of the two pricing alternatives.

stochastic dominance. The test is based on comparing weighted differences of frequency functions of two variables within given mutually exclusive fractiles. For each demographic strata, stochastic dominance of any order is rejected if one ratio is significantly positive for any single fractile. Table 3 reports for each demographic strata the maximum of these ratios among 20 fractiles in which the range of phone calls is divided (with approximately the same share of observations).

$\Longrightarrow$ INSERT TABLE 3: Tests of Stochastic Dominance $\Longleftarrow$

Results of Table 3 provides with strong evidence in favor of the suggested type–varying model, as *SOSD* of θ_0 over θ_1 is only rejected for two demographic categories in Bowling Green for very large consumption ranges (exceeding 90 calls per week).[26] *FOSD* of θ_0 over θ_1 is generally rejected in Bowling Green but never in Louisville. Therefore, expected profits are necessarily higher in Louisville under *OTPTs* than with a *SNLT*, while in Bowling Green such result is still ambiguous. Next section addresses this evaluation by means of simulations from using the empirical distributions of type components.

4.3 Welfare Simulations

I now evaluate the average expected consumer surplus, profits (revenues), and total welfare of screening local telephone customers through either a mandatory *ex post* pricing, *SNLT*, a continuum of optional two–part tariffs, *OTPT*, or a continuum of fully nonlinear options, *ONLT*. The empirical approach consists in first estimating the kernel distributions of θ_0, θ_1, and θ_2, and then evaluating each of the tariff solutions obtained in Section 2 for different realization of types from random draws generated by the estimated kernel distributions.[27] Table 4 presents the results of evaluating these tariffs and their associated welfare effects for the two Kentucky local exchanges where the tariff experiment was conducted.[28]

At this stage it may be worth discussing some empirical identification issues. Specific values of θ_0 and θ_1 are identified as the actual and expected number of calls during the spring months, when consumers faced a zero marginal charge. The existence of a positive charge per call could lead to a selection effect in Louisville where the flat rate was still an option later in the fall, and/or a suppression effect in Bowling Green (mandatory measured) and Louisville (optional measured) due to the negative slope of demand. During the fall months in which these tariffs applied, customers in Bowling Green made on average 134.33 local calls per month. This number identifies the number of calls of the *ex post* tariff in my base case for Bowling Green since it already includes the effect of a

[26] I furthermore checked that *SOSD* was never rejected for neither of the two cities in any single month, using 10 and 15 fractiles.

[27] I compute an adaptive Gaussian kernel with optimal bandwidth chosen to minimize the mean integrated square error of the estimation of the distributions of θ_0, θ_1, and θ_2 (actual or expected calls and estimation bias respectively) corresponding to each local exchange. The estimation procedure discretizes the ranges of θ_0, θ_1, and θ_2 around a 128 point grid to obtain the kernel estimation of each density by means of a fast Fourier transform. Estimation of $f_i(\cdot)$ and $F_i(\cdot)$ for intermediate values of θ_0, θ_1, or θ_2 is obtained by polynomial interpolation (with all 128 point estimates of the kernel) using Neville's algorithm. It should be noted that all estimated kernel distributions fulfill the *IHR* property. I computed Proschan and Pyke's (1967) nonparametric test of monotone failure rate. Test reject in all cases the hypothesis of constant hazard rate in favor of increasing hazard rate with p–values always below 0.01

[28] To compute *OTPT* we make use of the fact that the sample only includes active consumers, so that $F_2[\underline{\theta}_2(\theta_1)] = 0$ for all possible θ_1, and $E_2[\theta_2 \mid \theta_2 \geq \underline{\theta}_2(\theta_1)] = \theta_2^\star(\theta_1)$, which is straightforward to compute from the data because for the particular indirect utility function (4), $\theta_2^\star(\theta_1)$ equals the average of $\theta_0 - \theta_1$ for all values of θ_1. This is because equation (17a) ensures that each *ONLT* option is tangent only to one of the options of the *OTPT*.

positive marginal tariff. In Louisville this number is significantly higher as it averages the number of calls of 10% of the customers on optional measured service, 86.69, and the 189.28 monthly calls of the remaining 90% of customers on optional flat rate service in that exchange. The value of 179.02 is therefore used in the base case to identify the volume of demand under the *ex ante* pricing regime in Louisville.[29] Finally, the price elasticity of demand function (5) is given by $\varepsilon = -\alpha p$. The simulations are run for four alternative values of price elasticity (evaluated at the average p) as reported in four independent empirical studies of local telephone demand:[30] I choose $\varepsilon = -0.17$ as the base case common to the two cities. After comparing local tariffs and telephone usage patterns in the two local exchanges, I chose an average price per call of 7 cents per call as representative for the base case of the simulations.[31]

4.4 Results

Table 4 evaluates each particular nonlinear pricing solution and its associated welfare magnitudes: consumer surplus V, profits π, and total welfare W. All simulations in Table 4 are shown in 1986 dollars per month. Reported simulations are the average of 10,000 independent draws from the kernel estimation of the empirical distribution of types. I focus on the case where $\varepsilon = -0.17$. Thus, in Bowling Green, the optimal *ex post* tariff involves an average marginal rate of \$0.07, and an average monthly fee of \$44.07. Given the empirical distributions of types in that local exchange, consumers enjoy an average expected money surplus of \$11.25, the local monopolist expects to make \$44.92 in profits per customer, and total expected welfare amounts to \$56.17 per person.

$\Longrightarrow$ INSERT TABLE 5: Simulation Results $\Longleftarrow$

Average monthly fees are slightly higher under optional pricing than with *SNLT*, although almost no distinction is found between *OTPT* and *ONLT*. Marginal rates are 31% lower with *OTPT* than with *SNLT*, while under *ONLT* they rise 19%. These are however average magnitudes. Thus, the higher consumption under *ONLT* relative to *OTPT* could be explained by a likely reduction in the average marginal tariff under optional nonlinear tariff relative to *OTPT* as consumption increases for each chosen tariff. This increase in consumption explains the 19% increase in expected consumer surplus under optional nonlinear tariff due to a 5% expansion of demand relative to *ex post* pricing, as compared to the 1% expansion induced by *OTPT*.

Introduction of *OTPT* enhances welfare by about 2%, mostly due to a 4% increase in profits, because consumer surplus is reduced by 4% (of an initial smaller amount). *ONLT* reduce welfare

[29] In computing consumption for Louisville, as well as in estimating the representative price of a call in this exchange, I took into consideration that the sample is choice biased, with the proportion of users of optional measured service in the sample being three times that of the population.

[30] These studies are: Park, Wetzel, and Mitchell (1983), −0.1; Kling and Van Der Ploeg (1990), −0.17; Train, McFadden, and Ben–Akiva (1987), −0.45; and Hobson and Spady (1988), −0.7.

[31] The magnitude of the simulations reported in Table 4 depends on the assumed value for the average price of a call that may actually be priced in many other dimensions. It also depends on the assumed value of the price elasticity of demand. The assumed average cost of a call still remains representative of the actual situation in many local exchanges where metered calls vary from 5 to 10 cents. Subject to demand specification (5), simulations generate an average monthly fee that is close to current standards when $\varepsilon = -0.45$. Still, we do not know if this value for the elasticity of demand is appropriate for Louisville in 1986. The base case analyzed here is in between $\varepsilon = -0.45$ and $\varepsilon = 0.10$, which is commonly regarded as the most plausible one because of the quality of the data used for its estimation. Notice that simulated magnitudes in Table 4 do not intend to replicate those of any representative actual tariff, but rather to provide with relative measures of performance of the different welfare components using a roughly representative average cost of a call and the actual empirical distribution of types.

by 5%, but the distribution of its components is quite different from the *OTPT* case. The effect of the reduction of marginal rates for large consumers under *ONLT* dominates, and thus consumers benefit more from the introduction of *ONLT* than from the introduction of *OTPT*, although the latter one is the welfare maximizing pricing policy in expectation among the three analyzed here.

Finally, all magnitudes considered (with the exception of consumption) are inversely related to the absolute value of the elasticity of demand. Thus, the more inelastic is the demand, the higher is the average fixed fee as well as the average marginal tariffs. But also the average expected consumer surplus, profits and total welfare. The welfare analysis carried out for the reference scenario when $\varepsilon = -0.17$ is also valid for the others, so that the conclusion of optional two–part tariffs being the preferred pricing option appears to be robust to different values of the elasticity of demand.

For the case of Louisville, the reference case of *OTPT* is also characterized with an average marginal rate of \$0.07, but an average monthly fee of \$63.69.[32] Individual expected consumer surplus is \$10.03, expected profits per customer are \$24.56, and total expected welfare amounts to \$75.03 per person. The welfare analysis of the results of Louisville is very similar to that one of Bowling Green. There are two sources of differences between these two exchanges that affect the results of simulations. First, consumption patterns vary due to differences in demographics, socioeconomic variables, tariff options, and/or the size of the local network. The effect of all these variables have already been captured through the identification of exchange specific levels of telephone usage under different tariff regimes. The other source is the disparate behavior of type shocks in these two cities. Systematic underestimation of future consumption is the origin of the wider effects of welfare in Louisville relative to Bowling Green when comparing pricing alternatives. Thus, for instance, for the $\varepsilon = -0.17$ scenario, going from *SNLT* to *OTPT* reduces the expected consumer rents by 4% and increases expected profits by 4% in Bowling Green, while in Louisville the expected consumer surplus reduction is about 20% and the increase in expected profits reaches 7%. However, *OTPT* are again the welfare maximizing among the pricing strategies considered here.

Welfare increases in expectation when we implement optional two–part tariffs instead of *ex post* nonlinear pricing. The *SOSD* of θ_0 over θ_1 is the dominant factor driving this result. The *FOSD* of Louisville, with mean increasing effect on the usage level accounts for the stronger magnitude of the increase of expected profits (7% in Louisville *vs.* 4% in Bowling Green). Finally, the additional 4% increase in profits obtained when *ONLT* is offered instead of *OTPT* should be explained by the monopolist being able to discriminate consumers also with respect to θ_2 and not only θ_1. Expected profits increase with more sophisticated screening mechanisms that account for *ex post* differences.

5 Conclusions

Optional nonlinear pricing has not attracted much attention among economists until very recently. Traditionally, economists have incorrectly extended the application of results of the standard nonlinear pricing theory to situations where consumption and tariff choice were not simultaneous. The early treatment of Clay, Sibley, and Srinagesh (1992) studied the design of optimal two–part tariffs, but

[32] Observe that average marginal rates are normalized to \$0.07 both for the *OTPT* and *ONLT* cases. Since consumption (independent of ε) is also normalized across scenarios, the average marginal rate is always the same for these two alternative pricing strategies.

restricted their attention to discrete types. They also limited drastically the range of variation of θ_2 to ensure that the same *SCP* held both *ex ante* and *ex post*, so that the ordering of individual consumer preferences remained unaltered after the realization of the shock. Miravete (1996) extended this model to the case of a continuum of two–part tariff options with a continuum of types, independently of whether the ordering of consumer tastes changed or not after the realization of the shock. Miravete (2002) used a particular closed form solution of this model to analyze the estimation bias of not dealing with asymmetric information and self–selection issues in a cross–section framework. Finally, Courty and Li (2000) analyzed a general model of sequential screening where the uncertainty does not add up to the *ex ante* type but rather define one among a family of distributions of types.

Relative to all these works, the present paper contributes by characterizing a fully nonlinear tariff when consumers buy more than one unit, and by making explicit the role of the statistical assumptions on the existence of quantity discounts (*IHR* of the distribution of type components), and welfare effects (*FOSD* and *SOSD* of θ_0 over θ_1). This paper also compares different optimal nonlinear tariffs depending on whether they are designed *ex ante* or *ex post*, through the preservation of the *IHR* property of the distribution of type components through convolution. Finally, the paper also contributes to this literature by providing strong evidence in favor of the suggested type–varying model based on direct observation of consumer types. Using simulations from the kernel distributions of these types, the paper reports results that favor optional two–part tariffs as the expected welfare maximizing strategy in two local exchanges of Kentucky.

Appendix

• Derivation of the Ex Post Tariff

The corresponding Hamiltonian for the monopolist's ex post problem is:

$$(A.1) \qquad H[V,p,\theta] = [v(p(\theta),\theta) - V(\theta) - (p(\theta)-c)v_p(p,\theta) - K]\, f(\theta) + \lambda(\theta)v_\theta(p(\theta),\theta)$$

Using equation (5), the first order necessary conditions are:

$$(A.2) \qquad H_p : -(p(\theta)-c)v_{pp}(p,\theta)f(\theta) + \lambda(\theta)v_{p\theta}(p(\theta),\theta) = 0$$

$$(A.3) \qquad H_V : f(\theta) = \lambda'(\theta) \qquad ; \qquad \lambda(\overline{\theta}) = 0$$

There is not transversality condition at $\overline{\theta}$ since $V'(\overline{\theta}) > 0$ because the participation constraint is only binding at $\underline{\theta}$. Then:

$$(A.4) \qquad \lambda(\theta) = \int_{\overline{\theta}}^{\theta} f(z)dz = F(\theta) - 1$$

Equations $(7a) - (7b)$ follow from substituting this expression and the *SCP* into the first order necessary conditions H_p and H_V. The solution of the *ex ante* problem is similar to this one, although integrating out the effect of θ_2.

• Proof of Proposition 1

Differentiation of equation $(7a)$ with respect to θ_0 leads to:

$$(A.5) \qquad \hat{p}'(\theta_0) = -\frac{\alpha\,[r_0(\theta_0) + \theta_0 r_0'(\theta_0)]}{[\alpha\theta_0 r_0(\theta_0)]^2} \leq 0,$$

as long as $F_0(\theta_0)$ is *IHR*. This proves that the transfer function $\hat{T}(\theta_0)$ is concave in the *ex post* type θ_0. To prove the existence of a concave tariff I need to prove that:

$$(A.6) \qquad \hat{T}''[x(\theta_0)] = \hat{p}'[x(\theta_0)] = \frac{\hat{p}'(\theta_0)}{\hat{x}'(\theta_0)} \leq 0.$$

But:

$$(A.7) \qquad \hat{x}'(\theta_0) = \frac{\partial\{\theta_0 \exp[-\alpha\hat{p}(\theta_0)]\}}{\partial\theta_0} = [1 - \alpha\hat{p}'(\theta_0)]\exp[-\alpha\hat{p}(\theta_0)] > 0,$$

which is ensured by the *SCP* and the concavity of the transfer function as shown in $(A.5)$. ∎

• Proof of Proposition 2

Differentiation of equation $(10a)$ with respect to θ_1 leads to:

$$(A.8) \qquad \tilde{p}'(\theta_1) = -\frac{\alpha\,[r_1(\theta_1) + (\theta_1)r_1'(\theta_1)]}{[\alpha\theta_1 r_1(\theta_1)]^2} \leq 0,$$

which suffices to ensure the concavity of the lower envelope of *OTPTs* since, similar to Proposition 1, it is straightforward to prove that $\tilde{x}'(\theta_1+\theta_2)>0$ for all θ_2. ■

• Proof of Proposition 3

Provided that $\tilde{T}(\theta_1)$ is concave, in order to ensure that each nonlinear tariff $\tilde{\tilde{T}}(\theta_2\,|\,\theta_1)$ is concave, it only remains to analyze whether marginal tariffs $\tilde{\tilde{p}}(\theta_2\,|\,\theta_1)$ are decreasing in θ_2. Thus, for each particular nonlinear option $\{\tilde{\tilde{A}}(\theta_2\,|\,\theta_1),\tilde{\tilde{p}}(\theta_2\,|\,\theta_1)\}$ to be concave it is required that:

$$(A.9)\quad \frac{\partial\Delta\tilde{\tilde{p}}(\theta_2\,|\,\theta_1)}{\partial\theta_2}=-\frac{1}{\alpha(\theta_1+\theta_2)}\left[\frac{r_2'(\theta_2)}{r_2^2(\theta_2)}-\frac{[1-F_2(\mu)]f_2'(\theta_2)}{f_2^2(\theta_2)}\right]-\frac{1}{\alpha(\theta_1+\theta_2)^2}\left[\frac{1}{r_2(\theta_2)}-\frac{1-F_2(\mu)}{f_2(\theta_2)}\right]\leq 0.$$

The concavity of the nonlinear tariff option critically depends on the signs of the terms between brackets. The first term between brackets in equation $(A.5)$ is ensured to be positive only if $r_2'(\theta_2)>f_2'(\theta_2)[1-F_2(\theta_2^\star(\theta_1))]/[1-F_2(\theta_2)]^2$, while the second term between brackets in equation $(A.5)$ is negative only as long as the shock θ_2 does not exceed $\theta_2^\star(\theta_1)$. ■

• Proof of Proposition 4

First note that if any distribution function $F_i(\theta_i)$ is *IHR*, this is equivalent to the corresponding survival function $1-F_i(\theta_i)$ being log–concave:

$$(A.10)\qquad \frac{\partial^2\log[1-F_i(\theta_i)]}{\partial\theta_i^2}=\frac{\partial}{\partial\theta_i}\left[\frac{-f_i(\theta_i)}{1-F_i(\theta_i)}\right]\leq 0.$$

Second, note that by Definition 1, the survival function is twice continuously differentiable. Therefore, it is a Pólya Frequency function of order 2 (PF_2), *i.e.*, $\forall x_1<x_2\in X\subseteq\mathbb{R}$ and $\forall y_1<y_2\in Y\subseteq\mathbb{R}$:

$$(A.11)\qquad \begin{vmatrix}1-F_i(x_1-y_1) & 1-F_i(x_1-y_2)\\ 1-F_i(x_2-y_1) & 1-F_i(x_2-y_2)\end{vmatrix}\geq 0.$$

To realize the equivalence between $(A.10)$ and $(A.11)$, assume without loss of generality that $x_1<x_2$ and $0=y_1<y_2=\Delta$. Then, from the definition of PF_2 and making use of common properties of determinants, the following equivalent inequality holds:

$$(A.12)\quad \begin{vmatrix}1-F_i(x_1) & 1-F_i(x_1-\Delta)\\ 1-F_i(x_2) & 1-F_i(x_2-\Delta)\end{vmatrix}=\Delta\cdot\begin{vmatrix}\dfrac{1-F_i(x_1)-[1-F_i(x_1-\Delta)]}{\Delta} & 1-F_i(x_1-\Delta)\\ \dfrac{1-F_i(x_2)-[1-F_i(x_2-\Delta)]}{\Delta} & 1-F_i(x_2-\Delta)\end{vmatrix}\geq 0.$$

Since $\Delta>0$, we can take limits in the latter determinant to obtain:

$$(A.13)\qquad \lim_{\Delta\to 0}\begin{vmatrix}\dfrac{1-F_i(x_1)-[1-F_i(x_1-\Delta)]}{\Delta} & 1-F_i(x_1-\Delta)\\ \dfrac{1-F_i(x_2)-[1-F_i(x_2-\Delta)]}{\Delta} & 1-F_i(x_2-\Delta)\end{vmatrix}=\begin{vmatrix}-f_i(x_1) & 1-F_i(x_1)\\ -f_i(x_2) & 1-F_i(x_2)\end{vmatrix}\geq 0,$$

which leads to:

$$(A.14)\qquad \frac{f_i(x_1)}{1-F_i(x_1)}\leq\frac{f_i(x_2)}{1-F_i(x_2)},$$

i.e., $F_i(\cdot)$ is *IHR*. Thus, I have to prove that the survival function of the convolution distribution is log–concave, *i.e.*, for $x_1 < x_2$ and $y_1 < y_2$:

$$D = \begin{vmatrix} 1 - F_0(x_1 - y_1) & 1 - F_0(x_1 - y_2) \\ 1 - F_0(x_2 - y_1) & 1 - F_0(x_2 - y_2) \end{vmatrix} \geq 0. \tag{A.15}$$

Applying Definition 2 of the Fourier convolution to the survival function we get:

$$D = \begin{vmatrix} \int [1 - F_1(x_1 - z)] f_2(z - y_1) dz & \int [1 - F_1(x_1 - z)] f_2(z - y_2) dz \\ \\ \int [1 - F_1(x_2 - z)] f_2(z - y_1) dz & \int [1 - F_1(x_2 - z)] f_2(z - y_2) dz \end{vmatrix} \geq 0. \tag{A.16}$$

Using the commutative property of convolutions:

$$\int F_1(x - z) f_2(z - y) dz = \int f_1(x - z) F_2(z - y) dz, \tag{A.17}$$

equation $(A.16)$ becomes:

$$D = \begin{vmatrix} \int [1 - F_1(x_1 - z)] f_2(z - y_1) dz & \int f_1(x_1 - z)[1 - F_2(z - y_2)] dz \\ \\ \int [1 - F_1(x_2 - z)] f_2(z - y_1) dz & \int f_1(x_2 - z)[1 - F_2(z - y_2)] dz \end{vmatrix} \geq 0. \tag{A.18}$$

The final step involves the application of the *Basic Composition Formula* to convolutions as stated by Karlin (1968, §1.2):

$$D = \int\int_{z_1 < z_2} \begin{vmatrix} 1 - F_1(x_1 - z_1) & f_1(x_1 - z_2) \\ 1 - F_1(x_2 - z_1) & f_1(x_2 - z_2) \end{vmatrix} \cdot \begin{vmatrix} f_2(z_1 - y_1) & 1 - F_2(z_2 - y_1) \\ f_2(z_1 - y_2) & 1 - F_2(z_2 - y_2) \end{vmatrix} dz_1 dz_2 \geq 0. \tag{A.19}$$

Observe that for this expression to be positive and thus ensure that the distribution $F_0(\cdot)$ is *IHR*, each determinant has to be positive. Assuming without loss of generality that $0 = z_1 < z_2 = \Delta$, the first determinant is positive whenever:

$$[1 - F_1(x_1)] f_1(x_2 - \Delta) - [1 - F_1(x_2)] f_1(x_1 - \Delta) \geq 0, \tag{A.20}$$

which implies:

$$\frac{f_1(x_2 - \Delta)}{1 - F_1(x_2 - \Delta)} \cdot \frac{1 - F_1(x_2 - \Delta)}{1 - F_1(x_2)} \geq \frac{f_1(x_1 - \Delta)}{1 - F_1(x_1 - \Delta)} \cdot \frac{1 - F_1(x_1 - \Delta)}{1 - F_1(x_1)}. \tag{A.21}$$

But since $\Delta > 0$ and $x_1 < x_2$:

$$\frac{f_1(x_2 - \Delta)}{1 - F_1(x_2 - \Delta)} \geq \frac{f_1(x_1 - \Delta)}{1 - F_1(x_1 - \Delta)}, \tag{A.22}$$

which is just the hypothesis that $F_1(\cdot)$ is *IHR*. Similarly, comparing the other elements of inequality $(A.21)$, that is:

$$\frac{1 - F_1(x_2 - \Delta)}{1 - F_1(x_2)} \geq \frac{1 - F_1(x_1 - \Delta)}{1 - F_1(x_1)}, \tag{A.23}$$

which is equivalent to:

$$\begin{vmatrix} 1 - F_1(x_1) & 1 - F_1(x_1 - \Delta) \\ \\ 1 - F_1(x_2) & 1 - F_1(x_2 - \Delta) \end{vmatrix} \geq 0, \tag{A.24}$$

that is the condition for the survival function $1 - F_1(\cdot)$ to be log–concave, which we have proved to be equivalent to the assumption of $F_1(\cdot)$ being *IHR*. A similar argument proves that if $F_2(\cdot)$ is *IHR*, the second determinant in inequality $(A.19)$ is also positive. Thus, $F_0(\cdot)$ is *IHR*. ∎

• Proof of Proposition 5

By the definition of convolution, it follows that:

$$(A.25)\quad r_0(\theta) = \frac{\int_{\Theta_j} f_i(\theta - z) f_j(z) dz}{1 - \int_{\Theta_j} F_i(\theta - z) f_j(z) dz} = \frac{\int_{\Theta_j} f_i(\theta - z) f_j(z) dz}{\int_{\Theta_j} [1 - F_i(\theta - z)] f_j(z) dz}$$

$$= \frac{\int_{\Theta_j} r_i(\theta - z)[1 - F_i(\theta - z)] f_j(z) dz}{\int_{\Theta_j} [1 - F_i(\theta - z)] f_j(z) dz} \leq \frac{\int_{\Theta_j} r_i(\theta)[1 - F_i(\theta - z)] f_j(z) dz}{\int_{\Theta_j} [1 - F_i(\theta - z)] f_j(z) dz} = r_i(\theta),$$

because $r_i(\theta) \geq 0$ and $r_i'(\theta) \geq 0$, $\forall \theta \in \Theta$. ∎

• Proof of Proposition 6

Since $r_i(\theta_i) = -\partial \log[1 - F_i(\theta_i)]/\partial \theta_i$, solving the differential equation $r_i(\theta) = f_i(\theta)/[1 - F_i(\theta)]$ with initial condition $F_i(\underline{\theta}) = 0$ leads to the following inequality $\forall \theta \in \Theta$:

$$(A.26)\qquad 1 - F_0(\theta) = \exp\left[-\int_{\underline{\theta}}^{\theta} r_0(z) dz\right] \geq \exp\left[-\int_{\underline{\theta}}^{\theta} r_i(z) dz\right] = 1 - F_i(\theta),$$

and therefore $F_0(\theta) \leq F_i(\theta)$ $\forall \theta \in \Theta \subset \mathbb{R}$, which is the definition of first order stochastic dominance of θ_0 over θ_i. ∎

• Proof of Proposition 7

Since an IHR distribution of type and the SCP ensures that $\hat{x}'(\theta) \geq 0$ –see equation $(A.3)$ in the proof of Proposition 1–, it easily follows by pointwise differentiation that:

$$(A.27)\qquad \frac{\partial E_\theta\{[\hat{p}(\theta) - c] \cdot \hat{x}(\theta)\}}{\partial r(\theta)} = \frac{\partial}{\partial r(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{y} \left[\frac{\hat{x}(z)}{r(\theta)} \cdot \frac{v_{p\theta}(p(\theta), \theta)}{v_{pp}(p(\theta), \theta)}\right] dz dF(y) < 0. \qquad ∎$$

• Proof of Proposition 8

These results are a direct consequence of the classical conditions of Hadar and Russell (1969) to order outcomes under uncertainty. Under circumstances of part *(i)*, the difference of expected profits between *ex post* and *ex ante* tariffs is (integrating by parts):

$$(A.28)\qquad \int_{\Theta} T(x)[F_0(x) - F_1(x)] dx = -\int_{\Theta} T'(z)[F_0(z) - F_1(z)] dz \geq 0,$$

while for part *(ii)* the result is obtained integrating $(A.25)$ by parts again:

$$(A.29)\qquad \int_{\Theta} T''(z) \int_{\Theta} [F_0(y) - F_1(y)] dy dz - T'(z) \int_{\Theta} [F_0(y) - F_1(y)] dy \Big|_{z=\underline{\theta}}^{z=\overline{\theta}} \geq 0,$$

which completes the proof. ∎

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Table 1. Descriptive Statistics

		BOWLING GREEN	LOUISVILLE	TEST
CALLS, θ	Average actual number of weekly calls	32.0489	36.6112	-6.63
		(26.902)	(38.197)	
EXPCALLS, θ_1	Average expected number of weekly calls	31.4137	25.9329	8.02
		(36.123)	(30.827)	
BIAS, θ_2	EXPCALLS – CALLS	0.6352	10.6783	-12.64
		(37.179)	(39.966)	
log(INCOME)	Log of monthly income of the household	7.3097	7.0847	13.55
		(0.798)	(0.819)	
HHSIZE	Number of people who live in the household	2.7960	2.5381	9.02
		(1.266)	(1.493)	
TEENS	Number of teenagers (13–19 years)	0.3711	0.2309	10.31
		(0.713)	(0.619)	
AGE1	Head of the household is between 15 and 34 years old	0.0614	0.0625	-0.22
		(0.240)	(0.242)	
AGE2	Head of the household is between 35 and 54 years old	0.2524	0.2644	-1.34
		(0.434)	(0.441)	
AGE3	Head of the household is above 54 years old	0.6861	0.6730	1.37
		(0.464)	(0.469)	
COLLEGE	Head of the household is at least a college graduate	0.2803	0.2244	6.31
		(0.449)	(0.417)	
MARRIED	Head of the household is married	0.6926	0.5059	18.85
		(0.462)	(0.500)	
RETIRED	Head of the houseold is retired	0.1525	0.2550	-12.40
		(0.360)	(0.436)	
BLACK	Head of the household is black	0.0622	0.1168	-9.25
		(0.242)	(0.321)	
CHURCH	Telephone is used for charity and church purposes	0.2082	0.1692	4.88
		(0.406)	(0.375)	
BENEFITS	Household receives some federal or local benefits	0.2063	0.3152	-12.11
		(0.405)	(0.465)	
MOVED	Head of household moved in the past five years	0.4820	0.4074	7.34
		(0.500)	(0.491)	
ONLYMALE	Head of household is single and male	0.0452	0.1053	-10.99
		(0.208)	(0.307)	
MARCH	Dummy variable for March observations	0.3288	0.3325	-0.38
		(0.470)	(0.471)	
APRIL	Dummy variable for April observations	0.3318	0.3318	0.00
		(0.471)	(0.471)	
MAY	Dummy variable for May observations	0.3394	0.3357	0.38
		(0.474)	(0.472)	
Observations		5241	4349	

Mean and standard deviations (between parentheses) of demographics for the spring sample. The "TEST" column shows the test of differences of means for each variable in these two cities.

Table 2. Consumption Expectation Bias

BOWLING GREEN					LOUISVILLE			
Obs.	PAT	Avg.Bias	Std.Dev.	Strata	Avg.Bias	Std.Dev.	PAT	Obs.
5241	2652.59	0.6352	(37.179)	ALL	10.6783	(39.966)	2353.89	4249
1723	879.39	0.9765	(37.076)	MARCH	11.6001	(43.581)	758.78	1446
1739	903.94	0.6571	(37.014)	APRIL	10.5580	(39.119)	791.41	1443
1779	879.94	0.2834	(37.457)	MAY	9.8842	(36.946)	819.24	1460
1967	1029.82	2.9062	(39.662)	LOW INCOME	15.9668	(50.592)	917.78	1645
3274	1662.00	-0.7291	(35.541)	HIGH INCOME	7.4610	(31.388)	1484.04	2704
714	293.15	0.0920	(18.198)	HHSIZE=1	6.2131	(34.470)	597.57	1095
1774	1016.19	-1.1249	(30.470)	HHSIZE=2	6.4538	(27.637)	874.67	1502
1290	704.12	2.9518	(33.353)	HHSIZE=3	13.8281	(38.995)	426.18	776
980	562.48	-0.0021	(47.312)	HHSIZE=4	14.3265	(43.909)	336.77	582
483	281.00	3.0087	(59.734)	HHSIZE $\geq$ 5	27.6001	(71.748)	277.91	394
3798	1941.58	-0.3655	(29.838)	TEENS=0	7.5578	(35.786)	2060.40	3653
1029	611.62	0.9405	(54.873)	TEENS=1	23.4185	(47.131)	252.33	460
414	225.09	9.0571	(42.156)	TEENS $\geq$ 2	34.1479	(65.503)	164.79	236
322	217.03	-4.7589	(26.910)	AGE1=1	8.4026	(32.578)	205.51	272
1323	869.76	-2.7377	(42.171)	AGE2=1	9.0469	(38.949)	723.88	1150
3596	1677.65	2.3592	(35.866)	AGE3=1	11.5307	(40.955)	1514.95	2927
1469	828.09	-3.4543	(37.277)	COLLEGE=1	4.6580	(28.899)	524.11	976
3772	1878.68	2.2279	(37.024)	COLLEGE=0	12.4203	(42.480)	1908.92	3373
3630	1851.96	0.5463	(36.427)	MARRIED=1	10.6344	(32.603)	1243.15	2200
1611	835.40	0.8355	(38.830)	MARRIED=0	10.7232	(46.315)	1166.71	2149
799	338.42	1.3146	(28.672)	RETIRED=1	9.6512	(35.496)	561.92	1109
4442	2361.63	0.5130	(38.512)	RETIRED=0	11.0299	(41.384)	1844.82	3240
326	237.93	11.6811	(71.411)	BLACK=1	29.3614	(66.110)	454.15	508
4915	2488.20	-0.0974	(33.587)	BLACK=0	8.2073	(34.340)	1957.76	3841
1091	600.92	-1.8867	(45.088)	CHURCH=1	7.8696	(52.922)	329.06	736
4150	2107.23	1.2982	(34.779)	CHURCH=0	11.2505	(36.754)	2056.26	3613
1081	493.97	2.2926	(35.188)	BENEFITS=1	13.8292	(42.011)	726.25	1371
4160	2201.68	0.2046	(37.671)	BENEFITS=0	9.2277	(38.910)	1661.81	2978
2526	1334.84	0.0820	(40.646)	MOVED=1	10.7220	(39.305)	1100.09	1772
2715	1381.03	1.1500	(33.634)	MOVED=0	10.6482	(40.422)	1303.97	2577
237	145.27	-3.5797	(23.912)	ONLYMALE=1	4.6319	(27.237)	265.54	458
5004	2541.78	0.8349	(37.682)	ONLYMALE=0	11.3900	(41.151)	2127.43	3891

"PAT" column reports Pearson analog goodness of fit test for equality of the distribution of the expected and actual number of calls. This test is distributed as a $\chi^2(19)$, with 0.05 and 0.01" critical values at 30.14 and 36.19 respectively. All statistics have p–values lower than 0.01.

Table 3. Test of Stochastic Dominance

	BOWLING GREEN		LOUISVILLE	
Order:	*FOSD*	*SOSD*	*FOSD*	*SOSD*
ALL	2.72	0.51	-5.65	-8.44
MARCH	1.03	-0.26	-3.27	-4.37
APRIL	1.52	0.25	-3.56	-4.74
MAY	2.16	0.91	-2.94	-5.15
LOW INCOME	-0.08	-1.94	-6.15	-6.15
HIGH INCOME	3.91	2.06	-1.92	-4.77
HHSIZE=1	2.64	0.51	-0.65	-2.73
HHSIZE=2	5.66	4.09	-0.79	-3.37
HHSIZE=3	0.27	-0.93	-3.28	-3.70
HHSIZE=4	0.79	0.00	-1.63	-2.69
HHSIZE $\geq$ 5	0.00	0.00	-2.55	-2.55
TEENS=0	3.59	2.14	-1.75	-5.77
TEENS=1	1.27	-0.41	-2.12	-2.12
TEENS $\geq$ 2	-0.18	-0.45	-0.58	-0.58
AGE1=1	3.74	2.68	2.73	1.92
AGE2=1	3.64	2.70	-1.61	-1.83
AGE3=1	0.65	-1.44	-5.34	-8.58
COLLEGE=1	4.25	3.59	0.06	-1.63
COLLEGE=0	0.69	-1.48	-5.89	-8.60
MARRIED=1	2.46	0.59	-3.46	-4.90
MARRIED=0	2.16	0.03	-4.51	-6.69
RETIRED=1	1.73	0.57	-1.65	-4.28
RETIRED=0	2.99	0.38	-5.43	-5.94
BLACK=1	-2.16	-2.16	-3.72	-3.72
BLACK=0	4.27	2.31	-3.00	-6.49
CHURCH=1	2.01	1.23	0.09	-0.85
CHURCH=0	3.40	-0.08	-6.57	-7.41
BENEFITS=1	1.47	-0.01	-4.60	-6.68
BENEFITS=0	3.27	0.57	-3.81	-5.48
MOVED=1	3.61	-0.22	-2.59	-2.72
MOVED=0	4.10	1.86	-4.00	-6.94
ONLYMALE=1	3.10	2.39	0.66	-1.48
ONLYMALE=0	2.66	0.36	-5.28	-8.51

Maximum ratios by demographics of Anderson's (1996) test for a uniform 20–fractile division of the calling range. These ratios are distributed as a studientized maximum modulus distribution, Stoline and Ury (1979). With 20 multiple comparisons and infinite degrees of freedom the 5% and 1% one–tail critical values are 3.03 and 3.49 respectively.

Table 4. Simulation Results

BOWLING GREEN					
Tariff		$\varepsilon = -0.10$	$\varepsilon = -0.17$	$\varepsilon = -0.45$	$\varepsilon = -0.70$
SNLT	A	74.911	44.065	16.647	10.702
	p	0.119	0.070	0.026	0.017
	x	134.340	134.340	134.340	134.340
	V	19.127	11.251	4.250	2.732
	π	76.357	44.916	16.968	10.908
	W	95.484	56.167	21.219	13.641
OTPT	A	77.047	45.322	17.122	11.007
	p	0.079	0.046	0.018	0.011
	x	136.312	136.312	136.312	136.312
	V	18.371	10.807	4.083	2.625
	π	79.389	46.699	17.642	11.341
	W	97.760	57.506	21.724	13.966
ONLT	A	77.134	45.373	17.141	11.019
	p	0.141	0.083	0.031	0.020
	x	141.431	141.431	141.431	141.431
	V	21.868	12.863	4.859	3.124
	π	68.936	40.551	15.319	9.848
	W	90.803	53.414	20.179	12.972
LOUISVILLE					
Tariff		$\varepsilon = -0.10$	$\varepsilon = -0.17$	$\varepsilon = -0.45$	$\varepsilon = -0.70$
SNLT	A	100.530	59.135	22.340	14.361
	p	0.195	0.115	0.043	0.028
	x	174.076	174.076	174.076	174.076
	V	21.323	12.543	4.739	3.046
	π	103.002	60.590	22.889	14.715
	W	124.326	73.133	27.628	17.761
OTPT	A	108.266	63.686	24.059	15.467
	p	0.119	0.070	0.026	0.017
	x	179.017	179.017	179.017	179.017
	V	17.046	10.027	3.788	2.435
	π	110.498	64.999	24.555	15.785
	W	127.543	75.026	28.343	18.221
ONLT	A	115.539	67.964	25.675	16.506
	p	0.119	0.070	0.026	0.017
	x	179.017	179.017	179.017	179.017
	V	9.773	5.749	2.172	1.396
	π	114.385	67.286	25.419	16.341
	W	124.158	73.034	27.591	17.737

Average value of 10,000 random draws from Gaussian kernel estimates of the corresponding probability density functions.

Figure 1: Standard Nonlinear Tariff

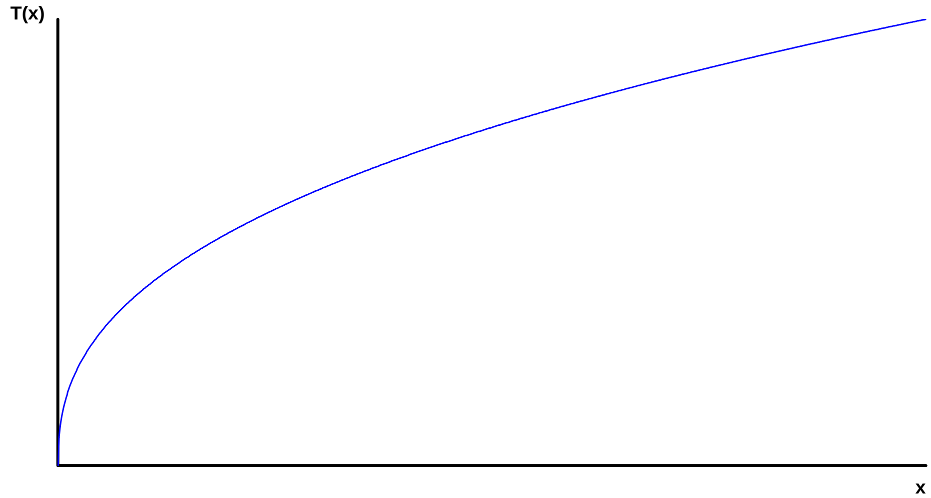

Figure 2: Optional Two-Part Tariffs

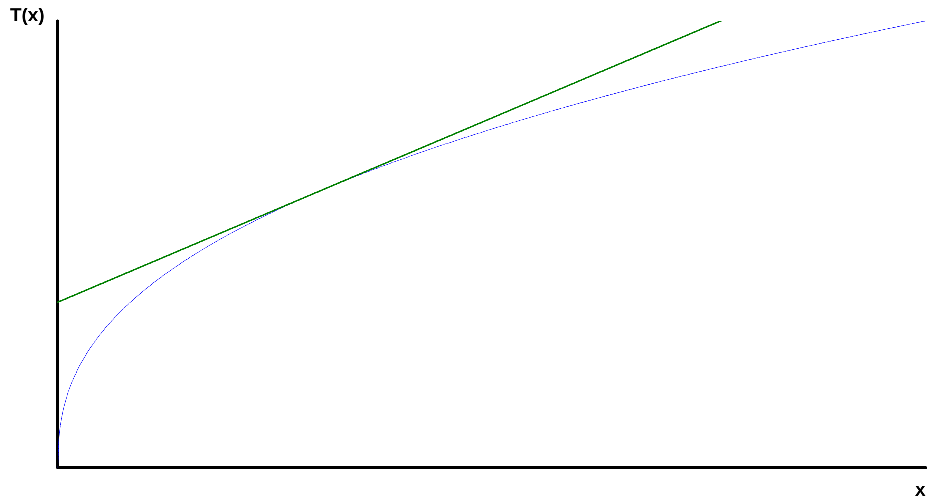

Figure 3: Optional Nonlinear Tariffs

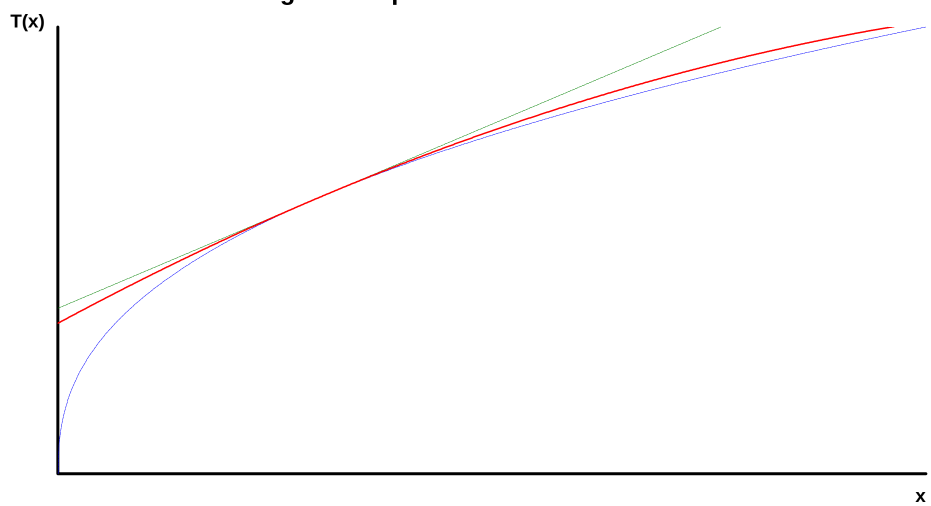

Figure 4. Empirical Distributions