THE WHITENED CUBE AS A PRECISION INTEGRATING PHOTOMETER.*

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SUMMARY.

The paper indicates how a whitened cube can be made to give accurate results when used in the determination of the average candle-powers of lamps having different polar distributions. As a preliminary, the relative effects of different portions of the interior of the cube in producing illumination on the window are determined, and the results used to calculate the illuminations produced by different types of light distribution. The ratios of these illuminations to that furnished by a uniform point source of unit intensity give apparent average candle-powers in the cube. It is then shown by experiment that candle-powers obtained by the usual methods of integrating photometry agree with these calculated values. Thus it is possible to determine by how much these apparent candle-powers differ from true candle-powers deduced by point-to-point methods, and hence to obtain true candle-powers from experimentally determined apparent values. In the special case when a comparison is made between lamps having identical polar distributions, it is shown that the usual method gives accurate results.

INTRODUCTION.

For several years past a 2-metre cube has been in use at the National Physical Laboratory, for determining the average candle-power of lamps whose polar distributions have not differed very much from that of the standard lamp with which they are compared. It has been recognized that such results are, to a certain extent, approximate, and it is the object of the present series of experiments to arrive at some estimate of the magnitude of the errors which may be involved in such determinations, and to derive principles which will tend to minimize these errors. The chief reasons which led to the use of the cube, especially of a large-sized one, rather than a sphere, are the greater ease of construction and the increased facility for installing and manipulating such heavy apparatus as arc-lamp projectors, as the whole of one side can be made to open for such purposes.

Dr. Sumpner † first suggested the use of a rectangular box instead of a sphere, and claimed that to a fairly close approximation it would give as good results as the sphere. In the discussion on Dr. Sumpner's paper Professor G. W. O. Howe described a box with 3-ft. edges which, he stated, acted satisfactorily. He also suggested the use of triangular pieces of cardboard to block up the corners so as to make the box approximate more closely to a sphere.

L. W. Wild,* as a result of this discussion, experimented with a rectangular box, and obtained photometric differences up to 4 per cent. His box was rather small, and his differences would probably be diminished by the use of a larger box.

Grondhal † in America has also described experiments with an oblong rectangular box, a box with the corners eliminated, and also a cube with the corners eliminated. He comes to the conclusion that for sources of similar distribution a rectangular box may be substituted for a sphere, and that a cubical box with the corners eliminated is for practical purposes a satisfactory substitute for a sphere, even for dissimilar and asymmetric sources.

THEORETICAL OUTLINE.

In the case of the Ulbricht sphere the illumination at the window is proportional to the total flux from the source, provided it receives no direct light from this source. In the case of the cube, however, this does not hold, the illumination on the window being a function of the first point of incidence of any part of the flux as well as of the total flux. For the same point of incidence the illumination on the window will be proportional to the total flux, but the proportionality factor, or "contribution coefficient" as it may be conveniently called, varies from point to point in the cube. This variation of the contribution coefficient has been determined for the whole of the cube. If, now, a source of known polar distribution and average candle-power be placed in the middle of the cube, we may calculate the flux falling on each area of the walls, and this, multiplied by the contribution coefficient for that area and summed over the whole surface of the cube, will give a quantity which, when divided by the same summation for a unit point source at the centre, will give the ratio of their apparent total fluxes or, what is proportional to it, the apparent average candle-power of the source when in the cube. We have thus a method of determining in a cube by how much the apparent average candle-powers differ from the actual average candle-power determined by a point-to-point method. This again can be compared with the average candle-power obtained by the usual experimental comparison of two lamps, for one of which this value is known.

THE PRESENT INVESTIGATION.

In the present investigation an Aldis daylight signalling lamp (Fig. 1) was used as the source of a constant flux. It consisted of a gas-filled lamp with twin spiral

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† *Illuminating Engineer* (London), 1910, vol. 3, p. 323.

* *Loc. cit.*, p. 549.

† *Transactions of Illuminating Engineering Society*, 1916, vol. 11, pt. 2, p. 152.

filaments at the focus of a 4-inch Mangin mirror. Only the light emitted towards the mirror was allowed to come from the projector, and this was effected by

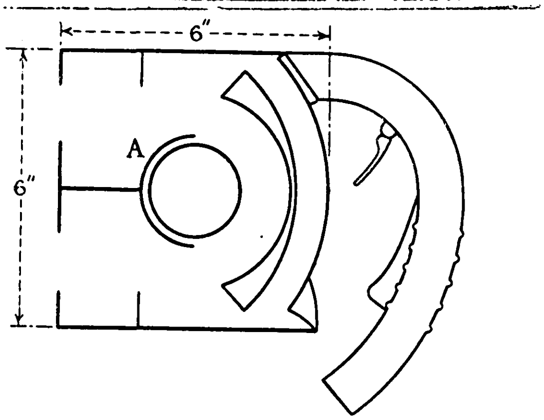

FIG. 1.—Aldis daylight signalling lamp (diagrammatic).

placing the blackened shield A over the half of the lamp away from the mirror. This resulted in a beam which had a diameter of about 17 cm. at a distance of 1 metre window; C, C′ the nearer halves of the same walls; and D, D′, D″, D‴ on the wall containing the window. The observations are the means of two sets of five settings of the photometer by each of two observers. The illumination on the window was not sufficiently large for convenience in working with a Lummer Brodhun photometer and comparison lamp of about 20 candle-power, without the use of a mirror in the photometer head. This can be seen diagrammatically in Fig. 3, where the mirror (M) is inclined at an angle of about 60° with the axis of the photometer bench. The effect of the mirror is that instead of matching the brightness of the photometer screen as illuminated by the window, against the bench comparison lamp, the image of the window in the mirror is used, this being six or seven times brighter. On the same side of the photometer as the comparison lamp a blue filter (Wratten photometric 78B of transmission approximately 54 per cent) was placed, and this gave a fairly good colour match between the gas-filled lamp working at an efficiency of 1·25 candles per watt and the tungsten-filament vacuum lamp at 0·70 candle per watt.

Check readings were taken from time to time with the beam incident on the middle of the wall facing

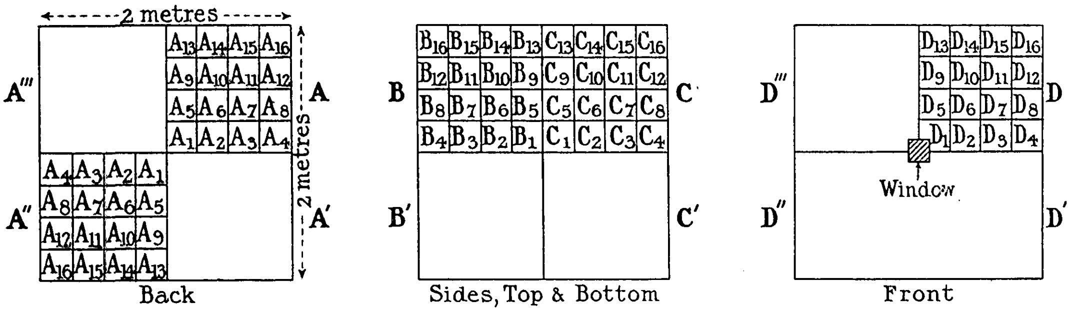

FIG. 2.—Types of wall, showing division into small squares.

from the projector. The exterior was whitened, as was also the retort stand which held it in position. Throughout the experiments the projector was in the middle of the cube, and it was oriented so that the beam from it fell approximately in the centre of each of the 16 squares into which each quarter side of the cube was divided. Experiments were made on the variations of candle-power of this lamp at different angles of tilt. It was found that these were extremely small and could be neglected. A small screen of about 15 cm. diameter was placed half-way between the projector and the window. It was not necessary in this case, as direct light from the source was never incident on the window, but, as in actual practice it would be necessary, it was placed in position and remained so during all the observations. These observations were not conducted over the whole of each side, as considerations of the symmetry of the system show this to be unnecessary. In Fig. 2, which shows the squares into which each type of wall is divided, it is obvious that the contribution coefficient will be the same for corresponding squares in A, A′, A″, A‴ which are on the wall facing the window; B, B′ the halves of the sides, top, and bottom further from the the window, and results for other places were corrected to correspond to the slight variation obtained due to changes in the projector lamp.

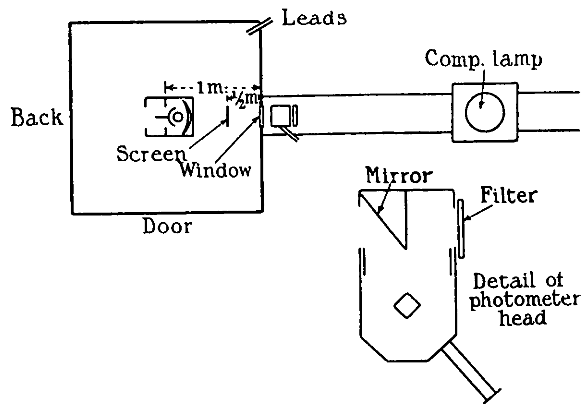

FIG. 3.—Diagram of apparatus.

Now, if L be the total flux in the beam of the projector, C the effective candle-power of the bench comparison lamp with colour filter, and x the distance of this lamp from the photometer at balance

$$\frac{C}{x^2} = bL, \text{ where } b \text{ is a constant}$$

$$\therefore b = C/Lx^2$$

so that the contribution coefficient, as b has been called, is proportional to $1/x^2$.

Table 1 gives the results of the survey of the cube over the back and front. Table 2 gives the results over the sides, top and bottom.

TABLE 1.

Position	Reading ($=x$), mm.	$\frac{1}{x^2} \times 10^7$	Position	Reading ($=x$), mm.	$\frac{1}{x^2} \times 10^7$
A_1	1 199	$6{\cdot}96=\alpha_1$	D_1	1 158	$7{\cdot}46=\delta_1$
A_2	1 022	$9{\cdot}57=\alpha_2$	D_2	1 156	$7{\cdot}48=\delta_2$
A_3	1 068	$8{\cdot}77=\alpha_3$	D_3	1 180	$7{\cdot}18=\delta_3$
A_4	1 092	$8{\cdot}38=\alpha_4$	D_4	1 197	$6{\cdot}98=\delta_4$
A_5	1 056	$8{\cdot}97=\alpha_5$	D_5	1 150	$7{\cdot}56=\delta_5$
A_6	1 031	$9{\cdot}41=\alpha_6$	D_6	1 168	$7{\cdot}33=\delta_6$
A_7	1 081	$8{\cdot}55=\alpha_7$	D_7	1 186	$7{\cdot}11=\delta_7$
A_8	1 110	$8{\cdot}12=\alpha_8$	D_8	1 210	$6{\cdot}83=\delta_8$
A_9	1 049	$9{\cdot}09=\alpha_9$	D_9	1 172	$7{\cdot}28=\delta_9$
A_{10}	1 061	$8{\cdot}88=\alpha_{10}$	D_{10}	1 171	$7{\cdot}28=\delta_{10}$
A_{11}	1 100	$8{\cdot}26=\alpha_{11}$	D_{11}	1 194	$7{\cdot}01=\delta_{11}$
A_{12}	1 118	$8{\cdot}00=\alpha_{12}$	D_{12}	1 220	$6{\cdot}72=\delta_{12}$
A_{13}	1 072	$8{\cdot}70=\alpha_{13}$	D_{13}	1 182	$7{\cdot}16=\delta_{13}$
A_{14}	1 088	$8{\cdot}45=\alpha_{14}$	D_{14}	1 181	$7{\cdot}17=\delta_{14}$
A_{15}	1 124	$7{\cdot}92=\alpha_{15}$	D_{15}	1 216	$6{\cdot}76=\delta_{15}$
A_{16}	1 136	$7{\cdot}75=\alpha_{16}$	D_{16}	1 221	$6{\cdot}71=\delta_{16}$

TABLE 2.

Position	Reading ($=x$), mm.	$\frac{1}{x^2} \times 10^7$	Position	Reading ($=x$), mm.	$\frac{1}{x^2} \times 10^7$
B_1	1 105	$8{\cdot}19=\beta_1$	C_1	1 098	$8{\cdot}29=\gamma_1$
B_2	1 108	$8{\cdot}14=\beta_2$	C_2	1 097	$8{\cdot}31=\gamma_2$
B_3	1 099	$8{\cdot}28=\beta_3$	C_3	1 120	$7{\cdot}98=\gamma_3$
B_4	1 105	$8{\cdot}19=\beta_4$	C_4	1 165	$7{\cdot}37=\gamma_4$
B_5	1 101	$8{\cdot}25=\beta_5$	C_5	1 101	$8{\cdot}25=\gamma_5$
B_6	1 108	$8{\cdot}14=\beta_6$	C_6	1 114	$8{\cdot}06=\gamma_6$
B_7	1 108	$8{\cdot}14=\beta_7$	C_7	1 129	$7{\cdot}84=\gamma_7$
B_8	1 098	$8{\cdot}29=\beta_8$	C_8	1 176	$7{\cdot}23=\gamma_8$
B_9	1 116	$8{\cdot}03=\beta_9$	C_9	1 126	$7{\cdot}88=\gamma_9$
B_{10}	1 115	$8{\cdot}05=\beta_{10}$	C_{10}	1 138	$7{\cdot}72=\gamma_{10}$
B_{11}	1 130	$7{\cdot}83=\beta_{11}$	C_{11}	1 152	$7{\cdot}53=\gamma_{11}$
B_{12}	1 110	$8{\cdot}12=\beta_{12}$	C_{12}	1 188	$7{\cdot}08=\gamma_{12}$
B_{13}	1 119	$7{\cdot}99=\beta_{13}$	C_{13}	1 138	$7{\cdot}72=\gamma_{13}$
B_{14}	1 121	$7{\cdot}96=\beta_{14}$	C_{14}	1 156	$7{\cdot}48=\gamma_{14}$
B_{15}	1 119	$7{\cdot}99=\beta_{15}$	C_{15}	1 191	$7{\cdot}05=\gamma_{15}$
B_{16}	1 125	$7{\cdot}90=\beta_{16}$	C_{16}	1 205	$6{\cdot}89=\gamma_{16}$

The variation of $1/x^2$, the contribution coefficient along several lines on the walls, is shown graphically in Fig. 4.

For any given polar distribution we can now find a quantity proportional to its apparent total flux or average candle-power in the cube. If F be the flux incident on any small square, and a the contribution coefficient for that square, then $\Sigma(Fa)$ taken over the whole cube is such a quantity. Let $l =$ semi-side of cube $= 8$ units; let the source be at the middle of the cube, i.e. distant l from the centre of each side, and let x_1y_1, x_2y_2, etc., be the co-ordinates of the mid-points of the small squares $A_1 \ldots A_{16}$, $B_1 \ldots B_{16}$, $C_1 \ldots C_{16}$, $D_1 \ldots D_{16}$, referred to an origin at the mid-point of the wall over which the summation is

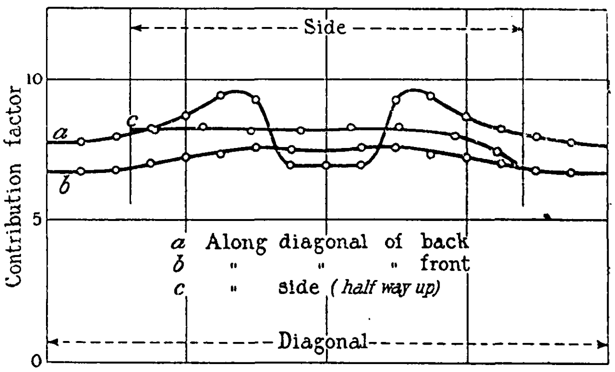

FIG. 4.—Variation of "contribution factor" across the cube.

being taken. Then the flux incident on any small square of area $(l/4)^2$

$$= \frac{\text{area} \times \cos(\text{angle of incidence})}{(\text{distance from source to centre of area})^2} \times (\text{c.p. in direction of centre of area})$$

$$= \frac{l^2}{4^2} \cdot \frac{l}{(l^2 + x^2 + y^2)^{1/2}} \cdot \frac{1}{l^2 + x^2 + y^2} \cdot \Phi_\theta$$

(See Fig. 5)

where Φ_θ is the candle-power of the source in a direction making an angle θ with the reference plane of the lamp.

$\Sigma(Fa)$ then becomes

$$\frac{l^3}{4^2}\sum\left\{\frac{a\Phi_\theta}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

The values of $(l^2 + x^2 + y^2)^{3/2}$ for the mid-points of each of the areas $A_1 \ldots A_{16}$, which are the same as those for corresponding areas in $B_1 \ldots B_{16}$, $C_1 \ldots C_{16}$, $D_1 \ldots D_{16}$, are given in Table 3.

TABLE 3.

Area	$(l^2 + x^2 + y^2)^{3/2}$	Area	$(l^2 + x^2 + y^2)^{3/2}$
A_1	536	A_9	854
A_2	636	A_{10}	969
A_3	853	A_{11}	1 218
A_4	1 218	A_{12}	1 621
A_5	637	A_{13}	1 218
A_6	742	A_{14}	1 346
A_7	969	A_{15}	1 621
A_8	1 346	A_{16}	2 062

In evaluating the values of θ, which is the angle between the reference plane and the direction of emission of the flux, two cases have to be considered :—

Case (1).—When the direction of emission is incident on a wall of the cube perpendicular to the reference plane.

Case (2).—When the direction of emission is incident on a wall parallel to the reference plane. The former is shown in Fig. 5, and it can be seen that

$$\theta' = \arc\sin \frac{y}{(l^2 + x^2 + y^2)^{1/2}}$$

In the latter case (see Fig. 5)

$$\theta'' = \arc\sin \frac{l}{(l^2 + x^2 + y^2)^{1/2}}$$

The values of θ' and θ'' are given in Table 4 for each of the areas $A_1 \ldots A_{16}$, which of course have corresponding squares on the other walls.

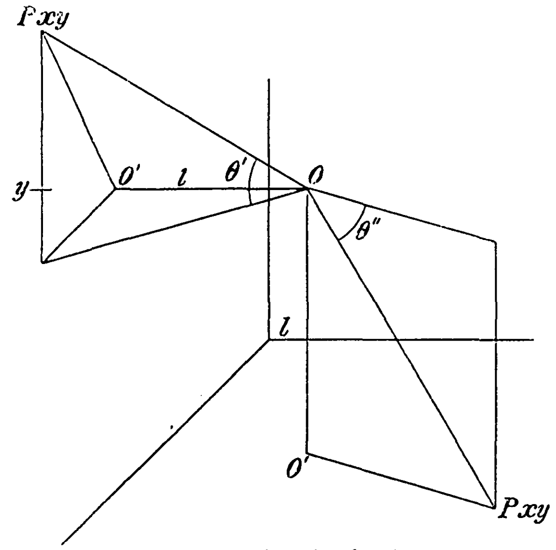

FIG. 5.—Geometry of angle of incidence, etc.

$\Sigma(Fa)$ FOR VARIOUS POLAR DISTRIBUTIONS.

(1) *A point source.*—In this case $\Phi_\theta =$ const. $= \Phi$, the candle-power of the source. The summation $\Sigma(Fa)$ is made up of three parts :—

(1) due to the back of the cube; this equals four times the summation over the small squares $A_1 \ldots A_{16}$

$$= 4\Phi\frac{l^3}{4^2}\sum\left\{\frac{a}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

(2) due to the front as above, but over the squares $D_1 \ldots D_{16}$

$$= 4\Phi\frac{l^3}{4^2}\sum\left\{\frac{\delta}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

(3) due to sides, top and bottom; this equals 8 times the summations over the small squares $B_1 \ldots B_{16}$, $C_1 \ldots C_{16}$,

$$= 8\Phi\frac{l^3}{4^2}\sum\left\{\frac{\beta + \gamma}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

Now $l = 8$ units

$$\therefore \Sigma(Fa) = 128\Phi\sum\left\{\frac{a + 2(\beta + \gamma) + \delta}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

$= 100 \cdot 3\ \Phi$ in the particular case of the measurements quoted in Tables 1 and 2.

TABLE 4.

Area	θ'	θ''	Area	θ'	θ''
A_1	7°	80°	A_9	32°	57°
A_2	6°	68°	A_{10}	30°	54°
A_3	6°	58°	A_{11}	28°	49°
A_4	5°	49°	A_{12}	25°	43°
A_5	20°	69°	A_{13}	41°	49°
A_6	19°	62°	A_{14}	40°	46°
A_7	18°	54°	A_{15}	37°	43°
A_8	16°	46°	A_{16}	33°	39°

(2) *A squirrel-cage metal-filament vacuum lamp.*—This lamp had a polar distribution as shown in Fig. 6. Measurements were made at the Russell angles (ten in each quadrant) whilst the lamp was rotating, and the average candle-power was deduced as 114·0.

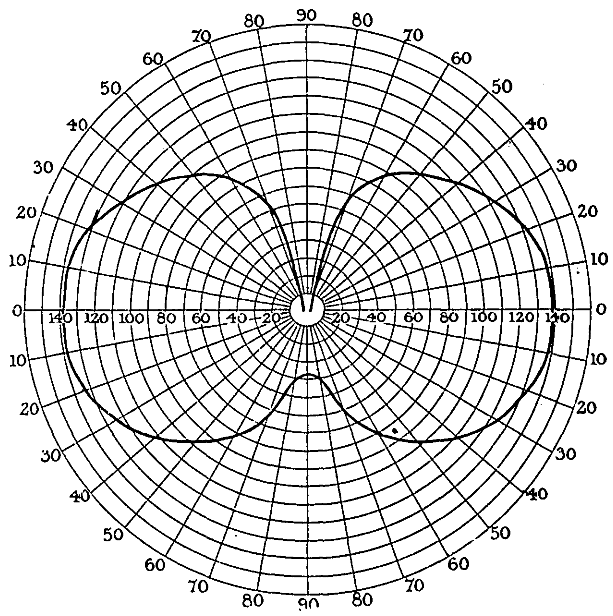

FIG. 6.—Polar curve of Osram reference lamp No. 2.

The summations in this case consist of four parts due respectively to

$$(1)\ \text{back} \quad = 4 \cdot \frac{l^3}{4^2}\sum\left\{\frac{\alpha\Phi_{\theta'}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

$$(2)\ \text{front} \quad = 4 \cdot \frac{l^3}{4^2}\sum\left\{\frac{\delta\Phi_{\theta'}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

$$(3)\ \text{sides} \quad = 4 \cdot \frac{l^3}{4^2}\sum\left\{\frac{(\beta + \gamma)\Phi_{\theta'}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

$$(4)\ \text{top and bottom} = 4 \cdot \frac{l^3}{4^2}\sum\left\{\frac{(\beta + \gamma)\Phi_{\theta''}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

$$\therefore\ \Sigma(F\alpha) = 128\sum\left\{\frac{(\alpha + \delta)\Phi_{\theta}}{(l^2 + x^2 + y^2)^{3/2}}\right\} + 128\sum\left\{\frac{(\beta + \gamma)(\Phi_{\theta'} + \Phi_{\theta''})}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

$$= 11\ 336$$

In calculating these summations the values of $\Phi_{\theta'}$ and $\Phi_{\theta''}$ are those read off directly from the polar diagram at the angles given in Table 4. The ratio of this to the summation for a unit point source gives a result of 113·0. Thus a lamp with the polar distribution shown in Fig. 6 behaves in the cube in the same way as does a point source (to a fairly close approximation) and may therefore be used instead of an ideal point source for the purpose of calibrating the cube for measurements of other lamps.

(3) *A squirrel-cage metal-filament vacuum lamp with close-fitting shade.* (*Axis of lamp vertical.*)—This lamp is illustrated in Fig. 7, and its polar distribution given in Fig. 8. It was set up with the centre of the edge of the shade at the mid-point of the cube. It was anticipated that a distribution of light of this kind, in which only the lower half of the cube received any direct light, would be one in which the value of $\Sigma(F\alpha)$ would show deviations from proportionality with the average candle-power. In this case the average candle-power as given by a point-to-point method was 422.

As before, the summation consists of four portions

$$(1)\ \text{back} \quad = 2\frac{l^3}{4^2}\sum\left\{\frac{\alpha\Phi_{\theta'}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

$$(2)\ \text{front} \quad = 2\frac{l^3}{4^2}\sum\left\{\frac{\delta\Phi_{\theta}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

$$(3)\ \text{sides} \quad = 2\frac{l^3}{4^2}\sum\left\{\frac{(\beta + \gamma)\Phi_{\theta'}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

$$(4)\ \text{bottom} = 2\frac{l^3}{4^2}\sum\left\{\frac{(\beta + \gamma)\Phi_{\theta''}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

$$\therefore\ \Sigma(F\alpha) = 40\ 525$$

On comparing this with the summation for a unit point source it can be seen that with this distribution the cube has the effect of making the apparent candle-power 404 instead of 422. Thus from theoretical considerations we may say that lamps having polar distributions similar to (1) that of a point source, (2) that of Fig. 6, and (3) that of Fig. 8, may be compared among themselves for average candle-power to an accuracy of about 4 per cent, if they are situated in the cube in the same way as the above calculations have assumed.

This statement is capable of experimental verification. The two lamps whose polar curves have been given were introduced into the middle of the cube, and photometric balance was obtained in the usual way. The mirror in the photometer was dispensed with, as the illumination of the window was satisfactory in each case. The means of five observations by each of two observers gave

(1) for the 114·0-c.p. lamp, 1 172 and 1 166 mm. .. .. .. Mean = 1 169
(2) for the 422-c.p. lamp, 622 and 616 mm... .. .. .. Mean = 619

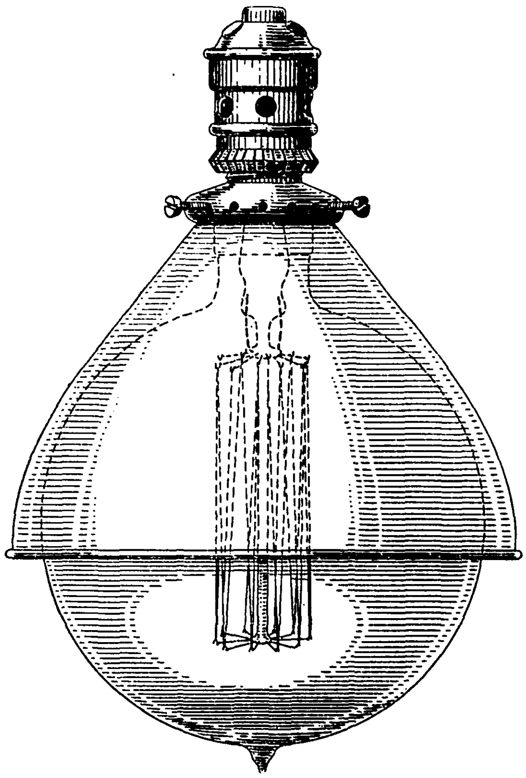

FIG. 7.—Standardized 1 000-watt, 240-volt lamp, and shade (shade whitened).

The comparison should of course be made with one lamp behaving as point source. The results of the last section, however, indicate that the 114·0-c.p. lamp is not very far from fulfilling such a condition.

$$\therefore\ \frac{\text{Average candle-power of 422-c.p. lamp}}{\text{Average candle-power of 114·0-c.p. lamp}} = \left(\frac{1\ 169}{619}\right)^2$$

$\therefore$ Average candle-power of 422-c.p. lamp

$$= (1\ 169 \div 619)^2 \times 114{\cdot}0$$
$$= 407$$

This result is very close to that which the summation leads one to expect.

Now the summation on the 114·0-c.p. lamp leads to the result that in the cube it behaves as if its candle-power were 113·0 and, if this value is used instead of 114·0, we get a still closer agreement.

$\therefore$ Average candle-power of 422-c.p. lamp

$$= (1\,169 \div 619)^2 \times 113{\cdot}0$$
$$= 403$$

(4) *A squirrel-cage metal-filament vacuum lamp with close-fitting shade.* (*Axis of lamp horizontal and lamp facing the back of the cube.*)—The summations are made for the same lamp as that used in the last section. In this position the back of the cube and the halves of 13; 2, 6, 10, 14; 3, etc., and the products $\beta\Phi_{\theta'}$ are taken $\beta_1\Phi_{\theta_1}$, $\beta_5\Phi_{\theta_2}$, $\beta_9\Phi_{\theta_3}$, etc.

$$\therefore\ \Sigma(Fa) = 128\sum\left\{\frac{a\Phi_{\theta''}}{(l^2 + x^2 + y^2)^{3/2}}\right\} + 256\sum\left\{\frac{\beta\Phi_{\theta'}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$
$$= 39\,800.$$

On comparing this with the summation for the point source, it is seen that the cube has the effect of making the apparent candle-power 397 instead of 422. This is about 6 per cent less than its known value. This

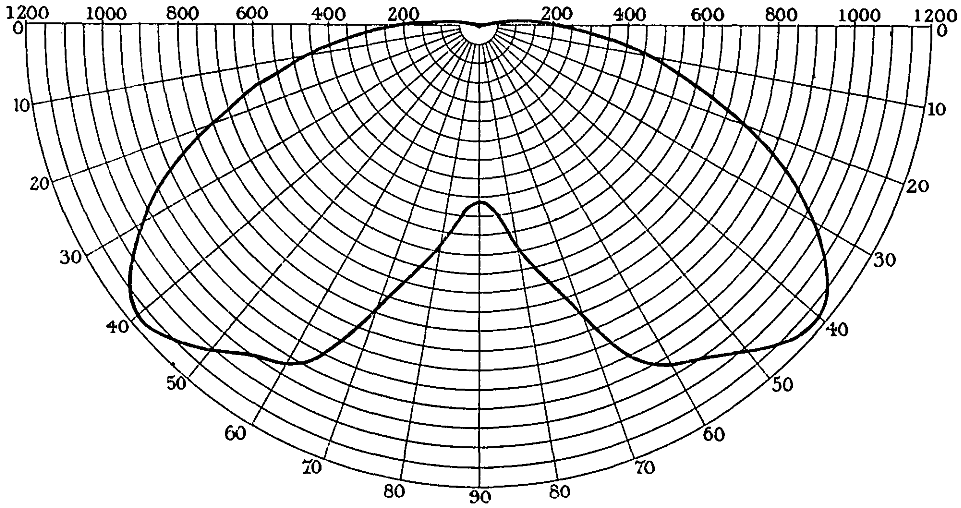

FIG. 8.—Polar curve of 1 000-watt lamp (Fig. 7).

the adjoining walls alone receive direct light. The polar curve of this lamp (Fig. 8) as thus situated in the cube is very similar to that of an arc, so that the results obtained in this case apply approximately to the case of the arc.

$\Sigma(Fa)$ consists of two portions (the C and D walls receive no direct light at all).

$$(1)\ \text{back} = 4\frac{l^3}{4^2}\sum\left\{\frac{a\Phi_{\theta''}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

θ'' taking the place of θ' because the back of the cube is now similarly situated with respect to the lamp as the bottom was previously.

(2) sides, top and bottom (the halves adjoining the back)

$$= 8\cdot\frac{l^3}{4^2}\sum\left\{\frac{\beta\Phi_{\theta'}}{(l^2 + x^2 + y^2)^{3/2}}\right\}$$

It should be noted in performing these summations that, since the axis of symmetry of the lamp is now horizontal, the numbering of the small squares from 1 to 16 does not correspond in the cases of the values of β and those of $\Phi_{\theta''}$. If the order 1 . . . 16 is maintained for the latter, then the order for β is 1, 5, 9, result was tested experimentally. The means of five observations by each of two observers gave

(1) for the 114·0-c.p. lamp, 1 173 and 1 175 mm. .. .. .. Mean = 1 174
(2) for the 422-c.p. lamp, 623 and 625 mm... .. .. .. Mean = 624

$\therefore$ Average candle-power of 422-c.p. lamp

$$= (1\,174 \div 624)^2 \times 114{\cdot}0$$
$$= 403$$

If, however, use is made of the result of section (2) which shows that the 114·0-c.p. lamp behaves in the cube as if it had an average candle-power of 113·0, this result is reduced to

$$(1\,174 \div 624)^2 \times 113{\cdot}0 = 400$$

This again is very close to the result indicated by theory, so that it can be assumed that a comparison of theory and practice for other positions at the middle of the cube will be accompanied by similar correspondence.

The preceding results and summations have applied only to lamps situated at the middle of the cube. It is important to find whether the same agreement will occur for an eccentric position. The position chosen was half-way between the top of the cube and the middle. It should be remembered, however, that the conditions in this eccentric position are not quite the same as for the central position, as the lamps used in the cube exert some disturbing influence on the "contribution coefficient." This can be seen in Fig. 4, where the big dip in curve a is due to the action of the projector in the middle of the cube, and also to the screen. With positions away from the centre it is to be anticipated that this drop in the contribution coefficient will take place roughly at the place where the line through the window and the lamp meets the cube again, and it is probable that the effect on the summations is appreciable in certain cases.

(5) *A point source.* (*Eccentric position.*)

$$\Sigma(Fa) = 100 \cdot 7$$

(6) *A squirrel-cage metal-filament vacuum lamp.* (*Eccentric position.*)

$$\Sigma(Fa) = 11\,360$$

On comparing this with the result for a unit point source it is seen that the apparent average candle-power has become $11\,360 \div 100 \cdot 7 = 112 \cdot 8$.

(7) *A squirrel-cage metal-filament vacuum lamp with close-fitting shade.* (*Eccentric position.*)

$$\Sigma(Fa) = 41\,280$$

On comparison with the result for a unit point source at the same position the apparent average candle-power becomes 410. On testing this experimentally, photometric balance was obtained at distances of

(1) for the 114·0-c.p. lamp, 1 165 and 1 158 mm. .. .. Mean = 1 161·5
(2) for the 422 c.p.-lamp, 610 and 606 mm. .. .. .. Mean = 608·5

∴ Average candle-power of 422-c.p. lamp

$$= (1\,161 \cdot 5 \div 608)^2 \times 114 \cdot 0$$
$$= 417$$

On comparing with the result 112·8 as the apparent candle-power of the 114·0-c.p. lamp given by the summation in section (6), we get

$$(1\,161 \cdot 5 \div 608)^2 \times 112 \cdot 8 = 413$$

and this again is in very close agreement with theory.

Conclusions.

These results make it possible for corrections to be applied to the determinations of average candle-power made in the cube in the usual way, i.e. by comparison of the illuminations produced on the window, by the lamp under test, and by a comparison lamp of known intensity. For, instead of the real candle-power of the comparison lamp, its apparent value in the cube is used, and this is given by the ratio of $\Sigma(Fa)$ for this lamp to the corresponding summation for a unit point source at the same place. The result after the application of the above is then multiplied by the ratio

$$\frac{\text{real candle-power}}{\text{apparent candle-power in the cube}}$$

where this ratio has been calculated for the type of polar distribution exhibited by the test lamp. Thus, if x and y are the distances on the photometer bench for photometric balance with the cube comparison lamp and test lamps respectively, P is the average candle-power of the cube comparison lamp, and Q that of *any* lamp having a similar polar distribution to the test lamp, then the average candle-power of the test lamp

$$= \frac{y^2}{x^2} \times \frac{\Sigma(Fa)_P}{\Sigma(Fa)_1} \times \frac{\Sigma(Fa)_1}{\Sigma(Fa)_Q} \times Q$$
$$= \frac{y^2}{x^2} \times \frac{\Sigma(Fa)_P}{\Sigma(Fa)_Q} \times Q$$

where the suffixes indicate for which lamp the summation is to be made.

If the polar distributions of the test lamp and comparison lamps are the same, then

$$\frac{\Sigma(Fa)_P}{\Sigma(Fa)_Q} = \frac{P}{Q}$$

and the average candle-power of the test lamp

$$= \frac{y^2}{x^2} P$$

This shows that the cube is quite accurate in comparing light sources of similar distribution.

An application of this theory can be made to the measurement of the total flux in a beam. It is obvious that if a narrow beam of light having a constant flux is incident on the walls of the cube, there will be different illuminations on the window which will vary directly as the contribution coefficient of the point of incidence. Now the greatest and least values of the contribution coefficient are proportional to 9·57 and 6·71 respectively, while the average for the whole cube, when used with a point source, is 7·96. If now the beam is incident at a point where the actual value is equal to the average, we may compare the fluxes in the beam and cube comparison lamp by taking 4π times the ratio of the inverse squares of the distances from the bench comparison lamp, multiplied by the ratio

$$\frac{\Sigma(Fa)_P}{\Sigma(Fa)_1}$$

Fig. 4 shows that the contribution coefficient varies considerably on the back of the cube but is fairly constant on the sides. Fig. 9 shows the same thing in terms of contours. Large portions have a value not far from 8·20, so that a convenient spot on one of these walls can be taken and the result multiplied by $796 \div 820 = 0 \cdot 97$.

Throughout this investigation the effects of foreign bodies in the cube have been assumed to be small, as they always had an area very small compared with the total area of the inside of the cube. It is proposed in a later paper to deal with this matter and also the effects of ventilating openings (such as have to be used in arc tests) and the non-uniformity of the cube as regards the whiteness of its interior.

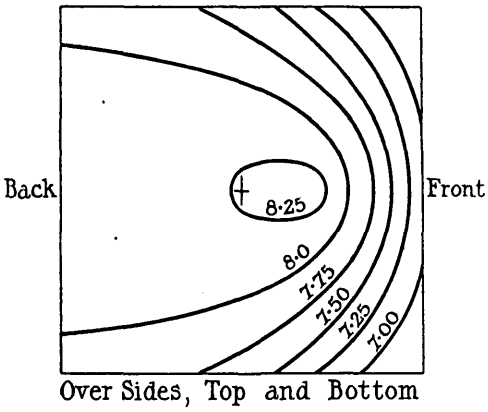

FIG. 9.—Variation of "contribution coefficient."

SYNOPSIS OF RESULTS.

(1) The variation in the contribution coefficient over the walls of a 2-metre cube has been determined.
(2) The apparent candle-powers of various light-distributions in the cube have been calculated and found to agree very closely with experiment.
(3) The cube has been shown to be quite accurate with sources having similar distributions.
(4) It has been shown how true average candle-powers may be deduced from the comparison of sources of different polar distribution, thus making the cube an instrument of precision.
(5) An application of the cube to the measurement of the total flux in beams has been indicated.

In conclusion I wish to express my indebtedness to Messrs. J. W. T. Walsh and H. C. Sturgeon of the National Physical Laboratory, the former for suggesting this work and for his interest in it, and the latter for invaluable assistance in taking the photometric readings and in the somewhat tedious task of performing the summations.

APPENDIX.

The complete calculations for $\Sigma(F\alpha)$ in section (6) are given here to show the method employed. In this case the lamp (Osram, Ref. No. 2, 114·0 candle-

TABLE 5.

Area	$[l^2+x^2+y^2]^{3/2}$	Area	$[(3l/2)^2+x^2+y^2]^{3/2}$	$[(l/2)^2+x^2+y^2]^{3/2}$
A_{17}	146	A_1	1 767	76
A_{18}	154	A_2	1 911	133
A_{19}	170	A_3	2 217	272
A_{20}	194	A_4	2 703	535
A_{21}	186	A_5	1 911	133
A_{22}	194	A_6	2 063	200
A_{23}	210	A_7	2 374	353
A_{24}	234	A_8	2 869	636
		A_9	1 217	272
Values for areas		A_{10}	2 374	353
$A_1 \ldots A_{16}$ are		A_{11}	2 703	535
given in Table 3		A_{12}	3 216	855
		A_{13}	2 703	535
		A_{14}	2 869	636
		A_{15}	3 216	855
		A_{16}	3 767	1 218

power) is midway between the middle of the cube and the top. The projection of the centre of the lamp

TABLE 6.

Values of θ.

Area	$\theta' = \text{arc sin} \frac{y}{\sqrt{(l^2+x^2+y^2)}}$	Area	$\theta'' = \text{arc sin} \frac{3l/2}{\sqrt{[(3l/2)^2+x^2+y^2]}}$	$\theta''' = \text{arc sin} \frac{\frac{1}{2}l}{\sqrt{[(\frac{1}{2}l)^2+x^2+y^2]}}$
A_{17}	48°	A_1	83°	71°
A_{18}	47°	A_2	79°	51°
A_{19}	44°	A_3	67°	38°
A_{20}	40°	A_4	60°	30°
A_{21}	54°	A_5	75°	51°
A_{22}	52°	A_6	70°	43°
A_{23}	49°	A_7	64°	34°
A_{24}	46°	A_8	58°	28°
		A_9	67°	38°
Values for areas $A_1 \ldots A_{16}$		A_{10}	64°	34°
are given in Table 4		A_{11}	60°	30°
		A_{12}	54°	25°
		A_{13}	60°	30°
		A_{14}	58°	28°
		A_{15}	54°	25°
		A_{16}	50°	22°

now comes at the common point of the four squares A_1 in Fig. 10, which also shows the new system of numbering squares that becomes necessary to avoid possible confusion on this account. A horizontal plane through the centre of the lamp cuts the wall at the double line, and it is about this line that symmetry occurs in the values of θ'. As regards the contribution factor, that is still symmetrical about a horizontal line through the centre of the wall. Thus the values of the contribution factor over the squares $A_1 \ldots A_8$ below the double line are the same as those over the former squares A_5, A_6, A_7, A_8; A_1, A_2, A_3, A_4, those over the squares $A_9 \ldots A_{24}$ are the same as over the former $A_1 \ldots A_{16}$, while those from $A_1 \ldots A_8$ above the double line are the same as over the former $A_9 \ldots A_{16}$. Similar considerations apply to all the vertical walls.

A_8	A_7	A_6	A_5	A_5	A_6	A_7	A_8
A_4	A_3	A_2	A_1	A_1	A_2	A_3	A_4
A_4	A_3	A_2	A_1	A_1	A_2	A_3	A_4
A_8	A_7	A_6	A_5	A_5	A_6	A_7	A_8
A_{12}	A_{11}	A_{10}	A_9	A_9	A_{10}	A_{11}	A_{12}
A_{16}	A_{15}	A_{14}	A_{13}	A_{13}	A_{14}	A_{15}	A_{16}
A_{20}	A_{19}	A_{18}	A_{17}	A_{17}	A_{18}	A_{19}	A_{20}
A_{24}	A_{23}	A_{22}	A_{21}	A_{21}	A_{22}	A_{23}	A_{24}

FIG. 10.—Division of sides of cube for summation described in appendix.

Now in the expression $\Sigma(Fa)$, F, the flux on an area, is given by

$$\frac{\text{area} \times \cos(\text{angle of incidence})}{(\text{distance from source to centre of area})^2} \times \text{(c.p. in direction of centre of area)}$$

$$= \frac{l^2}{4} \times \frac{l_1}{\sqrt{(l_1^2 + x^2 + y^2)}} \times \frac{1}{(l_1^2 + x^2 + y^2)} \times \Phi_\theta$$

where l_1 = perpendicular distance of source from a wall of the cube. In sections (1) to (4), $l_1 = l$, but now, if we consider the vertical walls, $l_1 = l$, the bottom $l_1 = 3l/2$ and the top $l_1 = \frac{1}{2}l$. The expressions for θ' and θ'' also involve l_1 thus :—

$$\theta' = \arc\sin \frac{y}{\sqrt{(l_1^2 + x^2 + y^2)}}$$

$$\theta'' = \arc\sin \frac{l_1}{\sqrt{(l_1^2 + x^2 + y^2)}}$$

where $l_1 = l$, $3l/2$ and $\frac{1}{2}l$ for different walls. This will result in modifications to Tables 3 and 4. These are shown in Table 5 and 6 respectively.

The summation $\Sigma(Fa)$ consists of four parts

(1) sides, back and front, below double line,

$$= 2\frac{l^2}{4^2} \cdot l \sum_1^{24} \left\{ \frac{(\alpha + \beta + \gamma + \delta)\Phi_{\theta'}}{(l^2 + x^2 + y^2)^{3/2}} \right\}$$

(2) sides, back and front, above double line,

$$= 2\frac{l^2}{4^2} \cdot l \sum_1^{8} \left\{ \frac{(\alpha + \beta + \gamma + \delta)\Phi_{\theta'}}{(l^2 + x^2 + y^2)^{3/2}} \right\}$$

(3) bottom

$$= 2\frac{l^2}{4^2} \cdot \frac{3l}{2} \sum_1^{16} \left\{ \frac{(\beta + \gamma)\Phi_{\theta''}}{\{(3l/2)^2 + x^2 + y^2\}^{3/2}} \right\}$$

(4) top

$$= 2\frac{l^2}{4^2} \cdot \frac{l}{2} \sum_1^{16} \left\{ \frac{(\beta + \gamma)\Phi_{\theta'''}}{\{(\frac{1}{2}l)^2 + x^2 + y^2\}^{3/2}} \right\}$$

$$\therefore \Sigma(Fa) = 64 \sum_1^{24} \left\{ \frac{(\alpha + \beta + \gamma + \delta)\Phi_{\theta'}}{(8^2 + x^2 + y^2)^{3/2}} \right\} + 64 \sum_1^{8} \left\{ \frac{(\alpha + \beta + \gamma + \delta)\Phi_{\theta'}}{(8^2 + x^2 + y^2)^{3/2}} \right\} + 96 \sum_1^{16} \left\{ \frac{(\beta + \gamma)\Phi_{\theta''}}{(12^2 + x^2 + y^2)^{3/2}} \right\} + 32 \sum_1^{16} \left\{ \frac{(\beta + \gamma)\Phi_{\theta'''}}{(4^2 + x^2 + y^2)^{3/2}} \right\}$$

$$(1) = 6\,400\Big(\frac{33{\cdot}02}{536} \times 1{\cdot}37 + \frac{32{\cdot}92}{636} \times 1{\cdot}37 + \frac{31{\cdot}64}{853} \times 1{\cdot}37 + \frac{30{\cdot}46}{1\,218} \times 1{\cdot}37 + \frac{31{\cdot}90}{637} \times 1{\cdot}31 + \frac{33{\cdot}39}{742} \times 1{\cdot}31$$
$$+ \frac{32{\cdot}20}{969} \times 1{\cdot}32 + \frac{30{\cdot}72}{1\,346} \times 1{\cdot}34 + \frac{31{\cdot}90}{854} \times 1{\cdot}20 + \frac{33{\cdot}39}{969} \times 1{\cdot}22 + \frac{32{\cdot}20}{1\,218} \times 1{\cdot}24 + \frac{30{\cdot}72}{1\,621} \times 1{\cdot}27$$
$$+ \frac{33{\cdot}02}{1\,218} \times 1{\cdot}10 + \frac{32{\cdot}92}{1\,346} \times 1{\cdot}11 + \frac{31{\cdot}64}{1\,621} \times 1{\cdot}15 + \frac{30{\cdot}46}{2\,062} \times 1{\cdot}19 + \frac{32{\cdot}27}{1\,767} \times 1{\cdot}01 + \frac{31{\cdot}93}{1\,911} \times 1{\cdot}03$$
$$+ \frac{30{\cdot}62}{2\,217} \times 1{\cdot}06 + \frac{29{\cdot}92}{2\,703} \times 1{\cdot}11 + \frac{31{\cdot}64}{2\,538} \times 0{\cdot}93 + \frac{31{\cdot}06}{2\,703} \times 0{\cdot}96 + \frac{29{\cdot}72}{3\,042} \times 1{\cdot}00 + \frac{29{\cdot}25}{3\,582} \times 1{\cdot}04\Big)$$
$$= 6\,400(0{\cdot}7861) = 5\,030$$

$$(2) = 6\,400\Big(\frac{32{\cdot}27}{536} \times 1{\cdot}37 + \frac{31{\cdot}93}{636} \times 1{\cdot}37 + \frac{30{\cdot}62}{853} \times 1{\cdot}37 + \frac{29{\cdot}92}{1\,218} \times 1{\cdot}37 + \frac{31{\cdot}64}{637} \times 1{\cdot}31 + \frac{31{\cdot}06}{742} \times 1{\cdot}31$$
$$+ \frac{29{\cdot}72}{969} \times 1{\cdot}32 + \frac{29{\cdot}25}{1\,346} \times 1{\cdot}34\Big)$$
$$= 6\,400 \times (0{\cdot}4239) = 2\,710$$

$$(3) = 9\ 600\Big(\frac{16\cdot48}{1\ 767}\times0\cdot22 + \frac{16\cdot45}{1\ 911}\times0\cdot33 + \frac{16\cdot26}{2\ 217}\times0\cdot68 + \frac{15\cdot56}{2\ 703}\times0\cdot84 + \frac{16\cdot50}{1\ 911}\times0\cdot45 + \frac{16\cdot20}{2\ 063}\times0\cdot60 + \frac{15\cdot98}{2\ 374}\times0\cdot75 + \frac{15\cdot52}{2\ 869}\times0\cdot87 + \frac{15\cdot91}{2\ 217}\times0\cdot68 + \frac{15\cdot77}{2\ 374}\times0\cdot75 + \frac{15\cdot36}{2\ 703}\times0\cdot84 + \frac{15\cdot20}{3\ 216}\times0\cdot93 + \frac{15\cdot71}{2\ 703}\times0\cdot85 + \frac{15\cdot44}{2\ 869}\times0\cdot88 + \frac{15\cdot04}{3\ 216}\times0\cdot93 + \frac{14\cdot78}{3\ 767}\times0\cdot99\Big)$$

$$= 9\ 600 \times 0\cdot0701 = 673$$

$$(4) = 3\ 200\Big(\frac{16\cdot48}{76}\times0\cdot58 + \frac{16\cdot45}{133}\times0\cdot97 + \frac{16\cdot26}{272}\times1\cdot13 + \frac{15\cdot56}{535}\times1\cdot22 + \frac{16\cdot50}{133}\times0\cdot97 + \frac{16\cdot20}{200}\times1\cdot08 + \frac{15\cdot98}{353}\times1\cdot18 + \frac{15\cdot52}{636}\times1\cdot24 + \frac{15\cdot91}{272}\times1\cdot14 + \frac{15\cdot77}{353}\times1\cdot18 + \frac{15\cdot36}{535}\times1\cdot22 + \frac{15\cdot20}{855}\times1\cdot27 + \frac{15\cdot71}{535}\times1\cdot22 + \frac{15\cdot44}{636}\times1\cdot24 + \frac{15\cdot04}{855}\times1\cdot27 + \frac{14\cdot78}{1\ 218}\times1\cdot29\Big)$$

$$= 3\ 200 \times 0\cdot9213 = 2\ 945$$

$$\therefore\ \Sigma(Fa) = (1) + (2) + (3) + (4) = 11\ 358$$

DISCUSSION ON

"MULTIPLE-UNIT SHUNTS FOR THE MEASUREMENT OF VERY HEAVY CURRENTS." *

Mr. A. E. Moore (*communicated*): I think there is much to be said for the author's discrimination between the "constant" and the "resistance" of a shunt. The constant is the important quantity, being the potential difference on the instrument or potential terminals per ampere through the shunt via the main connections. The author rightly draws attention to the fact that in the cases of all large shunts the resistance measured between the potential terminals is not the same as the constant or K value. Also, the resistance measured between the potential terminals is practically a constant, whereas the K value may vary very considerably in some shunts, according to the manner of making the main connections. Anyone who has had considerable experience in the accurate testing of ammeters which are provided with heavy-current shunts will appreciate the advantages of the author's multiple-unit shunt combination. The errors which are introduced by variations in the main-current connections to heavy-current shunts are, however, often lost sight of by the users. A quite common form of shunt for currents of from 1 000 to 10 000 amperes is shown in Fig. A. In this design two objects are aimed at: (1) To provide a large contact surface for the main connections in order to prevent undue heating, and (2) to effect a more or less uniform entry of the current into the shunt blocks. If every precaution were taken in the using and the testing of such shunts, better results than usual might be obtained. My experience, however, is that such shunts are very much misused. I recently had to test an ammeter provided with such a shunt for 1 500 amperes. The slots in the end blocks of the shunt had been partly filled in with strips of iron, so that connections to the main current leads had to be made on the faces AA and BB. For the purpose of obtaining some data for this contribution, I made tests with the main connections: (*a*), on the faces AA, and (*b*), on the faces BB. The results are given in the following table:

Reading on Shunted Ammeter	Main Amperes, AA	Percentage Error	Main Amperes, BB	Percentage Error
0	0	—	0	—
300	267	12·3 % high	300	nil
600	538·5	11·4 % high	606	1·0 % low
900	804	11·9 % high	902	0·2 % low
1 200	1 075	11·6 % high	1 202	0·2 % low
1 500	1 340	11·9 % high	1 498·5	0·1 % low

It may be urged that the shunt is not designed to have connections made in this way. I agree, but nevertheless connections are continually made in this manner, and it is hardly to be expected that every person who has to use a heavy-current shunt is fully aware of the large errors which may be introduced by improper connections. It is probable that, even when erected on the switchboard, all the slots are not filled with busbars, and whether or not the instrument attached to the shunt indicates correctly depends

* Paper by Mr. M. B. Field (see vol. 58, page 661).