Theoretical and empirical differentiations of phases in the modelling process

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Abstract: *The reconstruction of pupils modelling processes can be found in many empirical studies within the literature on modelling. The empirical differentiations of the phases, which includes putting statements and actions of the pupils in the right phase, has not been reconstructed from a cognitive psychological point of view on a micro level thus far. In this article different modelling cycles are discussed with attention to distinctions in the various phases. The "modelling cycle under cognitive psychological aspects" is specifically emphasized in contrast to the other cycles. On the basis of the results of the COM²-project (Cognitive psychological analysis of modelling processes in mathematics lessons, Borromeo Ferri) the phases of the modelling process* **are** *described empirically. Some difficulties in the process of distinguishing the various phases are also pointed out.*

Kurzreferat*: Die Rekonstruktion von Modellierungsprozessen bei Lernenden ist in vielen empirischen Studien innerhalb der Literatur zum Modellieren zu finden. Die empirische Unterscheidung der Phasen, was die Einordnung von Aussagen und Handlungen der Lernenden miteinbezieht, wurde bisher noch nicht aus kognitionspsychologischer Sicht auf einer Mikroebene rekonstruiert. In diesem Artikel wird nach einer Übersicht ausgewählter Modellierungskreisläufe der „Modellierungskreislauf unter kognitionspsychologischen Aspekten im Vergleich hervorgehoben. Somit liegt der Fokus auf einer kognitionspsychologischen Perspektive hinsichtlich des Modellierungskreislaufes. Auf der Basis von Ergebnissen des KOM²-Projekt (Kognitionspsychologische Analysen von Modellierungsprozessen im Mathematikunterricht, Borromeo Ferri) werden die Phasen empirisch beschrieben und auch Schwierigkeiten bei der Einordnung in diese Phasen verdeutlicht.*

ZDM-Classification: C30, D10

1. Different modelling cycles - different approaches and aims

Looking at the literature on modelling and applications one can find different modelling cycles. These cycles are different, because they are dependent on various directions and approaches of how modelling is understood and in some cases, if complex or non complex tasks are used. Kaiser (1995) gave a historical overview about different directions in the modelling discussion. In a new attempt (Kaiser et al., in press; Sriraman & Kaiser, 2006) to classify approaches and backgrounds of the national and international modelling discussion, it became much more evident, that different standpoints and views on modelling exist. I will not go into detail concerning the latter aspects, but discuss the various modelling cycles found in the literature. Most approaches, which will be presented belong to the German and Anglo-Saxon speaking countries. Current views of modelling in Romance speaking countries especially regarding modelling cycles could not be found.

However I will not give a whole historical overview about the discussion regarding modelling cycles itself. In the following the focus lies on the view of various modelling cycles with respect to the aspects of the differentiation of real situation (RS), situation model (SM) respectively mental representation of the situation (MRS), real model (RM) and mathematical model (MM). The terms situation model (SM) and mental representation of the situation (MRS) are used synonymously in this article. In the COM²-project the latter one is more suitable (see section 2.2).

In the modelling discussion in Germany there are arguments, whether or not it is necessary to distinguish between these phases mentioned before. Besides these differences it is obvious, that within the various cycles also the phase of validation and the phase of interpretation are sometimes understood as mixed or separately. This aspect is recognized but will not be discussed here in depth. So I divide four groups of modelling cycles concerning the first three phases and illustrate them as follows[1]:

Group 1: Distinction between situation model (SM)/mental representation of the situation (MRS) and real model (RM).

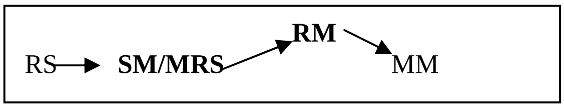

fig. 1: Group 1

Group 2: Mixed type of SM/MRS and RM.

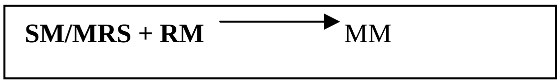

fig.2: Group 2

[1] My thanks go to Gabriele Kaiser and Werner Blum for constructive discussions.

Group 3: No distinction between SM/MRS and RM it is named understand as RM.

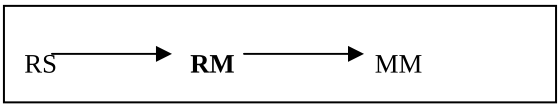

fig.3: Group 3

Group 4: From real situation (RS) to mathematical model (MM) without distinction in SM/MRS and RM.

fig.3: Group 4

1.1 Detailed description of the groups

In order to make this classification more transparent they are now described and illustrated with selected examples:

Group 1: The researchers who "work" with this (new) kind of modelling cycle focus especially on the cognitive processes of individuals during modelling processes. This is why the *situation model* is included in this cycle, because the researchers suppose that this phase is more or less run through by all individuals during modelling.
The well known term situation model is mainly used in connection with non complex modelling problems, to be precise, with word problems (see Kintsch & Greeno 1985, Nesher 1982, Verschaffel & Greer and a lot more) and has his origin in the text linguistics. A situation model can be described, without going into great detail here (see more in *Group 2*), as a mental representation of the situation, which is given in the word problem.
Blum/Leiss (2005) used the situation model leaning on Reussers approach (1997) and integrated it as new phase in their modelling cycle.

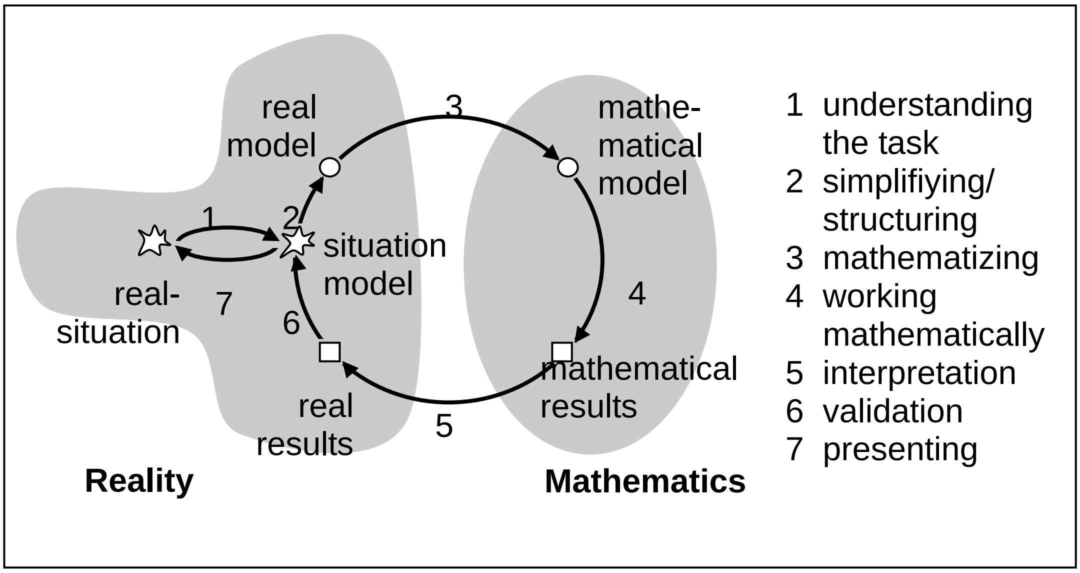

fig. 5: Modelling cycle from Blum/Leiss (in press)

The new thing done (in the DISUM-project (Blum/ Messner/Pekrun) is the idea, not taking word problems but complex modelling tasks, which is a new viewpoint in connection with the situation model. Blum/Leiss understand the situation model as an important phase during the modelling process, even as the most important one. That is because they describe the transition between real situation and situation model as a phase of understanding the task.

A similar approach (Borromeo Ferri, in press) in the COM²-project is using the phase of the situation model in adaptation of the modelling cycle of Blum/Leiss. However Borromeo Ferri used the name *m*ental *r*epresentation of the *s*ituation (MRS) instead of situation model, because this term better describes the kind of internal processes respectively the mental picture of an individual after/while reading the given (complex) modelling task. Besides this aspect, she uses this modelling cycle with these different phases to describe and to reconstruct these phases empirically. See the "modelling cycle under a cognitive psychological perspective" in section 2.2.

Group 2: A lot of research has been done in the field of word problems and how they can be solved: With theories of problem solving processes (e.g. Newell & Simon 1972); With theories of processes of text comprehensions (e.g. Anderson 1976); Analysis of the representation of the problems, particularly when text is used to present problem information or instructions (e.g. Hayes & Simon 1974). Kintsch & Greeno (1985) for instance developed a model, which main components are a set of knowledge structures and a set of strategies for using these knowledge structures in building a representation and in solving the problem. But not only in Kintsch' & Greeno's work, but also in many other investigations one can find, as mentioned before, the situation model, sometimes called as the problem model:

"The situation model includes inferences that are made using knowledge about the domain of the text information. It is a representation of the content of a text, independent of how the text was formulated and integrated with other relevant experiences. Its structure is adapted to the demands of whatever tasks the reader expects to perform."
(Kintsch & Greeno 1985, 110)

The phenomenon of the situation model seen as part of solving word problems brings out a kind of modelling cycle, in which the phase of building a situation model coincides with the phase building a real model. Blum & Niss (1991) pointed out that word problems already represent a real model and it therefore is about a reduced process of developing a model. The real situation in word problems is even simplified, so the mental representation of the situation or the situation model results directly from the real model. A legitimate question in this context could be from a theoretical point of view: Is a situation model really built before the real model is understood as a structured aspect in the word problem? Not answering this question here, I want to make clear, that the research works around solving and analysing word problems can be summarized regarding a modelling cycle, which combines situation model and real model. To this *Group 2* belong DeCorte & Verschaffel (1981), Nesher et al. (2003), Kintsch & Greeno (1985), Reusser (1997) and a lot more researchers.

Group 3: In this group researchers do not distinguish between SM/MRS and RM. So the situation model is not a phase in this kind of modelling cycle. Blum (1996) and Kaiser (1995) developed a cycle which is presenting this way of understanding the modelling process. What I mentioned about the cycle of Blum/Leiss in *Group 1*, however Blum is now arguing for the modelling cycle including the situation model, less for the cycle he developed in the 80's and 90's. So he sees more reasons for a difference between phases. Kaiser (2005) is more oriented in another direction at the moment, namely not to distinguish in a real model. That has also something to do with her experiences of many university series of seminars on mathematical modelling. But describing the next group afterwards it will be said more about that aspect. Also Maaß (2004) belongs to this group. On the basis of her empirical study she sees this kind of modelling cycle especially for 11^{th} to 13^{th} graders suitable for teaching modelling, but not for seven graders. That is why Maaß also is a "member" of Group 4.

See as an example for this group the modelling cycle from Kaiser (1995, 68) and Blum (1996, 18):

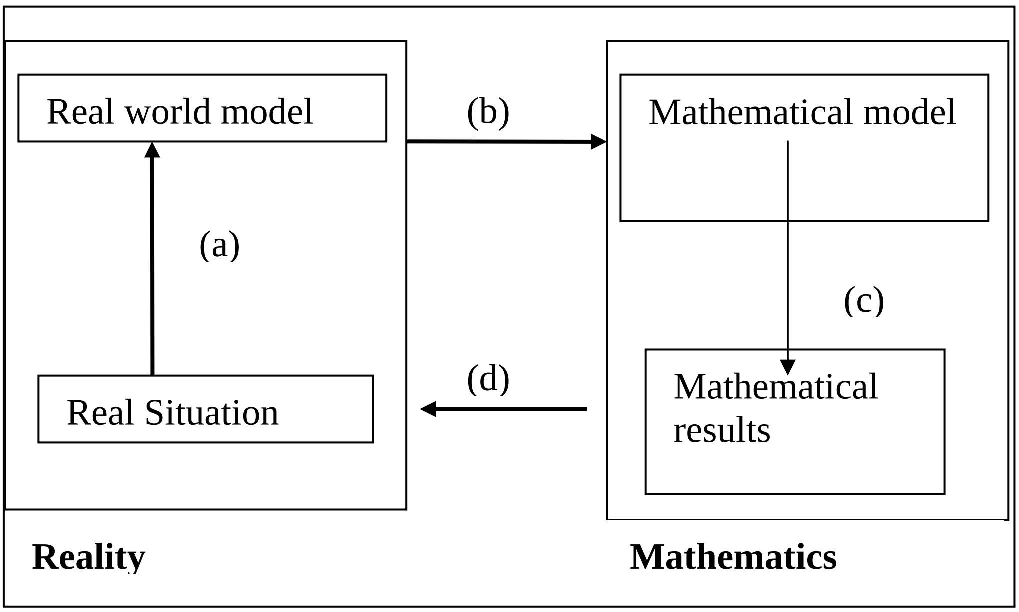

fig. 8: Modelling cycle from Kaiser (1995) and Blum (1996)

Group 4: Particularly in this group different directions of modelling are combined, which have one thing in common: for them there is no phase between real situation and mathematical model. Accordingly no distinction in real model or situation model is carried out. This has in part something to do with the kind of modelling problems which were used in this context. Mostly these are "realistic and complex" problems. So also the level of mathematics to solve these problems is another than in non complex problems. A prominent researcher, generally in the field of modelling, but especially in the way of considering modelling for understanding the real world better is of course Pollak (1979). Below one can see his modelling cycle:

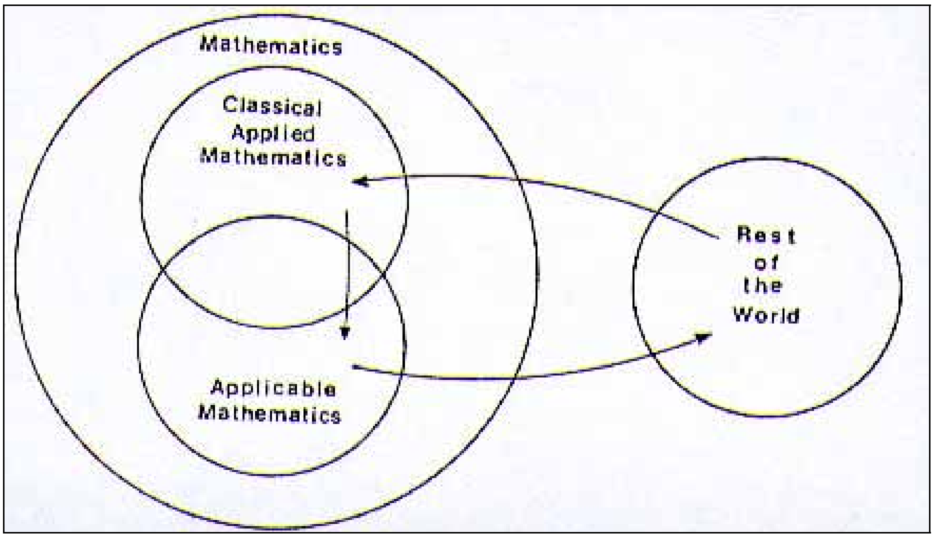

fig. 8: Modelling cycle of Pollak (1979, 233)

Further researchers belonging and using this kind of modelling cycle are for example Schupp (1989) and Ortlieb (2004).

As indicated earlier, Kaiser (2005) is more or less oriented in this direction, because her point of view lies more in the real complex problems itself and their solution process, less in a psychological view on modelling. In this context her experiences with seminars on mathematical modelling at university, which were based at school and were in cooperation with mathematicians and mathematics educators at school, play a role. Referring to Ortlieb (2004) she pointed out: "In applied mathematics one does not distinguish a real model from a mathematical model, but regards the transition from real life situation into a mathematical problem as a core of modelling." (Kaiser 2005, 101)

But not only "realistic" modelling can be a reason not to distinguish in a real model. Lesh et al. (2003) show a cycle without these differentiations. One reason for that could be their more pedagogical and psychological view on modelling concerning to every day school practise. Maaß (2004), mentioned before, is also a "member" of this group. She developed her cycle on the basis of her empirical study in school while teaching modelling with seven graders. She made clear, that for pupils' understanding of the modelling process a further differentiation in the real model is too difficult.

1.2 Summary of the groups and discussion

The presented classification of types of differentiations within the illustrated modelling cycles show, that different directions and viewpoints on modelling exist in the background. But there are also some more aspects to emphasize and to discuss:

- In general these modelling cycles and the description of the phases are normative and are seen as an ideal way of modelling. Through many empirical studies we know in the meantime, that the modelling process courses not in a linear way. But there is still a lack of research, how phases of modelling can be described on a micro-level and then be compared in contrast to the "ideal phases". This will be one aspect or rather one of the questions, which is analyzed in the COM²-project.
- Referring to the differentiations of the phases, especially between mathematical model and real model (and now also between situation model), there is a controversial discussion mainly in the German modelling community. The assumption

is that it is particularly difficult to prove these distinctions empirically. Shortly expressed: The more phases are differentiated theoretically the more difficult will be the empirical differentiation. One question in the COM²-project is if phases can be distinguished empirically, which will be discussed in the next chapter.

- An important point is of course to make a separation between modelling cycles used for research or for school aims. Concerning what was mentioned in *Group 4*, Maaß (2004) and Kaiser (2005) see a reduction of phases in a modelling cycle, which is taught in school as more useful. Maaß (2004) showed in her study with seven graders, that it was difficult for them to distinguish between real model and mathematical model in special. So it is legitimate to ask, if we need one kind of modelling cycle for school and one kind of cycle for research. Is it perhaps possible to have one for all purposes? This has to be discussed in the near future.
- One approach of combining aims for teaching modelling at school and aims for the research of modelling is given by Lesh & Sriraman (2005). They developed a conceptual scheme of models and modelling. But this schematic model is on another level a**s** the cycles presented before. However through this model Lesh & Sriraman made clear the following aspects, which can help to answer the above mentioned question:

"So, in essence there are three kinds of *complex systems*: (a) "real life" systems that occur (or are created) in everyday situations, (b) conceptual systems that humans develop in order to design, model, or make sense of the preceding "real life" systems, and (c) models that researchers develop to describe and explain students' modelling abilities. These three types of systems correspond to three reasons why the study of complex systems should be especially productive for researchers who are attempting to advance theory development in the learning sciences. In mathematics and science, conceptual systems that humans develop to make sense of their experiences generally are referred to as models. A naive notion of models is that they are simply (familiar) systems that are being used to make sense of some other (less familiar) systems - for some purpose. For example, a single algebraic equation may be referred to as a model for some system of physical objects, forces, and motions."
(Lesh &Sriraman 2005, 504)

2. From normative to descriptive – from theory to empiricism

In this section the COM²-project will be described shortly. Questions and goals of the study and also used methods, especially for the reconstructed phases of the modelling process will be presented.
After that, an example of a normative description of a modelling process will be shown. Coming from this illustration, the empirical description of the phases of the modelling cycle (process) will be pointed out on the basis of the results of the COM²-project.

2.1 Cognitive-psychological analysis of modelling processes in mathematics lessons (COM²-project)

Looking at modelling from a cognitive perspective has largely been neglected in the current discussion regarding modelling. This research project, in which context-bounded mathematics lessons are being analysed from a cognitive-psychological perspective, takes up issues from the PhD-thesis on mathematical thinking styles (Borromeo Ferri 2004a), in which learners' different individual mathematical thinking styles were reconstructed. The study also refers to an already completed follow-on case-study (Borromeo Ferri 2004b, 2004c), which discusses the influence of mathematical thinking styles on transition processes from real world to mathematics. Four different aspects will be analyzed from a cognitive perspective in the COM²-project:

1 Analysing learners and teachers in contextual mathematics lessons
2 Analysing micro-processes at an individual level
3 Analysing groups of pupils during the process
4 Considering the role of the teacher at the same time

The following questions were central to my study:

1. What influences do the mathematical thinking styles of learners and teachers have on modelling processes in context-bound mathematics lessons?

2. Can the differences between situation model, real model and mathematical model (as described in didactic literature on modelling) be reconstructed from the learners' ways of proceeding?

Concerning the differentiations of phases in the modelling cycle I will describe results of the study only regarding to question 2. In the following the design of the study including methods of analysing and reconstructing of the phases of modelling will be presented:
The project was carried out within the context of qualitative research. The investigation was conducted in three 10th grade classes from different *Gymnasien* (German Grammar Schools). The sample was comprised of 65 pupils and 3 teachers. Altogether three lessons are videotaped in one class. All pupils had to do a questionnaire on mathematical thinking styles. In addition, an interview is conducted with the teacher to reconstruct his or her mathematical thinking style which also includes biographical questions. In the three videotaped lessons the pupils were divided in groups of five and solved more or less complex modelling tasks. The problems are taken from the DISUM-project (Blum/Messner/Pekrun) and are of central importance as they delineate the field for the analysis.
One group in each class was videotaped while modelling. Not always having the same group within the three hours per class, the videotaped group changed with a new modelling problem. So, altogether 35 learners of the whole sample were in a special focus for reconstructing different phases in the modelling cycle and describing these phases empirically. Due to the fact, that there also were different modelling problems, many additional aspects could be generated from the data in dependence of the problem. In accordance with Grounded Theory (Strauss & Corbin 1996), codes were formed and used in order to break up and reassemble data. So besides others, statements of the pupils were coded and put into phases of the modelling process.

2.2 Empirical description of the phases and their transitions

As I mentioned in section 1.2 the description of the phases in the modelling process are normative and are seen as an ideal way of modelling. Firstly I will give a normative description of a modelling process, to make clear, that the empirical descriptions which will be shown afterwards, are different.
As an example for a normative description see Kaiser below (belonging to cycle in figure 8).

"A modelling process is done on the basis of the following ideal-typical procedure: A real world situation is the process' starting point. Then the situation is idealised (named (a) in figure 1), i.e. simplified or structured in order to get a real world model. Then this real world model is mathematised (b), i.e. translated into mathematics so that it leads to a mathematical model of the original situation. Mathematical considerations during the mathematical model produce mathematical results (c) which must be reinterpreted into the real situation (d). The adequacy of the results must be checked, i.e. validated. In the case of an unsatisfactory problem solution, which happens quite frequently in practice, this process must be iterated." (Kaiser 2005, 101)

Very helpful for the description of phases of modelling in the COM²-project were the reconstructed *individual modelling routes* of the learners in the groups while modelling (see Borromeo Ferri, in press). As a modelling route I use to denote the individual modelling process on an internal and external level. The individual starts this process during a certain phase, according to his/her preferences, and then goes through different phases several times or only once, focussing on a certain phase or ignoring others. To be precise from a cognitive viewpoint, one has to speak of visible modelling routes, as one can only refer to verbal utterances or external representations for the reconstruction of the starting-point and the modelling route. So, this made clear, that the modelling process is really not linear.
Combining all these reconstructed modelling routes of the pupils, it became clear, how phases and their transitions while modelling can be described empirically. I will not give examples in detail here, because they would only make sense in connection to a modelling problem used in the study. But the following descriptions are a result of all modelling routes of the pupils over all modelling problems used by the pupils.
In what I call the phases, they are the six areas an individual can go through while modelling, that means from real situation to real results. What I call the transition also means the transition from one phase to another phase.
As mentioned earlier, the following descriptions belong to the "modelling cycle under a cognitive perspective", which is shown below:

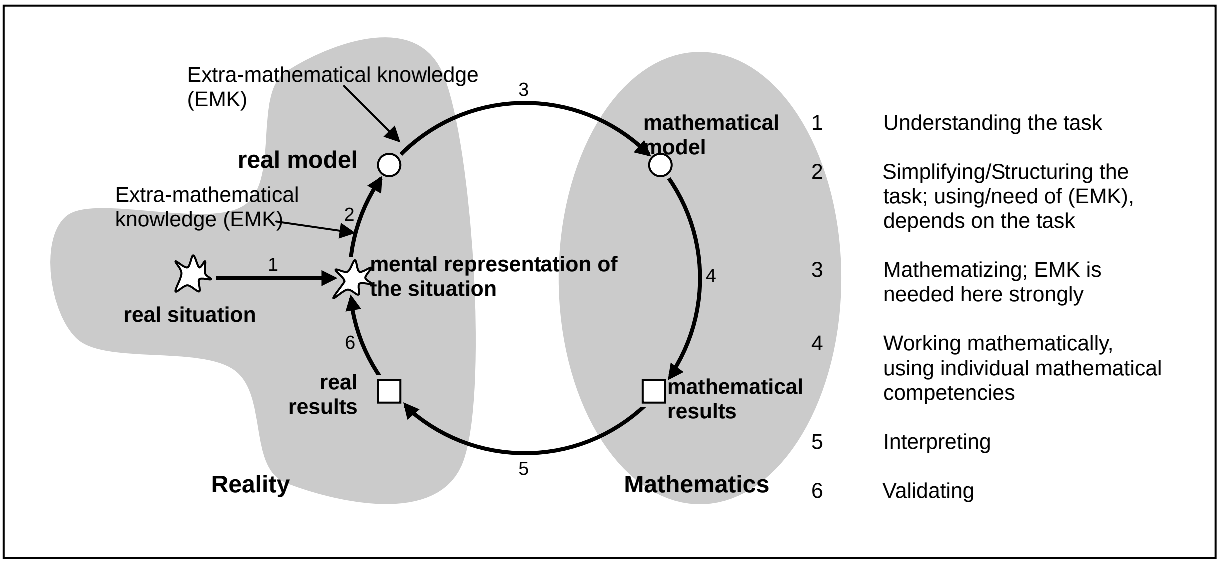

fig. 9: "Modelling cycle under a cognitive perspective"

Real Situation[2]

The real situation presents the situation, which is given in the problem. That can be a picture or only a text or both.

Within the *transition* from real situation to mental representation of the situation (MRS) the individual understands the problem more or less. A mental reconstruction of the situation given in the problem takes place, which is rather on an implicit level and mostly unaware for the individual. Even if the individual does not understand the problem, it can go on working on the task.

Mental representation of the situation (MRS)

The individual has a mental representation of the situation, which is given in the problem. This MRS can be very different, for example depending on the mathematical thinking style of the individual: visual imaginations in connection with strong associations to own experiences; or the focus lies more in the numbers and facts given in the problem, which the individual wants to combine or relate.

The difference between real Situation and MRS has two main aspects: 1) unaware simplifications of the task and in connecting with that, 2) the individual preference, how to deal with the problem in the upcoming modelling process.

Within the *transition* process from MRS to real model an idealisation and simplification of the problem takes place, which is more aware for the individual. This is because during the MRS the individual makes decisions, which influences the way of "filtering" the information in the problem. Depending what kind of problem is given, the question or the demand for extra-mathematical knowledge comes up.

Real Model

This phase has a strong connection to the MRS. That is why the real model is mostly built on an internal level of the individual. This also means that the level of external representations (sketches or formulae) can represent a real model as well. But this really depends on the verbal statements of the individuals while making an external representation.

The *transition* from real model to mathematical model is characterized as follows: the individual progress in mathematizing; moreover the extra-mathematical knowledge (depends on the task) is strongly demanded by the individuals and used to build a mathematical model.

Mathematical model

In this phase the individuals mainly make external representations in the sense of sketches or formulae. Now, verbal statements of the individuals are more on a mathematical level, less on a level referring to the reality. The transition into mathematics is completed here.

Within the *transition* from mathematical model to

[2] This phase is of course difficult to describe really in an empirical sense, because here the situation is given in a frame of a problem.

mathematical results the individuals use their mathematical competencies.

Mathematical results
The individuals mostly write down their results, which they get on the basis of the model.

The *interpretation* of the results takes place in the *transition* from mathematical results to real results. Also this phase, which is an important one, is often not done with awareness by the individuals.

Real results
The mathematical results are discussed by the individuals concerning their correspondence, if they can be real results.

While *validating* the individuals think about the correspondence of their real results and their MRS. This can be correct for them or not. On the basis of the data two different ways of validating of the learners could be generated:
"Intuitive Validation" (more unconscious):
The individual finds out for himself, that the results can be wrong for reasons that he can not really explain. Or he "feels" that the results are wrong, because they do not fit in the own frame of experiences and associations. So it is an intuitive decision, which is more unaware and not really ratio.

"Knowledge-based Validation" (more conscious)
Depending on how extra-mathematical knowledge is needed in the problem the different individuals deal with that. So "knowledge-based validation" means, that individuals either agree with their results on the basis of their extra-mathematical knowledge or not. There can be distinguished between two kinds of consciousness: 1) aware, but not knowledge-based and 2) aware, but knowledge-based.
Both "intuitive" and "knowledge-based" validation are connected with the previous reflections of the individual. The reason, why individuals mostly do not validate is the fact, that they make mostly an "inner-mathematical validation". Validating means for them "calculating" the mathematical model. They do not connect the results to the situation respectively to the reality, which is given in the situation.

Compared with a normative description of the phases, this gives more insights empirically from the perspective of the individual on what is happening from a cognitive perspective.

2.3 Discussion
The empirical differentiation of the phases was, besides others, one result of the current research study. Of course the distinction was and is not easy. Every statement of the learner has to be interpreted in connection with his/her external representation and with the problem. Here I will not give examples in detail regarding these statements. But I will point out some general aspects:
It became clear, that the way, how the problem is structured can have an influence on distinguishing between phases. In a pragmatic way I distinguish two kinds of structured problems (within the COM²-project):
1. Modelling problems which contain more information in connection with given numbers in the task and numbers, which have to be adding through extra-mathematical knowledge.
2. Modelling problems, which have less numbers given, but inner-mathematical knowledge **is** available on an implicit level and must be recognized and used for solving.
The analysis of the data showed, that concerning problems of category 1 compared to category 2 the distinction between real model and mathematical model was difficult. The reasons for that were the following[3]: Coming to a mathematical model of the problem, a lot of calculations had to be done before. But these calculations did not represent a mathematical model in the sense it is understood. On the other hand these calculations are not really a simplification of the problem, but only a meaningful combining of numbers, which are given. Because of these calculation the learners got intermediate results, which they interpreted backwards sometimes more realty-based or mathematically-based. So the individuals changed and stayed mainly between real model and mathematical model a long time. Analysing modelling processes of individuals, who solved problems of category 2, it was much easier to distinguish the phases. Reasons for that can be simply the fact, that not a lot of numbers were given. So, the problems were more open in this sense, although all problems were complex.
Besides this phenomenon one has to point out, that these analysis are done on a micro-level, otherwise it not possible to distinguish the phases. Also one has to add, that the interpretation could be seen from a subjective viewpoint. To guarantee a stronger reliability of the analysis, these were done at first independently by myself and by my research student, and then discussed together.

[3] I thank my research student Björn Wißmach for discussions especially about this phenomenon.

3. Final discussion

Given the variety of approaches and aims within the modelling discussion this area of research in mathematics education is growing rapidly.
One trend in the modelling discussion for the next years could be a cognitive viewpoint. This area of research has been neglected by and large in the past discussion. It is an important direction to gain insights into the minds of the pupils and teachers.

References

Anderson, J.R. (1976). *Language, memory, and thought*. Hillsdale, NJ: Erlbaum.

Blum, W. & Niss, M. (1991). Applied Mathematical Problem Solving, Modelling, Applications and Links to other Subjects – State, Trends and Issues in Mathematics Instruction. In *Educational Studies in Mathematics*, 22, No. 1, 37-68.

Blum, W. (1996). Anwendungsbezüge im Mathematik-unterricht – Trends und Perspektiven. In Kadunz, G. et al. (Eds.), *Trends und Perspektiven. Schriftenreihe Didaktik der Mathematik*, Vol. 23., Wien: Hölder-Pichler-Tempsky, 15 – 38.

Blum, W. & Leiss, D. (2005). *„Filling Up" – the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks*. Paper for the CERME4 2005, WG 13 Modelling and Applications.

Blum, W. & Leiss D. (in press). How do students and teachers deal with mathematical modelling problems? The example "Filling up". In Haines et al. (Eds.), *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics.* Chichester: Horwood Publishing.

Borromeo Ferri, R. (2004a). *Mathematische Denkstile*. Ergebnisse einer empirischen Studie. Hildesheim: Franzbecker.

Borromeo Ferri, R. (2004b). Mathematical Thinking Styles and Word Problems. In Henn, Hans-Wolfgang & Blum, Werner (Eds.), *Pre-Conference Proceedings of the ICMI Study 14, Applications and Modelling in Mathematics Education* (pp. 47-52), Dortmund.

Borromeo Ferri, R. (2004c). Vom Realmodell zum mathematischen Modell – Übersetzungsprozesse aus der Perspektive mathematischer Denkstile. In *Beiträge zum Mathematikunterricht* (p. 109-112), Hildesheim: Franzbecker.

Borromeo Ferri, R. (in press). Individual modelling routes of pupils – Analysis of modelling problems in mathematical lessons from a cognitive perspective. In Haines et al. (Eds.,) *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics.* Chichester: Horwood Publishing.

DeCorte, E, & Verschaffel, L (1981). Children's solution processes in elementary arithmetic problems: Analysis and imptovement. In *Journal of Educational Psychology*, vol. 6, p. 765-779.

Hayes, J.R. & Simon, H. (1974). Understanding written problem instructions. In L.W. Gregg (Ed.). *Knowledge and cognition* (pp. 167-200), Hillsdale, NJ: Erlbaum.

Kaiser, G. (1995). Realitätsbezüge im Mathematik-unterricht – Ein Überblick über die aktuelle und historische Diskussion. In Graumann, G. et al. (Eds.) *Materialien für einen realitätsbezogenen Mathematikunterricht*, (pp. 66 – 84) Bad Salzdetfurth: Franzbecker.

Kaiser, G. (2005). Mathematical Modelling in School – Examples and Experiences. In Kaiser, G. & Henn, H.-W. (Eds.), *Mathematikunterricht im Spannungsfeld von Evaluation und Evolution* (pp. 99-108) Hildesheim: Franzbecker.

Kaiser, G. et al. (in press). Mathematische Modellierung in der Schule. In *Beiträge zum Mathematikunterricht.* Hildesheim: Franzbecker.

Kintsch, W. & Greeno, J. (1985). Understanding word arithmetic problems. In *Psychological Review,* vol. 92, No. 1, p. 109-129.

Lesh, R. & Doerr, H. (Eds.) (2003). *Beyond Constructivismen – Models and Modeling Perspectives on Mathematics Problem Solving, Learning and Teaching.* Mahwah: Lawrence Erlbaum.

Lesh, R. & Sriraman, B. (2005). Mathematics Education as Design Science. In *Zentralblatt für Didaktik der Mathematik*, vol. 37 (6), p. 490-505.

Nesher, P. (1982). Levels of description in the analysis of addition and subtraction. In Carpenter, T.P. & Moser, J.M. & Romberg, T. (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 25-38) Hillsdale, NJ: Erlbaum.

Nesher, P & Hershkowitz, S. & Novotna, J. (2003). Situation model, text base and what else? Factors affecting problem solving. In *Educational Studies in Mathematics. An International Journal*, vol. 52 (2), p. 151-176.

Newell, A. & Simon, H.A. (1972). *Human problem solving*. Englewood Cliffs, NJ: Prentice-Hall.

Maaß, K. (2004). *Mathematisches Modellieren im Unterricht. Ergebnisse einer empirischen Studie*. Hildesheim: Franzbecker.

Ortlieb, C.P. (2004). Mathematische Modelle und Naturerkenntnis. In *mathematica didactica*, 27, 2004, 1, p. 23-40.

Pollak, H. (1979). The interaction between mathematics and other school subjects. In UNESCO (Eds.), *New trends in mathematics teaching IV* (pp. 232-248), Paris.

Schupp (1989). Mit oder ohne Computer?

Computergraphische Hilfen beim Konstruieren von Kegelschnitten. In *mathematiklehren* 1989, vol.. 37, p. 49.

Sriraman, B., & Kaiser, G. (2006). Theory usage and theoretical trends in Europe: A survey and preliminary analysis of CERME4 research reports. *Zentralblatt für Didaktik der Mathematik*, vol, 38, no. 1, pp. 22-51.

Strauss, Anselm L. & Corbin, Juliet (1996). *Grounded Theory, Grundlagen Qualitativer Sozialforschung.* Weinheim: Beltz.

Reusser, K. (1997). Erwerb mathematischer Kompetenzen. In: Weinert/Helmke (Eds.,) *Entwicklung im Grundschulalter.* (pp. 141-155) Weinheim: Beltz.

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