The wrinkling of thin membranes : part 2 : numerical analysis

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The Wrinkling of Thin Membranes: Part II—Numerical Analysis

Using a weighted residual method, a geometrically and physically nonlinear membrane element is derived, which can be used in the analysis of anisotropic membranes. What is special about the formulation is that the wrinkling behavior of the element is incorporated. If wrinkling occurs the stress situation in the element is determined by making use of a modified deformation tensor. A structure may have completely slack regions, leading to a singular stiffness matrix. Because of this we have chosen to use a restricted step method for the iterative solution procedure. A simple shear test is used to compare numerical and analytical results which show good agreement.

Introduction

Force transmission from muscle to bone near the elbow joint takes place by means of connective tissues which connect contractile elements from the muscle to the bones. They often consist of thin membrane-like structures. Because of their geometry, the membrane-like tissues will wrinkle easily. This wrinkling may have much influence on the stress state and the force transmission. In Roddeman et al. (1987) a mechanical model of wrinkling membranes has been presented. Wrinkling is accounted for by replacing a given deformation tensor, which would result in negative Cauchy stresses in the membrane, by a modified deformation tensor which results in the correct stress situation. With this model the wrinkling of anisotropic membranes in geometrically nonlinear analysis can be described. In this paper a membrane element will be derived which can be used for the modelling of thin structures.

The Equilibrium Conditions

The equilibrium conditions in local form may be formulated as:

$$\vec{\nabla}\cdot\sigma=\vec{0} \tag{1}$$

where $\vec{\nabla}$ is the gradient operator with respect to the deformed configuration and σ is the real Cauchy stress tensor which is symmetric: $\sigma=\sigma^c$.

For a finite element model an integral form of equation (1) is needed. Taking the inner product of the local equilibrium conditions with an arbitrary vector function $\vec{h}$ and integrating the result over the deformed volume V leads to:

$$\int_V (\vec{\nabla}\cdot\sigma)\cdot\vec{h}\,dV=0 \tag{2}$$

The integral form of equation (2) is equivalent to the equilibrium conditions (1), as long as $\vec{h}$ is arbitrary. Partial integration and the application of Gauss' theorem leads to:

$$\int_V \left[\sigma:(\vec{\nabla}\vec{h})^c\right]dV=\int_A \vec{n}_A\cdot[\sigma\cdot\vec{h}]\,dA \tag{3}$$

where $\vec{n}_A$ denotes the outward unit normal on the deformed surface A of the body. Since the volume V and the surface A are a priori unknown, equation (3) is transformed to the initial configuration, for which the volume V_0 and the surface A_0 are known:

$$\int_{V_0} [\sigma:((\mathbf{R}^{-c}\cdot\vec{\nabla}_0)\vec{h})^c]J\,dV_0=\int_{A_0} \vec{n}_A\cdot[\sigma\cdot\vec{h}]J^a\,dA_0 \tag{4}$$

with $J=dV/dV_0$; $J^a=dA/dA_0$; and $\vec{\nabla}_0$ is the gradient operator with respect to the initial configuration.

$\mathbf{R}$ denotes the real deformation tensor of the body. It should be noticed that in a wrinkled membrane with no flexural stiffness $\mathbf{R}$ is indefinite. It can be shown that an equivalent formulation of equation (4) is:

$$\int_{V_0} [(\mathbf{R}^{-1}\cdot J\sigma):(\vec{\nabla}_0\vec{h})^c]\,dV_0=\int_{A_0} \vec{n}_A\cdot[\sigma\cdot\vec{h}]J^a\,dA_0 \tag{5}$$

The term $\mathbf{R}^{-1}\cdot J\sigma$ in equation (5) is called the first Piola-Kirchhoff stress tensor π. If a membrane wrinkles (in a principal direction, for example given by unit vector $\vec{n}_1$) the exact shape in wrinkling direction is not known, and so the real deformation tensor $\mathbf{R}$ is unknown. It can be demonstrated however that $\mathbf{R}$ may be replaced by a similar deformation tensor $\mathbf{S}$, which is known.

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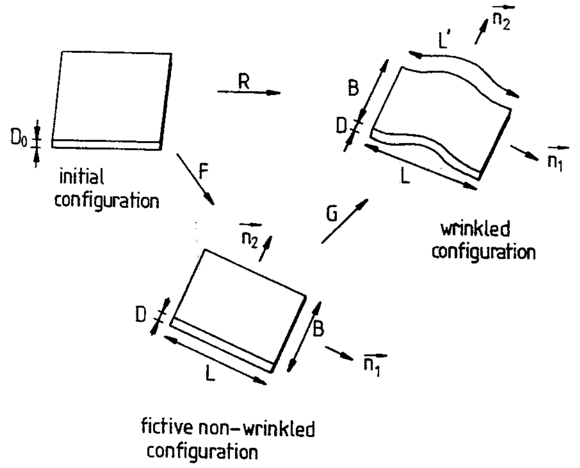

Fig. 1 The total deformation R is divided into F and G

Consider the deformation of a wrinkled membrane, as a deformation from the initial configuration to an imaginary nonwrinkled membrane (with corresponding deformation tensor **F**), followed by the wrinkling of the membrane, with corresponding deformation tensor **G** (Fig. 1). So, we write:

$$\mathbf{R}=\mathbf{G}\bullet\mathbf{F} \tag{6}$$

where:

$$\mathbf{G}=\vec{n}_2\vec{n}_2+g_{11}\vec{n}_1\vec{n}_1+g_{13}\vec{n}_1\vec{n}_3+g_{31}\vec{n}_3\vec{n}_1+g_{33}\vec{n}_3\vec{n}_3 \tag{7}$$

The frame $\vec{n}_1$, $\vec{n}_2$, $\vec{n}_3$ is the principal Cauchy frame where $\vec{n}_3$ is perpendicular to the plane of the fictive nonwrinkled membrane. The unknown terms $g_{ij}\vec{n}_i\vec{n}_j$ only cause deformations in the plane determined by $\vec{n}_1$ and $\vec{n}_3$ and define the wrinkles of the membrane part. Using the fact that there are only stresses in the $\vec{n}_2$ direction, it is obvious that:

$$J\sigma=(J\sigma_{22})\vec{n}_2\vec{n}_2 \tag{8}$$

and it can be shown that:

$$\begin{aligned}\pi&=\mathbf{R}^{-1}\bullet(J\sigma)=\mathbf{F}^{-1}\bullet\mathbf{G}^{-1}\bullet(J\sigma)\\&=\mathbf{F}^{-1}\bullet\{\vec{n}_2\vec{n}_2+g_{11}\vec{n}_1\vec{n}_1+g_{13}\vec{n}_1\vec{n}_3+g_{31}\vec{n}_3\vec{n}_1\\&\quad+g_{33}\vec{n}_3\vec{n}_3\}^{-1}\bullet(J\sigma_{22})\vec{n}_2\vec{n}_2=\mathbf{F}^{-1}\bullet(J\sigma_{22})\vec{n}_2\vec{n}_2=\mathbf{F}^{-1}\bullet(J\sigma)\end{aligned} \tag{9}$$

In a similar way the deformation tensor **F** can be regarded to be a deformation from the initial configuration to a nonwrinkled membrane, still with initial thickness D_0, with corresponding deformation tensor **S**, followed by a change of the thickness from D_0 to its real value D:

$$\mathbf{F}=(\vec{n}_1\vec{n}_1+\vec{n}_2\vec{n}_2+D/D_0\vec{n}_3\vec{n}_3)\bullet\mathbf{S} \tag{10}$$

After substituting equation (10) in (9) it is found that:

$$\begin{aligned}\pi=\mathbf{F}^{-1}\bullet(J\sigma)&=\mathbf{S}^{-1}\bullet(\vec{n}_1\vec{n}_1+\vec{n}_2\vec{n}_2+D/D_0\vec{n}_3\vec{n}_3)^{-1}\bullet(J\sigma_{22})\vec{n}_2\vec{n}_2\\&=\mathbf{S}^{-1}\bullet(J\sigma_{22})\vec{n}_2\vec{n}_2\end{aligned}$$

thus:

$$\pi=\mathbf{S}^{-1}\bullet(J\sigma) \tag{11}$$

It is easy to show that the last equation is also true in case the membrane does not wrinkle. The advantage of equation (11) is that **S** does not contain variations in the membrane thickness anymore, which will prove to be useful for the linearization of the equations to follow.

The integral form of the equilibrium conditions now can be written as:

$$\int_{V_0}\pi:(\vec{\nabla}_0\vec{h})^c dV_0=\int_{A_0}\vec{k}_0\bullet\vec{h}dA_0 \tag{12}$$

where π is given by equation (11) and $\vec{k}_0=J^a\vec{n}_A\bullet\sigma$, which is the force on the deformed surface transformed to the undeformed surface.

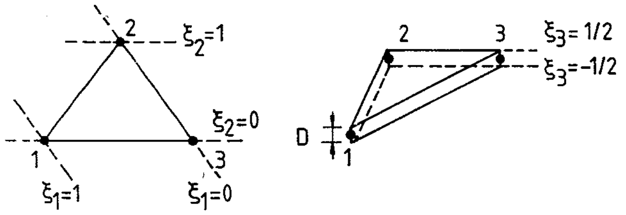

Fig. 2 A three-noded element. ξ_1, ξ_2, and ξ_3 are material coordinates.

An estimated solution of the equilibrium condition in integral form (12) is marked with a superscript *. In general the estimated solution will not satisfy equation (12) exactly, so a better solution is searched for. The difference between the estimated solution and a solution satisfying equilibrium condition (12) is indicated by a delta, so:

$$\pi=\pi^*+\delta\pi$$

$$\mathbf{S}=\mathbf{S}^*+\delta\mathbf{S}$$

, etc. Thus equation (12) may also be written as:

$$\int_{V_0}(\pi^*+\delta\pi):(\vec{\nabla}_0\vec{h})^c dV_0=\int_{A_0}(\vec{k}_0^*+\delta\vec{k}_0)\bullet\vec{h}dA_0 \tag{13}$$

Until now equation (13) is perfectly equivalent to the equilibium conditions (1). The difference between the displacement field satisfying the equilibrium conditions and the estimated displacement field is regarded to be the primary unknown of equation (13). Let us assume that the estimated solution is close to the real solution. Then equation (13) can be linearized with respect to the primary unknown.

According to equation (11), the difference in π in the estimated solution and equilibrium is approximately given by:

$$\delta\pi\approx\delta(\mathbf{S}^{-1})\bullet(J\sigma)^*+\mathbf{S}^{*-1}\bullet\delta(J\sigma) \tag{14}$$

Since:

$$\delta(\mathbf{S}^{-1})\approx-\mathbf{S}^{*-1}\bullet\delta\mathbf{S}\bullet\mathbf{S}^{*-1} \tag{15}$$

it is easily shown that linearization of equation (13) leads to:

$$\begin{aligned}&\int_{V_0}[\{(\vec{\nabla}_0\vec{h})^c\bullet\mathbf{S}^{*-1}\}:\{\delta(J\sigma)\}\\&\qquad-\{\mathbf{S}^{*-1}\bullet(J\sigma)^*\bullet(\vec{\nabla}_0\vec{h})^c\}:\{\mathbf{S}^{*-1}\bullet\delta\mathbf{S}\}]dV_0\\&-\int_{A_0}\delta\vec{k}_0\bullet\vec{h}dA_0\approx-\int_{V_0}\pi^*:(\vec{\nabla}_0\vec{h})^c dV_0+\int_{A_0}\vec{k}_0^*\bullet\vec{h}dA_0\end{aligned} \tag{16}$$

We will solve (16) by means of the Finite Element Method. The observed mechanical system is divided into a number of elements of finite dimensions. Often it is possible to define all kinds of elements and to give a general derivation of the equations without considering the type of the element. In the present case however it is convenient to have an element with constant strains and stresses. Otherwise the element might be divided into wrinkling and nonwrinkling zones, which would make the analysis unfeasible. Thus, we have decided to use a triangular, three-noded, constant strain element and specialize the derivation for this element. In Fig. 2 the element is shown. Equation (16) will be analyzed for one element (thus V_0 is the initial volume of the element and A_0 the initial surface).

The position of a material point of the element is given by:

$$\vec{x}=\psi_k\vec{x}_k+\xi_3 D\vec{n}_3 \tag{17}$$

where ψ_k are shape functions, ξ_3 is the material coordinate in the direction perpendicular to the plane of the membrane, and $\vec{x}_k$ are position vectors of the nodal points. D denotes the thickness of the membrane. The Einstein summation convention is used, i.e., when an index occurs twice in a product term, this implies summation with respect to all its possible values. Normally the possible values of an index are 1, 2, and 3. However, indices which are Greek characters only can take the values 1 and 2.

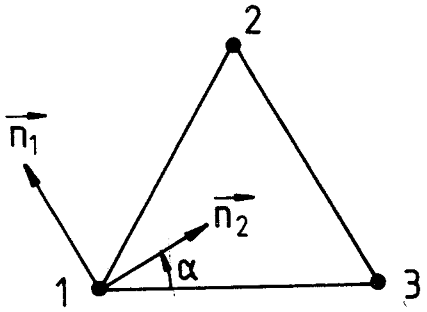

Fig. 3 The direction of the principal Cauchy frame is indicated by the angle α

The shape functions depend on the material coordinates ξ_1 and ξ_2:

$$\psi_1 = \xi_1; \psi_2 = \xi_2; \psi_3 = 1 - \xi_1 - \xi_2 \tag{18}$$

First, an expression will be derived for the increment $\delta\mathbf{S}$ (equation (16)). Base vectors in the initial configuration are given by:

$$\vec{c}_{01} = \partial\vec{x}_0/\partial\xi_1 = \vec{x}_{01} - \vec{x}_{03} \tag{19}$$

$$\vec{c}_{02} = \partial\vec{x}_0/\partial\xi_2 = \vec{x}_{02} - \vec{x}_{03} \tag{20}$$

$$\vec{c}_{03} = \partial\vec{x}_0/\partial\xi_3 = D_0\vec{n}_{03} \tag{21}$$

where the subscript 0 denotes values in the initial configuration. The reciprocal vectors are given by:

$$\vec{\gamma}_{01} = 1/c\vec{c}_{02} * \vec{c}_{03} \tag{22}$$

$$\vec{\gamma}_{02} = 1/c\vec{c}_{03} * \vec{c}_{01} \tag{23}$$

$$\vec{\gamma}_{03} = 1/c\vec{c}_{01} * \vec{c}_{02} \tag{24}$$

where

$$c = \vec{c}_{03} \bullet (\vec{c}_{01} * \vec{c}_{02}) \tag{25}$$

It can be derived that $\mathbf{F}$ is given by:

$$\mathbf{F} = \partial\psi_k/\partial\xi_\tau \vec{x}_k \vec{\gamma}_{0_\tau} + D\vec{n}_3\vec{\gamma}_{03} \tag{26}$$

From equations (10) and (26) it follows that:

$$\mathbf{S} = \partial\psi_k/\partial\xi_\tau \vec{x}_k \vec{\gamma}_{0_\tau} + D_0\vec{n}_3\vec{\gamma}_{03} \tag{27}$$

Using equation (27) leads to:

$$\delta\mathbf{S} = \partial\psi_k/\partial\xi_\tau \delta\vec{u}_k \vec{\gamma}_{0_\tau} + D_0\delta\vec{n}_3\vec{\gamma}_{03} \tag{28}$$

where $\vec{u}_k$ is the nodal displacement of node k. Secondly, expressions will be derived for the increment $\delta\vec{n}_3$ (equation (28)) and the increment $\delta(J\sigma)$ (equation (16)).

In Roddeman et al. (1987) it is shown that the real stresses in a wrinkled membrane can be determined by modifying the deformation tensor in the constitutive equation:

$$(J\sigma) = \mathbf{F}' \bullet \mathbf{H}(\mathbf{E}') \bullet \mathbf{F}'^c \tag{29}$$

with:

$$\mathbf{F}' = (\mathbf{I} + \beta\vec{n}_1\vec{n}_1) \bullet \mathbf{F} \tag{30}$$

where $\mathbf{I}$ is the unit tensor. The direction of the principal Cauchy frame is denoted by the angle α (see Fig. 3).

The parameters α and β and the element's thickness D are determined numerically by using the coupled conditions:

(*a*) there is no stress in wrinkling direction:

$$\vec{n}_1 \bullet (J\sigma) \bullet \vec{n}_1 = 0 \tag{31}$$

(*b*) the frame $\vec{n}_1$, $\vec{n}_2$, $\vec{n}_3$ is the principal frame:

$$\vec{n}_1 \bullet (J\sigma) \bullet \vec{n}_2 = 0 \tag{32}$$

(*c*) membranes are in a state of plane stress:

$$\vec{n}_3 \bullet (J\sigma) \bullet \vec{n}_3 = 0 \tag{33}$$

Using equations (29) to (33), it is possible to derive expressions for the increments in $\vec{n}_3$ and $J\sigma$, which can be represented by:

$$\delta\vec{n}_3 \approx \mathbf{N}^*_{3k} \bullet \delta\vec{u}_k \tag{34}$$

$$\delta(J\sigma) \approx {}^3\sigma^*_k \bullet \delta\vec{u}_k \tag{35}$$

Thus, increments in these terms are depending on increments in the primary unknown nodal displacements, via equations (34) and (35). The second order tensor $\mathbf{N}^*_{3k}$ (in equation (34)) and the third order tensor ${}^3\sigma^*_k$ (in equation (35)) depend on the current estimation of the nodal displacements. Using equations (34) and (35) the discretized form of equation (16) may be written as:

$$\int_{V_0} [\{(\vec{\nabla}_0\vec{h})^c \bullet \mathbf{S}^{*-1}\} : \{{}^3\sigma^*_k \bullet \delta\vec{u}_k\} - \{\mathbf{S}^{*-1} \bullet (J_\sigma)^* \bullet (\vec{\nabla}_0\vec{h})^c\} : \{S^{*1} \bullet (\partial\psi_k/\partial\xi_\tau \delta\vec{u}_k \vec{\gamma}_{0_\tau} + D_0\mathbf{N}^*_{3k} \bullet \delta\vec{u}_k\vec{\gamma}_{03})\}]dV_0 - \int_{A_0} \delta\vec{k}_0 \bullet \vec{h} dA_0 \approx - \int_{V_0} \pi^* : (\vec{\nabla}_0\vec{h})^c dV_0 + \int_{A_0} \vec{k}^*_0 \bullet \vec{h} dA_0 \tag{36}$$

Because of the approximation for the displacement field it is no longer possible to fulfill exactly equation (36) and the boundary conditions. However, an approximation can be made by choosing well determined weighting functions for $\vec{h}$. Often good results can be obtained by using the shape functions, introduced in equation (17), as weighting functions. So we introduce:

$$\vec{h} = \psi_l \vec{h}_l \tag{37}$$

where $\vec{h}_l$ is the value of the function $\vec{h}$ in the nodal point l.

Using equation (37) and the fact that all terms in the volume integral in expression (36) are constant over the volume, expression (36) may be replaced with:

$$\vec{h}_l \bullet [V_0\partial\psi_l/\partial\xi_v \vec{e}_p \vec{\gamma}_{0_v} \bullet S^{*-1} \bullet {}^3\sigma^{*rc}_k \bullet \vec{e}_p - V_0\partial\psi_l/\partial\xi_v \partial\psi_k/\partial\xi_\tau \vec{\gamma}_{0_\tau} \bullet \vec{e}_p (J\sigma)^* \bullet \mathbf{S}^{*-c} \bullet \vec{e}_p \vec{\gamma}_{0_v} \bullet \mathbf{S}^{*-1} - V_0 D_0 \partial\psi_l/\partial\xi_v \vec{\gamma}_{03} \bullet \vec{e}_p (J\sigma)^* \bullet \mathbf{S}^{*-c} \bullet \vec{e}_p \vec{\gamma}_{0_v} \bullet \mathbf{S}^{*-1} \bullet \mathbf{N}^*_{3k}] \bullet \delta\vec{u}_k - \int_{A_0} \delta\vec{k}_0 \bullet \vec{h} dA_0 \approx - \int_{V_0} \pi^* : (\vec{\nabla}_0\vec{h})^c dV_0 + \int_{A_0} \vec{k}^*_0 \bullet \vec{h} dA_0 \tag{38}$$

in which use has been made of the identity:

$$({}^3\mathbf{A}^{rc} \bullet \vec{w}) \bullet \vec{v} = ({}^3\mathbf{A} \bullet \vec{v}) \bullet \vec{w}$$

where $\vec{w}$ and $\vec{v}$ are arbitrary vectors. Furthermore $\vec{e}_1$, $\vec{e}_2$, and $\vec{e}_3$ denote an orthonormal set of vectors. The first term on the right of equation (38) will be written as:

$$\int_{V_0} \pi^* : (\vec{\nabla}_0\vec{h})^c dV_0 = \vec{h}_l \bullet [V_0 \vec{e}_p \partial\psi_l/\partial\xi_v \vec{\gamma}_{0_v} \bullet \pi^* \bullet \vec{e}_p] = \vec{h}_l \bullet \vec{f}^*_l \tag{39}$$

in which the vector $\vec{f}^*_l$ is defined, which is the equivalent nodal force in node l due to the stresses in the membrane. We only consider constant forces on the nodal points. Thus:

$$\int_{A_0} \delta\vec{k}_0 \bullet \vec{h} dA_0 = 0 \tag{40}$$

and:

$$\int_{A_0} \vec{k}^*_0 \bullet \vec{h} dA_0 = \vec{h}_l \bullet \vec{f}_l \tag{41}$$

where $\vec{f}_l$ is the prescribed nodal force on node l.

According to equations (39) to (41), expression (38) may be written as:

$$\vec{h}_l \bullet \mathbf{K}^*_{lk} \bullet \delta\vec{u}_k \approx \vec{h}_l \bullet [\vec{f}_l - \vec{f}^*_l] \tag{42}$$

where $\mathbf{K}^*_{lk}$ contains the term $[V_0 \ldots\ldots\ldots \mathbf{N}^*_{3k}]$ from equation (38). Equation (42) is satisfied by arbitrary $\vec{h}_l$ if:

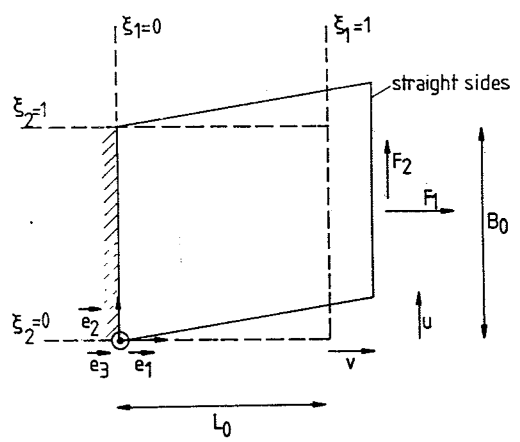

Fig. 4 Simple shear and stretching of a membrane

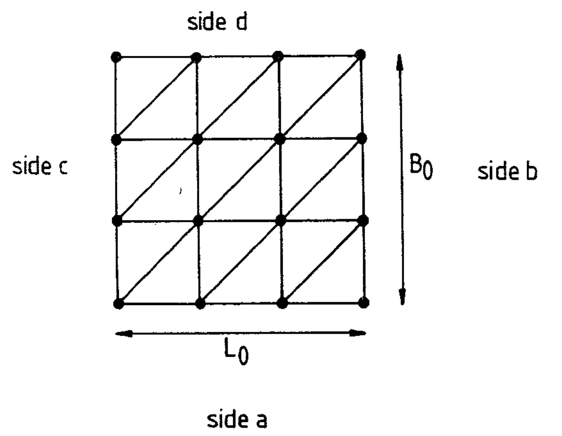

Fig. 5 An element mesh with 18 elements

$$\mathbf{K}^*_{lk}\cdot\delta\vec{u}_k \approx \vec{f}_l - \vec{f}^*_l \tag{43}$$

The matrix representation of the first term on the left side with respect to a certain base is called the stiffness matrix of the element. The right side contains the residual nodal forces.

Logical Structure of the Element

Given a new estimation of the nodal displacements, the element decides on the following criteria whether it is taut, wrinkled, or slack. If both the principal Cauchy stresses in an analysis without wrinkling are non-negative, the element is taut. In this situation the stiffness matrix and the equivalent nodal forces are determined by normal analysis (i.e., without wrinkling terms). If at least one of the principal Cauchy stresses in the analysis without wrinkling would be negative, and one principal Green-Lagrange strain in the analysis without wrinkling would be positive, the element is wrinkled. Then the new stiffness matrix and equivalent nodal forces are determined on the basis of wrinkling analysis. Otherwise the element is slack and the stiffness matrix and the equivalent nodal forces only contain zeros.

The Iterative Process

Choosing a base, assemblage of the equations (43) of all the elements of the body and elimination of prescribed nodal displacements leads to:

$$\underline{\underline{K}}^*\cdot\delta\underline{u} = \underline{f} - \underline{f}^* \tag{44}$$

with

$\underline{\underline{K}}^*$ the stiffness matrix of the structure;
$\delta\underline{u}$ the total column of incremental free nodal displacements;
$\underline{f}$ the total column of prescribed forces on the body;
$\underline{f}^*$ the total column of equivalent nodal forces due to the stresses in the elements.

Since there may be completely slack regions in the structure, the total stiffness matrix may be singular and Newton-Raphson iteration cannot be used. A globally convergent iterative procedure, based on a minimization problem, which can be used, for example, is a restricted step method where the column $\delta\underline{u}$ is the solution of:

$$(2\underline{\underline{K}}^{*T}\cdot\underline{\underline{K}}^* + \theta\underline{\underline{I}})\cdot\delta\underline{u} = 2\underline{\underline{K}}^{*T}\cdot(\underline{f} - \underline{f}^*) \tag{45}$$

The parameter θ is chosen each iteration step such that $(2\underline{\underline{K}}^{*T}\cdot\underline{\underline{K}}^* + \theta\underline{\underline{I}})$ is positive definite (which is true if θ is positive) and such that the norm of $(\underline{f} - \underline{f}^*)$ decreases. For a theoretical background see Fletcher (1980).

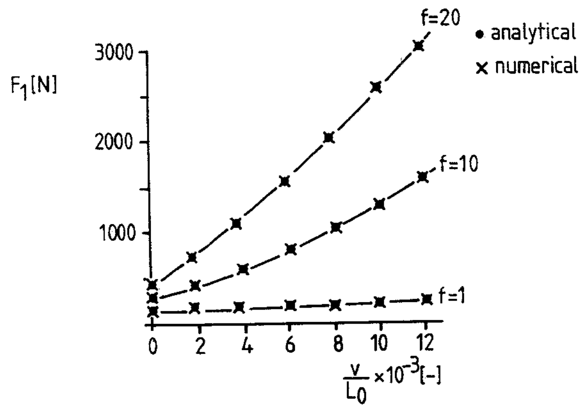

Fig. 6 The force F_1 as a function of the stretching distance v for different values of the anisotropy parameter f

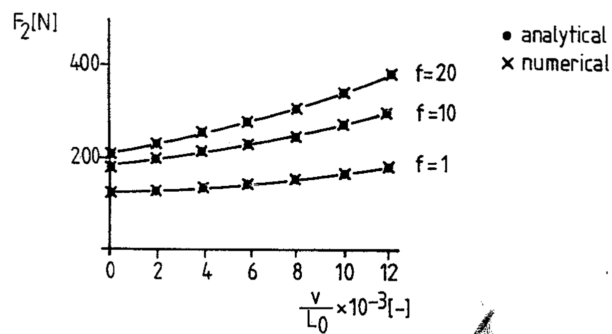

Fig. 7 The force F_2 as a function of the stretching distance v for different values of the anisotropy parameter f

Test Problems

In Roddeman et al. (1987) the analytical solution of the wrinkling of a membrane deformed by a simple shear and stretching is derived. In Fig. 4 the simple shear test is illustrated.

Again we have studied the behavior of the membrane when, with constant simple shear, the membrane is stretched. An element mesh with 18 elements is used (Fig. 5). The nodes on the sides of the mesh are prescribed displacements according to the analytical solution, so the nodal displacements in the inner area should converge to the analytical solution and the analytical results should be regained. Figure 6 and Fig. 7 compare analytical and numerical results. Tranversally isotropic material is used for the membrane. The parameter f determines the measure of anisotropy (see Roddeman et al., 1987). If $f=1$ then the membrane is isotropic, if $f>1$ then $\vec{e}_1$ is the stiffest direction. It can be seen that the agreement between the analytical and numerical results is very good.

Conclusions

The Finite Element Method can be used in geometrically nonlinear analysis of anisotropic membranes which wrinkle. Test problems show good agreement between analytical and numerical results. Since completely slack regions in the structure lead to a singular stiffness matrix, the iterative procedure should be chosen with care. Comparison between numerical and experimental results still has to be done. This is part of our present work and will be published later.

Acknowledgments

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