

Theoretical Analysis on Needle-Punched Carbon/Carbon Composites

Meng Han^{1,2} · Vadim V. Silberschmidt²

Abstract

Needle-punched carbon/carbon composites (NP-C/Cs) are advanced materials widely used in aerospace applications. The needle-punching technique improves the integrality of carbon-fibre plies, however, it also introduces many defects, affecting the mechanical behavior of NP-C/Cs. A theoretical model of irregular beams is suggested to investigate the mechanical behavior of unidirectional needle-punched carbon/carbon composites. Stress distributions in punched and squeezed fibres and an effect of the needle-punching technology are assessed.

Keywords Carbon/carbon composites · Mesomechanics · Theoretical analysis

1 Introduction

Carbon/Carbon composites (C/Cs) demonstrate excellent properties such as high specific stiffness and strength, low density, good wear resistance, high thermal resistance and low thermal expansion [1]. As a result, they are widely used in aerospace structures and braking systems.

The needle-punched (NP) structure, produced from fibre fabrics and nonwoven webs with a through-thickness needling technique, has advantages of the combined high inter-laminar properties and lower-cost processing [2, 3]. The NP C/Cs are prepared from the preforms by employing a chemical vapor-deposition (CVD) process, with heat-treatment at temperature of 2000 K for 2 h, followed by densification at 1300 K under the pressure of 1 kPa. Thanks to the NP process, interlaminar properties are strengthened; however, many defects are introduced into laminates inevitably.

Some efforts were taken to characterise the performance of C/Cs. Zhang et al. [4, 5] investigated bending properties and fracture mechanism of C/Cs with high-density preform and observed

Meng Han: hanmengmaker@163.com

¹ State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, People's Republic of China

² Wolfson School of Mechanical, Electrical and Manufacturing Engineering, Loughborough University, Loughborough, Leicestershire LE11 3TU, UK

delamination and interlaminar fracture. Li et al. [6–8] examined macro-fracture with scanning electron microscope (SEM) micrographs to understand the bending deformation and failure mechanism of 3D NP C/Cs at room and high temperatures. The obtained results showed that the load-deflection curves below 400°C exhibited a linear elastic behaviour and brittle fracture, while failure above 500°C was plastic. Static compressive behavior of such composites was studied by Zhang et al. [9, 10], demonstrating that both transverse and longitudinal compressive strengths for materials with dual matrix were higher than those with a single matrix. The failure modes under transverse and longitudinal compressive loading regions were shear and extension failure, respectively. Cai et al. [11–13] also investigated the bending properties of the 3D NP C/Cs, measuring the flexure strength of 98 MPa. Variety researchers [14–16] investigated effect of holes due to needling, e.g. properties of 3D z-pinned composites, including development model of their internal geometry and characterization of mechanical properties under static loading conditions.

Still, to the best of authors' knowledge, researches employing theoretical models and numerical simulations of mesoscopic features of NP C/Cs are rare. In this paper, the microscopic stress distributions are derived analytically for fibres and matrix; bending behavior of such composites is studied theoretical. Finally, some conclusions are drawn based on the obtained results.

2 Mesoscopic Features of NP C/Cs

The NP C/Cs studied in this paper were alternately stacked with 0° and 90° unidirectional (UD) carbon plies, as well as layers of short-cut-fiber web clothes between them, as schematically shown in Fig. 1(a). The composites contained ten layers and the thickness of each layer was 0.5 mm.

The preform [17, 18] was punched by regularly arranged needles on the needle plate. A distance between the neighboring needles was 2.4 mm along the X direction and 4 mm along the Y direction (Fig. 1(b)), the diameter of needle hole is 0.9 mm.

3 Theoretical Analysis of UD NP C/Cs

3.1 Mesoscale Mechanical Model of Squeezed Fibre Bundles

Fibres of UD NP C/Cs are squeezed in the X-Y plane as a result of application of the needle-punching technology [19, 20]. To simplify the analysis, the produced hole shape formed by squeezed fibres is considered as the irregular hexagon (Fig. 2). This is used as a representative

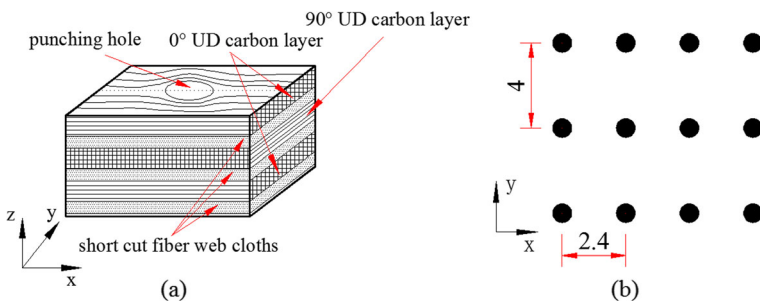


Fig. 1 Schematic diagram of geometry model: **a** NP C/Cs; **b** needle distribution

volume element (RVE) for analysis below. Hence, the shape equation for the matrix can be suggested as:

$$h_m(x) = \begin{cases} \frac{r}{l}x & (0 < x \leq l) \\ r & (l < x \leq (l+r)) \end{cases} \quad (1)$$

where r is the radius of the needle hole, l is the range of the punching effect.

The shape equation of a fibre bundle has the following form:

$$H(x) = \begin{cases} r\frac{x}{l} + \frac{h_y}{2} & (0 \leq x < l) \\ r + \frac{h_y}{2} & (l \leq x < l+r) \end{cases} \quad (2)$$

where h_y is the height of fibre bundles before punching.

According to the relationship of deformation between the fibre bundles and the matrix (Fig. 3) [21–23], the shear strain of the latter can be presented as:

$$\gamma_m = \frac{H(x)}{h_m(x)}y'(x) \quad (3)$$

where h_m is the height of matrix, H is the sum height of fibre bundle and matrix.

So, the shear stress of the matrix can be solved as:

$$q_m(x) = \tau_m(x) = G_m\gamma_m = G_m\frac{H(x)}{h_m(x)}y'(x) \quad (4)$$

where G_m is the shear modulus of matrix.

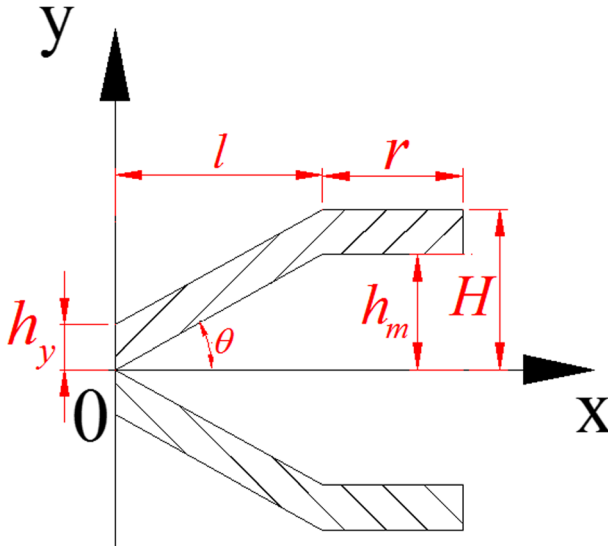


Fig. 2 Half of RVE of squeezed fibre bundles

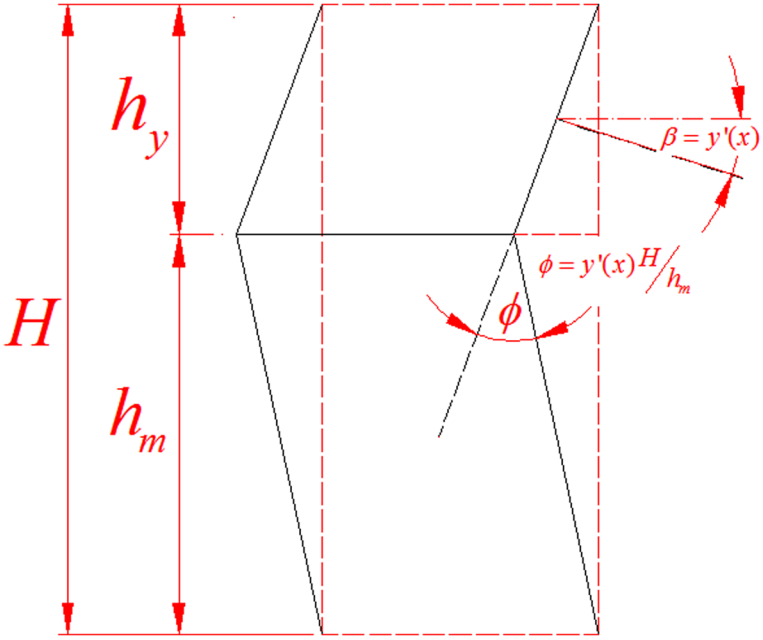


Fig. 3 Deformation relationship between fibre bundle and matrix

Then, equilibrium equations for fibre bundles can be written as (see Fig. 4):

$$\begin{cases} -M(x) + S_0x - \int_0^x q_f(x) \frac{h_y}{2} ds = M_0 \\ S(x) + \int_0^l q_f(x) \sin\theta ds = S_0 \\ q_f(x) \cos(\theta) = q_m(x) \end{cases} \quad (5)$$

where M_0 and S_0 are the moment and shear force at the end ($x=0$), respectively; $M(x)$ and $S(x)$ are the moment and shear force of the beam element representing the bundle, respectively; $q_m(x)$ and $q_f(x)$ are the shear stress of matrix and fibre domains, respectively.

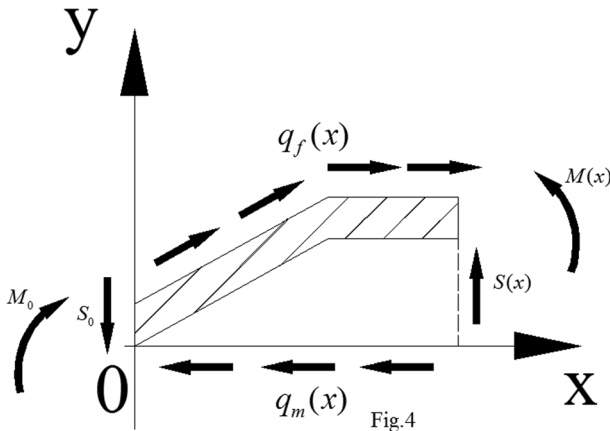


Fig.4

Fig. 4 Quarter of RVE of squeezed fibre bundles

The moment of fibre can be described by:

$$M(x) = \frac{E_{y11}h_y^3}{12}y''(x) \quad (6)$$

where $y(x)$ is the deflection shape equation of the beam element, E_{y11} is the elastic modulus of fibres along the 1 direction. Then, the equilibrium equations can be simplified as:

$$-\frac{E_{y11}h_y^3}{12}y_1''(x) + S_0x - \int_0^x \frac{G_m}{\cos(\theta)} \left(1 + \frac{h_y l}{2rx}\right) y_1'(x) \frac{h_y}{2} ds = M_0, \quad (0 \leq x < l), \quad (7a)$$

$$-\frac{E_{y11}h_y^3}{12}y_2''(x) + S_0x - \int_0^x \frac{G_m}{\cos(\theta)} \left(1 + \frac{h_y}{2r}\right) y_2'(x) \frac{h_y}{2} ds = M_0, \quad (l \leq x < l+r). \quad (7b)$$

The boundary conditions:

$$\begin{cases} y_1(0) = 0 \\ y_1'(0) = 1 \end{cases} \quad (8)$$

The Eq. (7a) cannot be solved analytically, so the forth-order approximation was used:

$$\begin{aligned} y_1(x) = & y_1(0) + y_1'(0)x - \frac{3}{2} \frac{G_m h_y^2 l y_1'(0) + 4M_0 r \cos\theta}{E_{y11} h_y^3 r \cos\theta} x^2 - \\ & \frac{1}{2} \frac{2G_m h_y^2 r^2 E_{y11} \cos\theta y_1'(0) - 4S_0 E_{y11} h_y r^2 \cos^2\theta - 3G_m^2 h_y^2 l^2 y_1'(0) - 12G_m l r M_0 \cos\theta}{E_{y11}^2 h_y^4 r^2 \cos^2\theta} x^3 + \\ & \frac{3}{8} \frac{G_m \left(\begin{aligned} & 6E_{y11} G_m h_y^2 l r^2 \cos\theta y_1'(0) - 4S_0 E_{y11} h_y l r^2 \cos^2\theta \\ & - 3G_m^2 h_y^2 l^3 y_1'(0) + 16E_{y11} M_0 r^3 \cos^2\theta - 12G_m l^2 r M_0 \cos\theta \end{aligned} \right)}{E_{y11}^3 h_y^5 r^3 \cos^3\theta} x^4 \end{aligned} \quad (9)$$

Using boundary conditions (8), the equations for moment $M_1(x)$ and shear force $S_1(x)$ can be solved ($0 \leq x < l$):

$$M_1(x) = \frac{E_y h_y^3}{12} y_1''(x) = \frac{E_y h_y^3}{12} \left[3(Q_1 + Q_2 x) + \frac{9}{2} Q_3 x^2 \right]. \quad (10)$$

where

$$\begin{cases} Q_1 = \frac{G_m h_y^2 l + 4M_0 r \cos\theta}{E_{y11} h_y^3 r \cos\theta}, \\ Q_2 = \frac{2G_m h_y^2 r^2 E_{y11} \cos\theta - 4S_0 E_{y11} h_y r^2 \cos^2\theta - 3G_m^2 h_y^2 l^2 - 12G_m l r M_0 \cos\theta}{E_{y11}^2 h_y^4 r^2 \cos^2\theta}, \\ Q_3 = \frac{G_m}{E_{y11}^3 h_y^5 r^3 \cos^3\theta} \left(\begin{aligned} & 6E_{y11} G_m h_y^2 l r^2 \cos\theta - 4S_0 E_{y11} h_y l r^2 \cos^2\theta \\ & - 3G_m^2 h_y^2 l^3 + 16E_{y11} M_0 r^3 \cos^2\theta - 12G_m l^2 r M_0 \cos\theta \end{aligned} \right). \end{cases} \quad (11)$$

The shear force $S_1(x)$ can be obtained directly:

$$S_1(x) = M_1'(x) = \frac{E_y h_y^3}{4} [Q_2 + 3Q_3 x] \quad (12)$$

The maximum axial stress is

$$\sigma_{1x}(x) = \pm \frac{M_1(x)d_y}{I_z} = \pm \frac{M_1(x)h_y}{4I_z} = \frac{E_y h_y^4 \left(3(Q_1 + Q_2 x) + \frac{9}{2} Q_3 x^2 \right)}{\frac{11bh_y^3}{2} + 24bh_y r^2 + 12bh_y^2 r}, (0 \leq x < l) \quad (13)$$

where b is the bundle's width.

The shear stress between the fibre bundle and the matrix is:

$$\tau_{1f}(x) = G_m \left(1 + \frac{h_y}{2r} \right) y_1'(x) = G_m \left(1 + \frac{h_y}{2r} \right) \left[1 - 3Q_1 x + \frac{3}{2} (Q_2 x^2 + Q_3 x^3) \right], (0 \leq x < l). \quad (14)$$

Equation (7b) can be solved exactly analytically as follows:

$$y_2(x) = C_1 e^{\omega x} + C_2 e^{-\omega x} + \frac{4S_0 r \cos \theta}{G_m h_y (2r + h_y)} x - \frac{4M_0 r \cos \theta}{G_m h_y (2r + h_y)} \quad (l \leq x < l + r) \quad (15)$$

Using the boundary conditions:

$$\begin{cases} y_2(l) = y_1(l) \\ y_2'(l) = y_1'(l) \\ y_2(L) = 0 \\ y_2'(L) = 0 \end{cases} \quad (16)$$

where $L = l + r$, the coefficients can be found as:

$$\begin{cases} C_1 = \frac{e^{\omega L} - \omega L - 1}{\omega(-2e^{2\omega L} + 4e^{\omega L} + \omega L e^{2\omega L} - \omega L - 2) - \omega L e^{2\omega L} + e^{2\omega L} - e^{\omega L}} \\ C_2 = \frac{e^{\omega L} - \omega L - 1}{\omega(-2e^{2\omega L} + 4e^{\omega L} + \omega L e^{2\omega L} - \omega L - 2) - \omega L e^{2\omega L} + e^{2\omega L} - e^{\omega L}} \\ S_0 = \frac{(1 - e^{\omega L}) G_m h_y (h_y + 2r)}{4r \cos \theta (-2e^{\omega L} + \omega L e^{\omega L} + 2 + \omega L)} \\ M_0 = \frac{G_m h_y (h_y + 2r) (\omega L e^{2\omega L} + \omega L - e^{2\omega L} + 1)}{4\omega r \cos \theta (-2e^{2\omega L} + 4e^{\omega L} + \omega L e^{2\omega L} - \omega L - 2)} \end{cases} \quad (17)$$

$$\text{where } \omega = \sqrt{\frac{6G_m}{E_{y11} h_y^2 \cos \theta} \left[1 + \frac{h_y}{(h_y + 2r)L} \right]}.$$

So, the moment $M(x)$ and shear force $S(x)$ equations can be obtained for $l \leq x < L$ using Eqs. (15) and (17):

$$M_2(x) = \frac{E_{y11} h_y^3}{12} y_2''(x) = \omega \frac{E_{y11} h_y^3}{12} \frac{e^{\omega L} - \omega L - 1 - \omega L e^{2\omega L} - 2\omega x + e^{2\omega L} - 2\omega x - e^{\omega L} - 2\omega x}{-2e^{2\omega L} + 4e^{\omega L} + \omega L e^{2\omega L} - \omega L - 2} e^{\omega x} \quad (18)$$

$$S_2(x) = M_2'(x) = \omega^2 \frac{E_{y11} h_y^3}{12} \frac{\omega L e^{2\omega L} - 2\omega x - e^{2\omega L} - 2\omega x + e^{\omega L} - 2\omega x + e^{\omega L} - \omega L - 1}{-2e^{2\omega L} + 4e^{\omega L} + \omega L e^{2\omega L} - \omega L - 2} e^{\omega x} \quad (19)$$

So that the maximum axial stress is:

$$\begin{aligned}\sigma_{2x}(x) &= \pm \frac{M_2(x)d_y}{I_z} = \pm \frac{M_2(x)h_y}{4I_z} \\ &= \pm \frac{\left(\omega E_{y11}h_y^3 \frac{e^{\omega L} - \omega L - 1 - \omega L e^{2\omega L - 2\omega x} + e^{2\omega L - 2\omega x} - e^{\omega L - 2\omega x}}{-2e^{2\omega L} + 4e^{\omega L} + \omega L e^{2\omega L} - \omega L - 2}\right) e^{\omega x}}{\left(\frac{11}{2}bh_y^3 + 24bh_yr^2 + 12bh_y^2r\right)} \quad (l \leq x < L)\end{aligned}\quad (20)$$

The shear stress between the fibre bundle and the matrix is:

$$\begin{aligned}\tau_{2f}(x) &= G_m \left(1 + \frac{h_y}{2r}\right) y_2'(x) = \\ G_m \left(1 + \frac{h_y}{2r}\right) &\frac{e^{\omega x + \omega L} - (\omega L + 1)e^{\omega x} + (\omega L - 1)e^{2\omega L - \omega x} + e^{\omega L - \omega x} - e^{2\omega L} + 2e^{\omega L} - 1}{-2e^{2\omega L} - 4e^{\omega L} + \omega L e^{2\omega L} - \omega L - 2}\end{aligned}\quad (l \leq x < L)\quad (21)$$

3.2 Mesoscale Mechanical Model of Deflected Fibre Bundles

Changing from the plane x-o-y (Fig. 4) to x-o-z (Fig. 5), another RVE can be introduced to analyse the in-plane deflection of fibre bundles caused by needle-punching.

Based on the RVE geometry, the relationship for deformation of the fibre bundle and the matrix can be obtained as:

$$\begin{cases} \gamma_{zm} = \frac{h_{zf}}{h_{zm}} z'(x) = \frac{h_{zf}}{h_{zm}} r_m \cos \alpha z'(\alpha) \\ h_{zf} = h_{zy} + h_{zm} \end{cases}\quad (22)$$

where h_{zy} , h_{zm} is the height of fibre bundle and matrix, respectively, γ_{zm} is the shear strain of matrix.

So, shear stress of the matrix is:

$$q_{zm}(\alpha) = G_m \gamma_{zm} = G_m \frac{h_{zf}}{h_{zm}} r_m z'(\alpha) \cos \alpha \quad (23)$$

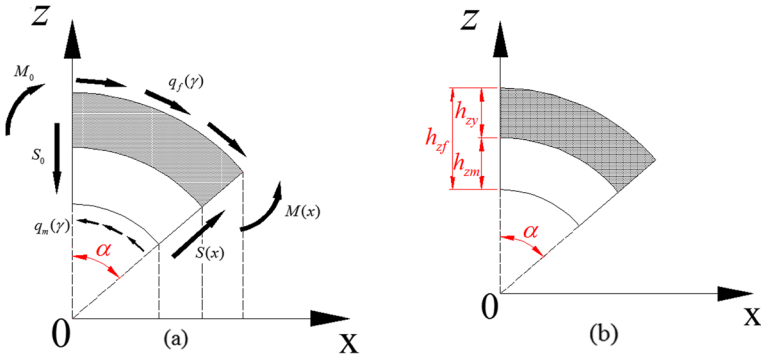


Fig. 5 RVE model of deflected fibre bundles: **a** analysis of internal force; **b** shape parameters

The differential equations of equilibrium in a polar coordinate system can be written

$$\begin{cases} -M_z(\alpha) + S_{z0}r_f \sin\alpha - \int_0^\gamma q_{zf}(\alpha) \frac{h_{zf}}{2} r_f \cos\alpha d\alpha = M_{z0} \\ S_z(\alpha) - \int_0^\gamma q_{zf}(\alpha) \cos\alpha d\alpha = S_{z0} \\ \int_0^\gamma q_{zf}(\alpha) r_f \gamma d\alpha = \int_0^\gamma q_{zm}(\alpha) r_m \gamma d\alpha \\ r_f = r_m + \frac{1}{2}h_{zf} + \frac{1}{2}h_{zm} \end{cases} \quad (24)$$

where $M_y(\alpha) = \frac{E_z h_{zf}^3}{12} z''(x) = -\frac{E_z h_{zf}^3}{12} r_f \sin(\alpha) z''(\alpha)$.

The shear stress in fibre bundle has the following form:

$$q_{zf}(\alpha) = \frac{G_m h_y}{h_m} r_m \cos\alpha \left[1 - \frac{h_{zf} + h_{zm}}{2r_f} \right] z'(\alpha) + c_1 \quad (c_1 = 0) \quad (25)$$

Equation (24) after simplification is:

$$z''(\alpha) - \left[\frac{3G_m(2r_f - h_{zf} - h_{zm})}{E_z h_{zf} h_{zm} r_f} \right] \frac{\cos^2\alpha}{\sin\alpha} z'(\alpha) = \frac{12(M_{z0} - S_{z0} r_f \sin\alpha)}{E_z h_{zf}^3 r_f \sin\alpha} \quad (26)$$

It has the following solution:

$$z(\alpha) = C_{z1} e^{\lambda\alpha} + C_{z2} e^{-\lambda\alpha} + \frac{4h_{zm} M_{z0}}{G_m h_{zf}^2 (2r_f - h_{zf} - h_{zm}) \cos^2\alpha} - \frac{4h_{zm} S_{z0} r_f \sin\alpha}{G_m h_{zf}^2 (2r_f - h_{zf} - h_{zm}) \cos^2\alpha} \quad (27)$$

where $\lambda = \sqrt{\frac{3G_m(2r_f - h_{zf} - h_{zm}) \cos^2\alpha}{E_z h_{zf} h_{zm} r_f \sin\alpha}}$.

Then, from Eq. (28) and the boundary conditions (29), parameters of Eq. (27) can be obtained as Eq. (30):

$$z'(\alpha) = \lambda C_{z1} e^{\lambda\alpha} - \lambda C_{z2} e^{-\lambda\alpha} + \frac{8M_{z0} h_{zm} \sin\alpha}{G_m h_{zf}^2 (2r_f - h_{zf} - h_{zm}) \cos^3\alpha} - \frac{4h_{zm} S_{z0} r_f (1 + \sin^2\alpha)}{G_m h_{zf}^2 (2r_f - h_{zf} - h_{zm}) \cos^3\alpha} \quad (28)$$

$$\begin{cases} z(0) = 0 \\ z'(0) = 1 \\ z(\gamma) = 0 \\ z'(\gamma) = 0 \end{cases} \quad (29)$$

$$\begin{cases} C_{z1} = \frac{e^{\lambda\gamma} - \lambda \sin\gamma \cos\gamma + \cos^2\gamma - 2}{D(\lambda, \gamma)} \\ C_{z2} = -e^{\lambda\gamma} \frac{-2e^{\lambda\gamma} + \cos^2\gamma e^{\lambda\gamma} + \lambda \sin\gamma \cos\gamma e^{\lambda\gamma} + 1}{D(\lambda, \gamma)} \\ M_{z0} = \frac{2 - \cos^2\gamma + \lambda \sin\gamma \cos\gamma - 2e^{2\lambda\gamma} + \cos^2\gamma e^{2\lambda\gamma} + \lambda \sin\gamma \cos\gamma e^{2\lambda\gamma}}{A \cdot D(\lambda, \gamma)} \\ S_{z0} = \frac{-2\lambda \cos^3\gamma e^{\lambda\gamma} - 2\sin\gamma e^{2\lambda\gamma} + \lambda \cos\gamma + 2\sin\gamma + \lambda \cos\gamma e^{2\lambda\gamma}}{B \cdot D(\lambda, \gamma)} \end{cases} \quad (30)$$

where

$$D(\lambda, \gamma) = -\lambda^2 \cos \gamma \sin \gamma + 2\lambda e^{\lambda \gamma} - 2\lambda e^{2\lambda \gamma} + \lambda \cos^2 \gamma e^{2\lambda \gamma} + 2\lambda \cos^3 \gamma e^{\lambda \gamma} + 2\sin \gamma e^{2\lambda \gamma} - \lambda \cos \gamma + \lambda^2 e^{2\lambda \gamma} \sin \gamma \cos \gamma - 2\lambda + \lambda \cos^2 \gamma - 2\sin \gamma - \lambda e^{2\lambda \gamma} \cos \gamma,$$

$$A = \frac{4h_{zm}}{G_m h_{zf}^2 (2r_f - h_{zf} - h_{zm})}, B = \frac{4h_{zm} r_f}{G_m h_{zf}^2 (2r_f - h_{zf} - h_{zm})}.$$

Please check displayed equation below if presented correctly.

So, the full form of deflection equation for the fibre bundles is solved:

$$z(\alpha) = Cz_1 e^{\lambda \alpha} + Cz_2 e^{-\lambda \alpha} + \frac{4h_{zm} M_{z0}}{G_m h_{zf}^2 (2r_f - h_{zf} - h_{zm}) \cos^2 \alpha} - \frac{4h_{zm} S_{z0} r_f \sin \alpha}{G_m h_{zf}^2 (2r_f - h_{zf} - h_{zm}) \cos^2 \alpha} \quad (31)$$

$$= \frac{S_2}{S_1 \cos^2 \alpha}$$

where,

$$\left\{ \begin{array}{l} S_1 = -\lambda^2 \sin \gamma \cos \gamma + 2\lambda e^{\lambda \gamma} - 2\lambda e^{2\lambda \gamma} + \lambda \cos^2 \gamma e^{2\lambda \gamma} + 2\lambda \cos^3 \gamma e^{\lambda \gamma} + 2\sin \gamma e^{2\lambda \gamma} \\ \quad - \lambda \cos \gamma + \lambda^2 \sin \gamma \cos \gamma e^{2\lambda \gamma} - 2\lambda + \lambda \cos^2 \gamma - 2\sin \gamma - \lambda \cos \gamma e^{2\lambda \gamma} \\ S_2 = (\lambda \sin \gamma \cos \gamma + 2 - \cos^2 \gamma - 2e^{2\lambda \gamma} + \lambda \sin \gamma \cos \gamma e^{2\lambda \gamma} + e^{2\lambda \gamma} \cos^2 \gamma) \\ \quad + (2e^{2\lambda \gamma} \sin \gamma - \lambda e^{2\lambda \gamma} \cos \gamma + 2\lambda \cos^3 \gamma e^{\lambda \gamma} - 2\sin \gamma - \lambda \sin \gamma) \sin \alpha \\ \quad + (-\lambda e^{\lambda \alpha} \sin \gamma \cos \gamma + 2e^{\lambda(2\gamma - \alpha)} - e^{\lambda(\gamma - \alpha)} + e^{\lambda(\alpha + \gamma)} - e^{\lambda(2\gamma - \alpha)}) \cos^2 \gamma - \\ \quad 2e^{\lambda \alpha} + e^{\lambda \alpha} \cos^2 \gamma) \cos^2 \alpha - \lambda \sin \gamma \cos \gamma e^{\lambda(2\gamma - \alpha)} \end{array} \right. \quad (32)$$

The moment and shear force of the fibre bundle are:

$$\left\{ \begin{array}{l} M_{yf}(\alpha) = \frac{E_{y11} h_{zf}^3}{12} z''(\alpha) = -\frac{E_{y11} h_{zf}^3}{12} r_f \sin \alpha z''(\alpha) = \frac{E_{y11} h_{zf}^3 r_f S_3}{12 S_1 \cos^4 \alpha} \\ S_f(\alpha) = M'_z(\alpha) = \frac{S_4}{S_1 \cos^5 \alpha} - \frac{S_5}{S_1 \cos^3 \alpha} \end{array} \right. \quad (33)$$

where

$$\left\{ \begin{array}{l} S_3 = -12 + 6\lambda \cos \gamma \sin \alpha + \lambda^2 e^{\lambda(\gamma - \alpha)} \cos^4 \alpha - 12\lambda e^{\lambda \theta} \cos^3 \gamma \sin \alpha + \\ \quad 12\sin \gamma \sin \alpha + 8\cos^2 \alpha + 6\cos^2 \gamma - 6\lambda \sin \gamma \cos \gamma - 6\lambda e^{2\lambda \gamma} \sin \gamma \cos \gamma \\ \quad + 6\lambda e^{2\lambda \gamma} \cos \gamma \sin \alpha - 12e^{2\lambda \gamma} \sin \gamma \sin \alpha - 6e^{2\lambda \gamma} \cos^2 \gamma + 12e^{2\lambda \gamma} - \\ \quad 2\lambda^2 e^{2\lambda \gamma - \lambda \alpha} \cos^4 \alpha - \lambda^2 e^{\lambda \alpha + \lambda \gamma} \cos^4 \alpha - \lambda e^{2\lambda \gamma} \cos \gamma \sin \alpha \cos^2 \alpha + \\ \quad \lambda^3 e^{2\lambda \gamma - \lambda \alpha} \sin \gamma \cos \gamma \cos^4 \alpha + \lambda^2 e^{2\lambda \gamma - \lambda \alpha} \cos^2 \gamma \cos^4 \alpha - \lambda \cos \gamma \sin \alpha \cos^2 \alpha \\ \quad - \lambda^2 \cos^2 \gamma e^{\lambda \alpha} \cos^4 \alpha + 2\lambda e^{\lambda \gamma} \cos^3 \gamma \sin \alpha \cos^2 \alpha + \lambda^3 e^{\lambda \alpha} \sin \gamma \cos \gamma \cos^4 \alpha \\ \quad + 2e^{2\lambda \gamma} \sin \gamma \sin \alpha \cos^2 \alpha + 4\lambda e^{2\lambda \gamma} \sin \gamma \cos \gamma \cos^2 \alpha + 4\lambda \sin \gamma \cos \gamma \cos^2 \alpha \\ \quad + 2\lambda^2 e^{\lambda \alpha} \cos^4 \alpha + 4e^{2\lambda \gamma} \cos^2 \gamma \cos^2 \alpha - 2\sin \gamma \sin \alpha \cos^2 \alpha \\ \quad - 8e^{2\lambda \gamma} \cos^2 \alpha - 4\cos^2 \gamma \cos^2 \alpha \\ S_4 = 48\sin \alpha + 48\lambda e^{\lambda \gamma} \cos^3 \gamma + 18\lambda \cos \gamma \cos^2 \gamma + 24e^{2\lambda \gamma} \cos^2 \gamma \sin \alpha + \\ \quad 8\cos^2 \gamma \sin \alpha \cos^2 \alpha - 2\lambda^3 e^{\lambda(2\gamma - \alpha)} \cos^5 \alpha + \lambda^3 e^{\lambda(\alpha + \gamma)} \cos^5 \alpha + \lambda^3 e^{\lambda(\gamma - \alpha)} \cos^5 \alpha \\ \quad - 2\lambda^3 e^{\lambda \alpha} \cos^5 \alpha - 36\lambda e^{\lambda \gamma} \cos^3 \gamma \cos^2 \alpha + 24\lambda \sin \gamma \cos \gamma \sin \alpha + \\ \quad 24\lambda e^{2\lambda \gamma} \sin \gamma \sin \alpha + 18\lambda e^{2\lambda \gamma} \cos \gamma \cos^2 \alpha - 24\lambda \cos \gamma - 2\sin \gamma \cos^4 \alpha - \\ \quad 16\sin \alpha \cos^2 \alpha - 48\sin \gamma - 24\cos^2 \gamma \sin \alpha + 40\sin \gamma \cos^2 \alpha - 48e^{2\lambda \gamma} \sin \alpha - \\ \quad 8e^{2\lambda \gamma} \cos^2 \gamma \sin \alpha \cos^2 \alpha + \lambda^4 e^{\lambda(2\gamma - \alpha)} \sin \gamma \cos \gamma \cos^5 \alpha - 36e^{2\lambda \gamma} \sin \gamma \cos^2 \alpha - \\ \quad 24\lambda e^{2\lambda \gamma} \cos \gamma + 48e^{2\lambda \gamma} \sin \gamma \\ S_5 = \lambda \cos \gamma \cos^2 \alpha + 8\lambda \sin \gamma \cos \gamma \sin \alpha - \lambda^3 e^{\lambda \alpha} \cos^2 \gamma \cos^3 \alpha + \\ \quad \lambda^4 e^{\lambda \alpha} \sin \gamma \cos \gamma \cos^3 \alpha - 2\lambda \cos \gamma + 4e^{2\lambda \gamma} \sin \gamma - 2e^{2\lambda \gamma} \sin \gamma \cos^2 \alpha - \\ \quad 2\lambda e^{2\lambda \gamma} \cos \gamma + \lambda e^{2\lambda \gamma} \cos \gamma \cos^2 \alpha - 2\lambda e^{\lambda \gamma} \cos^3 \gamma \cos^2 \alpha + 8\lambda e^{2\lambda \gamma} \sin \gamma \\ \quad \cos \gamma \sin \alpha - \lambda^3 e^{\lambda(2\gamma - \alpha)} \cos^2 \gamma \cos^3 \alpha + 4\lambda e^{\lambda \gamma} \cos^3 \gamma - 16e^{2\lambda \gamma} \sin \alpha \end{array} \right. \quad (34)$$

Using geometric neutral surface considerations of a circular beam (Fig. 6):

$$a = R - \rho = R - \frac{h}{\ln \frac{R_0}{R_1}} \quad (35)$$

Together with the Taylor series expansion omitted higher order term [24–26]:

$$a = \frac{h^2}{12R} \quad (36)$$

The axial stress of the beam element can be presented as:

$$\sigma_x(\alpha) = \frac{M_{yf}(\alpha)z}{abh(\rho + z)} \quad (37)$$

The shear stress of the fibre bundle has the following form:

$$\tau_{zf}(\alpha) = q_{zf}(\alpha) = \frac{G_m h_y}{h_m} r_m \cos \alpha \left[1 - \frac{h_{zf} + h_{zm}}{2r_f} \right] z'(\alpha) \quad (38)$$

where, the full form of $z'(\alpha)$ is given:

$$z'(\alpha) = \frac{S_6}{S_1 \cos^3 \alpha} \quad (39)$$

where,

$$\begin{aligned} S_6 = & -2e^{2\lambda\gamma} \sin\gamma \cos^2 \alpha + 4\sin\alpha + 4\lambda e^{\lambda\gamma} \cos^3 \gamma - 2\lambda e^{\lambda(2\gamma-\alpha)} \cos^3 \alpha + \\ & \lambda e^{\lambda(\alpha+\gamma)} \cos^3 \alpha + \lambda \cos\gamma \cos^2 \alpha - 2\lambda e^{\lambda\alpha} \cos^3 \alpha + \lambda e^{\lambda(\gamma-\alpha)} \cos^3 \alpha + \\ & 2e^{2\lambda\gamma} \cos^2 \gamma \sin\alpha - 2\lambda \cos\gamma - \lambda^2 e^{\lambda\alpha} \sin\gamma \cos\gamma \cos^3 \alpha + \lambda e^{\lambda\alpha} \cos^3 \alpha + \\ & \lambda^2 e^{\lambda(2\gamma-\alpha)} \sin\gamma \cos\gamma \cos^3 \alpha - 2\lambda e^{\lambda\gamma} \cos^3 \gamma \cos^2 \alpha + \lambda e^{2\lambda\gamma} \cos\gamma \cos^2 \alpha + \\ & \lambda e^{\lambda(2\gamma-\alpha)} \cos^2 \gamma \cos^3 \alpha + 2\lambda \sin\gamma \cos\gamma \sin\alpha + 2\lambda e^{2\lambda\gamma} \sin\gamma \cos\gamma \sin\alpha - \\ & 2\cos^2 \gamma \sin\alpha - 4\sin\gamma + 2\sin\gamma \cos^2 \alpha - 4e^{2\lambda\gamma} \sin\alpha - 2\lambda e^{2\lambda\gamma} \cos\gamma + 4e^{2\lambda\gamma} \sin\gamma \end{aligned} \quad (40)$$

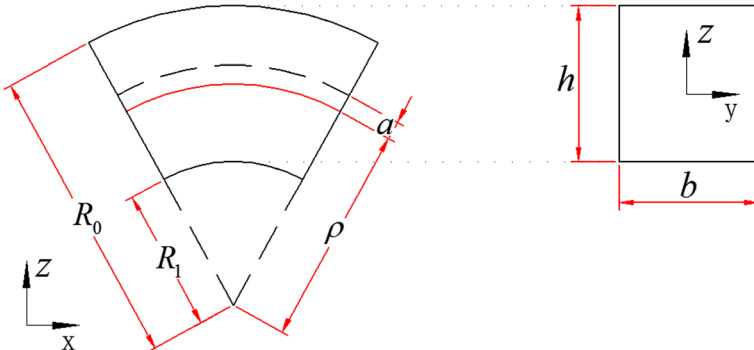


Fig. 6 Section parameters

4 Conclusion

The theoretical bending model was suggested for UD NP C/Cs to analyse the stress distribution of fibre bundles after the application of the needle-punching technology. The suggested analytical approach can be used for predictions of stress distribution in fibre bundles for design and optimization of C/Cs.

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