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Theoretical Analysis for a Damping Ratio of a Jointed Cantibeam*

By Masami MASUKO**, Yoshimi ITO***, and Keizo YOSHIDA****

The loss energy of a jointed cantibeam may be derived from a microscopic slip and a normal pressure on the interfaces. It is however very difficult to calculate theoretically the energy dissipation between the joint surfaces, because a characteristic of the micro-slip on the interfaces has not been fully clarified yet.

In this study the mechanism of the energy dissipation and also the ways of theoretically calculating the damping ratio of the jointed cantibeam have been investigated by considering experimental results of the characteristic of static micro-slip between the jointed surfaces.

From the results it is clarified that the damping ratio of the jointed cantibeam can be well calculated by using the equations proposed in this paper, and how the characteristic of micro-slip influences the behaviours of the damping ratio.

1. Introduction

Recently demand for machine tools having a high dynamic stiffness is increasing and it is widely recognized that the machine tool should have a high static stiffness together with a light weight. For this purpose the welded steel construction is very suitable, and then the use of machine tools made of welded structures has been examined frequently up to the present.

The steel plates used in the welded machine tool structures, however, have a low damping capacity as compared with the cast plates, therefore for the design of the welded machine tool structure it frequently comes into a difficult question how to increase the damping capacity. To increase the damping capacity of welded structures there are many methods, for example the use of a suitable welded joint, and the use of a jointed beam and a sandwiched beam.

Many studies have been made on the sandwiched beam, the jointed beam and the laminated beam, and some of these beams have been already used practically[1]. In the past authors also studied on the relationships between the form of welded joint and the damping capacity and reported that the steel plates welded with the plug joints, which are to be considered a kind of the jointed plates, show a very high damping capacity[2].

From the results of these previous studies[3] it is generally recognized that the damping capacity of the jointed beam and the sandwiched beam may be determined by the frictional loss energy caused by the slip between the interfaces of both steel plates or the slip between the steel plates and the sandwiched viscoelastic material. The slip found between the interfaces of jointed beam however is very small and shows very complicated characteristics, and moreover the coefficient of friction in this case may be considered smaller than the macroscopic coefficient of friction used widely in the field of wear[4],

By the reasons mentioned above, the mechanism of the frictional loss energy has not been clearly explained yet. In this paper, therefore the mechanism of the damping caused by the slip of the jointed beam, which is considered one of basic elements of the welded structure, has been investigated theoretically by using the previous experimental results[5][6], in which the behaviours of the static horizontal displacement in the bolted joint and the effects of the connecting conditions on the damping capacity of the bolted joint were dealt with.

Furthermore by applying these results to the jointed beam bolted together, the theoretical damping capacity of it has been calculated and compared with the experimental values and from this result a method how to calculate the damping capacity derived from the micro-slip between the interfaces has been also proposed.

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** Professor, Tokyo Institute of Technology, Meguro-ku, Tokyo.
*** Associate professor, Tokyo Institute of Technology.
**** Graduate Student, Tokyo Institute of Technology.

2. Nomenclature

A : unknown constant
a : unknown constant
a_n : n-th amplitude of damped free vibration
B : width of beam
C_{eq} : equivalent fraction of viscous damping
E : Young's modulus
E_{ne} : energy stored in system
E_{loss} : frictional loss energy in system
E_n : total energy stored in system at amplitude a
f_n : natural frequency of beam
$2h$: thickness of beam
I : cross-sectional moment of inertia of beam
k : bending stiffness of beam per unit width
l : length of cantibeam
l_p : influence area of interface pressure
m : integer constant
m_{eq} : equivalent mass of beam
n : integer constant
$p(x)$: interface pressure at the position x
t : time
$u(x,t)$: relative displacement at the position x when $t=t$
$u_r(x,t)$: residual micro slip at the position x when $t=t$
W : load
x : co-ordinate in the direction of neutral axis of beam
$y(x,t)$: bending deflection of beam at the position x when $t=t$
$Y(x)$: eigen function of the equation of motion for bending vibration of beam
(x,y,z) : Cartesian co-ordinates
α : slip ratio
δ : logarithmic damping decrement
ε : unknown constant
θ : angle of inclination of beam
μ : coefficient of friction
μ_{eq} : equivalent coefficient of friction
Ψ : damping ratio
φ_m : damping constant
ω : angular frequency

Suffix
i : i th influence area of interface pressure

3. Theoretical analysis and an example of theoretical calculation of the damping capacity derived from the micro-slip between the interfaces

3·1 Theoretical analysis

Let us consider now a jointed cantibeam, in which one end is in free condition and another end is in fixed condition as shown in Fig.1. In this jointed cantibeam it is assumed that the energy loss between the interfaces only yields within the influence area of the interface pressure given by the joint elements, such as a bolted joint or a plug welded joint, and that the influence area of interface pressure is separated in n places.

In order to simplify the theoretical analysis it is also assumed that each beam of the jointed cantibeam being vibrated has the equal bending stiffness and is in the same bending condition, and that each beam shows no extension of the neutral axis and no deformation of the cross-section.

If the jointed cantibeam is given an initial deflection at point A and vibrated freely, this beam shows a relative displacement at the interfaces as shown in Fig.2. Therefore, if the co-ordinates are defined as shown in Fig. 1, the relative displacement $u(x,t)$ at $x=x$ is nearly equal to the sum of Δu_1 and Δu_2 and then

$$u(x,t) \fallingdotseq \Delta u_1 + \Delta u_2 = 2h \tan\left(\frac{\partial y(x,t)}{\partial x}\right) \quad \cdots\cdots\cdots(1)$$

where $y(x,t)$ is the bending deflection of the jointed cantibeam in y direction.

The actual micro-slip $u_r(x,t)$ between the interfaces during the vibration is calculated by subtracting the elastic recovery part of the relative displacement from $u(x,t)$[6] and written as

$$u_r(x,t) = \alpha u(x,t) \quad \cdots\cdots\cdots(2)$$

where α is the unknown constant varied by the interface pressure and the condition of jointed surfaces. This mechanism is shown in Fig. 3 by using the hysterisis loop, and in the figure the

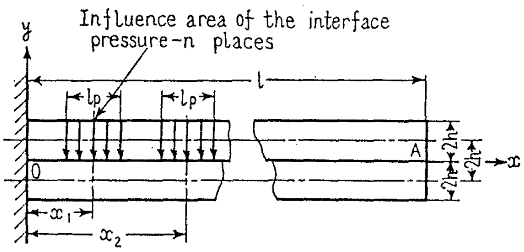


Fig. 1 Model of jointed cantibeam

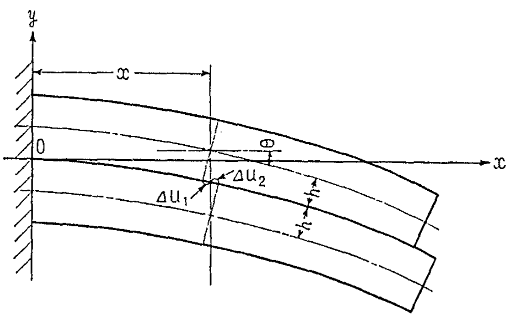


Fig. 2 Mechanism of the horizontal displacement

area OAB shows the loss energy dissipated by the micro-slip and the area ABH shows the elastic recovered energy.

By expressing the interface pressure at x as $p(x)$ and by assuming that $p(x)$ is constant in z direction (the direction to the width of beam), the normal load acting on the length of dx is $p(x)Bdx$, where B is the width of beam.

Thus the frictional force is given by $\mu p(x)Bdx$; therefore after an arbitrary time passed, the frictional energy loss per half cycle $E_{\rm loss}$ found within influence area of interface pressure may be written as

$$E_{\rm loss}=2\sum_{i=1}^{n}\int_{m\pi/2\omega}^{(m+1)\pi/2\omega}\int_{x_i-l_p/2}^{x_i+l_p/2}\mu Bp(x)\frac{\partial u_r(x,t)}{\partial t}dxdt \quad \cdots\cdots (3)$$

$(m=0, 1, 2\cdots\cdots)$

During the unloading process the energy to be introduced into the jointed cantibeam per half cycle E_{ne} is also given by the following equation after the arbitrary time passed

$$E_{ne}=\frac{3EI}{l^3}y^2\left(l,\frac{m\pi}{2\omega}\right) \quad \cdots\cdots (4)$$

$(m=0, 1, 2\cdots\cdots)$

where E : Young's modulus

I : cross-sectional moment of inertia of each cantibeam

l : length of jointed cantibeam

Defining here that the damping constant φ_m is the ratio of E_{ne} to $E_{\rm loss}$, φ_m is given by

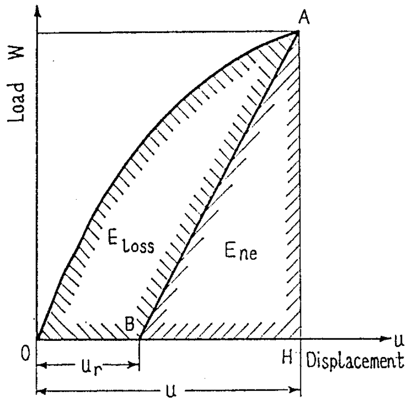
$$\varphi_m=\frac{E_{\rm loss}}{E_{ne}}=\frac{2\sum_{i=1}^{n}\int_{m\pi/2\omega}^{(m+1)\pi/2\omega}\int_{x_i-l_p/2}^{x_i+l_p/2}\mu Bp(x)\frac{\partial u_r(x,t)}{\partial t}dxdt}{\frac{3EI}{l^3}y^2\left(l,\frac{m\pi}{2\omega}\right)}$$

$$=\frac{4\sum_{i=1}^{n}\int_{m\pi/2\omega}^{(m+1)\pi/2\omega}\int_{x_i-l_p/2}^{x_i+l_p/2}\alpha\mu Bhp(x)\frac{\partial}{\partial t}\left\{\tan\left(\frac{\partial y(x,t)}{\partial x}\right)\right\}dxdt}{\frac{3EI}{l^3}y^2\left(l,\frac{m\pi}{2\omega}\right)} \quad \cdots\cdots (5)$$

It is reasonable to consider here that the damping ratio Ψ is expressed as the ratio between the total energy introduced into the system and the loss energy; therefore the damping ratio is written as

$$\Psi=\frac{E_{\rm loss}}{E_{\rm loss}+E_{ne}}=\frac{1}{1+\frac{1}{\varphi_m}} \quad \cdots\cdots (6)$$

While the logarithmic damping decrement δ is the ratio of a_n and a_{n+1}, namely $\delta=\ln\frac{a_n}{a_{n+1}}$, where a_n is an amplitude of vibration at certain time and a_{n+1} is the amplitude of vibration after



$u_r(x,t)=\alpha u(x,t)$

Fig. 3 Relationship between u_r and u

1 cycle passed. If the energy stored in system when amplitude of vibration is a_n, is denoted as E_n, it is easily known that $E_n=E_{ne}+E_{\rm loss}$ and $E_{n+1}=E_n-E_{\rm loss}$. Therefore, assuming that E_n is equal to $Ca_n{}^2$, the relationship between the damping ratio and the logarithmic damping decrement is written as follows.

$$\delta=\ln\left(\frac{E_n}{E_{n+1}}\right)^{1/2}=\frac{1}{2}\ln\left(\frac{1}{1-\Psi}\right) \quad \cdots\cdots (7)$$

In case of $\Psi\ll 1$, Maclaurin expansion of Eq. (7) is given by

$$\delta=\frac{1}{2}\left(\Psi+\frac{1}{2}\Psi^2\right) \quad \cdots\cdots (7\cdot a)$$

where the high order terms more than Ψ^3 are ignored. Furthermore by substituting Eq. (6) into Eq. (7) we can obtain

$$\delta=\frac{1}{2}\ln(\varphi_m+1) \quad \cdots\cdots (7\cdot b)$$

3·2 An example of numerical calculation

From the above analysis, it is easily considered that the value of α in Eq. (5), which shows the characteristics of the micro-slip between the interfaces, has large influences on the damping ratio. According to the previous study[5] the relationship between the interface pressure and the value of α for various conditions of jointed surfaces gives a curve as shown in Fig. 4 or Fig. 14, and this curve is assumed to be represented by

$$\alpha = u_r/u = Ae^{-ap^{\varepsilon}}$$

where A, a and ε are constants determined by the interfaces condition.

By considering here the condition that the interface pressure is uniformly spread over the overall contact area given by Bl, $p(x)$ yields p. Moreover $\partial y(x,t)/\partial x$ indicated the angle of inclination of beam is very small, and then $\tan[\partial y(x,t)/\partial x]$ may yield $\partial y(x,t)/\partial x$.

Putting $m=0$, Eq. (5) yields

$$\varphi_0 = \frac{4\mu Bhp\alpha}{(3EI/l^3)y^2(l,0)} \int_0^{\pi/2\omega} \int_0^l \frac{\partial^2 y(x,t)}{\partial x \partial t} dx dt \quad \cdots\cdots\cdots (8)$$

By making use of the boundary and initial conditions that are $y(l,0)=y_0$, $\partial y(l,0)/\partial t=0$, the bending deflection of beam being vibrated $y(x,t)$ can be written as

$$y(x,t) = Y(x)\frac{y_0}{Y(l)} \cos \omega t \cdots\cdots\cdots\cdots\cdots\cdots (9)$$

where $Y(x)$ is an eigen function given by

$$Y(x) = (\sinh\lambda_1 + \sin\lambda_1)(\cosh\lambda_1 x/l - \cos\lambda_1 x/l) - (\cosh\lambda_1 + \cos\lambda_1)(\sinh\lambda_1 x/l - \sin\lambda_1 x/l)$$

$$\lambda_1 = 1.875$$

Using Eq. (6), Eq. (8) and Eq. (9), we obtain

$$\Psi = \frac{1}{1 + k/4\mu\bar{y}pAe^{-ap^{\varepsilon}}} \cdots\cdots\cdots\cdots\cdots\cdots (10)$$

where $k = \frac{3EI}{l^3 B}$, $\bar{y} = \frac{h}{y(l,0)}$

From Eq. (10), the optimum value of interface pressure in which the damping ratio Ψ is in maximum, is obtained as

$$p = \sqrt[\varepsilon]{\frac{1}{a\varepsilon}} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (11)$$

In Figs. 5 ~ 7, the changes of the damping ratio Ψ are shown to relate to the interface pressure when the constants A, a and ε have various values. In these calculations, considering the results of previous study[(2)] we furthermore found that $k=3.33\times10^{-2}$ kg/mm^2 and $\bar{y}=20$.

From the results presented here, it is known that the constants A, a and ε, which show the characteristics of micro-slip between the jointed interfaces, have large influences on the damping ratio as mentioned below.

(1) The value of A has large effects on the absolute value of damping ratio, but has a very little effect on the form of curve, which shows the change of the damping ratio with an increasing interface pressure. Moreover the value of damping ratio is proportional to the value of A and the curve corresponding to a small value of A indicates a clear peak as compared with the curve for a large value of A.

(2) The values of a and ε have a little effect on the maximum value of damping ratio, but have a close relation to the form of curve. The

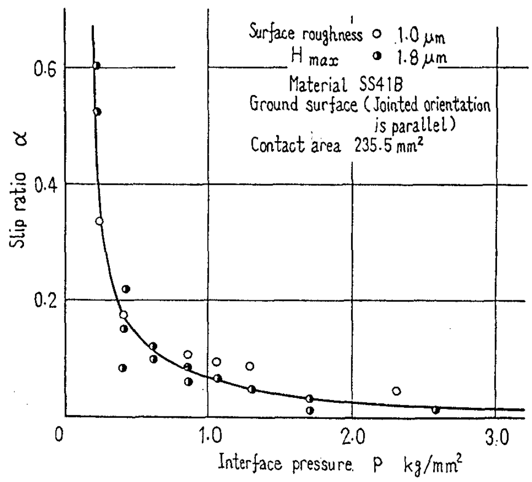


Fig. 4 Micro-slip characteristic between the jointed surfaces

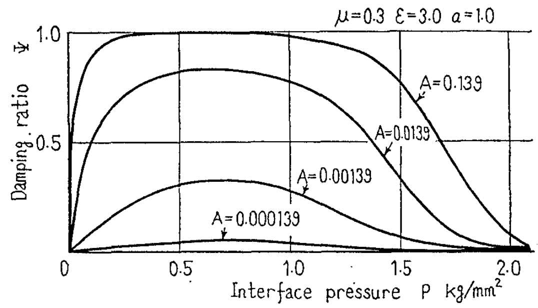


Fig. 5 Effects of A on the damping ratio Ψ

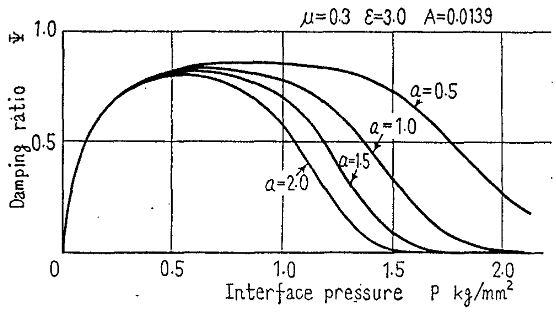


Fig. 6 Effects of a on the damping ratio Ψ

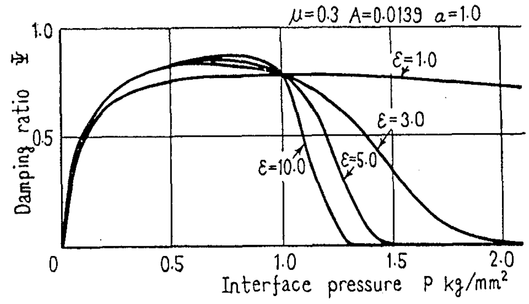


Fig. 7 Fffects of ε on the damping ratio Ψ

latter behaviours are clearly seen where the interface pressure is high. For example as known from Figs. 6 and 7, the curve corresponding to the small values of a and ε is gently sloping and with increasing values of a and ε the curve comes to indicate a clear peak.

From these results mentioned above it may be said that the damping ratio shows a clear peak and also a large value when the micro-slip between the jointed interfaces decreases steeply with an increased interface pressure. This phenomenon of micro-slip is called the negative derivative characteristic and for instance the interfaces having a low flow pressure and rough surfaces indicate a large negative derivative characteristic in its u-p curve.

The effect of the coefficient of friction on the damping ratio has been also calculated here and the results are shown in Fig. 8. From the presented figure it is known that the coefficient of friction has no effects on the form of Ψ-p curve, but the damping ratio Ψ increases with an increased coefficient of friction.

It should be noted, however, that there may be found a peak in the curve indicating the relation between the damping ratio and the coefficient of friction, because $E_{\rm loss}$ is proportional to $\mu p u_r$. Namely, for small values of μ, $E_{\rm loss}$ becomes nearly zero and for large values of μ, $E_{\rm loss}$ also becomes nearly zero, since in this case u_r may be nearly zero.

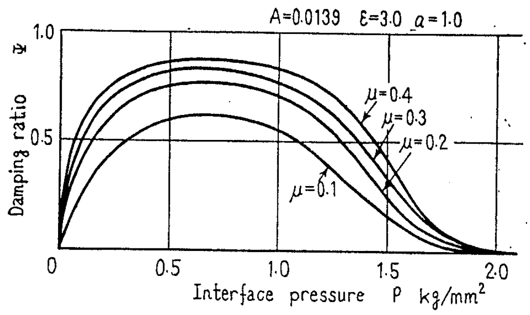


Fig. 8 Effects of the coefficient of friction μ on the damping ratio

In this analysis for the damping ratio of the jointed cantibeam this problem mentioned above has not been taken into consideration; therefore the practical use is considered to be limited to certain conditions. For this problem we think further detailed theoretical and experimental study will be necessary.

4. Experiment and experimental results

4·1 Experimental set-up and experimental techniques

In order to confirm the reliability of the theoretical analysis presented in the previous chapter experiments were carried out with the simple experimental set-up shown in Fig. 9. In this experimental set-up, the jointed beam should be held in fixed condition at its clamping place to measure accurately the damping capacity of the jointed cantibeam itself.

To meet this requirement the test specimen ①, namely the jointed cantibeam, clamping between two small blocks ②, is fixed by using the threaded shaft, and the clamping load is measured with the load cell. From the result of pre-experiment carried out with same techniques in the previous study[(2)], during experiments this clamping load is kept at two tons.

In all experiments, the jointed interfaces of the test specimen and also the contact surfaces

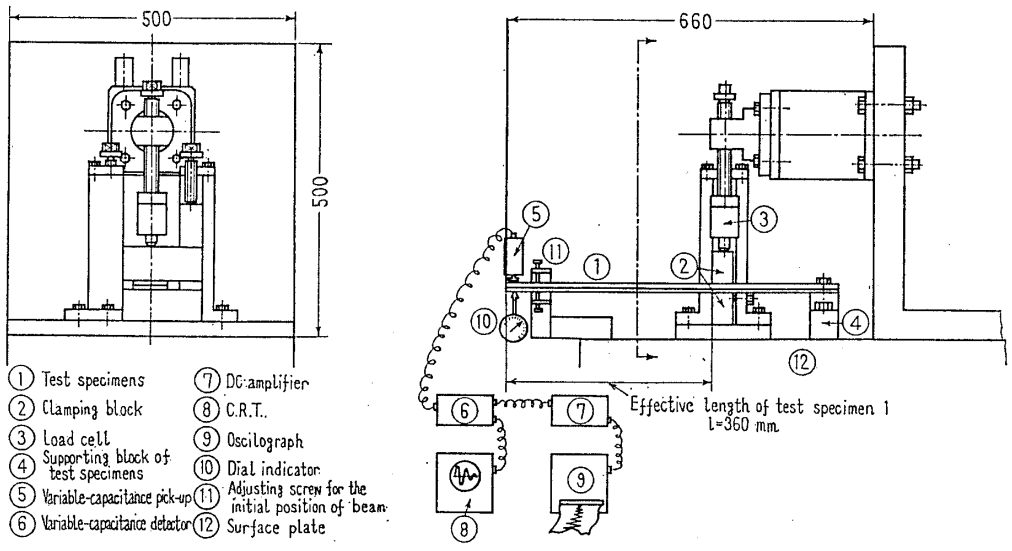


Fig. 9 Experimental set-up

between the blocks and the test specimen are degreased with trichlorethylene to obtain the real metal contact condition at the clamping device. The damped free vibrations of the jointed cantibeam were recorded on the oscillograph ⑨ and from the recorded curve the damping capacity has been measured as the logarithmic damping decrement, when the total amplitude of vibration is 100 μm.

The test specimens made of mild steel (J.I.S. SS41P) and examined here are shown in Fig. 10. Thicknesses of beams are 4 mm, 6 mm, and 10mm and the cantilength of beam is 360 mm.

The interface pressure acting on the interfaces of jointed cantibeam is controlled by changing the preload of connecting bolt M 8 from 50 kg to 600 kg, and these preloads were measured with strain gauges stuck on the stem of connecting bolt.

4·2 Experimental results and consideration

The experimental results are shown in Fig.11. In this figure the results were arranged by using the mean interface pressure on the horizontal axis.

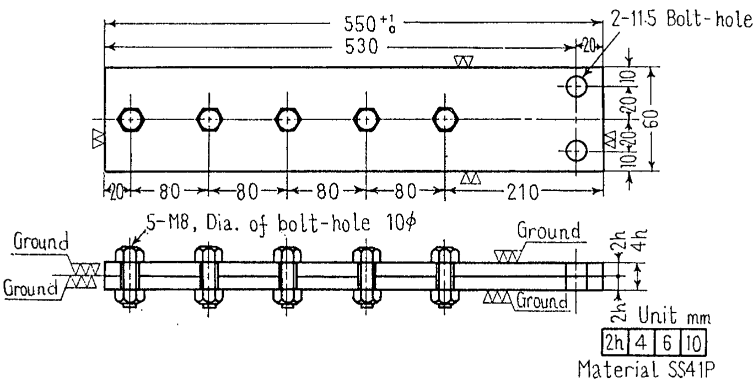


Fig. 10 Jointed cantibeam used in experiments

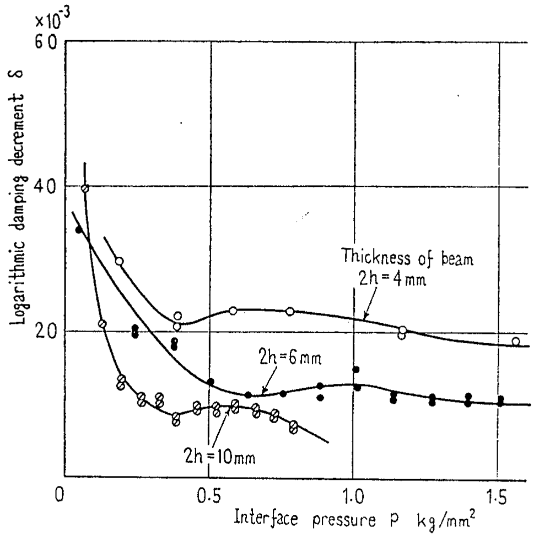


Fig. 11 Changes of the logarithmic damping decrement per 1 cycle with an increased thickness of beam (Total amplitude of vibration: 100μm)

The reasons are that the actual interface pressure changes with the change of the thickness of beam even if the preload of connecting bolt M 8 is held at a constant value for each jointed cantibeam having different thicknesses. As the mean interface pressure, here we use a mean pressure within the pressure cone proposed by Rötscher.

From the results shown in Fig. 11, it is known that the logarithmic damping decrement decreases with the increased interface pressure, and is at a maximum and a minimum during its decreasing process. For the reason of difficulties of experimental techniques we could not measure the peak in the δ-p curve, and then in Fig. 11 it is not seen, but the logarithmic damping decrement may have the peak value at a very low interface pressure as known by Eq. (10); therefore after passing through the peak value the logarithmic damping decrement shows the curve as presented in Fig. 11.

The logarithmic damping decrement means fundamentally the energy loss of beam per one cycle during its vibration and the natural frequency of beam is in proportion to its thickness, therefore the logarithmic damping decrement becomes small with the increased thickness of beam as shown in Fig. 11.

Figure 12 shows the results obtained under another arrangement, in which an equivalent viscous damping constant is considered to be expressed as the energy loss of beam per unit time. For structures subjected to an exciting force with a wide range of frequencies, it is suitable to consider the equivalent viscous damping constant in order to compare to each other in its damping capacity. By assuming a vibration system with one degree of freedom, which is equivalent to the jointed cantibeam examined here, the equivalent viscous damping constant C_{eq} is given by

$$C_{eq}=2m_{eq}\delta f_n \qquad (12)$$

where m_{eq} is an equivalent mass, δ is a logarithmic damping decrement and f_n is a natural frequency. As shown in Fig. 12 the value of C_{eq} increases with the increased thickness of beam and especially the jointed cantibeam with 10 mm thickness shows large values of C_{eq} under the low interface pressure less than about 0.2 kg/mm². This latter phenomenon is well explained by considering the mechanism of damping mentioned in the previous chapter, namely in this case the microslip is larger than in the other cases.

For the help of practical use in Fig. 13 the effects of the oil and MoS_2 applied to the interfaces on the damping capacity of the jointed cantibeam with 6 mm thickness are shown furthermore. It is known that the layer between the jointed interfaces has various effects on the logarithmic damping decrement according to the kind of layer itself. for example the damping capacity of the jointed beam with MoS_2 in its interfaces is about twice as large as that of the jointed beam having no layer between its interfaces.

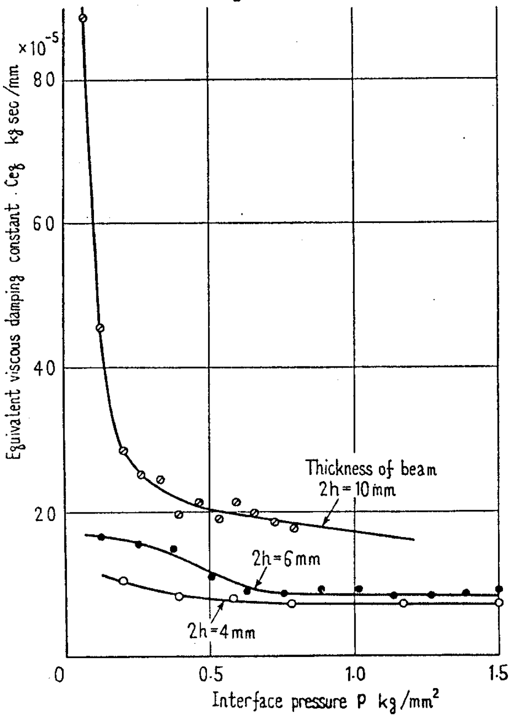


Fig. 12 Changes of the equivalent viscous damping constant with an increased thickness of beam (Total amplitude of vibration 100 μm)

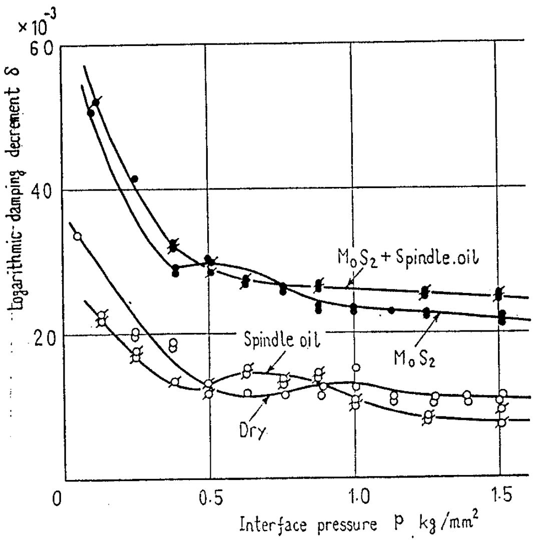


Fig. 13 Effects of the layers on the logarithmic damping decrement (Total amplitude of vibration: 100 μm)

4·3 Comparison between the numerical calculation and the experimental results

To compare the experimental values shown in Fig. 11 with the theoretical values calculated by Eq. (10), at first the relationship between the slip ratio α and the interface pressure must be known; therefore, the measurement of this relationship was carried out statically with the same techniques as in the previous study[5] and with the test specimens having nearly same jointed surfaces condition as the jointed cantibeam used in experiments.

The measured results are shown in Fig. 14 with semi logarithmic chart. In the figure the slip ratio α is in linear relation having a certain band width to the interface pressure p and the lines consist of three parts I, II and III having different gradients to each other corresponding with

Table 1 Measured values of the constants determining the slip ratio

Region		Interface pressure p kg/mm²	A	a	ε
Upper limit	I	0 ~0.15	1.0	0	1.0
	II	0.15~0.42	3.5	10.0	
	III	more than 0.42	0.25	1.3	
Lower limit	I	0 ~0.14	1.0	0	
	II	0.14~0.43	3.5	8.0	
	III	more than 0.43	0.15	1.5	

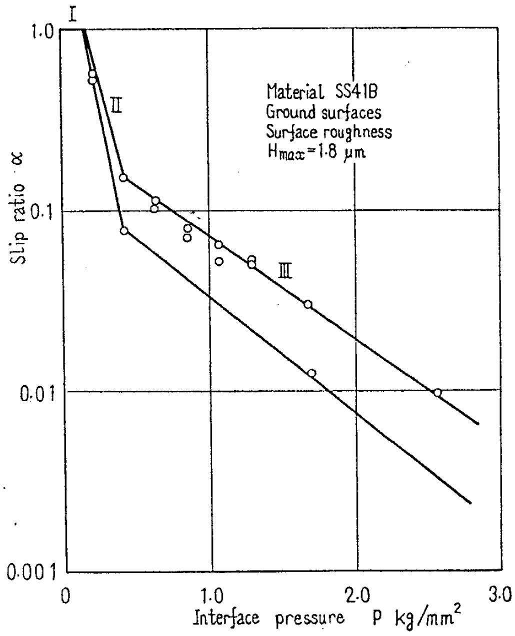


Fig. 14 Relationships between the slip ratio α and the interface pressure p

the value of interface pressure. From this figure the values of A, a and ε are calculated as shown in Table 1.

Let us consider here the coefficient of friction between the interfaces, which offers a problem frequently when the damping capacity derived from friction should be calculated. According to the previous studies, the micro-slip found on such a bolted joint is very small. This fact is also confirmed in this study by using the instrument shown in Fig. 15, by which the relative micro-slip between the upper and lower beams was measured with strain gauges made of semi-conductance.

From the results it is known that the micro-slip is very small, being about 0.01~0.03 μm when the total amplitude of vibration of the jointed cantibeam is 100 μm at its free end. Thus the coefficient of friction corresponding to the micro-slip condition such as bolted joint may not be considered as equal to the macroscopic coefficient of friction, which corresponds to the large slip and has been generally used in the field of wear and friction until now. In this study, therefore, to distinguish the difference between both coefficients of friction the equivalent coefficient of friction μ_{eq} has been considered.

That a true contact area is A_r when a micro-slip is u means that a true contact area reaches A_r when a microslip increasing from zero becomes u; therefore if a further micro-slip will be produced, a further horizontal load must be applied to the jointed surfaces. In this case, therefore, it may be reasonable to consider that the equivalent coefficient of friction μ_{eq} is given by the fraction between the horizontal and normal load applied to the jointed interface.

It has been recognized that indicating a sort of stick slip, such a micro-slip appears in the jointed interfaces. For example, the experimental results presented by Courtney-Pratt et al.,[4] in which they measured the static micro-slip with use of the optical interferometer, also verified these facts. By applying the idea of the equivalent coefficient of friction to the results presented by Courtney-Pratt et al., the values of μ_{eq} are obtained as shown in Fig. 16. It is also very interesting

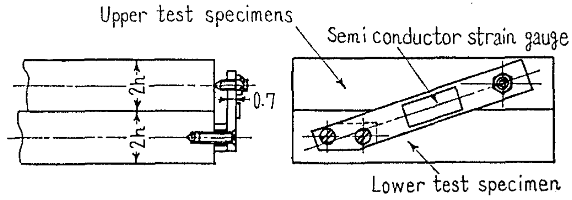


Fig. 15 Instrument for measuring the dynamic micro-slip

that μ_{eq} changes with the quantity of micro-slip.

From the results shown in Fig. 16 the equivalent coefficient of friction μ_{eq} may be estimated as its mean value at about 0.02 for the quantity of micro-slip in the jointed cantibeam examined here. However this value is a static one and here the dynamic equivalent coefficient of friction μ_{eqdyn} has to be considered. By considering the general relationships that the static coefficient of friction is (2~2.5) times of the dynamic coefficient of friction, we may write

$$\mu_{eqdyn}=\frac{1}{2\sim2.5}\mu_{eqst}$$

As the result of this calculation we estimated μ_{eqdyn} at 0.009 as its mean value.

Using the values of A, a, ε and μ_{eqdyn} measured and estimated above, and multiplying the ratio between the area subjecting to the preloads and the overall contact area to the logarithmic damping decrement calculated by Eq. (10), the theoretical logarithmic damping decrement yields as shown in Fig. 17, where the influence area of the preload of connecting bolt was calculated by using Rötscher's pressure cone.

In Fig. 17 the experimental values are also shown together and from these results it is known that for the jointed cantibeam with 4 mm thickness the theoretical values are in good agreement with the experimental values, and that the theoretical and experimental minimum values of the logarithmic damping decrement are also in good correspondence to each other, especially for the interface pressure in which the logarithmic damping decrement is minimum. The theoretical values of δ for the jointed cantibeam with 6 mm and 10 mm thickness, however, are slightly large to compared

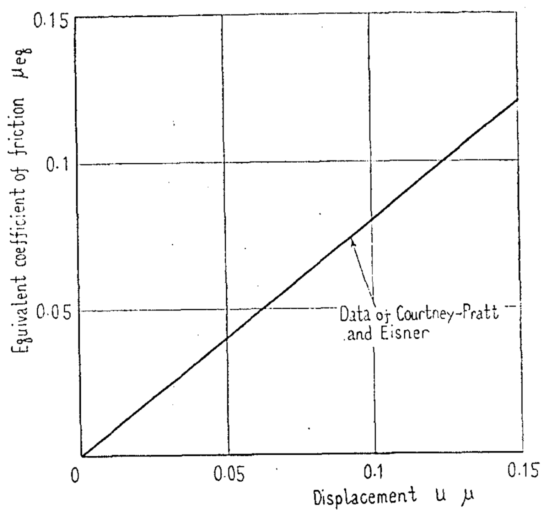
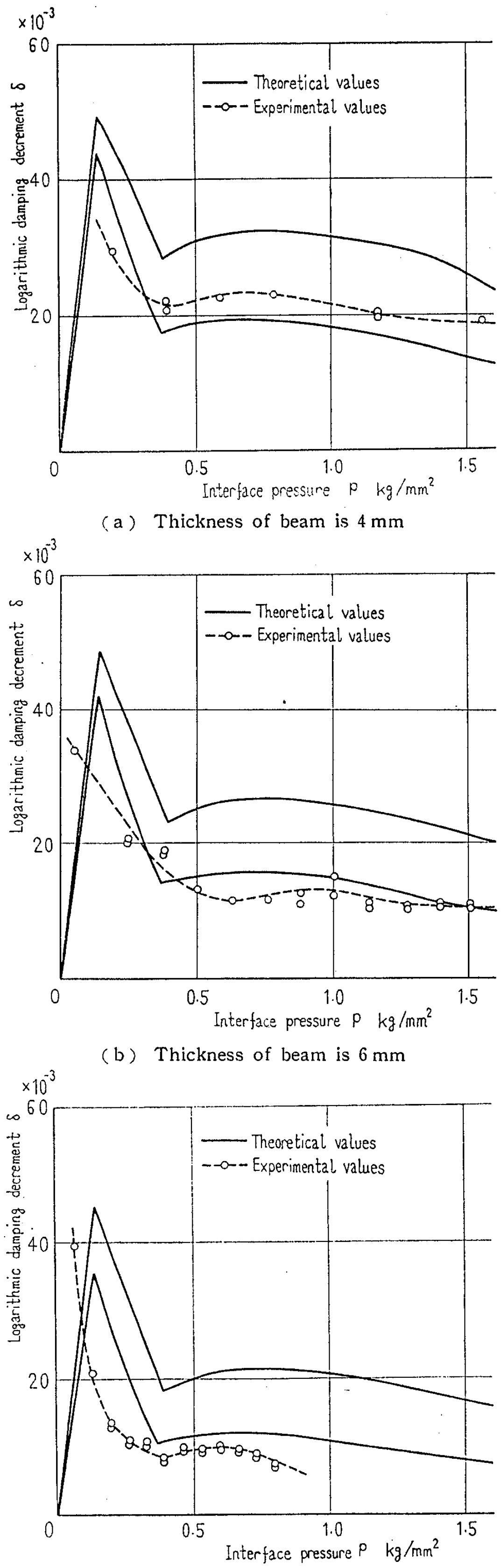


Fig. 16 Corelation of the equivalent coefficient of friction to the displacement



(a) Thickness of beam is 4 mm

(b) Thickness of beam is 6 mm

(c) Thickness of beam is 10 mm

Fig. 17 Comparison between the theoretical and experimental values

with their experimental values, as shown in Figs. 17(b) and (c), though the theoretical interface pressure where the logarithmic damping decrement shows the minimum value, is in comparatively good agreement with experimental one.

The reasons for these tendencies that the experimental values are lower than the theoretical values with the increasing thickness of the jointed cantibeam, may be explained as follows.

The actual slip ratio may be considered small as compared with slip ratio used in calculation, by the reasons of the increased bending stiffness and natural frequency of the jointed cantibeam with the increasing thickness of beam, namely though the horizontal displacement u is large, the micro-slip u_r remains small, and then the slip ratio α yields to the small value.

Furthermore in this study Rötscher's pressure cone has been used to calculate the influence area of the preload, however, there are some problems in this assumption. For the estimation of the influence area of preload, if we consider the facts that the pressure distribution of preload is high near the bolt-hole and low near the circumference of the pressure cone[7], the theoretical values come closer to the experimental values.

According to the results of comparison between the theoretical and experimental values mentioned above, it is clarified that the damping capacity of the jointed cantibeam caused by the frictional loss energy is estimated sufficiently and calculated theoretically by the theoretical analysis proposed in the previous chapter, and if we will solve the further problem mentioned above, the analysis may be expected to be more accurate.

For reference, some examples of the values of A, a, and ε for the jointed surfaces having various roughnesses are shown in Table 2.

5. Conclusions

To attempt to design machine tool structures

Table 2 Values of A, a and ε in $\alpha=u_r/u=Ae^{-ap^{\varepsilon}}$

Material	Machining method and surface roughness	Interface pressure p kg/mm²	A	a	ε
SS41B	Lapped surface $H_{max}=1.6\mu m$	0 ~0.5	0.95	5.0	1.0
		more than 0.5	0.11	1.2	
	Lapped surface $H_{max}=0.1\mu m$	0 ~0.48	1.0	4.5	
		more than 0.48	0.22	1.1	
	Lapped surface $H_{max}=0.04\mu m$	0 ~0.23	1.0	0	
		0.23~0.71	2.4	4.0	
		more than 0.71	0.4	1.3	

by using welded constructions, the low damping capacity of the steel plate, which is an important member in the welded construction, offers a problem. Such a sandwiched plate has been utilized in these cases to increase the damping capacity of steel plate itself; therefore in this study the damping mechanism and the damping capacity of the jointed cantibeam which is regarded as a fundamental construction of laminated and sandwiched beams, have been investigated theoretically and experimentally.

As the result we can conclude as follows.

(1) The mechanism for the damping capacity of the jointed cantibeam can be explained by considering the frictional loss energy between the jointed interfaces, and then by introducing the slip ratio α into the theoretical analysis, the effects of the characteristics of micro-slip on the damping capacity of jointed cantibeam can be also clarified.

It is known furthermore that there is an optimum value of the interface pressure as given by Eq. (11).

(2) As a concrete example, by using the slip ratio measured statically, the logarithmic damping decrement of the jointed cantibeam was calculated. Although the difference between the theoretical and experimental values becomes slightly large with the increasing thickness of beam, it may be said that the theoretical values are in good agreement with the experimental values.

Through the comparison between the theoretical and experimental values the following problems to be solved in the future are pointed out.

(i) The correlation of the dynamic slip ratio to the static slip ratio

(ii) The way how to determine the effective area of the preload

(iii) The estimation of the equivalent coefficient of friction at the interfaces being in micro-slip.

Acknowledgement

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Discussion

K. TANAKA (Kanazawa University):

(1) In this paper authors calculated that the micro-slip was from 100 Å to 300 Å when the total amplitude of vibration was 100 μm. In this condition the welding points in the real contact region are not in shear condition, but only indicate an elastoplastic deflection; therefore the energy loss between the jointed interfaces may be derived from the elastoplastic deflection of the welding points, namely the so-called internal friction, I think.

The authors, however, considered that the cause of energy loss is the external friction. How do authors think about this point?

(2) If the damping is caused mainly by the external friction, the free damped vibration shows a linear characteristic, and if the damping is caused by the internal friction, the free damped vibration shows a logarithmic characteristic, I think.

Please explain the experimental results for this problem.

(3) The results presented by Courtney-Pratt and et al. are equivalent to a stress-strain curve of welding points subjecting to the shear force. The relation expressed on page 1428, therefore, can not be applied to exchange the static coefficient of friction obtained from Fig. 16 for the dynamic one.

(4) From the facts that the value of μ_{eqdyn} is small, i. e., 0.009 as shown on page 1428, it is reasonable to consider that the damping is caused by the internal friction.

(5) The reasons why the logarithmic damping decrement increases due to the lubricant applied to the interfaces may be given by the damping due to the viscous resistance caused by the relative displacement under the high pressure, I think. How do authors think?

(6) It is emphasized that if the damping is caused by such a micro-slip proposed by authors, authors should be proved the exsistence of such a micro-slip with an electron microscope.

(7) For all the damping is caused by the internal friction, the numbers of welding points generally are in proportional to the contact load. Therefore, the analysis presented in this paper may hold at least formally.

(8) The discusser may well refer to the previous studies[(*1)~(*3)] on the dynamic contact.

H. Sato (Institute of Industrial Science, University of Tokyo):

(9) It is considered to be suitable that the damping related to the nature of material itself is measured with test pieces having a form of tuning fork or a symmetrical form to its support, in order to eliminate influences of inaccuracies caused by the dissipation to the foundation. I would like to know the reasons why authors used the cantibeam in this study and how authors consider the effects of the shape of test piece on its damping capacity.

(10) In the explanation of the results presented in Fig. 13, authors stated that the logarithmic damping decrement increase due to MoS_2 applied to the jointed interfaces, but if natures of MoS_2 as a lubricant are considered, the decreasing of the logarithmic damping decrement may be expected. Are there any reasons for this discrepancy?

(11) In Fig. 17 authors show that the logarithmic damping decrement calculated theoretically is zero when the interface pressure is zero. I presume that this theoretical value converges to the value of the logarithmic damping decrement of material itself.

Authors' closure

We interpret that Dr. Tanaka used the term of external friction as Coulomb friction and the term of internal friction as the friction derived from the elastoplastic deformation of welding points in contact surfaces, and considering this matter at first we reply to Dr. Tanaka's questions as a whole.

We should like to say that phenomena of friction fundamentally can not be considered to be able to separate into two regions such as an external friction and an internal friction. In this paper, therefore, we dealt with the energy loss between the jointed interfaces as the frictional energy loss including an elastoplastic deformation of microscopic welding points and a pure macro-slip. We have never considered that the frictional loss energy is only caused by the external friction.

(1) Question 1 is related to the question 4, so we reply here to both questions. Dr. Tanaka asked us about the reasons by what mechanism the micro-slip is caused, we think.

It is easily considered that such a questioned elastoplastic deformation of welding points is observed on a bolted joint, and that a so-called pure macro-slip is also observed on a bolted joint together with an elastoplastic deformation of welding points under low preloads.

For this problem, therefore, we have investigated and reported already in the reference (5). As the result in this paper we have used the term of micro-slip as the term indicating both cases together, that is, the elastoplastic deformation of welding points and the pure macro-slip between the interfaces.

(2) In all experiments we could not obtain the damped free vibration consisting of only one exponential curve. It can be seen that the damped free vibrations consist of two or three exponential curves, or one exponential curve having several linear parts. These results were also obtained in the study on the damping capacity of bolted cantibeam[(6)].

These facts mean that the damping between the jointed cantibeam can not be considered to be caused separately by the external friction or the internal friction.

(3) The relation of a static coefficient of friction to a dynamic one under contact conditions, in which a shear force is less than a static friction force determined by so-called macroscopic coefficient of friction used widely in the field of wear until now, has not been clarified yet.

It is, however, considered that a shear stress of jointed material τ subjected to dynamic forces decreases on account of heat generation at welding points, and then the dynamic coefficient of friction μ given as a ratio of the normal stress σ to the shear stress τ decreases as compared with the static one.

It may be one method to assume the relationship between the static and dynamic coefficient of friction mentioned on page 1428 in this paper. For this problem, we think the further investigation is necessary and now it is being investigated.

(5) Please refer to the author's closure (10).

(*1) Goodman, L. E. and Brown, C. B., *Trans. ASME*, Ser. E., Vol. 29, No. 1 (1962-3), p. 17.

(*2) Johnson, K. J., *Proc. Roy. Soc. Lond.*, Ser. A., Vol. 230 (1955), p. 531.

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(6) We investigated already the existence of such a micro-slip on a bolted joint, as mentioned in the author's closure (1), and the results presented in the reference (5) and elswhere. To confirm the micro-slip by using an electron microscope is very difficult from the viewpoint of experimental techniques, because the investigation should be done under vibrating conditions of the cantibeam.

(7) It is most desirable to do fully theoretical analysis based on the facts, after natures of damping were clarified perfectly, but if we considered that there are many problems to be investigated as mentioned in author's closure (1) and (2), it is impossible to do such an analysis, I think.

I emphasized that in the fields of engineering the analysis of damping capacity proposed here is very useful in the present.

(8) Thanks a lot for your kind information.

(9) Surely it is said that a torsion pendulum and a tuning folk are suitable to measure a small damping capacity such as an internal damping of material.

According to considerations to the measuring instruments of damping capacity of Enomoto(*4), the measuring instrument for damping is essentially required to be able to detect the damping caused by only the test specimen itself. One answer for this requirement is that only the test specimen is in vibration and other parts are fixed, namely the form of cantibeam corresponds to this condition.

From this reason we used the cantibeam type instrument in this study considering together the following reasons furthermore.

(i) For the reasons of easy experimental techniques such as machining of the test specimen.

(ii) To investigate clearly the effect of micro-slip on the damping capacity.

(iii) It can be presumed that shapes and dimensions of test specimens have influences on the damping capacity of machine members such as examined here.

The influences of shapes and dimensions of test specimens on their damping capacity may be conceivable from the results presented by Cochardt(*5), Yorgiadis(*6) and Hanks(*7). Hanks especially studied on the size effects on the damping capacity.

(10) For the questioned problem we have not studied yet, so we guess for following two reasons in the present.

(i) If solid lubricant MoS_2 lies between the jointed interfaces, the coefficient of friction decreases, but the micro-slip increases contrarily by the reasons of the prevention of welding at real contact points between the mating surfaces. Namely the effects of α in Eq. (5) are in reverse relation to the coefficient of friction, therefore it may not be said that the logarithmic damping decrement decreases with the decreasing coefficient of friction.

(ii) The jointed interfaces with MoS_2 show mixed lubricating conditions. In this condition the damping may yield the sum of the damping caused by the micro-slip at metal contact places and the damping caused by MoS_2 itself, since MoS_2 has such a crystal structure slipped easily at its cleavage surface.

(11) We apologize for the insufficient explanation. As pointed out, to be exact the logarithmic damping decrement reaches to the logarithmic damping decrement of material itself when the interface pressure is zero, but in this paper we assume that the material damping is negligible.

As reference the damping decrement δ_e of the equivalent cantibeam having the same thickness as the jointed cantibeam examined here was measured, and from the results it is known that the material damping can not be neglected for the thick beam, because the measured δ_e is 1.2×10^{-3} for the beam with 8 mm thickness and is 5×10^{-3} for the beam with 20 mm thickness.

Therefore, to increase the accuracy of calculated values and to clarify up to what conditions the proposed analysis can be applied, the investigations for the problems mentioned on page 1429 should be done in near future.

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