The Zero Lower Bound and Endogenous Uncertainty

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ABSTRACT

This paper documents that a strong negative correlation between various measures of macroeconomic uncertainty and real GDP growth only emerged since the onset of the Great Recession. Before that event the correlation was weak and in many cases not statistically less than zero, even when restricting the data sample to only include recessions. A major difference between the Great Recession and previous recessions is that the Fed has been constrained by the ZLB on the federal funds rate. We contend that the ZLB constraint contributed to the stronger negative correlation that emerged in mid-2008. To test our theory, we use a model where the ZLB occasionally binds. The model has the same key feature as the data—away from the ZLB the correlation is weak but strongly negative when the policy rate is close to or at its ZLB. Our model is also consistent with the stronger correlations that emerged in the data between real GDP growth and both inflation and nominal interest rate uncertainty in mid-2008.

Keywords: Monetary Policy; Uncertainty; Economic Activity, Zero Lower Bound, Survey Data
JEL Classifications: E32; E47; E58

1 INTRODUCTION

There is significant interest in understanding the relationship between uncertainty and economic activity. Several papers find a negative relationship in the data using various measures of uncertainty. For example, Bloom (2009) shows that unexpected increases in uncertainty, given by stock price volatility, are associated with declines in industrial production. Bekaert et al. (2013), Bloom et al. (2014), and Pinter et al. (2013) find similar relationships in the data. While those papers focus on financial market volatility, Jurado et al. (2015) use broader proxies for uncertainty and find that spikes in uncertainty are more infrequent but more persistent and more negatively correlated with hours, employment, and industrial production than the previous literature has typically estimated.[1] Leduc and Liu (2014) report that uncertainty shocks in the post-2008 period contributed to a much larger fraction of the observed unemployment fluctuations than in other recessions. They speculate that this finding is attributable to the zero lower bound (ZLB) constraint on monetary policy.

This paper documents that a strong negative correlation between uncertainty and real GDP growth only emerged since the Great Recession. We use forward-looking measures of uncertainty from survey and stock market data and estimates of realized volatility from a time-varying VAR with stochastic volatility. Before the Great Recession the correlation was weak and in many cases not statistically less than zero, even when restricting the data sample to only include quarters when the economy was in a recession. A major difference between the Great Recession and previous recessions is that the Fed has been constrained by the ZLB on the federal funds rate. We contend that the ZLB constraint contributed to the stronger negative correlation that emerged in mid-2008.

To test our theory, we use a New Keynesian model that imposes a ZLB constraint on the short-term nominal interest rate. The model predicts an increase in output uncertainty near and at the ZLB. When the nominal interest rate is far from its ZLB, uncertainty surrounding output is nearly constant and low. Therefore, the model has the same key feature as the data—away from the ZLB the correlation is weak but strongly negative when the short-term nominal interest rate is close to or at its ZLB. We also find that it is consistent with the stronger relationships that emerged in the data between real GDP growth and both inflation and nominal interest rate uncertainty in mid-2008.

There is an established literature that explores how uncertainty affects economic variables in structural models. This literature typically examines how endogenous variables respond to second moment shocks that exogenously increase uncertainty.[2] While the economic effects depend on the shocks, these models all produce negative relationships between uncertainty and economic activity.

Rational expectations models also contain uncertainty that is endogenous. In these models, households make predictions about the future realizations of both exogenous and endogenous variables. They also make forecasts about the degree of uncertainty surrounding those predictions. The measure of uncertainty in our theoretical model is equivalent to those forecasts, which in a mathematical sense is the expected volatility of the forecast errors regarding future variables.

Our results are important for the growing literature that links uncertainty and economic activity because they show that for particular states of the world uncertainty responds to what is

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[1] Others find a negative relationship between fiscal uncertainty and economic activity [Fernández-Villaverde et al. (2013), Born and Pfeifer (2014)] and oil price uncertainty and economic activity [Elder and Serletis (2010), Jo (2014)].

[2] A few recent examples include Bachmann and Bayer (2013), Bloom (2009), Bloom et al. (2014), Christiano et al. (2014), Fernández-Villaverde et al. (2011), Gilchrist et al. (2014), Justiniano and Primiceri (2008), and Mumtaz and Zanetti (2013). There is also an older literature that examines similar research questions to those posed in the stochastic volatility literature. See, for example, Leland (1968), Levhari and Srinivasan (1969), and Sandmo (1970).

happening in the economy. An increase in uncertainty occurs as the short-term nominal interest rate approaches its ZLB due to the restriction the constraint places on the ability of the central bank to stabilize the economy. Output becomes more responsive to shocks that hit the economy and, therefore, the distribution of future realizations of output become more dispersed when compared to the same distributions that exist when the short-term nominal interest rate is far from its ZLB. This result increases the expected volatility of the output forecast errors (i.e., uncertainty rises near or at the ZLB). Of course, these results do not rule out that economic activity can respond to uncertainty. It merely shows that in at least one case, when the short-term nominal interest rate is near or at its ZLB, uncertainty is responding to an event that is endogenous to the state of the economy.

It is well-known in the literature that the ZLB constraint has an important effect on the economy. Gust et al. (2013) estimate a nonlinear New Keynesian model with a ZLB constraint to quantify how much of the recent decline in output was due to the binding constraint. They find the constraint accounts for about 20% of the drop in U.S. real GDP from 2008 to 2009 and, on average, it caused output to be 1% lower from 2009 to 2011 than it would have been without the constraint. Nakov (2008) finds the optimal discretionary monetary policy leads to a more negative output gap at the ZLB when households face uncertainty about the real interest rate than when they have perfect foresight. Nakata (2012) also studies the effects of uncertainty when the ZLB binds by varying the standard deviation of discount factor shocks. He finds higher uncertainty increases the slope of the policy function for output, meaning positive discount factor shocks lead to a larger reduction in output when the ZLB binds. Basu and Bundick (2014) show cost and demand uncertainty shocks cause business cycle fluctuations, which become more pronounced when the ZLB binds. Specifically, they find that a positive demand uncertainty shock causes output to decline by 0.2% when the ZLB does not bind and by 0.35% when it binds. Moreover, they calculate that demand uncertainty shocks can account for one-fourth of the drop in output in mid-2008. These results show that the ZLB constraint increases the responsiveness of output to shocks, but we believe we are the first to explore what it means for expected second moments of endogenous variables.

Several other recent papers also study endogenous uncertainty but in different contexts. For example, Bachmann and Moscarini (2012) examine a model where uncertainty increases in recessions because it is less costly for firms to experiment with price changes to learn about their market power. Van Nieuwerburgh and Veldkamp (2006) argue that low production during a recession leads to noisy forecasts that impede learning and slow the recovery. In a related paper, Fajgelbaum et al. (2014) allow for self-reinforcing episodes of high uncertainty and low economic activity. In their model, firms learn about fundamentals by observing the investment activity of other firms. Investment is low in recessions and, since information flows slowly, uncertainty is high, which further discourages investment and causes an uncertainty trap. Gourio (2014) finds that the volatility of output is countercyclical because customers, suppliers, and workers expect larger losses when adverse shocks raise the probability that firms default. Navarro (2014) sets up a model where financial crises endogenously generate higher volatility. In our model, the constraint imposed by the ZLB reduces the effectiveness of monetary policy, which makes output more responsive to shocks that hit the economy and increases the expected forecast error volatility.

The rest of the paper is organized as follows. Section 2 describes our measures of uncertainty in the data and computes correlations between those measures and economic activity. Section 3 introduces our theoretical model, its calibration, and the solution method. Section 4 provides a theoretical explanation for the stronger negative correlations that emerged in mid-2008. Section 5 compares the correlations in the model to equivalent correlations in the data. Section 6 concludes.

2 RELATIONSHIP BETWEEN ECONOMIC ACTIVITY AND UNCERTAINTY

This section introduces three forward looking measures of macroeconomic uncertainty and shows how they are correlated with economic activity. We first compute the correlations using real GDP as our measure of economic activity and then compute equivalent correlations using industrial production. We also calculate similar correlations using a time-varying VAR with stochastic volatility.

2.1 DATA DESCRIPTION   Figure 1 shows three alternative measures of economic uncertainty: (1) the Chicago Board Options Exchange S&P 100 Volatility Index (VXO), (2) the dispersion in large manufacturers' forecasts of business activity from the Business Outlook Survey (BOS), and (3) the dispersion in forecasts of real GDP 1-quarter ahead from the Survey of Professional Forecasters (SPF). The shaded regions correspond to recessions, according to the National Bureau of Economic Research. We focus on these data series because they are forward looking measures of uncertainty and are able to capture changes in people's expectations over time, as opposed to predictions about future uncertainty that are based on statistical relationships (e.g., a GARCH model). We also believe macroeconomic uncertainty is an important factor that influences the behavior of all of these measures, even though they represent different segments of the economy.
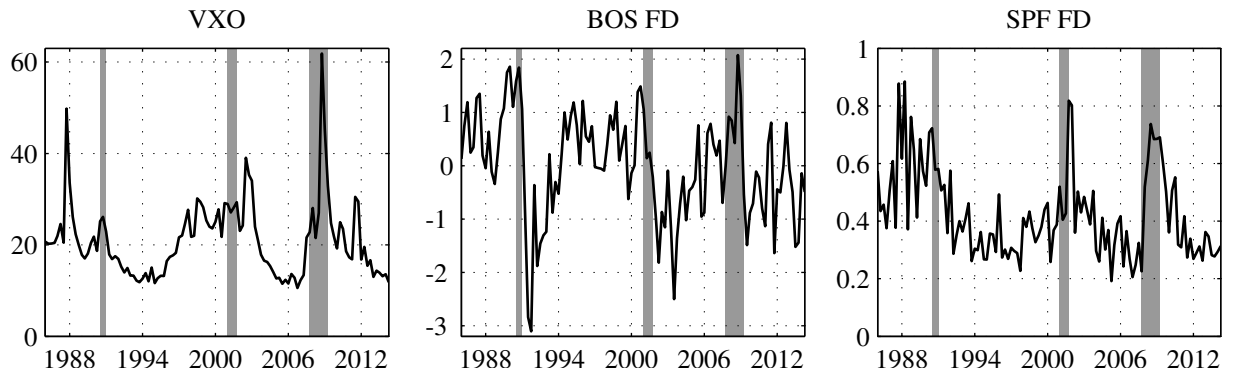

Figure 1: Alternative measures of economic uncertainty. Chicago Board Options Exchange Volatility Index (VXO): expected volatility in the S&P 100 over the next 30 days at an annualized rate; Business Outlook Survey Forecast Dispersion (BOS FD): dispersion in large manufacturers' forecasts of business activity over the next six months; Survey of Professional Forecasters real GDP Forecast Dispersion (SPF FD): dispersion in forecasts of real GDP 1-quarter ahead. The shaded regions correspond to recessions, according to the National Bureau of Economic Research.

The VXO measures the expected volatility in the S&P 100 stock market index over the next month at an annualized rate. For example, if the value on the vertical axis is $x\%$, then people expect there is a 68% chance the S&P 100 index will change by $\pm x/\sqrt{12}\%$ over the next month. We average the daily series each quarter so it is consistent with the frequency of real GDP releases.

The Business Outlook Survey (BOS), which is conducted monthly by the Federal Reserve Bank of Philadelphia, asks large manufacturing firms to forecast whether general business activity will increase, decrease, or remain unchanged over the next six months. Following Bachmann et al. (2013), the forecast dispersion (FD) in the firms' survey responses in month t is given by

$$\text{BOS FD}_t = \sqrt{\text{Frac}_t^+ + \text{Frac}_t^- - \left(\text{Frac}_t^+ - \text{Frac}_t^-\right)^2},$$

where an increase (decrease) in business activity is labeled as $+1$ (-1) and Frac^+ (Frac^-) is the fraction of firms who forecast that outcome. Thus, the BOS FD is the standard deviation of the

responses in each month. We average the monthly BOS FD series each quarter and then standardize the values so the vertical axis displays the number of standard deviations from the mean response.

The Survey of Professional Forecasters (SPF), which is conducted quarterly by the Federal Reserve Bank of Philadelphia, asks people who regularly make forecasts as part of their jobs to predict macroeconomic aggregates up to 4 quarters ahead (e.g., real GDP, inflation, interest rates). We focus on the forecasts of real GDP (y) 1 quarter ahead. The inter-quartile FD is given by

$$\text{SPF FD}_t = 100 \times (\log(y_{t+1|t-1}^{75}) - \log(y_{t+1|t-1}^{25})).$$

This value is the percent difference between the 75th and 25th percentiles of the quarter t forecasts of real GDP in quarter $t + 1$, given all observations in quarter $t - 1$ and earlier. We use the inter-quartile range, rather than more extreme percentiles, because on average only 41 firms complete the survey each quarter, which implies the tails of the distribution would have a very small sample.

Since our measures of uncertainty represent different segments of the economy (i.e., the stock market, manufacturing, and output), it is not surprising that they have different properties. The VXO is the least noisy measure, and it is only modestly higher during the 1991 and 2001 recessions. The three most prominent spikes in the index correspond to Black Monday (1987), the Enron scandal (mid-to-late 2002) and the second Gulf War (early 2003), and the 2008 financial crisis.[3] The SPF spikes around the time of Black Monday, after 9/11, and during the 2008 financial crisis. It also increases during the first Gulf War. The BOS rises right before the 1991 and 2001 recessions and during the Great Recession, which causes the largest spike in all three measures.

Sill (2012) computes a measure of uncertainty based on the standard deviation of the mean probability distribution assigned by the forecasters. Unfortunately, we could not use this measure because (1) the bins in the distribution changed in 1992Q1 and in 2009Q2, (2) the output variable was nominal GNP instead of real GDP before 1992Q1, and (3) individuals forecast the annual average GDP growth rate, which causes the information set to increase during the year. The last concern is the most problematic for our study because it restricts the sample to only one value per year, which leaves us with too small of a sample to calculate correlations. For those reasons, we use forecaster disagreement as a proxy for uncertainty, which is common in the literature. Moreover, several papers have shown that disagreement provides a good approximation for uncertainty [e.g., Bachmann et al. (2013), Bomberger (1996), Clements (2008), and Giordani and Soderlind (2003)].

2.2 CORRELATIONS IN THE U.S.   Table 1 shows the correlations between the growth rate of economic activity (i.e., quarter-over-quarter log differences in either real GDP or industrial production) and our measures of uncertainty with different samples.[4] The top row is based on data before the Great Recession (1986Q1-2008Q2) and the second row uses data since the Great Recession (2008Q3-2014Q2). Our data series begins in 1986Q1 rather than an earlier date because (1) there were major changes in monetary policy beginning in the 1980s, and (2) the VXO is only available since 1986Q1 and we wanted to draw connections to this commonly used measure of uncertainty.[5] We evaluate whether the correlations, ρ, are statistically less than zero using a one-tailed t-test.

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[3]See Bloom (2009) for a more detailed list documenting spikes in realized and expected stock price volatility.

[4]We also examined realized stock price volatility, but the results are similar to the VXO so they are not reported.

[5]There is also evidence of a structural break at the start of the Great Recession. The correlation between real GDP growth and the SPF FD before the Great Moderation (1968Q4-1985Q4) is -0.34, and the correlation since the Great Moderation (1986Q1-2008Q2) is -0.09, which are significantly different at a 5% level. The correlations with realized stock price volatility in the two samples are also significantly different at a 5% level. Although it is interesting to examine why the correlations changed during the Great Moderation period, we leave that exercise for future research.

Real GDP Growth vs. Uncertainty   The first three columns of table 1 use real GDP as a measure of economic activity. The results indicate that a strong negative relationship between our uncertainty measures and real GDP growth only emerged in recent data. The correlations based on the pre-Great Recession sample are weak, and they are not statistically less than zero when calculated with the VXO or SPF FD. Moreover, if we remove the quarters when the exogenous events identified in Bloom (2009) occurred in the pre-Great Recession sample, then none of the correlations are significant at a 5% level. In sharp contrast, all of the correlations based on data since the Great Recession are significant at a 1% level. We also use the Fisher z-transformation to test whether the correlations in these samples are statistically different. Those tests reveal that the correlations are different at a 1% level when calculated with the VXO and BOS and at a 5% level with the SPF. We obtain similar correlations between real GDP and the SPF FD over longer forecast horizons.[6]

	Real GDP Growth vs. Uncertainty			IP Growth vs. Uncertainty		
	VXO	BOS FD	SPF FD	VXO	BOS FD	SPF FD
Pre-Great Recession (1986Q1-2008Q2)	-0.04	-0.20**	-0.09	-0.09	-0.19**	-0.29***
Post-Great Recession (2008Q3-2014Q2)	-0.74***	-0.70***	-0.53***	-0.75***	-0.60***	-0.70***
Past Recessions (1968Q4-2007Q3)	$0.09^{\dagger}$	-0.16	-0.20	$0.00^{\dagger}$	0.01	-0.31*

Table 1: Correlations between the growth rate of economic activity (quarter-over-quarter log differences) and various measures of macroeconomic uncertainty. A † denotes a correlation calculated with realized stock price volatility as a measure of uncertainty. The values are statistically less than 0 at a ***1%, **5%, and *10% significance level.

The results in table 1 suggest that recessions may be a source of high uncertainty, but, perhaps counterintuitively, the bottom row shows there is little evidence for this relationship in the data. Both the SPF and BOS surveys began in 1968Q4. Between that date and the beginning of the Great Recession, the U.S. economy experienced 6 recessions, totaling 27 quarters. The correlations between real GDP growth and uncertainty in those quarters have roughly the same magnitude as the pre-Great Recession correlations. Moreover, none of the values are statistically less than zero, even at a 10% significance level. These results suggest there are unique features of the Great Recession that led to a strong negative relationship between uncertainty and real GDP growth.

Industrial Production Growth vs. Uncertainty   We focus on real GDP, but empirical work has often used industrial production as a measure of economic activity [e.g., Bloom (2009), Bekaert et al. (2013), and Jurado et al. (2015)]. To compare our findings with the literature, the last three columns of table 1 use industrial production growth instead of real GDP growth to compute correlations with uncertainty. Similar to our results based on real GDP growth, the correlations between industrial production growth and our uncertainty measures are stronger in the post-Great Recession sample than the pre-Great Recession sample. The Fisher z-transformation test shows the correlations in the two samples are statistically different at a 1% level when calculated with the VXO and SPF FD and a 5% level with the BOS FD. Interestingly, the correlations with the SPF FD are much stronger and statistically less than 0 in both the pre- and post-Great Recession samples.

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[6]The SPF computes growth rates of real GDP with the forecasts of real GDP in levels. The dispersion in the implied quarter-over-quarter growth rate of real GDP produces virtually identical correlations to those shown in table 1.

A benefit of industrial production is that it is released every month. Thus, we can calculate the same correlations at a monthly frequency since the BOS is conducted monthly and the daily VXO series can be aggregated at a monthly frequency. Our qualitative results are robust to using monthly data. The correlations in the pre-Great Recession sample are weak ($\rho(\text{IP Growth}, \text{VXO}) = -0.05$ and $\rho(\text{IP Growth}, \text{BOS FD}) = -0.14$), but more negative in the post-Great Recession sample ($\rho(\text{IP Growth}, \text{VXO}) = -0.43$ and $\rho(\text{IP Growth}, \text{BOS FD}) = -0.36$). Moreover, the correlations with the BOS FD (VXO) in the two samples are statistically different at a 5% (1%) level.[7]

The correlations in the pre-Great Recession sample are generally more negative than when we use real GDP, which suggests there is something different about the manufacturing sector that sets it apart from the overall economy. In our paper, we are interested in the connection between uncertainty and real GDP growth. It is beyond the scope of this paper and our theoretical model to formally explore why the relationship is stronger with manufacturing related variables, but we speculate that the difference may have to do with the fact that manufacturing activity is more volatile and more sensitive to changes in business conditions than other segments of the economy.

	Real GDP Growth			Industrial Production Growth		
	VXO	BOS FD	SPF FD	VXO	BOS FD	SPF FD
Uncertainty	-0.01	-7.26*	-1.18	-0.05	-12.50**	-6.82**
Post-GR	2.63**	23.17***	2.13	9.09***	54.61***	10.12***
Interaction	-0.20***	-36.21***	-9.51**	-0.48***	-81.65***	-28.86***

Table 2: OLS estimates of the growth rate of economic activity on various measures of uncertainty, an indicator for the post-Great Recession sample, and an interaction term. We compute Newey-West standard errors to correct for autocorrelation. The values are statistically different from 0 at a ***1%, **5%, and *10% significance level.

OLS Estimates One potential issue with the correlations in the previous section is that the standard errors are based on the assumption that the observations are independent. To provide additional evidence that the relationship between between economic activity and various measures of uncertainty changed in the post-Great Recession period, we estimate the following model:

$$\Delta \ln(y_t) = \beta_0 + \beta_1 \sigma_{y,t} + \beta_2 \text{Post-GR}_t + \beta_3 (\sigma_{y,t} \times \text{Post-GR}_t) + \varepsilon_t,$$

where $\Delta \ln(y)$ is the growth rate in economic activity, σ_y is a measure of uncertainty, and Post-GR is a binary variable that takes on a value of 1 during the post-Great Recession period and 0 otherwise. β_1 captures the association between uncertainty and economic activity during the pre-Great Recession period, β_2 measures the effect of the Great Recession on economic activity after controlling for uncertainty, and β_3 is the marginal effect on the relationship between uncertainty and economic activity in the post-Great Recession period. We estimate the model with OLS using two measures of economic activity—real GDP growth and industrial production growth—and each of our three measures of macroeconomic uncertainty: the VXO, BOS FD, and SPF FD. We test for significance using Newey-West standard errors to correct for autocorrelation in the data. This estimation is not intended to establish causation, but rather to provide support for previous results.

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[7]We also calculate correlations between both of our measures of economic activity and the SPF forecast dispersion for industrial production (SPF IP FD). This measure of uncertainty is analogous to the SPF FD for real GDP. The correlations are more negative in the post-Great Recession sample but statistically less than zero in both samples.

Table 2 displays the results of our estimation, which are qualitatively similar to the correlations in table 1. The standard deviation of the VXO (BOS FD, SPF FD) is 8.15 (0.06, 0.15), so the marginal effect of a 1 standard deviation increase in the VXO (BOS FD, SPF FD) during the post-Great Recession period is associated with a 1.6% (2.2%, 1.4%) decrease in real GDP growth. In the pre-Great Recession sample, the relationship between uncertainty and real GDP growth is weak, as none of the estimates are significant at a 5% level. The marginal effect of uncertainty in the post-Great Recession sample, however, is significant at a 5% level with all three uncertainty measures. When we use industrial production growth to measure economic activity, the effect of the BOS FD and SPF FD is significant at a 5% level in the pre-Great Recession sample, but the marginal effect of all of our uncertainty measures in the post-Great Recession period is significant at a 1% level. These estimates show that our results are robust to correcting for autocorrelation.

Correlations between Uncertainty Measures   We also investigate the correlations between the various uncertainty measures. All of them are positively correlated, but the correlations in the pre-Great Recession sample are relatively weak ($\rho(\text{SPF FD}, \text{VXO}) = 0.45$, $\rho(\text{SPF FD}, \text{BOS}) = 0.13$, $\rho(\text{BOS}, \text{VXO}) = 0.11$). In the post-Great Recession sample, the correlations are much stronger ($\rho(\text{SPF FD}, \text{VXO}) = 0.71$, $\rho(\text{SPF FD}, \text{BOS}) = 0.38$, $\rho(\text{BOS}, \text{VXO}) = 0.60$), which indicates that there was something happening in the economy that affected all of the uncertainty measures.

Time-varying VAR with Stochastic Volatility   To provide further evidence for how the correlations between real GDP growth and uncertainty changed in mid-2008, this section computes correlations based on a measure of uncertainty that does not rely on stock market or survey data.

Following Primiceri (2005), we use a time-varying VAR with stochastic volatility to estimate the volatility of real GDP growth each quarter and then show how those estimates are correlated with economic activity. One difference between Primiceri (2005) and our estimation is that we use real GDP growth instead of the unemployment rate so we can draw comparisons to equivalent statistics in our theoretical model. Another difference is we use data from 1958Q2 to 2014Q2, where the first ten years train the prior distributions of the parameters. The model, given by,

$$y_t = B_{0,t} + B_{1,t}y_{t-1} + B_{2,t}y_{t-2} + A_t^{-1}\Sigma_t\varepsilon_t, \quad t = 1, \dots, T,$$

has a 2-quarter lag, so our estimates are from 1968Q4 to 2014Q2, which is the same period as the survey data. y_t is a 3×1 vector that includes real GDP growth, the inflation rate, and the T-bill rate, $B_{0,t}$ is a 3×1 vector of time varying intercepts, $B_{1,t}$ and $B_{2,t}$ are 3×3 matrices of time varying coefficients, and ε_t is a normally distributed shock with an identity variance-covariance matrix. A_t and Σ_t are the result of a triangular reduction of the variance-covariance matrix, where

$$A_t = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{bmatrix}, \qquad \Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \sigma_{2,t} & 0 \\ 0 & 0 & \sigma_{3,t} \end{bmatrix}.$$

The model is estimated with Bayesian MCMC methods using code that accompanies Koop and Korobilis (2010). The code implements a correction to the algorithm outlined in Appendix A of Primiceri (2005), as explained by Del Negro and Primiceri (2015). The estimate for the volatility of real GDP growth, $\sigma_{1,t}$, is a proxy for macroeconomic uncertainty. We calculate correlations between real GDP growth and its estimated volatility with both the pre- and post-Great Recession samples. Our estimates are based on $100{,}000$ draws from the posterior probability distribution.
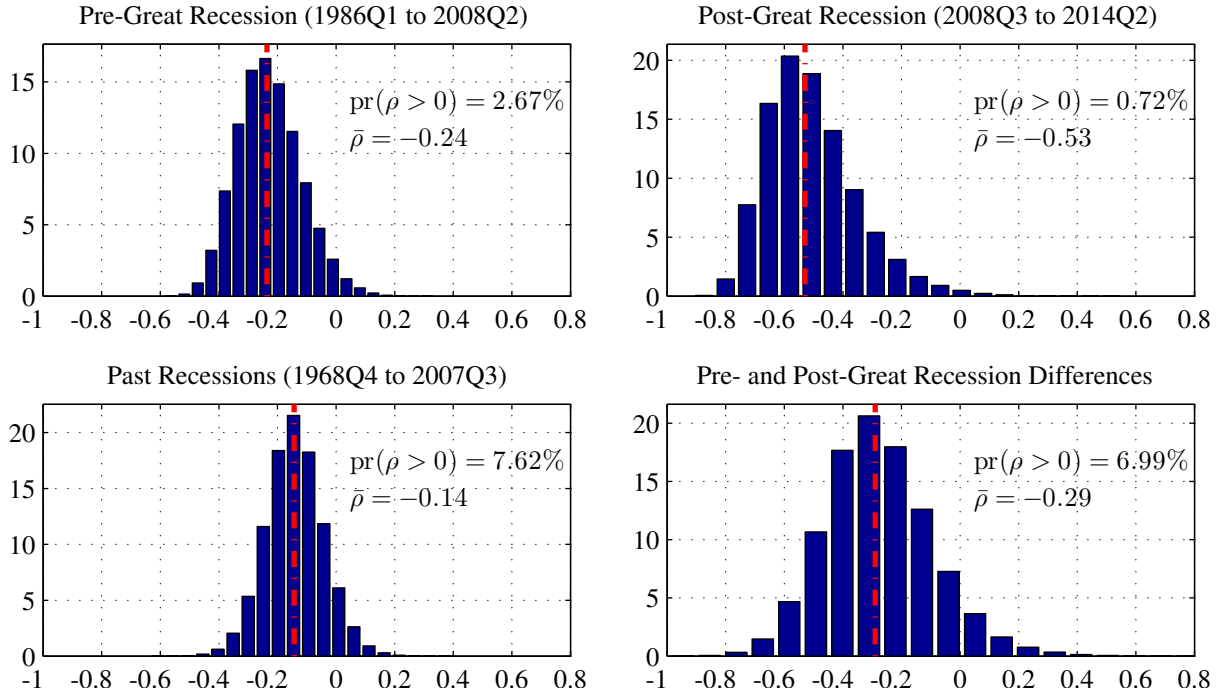
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Figure 2: Distributions of the correlation, ρ, between real GDP growth and its estimated volatility.

Figure 2 shows the distributions of the correlation, ρ, between real GDP growth and its volatility. With the pre-Great Recession sample, the correlation exceeds zero in 2.67% of draws and the median is -0.24. In contrast, the correlation with the post-Great Recession sample exceeds zero in only 0.72% of draws and the median is -0.53, which is more than twice as strong. When we only include quarters when the economy was in a recession, the correlation exceeds zero in 7.62% of draws and the median is closer to zero (-0.14). To give us a better sense for whether there is evidence that the correlations in the two samples are different, we plot the distribution of the post-Great recession correlations minus the pre-Great Recession correlations. A small fraction of positive values provides evidence for our previous finding that the correlation between real GDP growth and uncertainty has been more negative since mid-2008. Our estimation reveals that the difference between the correlations is positive in only 6.99% of the draws and the median difference is -0.29. These results are stronger if we remove the influence of the Gulf Wars and 9/11, which are arguably exogenous events. In that case, the difference between the correlations is positive in only 5.20% of draws and the median difference falls to -0.32. These results provide additional support for our findings in table 1, since they are based on an alternative measure of uncertainty.

2.3 SUMMARY OF U.S. CORRELATIONS   Thus far, we have examined correlations in the U.S between several proxies for uncertainty and two measures of economic activity. We found robust evidence of a strong negative correlation since the Great Recession. The correlations with real GDP before the Great Recession are weak, and often not statistically less than zero. The correlations with industrial production are negative in both periods but stronger since the Great Recession. Moreover, the correlations during other recessions are weak, which suggests that there is something unique about the post-Great Recession period that is causing the stronger relationship in the data.

A major difference between the Great Recession and previous recessions in the U.S. is that the

Fed has been constrained by the ZLB on the federal funds rate. Central banks typically conduct open market operations to stimulate demand during an economic downturn, but by 2008Q2 the Fed had cut the federal funds rate to 2% and economic conditions were sufficiently poor that a policy rate near zero was possible. In the following months, the Fed continued to reduce the federal funds rate and in 2008Q4 it hit its ZLB. More than six years after the ZLB was first hit, the Fed's target interest rate remains near zero. We contend that the binding constraint the ZLB placed on current and future monetary policy is one important factor that contributed to the negative correlation that emerged between real GDP growth and various measures of uncertainty since mid-2008.

Another potential source of the stronger correlations in the post-Great Recession sample is the rise in financial stress. One way to control for changes in financial stress is to add another variable to our VAR. Unfortunately, most measures of financial stress were developed in the early 1990s (e.g., Kansas City Financial Stress Index, Bloomberg Financial Conditions Index), which leaves us with too short of a sample to estimate the model. A proxy for financial stress that is included in all of the newer indices is an interest rate spread. The benefit of using an interest rate spread is that most spreads are strongly correlated with the financial stress indices and data is available over a longer period. We estimate our VAR using the Baa/Aaa and Aaa/10-year treasury spreads.[8]

Our qualitative results are robust to controlling for changes in the interest rate spread. When we include the Baa/Aaa spread in the VAR, the median correlation between real GDP growth and the estimated volatility of real GDP is -0.21 in the pre-Great Recession sample and -0.47 in the post-Great Recession sample. The difference between the correlations in the two samples is positive in only 12.54% of the draws and the median difference is -0.26. Alternatively, if we include the Aaa/10-year treasury spread in the VAR, the correlations in the pre- and post-Great Recession samples are -0.20 and -0.51, respectively. In this case, the difference between the correlations is positive in only 8.02% of draws and the median difference declines to -0.31. These results are significant because they show there is a high probability that a stronger negative correlation emerged in the mid-2008 even when we include a proxy for the recent increase in financial stress.

2.4 CORRELATIONS OUTSIDE THE U.S.   Finally, we examine the same correlations for other economies that faced constraints on monetary policy similar to the Fed during the Great Recession. The bank rate in the U.K. was reduced to 0.5% in 2009Q1. The Euro-zone deposit rate was cut from 3.25% in 2008Q4 to 0.25% by the end of 2009Q1, and was further reduced to 0% in 2012Q3. We find that the correlations between real GDP growth and uncertainty echo those for the U.S.

The European Central Bank (ECB) has conducted a survey of professional forecasters since 1999Q1. It asks participants to forecast Euro area real GDP growth over various horizons. For example, the survey conducted in 1999Q1 requests forecasts for 1999Q3, given the last GDP release is from 1998Q3. Similar to the U.S. SPF, we calculate the forecast dispersion as ECB SPF $\text{FD}_t = |\hat{y}_{t+2|t-2}^{75} - \hat{y}_{t+2|t-2}^{25}|$, where $\hat{y}_{t+2|t-2}^{x}$ is the xth percentile of the quarter t forecast of real GDP growth in quarter $t+2$, given observations in quarter $t-2$ and earlier. The correlation between Euro area real GDP growth and the ECB SPF FD with the pre-Great Recession sample (1999Q1-2008Q2) is -0.32, while the correlation with the post-Great Recession sample (2008Q3-2014Q2) is -0.47.[9]

The Bank of England also conducts a survey called the Survey of External Forecasters (BOE SEF), which has asked its participants to forecast real GDP growth since 1998Q1. Prior to 2006Q2,

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[8]See Hakkio and Keeton (2009) and Kliesen et al. (2012) for an overview of the various financial stress indices.

[9]We obtain similar results when we use the industry survey in the European Commission's Business and Consumer surveys. This survey asks manufacturers how they expect their production to develop over the next three months.

the survey asked for projections in quarter 4 of the survey year, quarter 4 1 year ahead, and the same quarter 2 years ahead. For example, the forecast dates in the 2006Q1 survey were 2006Q4, 2007Q4, and 2008Q1. Since 2006Q2, the survey has asked for projections for the same quarter 1, 2, and 3 years ahead. Unfortunately, we cannot calculate correlations with the pre-Great Recession sample because the forecast horizons change.[10] With the post-Great Recession sample, the correlation between real GDP growth and the dispersion in forecasts 1 year ahead is -0.46, which is similar in magnitude to the correlations we computed with the U.S. and Euro area surveys.[11]

Given the short sample of the survey data, we also computed correlations between real GDP growth and uncertainty using estimates of real GDP volatility from a time-varying VAR with stochastic volatility. Unfortunately, we cannot do the same exercise for the Euro area since data is only available from 1995Q1. For the U.K., we used the same data and sample period we used to estimate the model with U.S. data. With the pre-Great Recession sample, the correlation exceeds zero in 73% of draws and the median is 0.08. The post-Great Recession sample, however, is positive in only 0.05% of draws and the median is -0.67. Moreover, the difference between the correlations in the two samples is positive in only 0.05% of draws and the median difference is -0.74. These results indicate a strong negative correlation also emerged in the U.K. in mid-2008.

Finally, we estimate a time-varying VAR with stochastic volatility using Japanese data on real GDP growth, the inflation rate, and the T-bill rate from 1960Q2 to 2014Q2. Japan is a unique country to study because it has been constrained by the ZLB for a much longer period than the other countries we examined. By April 1995, the Bank of Japan had lowered its discount rate to 1% and the T-bill rate hit 0.37% by 1995Q3. When we use 1995Q1 to split the sample, we find that the median correlation between real GDP growth and its estimated volatility is 0.21 in the pre-1995 sample (1986Q1-1994Q4) and -0.20 in the post-1995 sample (1995Q1-2014Q2), which is positive in less than 1% of draws. The median difference between the correlations is -0.40, and it is positive in less than 5% of draws. The post-1995 sample includes quarters when Japan first hit its ZLB as well as the Great Recession. If we remove the Great Recession from the sample, then the median difference between the pre-1995 correlation and the post-1995 (1995Q1-2008Q2) correlation is -0.32, which is positive in less than 10% of draws. Thus, there is a high probability that the correlations in the two samples are different even when we remove the post-Great Recession sample. This result is particularly interesting because a financial crisis had been well under way in Japan since the early 1990s. Even if that crisis had lingering effects on uncertainty after 1995, a stronger negative relationship between real GDP growth and uncertainty still emerged during a period when the discount rate hit its ZLB and there was not a historic global financial crisis.

3 THEORETICAL MODEL AND MEASURE OF UNCERTAINTY

This section lays out our theoretical model and describes how we measure uncertainty. The model includes a ZLB constraint that occasionally binds due to discount factor and technology shocks.

3.1 MODEL   There are three actors in the model: (1) a representative household that has access to a one-period nominal bond, (2) a representative firm that bundles a continuum of intermediate inputs to produce a final good, and (3) a central bank that sets the short-term nominal interest rate.

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[10]See Boero et al. (2008) for more information about the BOE SEF and how it compares to similar surveys.

[11]There is also a survey of Japanese professional forecasters, but it began in mid-2004 and does not provide a large enough sample. See Komine et al. (2009) for details about the survey and analysis of the forecasters' performance.

Households   A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t [\log c_t - \chi n_t^{1+\eta}/(1+\eta)]$, where $\chi > 0$, $1/\eta$ is the Frisch elasticity of labor supply, c is consumption, n is labor hours, b is the real value of a 1-period nominal bond, E_0 is an expectation operator conditional on information available in period 0, $\tilde{\beta}_0 \equiv 1$, and $\tilde{\beta}_t = \prod_{j=1}^{t>0} \beta_j$. Following Eggertsson and Woodford (2003), β is a time-varying discount factor that follows

$$\beta_t = \bar{\beta}(\beta_{t-1}/\bar{\beta})^{\rho_\beta} \exp(\varepsilon_t), \tag{1}$$

where $\bar{\beta}$ is the steady-state discount factor, $0 \leq \rho_\beta < 1$, and $\varepsilon \sim \mathbb{N}(0, \sigma_\varepsilon^2)$. These choices are constrained by $c_t + b_t = w_t n_t + i_{t-1} b_{t-1}/\pi_t + d_t$, where $\pi_t = p_t/p_{t-1}$ is the gross inflation rate, w is the real wage rate, i is the gross nominal interest rate set by the central bank, and d is real dividends from intermediate firms. The optimality conditions to the household's problem imply

$$w_t = \chi n_t^{\eta} c_t, \tag{2}$$

$$1 = i_t E_t [\beta_{t+1}(c_t/c_{t+1})/\pi_{t+1}]. \tag{3}$$

Firms   The production sector consists of a continuum of monopolistically competitive intermediate firms and a final goods firm. Intermediate firm $f \in [0,1]$ produces a differentiated good, $y_t(f)$, according to $y_t(f) = z_t n_t(f)$, where $n_t(f)$ is the amount of employment used by firm f. z_t represents the level of technology, which is common across firms and evolves according to

$$z_t = \bar{z}(z_{t-1}/\bar{z})^{\rho_z} \exp(\upsilon_t), \tag{4}$$

where $\bar{z}$ is steady-state technology, $0 \leq \rho_z < 1$, and $\upsilon \sim \mathbb{N}(0, \sigma_\upsilon^2)$. Each intermediate firm chooses its labor supply to minimize its operating costs, $w_t n_t(f)$, subject to its production function.

The representative final goods firm purchases $y_t(f)$ units from each intermediate goods firm to produce the final good, $y_t \equiv [\int_0^1 y_t(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}$, according to a Dixit and Stiglitz (1977) aggregator, where $\theta > 1$ measures the elasticity of substitution between the intermediate goods. The final goods firm maximizes dividends to determine its demand function for intermediate good f, $y_t(f) = (p_t(f)/p_t)^{-\theta} y_t$, where $p_t = [\int_0^1 p_t(f)^{1-\theta} df]^{1/(1-\theta)}$ is the price of the final good.

Following Rotemberg (1982), each firm faces a cost to adjusting its price, $adj_t(f)$, which emphasizes the negative effect that price changes can have on customer-firm relationships. Using the functional form in Ireland (1997), $adj_t(f) = \varphi[p_t(f)/(\bar{\pi} p_{t-1}(f)) - 1]^2 y_t/2$, where $\varphi \geq 0$ scales the size of the adjustment cost and $\bar{\pi}$ is the steady-state gross inflation rate. Real dividends are then given by $d_t(f) = (p_t(f)/p_t) y_t(f) - w_t n_t(f) - adj_t(f)$. Firm f chooses its price, $p_t(f)$, to maximize the expected discounted present value of real dividends $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t (c_0/c_t) d_t(f)$. In a symmetric equilibrium, all firms make identical decisions and the optimality condition implies

$$\varphi \left(\frac{\pi_t}{\bar{\pi}} - 1\right) \frac{\pi_t}{\bar{\pi}} = (1-\theta) + \theta(w_t/z_t) + \varphi E_t \left[\beta_{t+1} \frac{c_t}{c_{t+1}} \left(\frac{\pi_{t+1}}{\bar{\pi}} - 1\right) \frac{\pi_{t+1}}{\bar{\pi}} \frac{y_{t+1}}{y_t}\right]. \tag{5}$$

Without price adjustments costs (i.e., $\varphi = 0$), the real marginal cost of producing a unit of output (w_t/z_t) equals $(\theta-1)/\theta$, which is the inverse of a firm's markup of price over marginal cost, μ.

Monetary Policy   The central bank sets the gross nominal interest rate, i_t, according to

$$i_t = \max\{\underline{\imath}, i_t^*\}, \qquad i_t^* = \bar{\imath}(\pi_t/\bar{\pi})^{\phi_\pi} (y_t/y_t^n)^{\phi_y} \exp(\nu_t) \tag{6}$$

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where $\underline{\iota}$ is the effective lower bound on the nominal interest rate, i_t^* is the notional interest rate (i.e., the policy rate the central bank would set if it was not constrained by the lower bound), ϕ_π and ϕ_y are the policy responses to the inflation and output gaps, $\bar{\iota}$ and $\bar{\pi}$ are the inflation and interest rate targets, which equal their steady-state values, $y_t^n = (\chi\mu)^{-1/(1+\eta)} z_t$ is the potential output target (i.e., the level of output when prices are flexible), and $\nu \sim \mathbb{N}(0, \sigma_\nu^2)$ is a monetary policy shock.

Competitive Equilibrium The resource constraint is given by $c_t = y_t - adj_t \equiv y_t^{gdp}$, where y_t^{gdp} includes the value added by intermediate firms, which is their output minus price adjustment costs. Thus, y_t^{gdp} represents real GDP in the model. A competitive equilibrium consists of sequences of quantities, $\{c_t, n_t, b_t, y_t, y_t^n\}_{t=0}^\infty$, prices, $\{w_t, i_t, \pi_t\}_{t=0}^\infty$, and exogenous variables, $\{\beta_t, z_t\}_{t=0}^\infty$, that satisfy the household's and firm's optimality conditions, (2), (3), and (5), the production function, $y_t = z_t n_t$, the monetary policy rule, (6), the solution for potential output, $y_t^n = (\chi\mu)^{-1/(1+\eta)} z_t$, the stochastic processes, (1) and (4), the bond market clearing condition, $b_t = 0$, and the resource constraint, given initial conditions, β_{-1} and z_{-1}, and sequences of exogenous shocks, $\{\varepsilon_t, \upsilon_t, \nu_t\}_{t=0}^\infty$.

3.2 MEASURE OF ENDOGENOUS UNCERTAINTY

A recent and growing segment of the literature introduces stochastic volatility (SV) into dynamic stochastic general equilibrium models to study the effects of uncertainty shocks. Our work differs from these papers in that we focus on how uncertainty about future variables endogenously responds to the state of the economy.

To illustrate our measure of uncertainty, it is useful to first describe how one measures uncertainty in a model with SV. As an example, suppose a model includes an exogenous variable, x, such as technology or government spending, that evolves according to $x_t = \rho_x x_{t-1} + \sigma_t \varepsilon_t$, where $0 \le \rho_x < 1$ and ε is white noise. SV is introduced by assuming the standard deviation of the shock is time-varying and follows an exogenous process, which relaxes the common assumption of homoscedastic innovations. Given the process for x, the expected forecast error, FE_x, equals

$$E_t[FE_{x,t+1}] = E_t[x_{t+1} - E_t x_{t+1}] = 0.$$

Although the forecast error is mean zero, there is uncertainty about its future value. One measure of that uncertainty is the expected standard deviation of the forecast error for x, which is given by

$$\sqrt{E_t[FE_{x,t+1}^2]} = \sqrt{E_t[(x_{t+1} - E_t x_{t+1})^2]} = \sqrt{E_t[(x_{t+1} - \rho_x x_t)^2]} = \sqrt{E_t \sigma_{t+1}^2}.$$

Models that allow for SV in various shocks are able to match features of the data that models with homoscedastic errors cannot match, but they do not explain why volatility changes over time because the uncertainty is exogenous. However, there is always uncertainty that is endogenous to the model (i.e., uncertainty due to the dispersion in the realizations of future variables instead of second moment shocks). We quantify the degree of endogenous uncertainty by following the logic of the SV literature. Specifically, the uncertainty surrounding y^{gdp}, 1 quarter ahead, is given by

$$\sigma_{y^{gdp},t} \equiv \sqrt{E_t[(y_{t+1}^{gdp} - E_t y_{t+1}^{gdp})^2]},$$

which varies over time like it does with SV shocks, except the fluctuations are now endogenous responses to the state of the economy. One example of such a state is a binding ZLB constraint. We focus on the uncertainty surrounding real GDP, but we can also calculate this measure of uncertainty for other variables in the model, including the inflation and nominal interest rates.

Steady-State Discount Factor	$\bar{\beta}$	0.9966	Monetary Policy Response to Inflation	ϕ_π	2
Frisch Elasticity of Labor Supply	$1/\eta$	3	Monetary Policy Response to Output	ϕ_y	0.1
Elasticity of Substitution between Goods	θ	6	Discount Factor Persistence	ρ_β	0.87
Rotemberg Adjustment Cost Coefficient	φ	160	Discount Factor Standard Deviation	σ_ε	0.002
Steady-State Labor	$\bar{n}$	0.33	Technology Persistence	ρ_z	0.9
Inflation Rate Target	$\bar{\pi}$	1.0055	Technology Shock Standard Deviation	σ_υ	0.0025
Nominal Interest Rate Lower Bound	$\imath$	1.00023	Monetary Policy Shock Standard Deviation	σ_ν	0.00225

Table 3: Calibrated parameters.

3.3 CALIBRATION   We calibrate our model at a quarterly frequency to match moments in the data from 1986Q1 to 2014Q2. The parameters are summarized in table 3. The steady-state discount factor, $\bar{\beta}$, is set to 0.9966, which equals the ratio of the average 3-month T-bill rate to the average quarter-over-quarter percentage change in the GDP implicit price deflator. The Frisch elasticity of labor supply, $1/\eta$, is set to 3, which is consistent with Peterman (2012). The leisure preference parameter, χ, is calibrated so that steady-state labor equals $1/3$ of the available time. The elasticity of substitution between intermediate goods, θ, is calibrated to 6, which corresponds to an average markup over marginal cost equal to 20%. The costly price adjustment parameter, φ, is set to 160, which matches the estimate in Ireland (2003). The lower bound on the gross nominal interest rate, $\imath$, is calibrated to 1.00023, which equals the average 3-month T-bill rate from 2009Q1 to 2014Q2.

The gross inflation rate target, $\bar{\pi}$, is calibrated to 1.0055 to match the average quarter-over-quarter percentage change in the GDP implicit price deflator. We set the monetary response to deviations from the inflation target, ϕ_π, equal to 2 and the response to deviations from potential output, ϕ_y, equal to 0.1. Those values are consistent with the estimates in Smets and Wouters (2007). The persistence of the discount factor, ρ_β, equals 0.87 and the standard deviation of the shock, σ_ε, equals 0.002, which are close to the values estimated in Gust et al. (2013). We set the persistence of the technology process equal to $\rho_z = 0.9$ and the standard deviation of the shock equal to $\sigma_\upsilon = 0.0025$. The standard deviation of the monetary policy shock, σ_ν, is set to 0.00225.

In choosing the parameters of the stochastic processes, our primary goal was to match the volatility of real GDP, since we focus on movements in its forecast error volatility in the model. In the data, the annualized standard deviations of quarter-over-quarter percent changes in real GDP, the GDP deflator inflation rate, and the 3-month T-bill rate are 2.45%, 0.95%, and 2.45% per year, respectively. To determine how these values compare to the data, we ran 10,000 simulations of the model that are each 114 quarters long (i.e., the length of our data from 1986Q1 to 2014Q2). We then compute the median standard deviations of real GDP, the inflation rate, and the interest rate. Those values and their 90% credible intervals are 2.22% (1.78%, 3.20%), 0.94% (0.67%, 1.41%), and 1.98% (1.60%, 2.45%) annually. All three credible intervals include the values in the data and the median standard deviations of real GDP and inflation are close to their historical averages.

The parameters of the stochastic processes, which affect the frequency and duration of ZLB events, are also chosen so our model is consistent with how long people *expected* the ZLB to bind in the U.S., rather than the *actual* ZLB duration. Figure 3 plots median forecasts of the T-bill rate q quarters ahead using data from the SPF (left panel) and Blue Chip (right panel) surveys. Specifically, we show the SPF (Blue Chip) forecasts made in the first quarter (February) from 2009 to 2013. Both surveys indicate that people did not initially expect the T-bill rate to remain near zero for a long period. For example, in 2009 and 2010 forecasters in both surveys predicted the T-bill rate would exceed 0.5% within 3 quarters, despite the severity of the recession. Swanson and Williams (2014) use options data to calculate the probability that the federal funds rate would be
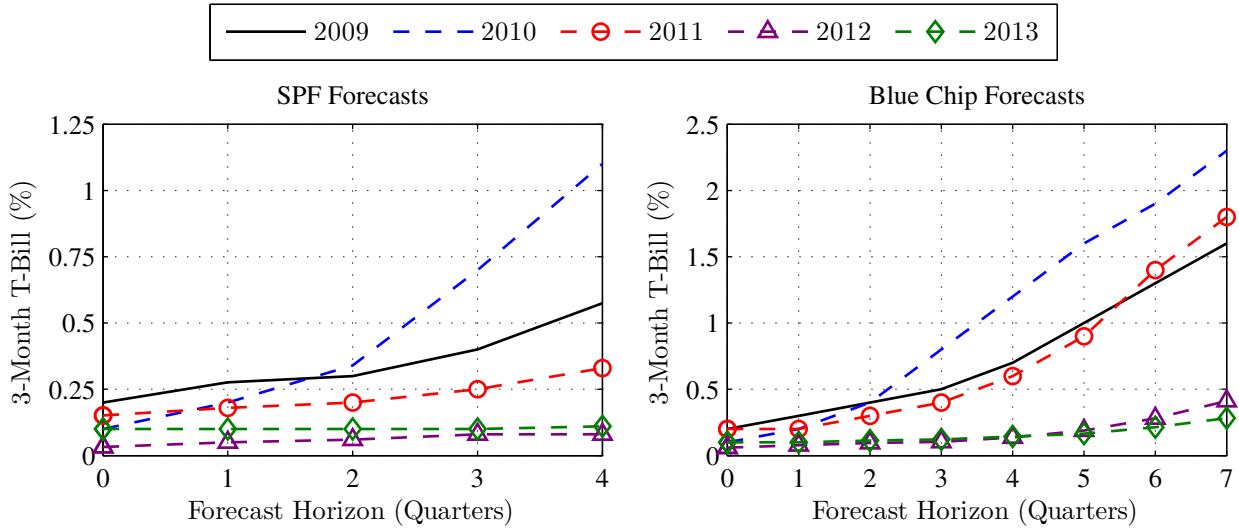
Figure 3: Median SPF (left panel) and Blue Chip (right panel) forecasts of the annual 3-month T-bill rate q quarters ahead. The SPF (Blue Chip) forecasts made in the first quarter (February) are shown every year from 2009 to 2013.
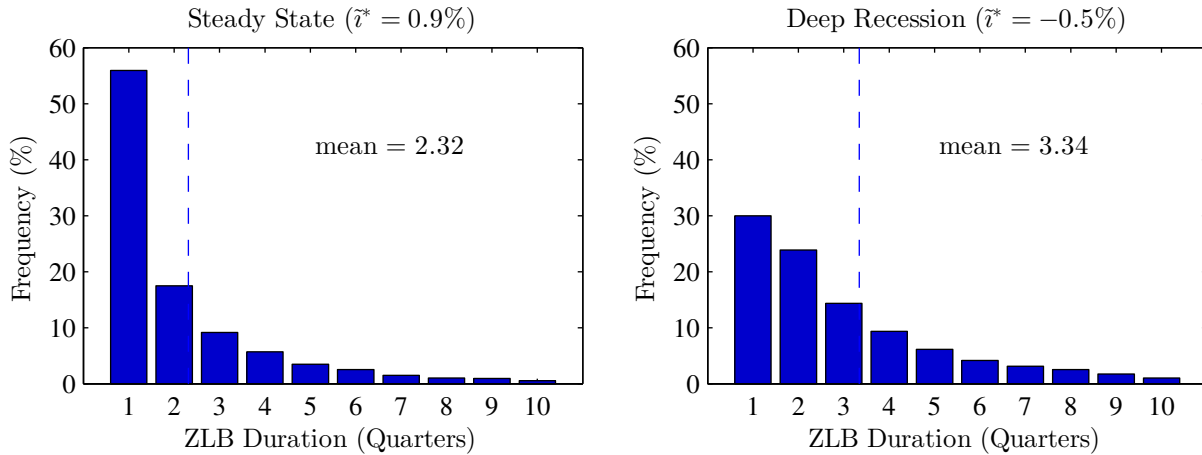

Figure 4: Distribution of ZLB events in our model. The vertical dashed lines are the average ZLB durations.

below 0.5% for the next 5 quarters. That probability ranged from 20% to 45% between 2008Q4 and 2010Q2 and was typically below 60% until 2011Q3, meaning households placed greater weight on leaving the ZLB than staying at the ZLB. Only after the Fed communicated date-based forward guidance in August 2011 did forecasters expect the T-bill rate to remain at zero for a longer period.

Figure 4 plots histograms of the durations of each ZLB event in our model. To compute these histograms, we initialize $10{,}000$ simulations at two alternative notional interest rates: $\tilde{\imath}^* = 0.9$ (left panel) and $\tilde{\imath}^* = -0.5$ (right panel).[12] We then count the number of quarters in the first ZLB event for each simulation and report the frequency of the ZLB event durations across the $10{,}000$ simulations. When the economy is initialized at its steady state, on average it takes 33 quarters until the ZLB binds and the average ZLB duration is 2.32 quarters. When we condition on a notional

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[12]Bauer and Rudebusch (2014), Gust et al. (2013), Krippner (2013), and Wu and Xia (2014) all estimate the notional federal funds rate when the ZLB binds and find that it was well below zero during and after the Great Recession.

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interest rate that is consistent with estimates during and after the Great Recession, the average ZLB event rises to 3.34 quarters. Therefore, our calibration produces ZLB events with a similar average duration to what households expected before the Fed communicated date-based forward guidance to the public. It is also possible for the model to generate much longer ZLB events. For example, 4.25% (1.35%, 0.50%) of the ZLB events are greater than 8 quarters (12 quarters, 16 quarters).

3.4 SOLUTION METHOD   We solve the model using the policy function iteration algorithm described in Richter et al. (2014), which is based on the theoretical work on monotone operators in Coleman (1991). This method discretizes the state space and iteratively solves for updated policy functions until the tolerance criterion is met. We use linear interpolation to approximate future variables, since it accurately captures the kink in the policy functions, and Gauss-Hermite quadrature to numerically integrate. See Richter et al. (2014) for a formal description of the algorithm.

The are two primary advantages of using a global solution method that depends on a continuum of future shocks. One, it allows for variation in endogenous variables when the ZLB binds, which is an artifact of the data and necessary to calculate correlations in the post-Great Recession period. Two, it permits recurring ZLB events and accounts for the expectational effects of going to and leaving the ZLB, which means uncertainty can vary gradually with changing economic conditions. A simplification that is commonly used in the literature is a two-state Markov chain on the discount factor. In that case, however, endogenous variables do not vary at the ZLB due to discount factor shocks, and the expectational effect of hitting the ZLB is constant when the interest rate is positive.

Benhabib et al.'s (2001) finding that constrained models have two deterministic steady-state equilibria has generated considerable discussion in the literature about whether there are conditions in which a unique MSV solution exists in stochastic models with a ZLB constraint. Specifically, they find two nominal interest rate/inflation rate pairs that satisfy the steady-state equilibrium system. In one steady state, the central bank meets its positive inflation target, whereas in the other steady state the economy experiences deflation. Richter and Throckmorton (2015) show that the numerical algorithm used in this paper converges to the inflationary equilibrium as long as there is a sufficient expectation of returning to a monetary policy rule that conforms to the Taylor principle. Our algorithm, however, never converges to the deflationary equilibrium. Although addressing questions related to the deflationary steady state is an interesting topic for future research, our analysis, like most macroeconomic research on the ZLB constraint, focuses on the solution centered around the inflationary steady state. For further information on this topic see Gavin et al. (2015).

4   MONETARY POLICY, THE ZLB CONSTRAINT, AND UNCERTAINTY

We first build intuition on why the ZLB generates a strong negative correlation between real GDP growth and uncertainty in the model by fixing technology at steady state, so that ZLB events are endogenous only due to positive discount factor shocks. We also show how the central bank's response to inflation affects this relationship. We then allow for stochastic changes in technology.

4.1 MODEL WITH CONSTANT TECHNOLOGY   We begin our analysis by holding technology constant (i.e., $z_t = \bar{z}$). In this case, the model contains one state variable, β_{-1}, which is exogenous and ranges from $\pm 1.6\%$ of its steady state. In all of our results, $\hat{x}$ denotes percent deviation from steady state (i.e., $\hat{x}_t \equiv 100(x_t - \bar{x})/\bar{x}$) where $x \in \{y, \beta, z\}$, $\hat{\sigma}_y$ denotes percent of steady state output (i.e., $\hat{\sigma}_{y,t} = 100(\sigma_{y,t}/\bar{y})$), and a tilde denotes a net interest rate (i.e., $\tilde{x} = 100(x - 1)$).
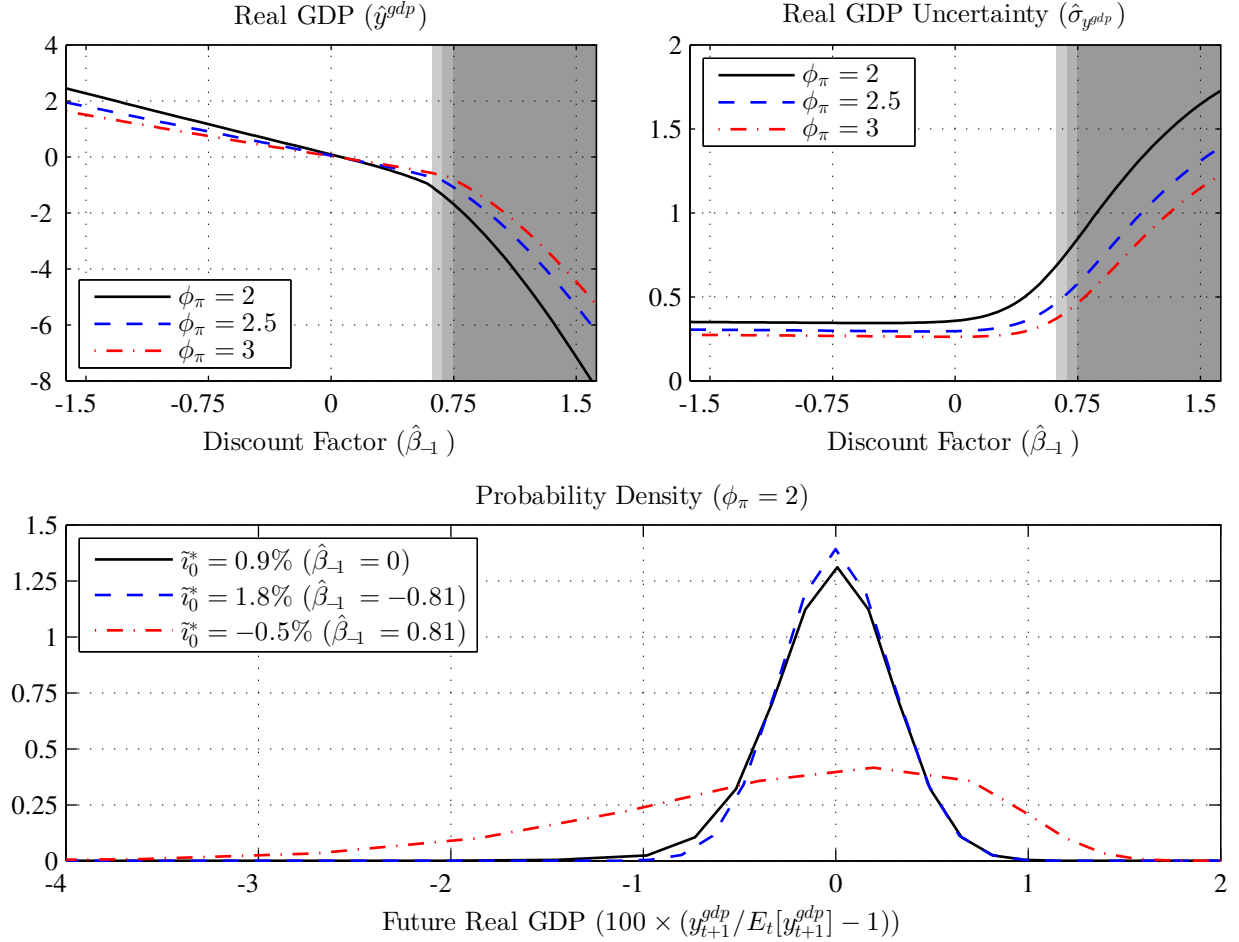
Figure 5: Policy function for real GDP (top left panel) and real GDP uncertainty (top right panel). The horizontal axes display the discount factor state in percent deviations from steady state. The vertical axis in the top left panel shows real GDP in percent deviations from steady state. The vertical axis in the top right panel shows the standard deviation of the forecast error for real GDP as a percent of steady-state real GDP. The shaded regions indicate where the ZLB binds. The bottom panel shows the density function of future real GDP at various notional interest rates (discount factors). The horizontal axis displays real GDP as a percent of its mean. The vertical axis shows the density value.

The top row of figure 5 plots the decision rules for real GDP, $\hat{y}^{gdp}$ (left panel), and real GDP uncertainty, $\hat{\sigma}_{y^{gdp}}$ (right panel), as a function of $\hat{\beta}_{-1}$ for three different values of the central bank's response to the inflation gap: $\phi_\pi = 2$ (solid line), $\phi_\pi = 2.5$ (dashed line), and $\phi_\pi = 3$ (dash-dotted line). The shaded regions indicate where the ZLB binds, which depends on the value of ϕ_π. When $\phi_\pi = 2$ ($\phi_\pi = 2.5$, $\phi_\pi = 3$), the ZLB binds in states where $\hat{\beta}_{-1} > 0.62$ ($\hat{\beta}_{-1} > 0.68$, $\hat{\beta}_{-1} > 0.75$). The bottom row plots the probability density function of future real GDP as a percent deviation from its mean, $100 \times (y_{t+1}^{gdp}/E_t[y_{t+1}^{gdp}] - 1)$. We display the density functions for three values of the initial notional interest rate: $\tilde{\imath}_0^* = 0.9\%$ (solid line), $\tilde{\imath}_0^* = 1.8\%$ (dashed line), $\tilde{\imath}_0^* = -0.5\%$ (dash-dotted line) when $\phi_\pi = 2$. Each notional interest rate is inversely related to the discount factor state ($\hat{\beta}_{-1}$), which is shown in the legend. The density functions are informative because they illustrate why macroeconomic uncertainty changes across different states and parameters.

We begin by discussing how uncertainty changes across discount factor states. The discount factor is a proxy for aggregate demand because it determines households' degree of patience.

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When the discount factor is low (high), households are impatient (patient), and less (more) willing to postpone consumption. Firms respond to the higher (lower) demand by increasing (decreasing) their prices and output. Therefore, the policy function for real GDP is downward sloping (top left panel). In discount factor states where the nominal interest rate is far from its ZLB, the slope of the policy function for real GDP is nearly constant, which means the distribution of future real GDP values is independent of the state of the economy. The probability density function is also narrower than in states where the ZLB binds (bottom panel). Thus, real GDP uncertainty is relatively low and nearly constant in states where the central bank is unconstrained by the ZLB (top right panel).

As the economy enters states where the nominal interest rate is close to or at its ZLB, demand continues to decline and firms further reduce their prices. In states where the central bank is not constrained, it is able to respond to the lower inflation by cutting its policy rate to dampen the effects of the fall in demand. However, given a large enough decline in demand, the ZLB will bind and the central bank will be unable to further reduce its policy rate. Thus, the economy becomes more sensitive to further declines in demand, which leads to lower real GDP than if the central bank was unconstrained. A steeper policy function for real GDP widens the distribution of possible real GDP values next period and skews it toward output losses. For example, when $\tilde{\imath}_0^* = 0.9\%$, a ± 1 standard deviation discount factor shock (i.e., $\pm 0.2\%$) causes real GDP to move from its steady state by $\pm 0.3\%$. When $\tilde{\imath}_0^* = -0.5\%$, the same change in the discount factor can cause real GDP to decrease by 1.1 percentage points or increase by 0.7 percentage points. The broader range of future real GDP values produces greater forecast error volatility and hence higher uncertainty.[13]

Figure 5 demonstrates that real GDP uncertainty endogenously increases in our model when the nominal interest rate is near or at its ZLB. We recognize that uncertainty also spikes in the data when the ZLB was not a concern (e.g., 1987: Black Monday, 1990: first Gulf War, 2001: 9/11, mid-2002 to early-2003: Enron scandal and second Gulf War), but we do not view those episodes as a problem for our theory because they are due to events that are exogenous to our model.

We next consider how the inflation coefficient in the monetary policy rule affects endogenous uncertainty. We conduct this exercise for two reasons. First, it provides excellent intuition for the relationship between the slope of the policy function for real GDP and macroeconomic uncertainty. Second, it illustrates how the ability of the central bank to stabilize the economy affects macroeconomic uncertainty. When the central bank places more emphasis on inflation stability (i.e., a higher ϕ_π), it affects the volatility of inflation and real GDP both away from and at the ZLB.

First consider the case where the nominal interest rate is far away from its ZLB. The top left panel of figure 5 shows that the slope of the policy functions flatten as ϕ_π increases. That reflects the central bank's success at stabilizing inflation around its target level. Real GDP becomes less responsive to discount factor shocks, and, as a result, the distribution of future real GDP becomes tighter around its expected value (i.e., uncertainty is lower when ϕ_π is higher). We show this effect graphically in the top panel of figure 6, where we plot the actual density functions of future real GDP across three values of ϕ_π. For each ϕ_π the discount factor equals its steady-state value.

Higher values of ϕ_π have similar impacts on real GDP uncertainty when the ZLB constraint binds. Greater inflation stability means the ZLB binds at higher discount factors states. It also means that when the nominal interest rate is at its ZLB, households expect inflation will be relatively more stable when the nominal interest rate rises. Since households always expect the nom-

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[13]Basu and Bundick (2015) compute impulse responses to an increase in the volatility of the discount factor. They show higher volatility arises endogenously at the ZLB through precautionary saving and contractionary bias channels.
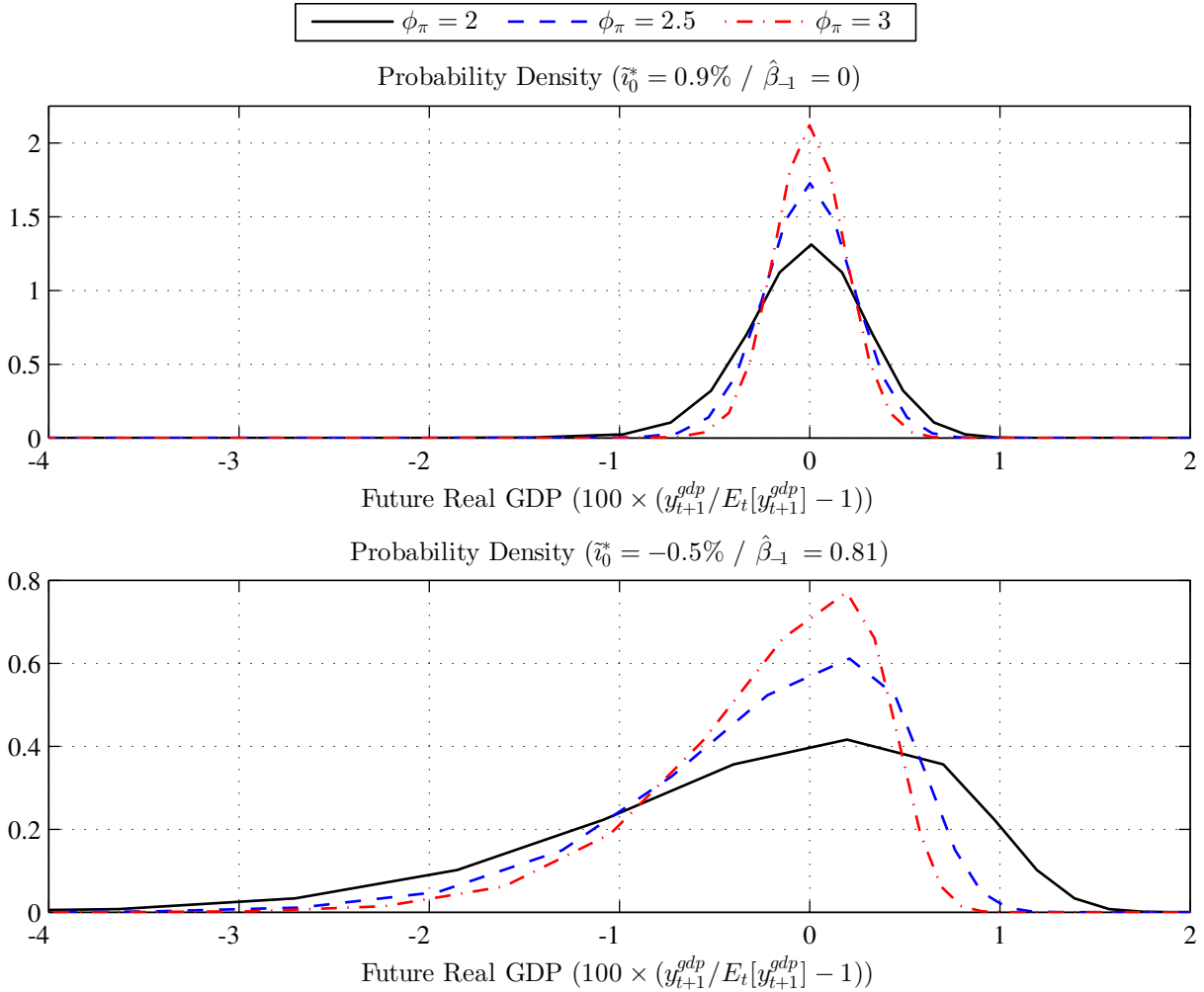
Figure 6: Density functions of future real GDP with various monetary policy responses to inflation. The top (bottom) panel shows the density conditional on the steady-state discount factor (a discount factor where the ZLB constraint binds). The horizontal axes display future real GDP as a percent of its mean. The vertical axes show the density value.

inal interest rate to exit its ZLB, more stable inflation away from the ZLB leads to more stable inflation and real GDP at the ZLB. For example, if $\tilde{\imath}_0^* = -0.5$, then a ± 1 standard deviation discount factor shock causes real GDP to range from -0.56% to -1.73% below its steady state when $\phi_\pi = 2$ and from -0.45% to -1.34% when $\phi_\pi = 3$. A flatter policy function for real GDP generates a narrower and more symmetric distribution for future real GDP (bottom panel), which suggests that one benefit of a higher ϕ_π is that it alleviates uncertainty near and at the ZLB. Regardless of the value of ϕ_π, however, uncertainty is unaffected by discount factor shocks when the nominal interest rate is far from its ZLB and increases sharply when it approaches and hits its ZLB.

Another way to compare the relationship between real GDP and uncertainty is with a generalized impulse response function (GIRF) of a shock to the discount factor following the procedure in Koop et al. (1996). The advantage of GIRFs is that they are based on an average of model simulations where the realization of shocks is consistent with households' expectations over time. To compute the GIRFs, we first calculate the mean of 10,000 simulations of the model conditional on random shocks in the every quarter (i.e., the baseline path). We then calculate a second mean
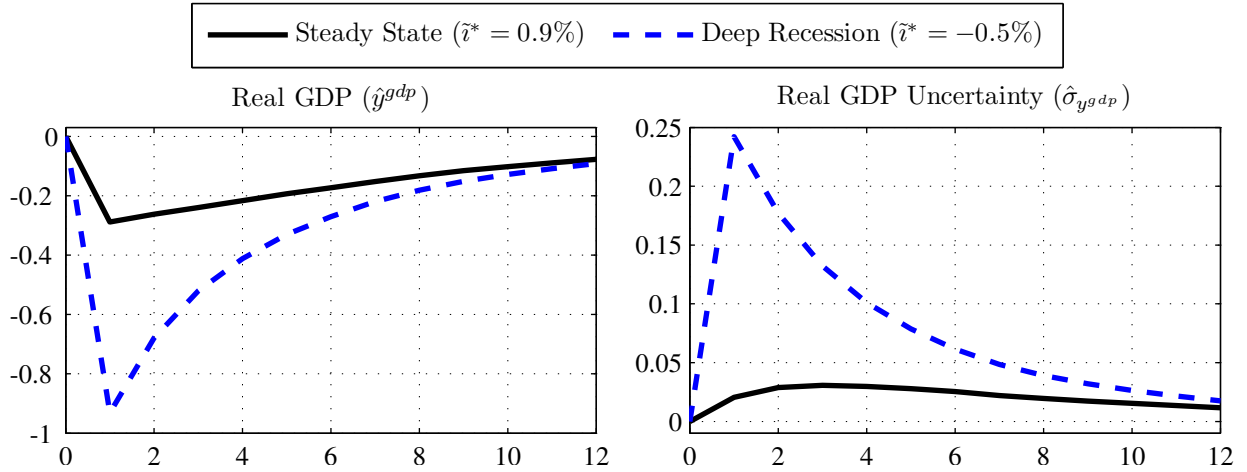
Figure 7: Generalized impulse responses to a 1 standard deviation positive discount factor shock at and away from the ZLB. The steady-state simulation (solid line) is initialized at the stochastic steady state. The deep recession simulation (dashed line) is initialized at a notional interest rate equal to -0.5%. The vertical axis is the percentage change in real GDP (or the difference in uncertainty) from the baseline. The horizontal axes displays the time period in quarters.

from another set of $10,000$ simulations, but this time the shock in the first quarter is replaced with the shock of interest. The GIRF reports the percent (or level) difference between the two paths.

Figure 7 plots the responses to a 1 standard deviation positive discount factor shock at and away from the ZLB. The steady-state simulation (solid line) is initialized at the stochastic steady state. The deep recession simulation (dashed line) is initialized at a notional interest rate equal to -0.5%. In both cases, the shock causes households to postpone consumption, which reduces real GDP. When the nominal interest rate is far from its ZLB, the drop in real GDP is damped by the monetary policy response. There is almost no change in uncertainty because households expect future shocks will have the same effect on real GDP regardless of the state of the economy. When the ZLB binds, however, the central bank cannot respond by lowering the nominal interest rate, which leads to larger declines in real GDP. In this case, uncertainty sharply increases since households expect a wider range of future real GDP values when the central bank is constrained.

Next, we run Monte Carlo simulations of the model that are 114 quarters long so that each simulation is the same length as our data from 1986Q1 to 2014Q2. Figure 8 plots one of the simulations with a single ZLB event that occurs at the end of the simulation and lasts for 24 quarters (i.e., the number of quarters in our post-Great Recession data period). The top panel plots the paths of real GDP growth, $\Delta \log y^{gdp}$ (left axis, solid line), and real GDP uncertainty, $\hat{\sigma}_{y^{gdp}}$ (right axis, dashed line), which provides another way to visualize the correlation between real GDP growth and uncertainty. The bottom panel plots the paths of the nominal (solid line) and notional (dashed line) interest rates. The shaded region indicates periods when the ZLB binds. In that region, the notional rate is negative. Outside of the ZLB region, the nominal and notional rates are equal.

There are three key takeaways from this simulation. One, uncertainty surrounding future real GDP is state-dependent. When the ZLB does not bind, uncertainty is essentially constant, except in quarters when the nominal interest rate is near its ZLB. In those situations, the high probability of hitting the ZLB next period leads to persistently higher uncertainty. The closer the nominal interest rate is to the ZLB, the higher the uncertainty, which underscores the importance of expectational effects. When the ZLB binds, $\hat{\sigma}_{y^{gdp}}$ is as much as three times larger than its value outside the ZLB.
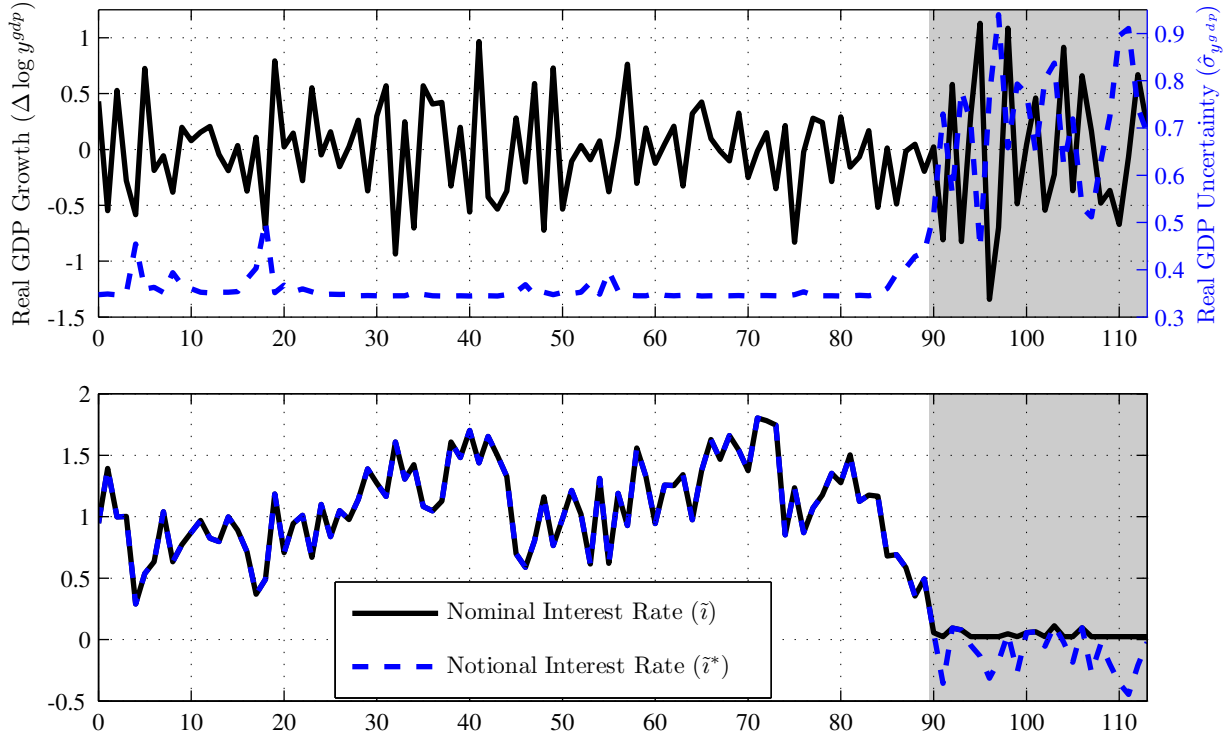
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Figure 8: Top panel: paths of real GDP growth (solid line) and real GDP uncertainty (dashed line). Bottom panel: paths of the nominal (solid line) and notional (dashed line) interest rates. The horizontal axes display the time period in quarters. In the top panel, the left vertical axis displays quarter-over-quarter real GDP growth. The right vertical axis shows real GDP uncertainty as a percent of steady-state output. In the bottom panel, the vertical axis displays the nominal/notional interest rates as a net percent. The shaded region indicates periods when the ZLB constraint binds.

The degree of uncertainty depends on the notional interest rate, which indicates how likely it is for the nominal interest rate to rise and exit the ZLB in the near-term. The smaller the notional interest rate, the less likely the nominal interest rate will exit the ZLB and the higher the uncertainty.

Two, there is a weak correlation between $\hat{y}^{gdp}$ and $\hat{\sigma}_{y^{gdp}}$ when the ZLB does not bind but a strong negative correlation between those variables when it does bind. The strength of those correlations depends on the likelihood of entering and staying at the ZLB. When the nominal interest rate is sufficiently far from its ZLB, the correlation is close to zero since uncertainty is nearly constant. In periods when the nominal interest rate is near or at its ZLB due to positive discount factor shocks, real GDP is well below its steady state and uncertainty is relatively high.

Three, it is possible for uncertainty to decline while the ZLB binds, which implies that uncertainty is time-varying at the ZLB. This outcome occurs whenever the notional interest rate is below zero and there is a negative discount factor shock. A lower discount factor means the household is more optimistic about the future economy, which increases the expected nominal interest rate and reduces real GDP uncertainty. This feature of the model is important because macroeconomic uncertainty continued to fluctuate in the data even after many central banks reduced their policy rates to the ZLB in mid-2008. In other words, our theory does not claim that uncertainty is always increasing when the ZLB binds, but rather that it is more strongly correlated with real GDP growth.

Our theoretical results are based on a model that does not consider the implications of un-conventional monetary policies, such as quantitative easing or forward guidance. Modeling such

policies is a difficult task and beyond the scope of this paper. We believe that such policies might have implications for the level of uncertainty predicted by the model. To the extent that those policies are successful, they will most likely reduce the expected volatility of future real GDP and therefore lower the level of uncertainty. However, we suspect that none of our qualitative findings (e.g., stronger negative correlations near and at the ZLB) would change. The reason is that unless these policies completely alleviate the constraint imposed by the ZLB, it will still be the case that real GDP is more responsive to shocks near and at the ZLB than it is away from the ZLB. As a result, the correlations between uncertainty and real GDP growth will also be stronger at the ZLB.
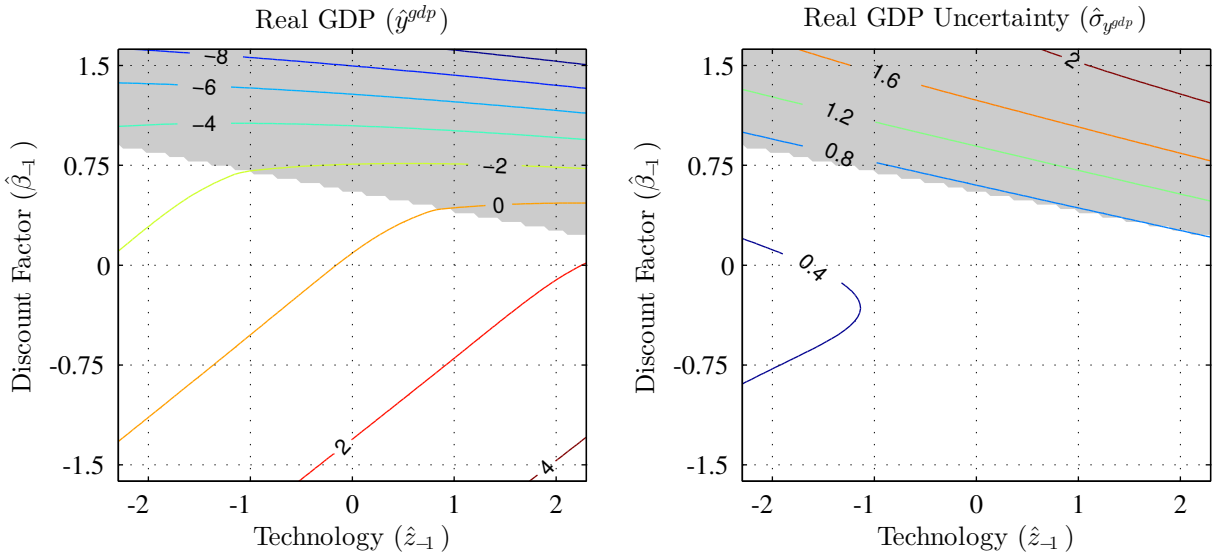

Figure 9: Policy function for real GDP (left panel) and real GDP uncertainty (right panel). The horizontal (vertical) axes in these panels display technology (the discount factor), which is in percent deviations from steady state. The contours in the left panel display real GDP in percent deviations from its steady state. The contours in the right panel show real GDP uncertainty as a percent of steady-state output. The shaded regions indicate where the ZLB binds.

4.2 MODEL WITH VARIABLE TECHNOLOGY

Now suppose technology is time-varying according to (4). The model contains two state variables, z_{-1} and β_{-1}, which range from ± 2.3 and $\pm 1.6\%$ of their steady-state values, respectively. Figure 9 shows the policy functions for real GDP (left panel) and real GDP uncertainty (right panel). The shaded regions indicate where the ZLB binds, which represents 34.4% of the state space. A low (high) level of technology increases (reduces) firms' marginal cost of production. Firms respond by decreasing (increasing) their production and raising (reducing) their prices. The central bank responds by increasing (decreasing) its policy rate. Thus, the nominal interest rate hits its ZLB given a sufficiently high level of technology. In those states, the dynamics are similar to what occurs in high discount factor states—lower inflation raises the real interest rate, causing a sharp decline in real GDP and a large increase in uncertainty.

The uncertainty surrounding future real GDP is mostly unaffected by the level of technology when the nominal interest rate is far from its ZLB. In that situation, uncertainty is stable and low, even when technology and the discount factor are both below their steady states. In contrast, when the nominal interest rate is close to or at its ZLB, regardless of whether it is due to unusually high technology or a high discount factor, forecast error volatility increases. As technology and

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the discount factor simultaneously increase and move away from their respective steady states, real GDP rapidly declines, which drives up uncertainty at the ZLB. Thus, variable technology represents another source of uncertainty but only when the central bank is constrained by the ZLB.
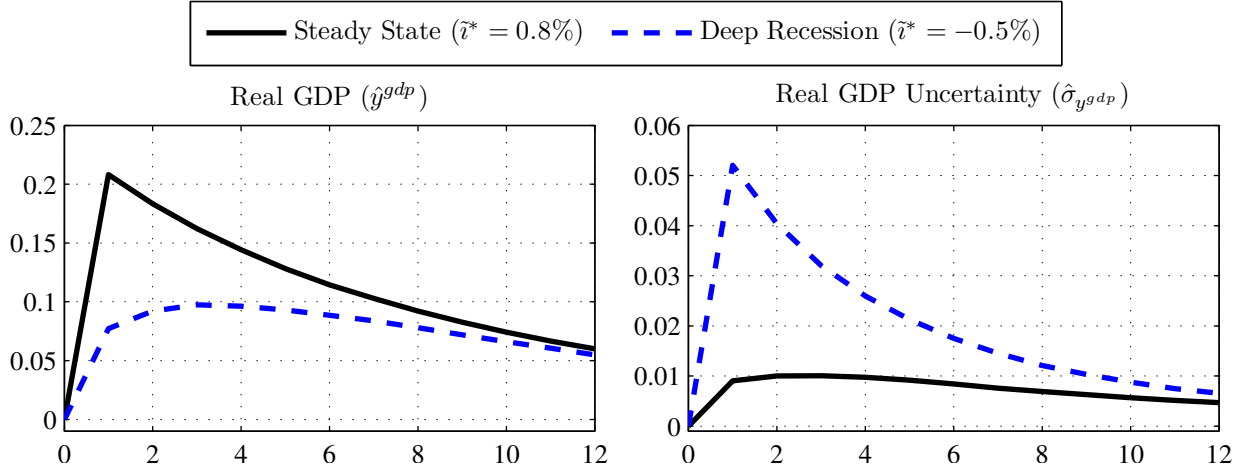

Figure 10: Generalized impulse responses to a 1 standard deviation positive technology shock at and away from the ZLB. The steady-state simulation (solid line) is initialized at the stochastic steady state. The deep recession simulation (dashed line) is initialized at a notional interest rate equal to -0.5%. The vertical axis is the percentage change in real GDP (or the difference in uncertainty) from the baseline. The horizontal axes displays the time period in quarters.

Although technology shocks increase the volatility of real GDP growth and the likelihood of spikes in uncertainty, the simulation properties of the model with constant technology continue to hold—uncertainty is time varying and strongly correlated with real GDP near and at the ZLB.

To obtain further insight for how technology shocks affect real GDP uncertainty and its relationship with real GDP, figure 10 plots generalized impulse responses to a 1 standard deviation positive technology shock at and away from the ZLB. There are two key takeaways from this simulation. One, it shows technology shocks sharply increase real GDP uncertainty when the ZLB binds, similar to positive discount factor shocks. Two, a positive technology shock generates a positive relationship between real GDP and real GDP uncertainty when the ZLB does not bind, whereas a positive discount factor shock (see figure 7) produces a negative relationship. Thus, the correlations between these variables are weaker in periods when the ZLB does not bind than in the model with constant technology. The next section shows that feature of the variable technology model is necessary to match the correlations in the data prior to 2008. A positive technology shock also produces a positive relationship between real GDP and real GDP uncertainty at the ZLB, but the policy functions in the variable technology model are also steeper, so the economy is more sensitive to discount factor shocks at the ZLB than in the model with constant technology. Thus, the correlations between these variables are similar across the two models when the ZLB binds.

5 MODEL PREDICTIONS AND THE DATA

This section compares correlations in the data to equivalent correlations in our theoretical model. We first outline our simulation procedure and then focus on the correlations between real GDP growth and real GDP uncertainty. We conclude by testing our model along two other dimensions and showing that our results are robust to adding habit formation and interest rate smoothing.

5.1 SIMULATION PROCEDURE   We perform simulations of the model to obtain distributions of the correlations between uncertainty and both real GDP and inflation. Similar to our data analysis, we examine the correlations conditional on a positive nominal interest rate and a rate close to or at its ZLB. Each simulation is initialized at the stochastic steady state and runs for 114 quarters, so the simulations are the same length as our data from 1986Q1 to 2014Q2. Unconditionally, ZLB events in the model are infrequent and as short as one quarter. However, the model can also produce simulations with isolated ZLB events that have a much longer duration similar to the data. We specify that each simulation must satisfy two criteria: (1) it must have a single ZLB event, and (2) the ZLB event must be at least 16 quarters. In simulations that meet these criteria, the average ZLB event is 20.2 (19.6) quarters in the model with variable (constant) technology. Therefore, the simulations are characteristic of countries' recent experiences at the ZLB. We first locate 10,000 simulations that satisfy these criteria and then compute correlations away from and near the ZLB.

	Data				Model	
Sample	VXO	BOS FD	SPF FD	VAR	$z = \bar{z}$	$z \sim AR(1)$
Away from ZLB	-0.04	-0.20**	-0.09	-0.24^{++}	-0.20^{++}	-0.13
Near the ZLB	-0.74***	-0.70***	-0.53***	-0.53^{+++}	-0.42^{+++}	-0.41^{+++}

Table 4: Comparison between correlations in the model and the data. The correlations in the model are between real GDP growth and the standard deviation of the real GDP forecast error 1-quarter ahead. The correlations in the data are between real GDP growth and various measures of uncertainty (VXO, BOS FD, SPF FD, VAR). The breakpoint between the two samples in the model ($\tilde{\imath}_t = 0.21\%$) matches the average T-bill rate in the last week of 2008Q3, which is the breakpoint in the data. The correlations in the data are significantly less than 0 at ***1%, **5%, and *10% levels. The correlations in the model and VAR are positive with probabilities less than $^{+++}$1%, $^{++}$5%, and $^{+}$10%.

5.2 REAL GDP UNCERTAINTY   Table 4 compares correlations in the model and the data. The correlations in the model are between quarter-over-quarter real GDP growth and the standard deviation of the real GDP forecast error 1-quarter ahead, which measures the uncertainty surrounding real GDP. The "Away from ZLB" ("Near the ZLB") correlations are based on the quarters in each simulation when $\tilde{\imath} > 0.21\%$ ($\tilde{\imath} \leq 0.21\%$). The correlations in the data are between real GDP growth and measures of uncertainty from survey data, stock market data, and VAR estimates of real GDP volatility. The "Away from ZLB" sample is from 1986Q1 to 2008Q2 and the "Near the ZLB" sample is from 2008Q3 to 2014Q2. The interest rate that separates the samples in the model equals the average T-bill rate in the last week of 2008Q3, which is the breakpoint in the data.[14]

There are several remarkable similarities between the correlations in the model and the data. One, the correlations with the "Near the ZLB" sample in both models are strongly negative and significantly less than zero, which is a robust feature of the data. Two, with the "Away from ZLB" sample, the model with constant technology predicts a weaker correlation than it does with the "Near the ZLB" sample, but it is stronger than the correlations with the SPF FD and VXO in the data. However, when the model includes technology shocks, its predictions better match the weak correlations in the data. Three, the correlations predicted by the model are closest in magnitude to those based on the SPF FD, which is a forward-looking measure of uncertainty for the entire economy, instead of a measure that is based on stock market volatility (VXO), a particular sector

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[14]If we set the cutoff in the model to $\underline{\imath}$ and the breakpoint in the data to 2008Q4, the correlations are very similar.

of the economy (BOS FD), or realized output volatility (VAR). In both the model and the SPF data, the correlations in the "Near the ZLB" sample are strongly negative and significantly less than zero, whereas in the "Away from ZLB" sample the correlations are weak and not statistically significant. We recognize that other sources of endogenous or exogenous uncertainty may have contributed to the stronger negative correlations in the data since the onset of the Great Recession, but our findings provide strong evidence that the ZLB constraint is at least one important factor.
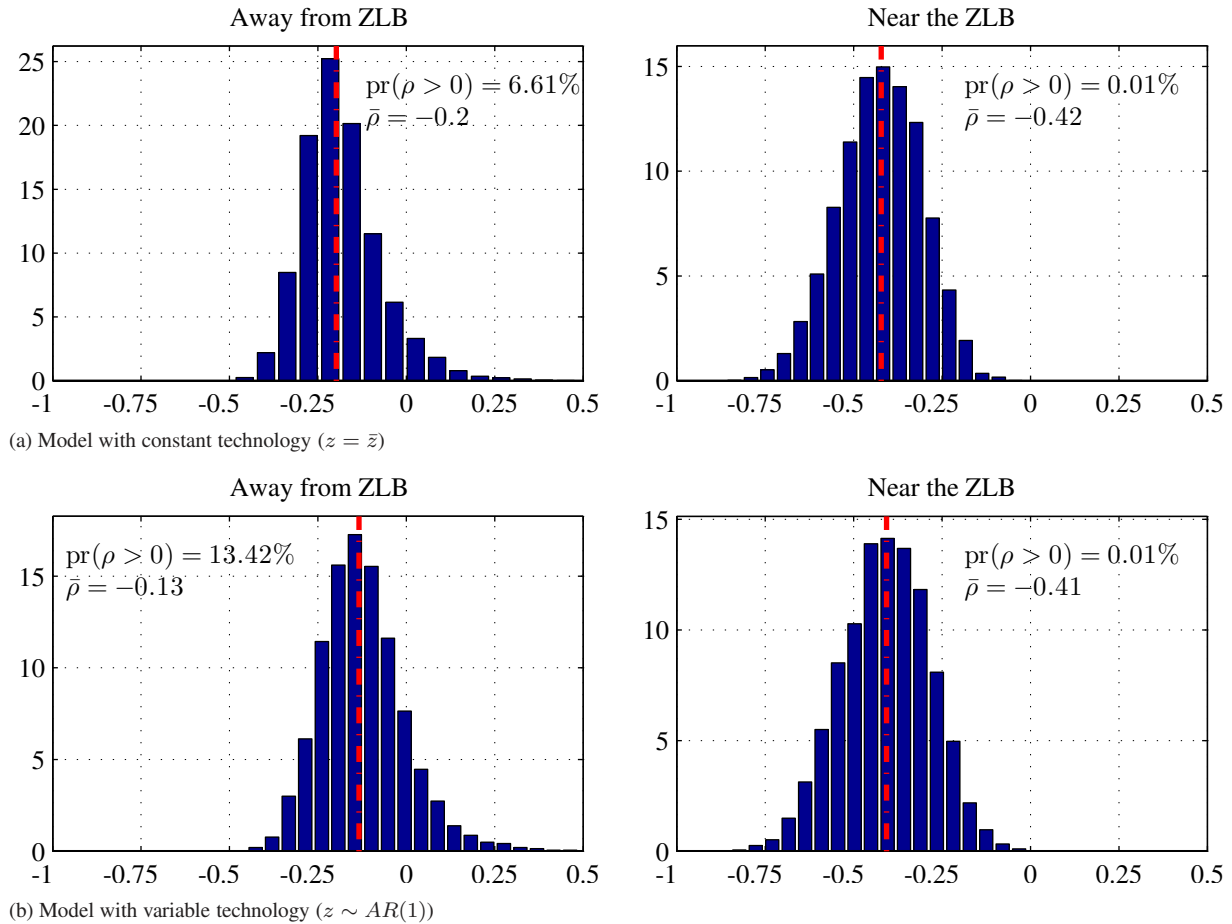

(a) Model with constant technology ($z = \bar{z}$)

(b) Model with variable technology ($z \sim AR(1)$)

Figure 11: Distributions of the correlation, ρ, between real GDP growth and real GDP uncertainty.

The previous discussion focused on the median correlations from the model. To see the range of correlations that are possible, figure 11 plots the distributions of the correlations from all $10,000$ simulations. The top (bottom) panel shows the correlations from the model with constant (variable) technology. The distributions from the model with variable technology are similar to the probability distributions from the time-varying VAR with stochastic volatility (figure 2). In the model with constant technology, the ZLB shifts the distribution to the left and skews the correlations towards more negative values, but the "Away from ZLB" distributions are also centered below zero.

Technology shocks change the "Away from ZLB" distributions. In that sample, the median estimate is less negative and far more simulations produce correlations that are greater than zero (13.4% compared to 6.6% in the model with constant technology). Similar to the VAR distributions, we can subtract the "Away from ZLB" distribution from the "Near the ZLB" distribution to

determine the probability that the correlations are more negative when the nominal interest rate is near or at its ZLB. The difference between the two samples is positive in only 3.9% (3.1%) of the simulations of the model with constant (variable) technology, which provides more evidence that the ZLB was a major source of the stronger negative correlations that emerged in mid-2008.
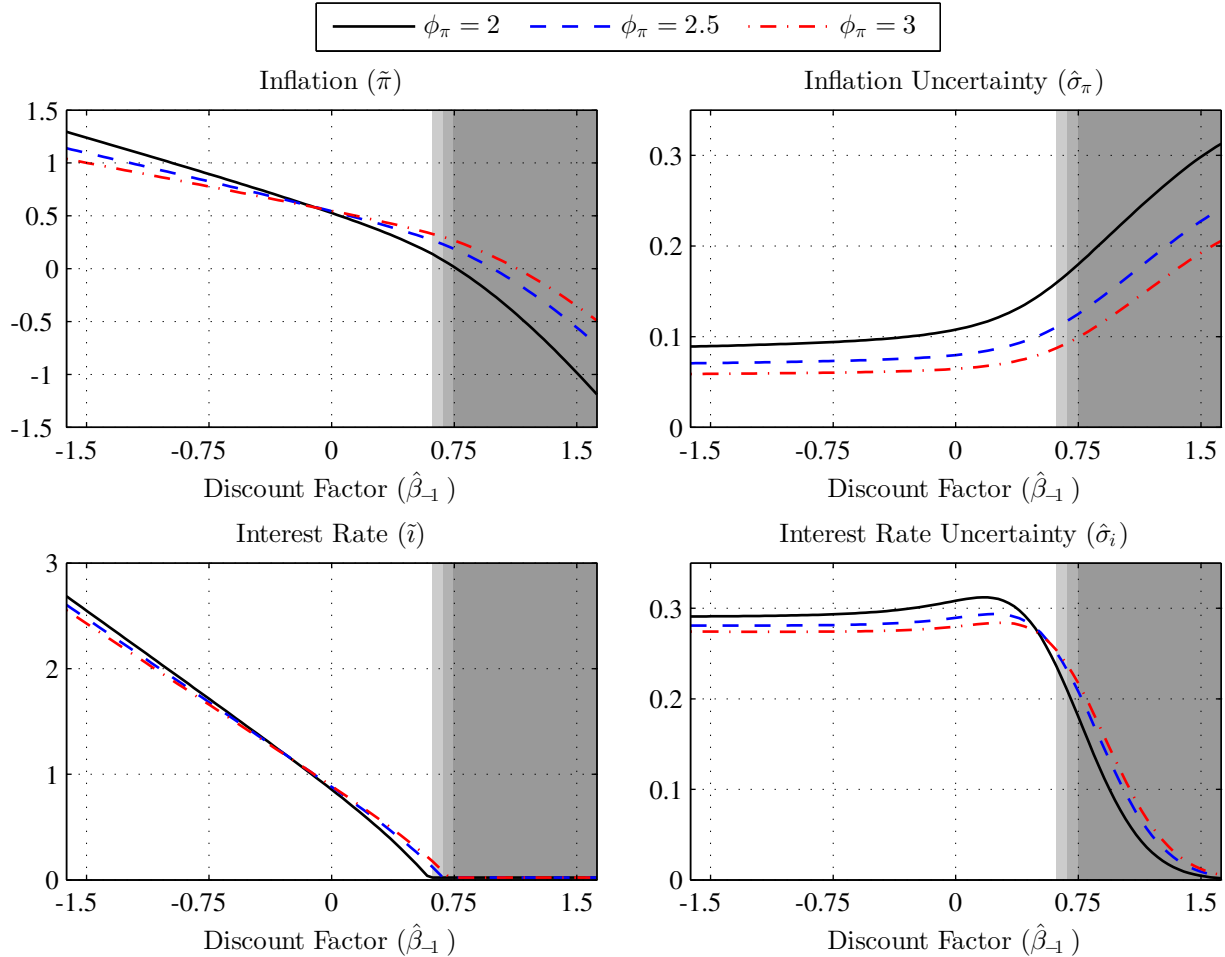

Figure 12: Policy function for inflation (top left panel), inflation uncertainty (top right panel), the nominal interest rate (bottom left panel), and nominal interest rate uncertainty (bottom right panel). The horizontal axes display the discount factor state in percent deviations from steady state. The vertical axes in the left panels display net rates. The vertical axis in the top (bottom) right panel shows the standard deviation of the forecast error for inflation (nominal interest rate) as a percent of steady-state inflation (interest rate). The shaded regions indicate where the ZLB binds.

5.3 INFLATION AND INTEREST RATE UNCERTAINTY

Thus far we have focused on the correlation between real GDP growth and real GDP uncertainty. This section tests our model along different dimensions by looking at the correlations between real GDP growth and the uncertainty surrounding both inflation and the nominal interest rate. We begin by examining the properties of our theoretical model and then turn to the data using SPF forecasts of the inflation and T-bill rates.

The top panels of figure 12 show the policy functions for inflation (left panel) and inflation uncertainty (right panel). The bottom panels show the policy functions for the nominal interest rate (left panel) and interest rate uncertainty (right panel). For simplicity, technology is fixed at

its steady state. The shaded regions indicate where the ZLB binds. The intuition for these results follows from our discussion about real GDP. When the discount factor is low, households are impatient and would like to increase current consumption. Firms respond to the higher demand by increasing their price level, which raises inflation and causes the central bank to increase its policy rate. As the discount factor increases, firms cut their prices because households become more willing to postpone consumption. When the central bank is unconstrained, it will reduce its policy rate to dampen the effects of the fall in demand. Given a sufficiently large decline in demand, the ZLB will bind and the central bank will be unable to respond to adverse shocks. As a consequence, positive discount factor shocks that occur at the ZLB cause larger declines in inflation (i.e., a steeper policy function). The more weight households place on those outcomes, the greater the dispersion in future inflation rates and the higher the uncertainty surrounding inflation.

Interest rate uncertainty, in contrast, is lower at the ZLB due to the constraint faced by the central bank. Any negative demand shock that occurs at the ZLB will not affect the nominal interest rate. It is also possible that positive demand shocks will not trigger a rate increase if the notional interest rate is sufficiently negative. Therefore, households place less weight on a positive nominal interest rate in discount factor states where the ZLB binds (i.e., the dispersion in the future policy rate declines), and the uncertainty surrounding the interest rate converges toward zero.

		Data		Model	
Sample	SPF IPD FD	SPF CPI FD	VAR	$z = \bar{z}$	$z \sim AR(1)$
Away from ZLB	-0.20**	-0.27***	-0.16^{++}	-0.23^{++}	-0.16^{+}
Near the ZLB	-0.44**	-0.61***	-0.35^{+}	-0.43^{+++}	-0.41^{+++}

Table 5: Comparison between correlations in the model and the data. The correlations in the model are between real GDP growth and the standard deviation of the inflation forecast error 1-quarter ahead. The correlations in the data are between real GDP growth and either the dispersion in forecasts of the inflation rate from the SPF or realized inflation volatility from the VAR. The correlations in the data are significantly less than 0 at ***1%, **5%, and *10% levels. The correlations in the model and VAR are positive with probabilities less than $^{+++}$1%, $^{++}$5%, and $^{+}$10%.

Table 5 compares correlations in our model to analogous correlations in the data. The correlations in the model are between real GDP growth and the standard deviation of the inflation forecast error 1-quarter ahead. The correlations in the data are between real GDP growth and the dispersion in forecasts of the inflation rate from the SPF. We use two measures of forecast dispersion: (1) the percent difference between the 75th and 25th percentiles of the individual forecasts of the GDP implicit price deflator (SPF IPD FD) and (2) the difference between the 75th and 25th percentiles of the individual forecasts of the CPI inflation rate (SPF CPI FD).[15] We also compute correlations using estimates of realized inflation volatility from our time-varying VAR with stochastic volatility.

A much stronger correlation between real GDP growth and inflation uncertainty emerged in mid-2008, similar to the correlations with real GDP uncertainty. All three measures of inflation uncertainty indicate that the correlations with real GDP growth were at least twice as strong from 2008Q3 to 2014Q2 as they were from 1986Q1 to 2008Q2. A z-transformation test shows the correlations with the SPF IPD FD are significantly different only at a 15% level, but the correlations with the SPF CPI FD are significantly different at a 5% level. Moreover, if we extend the sample back to 1968Q4, the correlations with the SPF IPD FD are significantly different at a 10% level.

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[15]We would prefer to use forecasts of core CPI inflation, but those were not included in the SPF until 2007Q1.

Both models have the same key feature as the data—the median correlation between real GDP growth and inflation uncertainty is much stronger when the nominal interest rate is close to or at its ZLB. Moreover, the difference between the correlations in the "Away from ZLB" sample and the "Near the ZLB" sample in the model with constant (variable) technology is positive in only 4.2% (3.2%) of the simulations, which provides further evidence that the correlations are more negative in the "Near the ZLB" sample. Neither model generates correlations in the "Near the ZLB" sample as negative as the correlations with the SPF CPI FD, but that is likely because they do not include an oil sector. The model with constant technology over-predicts the correlation with the SPF IPD FD and the VAR in the "Away from ZLB" sample, but with variable technology the correlation is between those values in the data. These results show that our model matches features of the data along another dimension. They also provide evidence that the ZLB contributed to the stronger negative correlations between real GDP growth and inflation rate uncertainty since mid-2008.
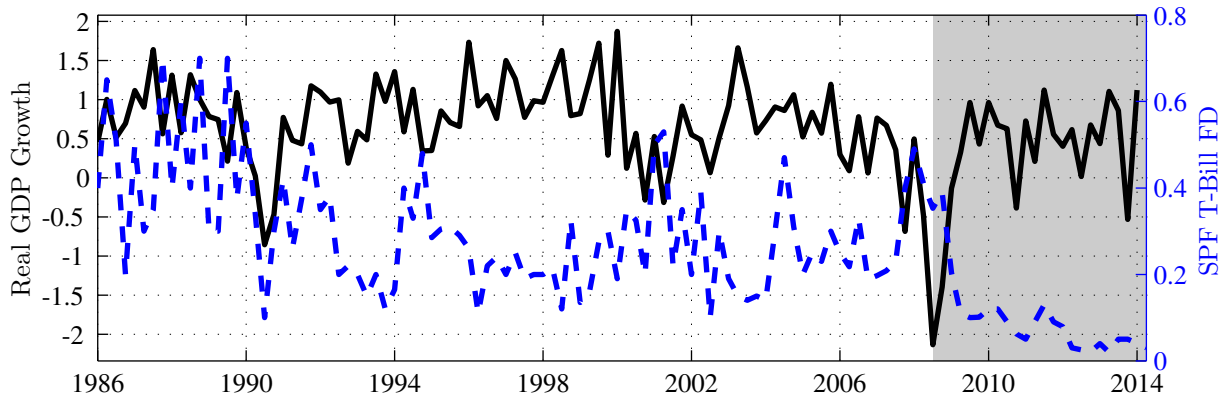

Figure 13: Real GDP growth (solid line) and 3-month T-bill uncertainty (dashed line). The horizontal axis displays the date. The left vertical axis displays quarter-over-quarter real GDP growth. The right vertical axis shows the dispersion in forecasts of the 3-month T-bill rate from the SPF. The shaded region indicates the post-Great Recession sample.

Figure 13 shows data on real GDP growth, $\Delta \ln(y_t/y_{t-1})$, (solid line, left axis) and nominal interest rate uncertainty (dashed line, right axis), which is measured by the difference between the 75th and 25th percentiles of the individual forecasts of the 3-month T-bill rate 1 quarter ahead (SPF T-bill FD). The post-Great Recession sample (2008Q3-2014Q2) is indicated by the shaded region.

The predictions of the model match two key features of the data. One, uncertainty about the T-bill rate converges toward zero as the economy moves into the post-Great Recession sample. Forecasters are aware that the short-term interest rate is constrained by the ZLB, which reduces their disagreement to its lowest level over the sample period. In the model, interest rate uncertainty approaches zero as the probability of exiting the ZLB declines. In other words, the longer households expect to remain at the ZLB, the more certain they are that the nominal interest rate will remain near zero in the future. Two, real GDP growth and T-bill uncertainty are positively correlated in the post-Great Recession sample. In the model, a drop in demand that causes the ZLB to bind decreases real GDP and simultaneously reduces uncertainty about the nominal interest rate. These results demonstrate that the model is consistent with the data along a third key dimension.

5.4 ROBUSTNESS OF THE MODEL CORRELATIONS   This section shows our results are virtually identical when we extend the model in section 3 to allow for habit formation in the household's

preferences and interest rate smoothing in the monetary policy rule—two features that are known to improve the empirical fit of the model [Christiano et al. (2005); Smets and Wouters (2007)].

A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t[\log(c_t - hc_{t-1}^a) - \chi n_t^{1+\eta}/(1 + \eta)]$, where c^a is aggregate consumption, which is taken as given by the household, and h is the degree of external habit persistence. The household's choices are constrained by $c_t + b_t = w_t n_t + i_{t-1}b_{t-1}/\pi_t + d_t$. The optimality conditions to the household's problem imply

$$w_t = \chi n_t^{\eta}(c_t - hc_{t-1}^a), \tag{7}$$

$$1 = i_t E_t[q_{t,t+1}/\pi_{t+1}], \tag{8}$$

where $q_{t,t+1} \equiv \beta_{t+1}(c_t - hc_{t-1}^a)/(c_{t+1} - hc_t^a)$ is the pricing kernel between periods t and $t + 1$ and $c = c^a$ in equilibrium. The production sector is unchanged, except firms now discount future dividends by $q_{t,k} \equiv \prod_{j=t+1}^{k>t} q_{j-1,j}$. The central bank sets the nominal interest rate according to

$$i_t = \max\{\underline{i}, i_t^*\}, \quad i_t^* = (i_{t-1}^*)^{\rho_i}(\bar{i}(\pi_t/\bar{\pi})^{\phi_\pi}(y_t/y_t^n)^{\phi_y})^{1-\rho_i}\exp(\nu_t) \tag{9}$$

where i_t^* is the notional interest rate and ρ_i controls the degree of interest rate smoothing. In the special case where both $h = 0$ and $\rho_i = 0$, the model is identical to one presented in section 3.

Sample	Real GDP Uncertainty		Inflation Uncertainty	
	$z = \bar{z}$	$z \sim AR(1)$	$z = \bar{z}$	$z \sim AR(1)$
Away from ZLB	-0.07	-0.16^+	-0.14^{++}	-0.20^{++}
Near the ZLB	-0.49^{+++}	-0.45^{+++}	-0.52^{+++}	-0.47^{+++}

Table 6: Correlations between real GDP growth and the standard deviation of the real GDP forecast error (cols. 1 & 2) and the standard deviation of the inflation forecast error (cols. 3 & 4) 1-quarter ahead in a model with habit persistence and interest rate smoothing. The correlations are positive with probabilities less than [+++]1%, [++]5%, and [+]10%.

Gust et al. (2013) use a particle filter to estimate a constrained nonlinear model that is similar to this model. Therefore, we set the habit persistence parameter, h, to 0.46629 and the persistence of the notional interest rate, ρ_i, to 0.95622, which equal their mean posterior estimates. The parameters of the stochastic processes are re-calibrated to match the volatilities of real GDP, the inflation rate, and the T-bill rate in the data. We set the persistence of the discount factor, ρ_β, to 0.92 and the standard deviation of the shock, σ_ε, to 0.0017. The standard deviation of the monetary policy shock, σ_ν, is set to 0.00125. All of the other parameters are set to the values shown in table 3.

The median standard deviations of real GDP, inflation, and the nominal interest rate as well as their 90% credible intervals are 2.25% $(1.96\%, 2.66\%)$, 1.02% $(0.80\%, 1.38\%)$, and 1.36% $(0.95\%, 1.95\%)$ annually, which are close to the values in the baseline model. When we initialize the economy in steady state, the average ZLB event is 3.66 quarters, and when we initialize at $i^* = -0.5\%$ it rises to 7.22 quarters, which is 4 quarters longer than the baseline model. Thus, this model also allows us to test whether a longer average duration of ZLB events affects our results.

Table 6 reports the correlations between real GDP growth and both real GDP and inflation uncertainty. The values are very similar to the values in tables 4 and 5. With both types of uncertainty, the correlations in the "Near the ZLB" sample are much stronger than the correlations in the "Away from ZLB" sample. The difference between the correlations with real GDP uncertainty in the two samples is positive in only 2.9% (6.7%) of the simulations of the model with constant (variable)

technology. When we calculate the correlations with inflation uncertainty, the equivalent value is 3.2% (7.5%). Therefore, our finding that the correlations are stronger when the economy is near or at the ZLB is robust to having habit formation, interest rate smoothing, and longer ZLB events.

6 CONCLUSION

This paper documents that a strong negative correlation between uncertainty and real GDP growth only emerged since mid-2008. Prior to that time, the correlation between those variables was weak and in many cases not statistically less than zero, even when restricting the data sample to only include recessions. Why did the Great Recession lead to a stronger negative correlation compared to previous recessions? One reason is the ZLB on the short-term nominal interest rate. During the Great Recession many central banks sharply reduced their policy rates and effectively hit the ZLB for the first time in their history. We contend that central banks' inability to further reduce their policy rates in response to adverse economic conditions contributed to the stronger correlation.

To test our theory, we use a model where the policy rate occasionally hits its ZLB. The model predicts an increase in output uncertainty near and at the ZLB. When the nominal interest rate is far from its ZLB, uncertainty surrounding output is nearly constant and low. This result occurs in the model because real GDP becomes more responsive to shocks that hit the economy when the central bank is constrained by the ZLB, which increases the dispersion of future real GDP. Therefore, the model has the same key feature as the data—away from the ZLB the correlation is weak but strongly negative when the short-term nominal interest rate is close to or at its ZLB. Our model is also consistent with the stronger relationships that emerged between real GDP growth and both inflation and interest rate uncertainty. While it is possible that the ZLB is not the only factor causing the stronger correlations, our results provide strong evidence that it is an important factor.

Our paper also presents several avenues for future research. One, it is possible to calculate our measure of uncertainty for other variables in both closed and open economy models, which could be used to explain the behavior of uncertainty in the data. Two, one could augment our model with stochastic volatility shocks to compare the role of those shocks in explaining movements in uncertainty. Finally, the ZLB affects the behavior of uncertainty because it introduces a kink in the policy functions. It stands to reason that other mechanisms that introduce nonlinearities, such as irreversible investment decisions, may have similar impacts on uncertainty that can explain data.

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