

The World According to Wavelets

The Story of a Mathematical Technique in the Making

Second Edition

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